NOTES AND CORRESPONDENCE

Low-Frequency Nonlinear Dynamics of Quasi-Geostrophic Waves in a Midlatitude Channel and the Effects of Tropical Influence

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ABSTRACT

Using a nonlinear model of stationary long waves which incorporates wave–wave and wave–mean flow interactions, we assess the variability of the stationary long waves in the stratosphere and troposphere. This note supplements the paper by Tung and Rosenthal, where the formulation was given in more detail, but the climatology and variability of stationary waves (No. 1 and No. 2) were not discussed.

1. Introduction

The debate over topography vs diabatic heating as the main source of stationary long waves has continued ever since the pioneering work of Charney and Eliassen (1949), who, using a barotropic midlatitude channel model, concluded that the large-scale quasi-stationary disturbances of the middle latitudes are mainly produced by flow over topography. Bolin (1950) and Smagorinsky (1953) found that thermal effects are important in producing the observed distribution of sea level pressure. Döös (1962), Sankar-Rao (1965a,b), Sankar-Rao and Saltzman (1969), and Saltzman (1965) concluded that differential heating is more important in accounting for the observed climatological stationary wave field in the troposphere. Manabe and Terpstra (1974) found from their GCM study that above the middle troposphere, topographic forcing is most important. Derome and Wiin-Nielsen (1971) found, using their two-level model, that neither form of forcing is negligible, but the response to orographic forcing is somewhat larger. Ashe (1979) used a two-level nonlinear model and found that the amplitude and scale of the observed climatology are predicted reasonably well by his topographic solution in the upper layer of his model, but better phase agreement was obtained when both forcings were included. Recently, Lin (1982) found topography to be more important, while Huang and Gambo (1982) found the opposite is true in their model. Alpert et al. (1983) found both to be important. Held (1983) found thermal effects to be dominant near the surface (but states, "However, the topographic response is not negligible at the surface either.") Jacqmin and Lindzen (1985) concluded that the stationary wave response to topographic forcing "strongly dominates" the response to thermal forcing both in the troposphere and the stratosphere. As one can see from our brief review, the issue is by no means settled at this point.

Some of the apparently conflicting conclusions can probably be reconciled by the argument [see Wallace (1983)] that orographic forcing is the dominant factor in determining the positions of the major ridges and troughs in the Northern Hemisphere stationary waves at the jet stream level, whereas thermal forcing makes an important contribution to maintaining the surface lows over the high-latitude oceans. The remaining discrepancy can, in part, be attributed to model differences and, in part, to the *variability* of the stationary waves in the atmosphere. Given the large month-to-month and year-to-year variability of the stationary long waves observed in both the stratosphere and troposphere [see Geller et al., (1984), Wallace and Blackmon (1983)], it is probably fortuitous that a certain numerical simulation under a single set of parameters produces a stationary wave field that is in good agreement with the observed climatology. Since in this case climatology is merely an ensemble average of several monthly means with large variance between each realization, a successful simulation of a particular climatology should not be taken to be a justification of the choice of model parameters (e.g., with and without mountains). Considerable caution should be exercised so as to avoid inferring too much from the supposed good agreement with climatology.

It thus appears that an understanding of the mechanisms that maintain the observed climatology of the stationary wave field cannot be separated from an understanding of the mechanisms that affect the variability of the stationary waves, and that, given the observed variability, it is probably more fruitful to undertake a study to understand the causes of variability than simply to make the simulation of observed climatology a final goal.

The present work is a mechanistic study of one of the many causes of low-frequency variability in the extratropical flow, namely, the effect of tropical influence. More specifically, we study the sensitivity of the large-scale flow in a midlatitude channel to changes in heat and momentum fluxes across its southern boundary.

One plausible mechanism that can account for the observed large variabilities in both the zonal mean flow and stationary waves in the extratropical stratosphere is the interaction between the forced stationary waves and the mean zonal flow. Given the large energy source contained in the stationary waves which are forced in the lower atmosphere, they are probably responsible for inducing the bulk of the variability in the mean flow in the stratosphere. Variability in the mean flow can, in turn, produce variability in the stationary wave amplitudes due to the possible sensitivity of wave response in the troposphere and stratosphere to the configuration of the wave guide determined by the mean flow [see Tung and Lindzen (1979) and Jacqmin and Lindzen (1985)]. In a nonlinear system like the atmosphere it is difficult, and perhaps fruitless, to separate the cause and effect of the coupling between the wave and the mean flow. We shall, therefore, treat the wave-mean flow system as an integral entity and instead assess the likely causes for the low-frequency variability of the system as a whole in extratropical latitudes.[1] We divide, in an admittedly artificial fashion, the causes of variability into "internal" and "external" ones. Internal mechanisms include transition between equilibria and vacillation cycles, while mechanisms external to the midlatitude system include eddy heat and momentum fluxes from the tropics[2] and the Hadley circulation forced in the tropics. A finding of significant sensitivity to the "external" influences will motivate future studies that treat both the tropics and the extratropics as an integral system.

2. The model

The model formulation has been discussed in detail in Tung and Rosenthal (1986). Only a brief summary is given here.

We consider quasi-geostrophic flows in a midlatitude channel governed by the following pseudopotential vorticity equation:

$$\frac{\partial}{\partial t} P\psi + J[\psi, P\psi + f] = f_0\left(\frac{\partial}{\partial z} - \frac{1}{H_0}\right)(Q/\Gamma) \quad (1)$$

where

$$P \equiv \nabla^2 + \frac{H_0 f_0^2}{R}\left(\frac{\partial}{\partial z} - \frac{1}{H_0}\right)\frac{\partial}{\partial z}\frac{1}{\Gamma}, \quad z \equiv H_0 \ln \frac{p_{00}}{p}$$

with

ψ streamfunction
Q diabetic heating
Γ static stability

To isolate the low-frequency portion of the flow, Tung and Rosenthal (1986) applied a running-time mean to (1). For simplicity in the present short presentation,[3] we shall omit such an operation here. In any case, if one is interested only in the low-frequency evolution of the stationary waves, it seems justified to drop the first term, $(\partial/\partial t)P\psi$, compared to a part of the second term involving the advection of the *wave* potential vorticity by the mean zonal flow, $(u\partial/\partial x)P\psi$. The same approximation cannot be adopted for the zonal mean equation, because the advection of the zonal mean potential vorticity by the mean zonal flow is zero. The zonal mean flow equation can be obtained by zonally averaging the vorticity Eq. (1) and then integrating it with respect to y twice and applying the lower boundary condition. Alternatively, one can use the zonal momentum equation and integrate it with respect to y once to obtain [see Tung and Rosenthal (1986)]

$$\frac{\partial}{\partial t} U = \frac{1}{\tau_E}(U^* - U) + T(U) \quad (2)$$

at the top of Ekman layer z_1, where

$$U \equiv \frac{1}{y_2 - y_1}[\psi|_{y1} - \psi|_{y2}] \quad (3)$$

is the zonal index between the latitidues y_1 and y_2. Here y_1 and y_2 are chosen to be the location of channel boundaries where the normal velocity due to the *stationary geostrophic* waves vanish, i.e.,

$$\frac{\partial}{\partial x}\psi = 0 \quad \text{at} \quad y = y_1, \text{and } y_2. \quad (4)$$

Therefore, U, as defined, is independent of x and y. Transient eddy fluxes of momentum and heat, as well as the ageostrophic mean Hadley circulation are not constrained by (4) and so are allowed to cross the southern boundary. Their effect on the angular mo-

[1] By concentrating only on the low-frequency portion of the variability, we will not be able to model the more rapid changes, such as sudden warmings, in adequate detail.

[2] By treating these wave fluxes across the southern boundary of the model (at 30°N) as "external" forcing, the feedback between the tropics and extratropics is not incorporated. Note also that this tropical influence is through the zonal mean budgets. Additional "teleconnection" via direct wave propagation from the tropics into the midlatitudes is not treated here.

[3] And also because we have not made an adequate assessment of the effect of the transient eddy-flux terms for the stratosphere.

mentum budget in (2) is included in the term $\tau_E^{-1} U^*$, which is defined as

$$\frac{1}{\tau_E} U^* = \frac{1}{(y_2 - y_1)} \overline{uv}|_{y_2}^{y_1}.$$

For an Ekman damping time τ_E of 6 days, we find that $U^* \sim 3$ m s^{-1} [see Tung and Rosenthal (1986)]. The parameter U^* is externally specified in our model.

In Eq. (2), T represents the deceleration by the mountain torque generated by pressure differences on the east and west sides of topography. It has to be calculated using the stationary wave solution to

$$J[\psi, P\psi + f] = f_0\left(\frac{\partial}{\partial z} - \frac{1}{H_0}\right)(Q/\Gamma) \qquad (5)$$

subject to the lateral boundary condition [(4) applied to (3)]

$$\psi|_{y2} = 0, \quad \psi|_{y1} = (y_2 - y_1)U(t, z) \qquad (6)$$

and the lower boundary condition [see Tung (1983)]

$$J\left[\psi, \frac{f_0 H_0}{R\Gamma}\psi_z + h\right] + \frac{H_0}{f_0 \tau_E}\nabla^2\psi = 0 \quad \text{at} \quad z = z_1. \qquad (7)$$

This includes forcing due to nonlinear flow over and around a realistic topography, but no diabatic forcing is included. Ekman pumping with a time scale of $\tau_E \sim 6$ days is incorporated. This is the only form of damping in the present form of our model.

The vertical variation of the zonal index is determined from the thermal wind relation according to

$$\frac{\partial}{\partial z} U = \frac{R}{(y_2 - y_1)H_0 f_0} \Delta\bar{T} \qquad (8)$$

where $\Delta\bar{T} \equiv \bar{T}|_{y_1} - \bar{T}|_{y_2}$ is the externally imposed temperature gradient in our model.

In the present calculation, radiative forcing and momentum damping on the *stationary waves* in the interior of the model are neglected. So, although the numerical domain spans from surface to 100 km, our results probalby are not too applicable to the real situation in the mesosphere and upper stratosphere. Our wave solutions in the upper atmosphere are, in general, too strong as a result of a lack of damping. The high upper boundary is adopted to faciliate the application of the upper boundary condition of either boundedness of energy density or upward wave radiation.

3. Results

Flow in the channel is forced by the two external parameters U^* and $\Delta\bar{T}$ previously mentioned, which are related to the flux of momentum and heat, respectively, into the midlatitude channel from the tropics. In addition $\Delta\bar{T}$ can change in response to changes in externally imposed radiative heating, but this aspect of the problem is not considered here. Thus, when we discuss the sensitivity of model response to tropical influence, we specifically mean the sensitivity to changes in the parameters U^* and $\Delta\bar{T}$ in a realistic range. In the absence of eddy momentum flux across our southern boundary at 30°N, we have $U^* \approx 0$ and the equilibrium solution in our channel has no net surface flow and no forced stationary wave. This situation is common in channel models with rigid lateral wall boundaries. The zonally symmetric flow (with zero surface velocity and a westerly upper-level flow forced by the temperature gradient) is referred to by some authors as the "Hadley regime." When a positive momentum flux consistent with observations (with $U^* \sim 3$ m s^{-1}) is imposed, a westerly surface flow (above the Ekman layer) is forced in the midlatitudes. Stationary waves are produced by this flow over the topography. Therefore, in this sense the observed zonally asymmetric westerly flow in the wintertime midlatitude region can be regarded as maintained largely by tropical westerly momentum sources and easterly momentum sinks.

By the thermal wind relations, it can be seen that the vertical shear of the zonal index in the channel is maintained by the gross horizontal temperature difference across the meridional channel [see Eq. 8].

In the first calculation, we specify $\Delta\bar{T}(z)$ to be such that it produces the "climatological" meridionally averaged vertical shear U_z taken from the analytic formula in Tung and Lindzen, (1979) (evaluated at 45°N). This, together with the surface value determined from (2), then yields $U(z)$. It should be noted that the actual zonal flow $\bar{u}(y, z)$ is, in general, not the same as $U(z)$, the latter being the channel average of the former. The meridional variation of $\bar{u}$ is to be determined as part of the nonlinear solution. In the set of calculations presented here, it is assumed that the meridional shear of $\bar{u}$ is induced entirely by the stationary waves. Specifically, we assume that in the *absence* of topography

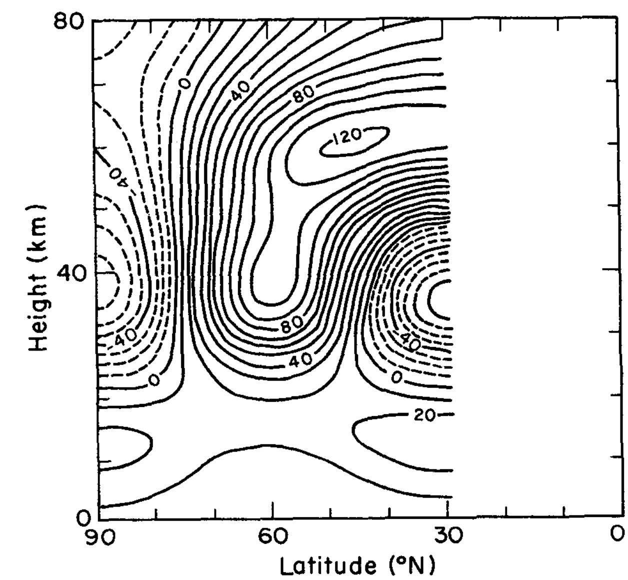

FIG. 1. Computed mean zonal wind $\bar{u}(y, z)$ in m s^{-1}. Surface zonal index is specified to be 3 m s^{-1}. Solid line denotes westerly flow and dashed line easterly flow.

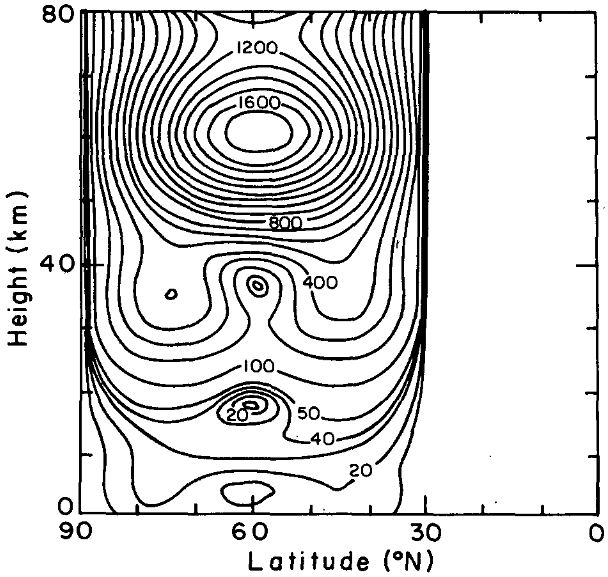

FIG. 2. Computed geopotential height in meters for zonal wavenumber 1 for the same condition as in Fig. 1.

(and, hence, zonal asymmetry in the present model) $\bar{u} = \bar{u}(z)$ and so, $\bar{u} = U(z)$.

In the presence of realistic topography, the calculated zonal mean flow $\bar{u}(y, z)$ is depicted in Fig. 1. Both the location and strength of the two jets are quite realistic, except that perhaps the easterlies over the high latitudes may be too strong. Figure 1 suggests that the stationary waves may play an important role in maintaining the horizontal jet structures in the real atmosphere. The wave amplitudes in the channel are shown in Fig. 2 for the zonal wavenumber 1 component and in Fig. 3 for zonal wavenumber 2 component. Compared with the "climatology" of van Loon et al. (1973) (with data available only below 30 km), we find that the wavenumber 1 amplitude in the lower stratosphere is slightly underpredicted by the model (400 m vs 600 m observed) and the calculated wavenumber 2 amplitude is less than half the observed (100 m vs 250 m observed). We will show in a moment that there is no difficulty in reproducing the amplitudes of van Loon et al. in the statosphere with a slight change of the external parameters. Our purpose here is to show the range of variability in the wave amplitudes and to point out that one should not demand each model simulation to produce the "observed climatology", because the latter is only a composite of many monthly means with large interannual variance.

However, this variability does not extend into the troposphere, where we always obtain underestimates for the stationary wave amplitudes (by a factor of about 2). It has often been suggested that diabatic differential heating has to be included in order to get the stationary wave amplitudes right for the troposphere. Perhaps this is so, but Jacqmim and Lindzen (1985) seem to disagree. Their calculation based on the linear primitive equations over the globe appears to have the right amplitudes in the troposphere even when topography is the only form of forcing for their stationary waves. Our experience based on our model has been that amplitudes calculated using the fully nonlinear version are always lower than those in the linear calculations; the magnitude of the difference varies depending on the meridional structure of the zonal mean wind used in the linear calculation, but in some cases can be large enough to account for the factor of 2 mentioned earlier. The problem of simulating the tropospheric stationary wave amplitudes is a difficult one and debates on topographic vs diabatic forcing are likely to continue for some time.

Given the large variability of the stratospheric wave amplitudes, it is not difficult to come up with a case that yields the so-called "correct climatological" wave amplitudes if that is what one desires. One such case is present here. This is obtained by decreasing the temperature gradient, $\Delta\bar{T}(z)$, at the statospheric jet level by about 20% (specifically, the parameter, U_2, in the analytic formula in Tung and Lindzen for the stratospheric jet strength is decreased from 150 to 120 m s^{-1}), and the momentum flux from the tropics, and hence the equilibrium U at the surface, is increased by 30%, (from 3 to 3.9 m s^{-1}). The effect of smaller $\Delta\bar{T}$ is to yield a weaker stratospheric jet, which turns out to be more favorable for the vertical propagation of wavenumbers 1 and, especially, 2. Wavenumber 2 was largely trapped in the previous case. The effect of a larger surface U is to create a stronger flow over the topography. This tends to increase the wave amplitudes at the surface and in the troposphere. The amplitudes for stationary wavenumbers 1 and 2 are presented in

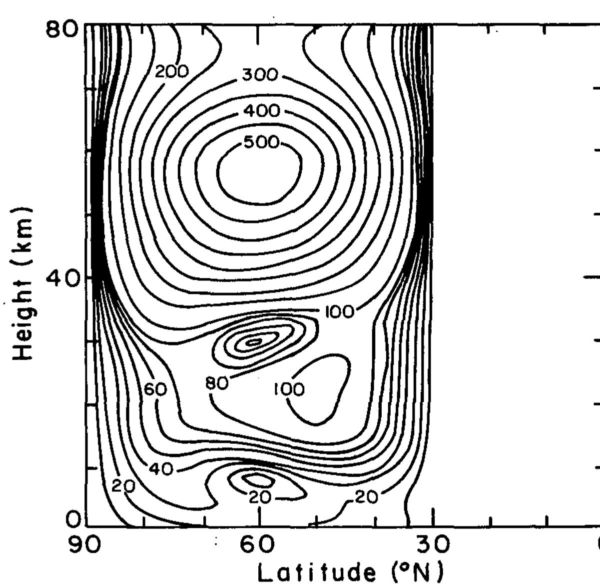

FIG. 3. As in Fig. 2 except for zonal wavenumber 2.

Figs. 4 and 5, respectively. It is seen that the wavenumber 1 amplitude now reaches 600 m and wavenumber 2 reaches 250 m near the 30 km level as observed (although the maximum occurs nearer the equator than in the climatology of van Loon et al.). The mean zonal flow induced is depicted in Fig. 6. The tropospheric jet is moved more poleward. The easterlies over the pole now largely disappear. The important point here is not how well we have simulated the climatology, but that by varying the momentum and heat fluxes ("tropical influence") by amounts reasonable within the observed range, we have produced a substantial change in the wave–mean flow system in midlatitudes.

The phase diagrams that describe the time variation of U are calculated according to Eq. (5) and shown in Fig. 7 for the $\Delta\bar{T}$ used in Fig. 1. The corresponding phase diagram (not shown) for the second case (with a 20% smaller $\Delta\bar{T}$ at the stratospheric jet level) has an almost identical shape except for the location of some of the small glitches. A different choice for U^* would move the horizontal axis (i.e., the location where $d/dt\ U = 0$) up or down without affecting the shape of the curve, but the location of the equilibrium would change. An important point is that there is only one equilibrium.

We are aware that many authors have argued for the existence of mulitple equilibria based on simpler (barotropic or two-layer) models. We have reexamined many of the earlier model results [see Tung and Rosenthal (1985); Cehelsky and Tung (1987)] and found that problems exist when these simple model results are applied to the atmosphere. For our model with only one equilibrium, stationary wave–mean flow systems whose net mass flow rate, U, is not the same as that equilibrium value will relax to the equilibrium value in an Ekman damping time scale. When the equilibrium is reached, the system will remain at the equilibrium until external parameters (U^* or $\Delta\bar{T}$) are changed.

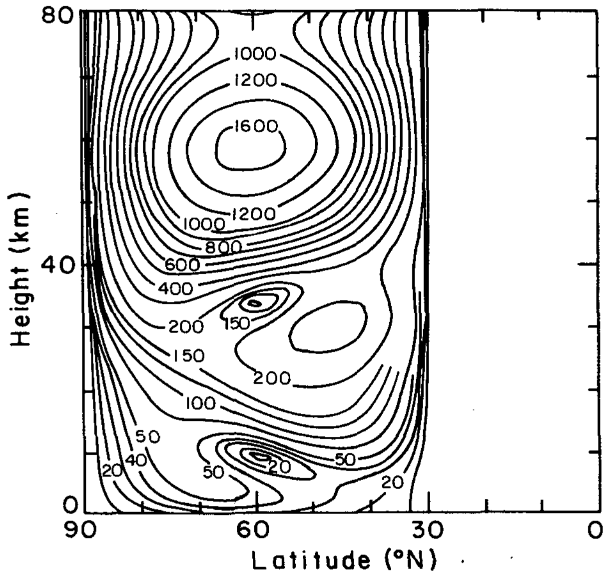

FIG. 5. As in Fig. 4 except for zonal wavenumber 2.

4. Conclusion

In conclusion, we have shown that our nonlinear model of stationary waves possesses substantial variability in both the stratospheric stationary wave am-

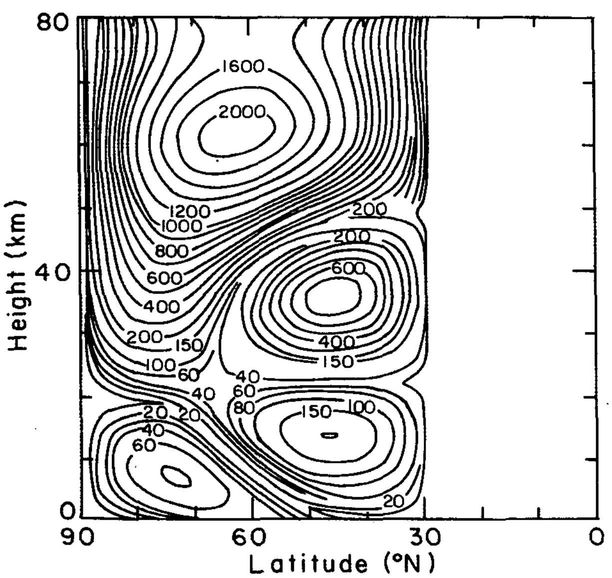

FIG. 4. Computed geopotential height in meters for zonal wavenumber 1 for a 20% decrease in temperature difference between the lateral boundaries, $\Delta\bar{T}$, at the stratospheric jet level and a 30% increase in surface zonal index.

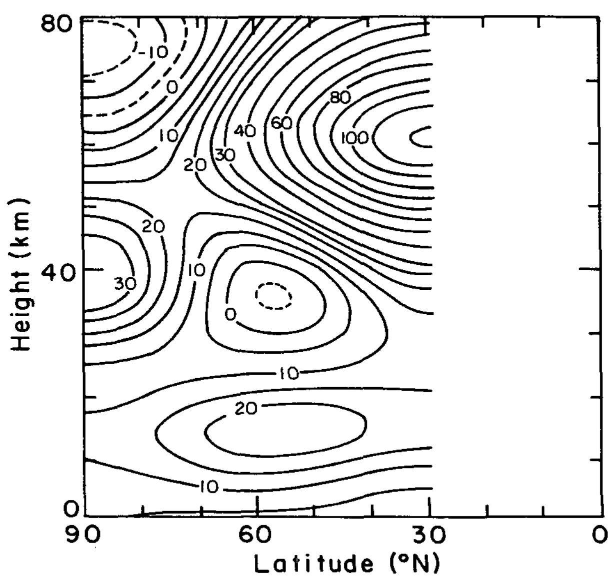

FIG. 6. As in Fig. 4 except for the zonal mean wind $\bar{u}(y, z)$ in m s^{-1}. Solid line denotes westerly flow and dashed line easterly flow.

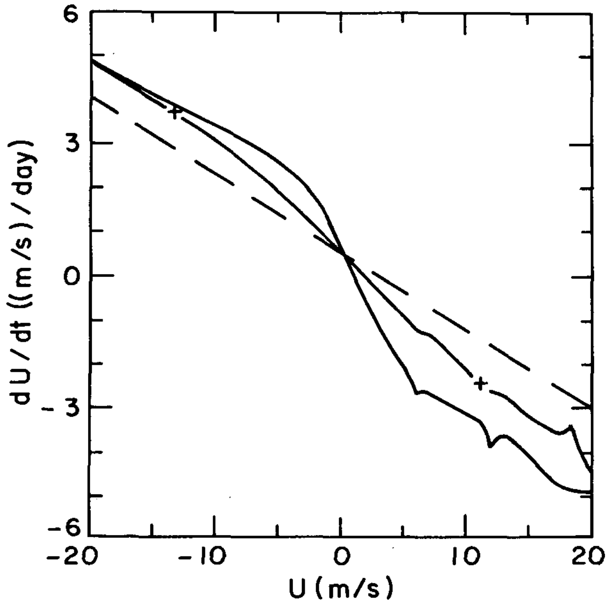

FIG. 7. Phase diagram for the surface zonal index U. Dashed straight line represents the function $1/\tau_E(U^* - U)$ in Eq. (5), which is the $\partial/\partial t U$ in the absence of mountains. The solid curve represents $\tau_E(U^* - U) + T(U)$ calculated without incorporating the nonlinear interactions. The curve with crosses is the same quantity calculated in the nonlinear model.

plitudes and the zonal mean wind and, importantly, that such variabilities can be caused by changes in eddy momentum and heat fluxes from the tropics into the midlatitudes (and, perhaps more importantly, from midlatitudes into the tropics) with magnitudes that lie within the observable ranges. It suggests that channel models which do not permit the fluxes of eddy momentum and heat across the southern boundary may underestimate the variability of the atmosphere.

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