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Tropospheric Wave Response to Descending Decelerations in the Stratosphere

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Abstract.

Baldwin and Dunkerton [1999] found that negative Northern Annular Mode (NAM) anomalies sometimes descend all the way from the stratosphere into the lower troposphere. A further analysis here shows that the character of the anomaly changes across the tropopause. Above the tropopause the NAM is approximately zonal and its descent represents the descent of decelerated zonal mean winds. This is explainable using theories similar to that for the descent of the zero-wind line associated with a major stratospheric sudden warming. Rarely can such a deceleration reach the denser troposphere. In the troposphere, however, the structure of the NAM has a large wave number one component. In some cases, this wave component appears to react to the decelerated wind configuration aloft. We also present a simple model calculation to show that tropospheric waves can react to changing stratospheric winds. These waves can project directly onto the tropospheric NAM and produce anomalies which appear connected to the negative stratospheric NAM anomalies.

1. Introduction

The Northern Annular Modes (NAMs) are a useful measure of vertical coupling between the extratropical stratosphere and troposphere. Negative anomalies in the stratosphere are shown to propagate downward to the lower-most stratosphere and influence the troposphere (*Baldwin and Dunkerton* [1999, 2001]; *Thompson and Wallace* [2001]). Here we show that although the stratospheric descents of NAM anomalies are well represented by changes in the zonal mean geopotential height field, the tropospheric NAM is sometimes better represented by the wave components of the geopotential height. Theoretically, Rossby wave drag on the mean flow and the descent of critical surfaces can account for the occurrence of stratospheric warming events and may explain the descent of negative NAM anomalies toward the troposphere. A simple analysis is presented which assumes the downward propagation of decelerated winds in the stratosphere and then uses linear quasi-geostrophic theory to describe the tropospheric wave reaction to the changing medium aloft. Some of the tropospheric anomalies seen to be connected to the descending stratospheric deceleration could actually be projections of planetary waves onto the tropospheric EOFs.

2. Observational Evidence

Baldwin and Dunkerton [1999, 2001] use the Northern Annular Modes to convincingly demonstrate that the stratosphere and troposphere are connected. The Northern Annular Modes (NAMs) consist of the first empirical orthogonal function (EOF) of geopotential height at each level — an EOF represents the spatial pattern which contains the most variance. These papers show that negative anomalies in the stratospheric NAM statistically coincide with negative anomalies in the tropospheric NAM. However, the spatial structure of the stratospheric and tropospheric EOFs are not the same. The NAM consists mainly of a zonal mean structure in the stratosphere but acquires wave components in the tropopause region. In the troposphere the NAM pattern has a large wave 1 component in addition to the zonal mean structure (see figure 1). As an example of descending negative anomalies, we consider the winter of 1998/1999 (figure 2). This winter contains two such events. The top of figure 2 shows the NAM anomalies from October 1998 to April 1999. It is an enlarged section of Plate 1 in Baldwin and Dunkerton's 2001 *Science* paper. It shows two large stratospheric events where negative anomalies (red) of the NAM descend toward the tropopause. In the first event (December), the tropospheric response is a positive NAM anomaly (blue) and, in the second event (March), the negative anomaly is seen to descend all the way down to the surface. Comparison of the top figure depicting changes in the atmospheric EOFs with the lower figure showing changes in the zonal mean flow for these same times is interesting. The two stratospheric events are evident in both the NAM anomalies and the projection of the geopotential height anomalies onto the zonal mean flow. However, the changes in the zonal mean field do not descend into the troposphere as they appear to do in the NAM figure.

figure 1

figure 2

In contrast, when we look at the wave-one component of the geopotential height (bottom of figure 3), the tropospheric anomalies seen in Baldwin and Dunkerton's figure become apparent. The wave-one component here is calculated by projecting the geopotential height (normalized by $\sqrt{density}$) onto a wave-one structure oriented so that it coincides

figure 3

with the wave-one component of the Arctic Oscillation (the surface EOF). In this way, the negative values (red) can be directly compared with the negative (red) anomalies of the NAM. Both the zonal mean regression and the wave-one regression are calculated with the geopotential height at 60°N, where the wave component of the NAM is large. This simple example demonstrates how the vertical coupling seen in the NAM can be viewed as changes in the stratospheric zonal mean influencing the wave-one component in the troposphere. Note also that the wave-one component exhibits precursors (in November and perhaps again in January and February) to the stratospheric events. This is when the wave, generated in the troposphere, propagates into the stratosphere, ostensibly breaking there and thus slowing the stratospheric mean flow as in the traditional theory. We then see the changes in the stratospheric mean flow. In this particular case, observations imply that there is a connection between descending deceleration of the stratospheric zonal mean flow and the tropospheric waves. Since it is less physically plausible to view the subsequent tropospheric wave anomaly as originating from the stratosphere, we consider the possibility that the observed wave anomaly is instead caused by tropospherically forced waves (due to topography and land-sea contrasts) which react to the descending deceleration aloft. A simple model calculation, outlined below, verifies the physics of this interaction.

3. Analytical Calculation

Vertically propagating Rossby waves generated in the troposphere propagate into the stratosphere where the density is low. As the wave amplitude increases beyond a certain threshold, it breaks. There it deposits easterly momentum to the stratospheric mean flow. In the winter, when the stratospheric winds are generally westerly, this tends to decelerate the winds. Subsequently, the critical threshold is lowered and planetary waves break lower in the atmosphere (*Matsuno* [1971], *Andrews, Holton and Leovy* [1987]). This wave-mean flow interaction seems to also account for the descent of negative NAM anomalies in the stratosphere, although the locations of negative anomalies are not necessarily the location of the zero-wind line. In the troposphere, where the air is much more dense, a substantial amount of energy is needed in order to affect the zonal mean circulation. However, as pointed out in the previous observations, the NAM anomalies in the troposphere also have a large wave component. The vertical connection seen in the NAM may be due to a troposphere wave response to the descent of stratospheric decelerations instead of a direct change in the tropospheric zonal mean wind. In the following analysis, the descent of stratospheric anomalies is assumed and linear quasi-geostrophic theory is used to describe the subsequent wave reaction in the troposphere.

The linearized Quasi-Geostrophic Potential Vorticity equation (eg. *Andrews, Holton and Leovy* [1987] page 175) can be written as

$$\epsilon\psi_{zz} + \psi_{yy} + F(z)\psi = 0 \tag{1}$$

$$F(z) = \frac{\beta - \epsilon(u_{0zz} - u_{0z}) - u_{0yy}}{u_0 - c} - k^2 - \frac{\epsilon}{4}$$

and the linearized thermodynamic equation gives us a relationship between the vertical velocity, w, and ψ

$$\frac{-wN}{ik\sqrt{\epsilon}}e^{-\frac{z}{2}}e^{-ik(x-ct)} = (u_0 - c)(\frac{1}{2}\psi + \psi_z) + u_{0z}\psi = 0. \tag{2}$$

u_0 is the background velocity, c is the phase velocity and k is the zonal wave number. Here ψ is related to the streamfunction by an exponential factor, $\psi' = \psi(y,z)e^{\frac{z}{2}}e^{ik(x-ct)}$ where z is the vertical coordinate, $z = -ln(\frac{p}{p_s})$, and ϵ is the ratio between the Coriolis force and the buoyancy forcing, $\frac{f_0^2}{N^2H^2}$. The bouancy frequency, N, which has a constant value of 0.01 in the troposphere and 0.02 in the stratosphere. At the tropopause, the value of ϵ is taken to be an average of its tropospheric value and its stratospheric value. The potential vorticity equation is separable for the case where u_0 is a function of z only and we can solve for the vertical and meridional functions. Using the boundary conditions $\psi = 0$ at the pole, $y = \pi a/2$, and at the equator, $y = 0$, $Y(y)$ is a sine series and $Z(z)$ satisfies the equation

$$\epsilon Z_{zz} + n(z)Z = 0.$$

where the index of refraction is

$$n(z) = \frac{\beta - \epsilon(u_{0zz} - u_{0z})}{u_0 - c} - (k^2 + l^2) - \frac{\epsilon}{4}. \tag{3}$$

Assuming that the forcing of the Rossby waves is due to topography, which is stationary, $c = 0$.

The idealization of decelerating winds will be represented by a profile shown in figure 3. The bottom of the decelerated stratosphere, z_2, will slowly descend from the top of the **figure 3** domain to 14 km, just above the extratropical tropopause height, and the tropospheric wave response will be calculated.

We call the upper part of the profile, region III. This is the deceleration region ($z > z_2$) where $u = u_{iii}$. To avoid the mathematical complications of a critical level, we shall take u_{iii} to be small but positive. The unperturbed stratospheric region is denoted region II, where the zonal wind is $u = u_{ii} > u_{iii}$. The lower tropospheric section, region I, where $z < z_1$, has a constant tropospheric wind speed $u = u_i$. The general solution in each region is

$$A_* \sin(\sqrt{n_*} z + \phi_*).$$

At the surface, $z = 0$, the vertical velocity $w = u^+ \frac{db}{dx}$ where b is the topography and u^+ is the zonal wind speed just above the surface, in this case, $u^+ = u_i$. If the topography is divided into its Fourier components, $b = he^{ikx} \sin(ly)$, and the vertical velocity at $z = 0$ is

$$w = u_i ikh \sin(ly) e^{ikx}.$$

h is the height of the mountain, which is taken to be 88 meters when the latitudinal wave number is two and 94 meters when the latitudinal wave number is one. These are the amplitudes of the waves associated with the combined topographic and thermal forcings as deduced from the observed wave fields (*Tung and Lindzen* [1979]). In all cases the zonal wave number is taken to be one to match the zonal wave number seen in the tropospheric NAM. Substituting the vertical velocity into the thermodynamic equation gives the lower boundary condition.

The upper condition is that the solutions are bounded as $z \rightarrow \infty$. This means that the solutions in region III must be either evanescent or radiating. Writing the uppermost solution in terms of exponentials simplifies this condition, $Z_{iii} = A_{iii}e^{\sqrt{n_{iii}z}} + B_{iii}e^{-\sqrt{n_{iii}z}}$. If $n_{iii} < 0$, then exponential solutions are found and the bounded condition can be satisfied by setting $B_{iii} = 0$. If $n_{iii} > 0$, then propagating solutions exist. The appropriate condition is then found by solving for the group velocity and ensuring that the energy propagates away from the domain of interest. In this case, that is true when the group velocity is positive. The vertical wave number is

$$m = \pm\sqrt{n_{iii}} = \sqrt{\frac{\beta}{u_0 - c} - (k^2 + l^2) - \frac{\epsilon}{4}}.$$

From this we can solve for the group velocity

$$c_g = \frac{\partial \omega}{\partial m} = \frac{2mk\beta}{k^2 + l^2 + m^2 + \frac{\epsilon}{4}}.$$

Assuming that the zonal wave number, k, is positive, the group velocity is positive when the vertical wave number, m, is positive. The solution in the upper part of the domain is then also found when $B_{iii} = 0$. By substituting the solutions for region I into the vertical velocity equation at the bottom boundary and evaluating, it is found that the coefficients must satisfy the relationship,

$$A_i(\frac{1}{2}\sin\phi_i + \sqrt{n_i}\cos\phi_i) + \frac{Nh}{\sqrt{\epsilon}} = 0.$$

Mathematically, we can treat each section separately and assume the solutions to be continuous at each discontinuity. Integration over the discontinuity gives the second constraint at the interface between regions. The matching conditions at $z = z_1$ are

$$A_i \sin(\sqrt{n_i}z_1 + \phi_i) = A_{ii}\sin(\sqrt{n_{ii}}z_1 + \phi_{ii})$$

and at $z = z_2$

$$A_{ii}\sin(\sqrt{n_{ii}}z_2 + \phi_{ii}) = A_{iii}e^{i\sqrt{n_{iii}}z_2}.$$

The second matching condition at the interfaces is found by integrating the original equation over the discontinuities. This condition gives the constraints

$$u_{ii}\sqrt{n_{ii}}A_{ii}\cos(\sqrt{n_{ii}}z_1 + \phi_{ii}) - u_i\sqrt{n_i}A_i\cos(\sqrt{n_i}z_1 + \phi_i) + (u_{ii} - u_i)A_i\sin(\sqrt{n_i}z_1 + \phi_i) = 0$$

and

$$iu_{iii}\sqrt{n_{iii}}A_{iii}e^{i\sqrt{n_{iii}}z_2} - u_{ii}\sqrt{n_{ii}}A_{ii}\cos(\sqrt{n_{ii}}z_2 + \phi_{ii}) + (u_{iii} - u_{ii})A_{iii}e^{i\sqrt{n_{iii}}z_2} = 0$$

Using these matching constraints, we can solve for the coefficients of the solutions in each region. In the troposphere, region I, both the phase and the amplitude depend on the depth of region II, $z2-z1$. At a critical depth, the tropospheric solution can change signs. If we think of the descending region as an absorption surface where the wave energy is transferred to the deceleration of the zonal mean, the phase of the tropospheric wave depends on the level of the absorption layer. Both the phase shift and the amplitude variations are associated with the changes in the medium above. Note that in this model there is no critical level, the absorption is instead simulated by radiation conditions in the upper layer.

Some examples of the type of behaviour that these analytic results can produce are shown here. These results are calculated at $y = 60°$.

Figure 5 and Figure 6 show enlargements of the two observed 1998/1999 negative NAM anomalies. In figure 5, the observed negative NAM descends all the way down to the **Figure 5** **Figure 6** surface. This corresponds with idealized calculations, like those shown to the right of the enlarged observation. The theoretical calculation uses background wind speeds of u_i = 27 m/s, u_{ii} = 60 m/s, u_{iii} = 44 m/s and latitudinal wave number one. The geopotential wave component is weighted by the square root of the density and in the stratosphere the wind anomaly is shown. The height of the wind anomaly is shown along the x-axis, which is time-like, as the wind descends at a uniform speed. In the troposphere the weighted wave amplitude is initially -0.3 meters and decreases to -50 meters, when the negative wind anomaly nears the tropopause. Contour intervals are 1.5 meters. *Baldwin and Dunkerton's* 1999 figure shows the NAM index, which has contour intervals of ± 0.5. To compare the values, this index should be multiplied by the values of the EOF pattern and then normalized by the square root of the pressure/1000 hPa. The anomaly values are of the same order of magnitude in both the observed and in the simple model. At 700 hPa, the value of the comparable negative geopotential height anomaly is also about -50 meters.

In figure 6, another type of tropospheric wave reaction is shown. To the left is the observed December, 1998 event, where we see negative stratospheric anomalies but positive tropospheric anomalies. On the right of the figure is a calculation which shows similar results. Here u_1 = 8 m/s, u_2 = 13 m/s, u_3 = 2 m/s, and the latitudinal wave number is two. This produces a positive periodic tropospheric wave response to the decending stratospheric deceleration.

The variation of amplitudes and phases seen in these figures are entirely due to changes in the height of the decelerating region and to the variations in the background wind speeds. The first example demonstrates a situation where the tropospheric waves remain relatively unaffected by the descent in the stratospheric deceleration until the descending region reaches about 3 scale heights. At this point there is a phase shift and a related increase in amplitude. In this case, the lower stratospheric value of the winds are relatively large so that the troposphere does not feel the stratospheric changes until the deceleration region is quite close to the tropopause. In the second example, the lower stratospheric winds are weaker allowing easy communication between the troposphere and the absorption region. Here we see large changes in the tropospheric waves in response to the descent of the absorbing region. Changing the height of the absorption layer changes both the phase and amplitude of the tropospheric wave.

This calculation demonstrates that changes in the zonal mean of the light stratosphere can affect wave reactions in the dense troposphere below by modifying the index of refraction through which these waves propagate.

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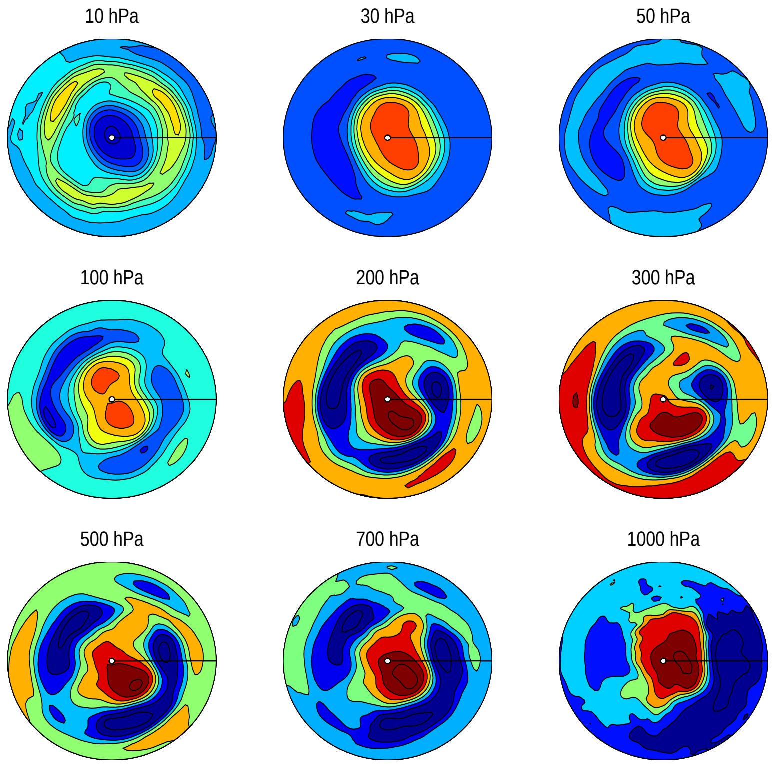

Figure 1. Winter EOFs from NOAA-CIRES Climate Diagnostic Center, Boulder CO. This shows that the largest variance in the stratosphere is relatively zonally symmetric, that the EOFs in the tropopause region are wavy and that in the troposphere there is a large wave-one component.

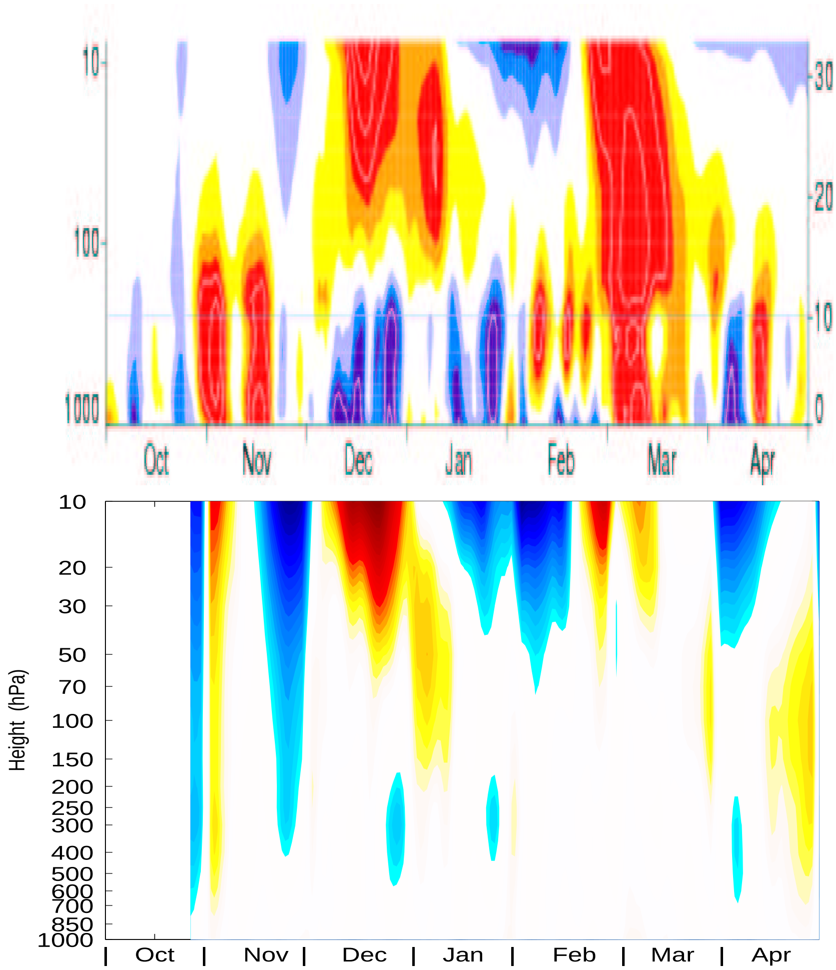

Figure 2. The NAM anomalies (top) for Oct. 1998 to April 1999 are compared to the projection of the geopotential height anomalies at 60°N onto the zonal mean (bottom).

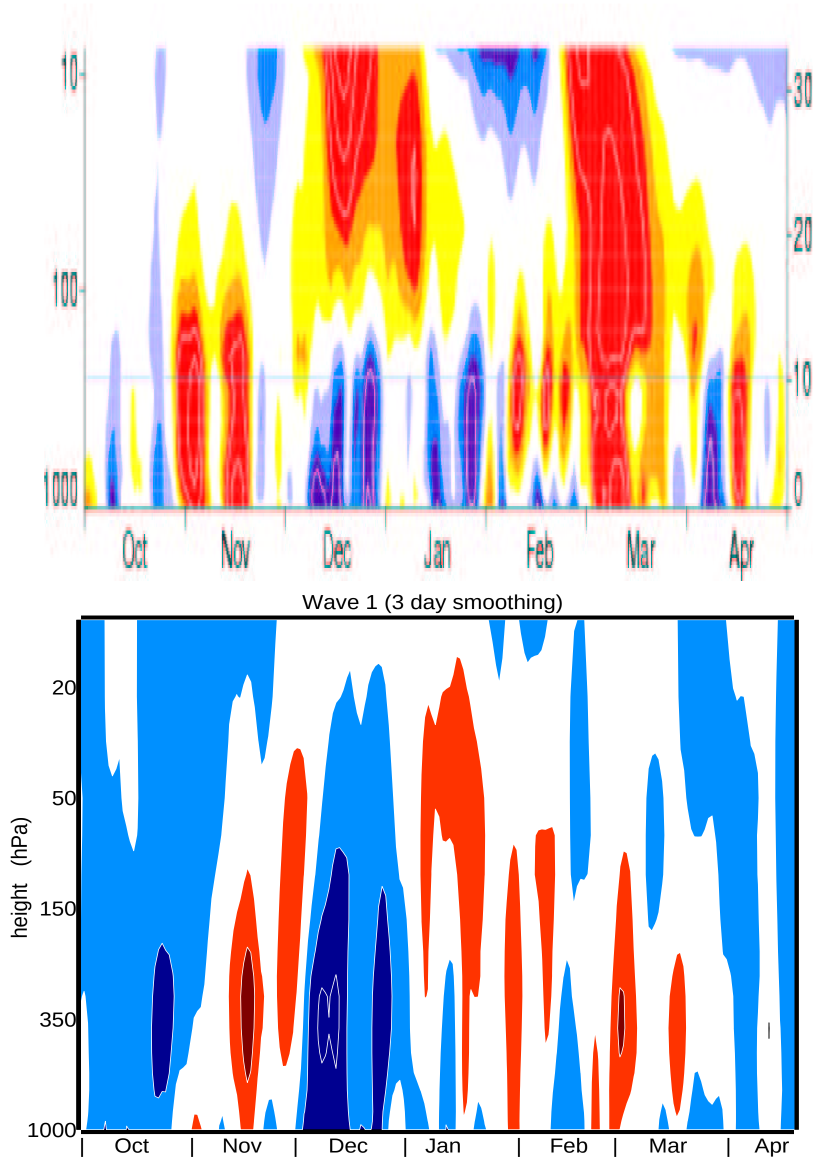

Figure 3. The NAM (top) from October 1998 to April 1999 are compared to the projection of the geopotential height anomalies at 60°N onto the wave-one component (bottom). Wave-one is oriented so that it coincides with the wave-one component of the AO. In this way, the negative values (red colors) of the projection onto wave-one can be compared with the negative (red) anomalies of the NAM.

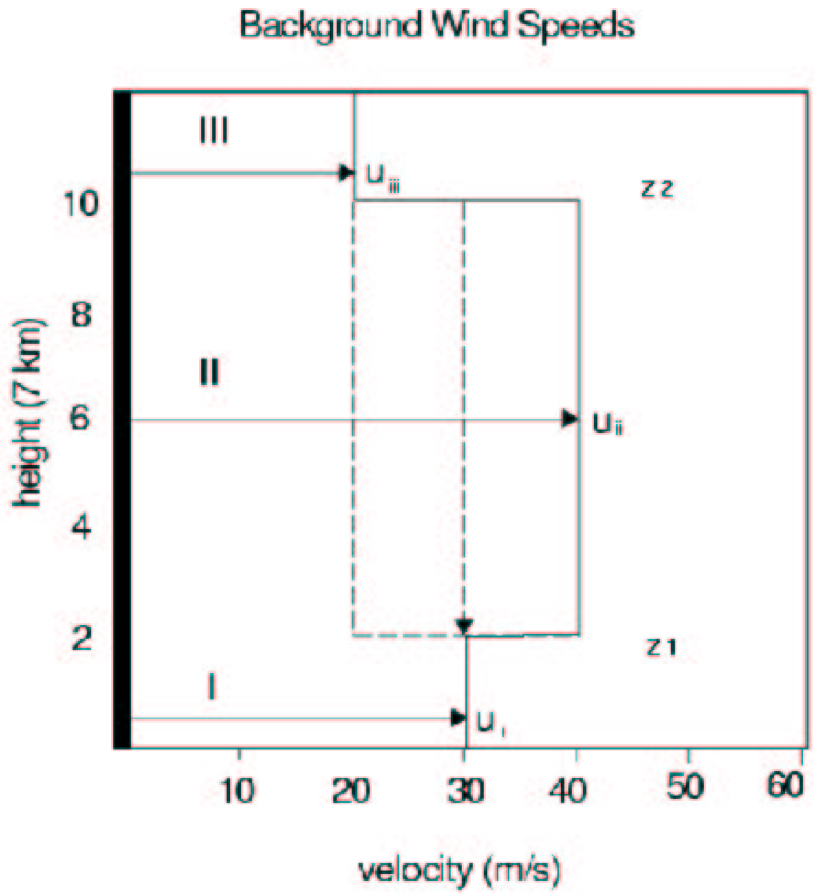

Figure 4. Tropospheric Jet Profile with Height

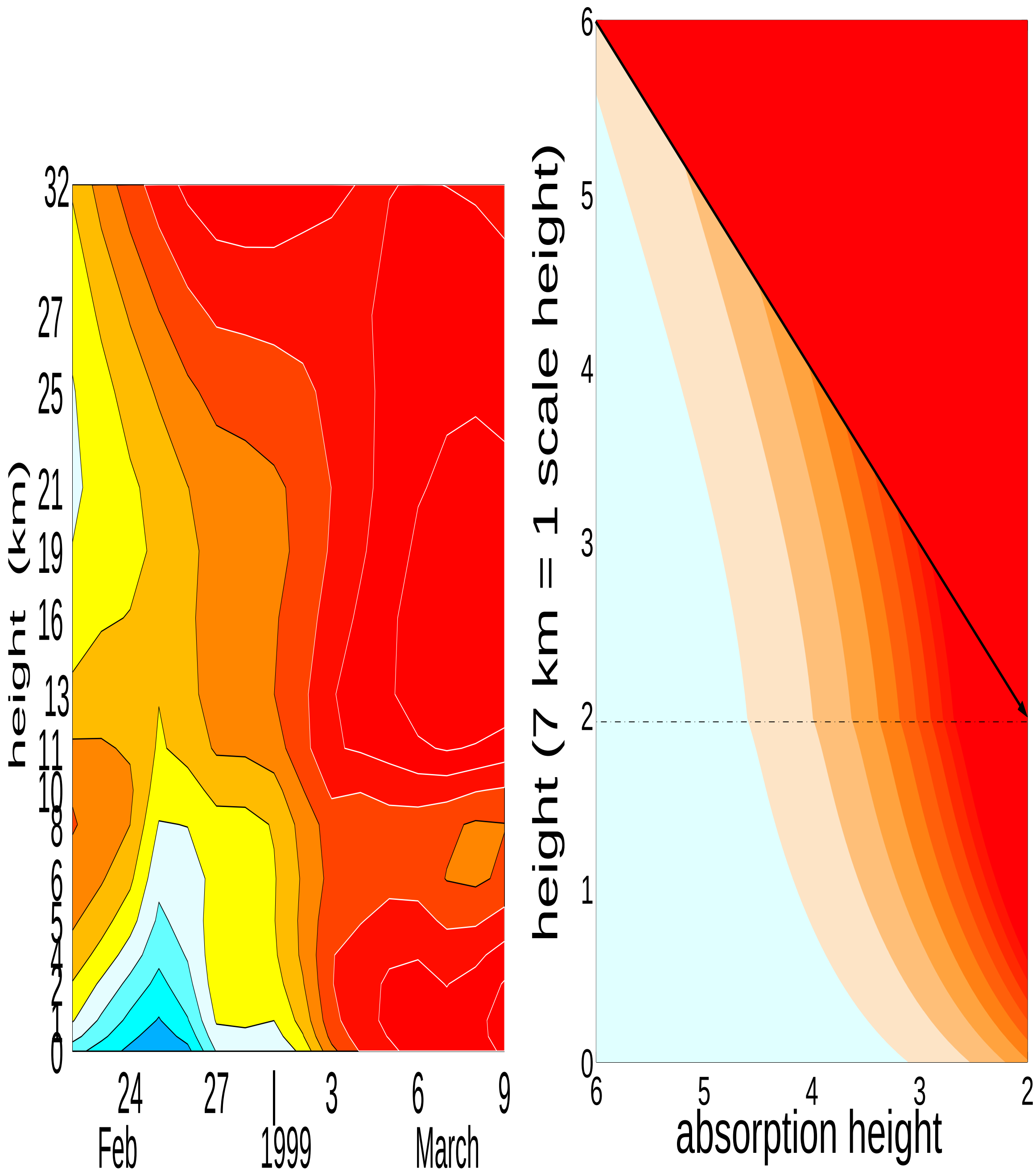

Figure 5. The NAM (left) during late winter of 1999 exhibits negative anomalies descending from the stratosphere to the troposphere. A theoretical calculation (right) shows the descent of decelerating winds in the stratosphere and the wave reaction in the troposphere.

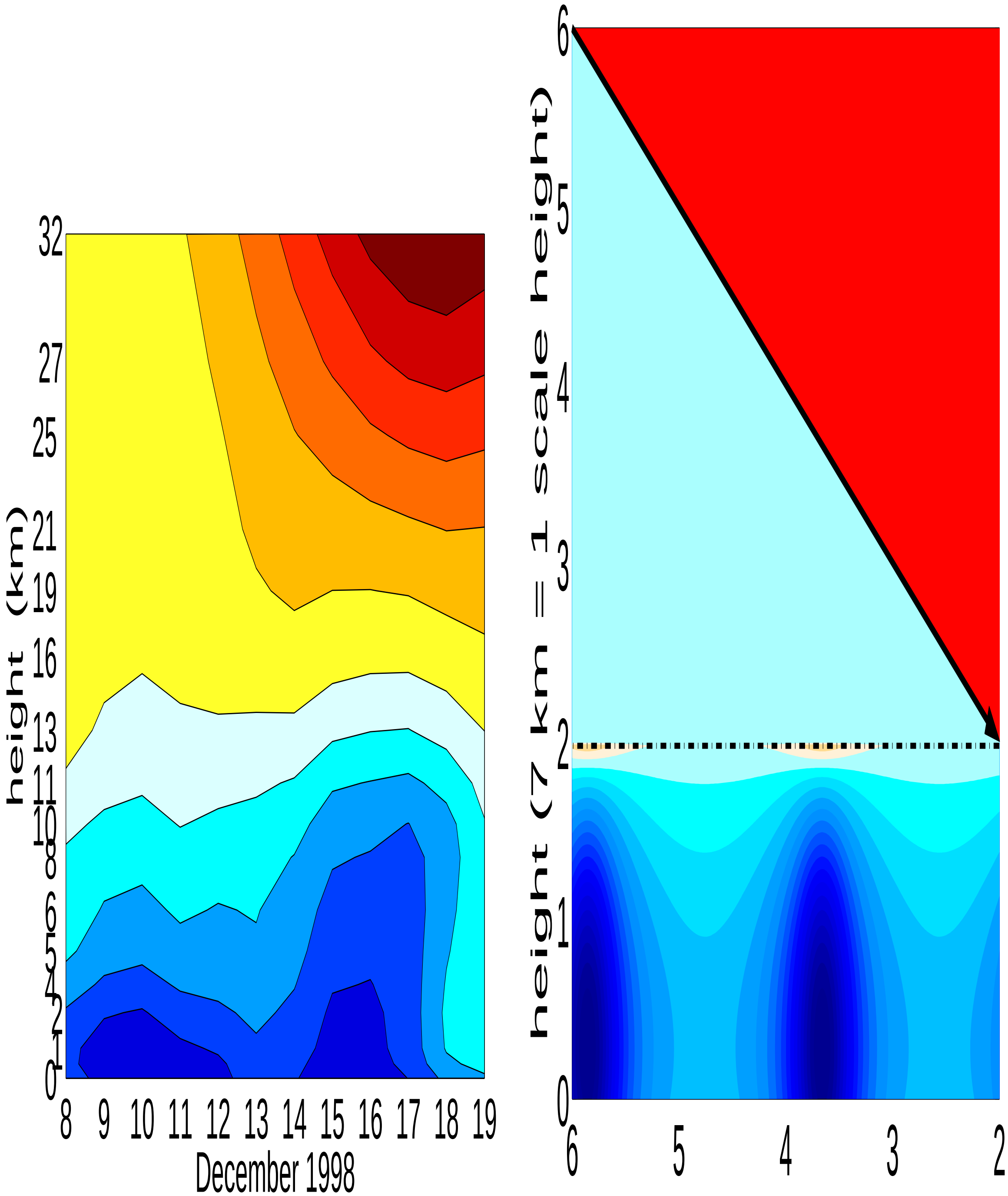

Figure 6. The NAM (left) during late winter of 1999 exhibits negative anomalies in the stratosphere and positive anomalies in the troposphere. A theoretical calculation (right) shows the descent of decelerating winds in the stratosphere and the wave reaction in the troposphere. In this case, the latitudinal wave number is two and the background velocities are u_1=8 m/s, u_2=13 m/s, and u_3=2 m/s.