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MODELING OF TRACER TRANSPORT IN THE MIDDLE ATMOSPHERE

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ABSTRACT

The discussion is concerned with zonally averaged (or the so-called 2-D) models of transport of trace gases in the middle atmosphere. A brief review of processes affecting transport and an order of magnitude estimate of various eddy transport terms are given.

1. INTRODUCTION

For various reasons (mostly economic ones) it is sometimes desirable to calculate the zonally averaged distribution of various minor constituents in the atmosphere directly from the zonally averaged equations of transport. Since motion in our atmosphere is not zonally symmetric, there arise in the averaged equations terms that represent transports by nonsymmetric motion fields (the "eddies") which cannot be determined consistently within the framework of the 2-D models. These eddy transport terms have to be parameterized in terms of the zonally averaged fields. The situation here is similar to the closure problem in turbulence theory. It is by now known, however, that the large-scale atmospheric waves in the middle atmosphere are largely organized and their transports do not resemble those of turbulence.

Given the fact that some form of parameterization of the eddy transports is unavoidable in 2-D Eulerian models, it is desirable to have a formulation in which the role played by the eddies can be made as small as possible, so that the degree with which our model depends on our ability to accurately parameterize the eddy transports can be reduced. That this is at least conceptually feasible is demonstrated by the generalized Lagrange mean formulation of Andrews and McIntyre (1978). In this formulation, the transport of a tracer is accomplished by advection of the zonally averaged flow only (with the average taken with respect to the "displaced" position), and no eddy transport terms appear explicitly. Practical problems encountered in the application of the theory of the Lagrangian mean to the tracer transport problem (as pointed out by e.g. McIntyre (1980)) prompted the development of alternate Eulerian mean models which can retain some of the positive attributes of the Lagrangian mean theory.

Two Eulerian formulations, the residual mean circulation (Andrews and McIntyre, 1976; Boyd, 1976; Dunkerton, 1978; Holton, 1980; Matsuno, 1980; Holton, 1981) and the formulation in isentropic coordinates (Mahlman et al, 1981; Tung, 1982) both have the property that for steady adiabatic small amplitude waves, the mean circulation reduces to the Lagrangian mean circulation. However, the manner in which this is accomplished is very different. In the residual mean formulation, the advective transport by the steady adiabatic waves is subtracted from the mean velocities in the definition of a residual circulation. In isentropic coordinates, the reduction comes from a simplification of air trajectories as compared to those in height or pressure coordinates.

In section 2, a brief review of the eddy transport processes is given from the viewpoint of air trajectories. This is followed by a parameterization of nonadiabatic and nonsteady waves in the stratosphere. This parameterization allows us to give an order of magnitude estimate of various eddy transports terms. Results are summarized in section 6.

2. AIR PARCEL TRAJECTORY AND EDDY TRANSPORT

The type of transport produced by the eddy field is intimately related to the nature of air displacement trajectory. If eddy displacements occur horizontally, only horizontal diffusion (or dispersion) of tracers can result. In this case the eddy transport tensor, $\mathbb{K}$, can have only one nonzero component — K_{yy}. If the eddy field involves both vertical and horizontal air displacements in the meridional plane, but the motion in the vertical direction is uncorrelated with that in the horizontal direction, the tensor $\mathbb{K}$ then has two nonzero components — K_{yy} and K_{zz}, and no off-diagonal elements. This diagonal transport tensor is relevant to random turbulence-like eddy processes. The resulting transport is diffusive, and as such the direction of transport is down-gradient, that is, from a region of high zonal mean tracer concentration to a region of low mean concentration.

A subtle generalization of the above-mentioned picture of Fickian diffusion is the parameterization proposed by Reed and German (1965). While still retaining the interpretation that the eddy processes in the stratosphere is turbulence-like, Reed and German suggested that the diagonal transport tensor $\mathbb{K}$ for Fickian diffusion can be generalized to include the off-diagonal components, K_{yz} and K_{zy}, if a sloping principal axis system for eddy displacements is adopted. The eddy motion along one axis is still assumed (albeit implicitly in their paper) to be uncorrelated with that in the other axis. However, when projected into the regular y-z axes, there would now be some correlation between the displacement in y-direction and that in the z-direction, hence the presence of the off-diagonal components. Consistent with this interpretation is the assumption that the tensor $\mathbb{K}$ be symmetric, i.e. $K_{yz} = K_{zy}$. (This guarantees that the tensor can be diagonalized by a real orthogonal transformation. In other words, it is assumed that there exists a (sloped) principal axis system in which the displacements along different directions are uncorrelated.) This generalization allows the possibility that diffusion along a coordinate axis (e.g. the y-axis) be countergradient. Countergradient northward eddy transports have been observed in the lower stratosphere during winter. This evidence, in fact, was the primary motivation for Reed and German to generalize the Fickian diffusion tensor.

There is no reason to believe that large-scale eddy displacements in the atmosphere should be uncorrelated in _any_ two directions. There has been increasing evidence to suggest that large-scale atmospheric waves possess coherent structures and do not behave at all like turbulence. Matsuno (1980) showed that fluid particle trajectories induced by forced stationary planetary waves are elliptical when projected onto the meridional plane. The transport tensor $\mathbb{K}$ for such an eddy field cannot be symmetric, because the displacements that make up the elliptical trajectories are strongly correlated in any two (_fixed_) directions. For the case of adiabatic steady planetary waves studied by Clark and Rogers (1978), Plumb (1979) and Matsuno (1980), the transport tensor turns out to be antisymmetric (giving an advective transport), just the opposite from what one would expect for a diffusive process. Although the tensor becomes full when transient waves are taken into consideration, the above-mentioned results nevertheless serves to point out that a substantial component of the eddy field in the stratosphere produces transports that are mainly nondiffusive in nature.

There is an interesting modification to what is said in the preceding paragraph. We have noted that the eddy transport in the stratosphere is more complicated than what one would infer from a turbulent diffusion model. This is a result of the strong coherence in the horizontal and vertical displacements associated with the air trajectories, which are nonrectilinear. This fact cannot be altered by a simple coordinate transformation (and so $\mathbb{K}$ cannot be diagonalized in general). It will be a different story if _dynamical_ coordinate transformations are allowed. In particular, if the isentropic coordinate system, with potential temperature (a dynamical variable) as the vertical coordinate, is adopted, then all _adiabatic_ eddy displacements, including the elliptical trajectories deduced by Matsuno (1980), become rectilinear — in the horizontal direction (i.e. along isentropes) only. Of the four components in the eddy transport tensor $\mathbb{K}$, only one, K_{yy}, is nonzero. The resulting transport resembles the classical Fickian diffusion in one dimension. Air mixes along isentropic surfaces, serving to smooth out gradients of mean tracer concentration along the isentropes.

If one further assumes that the eddy field is _steady_, even this last component of the transport tensor vanishes. No eddy transport is accomplished by an adiabatic, steady eddy field. For this idealized case, tracers are transported solely by the advection of mean flow in isentropic coordinates (see Tung (1982)).

The eddy field in the atmosphere is in general nonadiabatic and often nonsteady. Such nonconservative processes are difficult to parameterize, but it appears valid in assuming that in the stratosphere (below the breaking height for gravity waves), a large component of the atmospheric eddy field contributing to tracer transport is not turbulence-like, but coherent waves forced in the lower atmosphere.

3. DEFINITION OF THE EDDY TRANSPORT TENSOR

Surprisingly, there does not seem to be an agreed upon definition for the eddy transport tensor $\mathbb{K}$. Consequently, there has been some confusion, in particular concerning its dependence on the chemical species being transported.

The original definition of Reed and German (1965), which is also adopted here[1], is that the four components of the $\mathbb{K}$ tensor are given by

$$K_{yy} \equiv \overline{\eta' v'},\ K_{zy} \equiv \overline{\eta' w'},\ K_{yz} \equiv \overline{\zeta' v'},\ \text{and}\ K_{zz} \equiv \overline{\zeta' w'} \quad . \tag{3.1}$$

Here η' and ζ' are the horizontal and vertical eddy air displacement fields, respectively, and v' is the perturbation horizontal air velocity and w' is the perturbation vertical air velocity. (The same definition will be retained in isentropic coordinates, although the meaning of vertical "velocity" has to be reinterpreted.) It is important to note that the eddy field entering into the definition in (3.1) is independent of the particular minor species being transported.

For the conservative tracers that Reed and German were primarily concerned with, the eddy fluxes of a species with mass concentration χ is given by

$$\begin{pmatrix} \overline{v'\chi'} \\ \overline{w'\chi'} \end{pmatrix} = -\,\mathbb{K}\cdot\vec{\nabla}\overline{\chi} \quad . \tag{3.2}$$

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[1]Eve. though the definition in (3.1) is the same as in Reed and German (1965), the interpretation of the eddy displacement fields η' and ζ' is not the same. Reed and German assumed η' and ζ' as mixing lengths. This permits the further assumption that $w'/v' = \zeta'/\eta'$, which then implies $K_{yz} = K_{zy}$. Here, however, the displacements are related to the velocities according to $(\partial/\partial t + \overline{u}\,\partial/\partial x)\eta' = v'$, and $(\partial/\partial t + \overline{u}\,\partial/\partial x)\zeta' = w'$. In general the tensor is neither symmetric nor antisymmetric.

When the tracers under consideration are not inert, the eddy fluxes can no longer be expressed in the form of (3.2) with the transport tensor $\mathbb{K}$ given by (3.1). This is due to the possible presence of an eddy source term, S', in the equation for χ', the eddy species concentration (see e.g., Tung (1982)). According to perturbation theory, the appropriate modification to (3.2) should be the addition of an extra source term on the right hand side of (3.2), as

$$\begin{pmatrix} \overline{v'\chi'} \\ \overline{w'\chi'} \end{pmatrix} = - \mathbb{K}\cdot\vec{\nabla}\overline{\chi} + \begin{pmatrix} \overline{v'\sigma'} \\ \overline{w'\sigma'} \end{pmatrix} \quad (3.3)$$

where σ' is defined through

$$\left(\frac{\partial}{\partial t} + \overline{u}\frac{\partial}{\partial x}\right)\sigma' = S' \quad . \quad (3.4)$$

The definition for $\mathbb{K}$ should remain unchanged. In particular, it should not depend on the chemistry or concentration of the minor species being transported.

Some authors perfer to absorb the new term (the last term in Eq. (3.3)) into a generalized $\mathbb{K}$ so that the same expression (3.2) can be used for both conservative and reactive species. However, as pointed out by Strobel (1981), the eddy fluxes of some important chemical species, such as ozone, have terms in their eddy source fluxes that are proportional to the mean concentration $\overline{\chi}$, and consequently cannot be conveniently expressed in the form of (3.2), since (3.2) is a statement of the proportionality between the eddy fluxes and the gradient of the mean concentration.

The parameterization of the eddy source fluxes is difficult to do in general. We will postpone its discussion until section 5. In the next section, the components of the tensor $\mathbb{K}$ as defined in (3.1) (and therefore as independent of chemistry) are parameterized and their magnitudes quantitatively estimated.

4. QUANTITATIVE ESTIMATES OF $\mathbb{K}$

In the formulation of tracer transport in isentropic coordinates, or in the residual mean formulation in pressure coordinates, it is the deviation of the atmospheric eddy field from being strictly steady and adiabatic which contributes to the diffusive and advective transports of the tracers; the eddy transport tensor vanishes for adiabatic and steady eddy fields. Order of magnitude estimates of the degree of such deviations for typical eddy fields in the middle atmosphere and their effects on the diffusive and advective transports will be given in this section. The calculations are somewhat more direct in the isentropic coordinate formulation than in the residual mean formulation, and therefore the former formulation will be used first. Corresponding estimates in pressure coordinates will also be given for the purpose of comparing the two formulations.

Following a common practice, I will write the transport tensor $\mathbb{K}$ as a sum of a symmetric tensor $\mathbb{D}$ and an antisymmetric tensor ψ:

$$\mathbb{K} \equiv \mathbb{D} + \psi , \tag{4.1}$$

where

$$\mathbb{D} = \begin{pmatrix} D_{yy} & D_{y\theta} \\ D_{y\theta} & D_{\theta\theta} \end{pmatrix} , \text{ and } \psi \begin{pmatrix} 0 & -\psi \\ \psi & 0 \end{pmatrix} , \tag{4.2}$$

Using the definition in (3.1), but replacing z now by the new vertical coordinate, θ, and w' by the "vertical velocity" $\dot{\theta}'$ in isentropic coordinates, one can show that (see Tung (1983)[2]):

$$D_{yy} = \frac{\partial}{\partial t} \frac{1}{2} \overline{\eta'\eta'} , \; D_{\theta\theta} = \frac{\partial}{\partial t} \frac{1}{2} \overline{\zeta'\zeta'} , \; D_{y\theta} = \frac{\partial}{\partial t} \frac{1}{2} \overline{\eta'\zeta'} \tag{4.3}$$

and

$$\psi = - \frac{\partial}{\partial t} \frac{1}{2} \overline{\eta'\zeta'} + \overline{\eta'\dot{\theta}'} . \tag{4.4}$$

Here ζ' is understood to be the "vertical displacement" in isentropic coordinates, and is given by

$$\left(\frac{\partial}{\partial t} + \overline{u} \frac{\partial}{\partial x}\right)\zeta' = \dot{\theta}' . \tag{4.5}$$

4.1 The Diffusive Transport

The transport by the symmetric $\mathbb{D}$ tensor is diffusive and down-gradient in nature provided that $D_{yy}, D_{\theta\theta} \geq 0$ and $D_{yy} D_{\theta\theta} \geq D_{y\theta}^2$ (Matsuno, 1980). In Tung (1982) it was mentioned that if the eddy field can be assumed to be adiabatic, then this $\mathbb{D}$ tensor becomes diagonal. In fact, only the first component D_{yy} remained. It will be shown here that D_{yy} remains to be the dominant term for non-adiabatic eddies under typical conditions in the stratosphere in the presence of Newtonian cooling.

Due to the difference in the scales of vertical and horizontal

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[2]There are actually some density perturbations in the definition of these quantities if the isentropic coordinates are adopted, but these contribution were found to be small in Tung (1982) (see (D.11) and (D.12) there).

motions in the atmosphere, a simple ratio such as $|D_{\theta\theta}/D_{yy}|$ is not indicative of the ratio of vertical vs. horizontal eddy transports. A more appropriate ratio is obtained when the vertical displacement is normalized by a vertical scale "height", L_v, and horizontal displacement by a horizontal scale L_h. The more appropriate ratios that we need to consider are

$$r_1 \equiv \left|\frac{L_h^2 D_{\theta\theta}}{L_v^2 D_{yy}}\right| \quad \text{and} \quad r_2 \equiv \left|\frac{L_h D_{y\theta}}{L_v D_{yy}}\right| \quad . \tag{4.6}$$

The horizontal scale L_h can be chosen, for our purpose of studying large-scale transports, to be the radius of the earth, a. The vertical scale L_v is more difficult to fix, as it may depend on the scale of the mean stratification of the particular species under consideration. Nevertheless, since only an order of magnitude is needed here, we will take it to be $L_v \sim 0.3\theta$ in potential temperature, approximately equivalent to a density scale height H in height coordinates.

As a reference, we note that in the Reed and German formulation, typical values of the ratios are

$$r_1 \equiv \left|\frac{a^2 K_{zz}}{H^2 K_{yy}}\right| \sim 1 \tag{4.7}$$

and

$$r_2 \equiv \left|\frac{a}{H}\frac{K_{yz}}{K_{yy}}\right| \sim 1 \tag{4.8}$$

using typical orders of magnitude from Reed and German (1965) or Luther (1973):

$$K_{yy} \sim 10^{10}\ \text{cm}^2/\text{s},\ K_{zz} \sim 10^4\ \text{cm}^2/\text{s},\ K_{yz} \sim 10^7\ \text{cm}^2/\text{s} \quad .$$

Thus in the diffusion type parameterization of Reed and German that is in common use, the eddy vertical and horizontal transports are roughly comparable. The situation appears to be different in isentropic coordinates using a more rational approach to eddy dynamics.

4.1.1 Newtonian Damping

In isentropic coordinates, nonadiabatic eddies have a nonvanishing perturbation "vertical velocity", $\dot{\theta}'$, which can be calculated directly from the thermodynamic equation

$$\dot{\theta} = \theta Q/T \quad , \tag{4.9}$$

where Q is the diabatic heating range per unit mass divided by the specific heat c_p, and T is the temperature. The perturbation form of (4.9) is

$$\dot{\theta}' = \frac{\theta}{\overline{T}}[Q' - \frac{\overline{Q}}{\overline{T}}T']. \tag{4.10}$$

[Note that θ is not perturbed in (4.10) because it is an independent variable.]

Following Dickinson (1973), we adopt here the Newtonian damping parameterization, which gives

$$Q' = -\alpha_N T',$$

where α_N is the coefficient of Newtonian damping. Thus (4.10) gives

$$\dot{\theta}' = -\frac{\alpha\theta}{\overline{T}} \cdot T', \tag{4.11}$$

where

$$\alpha \equiv \alpha_N + \frac{\overline{Q}}{\overline{T}}$$

is the total Newtonian damping coefficient in isentropic coordinates. The extra term, $\overline{Q}/\overline{T}$, is typically one to two orders of magnitude smaller than α_N. A commonly used value for the Newtonian damping coefficient in the lower stratosphere is about 1/10 days. Its value is likely to be larger with increasing altitude, although there is no general agreement on the exact value. Blake and Lindzen (1973) gave a thermal relaxation time of about 3-7 days at 35 km and 1.2-2.5 days at 50 km. For our order of magnitude estimate I will adopt here a damping time scale of $\sim$5 days for the 30-40 km region and $\sim$10 days in the lower stratosphere.

4.1.2 The Ratio r

Using

$$ik(\overline{u}-c)\eta' = v'$$

and

$$ik(\overline{u}-c)\zeta' = \dot{\theta}'$$

where $\overline{u}$ is the speed of the mean flow and c is the phase speed of the wave, one finds

$$r \equiv \left|\frac{(\zeta'/L_v)}{(\eta'/L_h)}\right| \sim \frac{1}{(L_v/\theta)} \left|\frac{T'/\overline{T}}{v'/(a\alpha)}\right| . \qquad (4.12)$$

In (4.12), the parameterization (4.11) is used to express the eddy vertical velocity in terms of temperature fluctuations. van Loon et al. (1973) and van Loon et al (1975) found largest stratospheric temperature oscillation to occur in wavenumber 1 during winter with an amplitude of about 10-12°C. A value of $v' \sim 13$ m/s is found for the same wave using

$$f_o v' = ik\Phi' \quad ,$$

and adopting the value $|\Phi'/g| \sim 600$ m, also from van Loon et al. Thus

$$r \sim \begin{array}{l} 0.07 \text{ for } \alpha = 1/10 \text{ days} \\ 0.14 \text{ for } \alpha = 1/5 \text{ days} \end{array} \qquad (4.13)$$

4.1.3 An Alternative Estimate

In arriving at (4.13), I have used the observed values for T' and Φ' in pressure coordinates, since the corresponding values in isentropic coordinates are not readily available. Although such a procedure appears to be sufficient for our purpose of obtaining an order-of-magnitude estimate, it would be reassuring if an alternative estimate can be given that does not rely on our knowing these quantities directly.

To give a more deductive estimate for the ratio r, I use the equation for hydrostatic equilibrium

$$T' = \frac{\theta}{c_p} \frac{\partial}{\partial \theta} \Phi' \quad . \qquad (4.14)$$

The vertical displacement is then estimated using (4.14) together with (4.11) and (4.5) as

$$\zeta' \sim - \frac{\alpha \theta^2}{c_p \overline{T} \, ik(\overline{u}-c)} \frac{\partial}{\partial \theta} \Phi' \quad , \qquad (4.15)$$

while the horizontal displacement is, for geostropic waves,

$$\eta \sim \frac{\Phi'}{(\overline{u}-c) f_0} \quad . \qquad (4.16)$$

Thus the normalized ratio of vertical to horizontal displacements is

$$r \sim \frac{\alpha f_0 L_h}{(\Delta\theta/\theta)(L_v/\theta)k\ c_p\overline{T}} \tag{4.17}$$

In (4.17), we have written $|\partial/\partial\theta\ \Phi'| \sim |\Phi'/\Delta\theta|$, where $\Delta\theta$ is the vertical scale of the wave in isentropic coordinates. Note that the wave amplitudes cancel and therefore do not appear in (4.17). The same is true also for the factor $(\overline{u}-c)$ in (4.15) and (4.16), although $\Delta\theta$ does depend on $(\overline{u}-c)$ through the dispersion relation.

A WKB solution of the vertical structure equation for quasi-geostrophic waves gives

$$\Phi'(\theta)/\Phi'(\theta_0) = (\frac{\theta}{\theta_0})^{\lambda} \quad ,$$

where

$$\lambda = \frac{1}{\kappa}\{\frac{1}{2} + i[\frac{\hat{\beta}\ RH\Gamma^{(0)}}{f_0^2}(\frac{1}{\overline{u}-c} - \frac{1}{U_T})]^{1/2}\} \quad , \tag{4.18}$$

$\kappa = R/c_p = 2/7$

$\Gamma^{(0)} \simeq \partial T_e/\partial z + g/c_p$, the static ability parameter

and $U_T \simeq \hat{\beta}/[(k^2+\ell^2) + f_0^2/4RH\Gamma^{(0)}]$ is the Doppler-shifted speed at which the wave ceases to propagate vertically ($\hat{\beta}$ is the mean potential vorticity gradient).

For stationary ultralong waves, whose vertical wavelengths are typically larger than two scale heights, the square root in λ is typically much smaller than 1/2. Therefore

$$|\lambda| \sim \frac{1}{2\kappa} \sim 7/4 \quad ,$$

and

$$|\frac{\Delta\theta}{\theta}| \sim |\Phi'/(\theta\frac{\partial}{\partial\theta}\Phi')| \sim \frac{1}{|\lambda|} \sim 4/7 \quad ,$$

which gives

$$\begin{aligned} r &\sim 0.07 \quad \text{for } \alpha = 1/10 \text{ days} \\ r &\sim 0.14 \quad \text{for } \alpha = 1/5 \text{ days} \quad . \end{aligned} \tag{4.19}$$

These figures are remarkably close to the ones estimated earlier in (4.13).

The estimates in (4.19) are for wavenumber 1. The corresponding figures for wavenumber 2 should be reduced by approximately a factor of 1/2.

4.1.4 Critical Levels

The above estimates are based on the assumption that the wave encounters no critical level in its vertical propagation (in fact I have used in the estimates, $\bar{u}-c = \bar{u} \sim 30$ m/s.) It would appear from (4.18) that if there exists a region in which $\bar{u}-c$ approaches zero, then the vertical scale of the wave should vanish, yielding an infinite r in (4.17). This, of course, does not happen when damping is present, and $\bar{u}-c$ in (4.18) should be replaced by $\bar{u}-c + \alpha/ik$ when $\bar{u}-c$ is not much larger than α/k. The quantity α/k is about 5 m/s for α=1/10 days and wavenumber 1. For $\bar{u}-c$ small, the limit for $|\lambda|$ is finite and is approximately

$$|\lambda| \sim \frac{1}{\kappa} \left[\frac{\hat{\beta} R H \Gamma^{(0)}}{f_0^2 \; \alpha/k} \right]^{1/2} \sim 9.$$

This yields

$$r \sim 35\% \text{ for } \alpha = 1/10 \text{ days}$$
$$50\% \text{ for } \alpha = 1/5 \text{ days} \qquad (4.20)$$

It appears that near critical levels, the effect of vertical displacement is about 4 to 5 times larger than the case away from critical levels, although it is still smaller than the effect of horizontal displacement since r is still less than one.

For forced stationary waves that propagate to the stratosphere, critical levels (i.e. zero wind lines) do not seem to occur frequently, away from the tropics. Exception occurs during a major sudden warming, when easterlies descend into the stratosphere. To the extent that major warmings are rare, our estimate of

$$r \sim 10^{-1} \qquad (4.21)$$

in (4.19) appears to hold most of the time in mid to high latitude regions.

4.1.5 Gravity Waves

We now repeat the estimate of r for gravity waves. To estimate the horizontal displacement for gravity waves, I use

$$\left(\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x} \right) v' = - \frac{\partial}{\partial y} \Phi'.$$

This gives

$$\eta' \sim i\ell\Phi'/\omega^2,$$

where ℓ is the meridional wavenumber of the gravity wave, $\omega = k(\overline{u}-c)$, its Doppler-shifted frequency. Under hydrostaticity, the vertical displacement for gravity waves is also given by (4.15). Thus the normalized ratio between vertical and horizontal displacements is

$$r \sim \left| \alpha \frac{(\omega/\ell)\, L_h}{(\Delta\theta/\theta)(L_v/\theta)\, c_p\overline{T}} \right| . \qquad (4.22)$$

The vertical wave scale $|\Delta\theta/\theta|$ can be estimated from the dispersion relation, giving

$$|\theta/\Delta\theta| \sim \frac{1}{\kappa} \left| \frac{1}{2} + i \left[\frac{RH\Gamma^{(0)}(1+\ell^2/k^2)}{(\overline{u}-c)^2}\right]^{1/2} \right| . \qquad (4.23)$$

$$\sim \frac{[RH\Gamma^{(0)}(1+\ell^2/k^2)]^{1/2}}{\kappa|\overline{u}-c|} ,$$

since the square root in (4.23) is typically much larger than 1/2. [This is not true for the fast periodic tides, which we are not concerned with, since they seem to contribute little to transient eddy dispersion.] Thus (4.22) becomes

$$r \sim \frac{a\alpha|k/\ell|(1+\ell^2/k^2)^{1/2}}{(L_v/\theta)(c_p\overline{T})^{1/2}} . \qquad (4.24)$$

Note that the dependence on $\overline{u}-c$ cancels in (4.24). This is fortunate, as this quantity can vary over a range of values for gravity waves. Furthermore, (4.24) should hold even near critical levels.

For $|\ell/k| \sim 1$, (a "guesstimate" from Lindzen (1981)), (4.24) gives

$$r \sim 0.07 \quad \text{for } \alpha = 1/10 \text{ days}$$
$$r \sim 0.14 \quad \text{for } \alpha = 1/5 \text{ days} , \qquad (4.25)$$

indicating also for gravity waves that the effect of vertical displacement is small compared with that of horizontal displacement.

4.1.6 Pressure Coordinates

As mentioned earlier, there are some indications that, unlike the case in isentropic coordinates, the normalized ratio r is of order unity in pressure coordinates. To illustrate the difference in the two systems, I now redo the calculation for r in pressure coordinates.

The vertical velocity w' in pressure coordinates is given by

$$(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x})T' + \Gamma^{(0)}w' = Q' \quad . \tag{4.26}$$

Note that in isentropic coordinates the temperature advection term does not appear as it does in (4.26).

Using as before the Newtonian damping parameterization:

$$Q' = -\alpha_N T'$$

and the definition for vertical displacement ζ':

$$(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x})\zeta' = w' \quad ,$$

one finds

$$\zeta' = -\frac{(\bar{u}-c+\frac{\alpha_N}{ik})T'/\Gamma^{(0)}}{(\bar{u}-c)} \tag{4.27}$$

The horizontal displacement is

$$\eta' = \frac{v'}{ik(\bar{u}-c)} \quad .$$

The normalized ratio in pressure coordinates is

$$(r_p) \equiv \left|\frac{\zeta'/H}{\eta'/a}\right| = \frac{\bar{T}}{H\Gamma^{(0)}}\left|(1+\frac{ik(\bar{u}-c)}{\alpha_N})\right| \left|\frac{T'/\bar{T}}{v'/(a\,\alpha_N)}\right| \quad . \tag{4.28}$$

Using the same value for T' and v' as in section 4.1.2, one finds that the ratio of the quantity r in pressure coordinates and that in isentropic coordinates is essentially given by ($H\Gamma^{(0)}/\bar{T}$ is about the same as L_v/θ):

$$\frac{(r)_p}{(r)_\theta} \sim \left|1+\frac{ik(\bar{u}-c)}{\alpha_N}\right| \quad . \tag{4.29}$$

This ratio is always greater than one, indicating that the effect of vertical displacement is more significant in pressure coordinates.

The quantity, $(\bar{u}-c)/(\alpha_N/k)$, in (4.29) measures the relative importance of the horizontal temperature advection vs. the diabatic heating term (viz. the first vs. the third term in Eq. (4.26)). For a typical value of $\overline{u}-c = u \sim 30$ m/s. This ratio is

$$(\overline{u}-c)/(\alpha_N/k) \sim \begin{matrix} 6 \text{ for wavenumber } 1 \\ 12 \text{ for wavenumber } 2 \end{matrix}$$

in the lower stratosphere, where $\alpha = 1/10$ days is used. This then suggests that r in pressure coordinates is about one order of magnitude larger than the corresponding quantity in isentropic coordinates, implying that vertical and horizontal diffusion are about comparable in pressure coordinates.

4.1.7 Ratio of Vertical to Horizontal Transports

If it can be assumed that the time dependence in the vertical displacement field is the same as that in the horizontal displacement field, in particular, that they have the same time scales (this assumption appears to be valid for the organized wave motion under consideration), then the smallness of the ratio of normalized vertical and horizontal displacements implies directly the smallness of vertical transport as compared to the horizontal transport. Specifically, the normalized ratios of vertical vs. horizontal transports defined in (4.6) are estimated from

$$r_1 \sim r^2 \quad \text{and} \quad r_2 \sim r \,. \tag{4.30}$$

This gives

$$r_1 \sim 0(10^{-2}), \quad r \sim 0(10^{-1}) \tag{4.31}$$

implying that $D_{\theta\theta}$ can be neglected when compared with D_{yy}, and $D_{y\theta}$ can be approximately neglected also when compared to D_{yy}. In other words, our estimates suggest that the dominant diffusive transport is horizontal diffusion along horizontal gradients of mean tracer concentration. Therefore it appears justified in approximating the diffusion tensor by a scalar:

$$\mathbb{D} \simeq \begin{bmatrix} D_{yy} & 0 \\ 0 & 0 \end{bmatrix} \,. \tag{4.32}$$

We will next estimate the magnitude of D_{yy}.

4.1.5 Maximum Magnitude of D_{yy}

Since the diffusion coefficient is given in terms of a time derivative, i.e., $D_{yy} = \partial/\partial t \; 1/2 \; \overline{\eta'\eta'}$, it is clear that periodic wave motions do not give rise to net transport. Net transport is produced by irreversible processes, either caused by direct dissipation of the wave or through cascade of wave energy to smaller scales and ultimate dissipation of the small scale waves. Since wave amplification is necessary in order to give positive diffusion, the wave must obtain its energy from somewhere, either through forcing, wave-mean flow interaction, or wave-wave interaction.

In terms of kinetic energy, a large component of the eddy field in the middle atmosphere is due to the stationary planetary waves forced by topography and differential heating in the lower atmosphere. These waves occasionally amplify in a time scale of about four or five days, reaching an equilibrated maximum amplitude when forcing balances damping. A dramatic example of such events is the so-called sudden warming phenomenon, when wave amplification of ∿ 1000 meters in geopotential height is observed to occur in less than a week in the stratosphere (and possibly larger values higher up). The time dependence of such an amplification episode can be modelled approximately by

$$\eta' \simeq \eta_0' \cdot (1-e^{-(t/\tau)}) \quad . \tag{4.33}$$

The time behavior of (4.33) is depicted in Fig. 1a. Essentially it consists of an approximately linear growth initially (for t/τ small), reaching an equilibrated maximum amplitude η_0' when t is larger than the amplification time scale τ. (This time behavior was deduced by Tung and Lindzen (1978a,b) for the resonant amplification of forced stationary wave in the presence of damping with a time scale τ. The diffusion coefficient D_{yy} induced by such an eddy field is found to be

$$D_{yy} = \overline{\eta_0'^2}\ \frac{1}{\tau}\ e^{-(t/\tau)}(1-e^{-(t/\tau)}); \tag{4.34}$$

its behavior with time is plotted in Fig. 1b. Note that the maximum

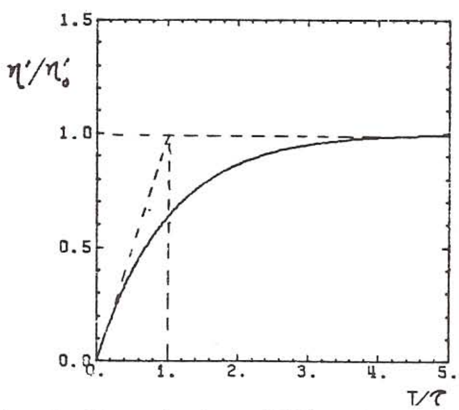

Fig. 1a Transient amplification of a forced wave in the presence of damping

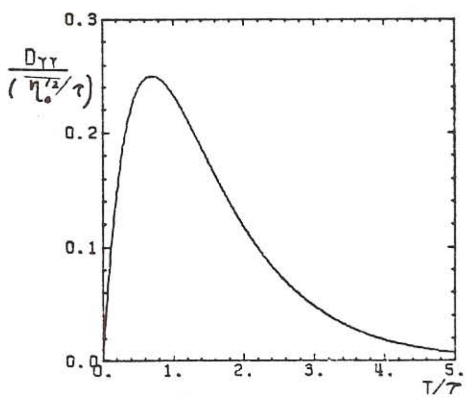

Fig. 1b Contribution to the eddy diffusion coefficient from a single episode of wave amplification

value achieved for D_{yy} for the episode is

$$(D_{yy})_{max} = \frac{\overline{\eta_0'^2}}{4\tau} \quad . \tag{4.35}$$

To estimate this value, the eddy horizontal displacement in isentropic coordinates is needed but unfortunately it is not readily available. I will instead use the values in pressure coordinates, hoping that there is not much difference in the horizontal quantities. For the 1973 sudden warming, Kanzawa (1980) gave a peak value for the geopotential height of wave number 1 to be 1,100 meters in the stratosphere (although for this particular warming event there later developed a second peak in the mesosphere with a peak of 2,200 meters. This larger value is not used here because it occurs outside the photochemical region of primary interest here). Thus roughly $\eta_0' \sim \Phi/f_0\bar{u} \sim 3{,}000$ km, achieved in approximately five days, and

$$(D_{yy})_{max} \sim 5 \times 10^{10} \ cm^2/s \quad . \tag{4.36}$$

This value is comparable to that of K_{yy} in the mixing-length formulation of Reed and German (1965). Note, however that our estimate in (4.36) is likely to be an upperbound for the typical values of D_{yy}, because major sudden warmings are rare over a period of a year. Furthermore, the peak value quoted in (4.36) is attained only for a short period (see Fig. 1b). Over longer periods of time, it is the less dramatic, but more frequent, events that contribute more to the diffusive process.

4.1.6 A Parameterization of D_{yy}

We envisage a series of amplification episodes each of the form (see Fig. 2):

$$\eta' \simeq \Delta\eta' \cdot (1 - e^{-(t-t_0)/\tau}) + \eta_m', \ t \geq t_0, \tag{4.37}$$

where η_m' is the time mean eddy field, $\Delta\eta'$ the (maximum) amplification amplitude from the time mean value and t_0, the starting time of amplification. Each such episode contributes to D_{yy} terms:

$$d_{yy} \equiv \overline{\Delta\eta'^2} \frac{1}{\tau} e^{-(t-t_0)/\tau} \left[1 - e^{-(t-t_0)/\tau}\right] + \overline{\eta_m'\Delta\eta'} \ \frac{1}{\tau} e^{-(t-t_0)\tau} \tag{4.38}$$

The superposition of contributions from all episodes gives

$$D_{yy} \simeq \frac{1}{\tau} \int_{\infty}^{t} d_{yy} \, dt_0 = \frac{1}{\tau}\left[\overline{\eta_m'\Delta\eta'} + \frac{1}{2}\overline{\Delta\eta'^2}\right] \quad , \tag{4.39}$$

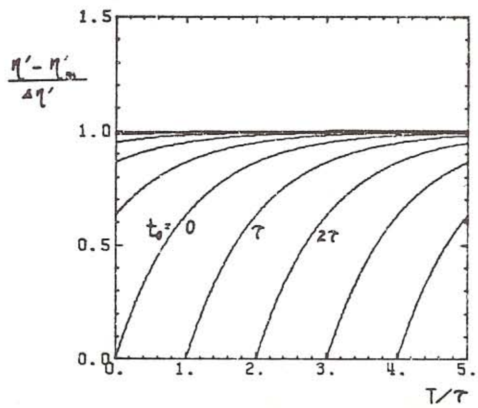

Fig. 2a A series of wave amplification episodes each starting at a different time t_0.

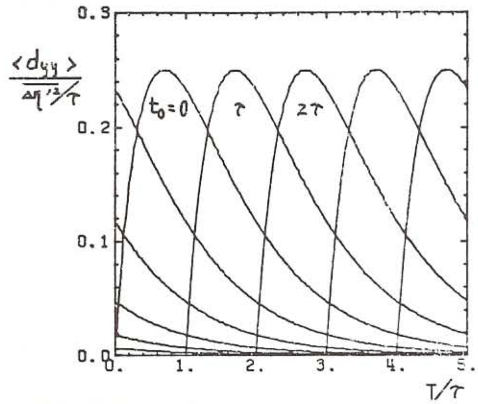

Fig. 2b Contribution to the eddy diffusion coefficient from a series of wave amplification episodes

assuming the episodes are uncorrelated with each other. [By assuming that t_0 is continuous, we admittedly are overestimating the value of D_{yy}]. Note that the expression obtained is independent of the fast amplification time (t/τ), as desired, but may vary slowly on a seasonal time scale as the amplitudes change. Note also that the first term, $\overline{\eta_m'\Delta\eta'}$, may be of either sign depending on the correlation of the phases of $\Delta\eta'$ and η_m'. However, this term disappears when (4.39) is time averaged over periods longer than τ (as denoted by $< >$) (since by definition $<\Delta\eta'> = 0$):

$$<D_{yy}> \simeq \frac{1}{\tau} <\overline{\Delta\eta'^2}> \quad . \tag{4.40}$$

If $<\overline{\Delta\eta'^2}>$, the distribution of the variance of the transient eddies[3], can be obtained from observed data, one then has the required estimate of the diffusion coefficient D_{yy}.

Lau and Oort (1982) obtained transient eddy statistics using GFDL and NMC analyses of data below 100 mb for six winter and summer seasons. Latitude-pressure distribution of the following quantity is displayed:

$$(<\overline{(\Delta\Phi/g}^2>)^{1/2}$$

---

[3]Periodic travelling waves should first be removed from data.

A maximum of $\sim$ 200 m is found near the 300 mb level during the winter season. This is not exactly what we wanted, as the zonal mean geopotential height was not removed from $\Delta\Phi/g$. I will use the figure, 200 m, as the upperbound for

$$(\overline{<(\Delta\Phi'g)^2>})^{1/2} \quad .$$

So we have

$$(\overline{<\Delta\eta'^2>})^{1/2} \lesssim 600 \text{ km}$$

and

$$<D_{yy}> \lesssim 4 \times 10^9 \text{ cm}^2/\text{s} \quad . \qquad (4.41)$$

This value is consistent with that estimated by Kida (1983) from his GCM. He found

$$K_{yy} \sim 3 \times 10^9 \text{ cm}^2/\text{s} \quad .$$

At the value given by (4.41), the diffusion process does not appear to be competitive with mean advection in transporting tracers over global scales. A time scale for global diffusion, a^2/D_{yy}, is about three years, while the advection time scale, $a/\bar{v}$, is about four months using $\bar{v} \sim 1/2$ m/s. Nevertheless, the diffusion process may be more effective over smaller scales and in high gradient regions.

Our result concerning large-scale diffusion appears to be at variance with the conclusion of Pyle and Rogers (1980a), who found insufficient transport of columnar ozone into the high latitudes when all the eddy diffusion terms were dropped (but with the mean circulation replaced by the approximate residual mean circulation in the Oxford 2-D model of Harwood and Pyle (1975)). It is possible that the numerical scheme used in the original Oxford 2-D model becomes unstable when the diffusion term is removed and numerical instability may cause the breakdown of the large scale circulation into small cells, which are ineffective in transporting over large distances.

4.2 The Advective Transport

The advective eddy transport is caused by the antisymmetric part of the transport tensor. The eddy induced advective velocities are given by

$$\bar{v}_E \simeq -\frac{1}{\bar{\rho}_\theta}\frac{\partial}{\partial\theta}\bar{\rho}_\theta\psi, \quad \bar{\dot{\theta}}_E \simeq \frac{1}{\bar{\rho}_\theta}\frac{\partial}{\partial y}\bar{\rho}_\theta\psi, \qquad (4.42)$$

where

$$\psi = \overline{\eta'\dot{\theta}'} - \frac{\partial}{\partial t}\frac{1}{2}\,\overline{\eta'\zeta'} \quad .$$

These eddy advective velocities are to be compared with the advection by the mean velocities $\overline{v}$ and $\dot{\overline{\theta}}$. In isentropic coordinates, the mean vertical velocity $\overline{\dot{\theta}}$ can be directly estimated from the thermodynamic equation (4.9) as

$$\overline{\dot{\theta}} \simeq \theta \frac{\overline{Q}}{\overline{T}} \quad . \qquad (4.43)$$

Since the eddy advective transport adds to the mean transport, the combined transport can be viewed as given by a combined diabatic heating rate $\overline{Q}*$ (Mahlman, personal communication):

$$\overline{\dot{\theta}} + \overline{\dot{\theta}}_E = \frac{\theta}{\overline{T}}\,\overline{Q}* \quad , \qquad (4.44)$$

where

$$\overline{Q}* \equiv \overline{Q} + \overline{Q}_E$$

$$\overline{Q}_E \equiv \frac{\overline{T}}{\theta}\,\overline{\rho}_\theta^{\,-1}\,\frac{\partial}{\partial y}\,\overline{\rho}_\theta\psi \simeq \frac{\overline{T}}{\theta}\frac{\partial}{\partial y}\,\psi \; . \qquad (4.45)$$

Since the transient part in ψ: $-\,\partial/\partial t \;\; 1/2\,\overline{\eta'\zeta'}$, can be shown to be smaller than $\overline{\eta'\dot{\theta}'}$ by one order of magnitude, we use

$$\psi \sim \overline{\eta'\dot{\theta}'} = -\,\alpha\,\frac{\theta}{\overline{T}}\,\overline{\eta'T'}$$

and so

$$|\overline{Q}_E| \sim |\frac{1}{a}\,\alpha\,\overline{\eta'T'}| \sim \begin{matrix} 0.2°\mathrm{C/day\ for\ } \alpha=1/10 \mathrm{\ days} \\ 0.5°\mathrm{C/day\ for\ } \alpha=1/5 \mathrm{\ days}\end{matrix} \qquad (4.46)$$

using $T' \sim 10°C$ (van Loon et al., 1973) and $\eta' \sim 1500$ km. These values of $\overline{Q}_E$ are not negligible compared to $\overline{Q}$, which is about $\sim 1°C/day$ according to Murgatroyd and Singleton (1962), or $\sim 0.4°C/day$ according to Dopplick (1979). Nevertheless, the contribution from Q_E is within the range of uncertainty in our ability to calculate $\overline{Q}$ at the present time, and so an accurate calculation of $\overline{Q}_E$ does not seem warranted, at least for the time being.

The eddy advective transport appears to be also important in pressure coordinates. Rood and Schoeberl (1983) have found that in the presence of Newtonian damping the residual mean circulation calculated in their model tends to overestimate the "Lagrangian mean" circulation by as much as 30% with $\alpha=1/10$ days implying that the advective eddy transport by nonadiabatic stationary planetary waves accounts for about ∿ 30% of the transport in pressure coordinates.

5. CHEMICAL EDDY TERMS

Eddy fluxes in 2-D models in the chemical source term have generally been ignored in the past even for nonconservative species, until recently, when studies, first by Pyle and Rogers (1980b) and later by Strobel (1981), Garcia and Solomon (1983), and Rood and Schoeberl (1983), demonstrated the importance of these chemical eddy terms.

As mentioned previously in section 3, the presence of a chemical source term S in the species transport equation

$$\frac{d}{dx}\chi = S \tag{5.1}$$

invariably gives rise to a chemical eddy source, S', when the air motion is zonally asymmetric. The perturbation equation:

$$D_0\chi' + v'\frac{\partial}{\partial y}\overline{\chi} + w'\frac{\partial}{\partial z}\overline{\chi} = S' \tag{5.2}$$

can be "solved" to yield

$$\chi' = -\eta'\frac{\partial}{\partial y}\overline{\chi} - \zeta'\frac{\partial}{\partial z}\overline{\chi} + \sigma' \quad , \tag{5.3}$$

where

$$D_0 \equiv \frac{\partial}{\partial t} + \overline{u}\frac{\partial}{\partial x} \ , \quad D_0 \cdot \sigma' = S' \ .$$

Strictly speaking, (5.3) is not a solution for χ', because the chemical eddy term σ' may depend on χ' (and even the concentration of other species partaking in the reaction). In general no closed form solution can be found for χ', and hence also for the eddy fluxes of χ (see (3.3)).

To fix ideas, let us consider the following simple example of a conservative tracer (see also Matsuno (1980)):

$$\frac{d}{dt}\chi = -\frac{1}{\tau_0}(\chi - \overline{\chi}_0) \quad . \tag{5.4}$$

The "source" term on the right-hand side of (5.4) describes a relaxation process about a mean equilibrium distribution $\overline{\chi}_0$, involving a relaxation time τ_0. τ_0 is here treated as a mean quantity dependent on the mean photochemistry involved. The perturbation source term is thus

$$S' = -\frac{1}{\tau_0}\chi' \quad . \tag{5.5}$$

Even for this simple eddy source term, (5.2) cannot be solved explicity. We shall instead attempt to solve it asymptotically assuming that the chemical relaxation time, τ_0, is much longer than the dynamical advection time scale: $a/\bar{u}$ (about three days). Therefore,

$$\begin{aligned}\chi' &= -\eta'\frac{\partial}{\partial y}\bar{\chi} - \zeta'\frac{\partial}{\partial z}\bar{\chi} + D_0^{-1}\left(-\frac{1}{\tau_0}\chi'\right) \\ &\simeq -\eta'\frac{\partial}{\partial y}\bar{\chi} - \zeta'\frac{\partial}{\partial z}\bar{\chi} + \frac{1}{\tau_0}\left[D_0^{-1}\eta'\frac{\partial}{\partial y}\bar{\chi} + D_0^{-1}\zeta'\frac{\partial}{\partial z}\bar{\chi}\right] + O\left(\left(\frac{a}{\bar{u}\tau_0}\right)^2\right)\end{aligned} \tag{5.6}$$

The eddy fluxes of χ' are then found to be

$$\begin{aligned}\overline{v'\chi'} &\simeq -K_{yy}\frac{\partial}{\partial y}\bar{\chi} - K_{yz}\frac{\partial}{\partial z}\bar{\chi} - \frac{1}{\tau_0}\left[K_{yy}^{(1)}\frac{\partial}{\partial y}\bar{\chi} + K_{yz}^{(1)}\frac{\partial}{\partial z}\bar{\chi}\right] \\ \overline{w'\chi'} &\simeq -K_{zy}\frac{\partial}{\partial y}\bar{\chi} - K_{zz}\frac{\partial}{\partial z}\bar{\chi} - \frac{1}{\tau_0}\left[K_{zy}^{(1)}\frac{\partial}{\partial y}\bar{\chi} + K_{zz}^{(1)}\frac{\partial}{\partial z}\bar{\chi}\right] ,\end{aligned} \tag{5.7}$$

where the K's are the components of the eddy transport tensor defined in section 3 and the $K^{(1)}$'s are defined here as

$$\begin{aligned}K_{yy}^{(1)} &\equiv -\overline{v'D_0^{-1}\eta'} , \qquad K_{yz}^{(1)} \equiv -\overline{v'D_0^{-1}\zeta'} \\ K_{zy}^{(1)} &\equiv -\overline{w'D_0^{-1}\eta'} , \qquad K_{zz}^{(1)} \equiv -\overline{w'D_0^{-1}\zeta'} \quad .\end{aligned} \tag{5.8}$$

Note that, like the K's, the $K^{(1)}$'s are also independent of chemistry or species concentration. They depend only on the air displacements and have to be parametrized. The dependence on the mean chemistry, which is in principle determinable within the framework of 2-D theory, has been separated out in (5.7) as a factor, $1/\tau_0$, multiplying the empirical parameters, $K^{(1)}$'s, which cannot be determined within the 2-D theory. It has been shown in Tung (1982) that such a separation can in general be done.

Unlike the $\mathbb{K}$ matrix, whose symmetric components are caused entirely by transient eddies, the symmetric part of the $\mathbb{K}^{(1)}$ matrix has a significant stationary component. This can be seen more clearly if we reexpress the $K^{(1)}$'s using the identities

$$D_0\overline{\alpha'\cdot\beta'} = \frac{\partial}{\partial t}\overline{\alpha'\beta'} - \overline{\alpha' D_0 \beta'}, \text{ and } D_0 D_0^{-1} = 1,$$

so that

$$K_{yy}^{(1)} = \overline{\eta'\eta'} - \frac{\partial}{\partial t}\overline{\eta' D_0^{-1}\eta'} = \frac{1}{2}\overline{\eta'\eta'}$$

and similarly

$$K_{zz}^{(1)} = \frac{1}{2}\overline{\zeta'\zeta'}$$

$$K_{yz}^{(1)} = \overline{\eta'\zeta'} - \frac{\partial}{\partial t}\overline{\eta' D_0^{-1}\zeta'}, \quad K_{zy}^{(1)} = \overline{\eta'\zeta'} - \frac{\partial}{\partial t}\overline{\zeta' D_0^{-1}\eta'} .$$

The $\mathbb{K}^{(1)}$ matrix is actually symmetric for steady wave fields. Comparing the magnitudes of the components of $1/\tau_0\, \mathbb{K}^{(1)}$ with the corresponding components of $\mathbb{D}$, the symmetric part of $\mathbb{K}$, we see that the ratios are principally determined by the strength of the stationary eddy field vs. that of the transient eddy field in the atmosphere. For example, using the parameterization of transient eddies (4.30), we find

$$\left|\frac{1}{\tau_0} K_{yy}^{(1)} / D_{yy}\right| \sim \frac{\tau}{\tau_0} \frac{\overline{\langle\eta'\eta'\rangle}}{\overline{\langle\Delta\eta'\Delta\eta'\rangle}} . \qquad (5.9)$$

This ratio is in general not small despite our assumption that τ_0 be much greater than $a/\bar{u} \sim$ 2-3 days.

Nevertheless, the $\mathbb{K}^{(1)}$ matrix itself can be simplified if we adopt the isentropic coordinate system. Using our earlier result that the normalized ratio between the vertical and horizontal displacements in isentropic coordinates is small (i.e. $r \ll 1$ according to (4.13)), the $\mathbb{K}^{(1)}$ matrix can be approximated by

$$\mathbb{K}^{(1)} \simeq \begin{bmatrix} D_{yy}^{(1)} & 0 \\ 0 & 0 \end{bmatrix}, \qquad D_{yy}^{(1)} \equiv \frac{1}{2}\overline{\eta'\eta'} \qquad (5.10)$$

This approximation is the same as that used in Tung (1982).

Using data from van Loon et al (1973), who gave a value of $\sim$600 m for the geopotential height for zonal harmonic standing wave 1 in January around 10 mb level, we estimate the value for $D_{yy}^{(1)}$ to be

$$D_{yy}^{(1)} \sim 10^{16}\ \text{cm}^2. \tag{5.11}$$

At this value, the chemical eddy team will become important compared with the dynamical diffusion term when the chemical relaxation time is less than 30 days. That is

$$\frac{1}{\tau_0} D_{yy}^{(1)} \gtrsim D_{yy} \quad \text{if} \quad \tau_0 \lesssim 30 \text{ days.}$$

There are a few more points that are worth making concerning this simple example. First, that the eddy fluxes in (5.7) can be expressed in terms of the mean species gradients is entirely a consequence of our assumption that τ_0 contains no eddy term. In more general systems, the photochemical relaxation time is rather temperature-dependent and perturbations in the temperature field should cause perturbations in τ_0, which then leads to an additional term in (5.7) proportional to $\overline{\chi}$. Secondly, our asymptotic solution procedure requires the assumption that the photochemical time scale is longer than the dynamical time scale, $a/\overline{u}$. This assumption is probably correct in the lower stratosphere but becomes increasingly invalid higher up. In regions where the relaxation time is very short, a different assumption -- the photochemical equilibrium assumption -- can be used, namely, the dynamical transports can be neglected and (5.4) approximated by

$$0 \simeq -\frac{1}{\tau_0}(\chi - \overline{\chi}_0), \tag{5.12}$$

yielding the equilibrium solution $\chi \simeq \overline{\chi}_0$. The troublesome region is the so-called transition region where the photochemical and dynamical time scales are comparable. The method presented here cannot handle this problem and further study is needed.

6. SUMMARY

A main problem in the formulation of zonally averaged models of tracer transport is the treatment of eddy fluxes. As these eddy terms cannot be determined consistently within zonally averaged models, their magnitudes need to be calculated from a knowledge of the wave field in the atmosphere. I have attempted in this paper to give an order of magnitude estimate of various eddy transport teams, assuming that the eddy field in the stratosphere consists predominantly of organized (but nonadiabatic and possibly nonsteady) planetary and nonbreaking gravity waves forced in the lower atmosphere (gravity wave breaking level appears to occur higher up in the mesophere, above the photochemical region of interest here (see Lindzen (1981)). Though the estimates obtained depend somewhat on the parameters chosen, the following features appear to be typical:

(1) In isentropic coordinates, the effect of vertical displacement is found to be about one order of magnitude smaller than that of horizontal displacement (even after the difference in vertical vs. horizontal scales is taken into account). The eddy displacements are, therefore, approximately rectilinear in isentropic coordinates.

(2) As a consequence, the dominant eddy diffusion is horizontal (i.e. along isentropes) across horizontal gradient of mean tracer concentration (the K_{yy} term in the transport equation). Contrast this with the situation in pressure coordinates where the normalized ratio of the vertical and horizontal diffusion coefficients is about order unity (see (4.7) using Reed and German's K_{yy} and K_{zz}). This ratio is found also by Kida (1983) to be of order one in his GCM, although both of his calculated K_{yy} and K_{zz} are about one order smaller than the values used by Reed and German.

(3) Assuming that the transient eddies are due to damped forced waves, I have given a parameterization of the coefficient K_{yy} (or called D_{yy} here), and estimated its climatological magnitude to be

$$K_{yy} < 4 \times 10^9 \ cm^2/s.$$

At this magnitude, the diffusive transport appears to be one order of magnitude smaller than the mean advective transport over scales comparable to the radius of the earth, but may be more effective over scales less than a thousand kilometers.

(4) Occasionally during major sudden warmings, the diffusion coefficient can achieve a maximum value of

$$K_{yy} \sim 5 \times 10^{10} \ cm^2/s,$$

for a short period of time (a few days). Though at this value the diffusive process is competitive with the mean advection in transport, the short duration at which this larger value is achieved and the infrequency of major sudden warmings tend to make the former a less effective means of systematic transport than the latter over seasonal time scales.

(5) The eddy advective transport is found to be dominated by the stationary planetary wave component in the presence of

eddy diabatic heating Q'. Assuming Q' is about 1-2°C/day, I have estimated the magnitude of this transport to be equivalent to that arising from a mean diabatic heating rate of about a fraction of a degree per day. Though not clearly negligible compared to the mean diabatic circulation, the eddy advective contribution is within the range of uncertainty in our ability to deduce the diabatic heating rate from observed data.

(6) For nonconservative tracers, chemical eddy terms may also be present. Since perturbations in the minor chemical species arise ultimately from perturbations in air, the chemical eddy terms should in principle be expressible in terms of air displacement correlations, which can be parameterized; the parameterization would then be independent of the minor species within the same atmosphere. We have shown here using a simple example how this can be done for the case when the photochemical time scale is longer than the advection time scale, $a/\bar{u}$, which is about 2-3 days. It is found that the chemical eddy term in our example is important when the photochemical relaxation time is less than about 30 days.

As far as the dynamical transports are concerned, it now appears that the dominant mechanism for systematic tracer transport is advective in nature, with horizontal diffusion playing a secondary role and vertical diffusions even less significant. The approximate transport equation is then:

$$\frac{\partial}{\partial t}\bar{\chi} + \frac{\bar{V}^*}{\bar{\rho}_\theta}\frac{\partial}{\partial y}\bar{\chi} + \frac{\bar{W}^*}{\bar{\rho}_\theta}\frac{\partial}{\partial \theta}\bar{\chi} - \frac{\partial}{\partial y}\left(D_{yy}\frac{\partial}{\partial y}\bar{\chi}\right) = \bar{P} \quad , \qquad (6.1)$$

where $\bar{P}$ involves various chemical source terms. In (6.1) the advective velocities are found from (see Tung (1982) with $\bar{W}^* \equiv \bar{W} + W_E$):

$$\bar{W}^* \simeq \bar{q}^*/\Gamma^{(0)} \qquad (6.2)$$

$$\frac{\partial}{\partial y}\bar{v}^* + \frac{\partial}{\partial \theta}\bar{W}^* \simeq 0 \qquad (6.3)$$

with

$$\bar{q}^* \equiv \bar{q} + \Gamma^{(0)}\frac{\partial}{\partial y}\bar{\rho}_\theta\,\psi$$

$\bar{q}$: the mean diabatic heating rate per unit volume.

The mean density $\bar{\rho}_\theta$ appearing in (6.1) can be approximated by the basic state density $\bar{\rho}_\theta^{(0)}$, which is a function of vertical coordinates only:

$$\bar{\rho}_\theta^{(0)}(\theta) \simeq \bar{\rho}_\theta^{(0)}(\theta_0)\cdot\left(\frac{\theta}{\theta_0}\right)^{-9/2} \tag{6.4}$$

Thus from a diagnostic point of view, $\bar{q}^*$ is the only quantity that needs to be specified in determining the advective transports. Because of the uncertainties in $\bar{q}$, one can replace $\bar{q}^*$ by $\bar{q}$ at the present stage of model development.

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