

# Dynamic Variability of Column Ozone

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We give a first-order, analytic estimate of the dynamical change in column ozone density as a function of temperature change in the lower stratosphere. It is found that for each 1% ( $\sim 2^\circ\text{C}$ ) cooling averaged over the lower stratosphere, the column ozone density can be expected to decrease by approximately 7%, if it can be assumed that such a temperature change is induced by changes in wave transports.

## INTRODUCTION

Since the early days when Dobson's measurements of column density of ozone (i.e., total amount of ozone in a vertical column of unit area) were becoming available, researchers have noted the so-called "ozone-weather relationships" [Dobson *et al.*, 1928; 1946; Meetham, 1937; Reed, 1950]: A surface pressure system, when it is deep enough to affect the vertical motion in the lower stratosphere, could displace ozone mixing ratio isopleths up (or down), thus decreasing (or increasing) the column amount as measured by the Dobson instrument [Reed, 1950]. Changes in column amount of ozone are often found to be correlated with the changes in lower stratospheric temperature, with colder (warmer) temperatures correlating with lower (higher) column ozone amounts.

The mechanism responsible for ozone-weather correlation is likely to also apply to the case of larger-scale, and longer-term, temperature changes in the lower stratosphere. However, for time scales longer than those of weather systems (approximately a few days), the cause and effect of ozone-temperature changes are more difficult to discern based only on the observation that the temperature and ozone changes are correlated. This is because of the fact that temperature and ozone changes are intimately related, both dynamically and radiatively. A given ozone change (via chemical depletion, for example) can radiatively induce a temperature change in the lower stratosphere over a time scale longer than the radiative relaxation time. On the other hand, a given temperature change produced by, say, a change in large-scale wave transport of heat, can induce a change in column ozone through changes in the dynamical transports of ozone. Both mechanisms can produce temperature-ozone correlations.

Although the simulation of such temperature-ozone correlations requires a coupled radiative-dynamical-chemical model, it is sometimes useful, for physical understanding of

the dominant mechanism involved, to have answers to some simple "what if" type of questions:

1. How much of the observed ozone change can be attributed to the observed temperature change if one assumes that the temperature change is caused entirely by dynamical causes (namely, changes in wave Eliassen-Palm flux divergence)?

2. How much of the observed temperature change can be thought of as produced by the radiative effects of the observed ozone change?

It is the purpose of the present paper to address the first question and to derive some simple approximate analytic formula to help one estimate easily the dynamical component of ozone change. We believe that this is a necessary first step before one can understand and isolate the long-term trend in atmospheric ozone due to chemical causes.

Tung [1986] made an attempt to derive some simple relations between ozone and temperature changes. No general formula was obtained because of the difficulties associated with accounting for the horizontal as well as vertical transports of ozone. We have since overcome most of these difficulties and have obtained an approximate expression relating column ozone change to temperature change incorporating the effects of both vertical and horizontal transports.

Mathematically, since column density is a vertically integrated quantity measuring the total number of molecules of the species under consideration, only horizontal divergence (or convergence) of air out of (or into) the column can change the total column content. Thus our derived expression for the rate of change of column density,  $\Omega$ , involves only horizontal transports. Nevertheless, since by conservation of mass the horizontal and vertical velocities are related, the same change can be diagnosed from a consideration of the vertical displacement of tracer mixing ratio, provided it is understood that the difference in the mass of the species in the column is assumed to have been transported out of the column by the implied horizontal motion. Since the effect of vertical displacements on column density is easier to understand, we will begin our presentation by considering vertical displacements. We will then show that the results thus obtained are consistent with the more rigorous, but still approximate, formula derived by incorporating both horizontal and vertical transports.

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It should also be pointed out that since column density of ozone is affected more effectively by changes in the lower stratosphere, where its peak concentration is located, we will restrict our attention to the effect of temperature change in the lower stratosphere and to the dynamical effects of such a temperature change on column ozone. The effects of temperature feedback on the photochemistry of ozone, which may be more important higher up, is not studied here.

#### VERTICAL DISPLACEMENTS

Using the energy equation

$$\frac{d}{dt}T + \frac{g}{c_p}w = Q \quad (1)$$

where  $T$  is the temperature,  $w$  is the vertical velocity in the log-pressure coordinate  $z \equiv H \ln(p_{00}/p)$ , and  $Q$  is the net radiative heating rate, one can estimate the change in vertical displacement,  $\delta z$ , induced by a change in temperature  $\delta T \equiv T - T_1$ , from

$$\frac{d}{dt}\delta z = \delta w$$

and

$$\frac{d}{dt}\delta T + \frac{g}{c_p}\frac{d}{dt}\delta z = \delta Q \quad (2)$$

For the case where the given change in temperature,  $\delta T$ , is assumed to be caused by changes in wave driving and not by changes in the radiative properties of the atmosphere,  $\delta Q$  is given by

$$\delta Q = -\alpha\delta T$$

where  $\alpha = -\partial Q/\partial T$  is the coefficient of Newtonian cooling. For the more general case,  $\delta Q$  can be approximated by

$$\delta Q \simeq \alpha(\delta T_e - \delta T)$$

where  $T_e$  is the radiative equilibrium temperature. The formulae to be obtained are all under the assumption that  $\delta T_e = 0$ . That is, the change in temperature is not radiatively induced. This assumption is further discussed in a later section.

Equation (2) is integrated in time from  $t = t_1$ , when  $T = T_1$  and  $\delta T = 0$ , to  $t = t_2$ , when  $T = T_2$ ,  $\delta T = \Delta T \equiv T_2 - T_1$ , and  $\delta z = \Delta z$ , to yield approximately

$$\Delta z = -\frac{c_p}{g}\Delta T - \frac{c_p}{g}\alpha \int_{t_1}^{t_2} \delta T dt \quad (3)$$

Assuming that the temperature change occurs rapidly from  $T_1$  to  $T_2$  near  $t = t_1$  and that  $T$  persists at  $T_2$  for a period  $\Delta t \simeq t_2 - t_1$ , one can approximate the integral in (3) as

$$\int_{t_1}^{t_2} \delta T dt \simeq \Delta T \Delta t$$

Thus (3) is simplified to

$$\frac{\Delta z}{H} = -\kappa^{-1}(1 + \alpha\Delta t)\Delta T/T \quad (4)$$

where  $\kappa \equiv R/c_p = 2/7$ . The quantity  $\Delta t$  is understood to be the duration during which the temperature change  $\Delta T$  persists.

These approximate expressions for vertical displacements are not new. The first term on the right-hand side of (3) is understood to be due to the vertical lifting of the isentropes caused by the temperature change. The second term is the additional lifting relative to the isentropes when air is heated diabatically.

To see this, we can derive (3) alternatively in the following manner. Let  $\theta$  be the potential temperature defined by

$$\theta = T\left(\frac{p_{00}}{p}\right)^{R/c_p}$$

Let  $p_1$  be the pressure of the isentrope associated with the temperature  $T_1$  and  $p_2$  be associated with the temperature  $T_2$  for the same isentrope. Then,

$$p_1 = p_{00}\left(\frac{T_1}{\theta}\right)^{c_p/R}$$

and

$$p_2 = p_{00}\left(\frac{T_2}{\theta}\right)^{c_p/R} = p_1\left(1 + \frac{\Delta T}{T_1}\right)^{c_p/R}$$

Thus when  $T$  changes from  $T_1$  to  $T_2$ , the isentrope  $\theta$  will be shifted by an amount

$$\begin{aligned} (\Delta z)_{\text{adiabatic}} &= H \ln\left(\frac{p_1}{p_2}\right) = -H\kappa^{-1} \ln\left(1 + \frac{\Delta T}{T_1}\right) \\ &\simeq -H\kappa^{-1}\Delta T/T_1 \simeq -\frac{c_p}{g}\Delta T \end{aligned}$$

for small temperature change. This is the same as the first term on the right-hand side of (3).

A change of  $\delta T$  in temperature, as measured on isentropic surfaces, will induce a change in net diabatic heating rate approximately as

$$\delta Q = -\alpha\delta T.$$

There is a diabatic change in the vertical "velocity,"  $\dot{\theta}$ , relative to the isentropes, which can be estimated from the energy equation

$$\dot{\theta} = \frac{\theta}{T}Q$$

as

$$\delta\dot{\theta} = -\alpha\theta\frac{T_e}{T}\frac{\delta T}{T} \simeq -\alpha\theta\delta T/T$$

A tracer originally at  $\theta = \theta_1$  will be moved to  $\theta_2$  by the diabatic heating. At equilibrium with the new temperature  $T_2$ , the two isentropes  $\theta_1$  and  $\theta_2$  will be separated by a distance (in log-pressure) of

$$(\Delta z)_{\text{diabatic}} = H\frac{c_p}{R} \ln\left(\frac{\theta_2}{\theta_1}\right) - H\frac{c_p}{R} \ln\left(\frac{\theta_1}{T_2}\right) \simeq H\frac{c_p}{R}\frac{\Delta\theta}{\theta}$$

Since

$$\Delta\theta = \int_{t_1}^{t_2} \delta\dot{\theta} dt$$

one has

$$(\Delta z)_{\text{diabatic}} \simeq -\frac{c_p}{g}\alpha \int_{t_1}^{t_2} \delta T dt$$

which is the same as the second term in the right-hand side of (3).

From (4) we see that for the case  $\alpha\Delta t \sim 1$  the adiabatic and diabatic displacements are comparable in magnitude. For a 1% decrease in temperature, i.e.,

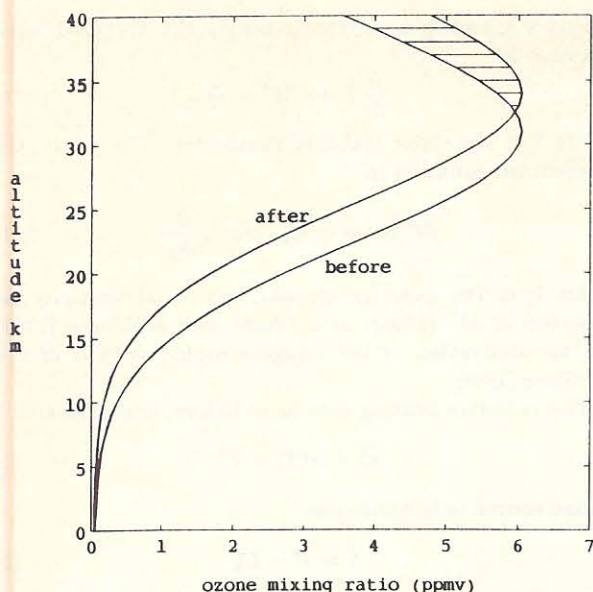


Fig. 1. Mass mixing ratios,  $\chi_1$  and  $\chi_2$ , of a tracer before and after vertical displacement, respectively.

$$\Delta T/T = -1\%$$

there will be a corresponding upward shift of the isentrope by

$$(\Delta z)_{\text{adiabatic}}/H = 3.5\%$$

and an additional lift relative to the rising isentrope by

$$(\Delta z)_{\text{diabatic}}/H \simeq 3.5\%$$

Thus the total vertical displacement due to a 1% decrease in temperature ( $\sim 2^\circ K$ ) is, from (4),  $\Delta z/H \simeq 7\%$ , or  $\Delta z = 0.5 \text{ km}$ .

#### EFFECT ON COLUMN DENSITY

As temperature is lowered by  $-\Delta T$ , air tends to be displaced upward by  $\Delta z$ , as given by (4). The mass mixing ratio of ozone, as well as other inert tracer, is also moved up by  $\Delta z$ . In doing so, the same mixing ratio is now located in an altitude with a lower air density. Therefore the column density of ozone, which is the vertical integral of the mass mixing ratio weighted by air density, is lowered [Reed, 1950]. The difference between the "before" and "after" column amounts, the "depletion," is accounted for by the implied horizontal transports away from the column. We defer the explicit calculation of horizontal transports until the next section. Here we diagnostically calculate the net change in column density by vertical displacement in the following manner.

Let  $\chi$  be the mass mixing ratio of an inert tracer and  $\Omega$  be its column density, with

$$\Omega \equiv \int_0^\infty \rho_o(o) e^{-z/H} \chi(z) dz \quad (5)$$

Let  $\chi_1$  and  $\Omega_1$  be the before profile and column density and  $\chi_2$  and  $\Omega_2$  be the after profile and column density,

respectively, where  $\chi_2$  is simply  $\chi_1$  displaced vertically by a distance  $\Delta z$  (see Figure 1), i.e.,

$$\chi_2(z) = \chi_1(z - \Delta z) \quad (6)$$

Equation (6) can be used to calculate  $\Omega_2$  in terms of  $\Omega_1$ :

$$\begin{aligned} \Omega_2 &\equiv \int_0^\infty \rho_o(o) e^{-z/H} \chi_2(z) dz \\ &= \int_0^\infty \rho_o(o) e^{z/H} \chi_1(z - \Delta z) dz \\ &= \int_0^\infty e^{-\Delta z/H} \rho_o(o) e^{-z'/H} \chi_1(z') dz' \quad (7) \end{aligned}$$

as a result of a change of independent variable  $z' = z - \Delta z$  and the assumption that  $\chi_1$  is negligible near the ground. For small temperature change,

$$e^{-\Delta z/H} \simeq 1 - \Delta z/H$$

and so (7) becomes

$$\frac{\Omega_2 - \Omega_1}{\Omega_1} \simeq -\frac{\widetilde{\Delta z}}{H} \quad (8)$$

where

$$\widetilde{\Delta z} \equiv \frac{\int_0^\infty (\Delta z) e^{-z/H} \chi_1(z) dz}{\int_0^\infty e^{-z/H} \chi_1(z) dz}$$

is the vertical average of  $\Delta z$  weighted by the preexisting partial pressure of the tracer under consideration. For an uniform temperature change

$$\widetilde{\Delta z} = \Delta z$$

and so

$$\frac{\Omega_2 - \Omega_1}{\Omega_1} \simeq -\frac{\Delta z}{H} = \kappa^{-1} (1 + \alpha \Delta t) \Delta T/T \quad (9)$$

In other words, a 7% decrease in column density can be expected from a 1% decrease in temperature, if the change in temperature is uniform vertically. If the temperature change is nonuniform, the effect of such a change will be weighted by the partial pressure of the tracer under consideration, and therefore the temperature change in the lower stratosphere is emphasized for a species such as ozone.

Ozone of course is not inert, and a part of its column in high altitudes is located in the photochemical equilibrium region. In this region the excess mass mixing ratio brought up from below by the rising isentrope (the shaded portion in Figure 1) will cause an enhanced photochemical destruction and will be "depleted" in the atmosphere's attempt to reestablish local photochemical equilibrium. Thus a dynamical process of mass redistribution can in principle lead to an irreversible chemical depletion, which can alter the global amount of ozone.

In conclusion, we state that for ozone, a 1% decrease in lower stratosphere temperature can displace ozone mass mixing ratio isopleths up by approximately half a kilometer. Such a vertical displacement can potentially cause at least a 7% decrease in its column density, if the effect of horizontal displacements can be neglected.

## HORIZONTAL TRANSPORTS

It is clear from the mass conservation principle that a purely vertical displacement of a tracer mixing ratio profile cannot change its total column content without the accompanying horizontal transport that actually takes the tracer molecules away (or into) the column. It is also clear (from equation (12) to follow) that it is not possible to have a purely vertical flow of air without inducing horizontal flows, if we assume that air masses are not escaping from the top of the atmosphere. Thus although we have considered only vertical displacements in the previous section, the effect on the column density of tracers displaced is in fact ultimately caused by the implied horizontal transports. This point will be made clear by the equation governing the change in column densities (equation (17) to be derived), which involves only the horizontal velocities. Nevertheless, since the horizontal velocity is related to the vertical velocity as a result of mass conservation, the result we have obtained earlier by specifying the vertical velocity without deducing the implied horizontal velocity turns out to be the same, when appropriately averaged, as the result we will obtain from specifically calculating such a horizontal velocity from the vertical velocity.

We will discuss the effects of horizontal transports only within the framework of a zonally averaged model. The equation of transport for ozone, whose mass mixing ratio is  $\chi$  and whose source is  $S$ , is given by

$$\frac{d}{dt}\chi = S \quad (10)$$

When zonally averaged (the overbar denotes averaging over the longitudes) the approximate two-dimensional equation is, when expressed in log-pressure coordinates,

$$\begin{aligned} \frac{\partial}{\partial t}\rho_o\bar{\chi} + \frac{\partial}{\partial y}(\rho_o\bar{v}^* \cos\phi\bar{\chi}) + \frac{\partial}{\partial z}(\rho_o\bar{w}^*\bar{\chi}) \\ - \frac{\partial}{\partial y}\rho_o K_{yy} \cos^2\phi \frac{\partial}{\partial y}\bar{\chi} = \rho_o\bar{S} \end{aligned} \quad (11)$$

where

$$\begin{aligned} z &= H \ln \frac{p_{o0}}{p} \\ \rho_o &= \rho_o(o)e^{-z/H} \\ y &= a \sin\phi \end{aligned}$$

and where  $\phi$  is equal to the latitude,  $K_{yy}$  is the isentropic transient eddy diffusion coefficient, and  $(\bar{v}^*, \bar{w}^*)$  is the residual mean circulation.

The approximation (11) is valid in the lower stratosphere in the absence of significant cross-isentropic mixing, except that strictly speaking (11) should be stated in isentropic coordinates ([see *Mahlman et al.*, 1984; *Tung*, 1982; 1984]). However, since we will be dealing with vertically integrated quantities, the choice of vertical coordinates becomes secondary.

The advective velocities are related through the continuity equation

$$\frac{\partial}{\partial y}\rho_o\bar{v}^* \cos\phi + \frac{\partial}{\partial z}\rho_o\bar{w}^* = 0 \quad (12)$$

Under geostrophic approximation, the thermodynamic equation is

$$\frac{\partial}{\partial t}\bar{T} + \Gamma\bar{w}^* = \bar{Q} \quad (13)$$

where  $\Gamma$  is the static stability parameter. The zonal mean momentum equation is

$$f\bar{v}^* \cos\phi = K_{yy} \cos^2\phi \frac{\partial}{\partial y}\bar{q} \quad (14)$$

where  $\bar{q}$  is the quasi-geostrophic potential vorticity (see *Newman et al.* [1986], and *Plumb and Mahlman* [1987]). For the derivation of the nongeostrophic version of (14), see *Tung* [1986].

The radiative heating rate is, as before, approximated by

$$\bar{Q} = \alpha(T_e - \bar{T}) \quad (15)$$

Ozone source is linearized as

$$\bar{S} = P - L\bar{\chi} \quad (16)$$

where  $P$  is the rate of production, and  $L$  is the loss frequency of ozone.

In terms of column density,

$$\bar{\Omega} = \int_o^\infty \rho_o\bar{\chi} dz$$

(11) becomes

$$\begin{aligned} \frac{\partial}{\partial t}\bar{\Omega} + \frac{\partial}{\partial y} \int_o^\infty dz \bar{v}^* \cos\phi [\rho_o\bar{\chi} - \frac{f}{\partial\bar{q}/\partial y} \rho_o \frac{\partial}{\partial y}\bar{\chi}] \\ = \int_o^\infty dz [\rho_o P - L\rho_o\bar{\chi}] \end{aligned} \quad (17)$$

Let

$$\langle ( ) \rangle = \int_{y_0}^{y_1} ( ) dy$$

be the latitudinal average over two latitudes  $y = y_0$  and  $y = y_1$ . We choose  $y_0$  to be the latitude where  $\bar{v}^* \cos\phi = 0$ . This is true if  $y_0 = \pm a$ , i.e., one of the poles. Thus (17) becomes

$$\begin{aligned} \frac{\partial}{\partial t} \langle \bar{\Omega} \rangle + \int_o^\infty dz \bar{v}^* \cos\phi [\rho_o\bar{\chi} - \frac{f}{\partial\bar{q}/\partial y} \rho_o \frac{\partial}{\partial y}\bar{\chi}] \Big|_{y_1} \\ = \int_o^\infty dz [\langle \rho_o P \rangle - \langle L\rho_o\bar{\chi} \rangle] \end{aligned} \quad (18)$$

Let  $\delta\bar{T} \equiv \bar{T} - \bar{T}_1$  be the temperature change. It is assumed that such a temperature change occurs between  $y_0$  and  $y_1$  so that  $\delta\bar{T} = 0$  at  $y = y_1$ . It is further assumed that  $\delta\bar{T}$  is deep vertically. Specifically, the temperature change occurs in the lower stratosphere over a scale larger than a scale height. From (13) the change in vertical velocity is

$$\delta\bar{w}^* = -(\frac{\partial}{\partial t}\delta\bar{T} + \alpha\delta\bar{T})/\Gamma \quad (19)$$

From (12) we have

$$\begin{aligned} \rho_o\delta\bar{v}^* \cos\phi|_{y_1} &= \frac{\partial}{\partial z} \int_{y_0}^{y_1} \rho_o(\frac{\partial}{\partial t}\delta\bar{T} + \alpha\delta\bar{T})/\Gamma dy \\ &\simeq -\langle \frac{\partial}{\partial t}\delta\bar{T} + \alpha\delta\bar{T} \rangle / (H\Gamma) \end{aligned} \quad (20)$$

assuming the temperature change is "deep" vertically, so that the dominant vertical variation is given by  $\rho_o(z)$ . Note that  $\delta\bar{v}^*$  locally at  $y_1$  is affected by  $\delta\bar{T}$  in the whole region between  $y_o$  and  $y_1$ .

From (18) the ozone change corresponding to the temperature change is given by

$$\frac{\partial}{\partial t} \langle \delta\bar{\Omega} \rangle + \int_o^\infty dz \langle L\rho_o\delta\bar{\chi} \rangle + \int_o^\infty dz \delta\bar{v}^* \cos\phi[\rho_o\bar{\chi} - \frac{f}{\partial\bar{q}/\partial y} \rho_o \frac{\partial}{\partial y} \bar{\chi}] \Big|_{y_1} = 0 \quad (21)$$

In (21) the change in ozone at  $y_1$ , where  $\delta\bar{T} = 0$ , is ignored. Combining (20) and (21), we have

$$\frac{\partial}{\partial t} \langle \delta\bar{\Omega} \rangle + \int_o^\infty dz \langle L\rho_o\Delta\bar{\chi} \rangle = \int_o^\infty dz \langle \frac{\partial}{\partial t} \delta\bar{T} + \alpha\delta\bar{T} \rangle (H/\Gamma)^{-1} [\rho_o\bar{\chi} - \frac{f}{\partial\bar{q}/\partial y} \rho_o \frac{\partial}{\partial y} \bar{\chi}] \Big|_{y_1} \quad (22)$$

Again letting the tilde denote the vertical average weighted by the preexisting ozone partial pressure, we have

$$\tilde{L} \equiv \int_o^\infty (L)e^{-z/H} \chi_1(z) dz / \int_o^\infty e^{-z/H} \chi_1(z) dz$$

The second term on the left-hand side of (22) is approximated, assuming small changes, as

$$\int_o^\infty dz \langle L\rho_o\delta\bar{\chi} \rangle \simeq \tilde{L} \int_o^\infty dz \langle \rho_o\delta\bar{\chi} \rangle$$

where  $\tilde{L}$  is the typical loss frequency of ozone in the lower stratosphere. For the right-hand side of (22) we assume for the moment that the horizontal gradient of  $\bar{\chi}$  at  $y_1$  is negligible. The effect of nonzero  $\partial\bar{\chi}/\partial y|_{y_1}$  will be incorporated later. Thus for deep  $\delta\bar{T}$  (22) becomes

$$\frac{\partial}{\partial t} \langle \delta\bar{\Omega} \rangle + \tilde{L} \langle \delta\bar{\Omega} \rangle = \left[ \frac{\partial}{\partial t} \langle \delta\bar{T} \rangle + \alpha \langle \delta\bar{T} \rangle \right] \bar{\Omega}(y_1)/(H\Gamma) \quad (23)$$

Equation (23) can be integrated between  $t = 0$  to  $t$ , to yield

$$\frac{\langle \delta\bar{\Omega} \rangle}{\bar{\Omega}(y_1)} = (H\Gamma)^{-1} \langle \delta\bar{T} \rangle + (\alpha - \tilde{L})(H\Gamma)^{-1} e^{-\tilde{L}t} \int_o^t \langle \delta\bar{T} \rangle e^{\tilde{L}t'} dt' \quad (24)$$

The first term on the right-hand side of (24) can be recognized as the change in column density due to adiabatic uplifting of the isentropes, while the second term is due to the diabatic upward movement of air relative to the isentropes, modified by the presence of chemical loss.

We evaluate (24) at  $t = \Delta t$ , where  $\Delta t$  is the persistence time for the temperature change defined earlier. At  $t = \Delta t$ ,  $\delta\bar{\Omega} = \Delta\bar{\Omega}$ ,  $\delta\bar{T} = \Delta\bar{T} \equiv \bar{T}_2 - \bar{T}_1$ . If it is assumed that the photochemical relaxation time,  $\tilde{L}^{-1}$ , is much longer than the persistence time  $\Delta t$  for the temperature change, this yields

$$\frac{\langle \Delta\bar{\Omega} \rangle}{\bar{\Omega}(y_1)} \simeq (H\Gamma)^{-1} (1 + \alpha\Delta t) \langle \Delta\bar{T} \rangle \quad (25)$$

The terms on the right-hand side of (25) can be identified with the corresponding terms on the right-hand side of (9) derived in the previous section, except that the appropriate quantities should be averaged over the latitude of temperature change.

In regions when the photochemical time  $\tilde{L}^{-1}$  is shorter than the persistence time  $\Delta t$ , (24) becomes

$$\frac{\langle \Delta\bar{\Omega} \rangle}{\bar{\Omega}(y_1)} \simeq (H\Gamma)^{-1} (\alpha/\tilde{L}) \langle \Delta\bar{T} \rangle \quad (26)$$

In this case the change in column density of ozone is mainly caused by enhanced photochemical destruction of air lifted by vertical motion into the photochemical equilibrium region. Since  $\tilde{L}^{-1}$  is typically of the order of 100 days at high latitudes and is longer than  $\Delta t$ , it is (25) that applies in practice.

If the temperature change  $\Delta\bar{T}$  is not deep, then the quantity  $\Delta\bar{T}$  in the formula (25) should be reinterpreted as the averaged temperature change weighted by the preexisting vertical profile of ozone partial pressure.

If the horizontal gradient of  $\bar{\chi}$  near the edge of temperature change, i.e.,  $\partial\bar{\chi}/\partial y|_{y_1}$  is not small, an additional factor

$$\left\{ 1 - \frac{f}{\partial\bar{q}/\partial y} \frac{\partial}{\partial y} \frac{\bar{\Omega}(y_1)}{\bar{\Omega}(y_1)} \right\} \quad (27)$$

should be multiplied to the right-hand side of the formula in (25). The factor is less (greater) than 1 if the sign of  $f\partial\bar{\Omega}/\partial y$  is positive (negative), assuming  $\partial\bar{q}/\partial y \simeq 2\Omega/a$  is positive. (Here and only here  $\Omega$  denotes Earth's rotational frequency). For ease of interpretation, one should pick  $y_1$  to lie in a region of small horizontal gradient of  $\Omega$  so that (27) would be close to 1 and (25) applies.

In summary, we state that a 1% decrease in lower stratosphere temperature averaged over a deep vertical extent can cause a 7% decrease in column density of ozone, averaged over the horizontal extent of the temperature change. At any particular location however, the local (unaveraged) change can be higher (or lower) than 7% if horizontal displacement into the region is down (or up) the gradient of the preexisting ozone mixing ratio.

#### OTHER CONSIDERATIONS

In the real atmosphere, temperature changes can sometimes be induced by a combination of both dynamical and radiative causes. In the extreme case where the temperature change  $\delta T$  is simply a response to  $\delta T_e$ , there would be no change in diabatic heating, i.e.

$$\delta Q \simeq \alpha(\delta T_e - \delta T) = 0$$

and so  $(\Delta z)_{\text{diabatic}} = 0$ . This is the case for the doubling  $\text{CO}_2$  experiment of *Fels et al.* [1980], where temperature in the stratosphere is changed in response to changes in radiative equilibrium temperature. No appreciable change in diabatic circulation is found in the General Circulation Model by *Fels et al.* [1980]. However, the cooling of the lower stratosphere as a result of  $\text{CO}_2$  increase can affect the global column abundance of ozone by the adiabatic process. So, for this case there will be a 3.5% decrease in column density of ozone for each 1% cooling of the lower stratosphere temperature.

## CONCLUSIONS

The observed latitudinal distribution (see e.g. *Bowman and Krueger* [1985]) of column density of ozone is a testimony to the importance of dynamical transport in controlling the abundances of ozone in the high latitudes. In the tropical region, where ozone is predominantly produced, the column abundance is actually very low all year around. Ozone produced here is transported to middle to high-latitudes in both hemispheres, where maximum column densities are found.

It can be said that since high-latitude ozone is there because it is brought there from low and mid-latitudes by the dynamical transports, it is therefore not surprising to find that high-latitude ozone is very sensitive to dynamical variability in the transporting circulation. We have quantified, in a first-order approximation, such a sensitivity to be approximately -7% change in column ozone density to -1% change in temperature in the lower stratosphere, if it is assumed that the temperature change is dynamically induced. This conclusion is not restricted to high latitudes and appears to apply to other parts of the globe. However, since large seasonal and year-to-year changes of temperature tend to occur near high latitudes, the magnitude of dynamical variability there appears to be higher than, say, over the tropics.

This estimate appears to be quantitatively consistent with the observational evidence presented by *Mahlman and Fels* [1986] concerning the year-to-year declines in the October mean column ozone changes over Antarctica. They found a roughly 10 Dobson unit decrease in column ozone associated with each degree drop in the lower stratospheric temperature. It should however be noted that there may be some fraction of the total Antarctic ozone decrease since 1979 that is unrelated to temperature changes [*Schoeberl et al.*, 1986; *Newman and Randel*, 1988].

The virtue of our analytic, approximate, formula lies in its simplicity. It is, however, not intended to be used in place of the more accurate numerical model simulations. It is useful in aiding our physical understanding of the dominant processes involved by giving us an order-of-magnitude estimate of the dynamical component of ozone change. If such an estimate is very different from the observed change, then other, nondynamical, causes of ozone change should be sought.

It is also useful to point out that we have been concerned here only with changes between two atmospheric states. We have shown that a positive difference in vertical displacement can cause a negative change in column density of ozone. There is no requirement that the actual vertical transports in either of the two atmospheric states be positive.

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