Linear vs. Nonlinear Neutralization of Baroclinic Flows

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1. Introduction

In a series of papers (Lindzen and Farrell (1980), Lindzen (1993), Sun and Lindzen (1994), and Roe and Lindzen (1996)), Lindzen and co-workers constructed various model mean states which are neutral to baroclinic instabilities. The studies were in part motivated by the suggestion that such neutral states may in fact be relevant to the atmosphere's actual state (Lindzen 1993). As the simpler neutral profiles considered did not in fact resemble the atmosphere, more complexity was added. The latest is Roe and Lindzen (1996), hereafter referred to as RL. It considered the baroclinic instability in a two-level model including barotropic (meridional) shear, and showed that the inclusion of meridional shear can be stabilizing, consistent with the earlier result of James (1987).

We wish in the present note to clarify a misleading statement made by RL concerning equilibration in nonlinear models, and in the process we hope to highlight the difference between *equilibration* and *neutralization*. In particular, we point out that the *equilibrated* states in nonlinear models are *highly* unstable to a band of zonal wavenumbers around 7-13 in a linear stability analysis. We argue that this is also true in the real atmosphere. The apparent paradox is not a conceptual one, but rather a result of the faulty diagnostics commonly used, i.e. the linear stability analysis of the zonal mean flow. We then conclude that the suggestion that the *equilibrated* state be linearly *neutral* to all baroclinic instability may be too stringent a requirement.

2. Linear Stability of Initial vs. Equilibrated Flow

We agree with RL that the development of barotropic shear may reduce the instability of an initial baroclinic flow. In fact, nonlinear models do calculate and incorporate this mean flow modification. We also agree with RL that calculations of the stability of an equilibrated baroclinic flow must be based on the *modified* zonal mean flow, including its meridional profile. This was, in fact, incorporated into the work of Cehelsky and Tung (1991), as can be seen from their statement (p. 1944): "...the mean meridional temperature difference equilibrates near the critical value of the main heat-transporting eddy, as predicted by a linear stability theory based on the *modified* flow." Fig. 16 of their work showed the meridional structure of the modified zonal flow in various cases. Extensive discussion was on given on p. 1943 of how the modified flow can be made less unstable than the initial flow, similar to RL. Note that to obtain the modified flow requires a nonlinear model. This diminishes the predictive value of the above quoted statement.

It was also pointed out by Cehelsky and Tung (1991) that nonlinear runs produce less meridional modification than quasi-linear runs. They found that in the parameter regimes of relevance and for the dominant heat transporting wave, the stability properties were not altered too significantly by the barotropic modification. For these cases, a more predictive, though less precise, statement of the theory was given (p. 1945): "If only the *preexisting* flow is known, our proposed theory can be stated as follows: The mean meridional temperature difference equilibrates near the critical value of the main heat-transporting eddy as predicted by linear stability theory based on the preexisting flow..."

Roe and Lindzen (1996) take this work out of context when they say that (p. 2750): "Cehelsky and Tung (1991) calculated the instability of modes with respect to the initial state rather than the equilibrated, final state."

Note also that the statement in Cehelsky and Tung concerns the neutralization of the "main heat-transporting eddy", and not the neutralization of all baroclinic waves. This is an important distinction which we will elaborate later.

3. Linear vs. Nonlinear Neutrality

The proposal that the observed atmospheric state should be near neutral to all waves is an interesting one. The problem lies not in this idea but in the choice of a diagnostic tool for testing "neutrality". This has been the source of much confusion in the literature. The diagnostic tool commonly used to determine the stability of modeled and observed results is linear stability analysis of the *zonal mean* flow. If one takes a realistic atmospheric state, averages it zonally, and then performs a linear stability analysis of the resultant two-dimensional flow, one will find that the mean atmosphere is highly unstable and that the most unstable waves have zonal wavenumbers 12-15. (This diagnostic has in fact been done by Gall (1976), using a General Circulation Model.[1]) However, the observed dominant baroclinic waves are of much longer scale, specifically wavenumbers 4-7 (Randel and Held 1991).

One can pinpoint better the source of the discrepancy using a nonlinear numerical model which permits baroclinic and barotropic instability. Run such a model until it *equilibrates.* Then take the zonal average of the equilibrated zonal flow and perform a linear stability analysis on it. One would have expected *neutrality.* After all, the system has already equilibrated, and thus the instabilities should have modified the mean flow, i.e. "neutralized" it. However, this is *not* what one finds. This fact was pointed out by Cehelsky and Tung (1991), Welch and Tung (1997a, 1997b), and also by Salmon (1980) and Vallis (1988) earlier. In each case a nonlinear model run to equilibrium yields a final state which is supercritical to the most unstable wave, *if* instability is determined according to a linear stability analysis of the zonal mean flow. One such result is shown in Fig. 1, which displays the marginal stability curves for the initial (Hadley) and equilibrated flows of a nonlinear two-level model (from Welch and Tung (1997b)). The horizontal dashed line shows the equilibrated temperature gradient (or equivalently, vertical shear). It is above the stability boundary of both the initial flow (dashed curve) and the modified flow (solid curve), and so the model atmosphere is still unstable, according to this analysis. Note that friction is included in these stability calculations. This problem is *not* associated with the use of the initial instead of the modified flow, as RL implied. With respect to the modified flow, wavenumbers 6-12 are still "unstable" by this measure.

It is also worth pointing out that in this model, as is also the case in the atmosphere, the

[1] Gall calculated maximum growth rates of 0.6 day^{-}1, which is equivalent to a doubling time of 1.2 days. Although no damping was included in Gall's stability analysis, his result on these fast growing waves should not be affected significantly by it.

dominant heat-transporting wave, at wavenumber 5, is much longer than the linearly most unstable wave, which would be wavenumber 11-12, according to Fig. 1. If linear stability theory works it should be wavenumbers 11-12 which grow the fastest and hence become the dominant waves.

The discrepancy is highlighted by performing another analysis, in which we take the model equilibrated *wavy* flow, perturb it randomly with small perturbations, and see if these secondary perturbations grow. If they do, then the flow is unstable. If they don't, we would conclude that the flow is stable to small amplitude perturbations. The result is that there is no amplification of the small perturbations. This should not be surprising, because the flow that we use has already equilibrated.

The above discussion points to the conclusion that it is the linear stability analysis of the zonal mean flow which is giving us the false diagnostics about the stability of the flow. The equilibrated model flow, and perhaps also the atmosphere, may indeed be "neutral" (or near neutral), as suggested by Lindzen (1993), but neutrality should not be defined according to linear stability analysis of the zonal mean flow, whether preexisting or modified.

4. Nonlinear Neutralization

A careful energetics analysis of the nonlinearly equilibrated model state has been done, and is discussed in more detail in Welch and Tung (1997a, 1997b). Briefly, the waves and mean flow evolve towards equilibrium in the following manner. The linearly most unstable waves (with wavenumbers 11-12) do grow initially. They quickly saturate, however, and cannot grow further without breaking. The next longer wave then grows and transports heat until it also saturates. Such a process of upscale energy cascade continues all the way to wavenumber 5, such that at equilibrium wavenumbers 6-12 are equilibrated nonlinearly, although they may still appear to be unstable linearly. Wavenumber 5 does not equilibrated in this manner. It grows by extracting potential energy from the zonal mean state and in the process lowers the temperature gradient and "neutralizes" the mean flow. This last process is predominantly a quasi-linear one, and thus a linear stability analysis here gives the correct diagnostics: this wave is neutral, whether defined in the linear or the nonlinear sense. Fig. 1 verifies that wavenumber 5 is indeed near linear neutrality.

The scale of the dominant heat-transporting eddy is determined by the level of radiative forcing. A lower radiative forcing than that used to produce Fig. 1 would yield a dominant wave shorter than wavenumber 5, and a higher forcing would yield a longer dominant wave.

The atmosphere is probably not close to a state in which *all* modes have been linearly neutralized, but to a state in which only the longer modes have been, while the shorter remain linearly unstable but have been neutralized by a nonlinear transfer of energy to lower (linearly stable) wavenumbers. The mechanism is conceptually simple, and hence we believe it to be encouraging, in contrast to the opinion of RL (p. 2750).

5. Conclusions

The idea that the mid-latitude troposphere could be close to a state in which all zonal waves have been linearly neutralized is an interesting one. The simplicity of the concept is alluring, but it appears to be inconsistent with observations and nonlinear numerical simulations. Instead of simple linear stability analysis, a nonlinear upscale cascade must be included in the theory to account for the supercritical state of the baroclinic atmosphere. A better diagnostic of the stability of the equilibrated flow to perturbations of a certain wavelength would be based on the mean flow plus a finite amplitude wave of that wavelength, i.e. a linear analysis of a wavy state. The more common linear stability analysis of the zonal mean flow alone is shown here to yield an incorrect result.

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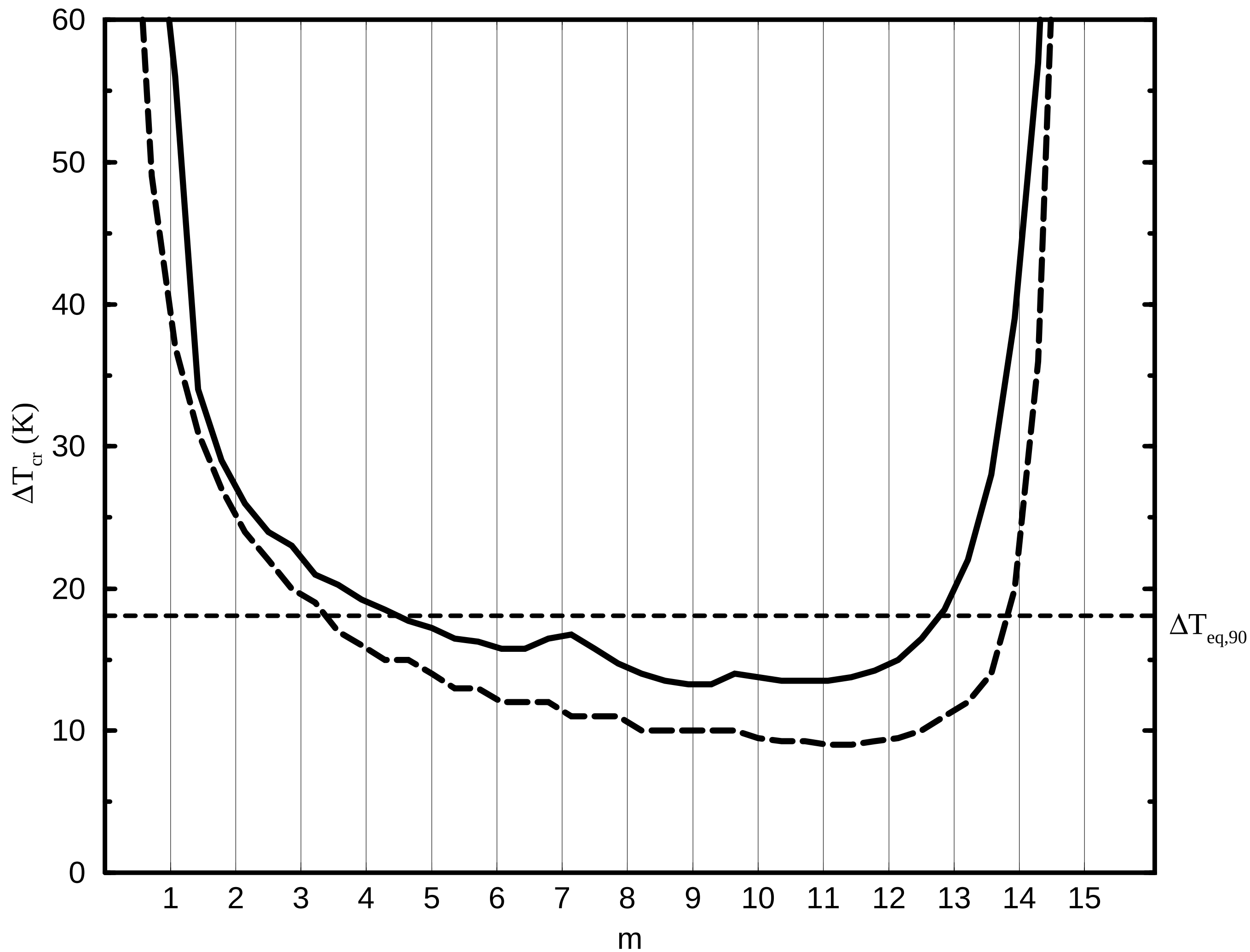

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