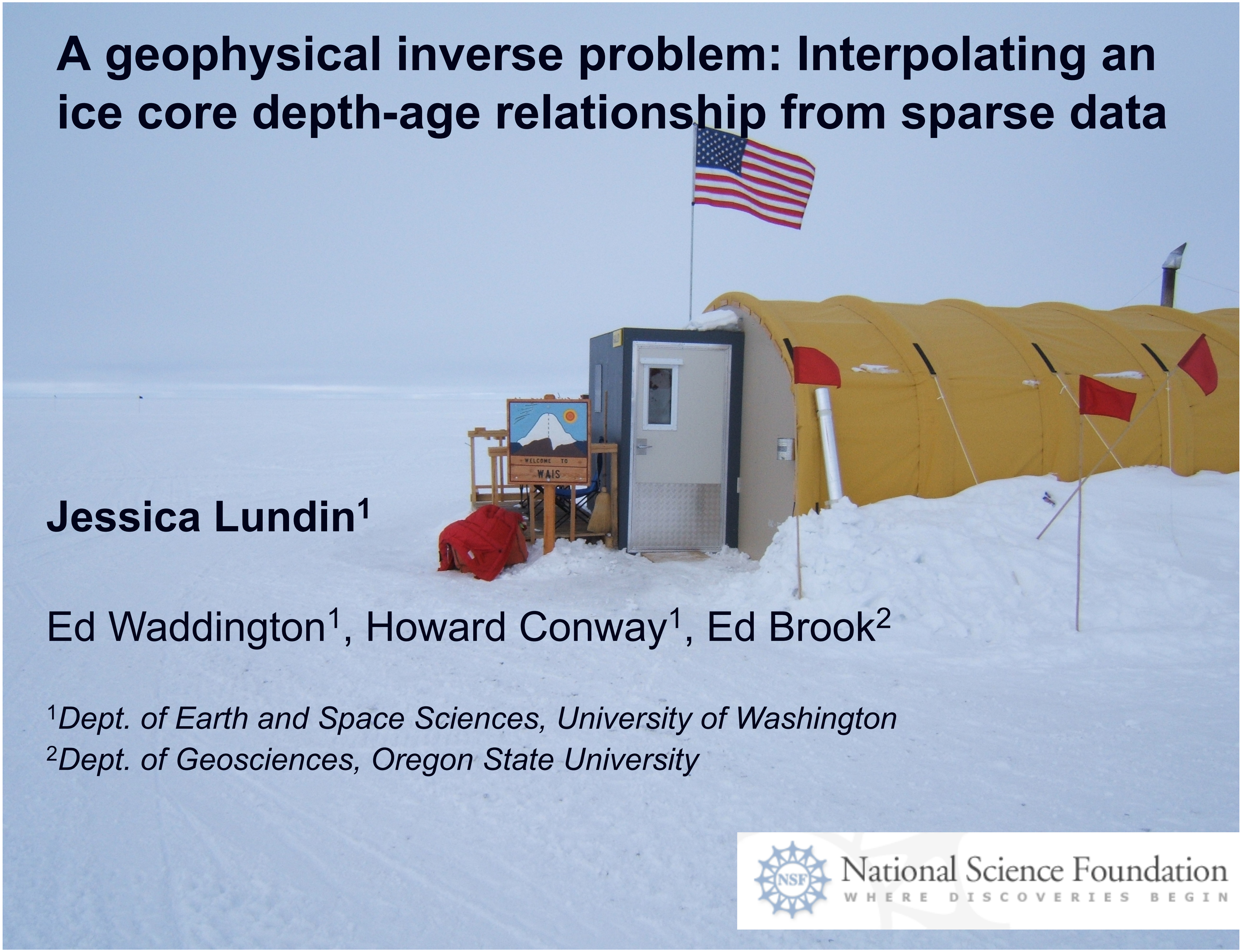
A geophysical inverse problem: Interpolating an ice core depth-age relationship from sparse data



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National Science Foundation
WHERE DISCOVERIES BEGIN

Ice Cores and depth-age relationship

- One of the best paleoclimate proxies available
  - Only record of past atmospheric gas

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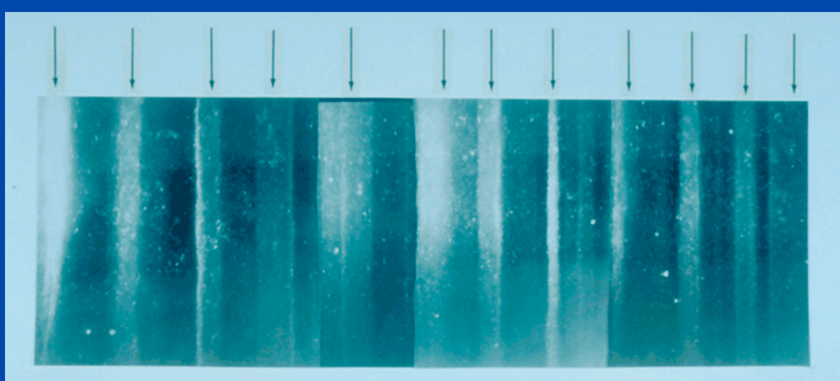
  - Temperature record from oxygen isotopes in water
  - 1-d spatial, high resolution temporal record (800ka)



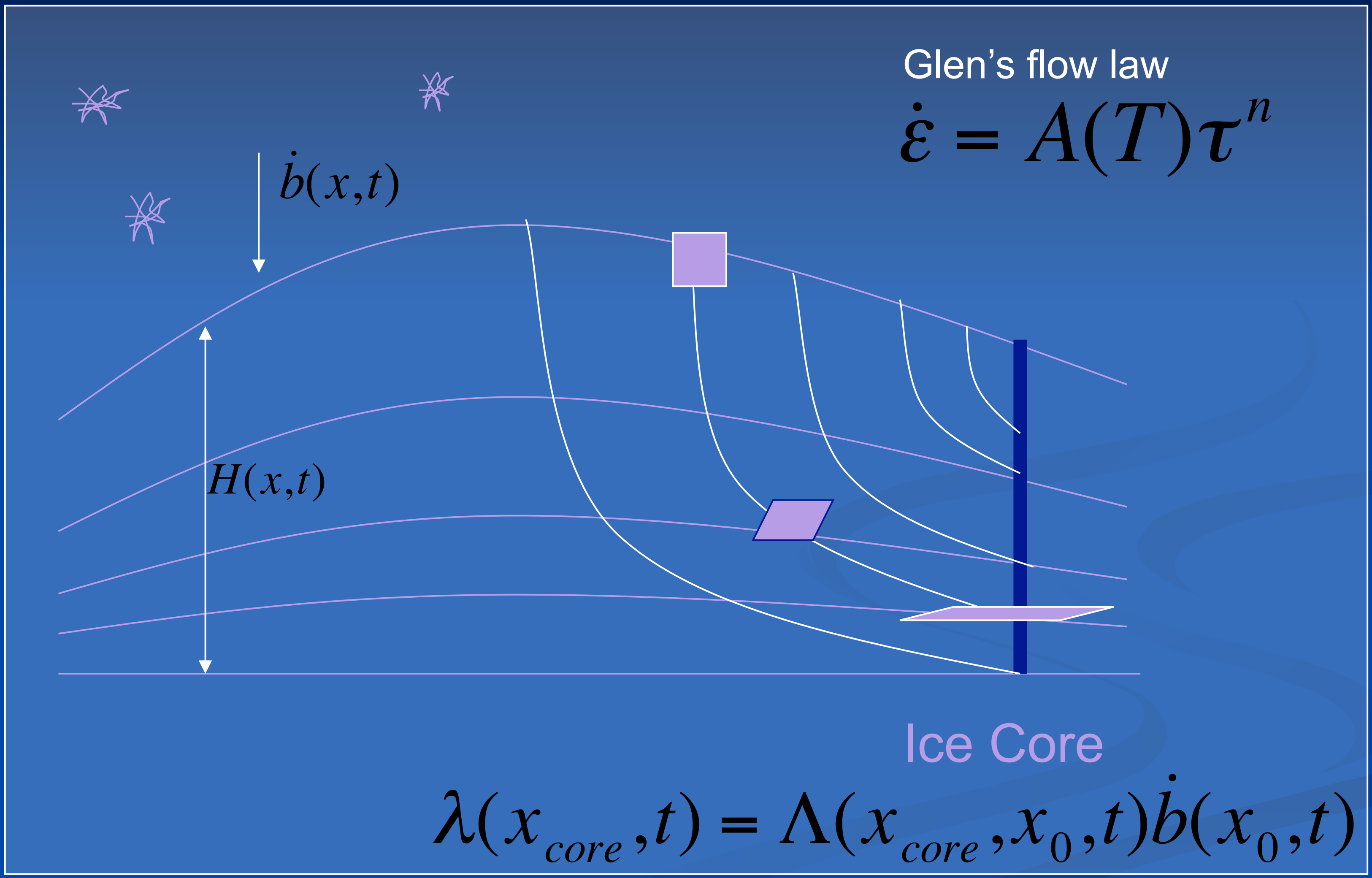
GISP2 ice core
http://upload.wikimedia.org/wikipedia/commons/3/31/GISP2_1855m_ice_core_layers.png

Photos: left-Ben Smith, right-Jessica Lundin

Ice Cores and depth-age relationship

- One of the best paleoclimate proxies available
  - Only record of past atmospheric gas
  - Temperature record from oxygen isotopes
  - 1-d spatial, high resolution temporal record (800ka)
  - Record for past ice sheet evolution (thickness, dynamics)

Ice Dynamics



Depth-age relationship of ice core is a function of **ice dynamics** and **accumulation**

Example: what can go wrong

Create synthetic depth-age data from
1-d steady-state flow model

Make linear and cubic interpolations

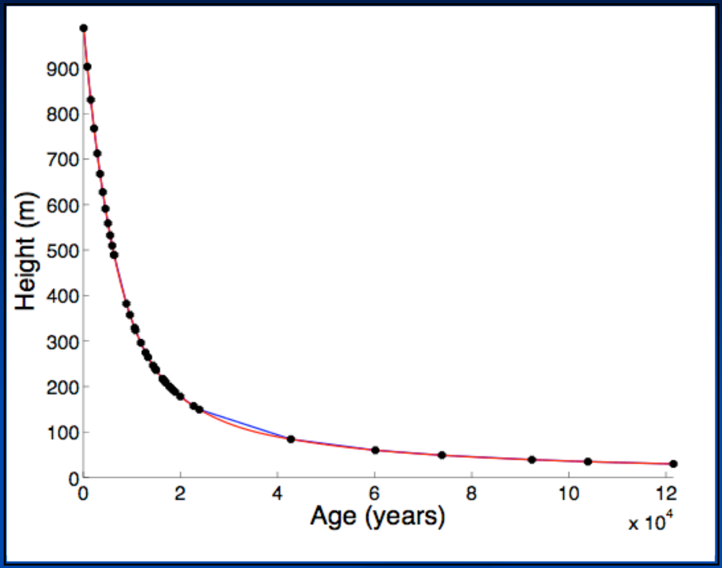
Determine layer thickness $\lambda(t)$ derivative of depth-age interpolations

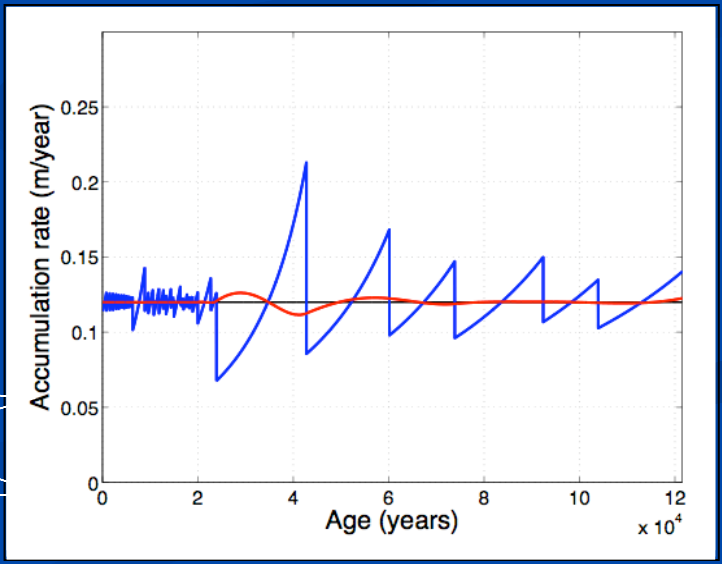
Determine thinning function $\Lambda(t)$ from 1-d flow model

Determine accumulation rate

$$\lambda(t) = \Lambda(t)\, b(t)$$

Create your own climate events or an artifact of a bad interpolation?!





Why an inverse approach?

Linear and cubic-spline interpolations of sparse depth-age data can lead to unphysical results

- Don't properly account for the variation of dynamical strain with depth
- Don't account for temporal changes in accumulation rate.

A physically-based interpolation scheme is necessary

An inverse approach:

- applies known physics,
- incorporates data uncertainty
- Can prevent model from overfitting the data.

Inverse Approach

Determine boundary and initial conditions and other parameters from observable data and known physics.

d = Gm

Known:

d is data, which are depth, age, and their uncertainties

G is the physics, 1-d transient kinematic flow model

Unknown:

m is the model vector, accumulation-rate histories with time

Inverse Approach

$$H - z(A) = \int_0^A \Lambda(A) b(A) dt$$

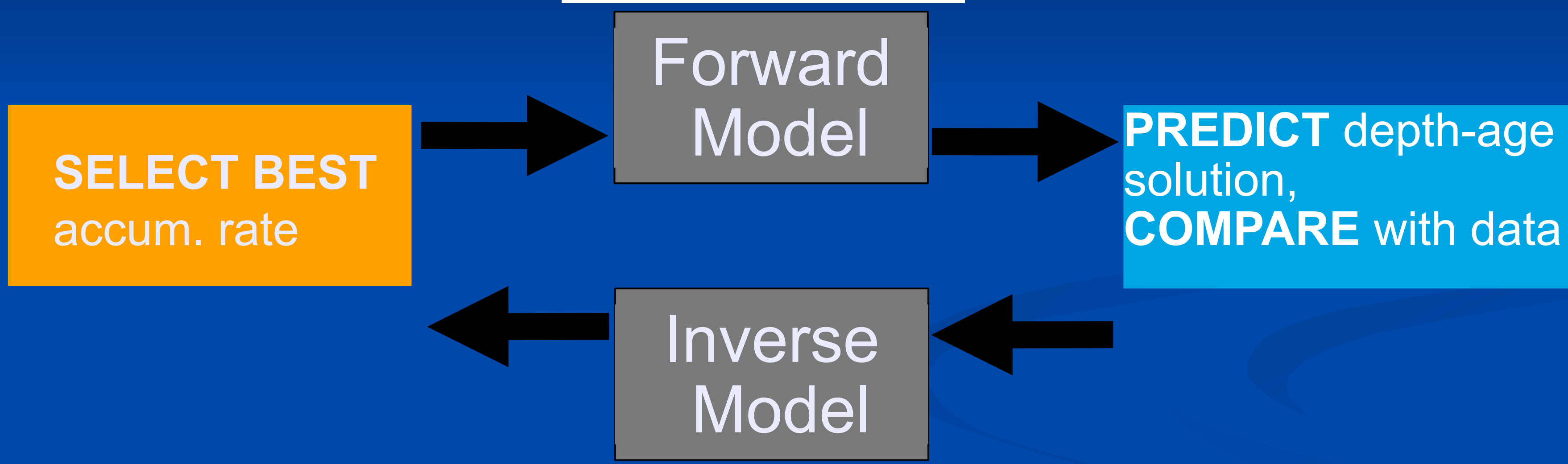
ESTIMATE accum. rate

→ Forward Model →

PREDICT depth-age solution,
COMPARE with data

Inverse Approach

$$H - z(A) = \int_0^A \Lambda(A) b(A) dt$$



Known:

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Unknown:

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Inverse Approach: Gradient method

Performance Index

$$I = \|m\|_2 + \nu(\|d\|_2 - T^2)$$

Model norm

$$\|m\|_2 = \int_{-A_m}^{0} \left(\frac{d^2 b}{dt^2}\right)^2 dt$$

Data norm

$$\|d\|_2 = \sum_{j=1}^{N} \left(\frac{z_j^d - z_j^m}{\sigma_j}\right)^2$$

Tolerance

$$T = N^{1/2}\left[1 - \frac{1}{4N} - \frac{13}{32N^2} + O(N^{-3})\right]$$

Parker, R. 1994. *Geophysical Inverse Theory*

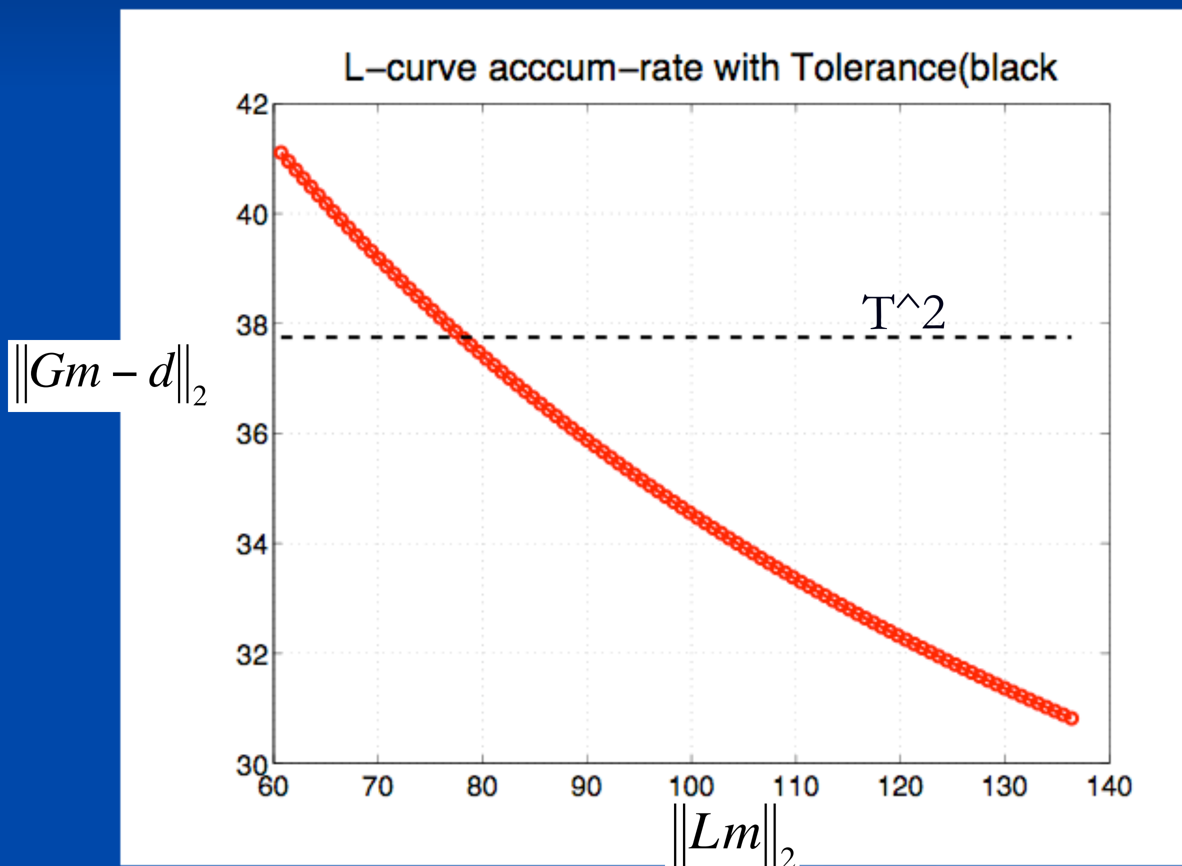
Higher-Order Tikhonov Regularization

$$\begin{bmatrix} \nu\sigma^{-1}G \\ L \end{bmatrix} m = \begin{bmatrix} \sigma^{-1}d \\ 0 \end{bmatrix}$$

- Model determined using Matlab least-sqares non-negative: *lsqnonneg(LHS, RHS)*
- L is second derivative roughening matix
- G is the problem physics
- ν is the Lagrangian tradeoff parameter
- d is the data and σ the data uncertainty

Trade off parameter ν

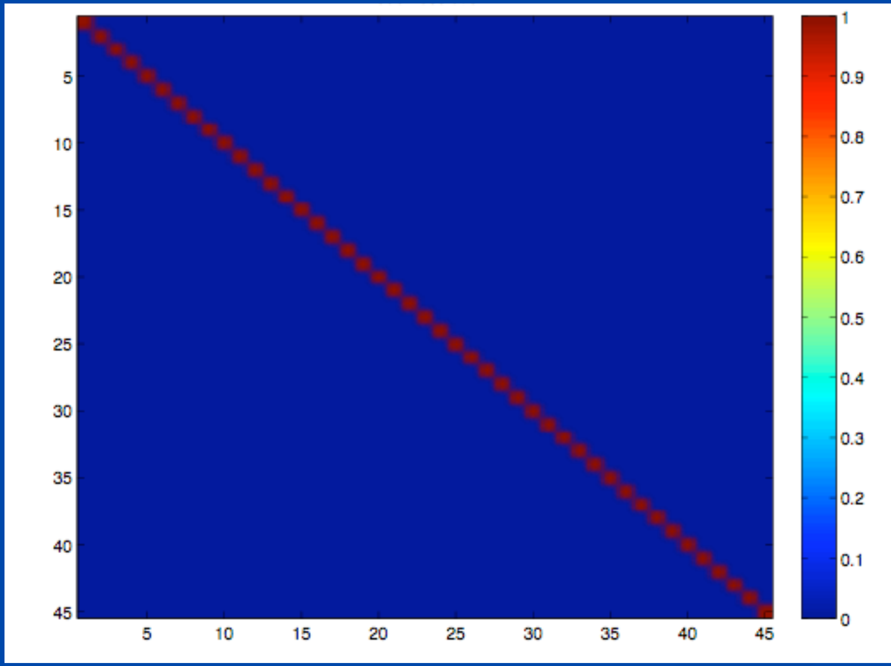
Choose ν from L-curve plot of model and data norms



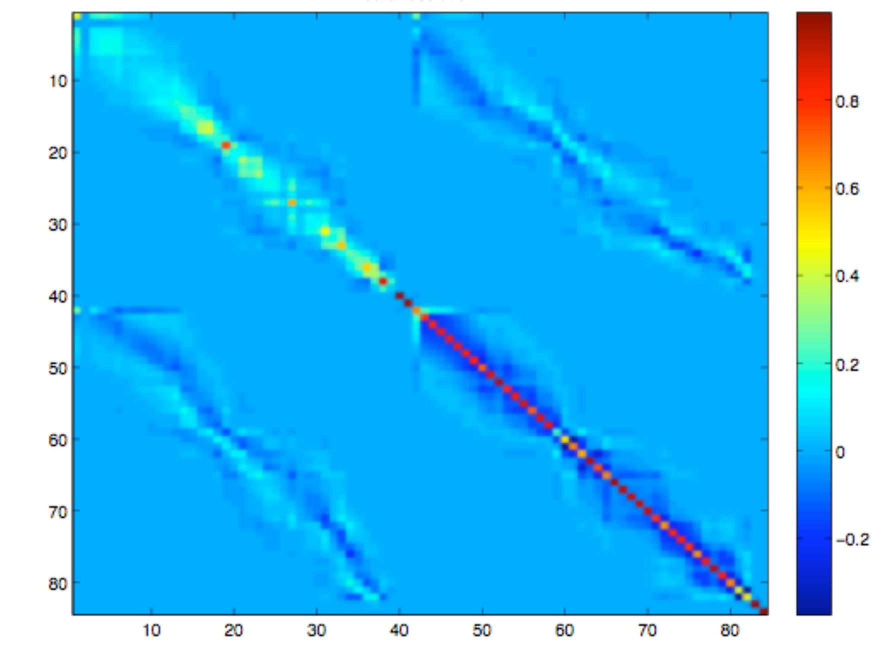
Model and Data Resolution

$$svd\begin{bmatrix} \nu\sigma^{-1}G \\ L \end{bmatrix}$$

Model Resolution $\mathbf{V}_n\mathbf{V}_n^T$



Data Resolution $\mathbf{U}_p\mathbf{U}_p^T$



Model uncertainty

- Add red noise to data, based on data uncertainty

$$f(t+\Delta t) = \alpha f(t) + \sigma n(t)$$

- Create realizations of model

- Consider variance of solutions, characterize model behavior

Model application to Siple Dome

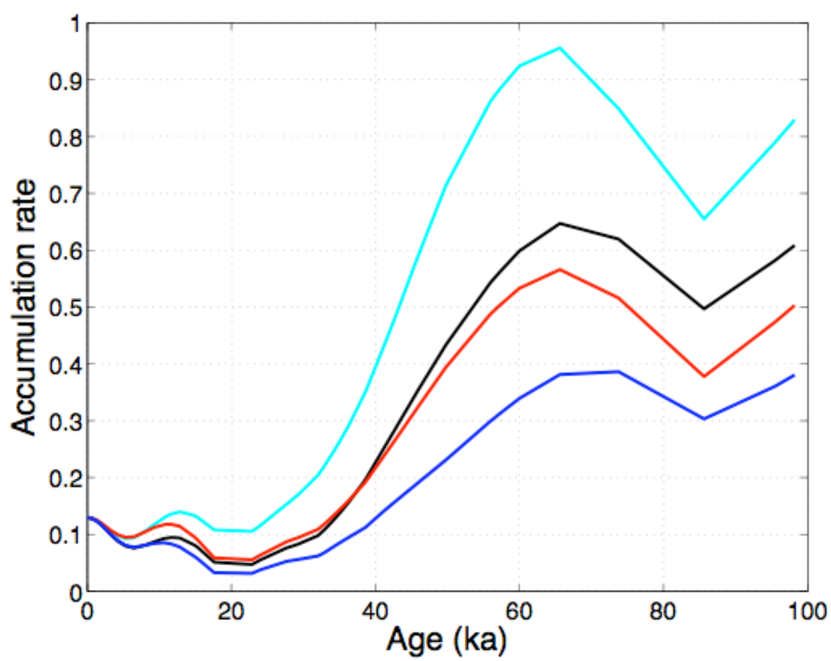
High (annual) resolution dating via physical and chemical markers

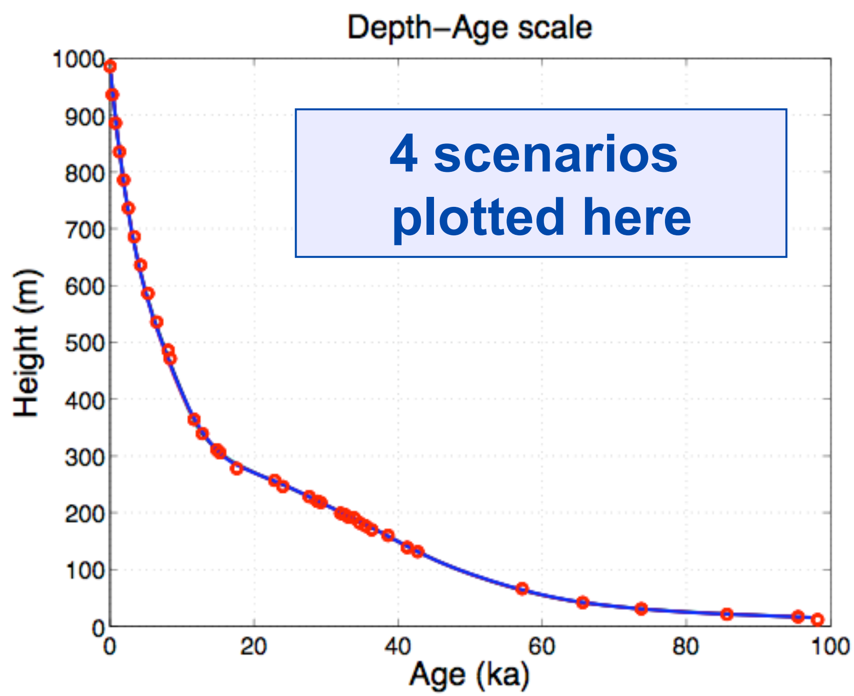
Sparsely dated
CH_4 and O_2 tie in points with Greenland cores

Taylor et al., 2004; Blunier and Brook 2001; Brook et al., 2005 NSIDC

Consider 4 evolution histories for Siple Dome

1. Black: Flank flow (*h*=0.25) and 350m thinning 15-14ka
2. Cyan: Transitional flow (h=0.5) and thinning
3. **Blue**: Flank flow, no thinning
4. Red: Transitional flow, no thinning





Result: **4 senarios have identical depth-age relationship**

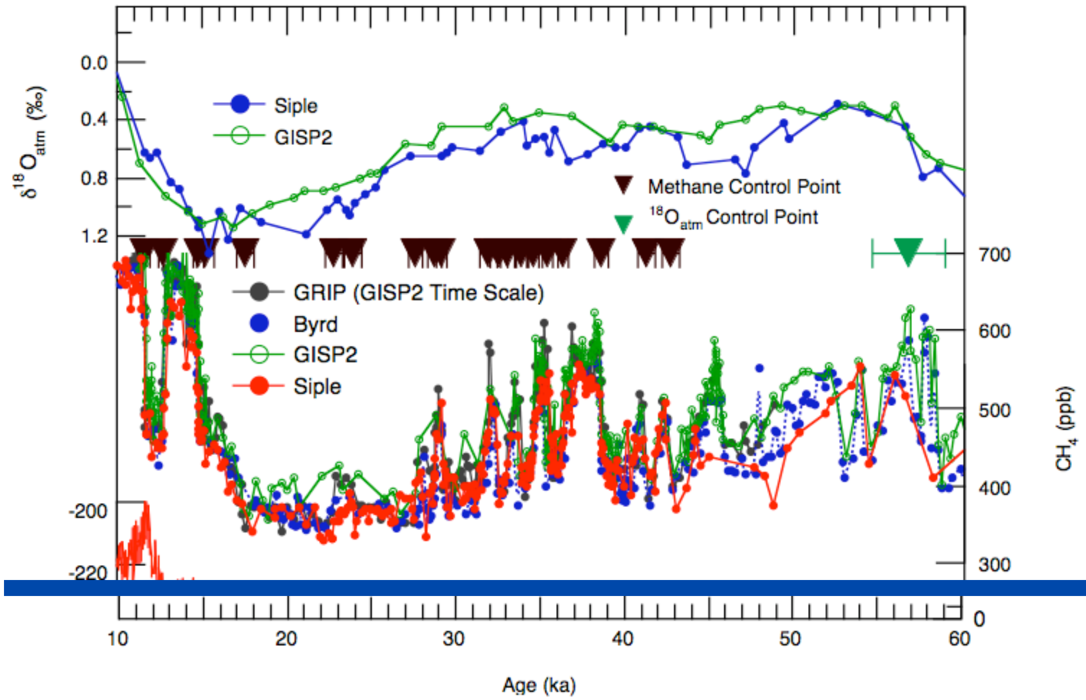
Conclusions

A novel method for determining a physically-based depth-age relationship using an inverse approach has been developed.

The depth-age profile for Siple Dome has been determined, using our model selection criteria,e.g., smooth accumulation history

Pairs of thinning function and accumulation rate are found for different ice evolution scenarios

336 E.J. Brook et al. / Quaternary Science Reviews 24 (2005) 1333–1343



control points

If we know the Greenland depth-age relationship, and can relate the

Table 1
Control points for Siple Dome gas-age time scale, Δage, and estimated uncertainties

Depth	Corr. variable	Gas age (ka)	Δage (yr)	Ice age (ka)	Δage unc. (ka)	Correlation unc. (ka)	GISP vs. Siple, total unc	GISP unc. (ka)	Absolute unc. (ka)
514.78	CH_4	8.33	282	8.61	0.08	0.20	0.38	0.17	0.55
621.73	CH_4	11.63	243	11.88	0.07	0.12	0.30	0.23	0.53
646.71	CH_4	12.79	351	13.14	0.11	0.17	0.38	0.26	0.63
674.88	CH_4	14.78	381	15.16	0.11	0.13	0.35	0.30	0.64
680.38	CH_4	15.21	442	15.65	0.13	0.12	0.35	0.30	0.66
708.08	CH_4	17.56	645	18.20	0.19	0.32	0.61	0.35	0.97
729.15	CH_4	22.81	815	23.63	0.24	0.27	0.61	0.46	1.07
739.72	CH_4	23.95	719	24.67	0.22	0.27	0.58	0.48	1.06
757.84	CH_4	27.61	729	28.34	0.22	0.26	0.58	0.55	1.13
765.48	CH_4	28.72	670	29.39	0.20	0.15	0.45	0.57	1.03
767.89	CH_4	29.21	689	29.90	0.21	0.18	0.48	0.58	1.07
786.93	CH_4	31.97	633	32.60	0.19	0.06	0.35	0.64	0.99
789.02	CH_4	32.52	683	33.21	0.20	0.30	0.60	0.65	1.25
793.57	CH_4	33.05	629	33.68	0.19	0.33	0.62	0.66	1.28
795.20	CH_4	33.85	599	34.45	0.18	0.21	0.49	0.68	1.17
803.82	CH_4	34.60	672	35.28	0.20	0.14	0.44	0.69	1.13
809.26	CH_4	35.45	599	36.05	0.18	0.13	0.41	0.71	1.12
815.94	CH_4	36.30	572	36.87	0.17	0.10	0.37	0.73	1.09
825.87	CH_4	38.56	496	39.05	0.15	0.20	0.44	0.77	1.22
846.92	CH_4	41.21	572	41.78	0.17	0.29	0.56	0.82	1.38
854.53	CH_4	42.65	482	43.13	0.14	0.32	0.57	0.85	1.42
919.88	$\delta^{18}O_{atm}$	56.70	463	57.16	0.14	2.00	2.24	1.13	3.37

References

Aster et al., 2005. *Parameter Estimation and Inverse Problems*

Blunier and Brook 2001. *Science*

Brook et al., 2005. *Quaternary Science Reviews*

Conway et al., 1999. *Science*

Nereson et al., 1998. *Annals of glaciology*

Price et al., 2007. *JGR*

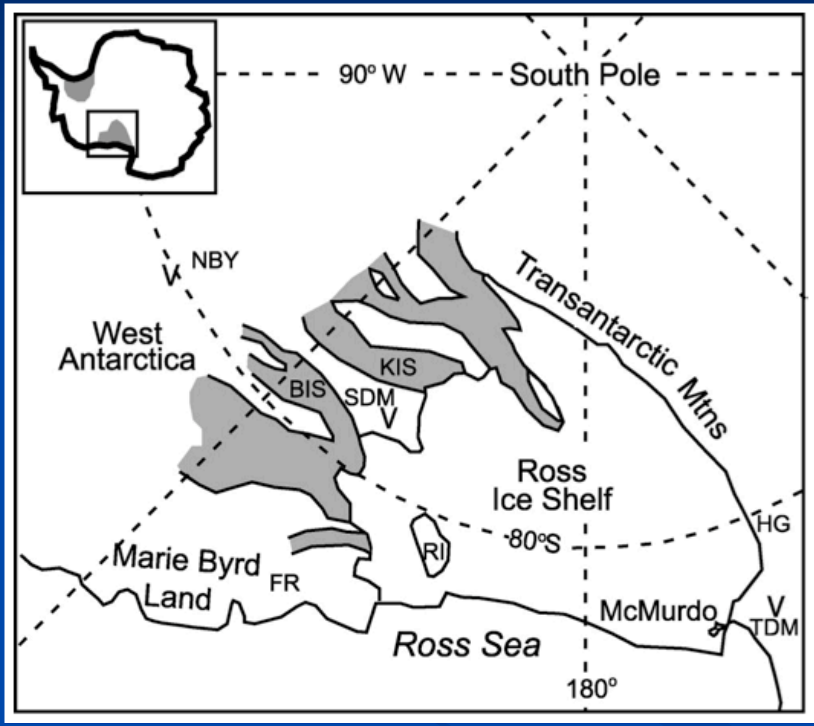
Taylor et al., 2004. *J. Glaciology*

Waddington et al., 2005. *Geology*

Model application to Siple Dome

Ice dynamics background

- Thickness history:
  - Forward model of thinning function and accumulation-rate pairs to match depth-age data; result: thinning of 200-400m since the LGM (Waddington et al, 2005)
  - Full-stress ice flow model; result: 350m thinning 15-14ka (Price et al, 2007)
- Divide migration history
  - Matching radar layers; result: transitional flow (h=0.5) for last 4ka with divide flow 3ka. (Nereson et al, 1998)



Siple Dome ice evolution histories

The thinning function $\Lambda(t)$ is determined by a particle tracking scheme, and is affected by:

- thickness change
- ice divide migration

The accumulation rate is inferred for each thinning function, posed as an inverse problem

Depth-age relationship for Antarctic ice cores

Motivation

Depth-age relationships for ice cores are important for:

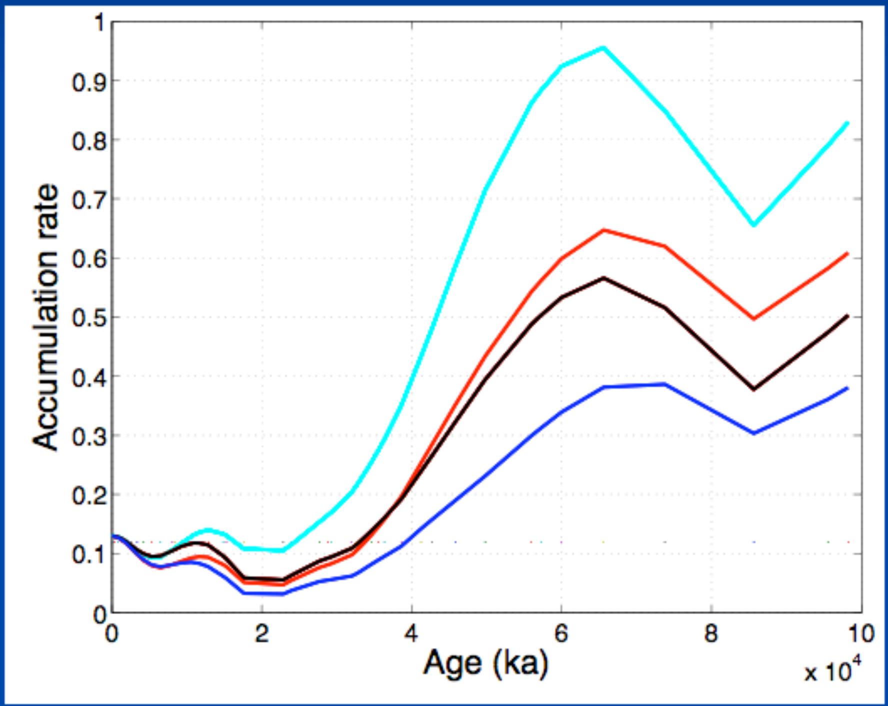
- Paleoclimate studies
- Past ice sheet evolution

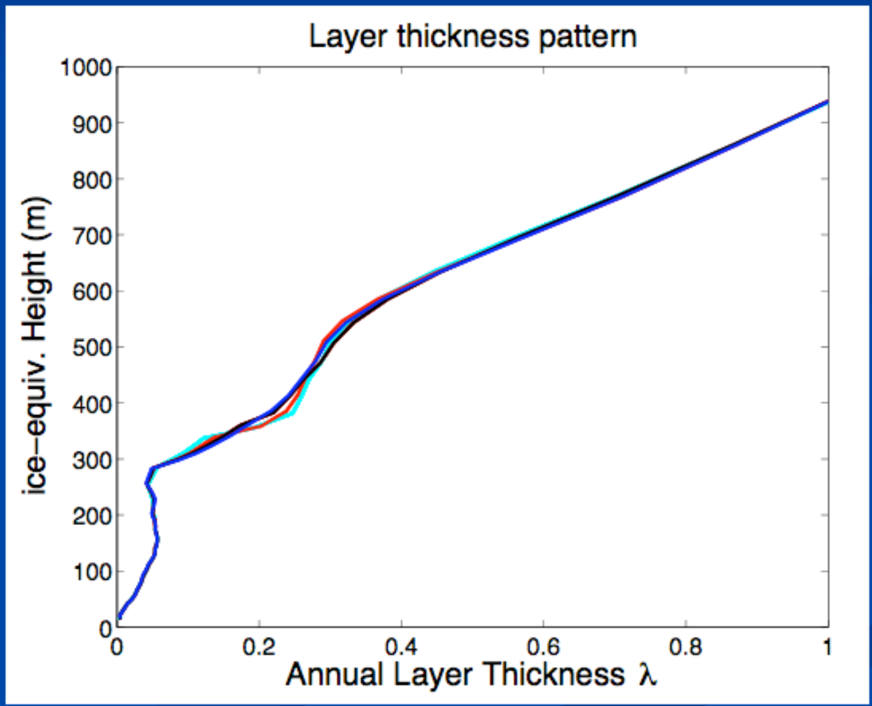
Antarctic ice cores are challenging to date due to low accumulation rates, and may be correlated to Greenland cores through inflection points (control points) in the methane record.

How can we interpolate between sparse control points to obtain a continuous depth-age relationship?

Consider 4 evolution histories for Siple Dome

1. Black: Flank flow (*h*=0.25) and no ice sheet thinning
2. Cyan: Flank flow and 350m of thinning between 15-14ka
3. Blue: Transitional flow (*h* = 0.5) and no thinning
4. Red: Transitional flow and 350m thinning 15-14ka.





Recent divide history from Nereson, et al 1998 and 350m thinning from Price, et al., 2007