

Step 1. Take a multistroke gesture *strokes* and generate unistroke permutations. For gestures serving as templates, Step 1, which uses Steps 3-6, should be carried out once on the input points. For candidates, Steps 2-7 should be applied to the input points. For constants we use $N=96$, $size=250$, $\hat{c}=30$, $O=(0,0)$, and $I=12$.

```

GENERATE-UNISTROKE-PERMUTATIONS(strokes)
1  for i from 0 to |strokes| do  $order_i \leftarrow i$ 
2  HEAP-PERMUTE(|strokes|, order, out orders)
3   $M \leftarrow$  MAKE-UNISTROKES(strokes, orders)
4  foreach unistroke U in M do
5     $U_{points} \leftarrow$  RESAMPLE( $U_{points}$ , N) // step 3
6     $\omega \leftarrow$  INDICATIVE-ANGLE( $U_{points}$ ) // step 4
7     $U_{points} \leftarrow$  ROTATE-BY( $U_{points}$ ,  $-\omega$ )
8     $U_{points} \leftarrow$  SCALE-DIM-TO( $U_{points}$ , size,  $\hat{c}$ ) // step 5
9     $U_{points} \leftarrow$  CHECK-RESTORE-ORIENTATION( $U_{points}$ ,  $+\omega$ )
10    $U_{points} \leftarrow$  TRANSLATE-TO( $U_{points}$ , O)
11    $U_{vector} \leftarrow$  CALC-START-UNIT-VECTOR( $U_{points}$ , I) // step 6
HEAP-PERMUTE(n, order, out orders)
1  if n = 1 then APPEND(orders, order)
2  else
3    for i from 0 to n do
4      HEAP-PERMUTE(n-1, order, out orders)
5      if IS-ODD(n) then SWAP(order0, ordern-1)
6      else SWAP(orderi, ordern-1)
MAKE-UNISTROKES(strokes, orders)
1  foreach order R in orders do
2    for b from 0 to  $2^{|R|}$  do
3      for i from 0 to |R| do
4        if BIT-AT(b, i) = 1 then // b's bit at index i
5          APPEND(unistroke, REVERSE(strokesR))
6        else APPEND(unistroke, strokesR)
7      APPEND(unistrokes, unistroke)
8  return unistrokes

```

Step 2. Combine candidate *strokes* into one unistroke *points* path.

```

COMBINE-STROKES(strokes)
1  for i from 0 to |strokes| do
2    for j from 0 to |strokesi| do
3      APPEND(points, strokesij) // append each point
4  return points

```

Step 3. Resample a *points* path into *n* evenly spaced points.

```

RESAMPLE(points, n)
1   $I \leftarrow$  PATH-LENGTH(points) / (n - 1)
2   $D \leftarrow 0$ 
3   $newPoints \leftarrow points_0$ 
4  foreach point  $p_i$  for  $i \geq 1$  in points do
5     $d \leftarrow$  DISTANCE( $p_{i-1}$ ,  $p_i$ )
6    if ( $D + d$ )  $\geq I$  then
7       $q_x \leftarrow p_{i-1,x} + ((I - D) / d) \times (p_{i,x} - p_{i-1,x})$ 
8       $q_y \leftarrow p_{i-1,y} + ((I - D) / d) \times (p_{i,y} - p_{i-1,y})$ 
9      APPEND(newPoints,  $q$ )
10   INSERT(points, i,  $q$ ) //  $q$  will be the next  $p_i$ 
11    $D \leftarrow 0$ 
12  else  $D \leftarrow D + d$ 
13  return newPoints

```

```

PATH-LENGTH(A)
1   $d \leftarrow 0$ 
2  for i from 1 to |A| step 1 do
3     $d \leftarrow d +$  DISTANCE( $A_{i-1}$ ,  $A_i$ )
4  return  $d$ 

```

Step 4. Find and save the indicative angle ω from the *points*^{*} centroid to first point. Then rotate by $-\omega$ to set this angle to 0° .

```

INDICATIVE-ANGLE(points)
1   $c \leftarrow$  CENTROID(points) // computes  $(\bar{x}, \bar{y})$ 
2  return ATAN( $c_y - points_{0,y}$ ,  $c_x - points_{0,x}$ ) // for  $-\pi \leq \omega \leq \pi$ 
ROTATE-BY(points,  $\omega$ )
1   $c \leftarrow$  CENTROID(points)
2  foreach point p in points do
3     $q_x \leftarrow (p_x - c_x) \cos \omega - (p_y - c_y) \sin \omega + c_x$ 
4     $q_y \leftarrow (p_x - c_x) \sin \omega + (p_y - c_y) \cos \omega + c_y$ 
5    APPEND(newPoints,  $q$ )
6  return newPoints

```

Step 5. Scale dimensionally-sensitive based on threshold $\hat{c}=30$. Next, *if* using bounded rotation invariance, restore drawn orientation by rotating $+\omega$. Then translate to the origin $O=(0,0)$.

```

SCALE-DIM-TO(points, size,  $\hat{c}$ )
1   $B \leftarrow$  BOUNDING-BOX(points)
2  foreach point p in points do
3    if  $\min(B_{width} / B_{height}, B_{height} / B_{width}) \leq \hat{c}$  then // uniform
4       $q_x \leftarrow p_x \times size / \max(B_{width}, B_{height})$ 
5       $q_y \leftarrow p_y \times size / \max(B_{width}, B_{height})$ 
6    else // non-uniform
7       $q_x \leftarrow p_x \times size / B_{width}$ 
8       $q_y \leftarrow p_y \times size / B_{height}$ 
9    APPEND(newPoints,  $q$ )
10 return newPoints

```

```

CHECK-RESTORE-ORIENTATION(points,  $\omega$ )
1  if using bounded rotation invariance then
2    points  $\leftarrow$  ROTATE-BY(points,  $\omega$ )
3  return points

```

```

TRANSLATE-TO(points, k)
1   $c \leftarrow$  CENTROID(points)
2  foreach point p in points do
3     $q_x \leftarrow p_x + k_x - c_x$ 
4     $q_y \leftarrow p_y + k_y - c_y$ 
5    APPEND(newPoints,  $q$ )
6  return newPoints

```

Step 6. Calculate the start unit vector *v* for *points* using index $I=12$.

```

CALC-START-UNIT-VECTOR(points, I)
1   $q_x \leftarrow points_{I,x} - points_{0,x}$ 
2   $q_y \leftarrow points_{I,y} - points_{0,y}$ 
3   $v_x \leftarrow q_x / \sqrt{(q_x^2 + q_y^2)}$ 
4   $v_y \leftarrow q_y / \sqrt{(q_x^2 + q_y^2)}$ 
5  return v

```

Step 7. Match candidate *points* having start unit vector *v*, processed from the raw *strokes* in Step 2, where now $S = |strokes|$, against unistroke permutations *U* within each multistroke *M*. We use $\Phi = 30^\circ$ for the start angle similarity threshold and $size=250$. The symbol ϕ equals $\frac{1}{2}(-1 + \sqrt{5})$. We pass $\theta = \pm 45^\circ$ and $\theta_\Delta = 2^\circ$.

```

RECOGNIZE(points, v, S, multistrokes)
1   $b \leftarrow +\infty$ 
2  foreach multistroke M in multistrokes do
3    if  $S = |M_{strokes}|$  then // optional: require same # strokes
4      foreach unistroke U in M do
5        if ANGLE-BETWEEN-VECTORS(v,  $U_{vector}$ )  $\leq \Phi$  then
6           $d \leftarrow$  DISTANCE-AT-BEST-ANGLE(points, U,  $-\theta$ ,  $\theta$ ,  $\theta_\Delta$ )
7          if  $d < b$  then  $b \leftarrow d$ ,  $M' \leftarrow M$ 
8           $score \leftarrow 1 - b / [\frac{1}{2}\sqrt{(size^2 + size^2)}]$ 
9          return  $\langle M', score \rangle$ 
ANGLE-BETWEEN-VECTORS(A, B)
1  return  $\text{ACOS}(A_x \times B_x + A_y \times B_y)$ 

```

DISTANCE-AT-BEST-ANGLE($points, T, \theta_a, \theta_b, \theta_\Delta$)

```

1   $x_1 \leftarrow \varphi\theta_a + (1 - \varphi)\theta_b$ 
2   $f_1 \leftarrow \text{DISTANCE-AT-ANGLE}(points, T, x_1)$ 
3   $x_2 \leftarrow (1 - \varphi)\theta_a + \varphi\theta_b$ 
4   $f_2 \leftarrow \text{DISTANCE-AT-ANGLE}(points, T, x_2)$ 
5  while  $|\theta_b - \theta_a| > \theta_\Delta$  do
6    if  $f_1 < f_2$  then
7       $\theta_b \leftarrow x_2$ 
8       $x_2 \leftarrow x_1$ 
9       $f_2 \leftarrow f_1$ 
10      $x_1 \leftarrow \varphi\theta_a + (1 - \varphi)\theta_b$ 
11      $f_1 \leftarrow \text{DISTANCE-AT-ANGLE}(points, T, x_1)$ 
12   else
13      $\theta_a \leftarrow x_1$ 
14      $x_1 \leftarrow x_2$ 
15      $f_1 \leftarrow f_2$ 
16      $x_2 \leftarrow (1 - \varphi)\theta_a + \varphi\theta_b$ 
17      $f_2 \leftarrow \text{DISTANCE-AT-ANGLE}(points, T, x_2)$ 
18   return  $\text{MIN}(f_1, f_2)$ 

```

DISTANCE-AT-ANGLE($points, T, \theta$)

```

1   $newPoints \leftarrow \text{ROTATE-BY}(points, \theta)$ 
2   $d \leftarrow \text{PATH-DISTANCE}(newPoints, T_{points})$ 
3  return  $d$ 

```

PATH-DISTANCE(A, B)

```

1   $d \leftarrow 0$ 
2  for  $i$  from 0 to  $|A|$  step 1 do
3      $d \leftarrow d + \text{DISTANCE}(A_i, B_i)$ 
4  return  $d / |A|$ 

```