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Misleading patterns in correlation maps

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[1] We point out the pitfall of using spatial patterns of correlation coefficients obtained from filtered data to infer physical mechanisms. Filtering is often used to emphasize a signal of interest by removing unrelated variability, but it alters subsequent correlation maps when the removed variance contains spatial structure, as is most often the case in geophysical applications. This then results in misleading patterns of correlation. In the case where filtering involves the removal of independent signals, we show that the resulting increase in correlation is entirely due to the removed signal. We include a short discussion of alternative methods that can be used to generate more consistent maps of statistical relationships.

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1. Introduction

[2] This note addresses the problem of filtering a time series prior to calculating correlations with that time series. Because correlation calculations are nonlinear, the effect of the filtering is not obvious.

[3] We are motivated by the field of solar-climate interactions where isolating the effect of solar variations on the atmosphere invariably involves extracting a small signal from much larger atmospheric variations. This means that some type of filtering is essential and, historically, the examination of correlation patterns is used to discern the solar influence on climate (*Rheita* [1645], *Riccioli* [1651], *Herschel* [1801], *Flaugergues* [1818], *Koppen* [1914, 1873a, 1873b], *Shaw* [1928], *Labitzke and van Loon* [1988], and *Reid* [1991] as quoted by *Hoyt and Schatten* [1997]). It is therefore important to understand the effect of combining these techniques. Here we use multiple regression as an example of one way to filter a time series but it should be kept in mind that our results apply generally to any filtering process where the removed signal is approximately orthogonal to the remaining time series.

[4] Multiple regression analysis can be used to model changes in the atmosphere by determining the dependence of the variability on a number of known predictors. The dependence can be determined in a number of different ways but here we focus on the simplest and most used method, linear least squares. In this case, the variability, T(t), is assumed to be a linear combination of predictors. For example, *Gleisner and Thejll* [2003] (hereinafter referred to as GT), an El Niño index, a time series of volcanic aerosols, the 10.7 cm solar flux and a linear trend were chosen to be the predictors for atmospheric variability. In this way, a time series can be written as

$$T(t) = k_1 NINO3(t) + k_2 F10.7(t) + k_3 VOLCANO(t) + k_4 TREND(t) + Residual.$$

The coefficients, k_1 to k_4, are found by fitting the time series at each grid point to this model and minimizing the square of the Residual.

[5] In order to emphasize a small signal amongst a noisy atmospheric time series, one might consider subtracting out the terms associated with the other predictors in order to emphasize the remaining signal. In this case, GT removed all terms except the solar cycle, yielding what they call a "corrected" time series,

$$T^*(t) = k_2\, F10.7(t) + Residual.$$

We will show that the correlation of an independent time series, $S(t)$, with $T^*(t)$ will always be larger than its correlation with the original time series, $T(t)$, even if there is no signal related to $S(t)$ present in $T(t)$. This is always true as long as $S(t)$ is uncorrelated with the removed signal. This deceptive increase in correlation may then give false confidence in an unfounded relationship between $S(t)$ and $T(t)$. When applied to data which also varies in space, the corrected correlations will also contain misleading structures which are due to the removed signals and not to any inherent relationship with the original field. In section 3, we briefly mention some alternative methods that can produce more reliable results.

2. Patterns of Correlations

[6] To give a simple and obvious illustration of the problem, we consider carrying the filtering procedure to the extreme by removing all other variability except the solar cycle. The correlation between this "corrected" time

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series and the solar cycle itself would then yield $r = 1$ everywhere. This flat spatial pattern does not suggest that the atmosphere's response to the solar cycle is perfectly uniform. The remaining signal still contains spatial variation which can be seen in the solar regression coefficient, k_2. However, this is normalized out in the calculation of the correlation coefficient, r, and hence the spatial information about the solar cycle is lost. This illustrates the pitfall of using correlations to infer a spatial pattern when the data have been filtered.

[7] Below we show that the correlation values obtained with corrected fields are determined as much by the variance in the removed signal as by any actual relationship between the independent time series (the 10.7 cm solar flux in the above example) and the original data. By comparing these "corrected" correlation coefficients with the correlation coefficients that one obtains without any correction, we can reveal exactly where the two correlations differ and why.

[8] The calculations below are shown in terms of S, T, T^* and T_c. S is the independent predictor, which means that S is independent of the removed signal. S may be one of the original predictors which remains in the filtered time series (this is the case given by GT where $S = F10.7$) or S may be an entirely new time series which is orthogonal to the removed signal. T is the original time series, T^* is defined to be the corrected observable ($T^* = T - T_c$) and T_c is the correction ($T_c = k_1\ NINO3(t) + k_3 VOLCANO(t) + k_4\ TREND(t)$ in the example above).

[9] The correlation between the independent predictor and the original data is defined to be

$$Corr(S,T) = \frac{Cov(S,T)}{\sqrt{var(S)var(T)}}$$

and the correlation between the independent predictor and the corrected data is

$$\begin{aligned} Corr(S,T^*) &= Corr(S, T - T_c) \\ &= \frac{Cov(S,T) - Cov(S,T_c)}{\sqrt{var(s)[var(T) + var(T_c) - 2Cov(T,T_c)]}}. \end{aligned}$$

If S is independent of the removed signal, then $Cov(S, T_c) = 0$. This will be true if S is one of the retained predictors from a multiple regression analysis where the predictors are unrelated. Generally, if the covariance between predictors is small, $|Cov(S, T_c)| \ll |Cov(S, T)|$, then this approximation holds.

[10] GT tested the colinearity of their regression variables and came to the conclusion that they were linearly unrelated using a test developed by *Belsley* [1991]. This test or an examination of the condition number [*Weisberg*, 1985] can be used to verify that the predictors are not collinear, and unless complex interactions between predictors is suspected, a simple calculation of the correlation between predictors is often sufficient to determine the extent of their orthogonality.

[11] To compare the corrected and original correlations, consider the ratio:

$$\begin{aligned} \frac{Corr(S, T - T_c)}{Corr(S,T)} &= \frac{Cov(S,T)\sqrt{var(S)}}{Cov(S,T)\sqrt{var(S)}} \frac{\sqrt{var(T)}}{\sqrt{var(T) + var(T_c) - 2Cov(T,T_c)}} \\ &= \frac{\sqrt{var(T)}}{\sqrt{var(T) + var(T_c) - 2Cov(T,T_c)}}. \end{aligned} \quad (1)$$

Cancelling out the common terms, which are the only terms involving the time series, S, we see that the change in correlation has nothing to do with S or the relationship between it and the original observable. The change in correlation depends only on the variance and covariance of the removed signals and the original time series.

[12] When the removed signal (T_c) is uncorrelated with the remaining signal (T^*), a correlation with the corrected signal will always be greater than the correlation with the total signal. A proof of the increase in correlation is shown below.

[13] Consider the covariance of T with T_c (the last term in the denominator of equation (1)):

$$\begin{aligned} Cov(T,T_c) &= Cov(T_c + T^*, T_c) = Cov(T_c, T_c) + Cov(T^*, T_c) \\ &= var(T_c) \end{aligned}$$

because T^* and T_c are uncorrelated.When this is substituted into the ratio of correlations,

$$\frac{Corr(S,T^*)}{Corr(S,T)} = \frac{\sqrt{var(T)}}{\sqrt{var(T) - var(T_c)}} > 1. \quad (2)$$

This implies that as long as the variability removed from the original time series is uncorrelated with the remaining signal, the corrected correlation will always be greater than the original correlation, regardless of the relationship (or lack of relationship) with S.

[14] This can also be written in terms of the original regression coefficients. Without loss of generality, the variances of the original time series, $T(t)$, and the original predictors ($NINO3(t)$, $Volcano(t)$, $F10.7(t)$ and the $Trend(t)$ given by GT) can be normalized to 1. When the variables are normalized in this way, the regression coefficients are commonly referred to as β coefficients. In this case, the ratio reduces to

$$\frac{Corr(S,T^*)}{Corr(S,T)} = \frac{1}{\sqrt{1 - \left(k_1^2 + k_3^2 + k_4^2\right)}}. \quad (3)$$

Here we see that the ratio is completely determined by the variance of the removed signal and is unrelated to the solar cycle coefficient, k_2. Because these coefficients change with height and latitude as the influence of the predictors change with height and latitude, the resulting patterns will be

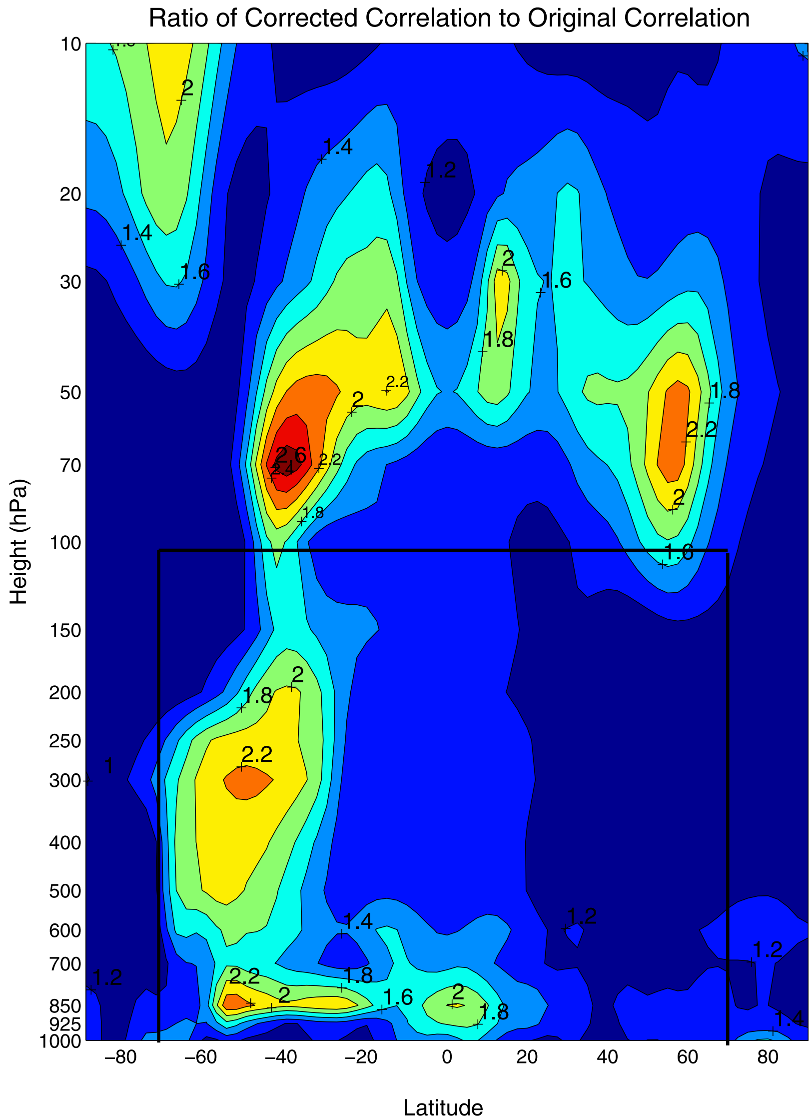

Figure 1. Ratio of correlations of the solar flux and corrected versus original temperatures. The corrected temperature correlations are calculated with the annually averaged 10.7 cm solar flux and the annual average of the corrected temperature time series. The standard temperature correlations are calculated with the annually averaged 10.7 cm solar flux and the annual average of the original National Centers for Environmental Prediction/National Center for Atmospheric Research temperature reanalysis. The bold black box encompasses the area of the corrected temperature correlations shown by *Gleisner and Thejll* [2003] on the left side of their Figure 1c.

misleading if one is looking for a spatial connection between $S(t)$ and the original time series, $T(t)$.

[15] Figure 1 shows this ratio for annually averaged correlations with the NCEP/NCAR temperature and the solar cycle. It contains some of the features highlighted by GT from their enhanced temperature correlation, including maxima over the equator and in the Southern Hemisphere midlatitudes. These patterns, however, are simply the result of the variances removed. The increases in correlation are due to temperature changes related to the time series of *NINO*3, the volcanic aerosols and the *TREND*. They do not reveal anything new about the solar cycle signal in temperatures. Here an annual average was taken for comparison with the correlation patterns of GT. Note that the effect of taking an annual average of these terms is linear, $\langle T^* \rangle = \langle T - T_c \rangle = \langle T \rangle - \langle T_c \rangle$, and does not change the general mathematical relationships described above.

3. Alternative Analysis

[16] A correlation is composed of the covariance of two time series normalized by the square root of the variance of the individual signals involved. One way to avoid the problem of polluting the correlation with the variances of a removed signal would be to consider the regression coefficient on its own merits. When the predictors of a regression analysis are uncorrelated, this is just the covariance of the predictor with the original variability divided by the variance of the predictor. The regression coefficient is then the amount that the observable changes for every

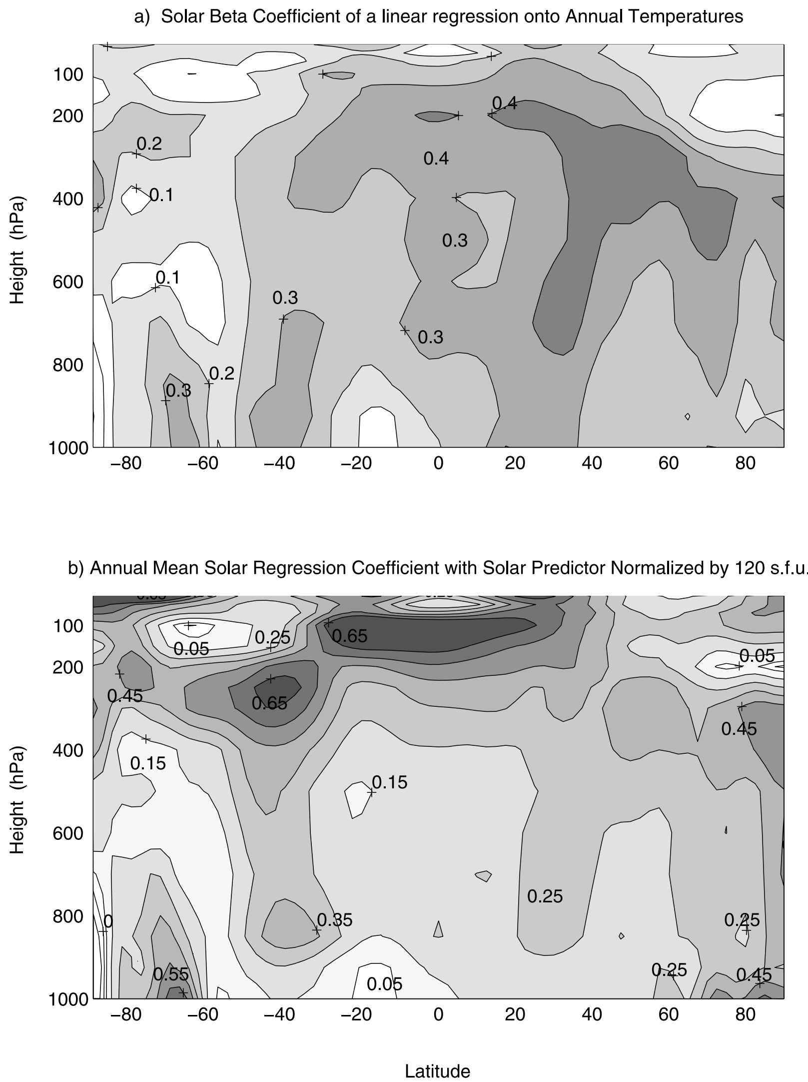

Figure 2. Annual regression on zonally averaged air temperature anomalies with 10.7 cm solar flux, 1958–2001. (a) The β coefficient shows the fraction of one standard deviation change in temperature (K) for every standard deviation change in solar flux. (b) This regression coefficient shows the change in temperature (K) for every 120 sfu (maximum-minimum) change in solar flux. A solar flux unit (sfu) is 10^{-22} W m^{-2} Hz^{-1}. These regression coefficients all show a small increase in temperature associated with an increase in solar flux.

standard deviation change in the predictor and it remains unchanged when other, orthogonal terms are filtered out of the time series:

$$\frac{Cov(S,T)}{Var(S)} = \frac{Cov(S,T^*)}{Var(S)} = k_2. \qquad (4)$$

GT did not show their regression coefficients. They showed only solar correlation coefficients calculated from the corrected time series, T^*.

[17] Figure 2 shows the NCEP/NCAR temperature reanalysis [*Kalnay et al.*, 1996] regressed onto the F10.7 cm solar flux (provided as a service by the National Research Council of Canada) for the same years as given by GT, 1958–2001, courtesy of the NOAA-CIRES Climate Diagnostics Center in Boulder, Colorado [*National Climate Data Center*, 1994]. Figure 2a shows the β coefficient which is produced by standardizing (subtracting the mean and dividing by the standard deviation) both the observable and the predictors before calculating the regression coefficients. The β coefficient shown here is then the fraction of one standard deviation change in temperature that coincides with one standard deviation change in the solar flux. Multiplying these values by the standard deviation of the temperature at each point, produces the regression coefficients referred to in equation (4). To see the change in temperature (K) for every 120 sfu, which is roughly the difference between the maximum and minimum 10.7 cm flux values, the solar predictor is normalized by 120 sfu, instead of one standard deviation of the solar flux time series. This is shown in Figure 2b.

[18] Comparing the pattern seen in this regression (Figure 2b) to those emphasized by GT (left part of their Figure 1c), we see some important differences. The equatorial maxima seen

in GT's temperature correlation, which they link to tropical convection, is completely missing in this pattern. Also, the structure in the subtropical troposphere is less pronounced.

[19] The main difference, though, is that these regression maps are robust. These maps provide direct information about the observable (in this case the annual mean NCEP/NCAR reanalysis temperatures) and they will not change with the addition of independent predictors.

[20] If, however, correlated predictors are necessary, then interaction terms become important and the linear regression model should be rethought. If the relationship between variables is known, these can be explicitly added to a linear regression model and this is the first thing that should be done. Curvature in the relationship can also be dealt with by transforming the variables or explicitly allowing for nonlinear components, or interaction terms, in the regression analysis. However, when the method of least squares is applied to nonorthogonal data, very poor estimates of the regression coefficients are obtained. *Mela and Kopalle* [2002] discuss the impact of collinearity on regression analysis. In general, the estimates of the regression coefficients may be inflated and very unstable; that is, their magnitudes and signs may change given different sampling of the data. Instead, *Montgomery and Peck* [1992] suggest using a bias estimator of the regression coefficients in a method called ridge regression, which is closely related to Bayesian estimation. See *Montgomery and Peck* [1992] for more details.

[21] Besides ridge regression, partial correlation may also be a useful method to derive a better conditioned regression model. It is a stepwise procedure where predictors are one by one added to or removed from the regression model in order of their partial F static. The procedure stops when the addition or subtraction of terms no longer improves the model (as defined by a set limit on the F statistic). Although the forward version of this procedure is unstable, the backward version (where all predictors are included and then one by one removed) and the stepwise regression (where previously added predictors can be removed from the forward version) tend to be reliable. Details of this method can be found starting on page 290 of *Montgomery and Peck* [1992].

4. Conclusions

[22] It is a challenge to extract a small signal amid larger noise. While at first glance, it may seem reasonable to filter out other signals in order to emphasize and examine the remaining signals, the impact of removing these signals needs to be appreciated. We show here that the resulting enhancements in correlation have nothing to do with the signature of the independent predictor in the original observations. Instead, the augmented spatial patterns are the result of the removed signals. The patterns are therefore misleading and, as shown by comparison with GT's solar example, not reliable as an indicator of a relationship with the original field.

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