Beyond Janus & Epimetheus: Momentum Trading Among Co-Orbiting Satellite Groups

DOUG BALCOM

UNIVERSITY OF WASHINGTON

APPLIED MATHEMATICS

Special Thanks to…

Sasha Malinsky

Janus and Epimetheus: Momentum Trade in Orbit Around Saturn

MS Final Presentation

Winter 2011 MS Symposium

Applied Mathematics Department

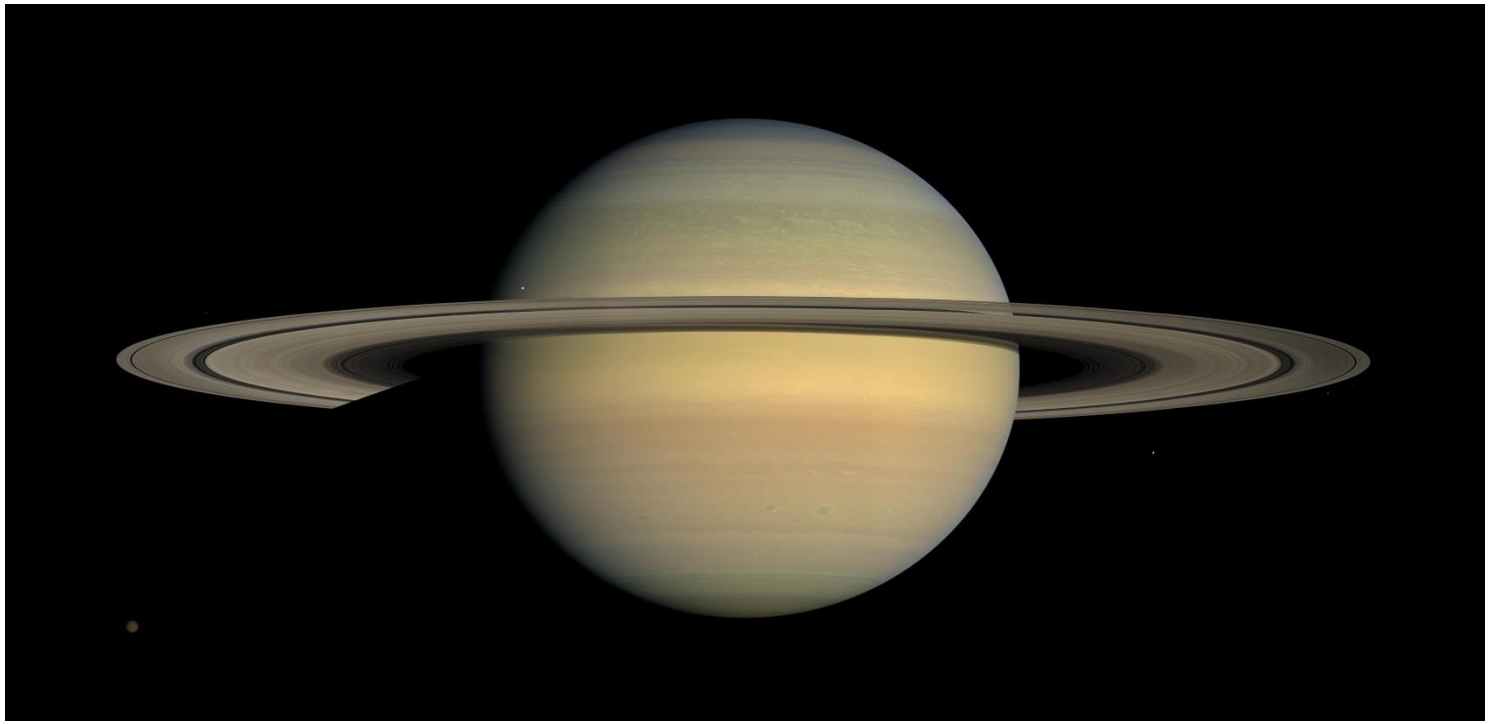
University of Washington

Topics

- The Players
  - The Phenomenon
    - The Questions
      - The Mathematics
        - The Model
          - The Experiments
            - The Conclusions
              - The Next Steps
                - References

The Players:

Saturn



Mass:	5.6846×10^{26} kg
Equatorial radius:	60268 km

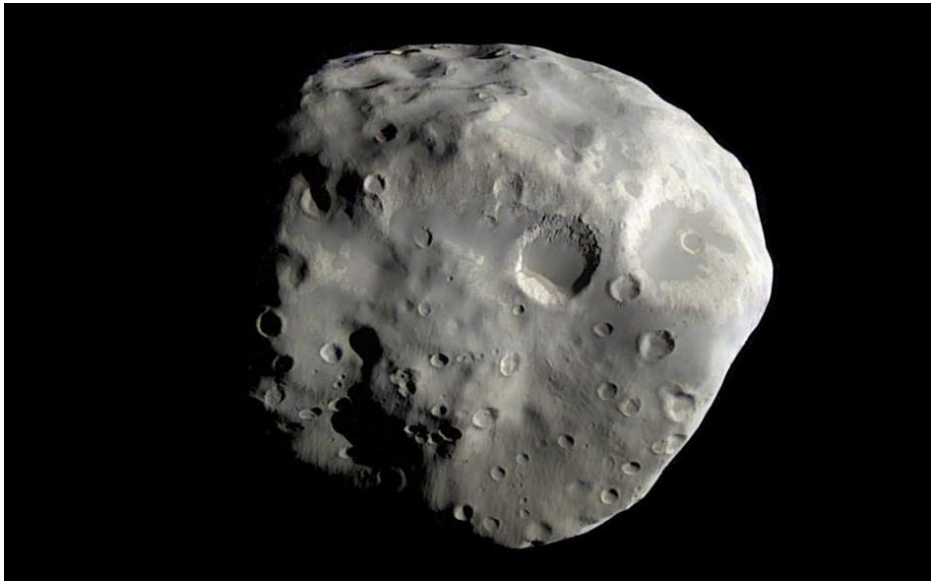
The Players:

Janus

Mass:	1.8975×10^{18} kg
Mean diameter:	179 km
Semi-major axis (outer orbit):	151472 km
Semi-major axis (inner orbit):	151450 km

The Players:

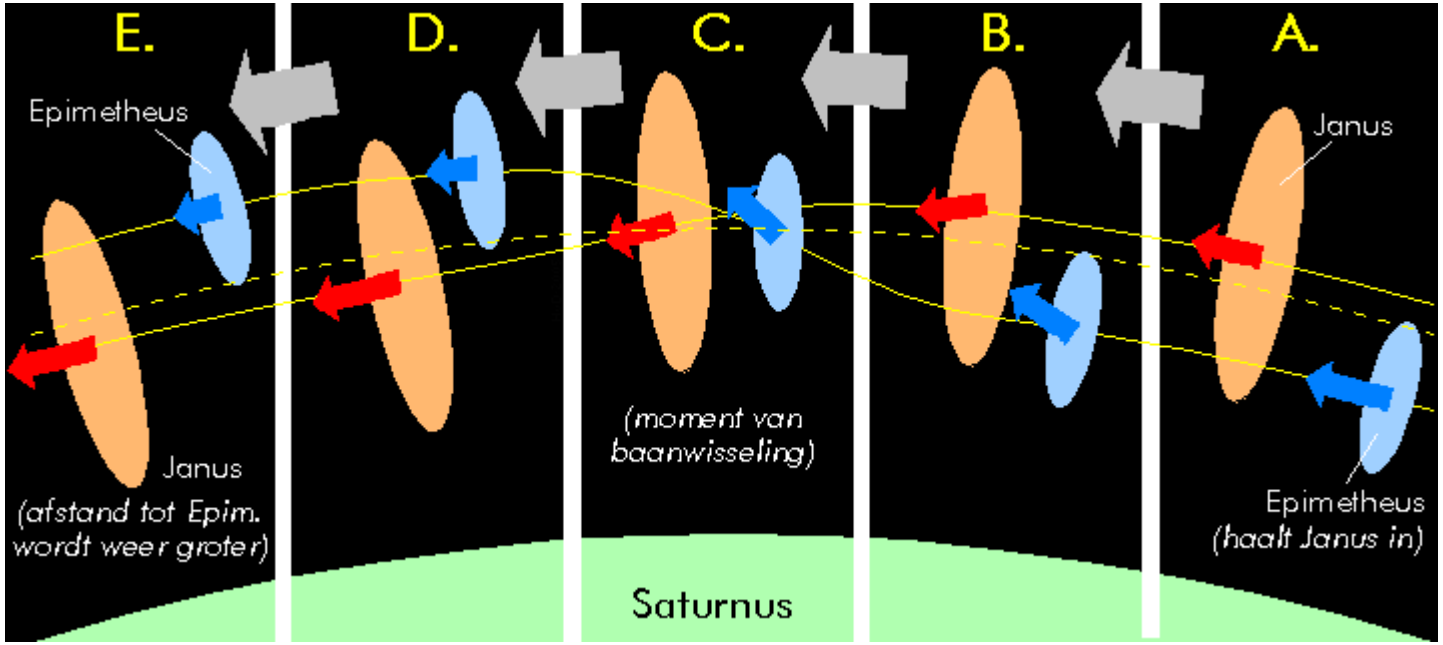
Epimetheus



Mass:	5.266×10^{17} kg
Mean diameter:	113 km
Semi-major axis (outer orbit):	151500 km
Semi-major axis (inner orbit):	151422 km

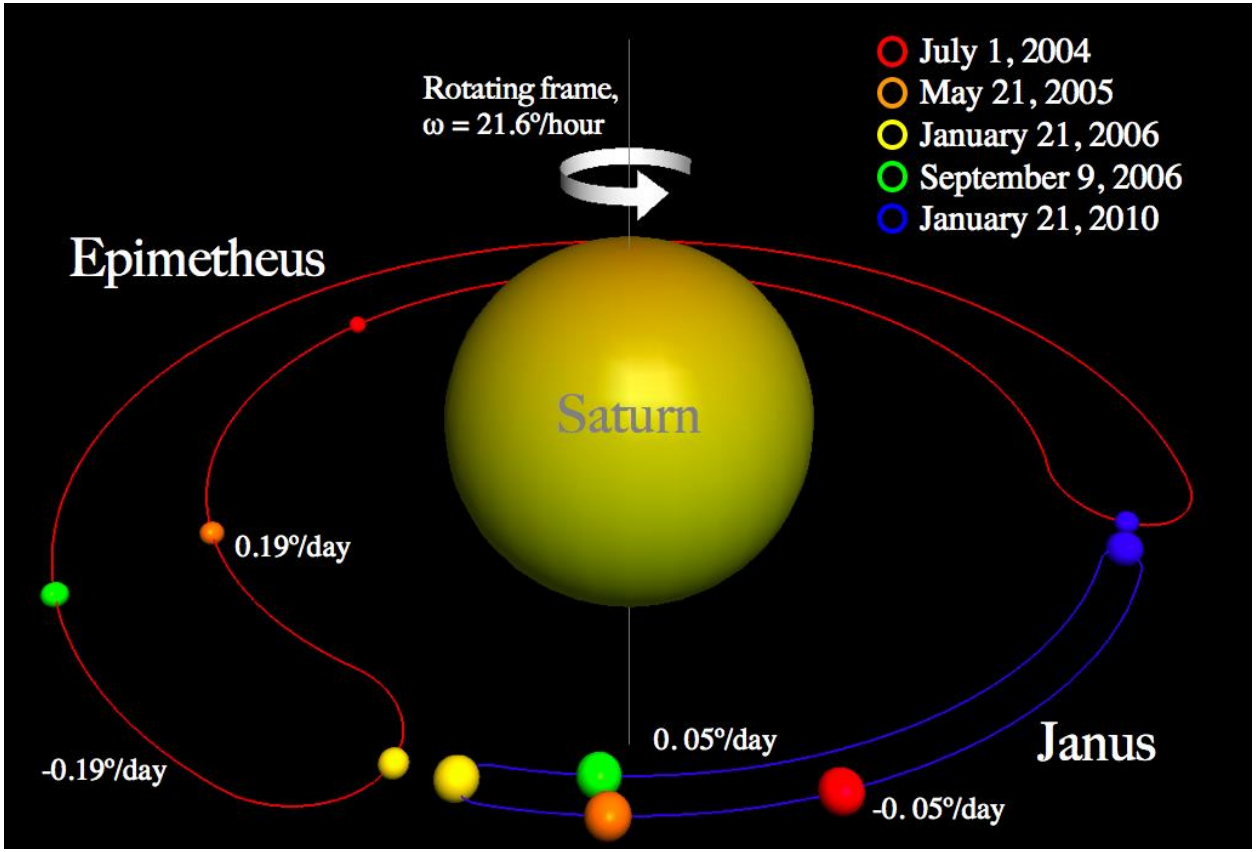
The Phenomenon:

Momentum Exchange



The Phenomenon:

Horseshoe Orbits



The Questions:

Could this work with 3+ moons?

If 3 or more satellite bodies shared an orbit & exchanged momentum in this manner:

- Could such a system remain stable over time? Or would the dynamics cause bodies to escape the momentum exchange pattern?
- How would the relationships between initial orbits, and between masses of the bodies, affect the stability of the system?
- If such a system began with axial or radial symmetries, would these symmetries be preserved over time? How would such symmetries affect the stability of the system?

The Mathematics:

Assumptions

- Each body is a point mass
- Neglect the following:
  - Drag
  - Relativistic effects
  - Gravity of other bodies (besides the ones being modeled)
- Saturn's position fixed as center of our coordinate frame
- Neglect the miniscule perturbations of Saturn's position caused by the satellites

The Mathematics:

Equations of Motion

From Newton's second law of motion:

$$\frac{d^2\vec{r}_J}{dt^2} = -\frac{\mu}{\|\vec{r}_J\|^3}\vec{r}_J + \frac{Gm_E}{\|\vec{r}_{je}\|^3}\vec{r}_{je}$$

$$\frac{d^2\vec{r}_E}{dt^2} = -\frac{\mu}{\|\vec{r}_E\|^3}\vec{r}_E + \frac{Gm_J}{\|\vec{r}_{ej}\|^3}\vec{r}_{ej}$$

- $\vec{r}_J$ & $\vec{r}_E$ = spatial vectors from Saturn to each satellite
- $\vec{r}_{je}$ & $\vec{r}_{ej}$ = spatial vectors from Janus to Epimetheus and vice versa
- m_J & m_E = masses of the two satellites
- G = gravitational constant $\approx 6.67428 \times 10^{-11} \frac{m^3}{kg \cdot sec^2}$
- $\mu = GM_s$, where M_s is the mass of Saturn.

The Mathematics:

Conversion to 1st-order ODEs

$$\frac{d\vec{r}_J}{dt} = \vec{v}_J$$

$$\frac{d\vec{v}_J}{dt} = -\frac{\mu}{\|\vec{r}_J\|^3}\vec{r}_J + \frac{Gm_E}{\|\vec{r}_{je}\|^3}\vec{r}_{je}$$

$$\frac{d\vec{r}_E}{dt} = \vec{v}_E$$

$$\frac{d\vec{v}_E}{dt} = -\frac{\mu}{\|\vec{r}_E\|^3}\vec{r}_E + \frac{Gm_J}{\|\vec{r}_{ej}\|^3}\vec{r}_{ej}$$

The Mathematics:

Extension to n satellites

For each satellite body b_j $(j = 1, \ldots, n)$:

$$\frac{d\vec{r}_{b_j}}{dt} = \vec{v}_{b_j}$$

$$\frac{d\vec{v}_{b_j}}{dt} = -\frac{\mu}{\left\|\vec{r}_{b_j}\right\|^3}\vec{r}_{b_j} + \sum_{\substack{k=1,\\ k\neq j}}^{n} \frac{Gm_{b_k}}{\left\|\vec{r}_{b_j b_k}\right\|^3}\vec{r}_{b_j b_k}$$

The Mathematics:

2-body Analog

To evaluate solver accuracy, consider the case with 1 satellite:

$$\frac{d\vec{r}}{dt} = \vec{v}$$

$$\frac{d\vec{v}}{dt} = -\frac{\mu}{\|\vec{r}\|^3}\vec{r}$$

The Mathematics:

Keplerian 2-body Solution

Kepler's analytical solution to the 2-body case:

Path: $$r(\theta) = \frac{a(1-e^2)}{1+e\cos\theta}$$

Speed: $$v = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}$$

Period: $$T = \sqrt{\frac{4\pi^2 a^3}{\mu}}$$

- r = distance from Saturn to satellite
- θ = angle from periapsis direction to the Saturn-satellite vector
- a = semi-major axis of the orbital ellipse
- e = eccentricity of the orbital ellipse
- v = magnitude of the satellite's velocity
- T = orbital period

The Model:
Simplifying Assumptions

- All orbits are **coplanar** (zero inclination)

- All orbits start out **circular** (zero initial eccentricity)

Both of the above are *close* to reality for Janus & Epimetheus

The Model:

Solver Evaluation Phase (1 Satellite)

Approach:

- Janus only
- Initial conditions:
  - Position: $\theta = 0; r = r_J$ (standard low-orbit Janus altitude)
  - Velocity: Keplerian v_J, angular direction $\frac{\pi}{2}$ (orthogonal to radial vector)
- Integrate forward an integer number of Keplerian periods
- Evaluate errors in final position (r_f, θ_f) and velocity magnitude v_f:
  - Final angular error = θ_f (exact: $\theta_f = 0$)
  - Final radial error = $r_f - r_J$ (exact: $r_f = r_J$)
  - Maximum radial error = maximum $|r - r_J|$ (exact: $r = r_J$ throughout)
  - Final velocity magnitude error = $v_f - v_J$ (exact: $v_f = v_J$)

The Model:

Solver Evaluation Results

- Evaluated ode23, ode45, ode113, ode15s
- Relative tolerance (RelTol) setting: minimum allowed (2.3×10^{-14})
- Results:
  - ode23: very good accuracy, way too slow
  - ode15s: not accurate enough
  - ode45: very good accuracy, too slow
  - **ode113**: good accuracy, fast enough ← *The winner!*
- By tweaking AbsTol & MaxStep settings as well, achieved the following accuracy in 1000-year integration (run time < 3 hours):
  - Radial errors: Final error < 0.002 m; maximum error < 0.014 m (!)
  - Final angular error: < 6.5×10^{-5} radians
  - Final overall position error: < 1 km (nearly all from angular error)
  - Final velocity magnitude error: ~ 1 m/sec (≈ 0.0065 %)

The Model:

Experiment Phase (2+ Satellites)

Approach:

- Multiple satellites (masses of Janus, Epimetheus & others)
- Initial conditions:
  - Radial positions r: standard altitudes for Janus & Epimetheus (or other experimental ones); corresponding altitudes for other satellites
  - Angular positions θ: distributed around Saturn
  - Velocities: determined by Keplerian equations (for circular orbit case)
- Integrate forward $\sim$ 1000 years (using ode113, with tolerance & step size settings determined during Solver Evaluation phase)
- Process & plot results

The Experiments:

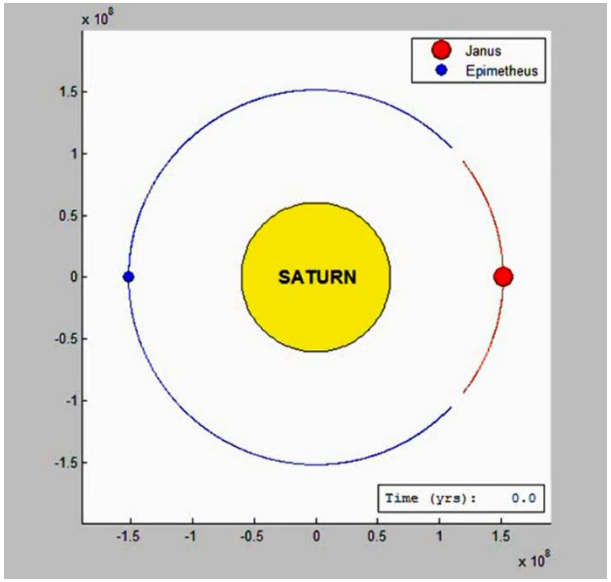
Data Analysis Notes

- When plotting orbital distance r, orbital eccentricities show up as high-frequency oscillations with a period $\sim$ 16.7 hours (the mean orbital period in the Janus-Epimetheus neighborhood)
- To cleanly analyze changes in r, I employed a 4^{th}-order Butterworth filter with a cutoff frequency corresponding to a period of 2×16.7 hrs. This produces:
  - High-pass dataset consisting of eccentricity-based oscillations about a local mean
  - Low-pass dataset showing longer period variations without the eccentricity "noise"
- Fourier analysis on these datasets yields power spectra & autocorrelations to help discern dominant cycles at these scales
- Local min-max extraction: helps expose trends in the limits & amplitudes of these cycles

The Experiments:

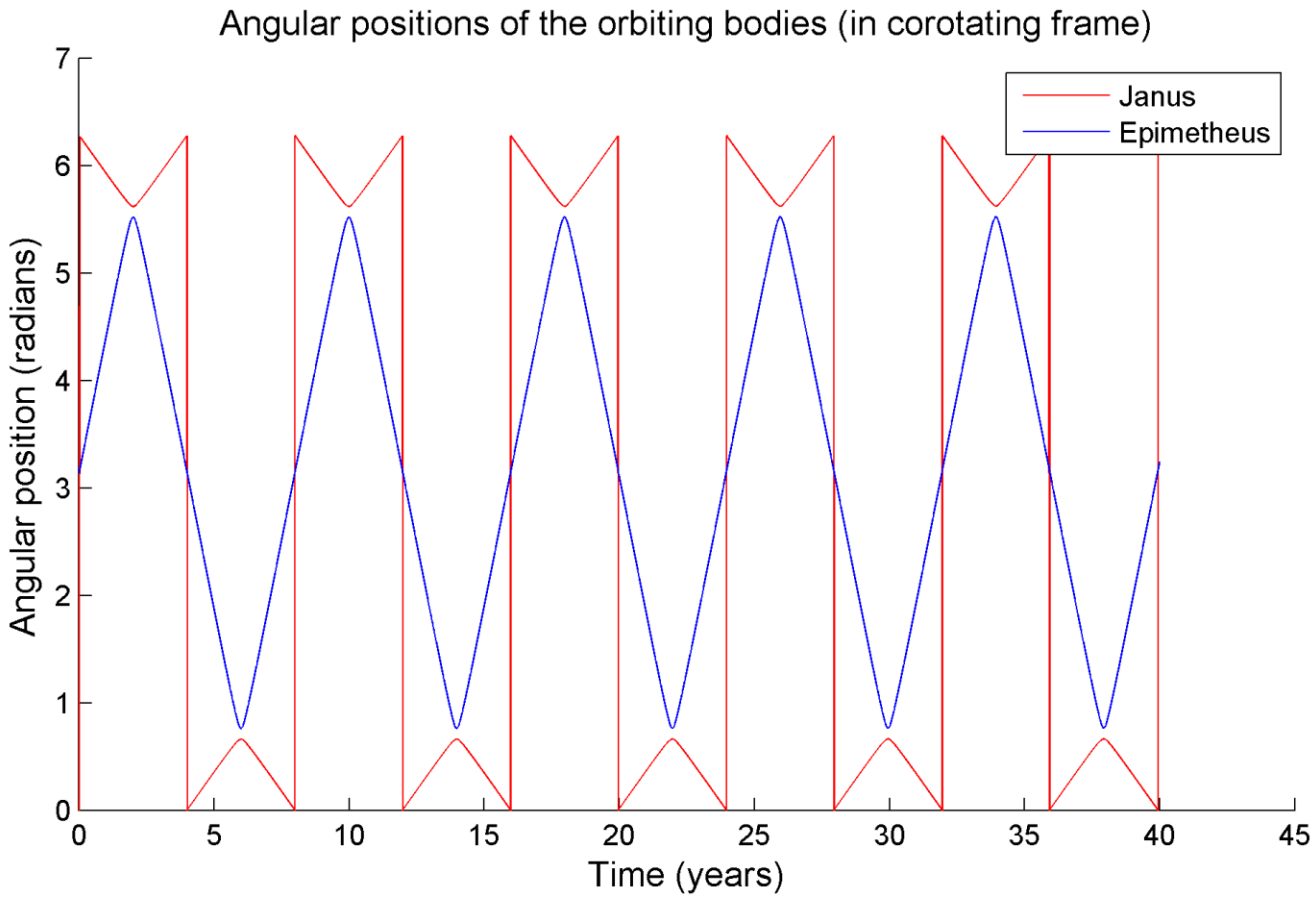
Janus & Epimetheus

- Goal: Verify the model by recreating the actual Janus-Epimetheus system
- Initial conditions:
  - Janus in low orbit
  - Epimetheus in high orbit
  - Angular spacing: π radians
- Results:
  - One repeated momentum exchange pattern with an 8-year cycle (as expected), regular as clockwork
  - Exceedingly stable



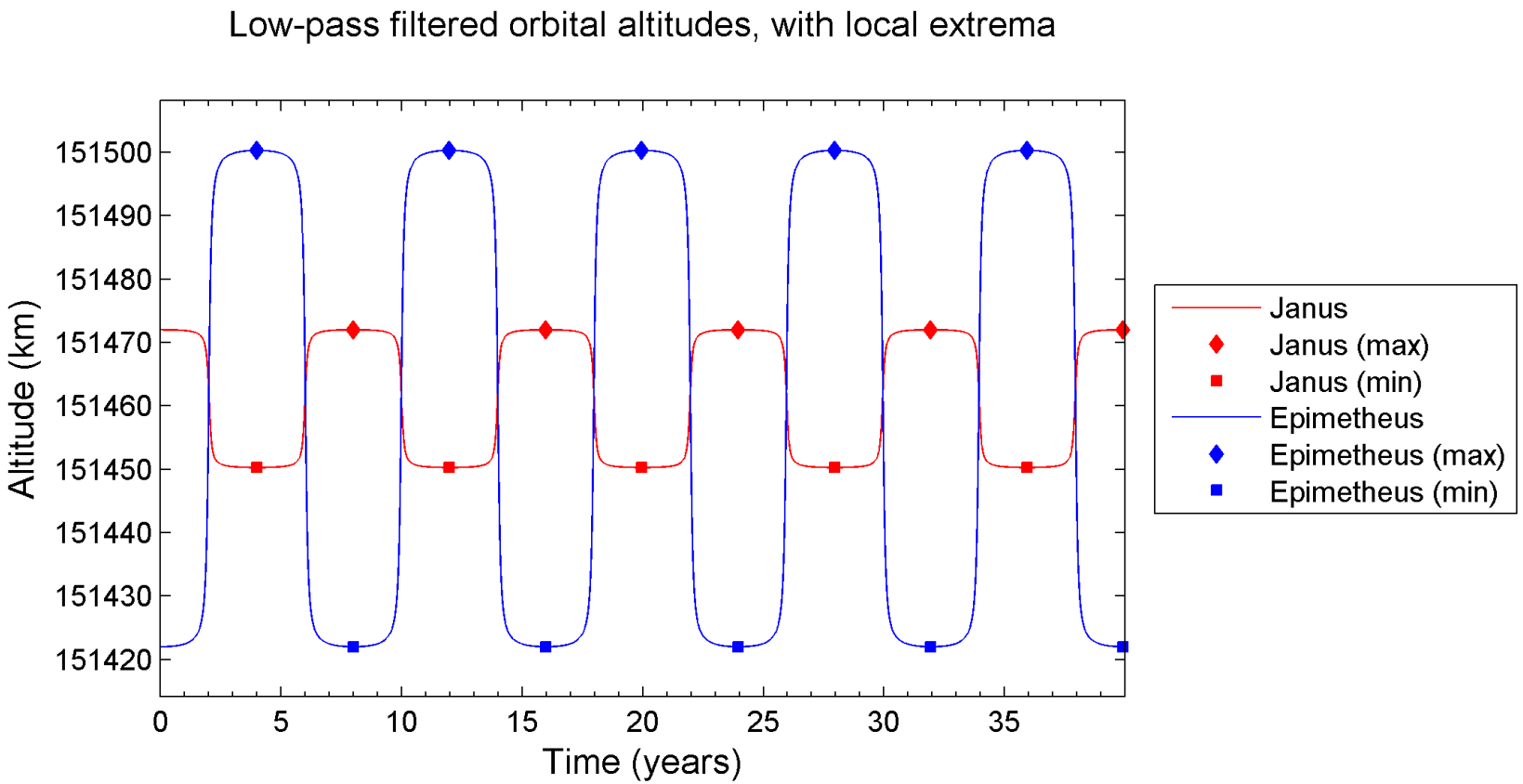
The Experiments:

Janus & Epimetheus



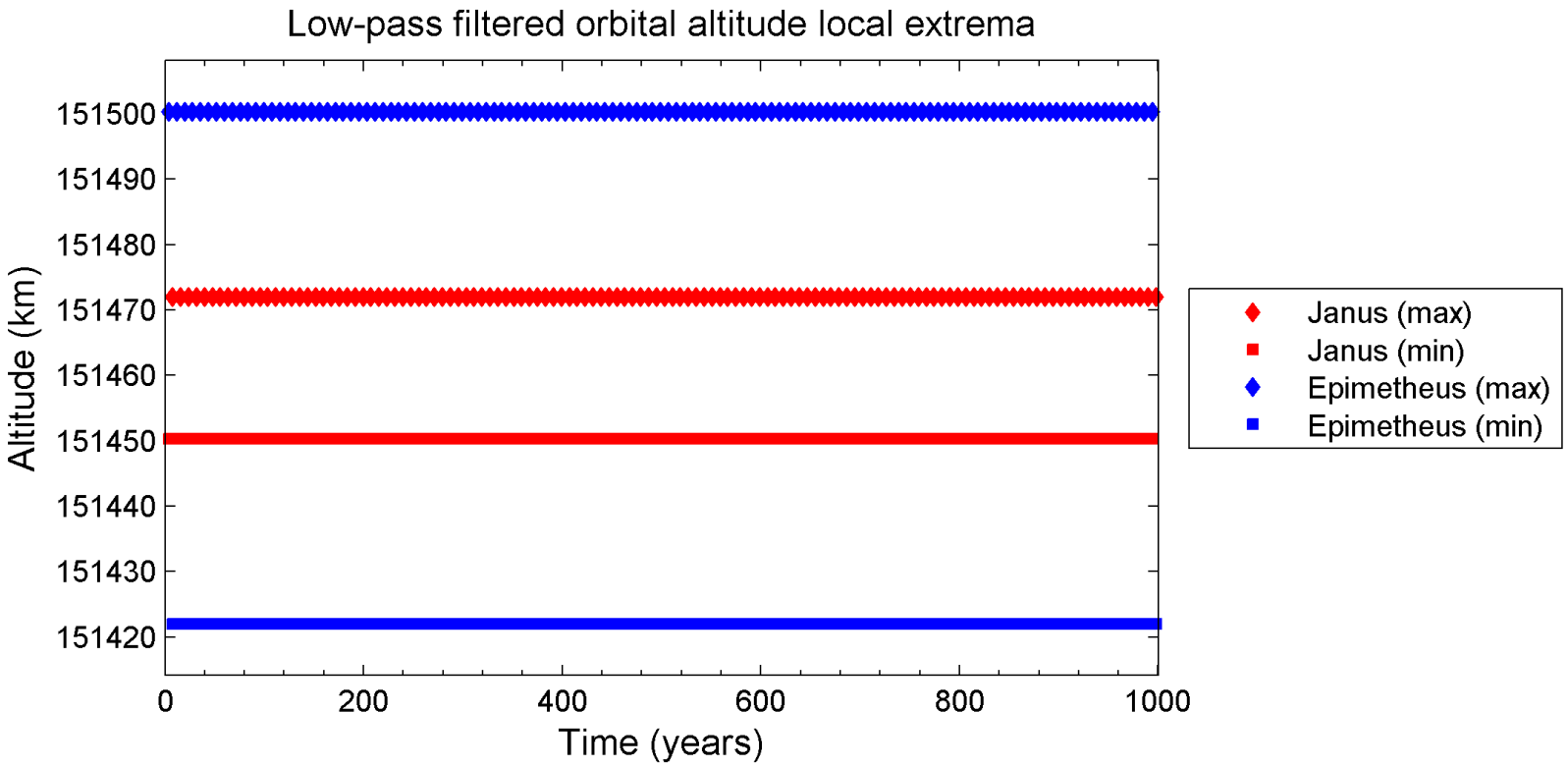
The Experiments:

Janus & Epimetheus



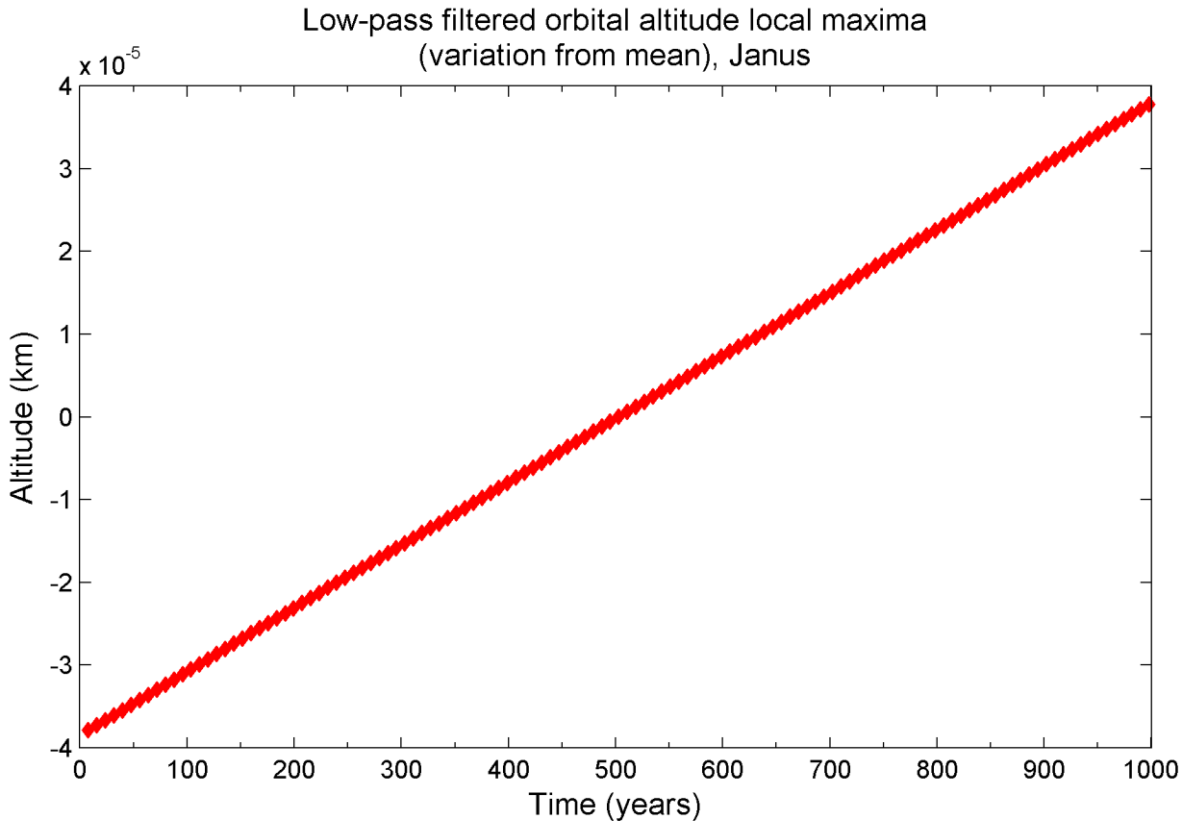
The Experiments:

Janus & Epimetheus



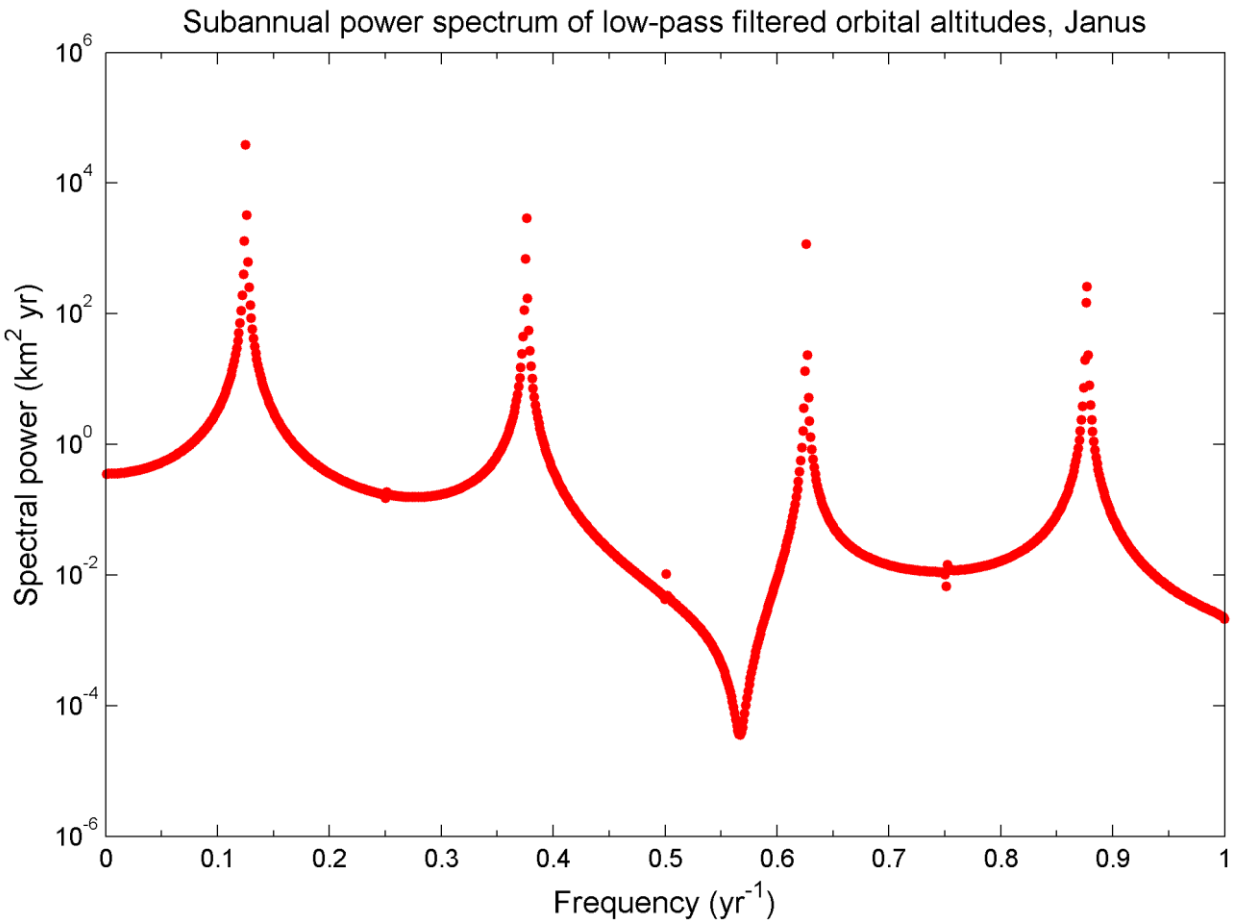
The Experiments:

Janus & Epimetheus



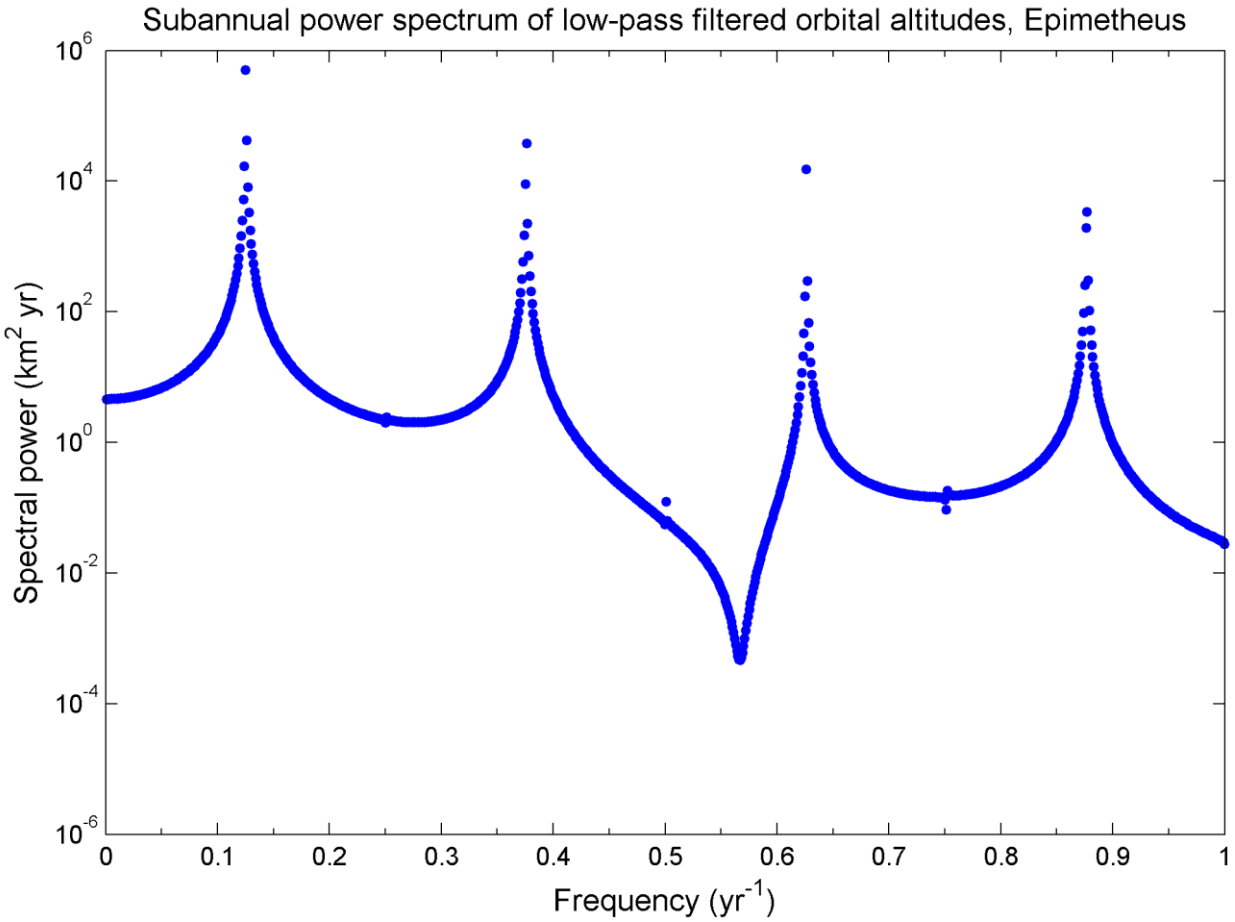
The Experiments:

Janus & Epimetheus



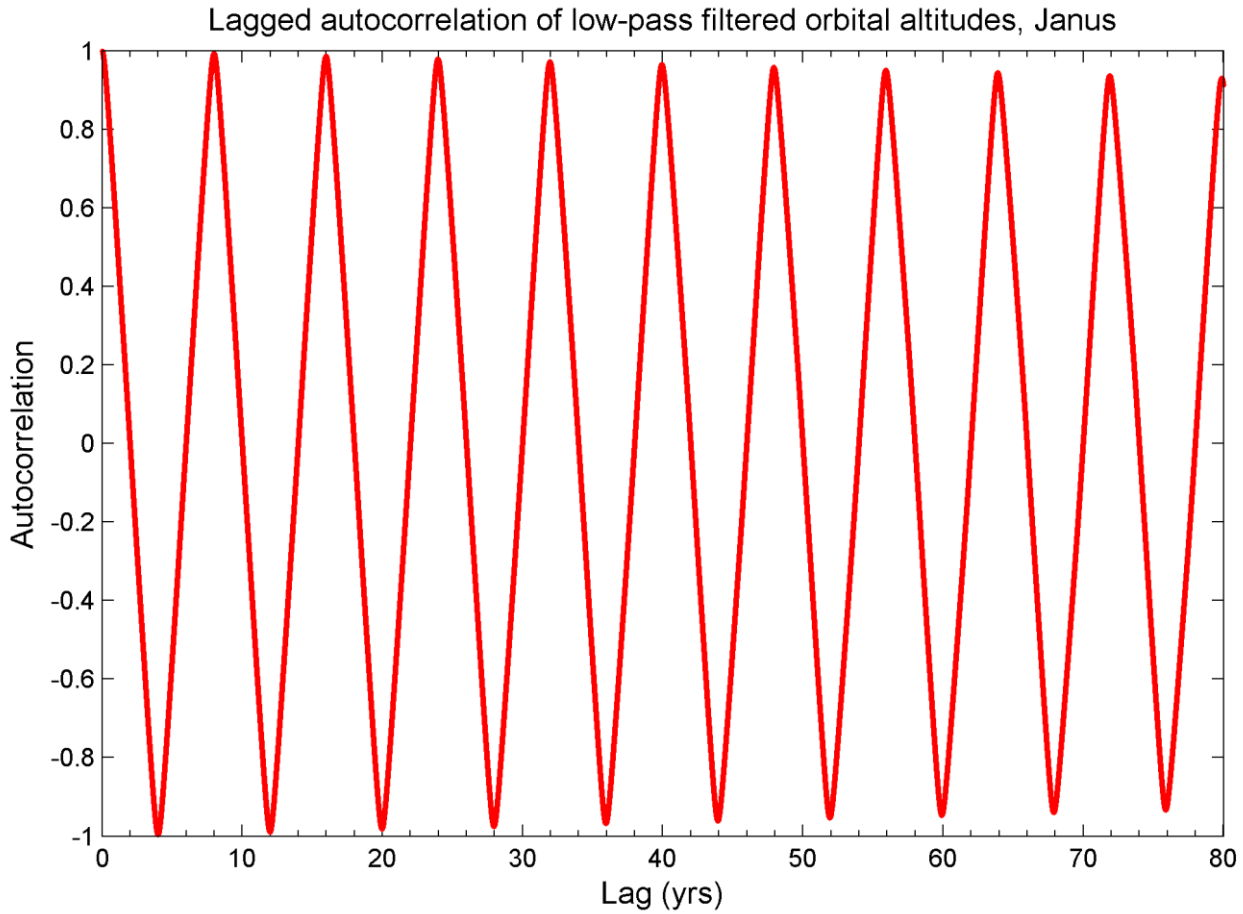
The Experiments:

Janus & Epimetheus



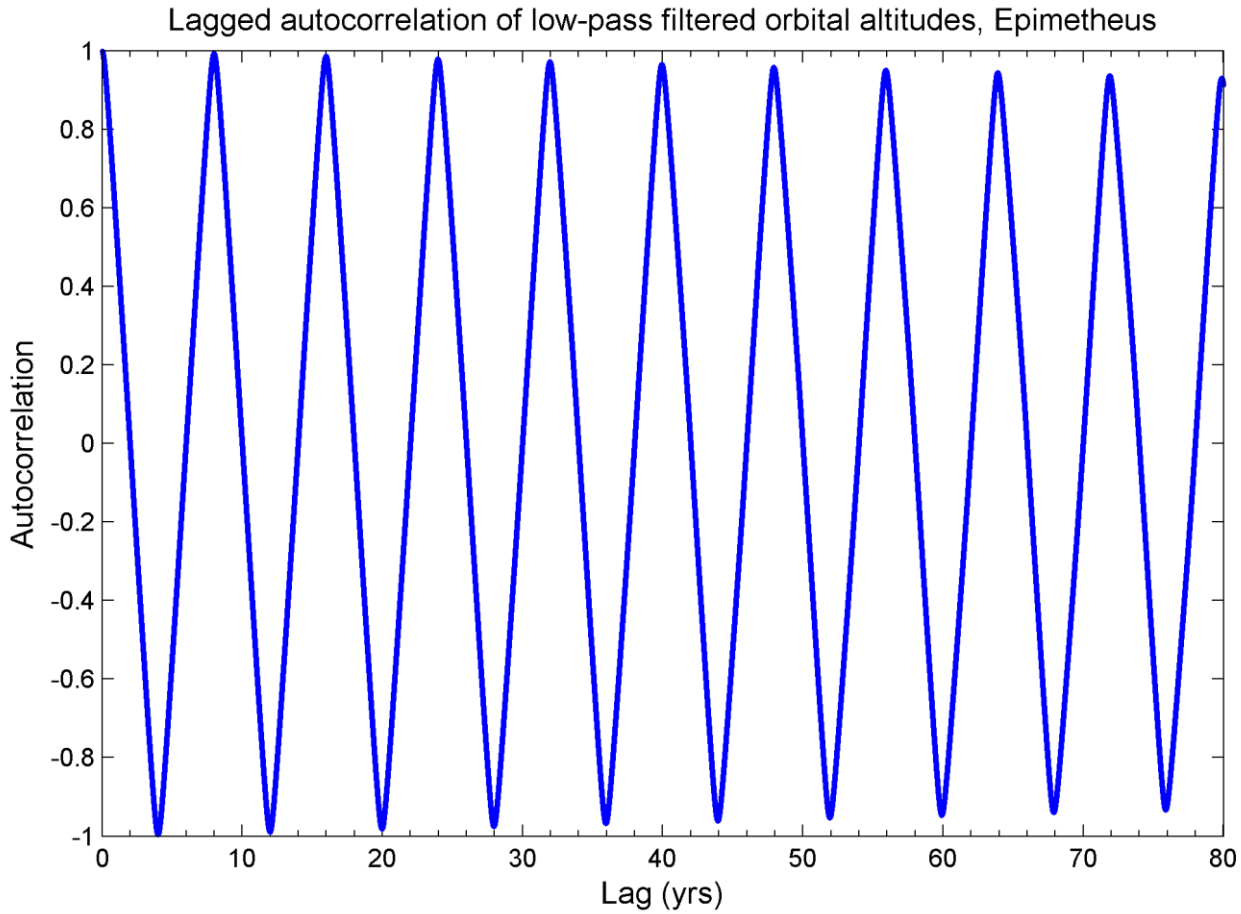
The Experiments:

Janus & Epimetheus



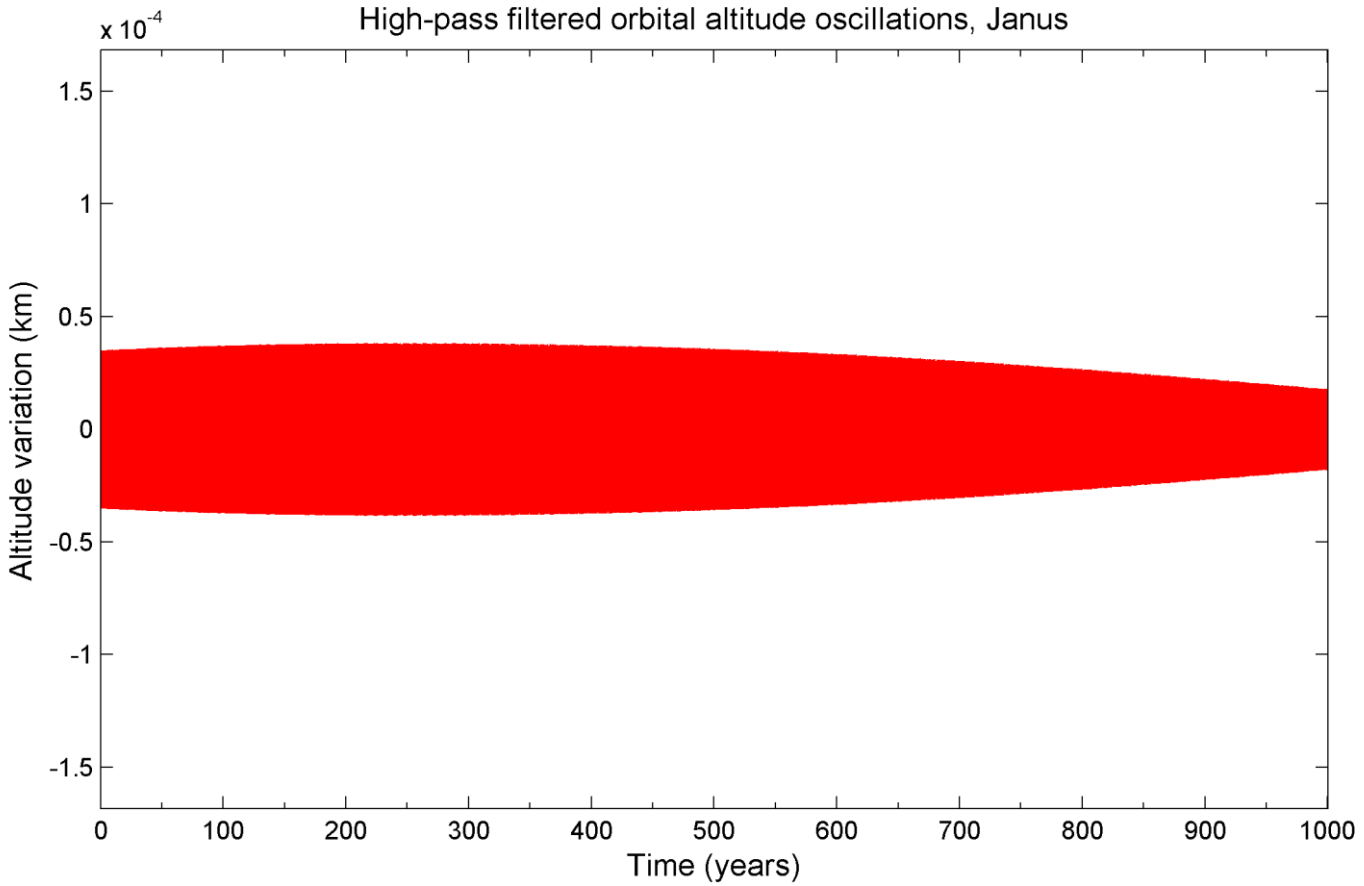
The Experiments:

Janus & Epimetheus



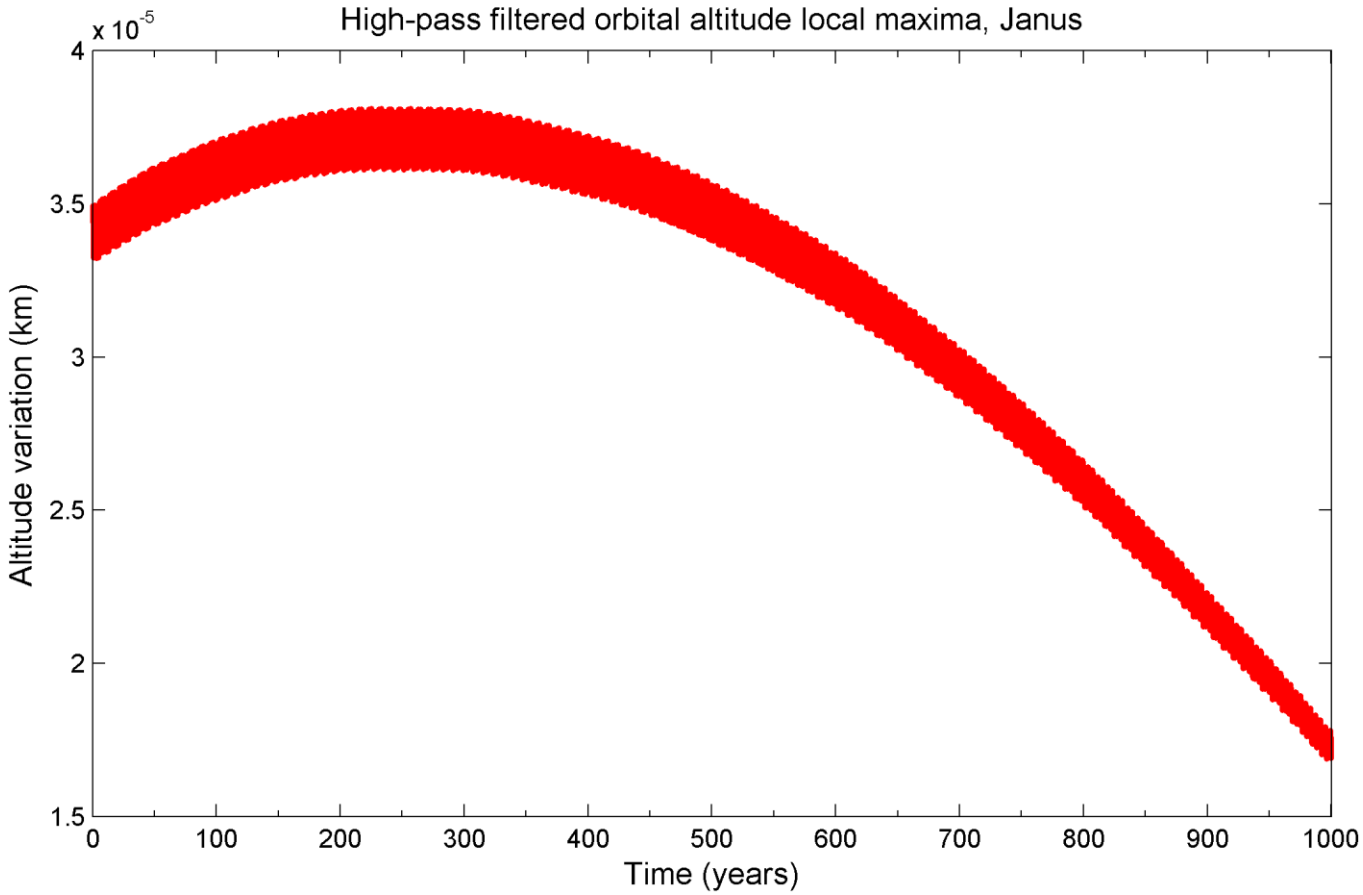
The Experiments:

Janus & Epimetheus



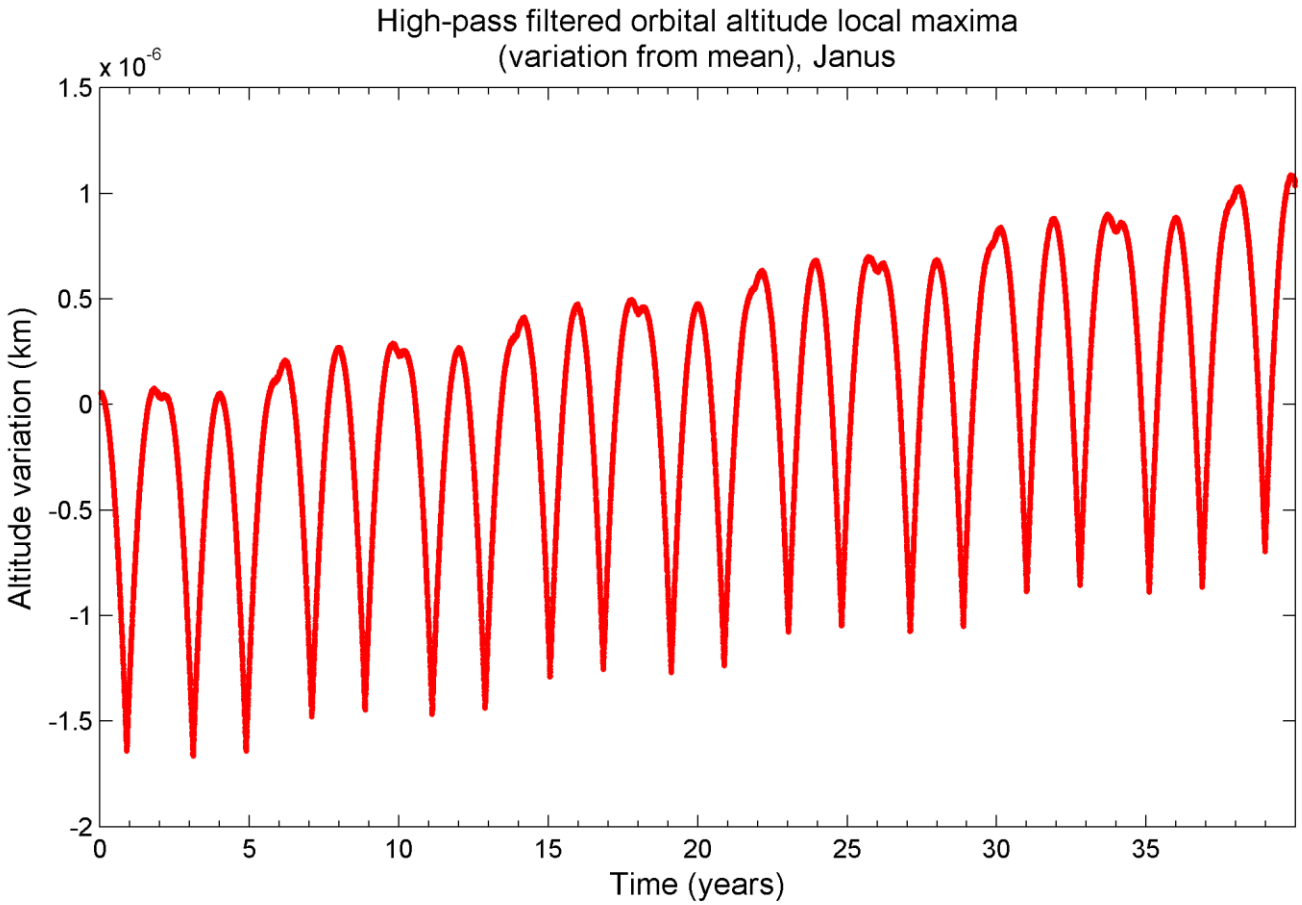
The Experiments:

Janus & Epimetheus



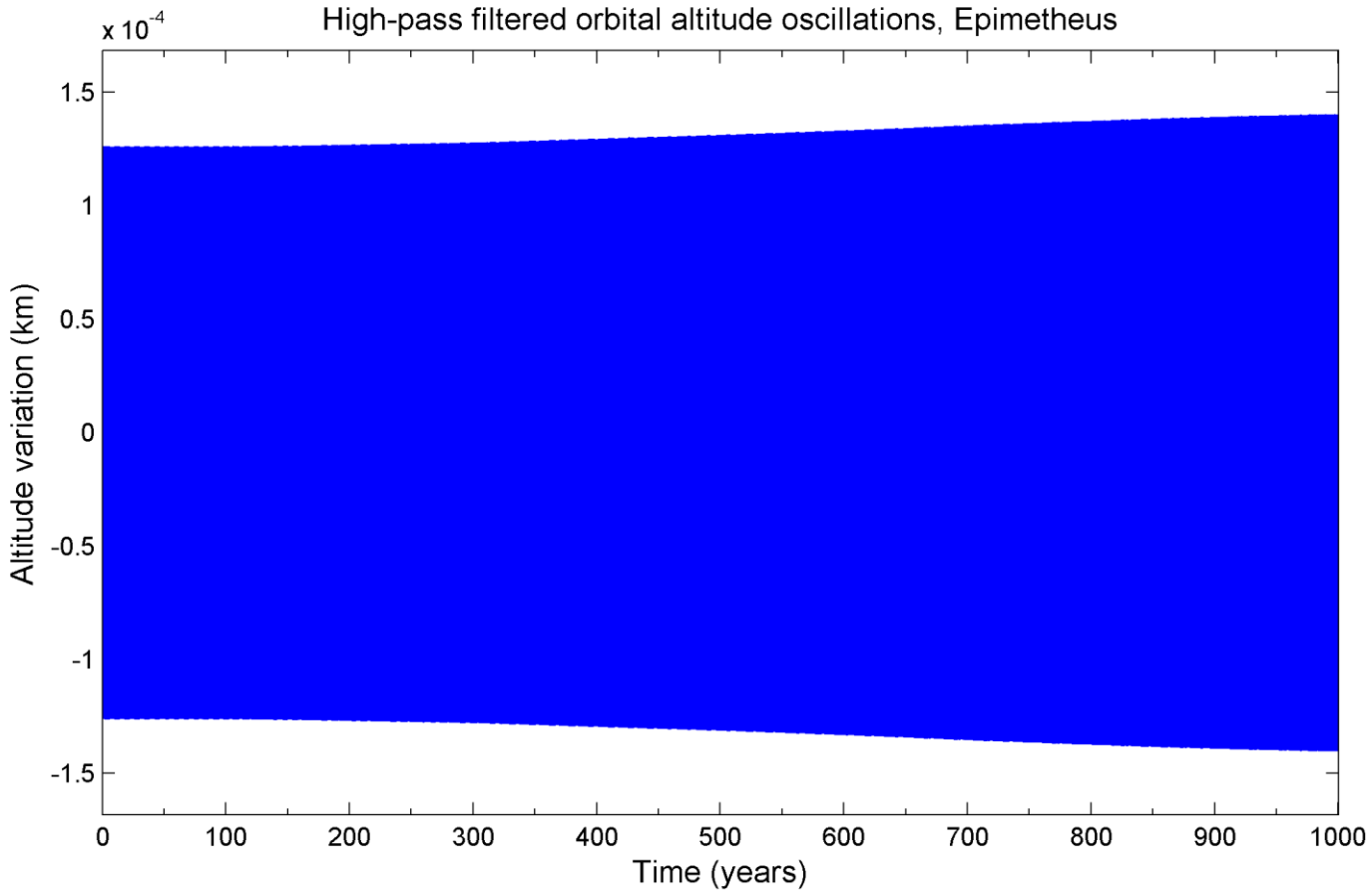
The Experiments:

Janus & Epimetheus



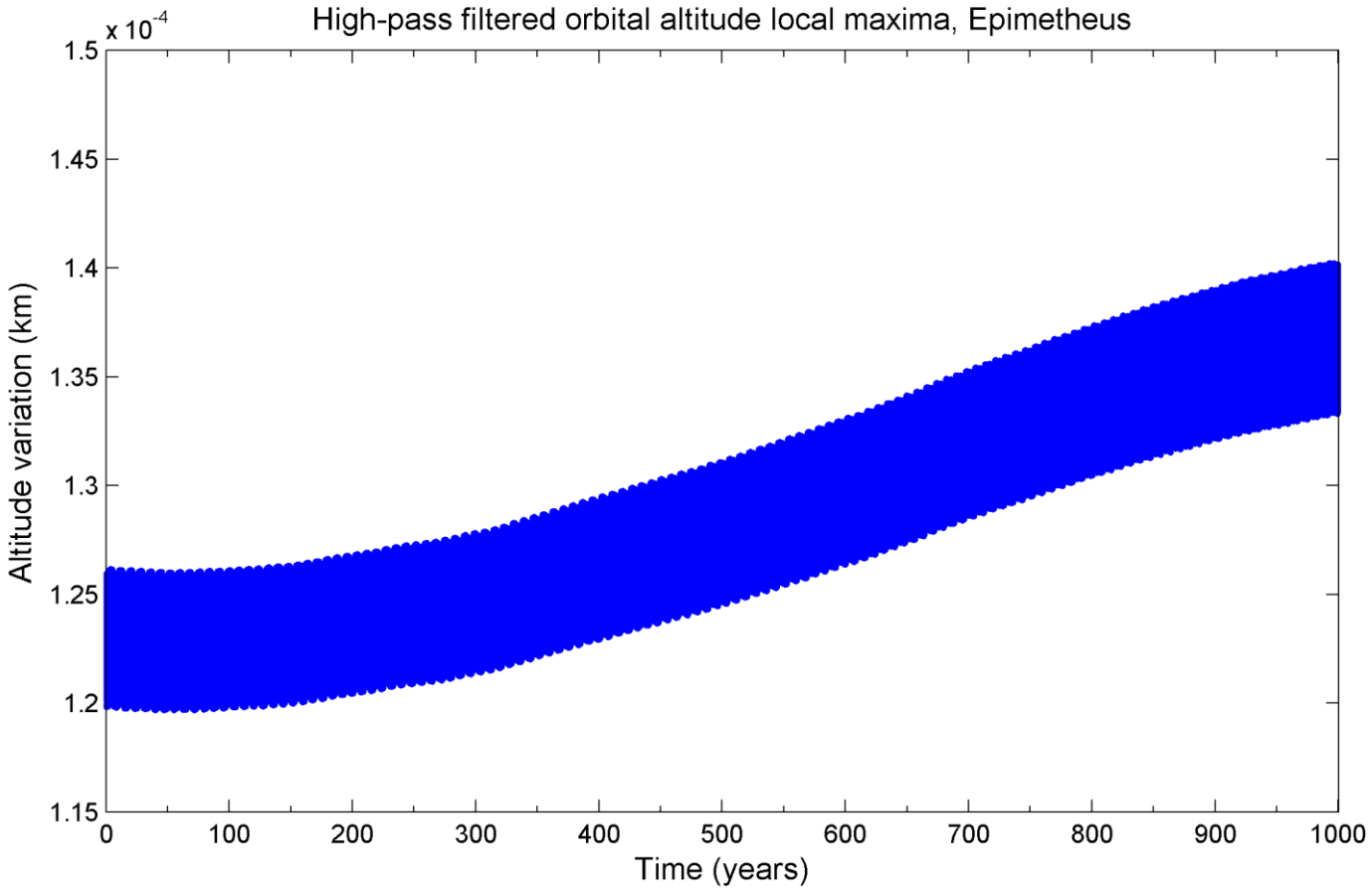
The Experiments:

Janus & Epimetheus



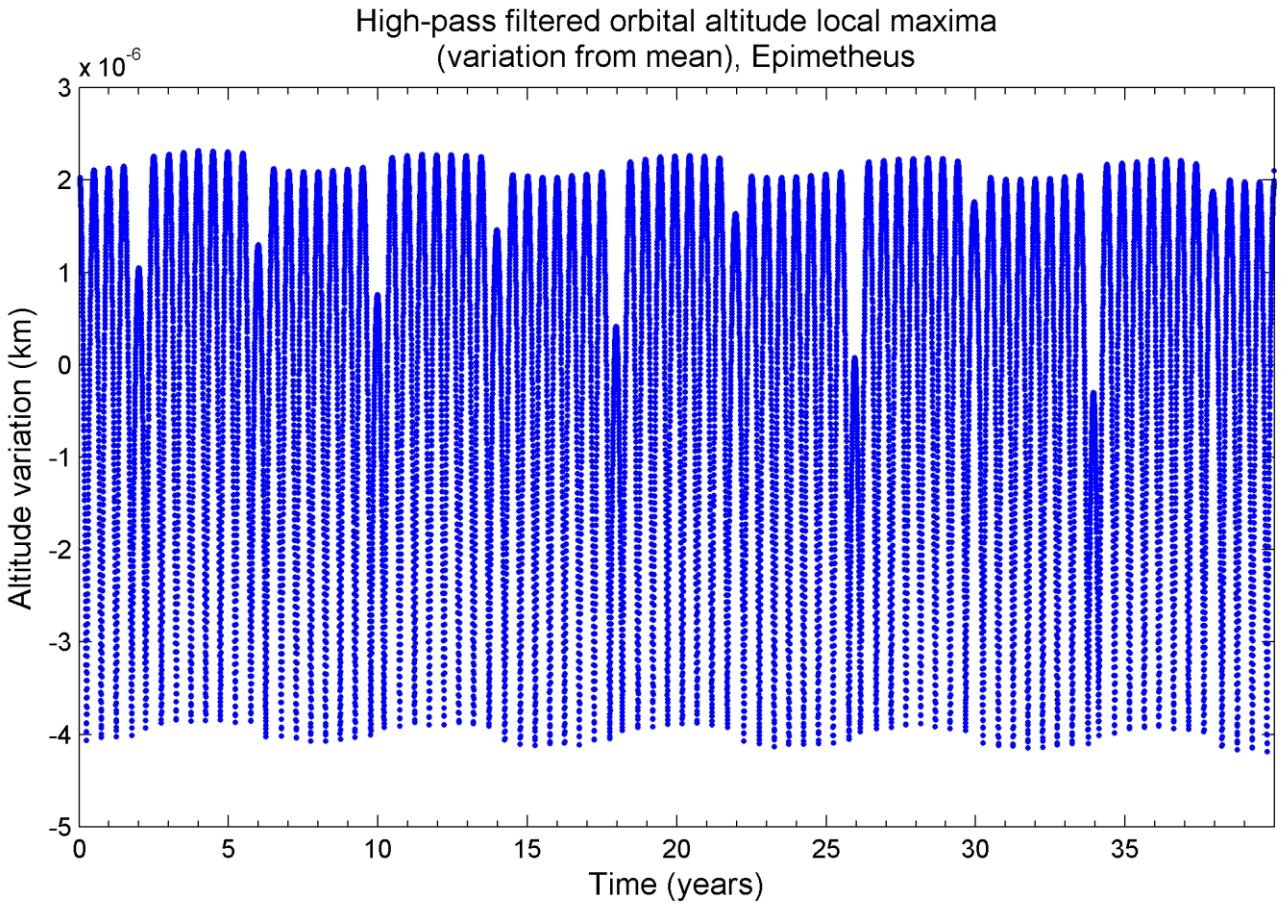
The Experiments:

Janus & Epimetheus



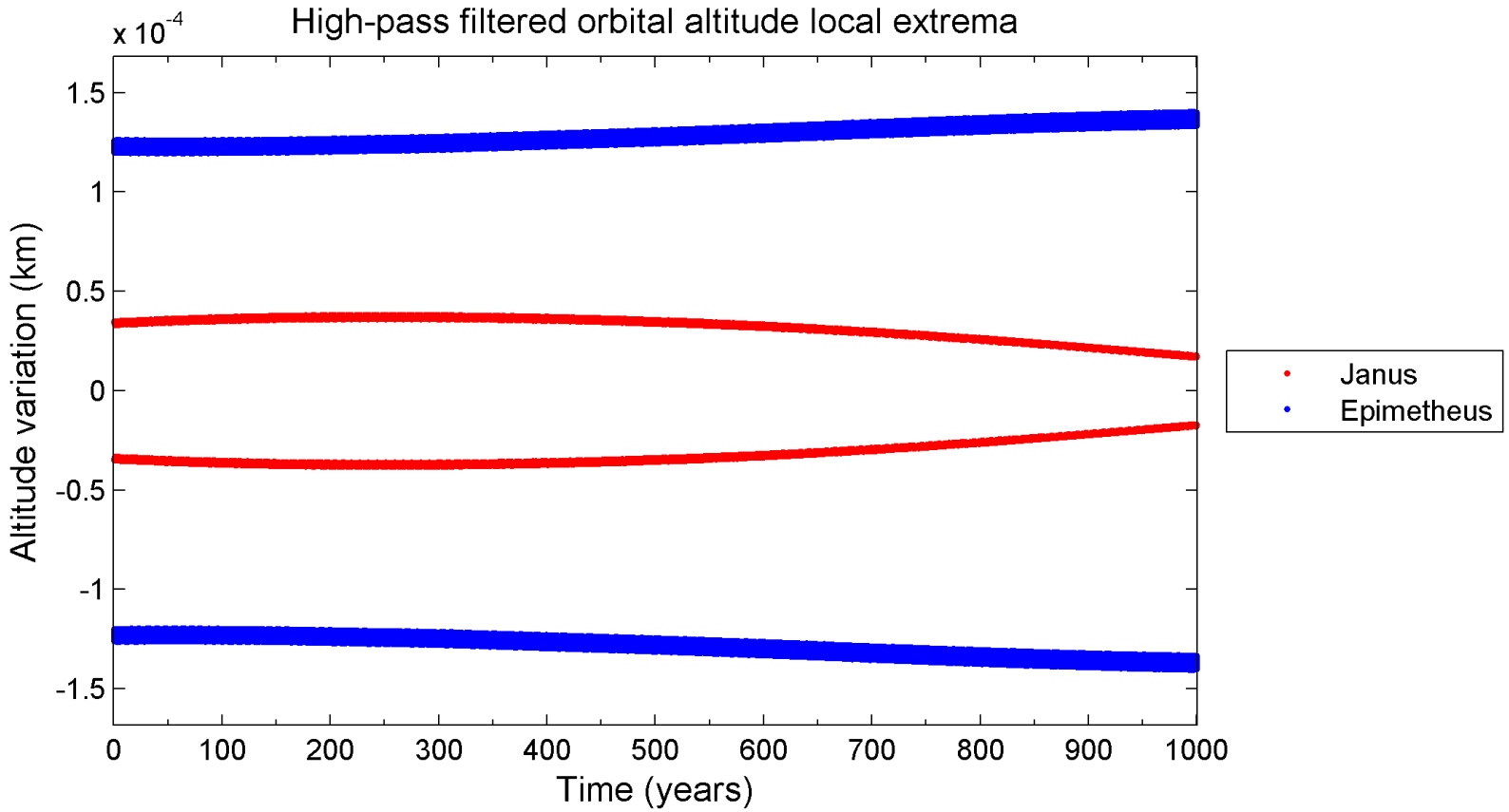
The Experiments:

Janus & Epimetheus



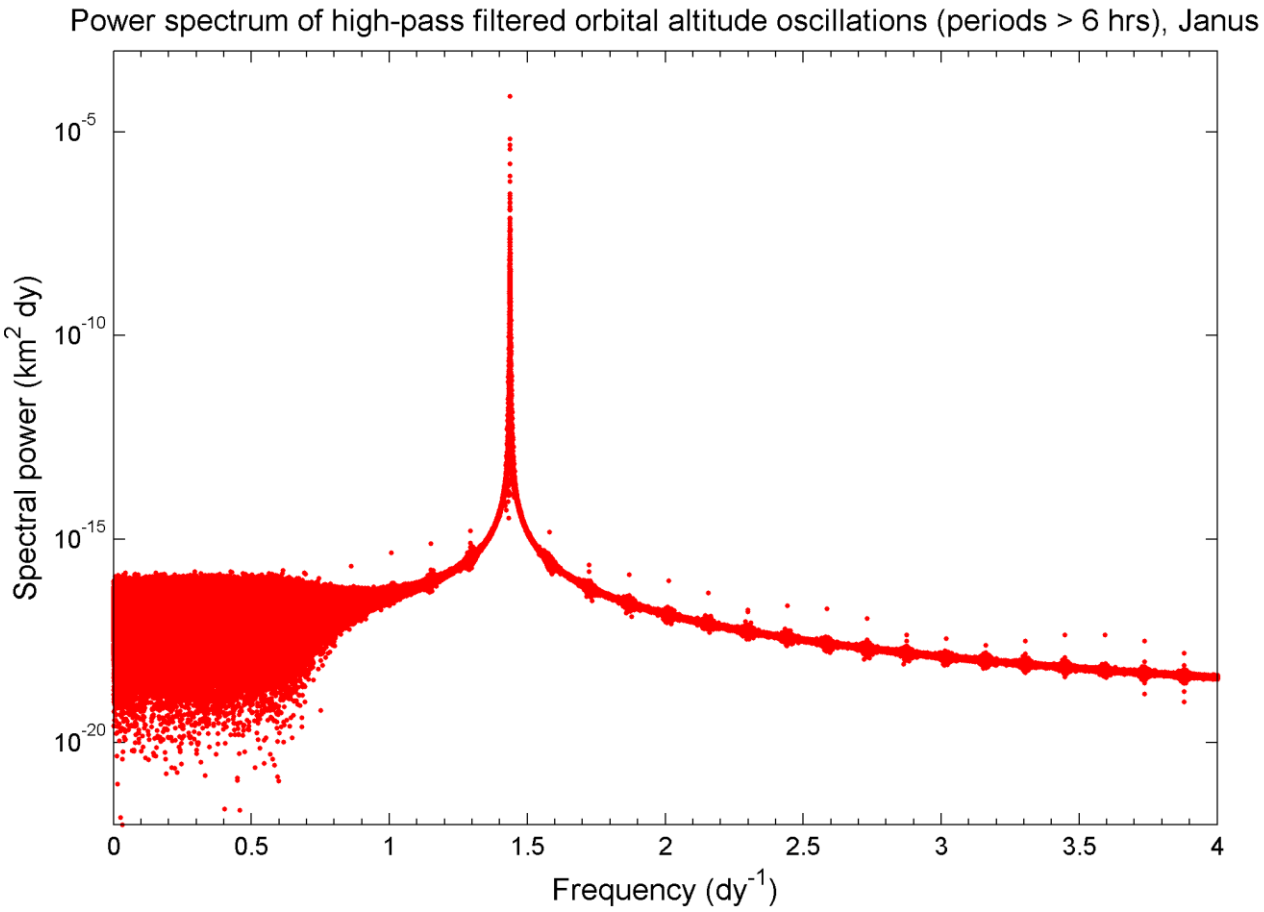
The Experiments:

Janus & Epimetheus



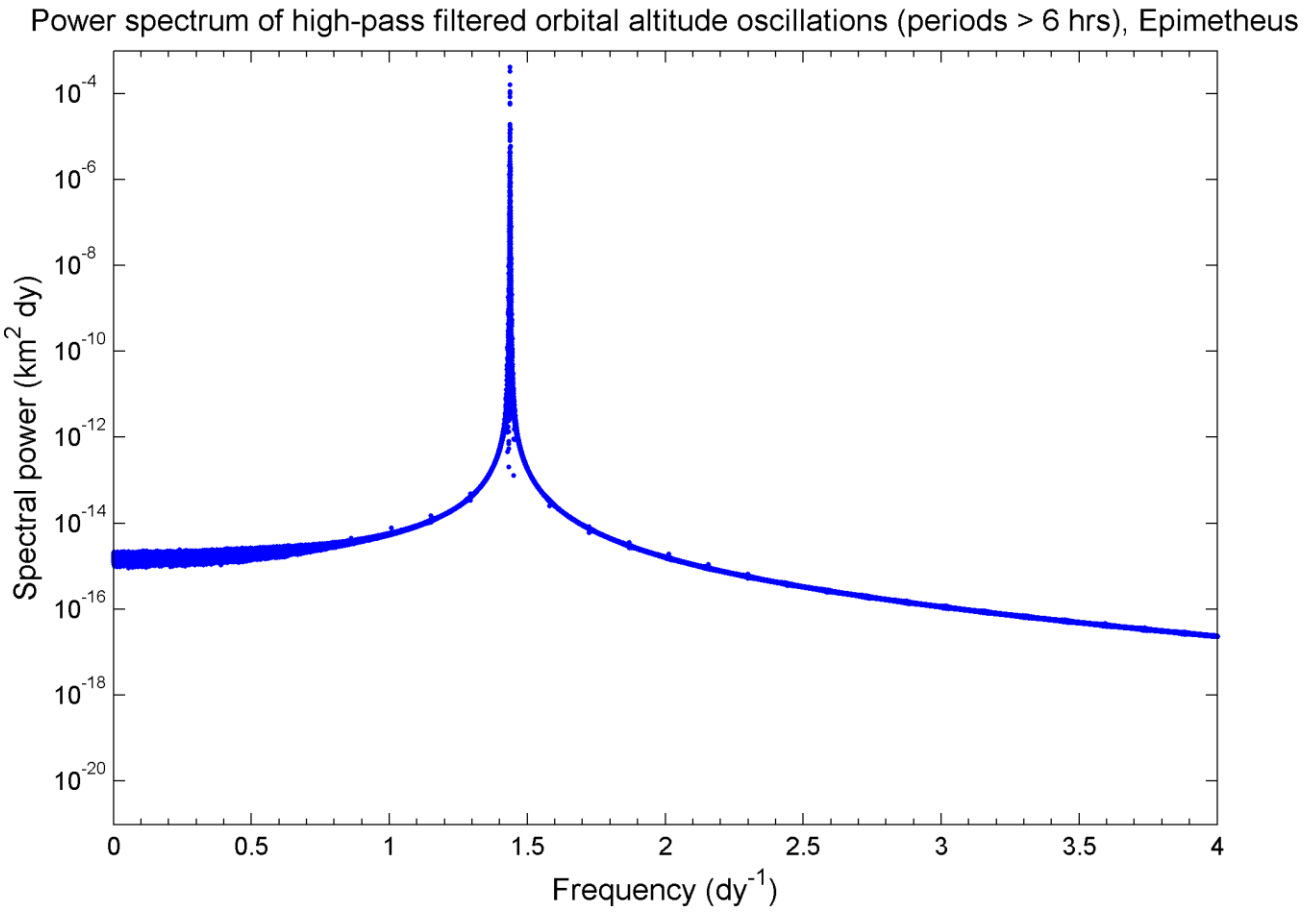
The Experiments:

Janus & Epimetheus



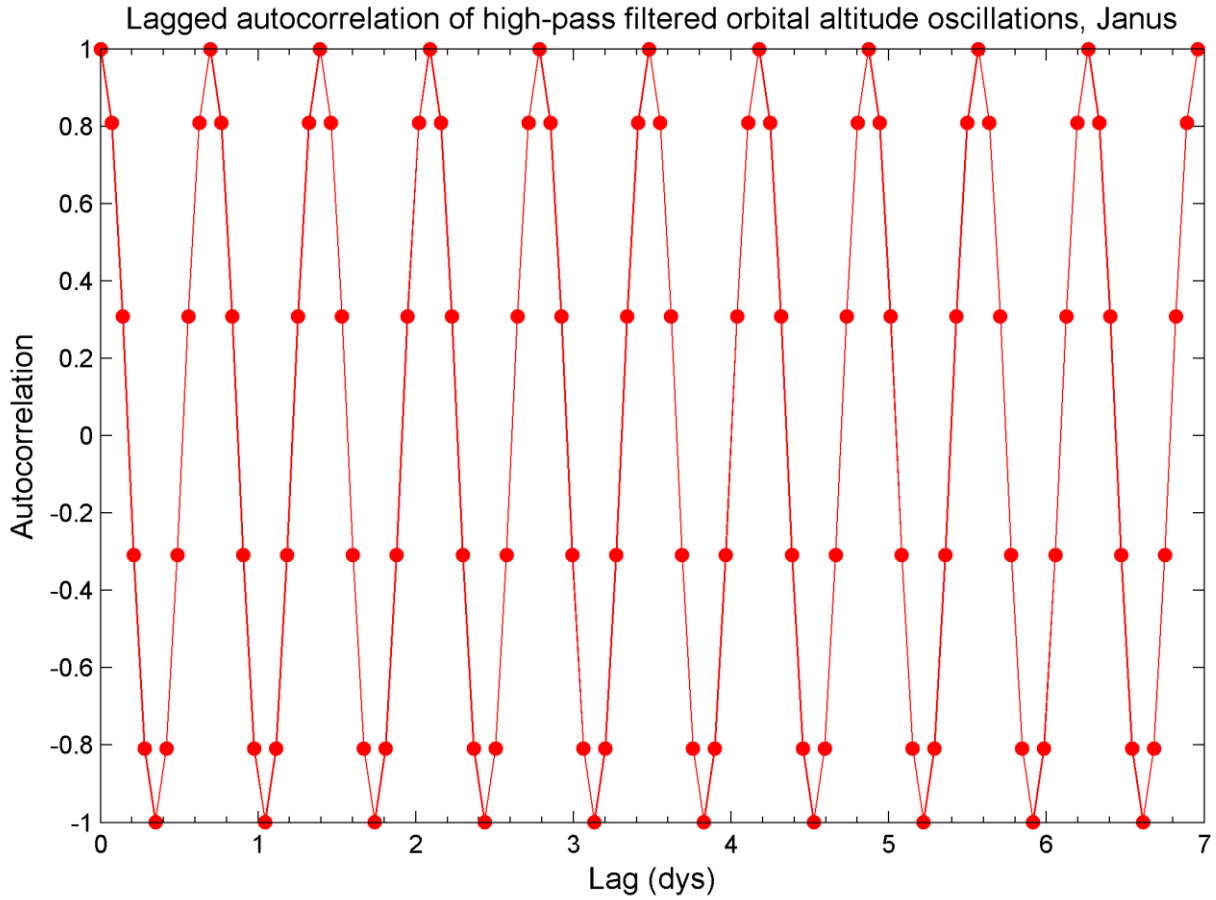
The Experiments:

Janus & Epimetheus



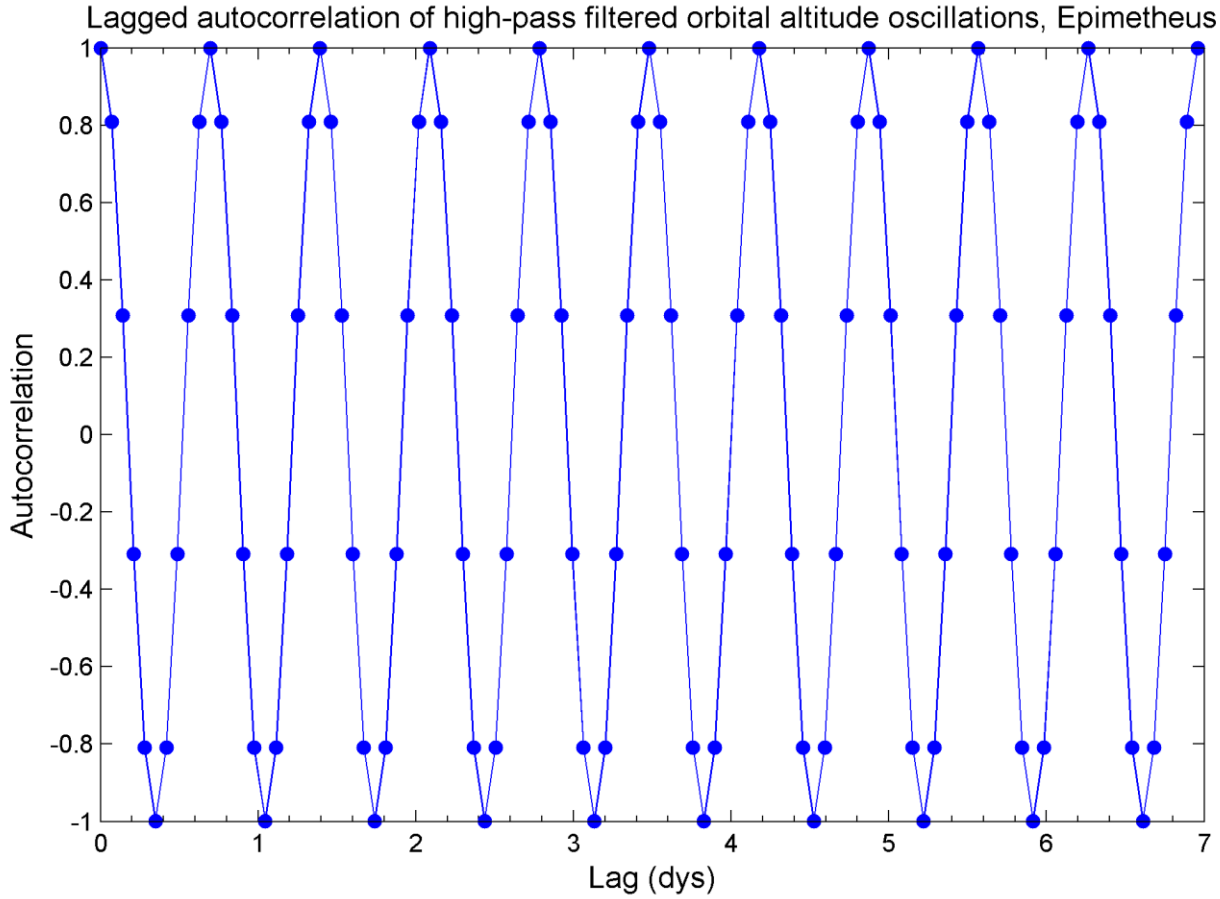
The Experiments:

Janus & Epimetheus



The Experiments:

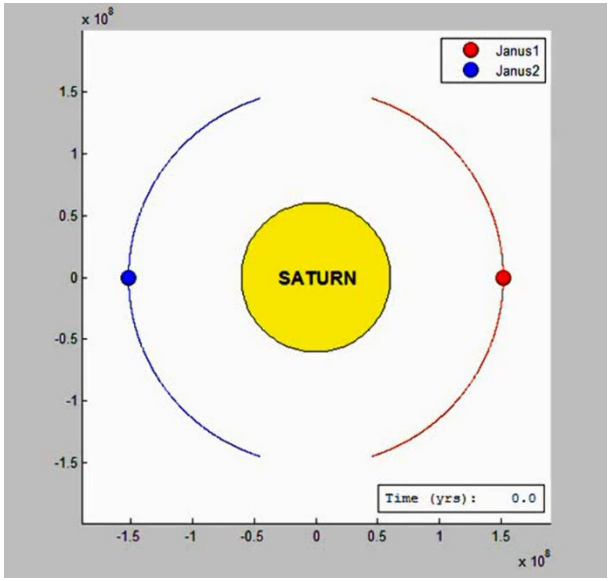
Janus & Epimetheus



The Experiments:

Janus × 2

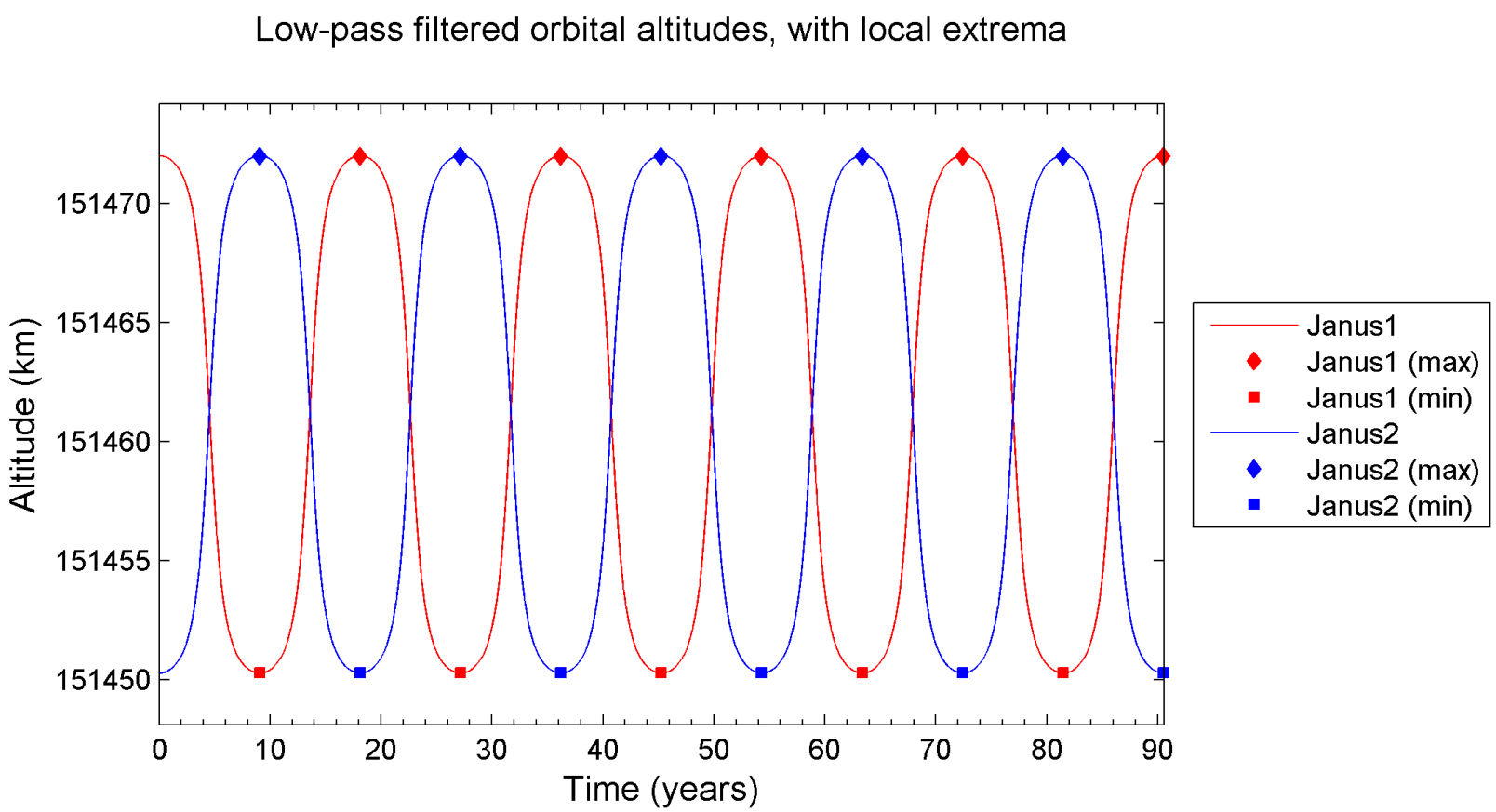
- Goal: See how the model treats the symmetry of 2 identical moons, before moving on to 3 or more



- Initial conditions:
  - Janus1 in standard low orbit
  - Janus2 in standard high orbit
  - Angular spacing: π radians
- Results:
  - One repeated momentum exchange pattern with an 18-year cycle
  - Totally symmetric trajectories
  - Exceedingly stable

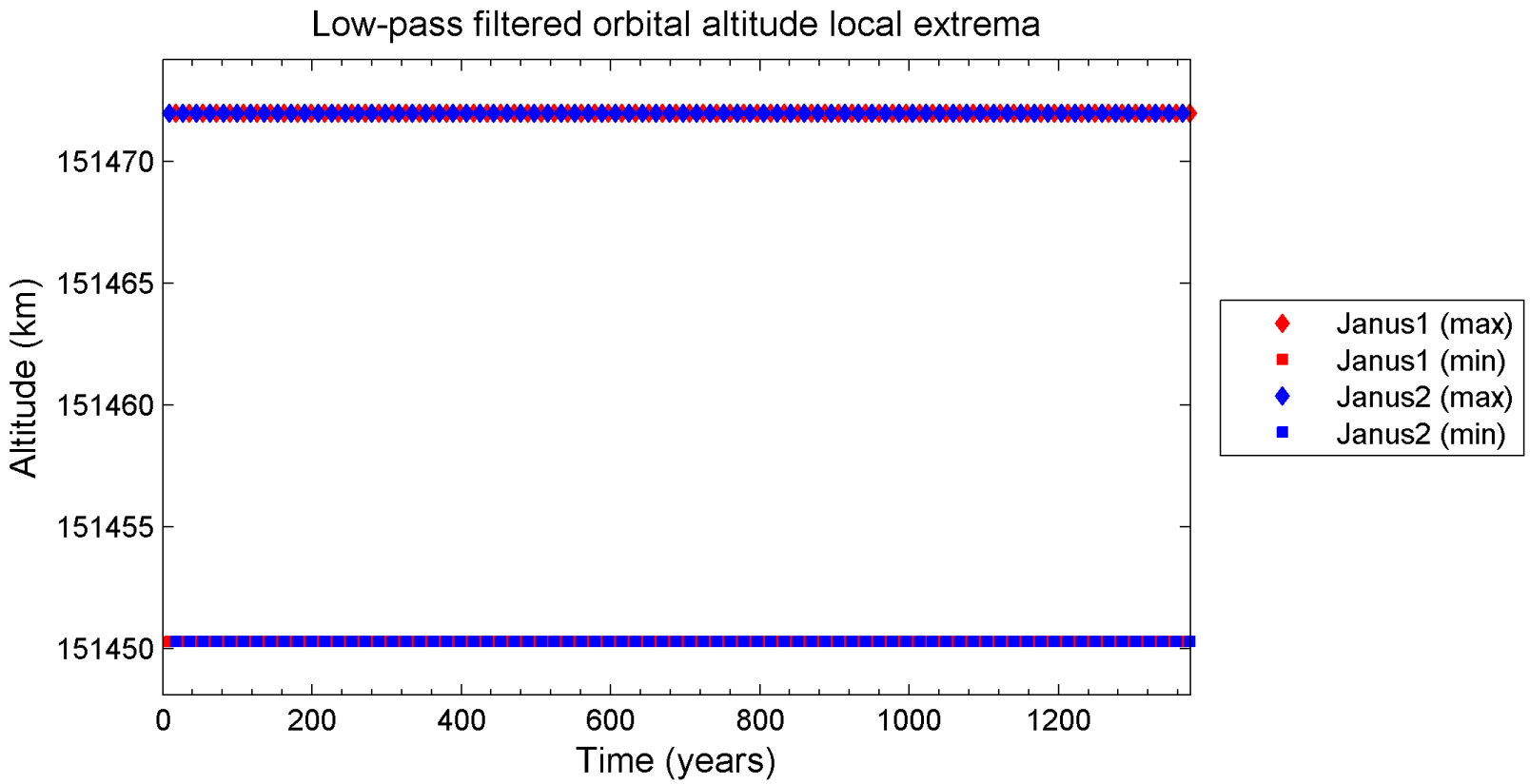
The Experiments:

Janus × 2



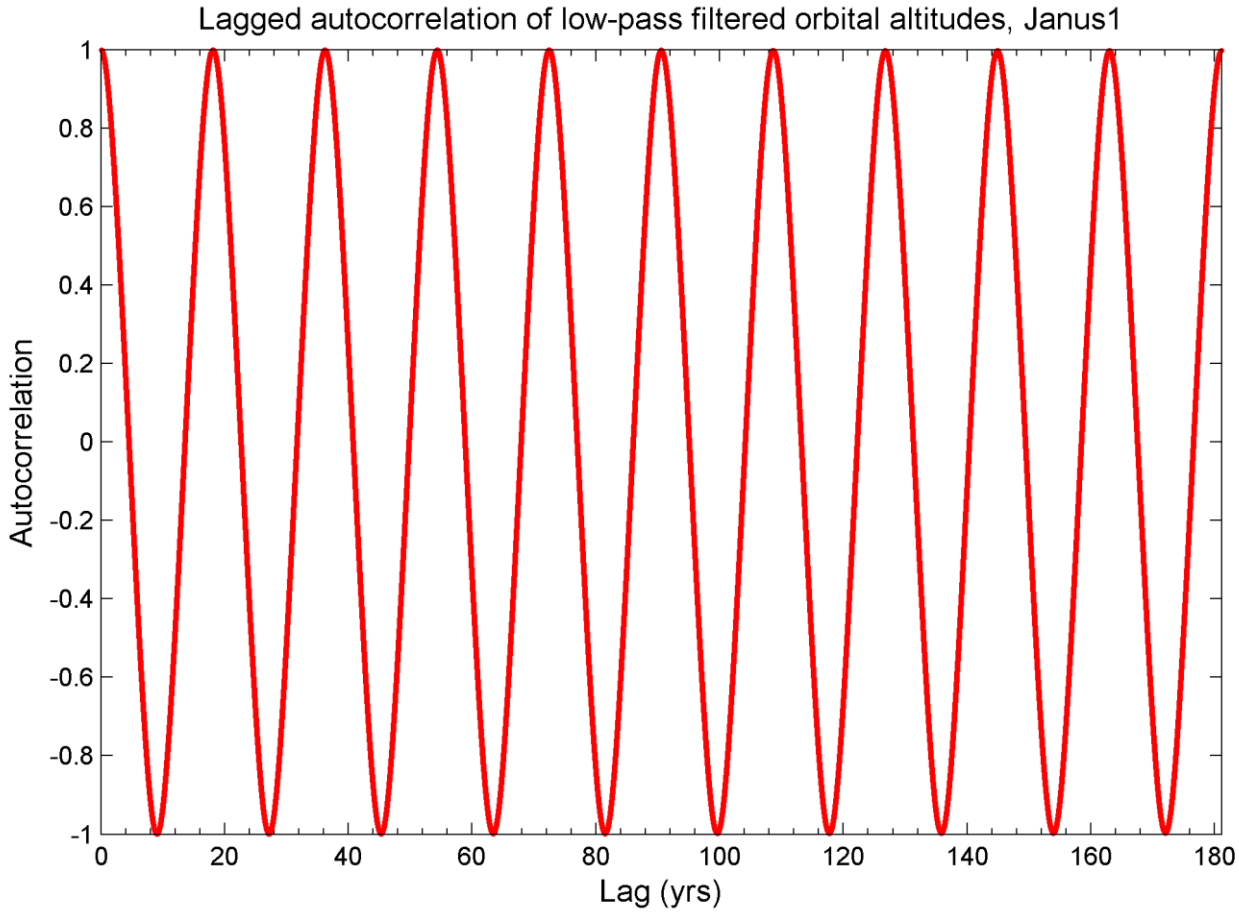
The Experiments:

Janus × 2



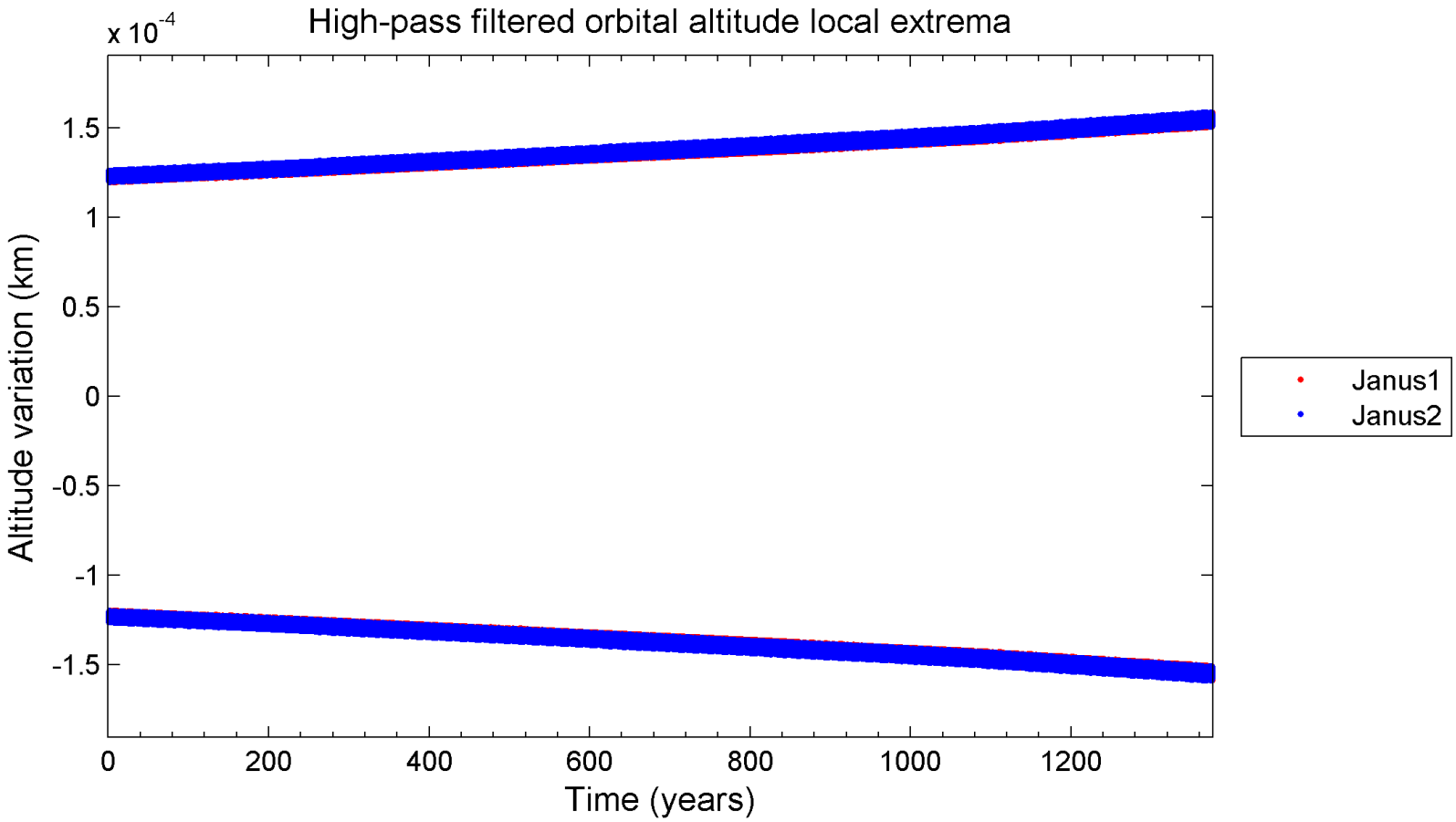
The Experiments:

Janus × 2



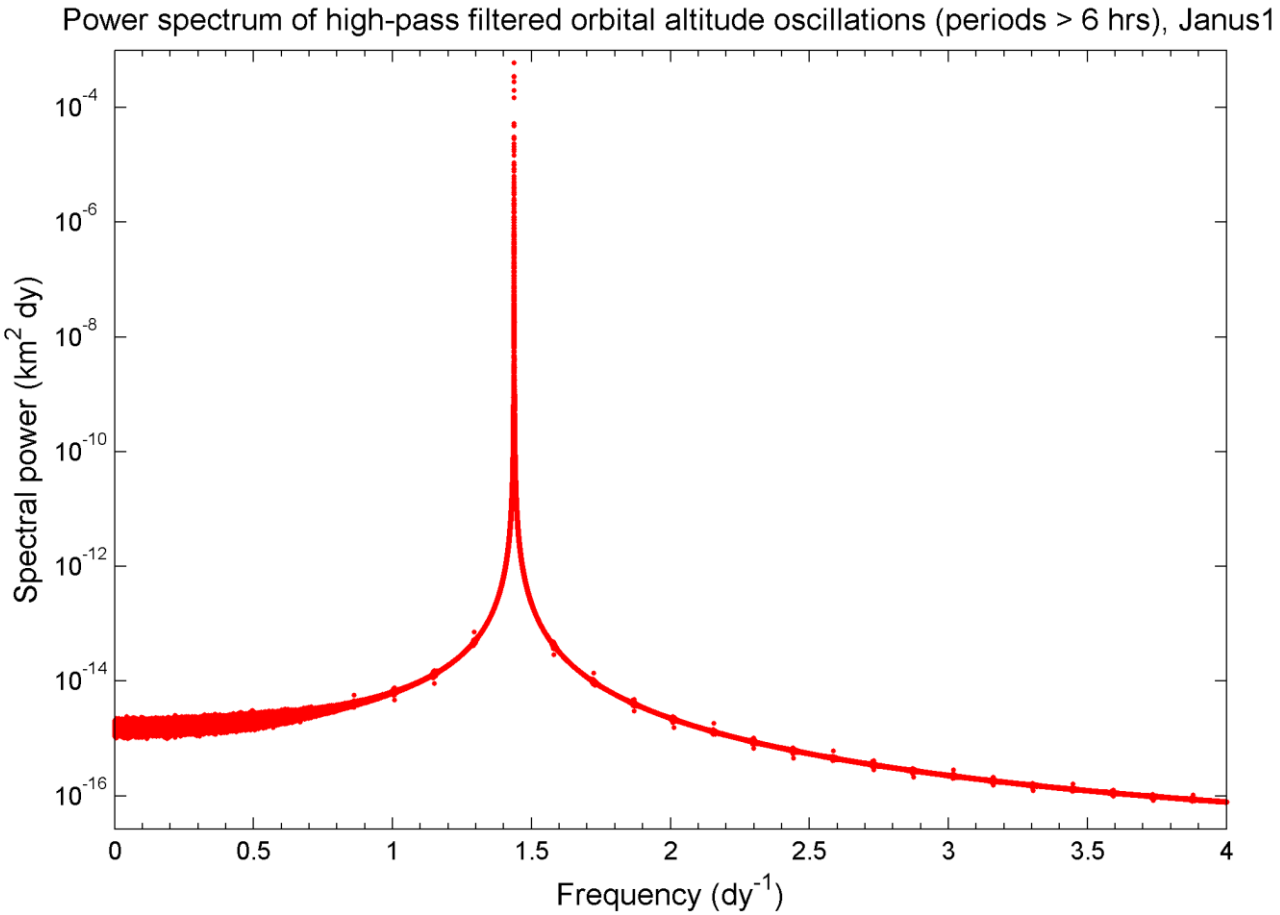
The Experiments:

Janus × 2



The Experiments:

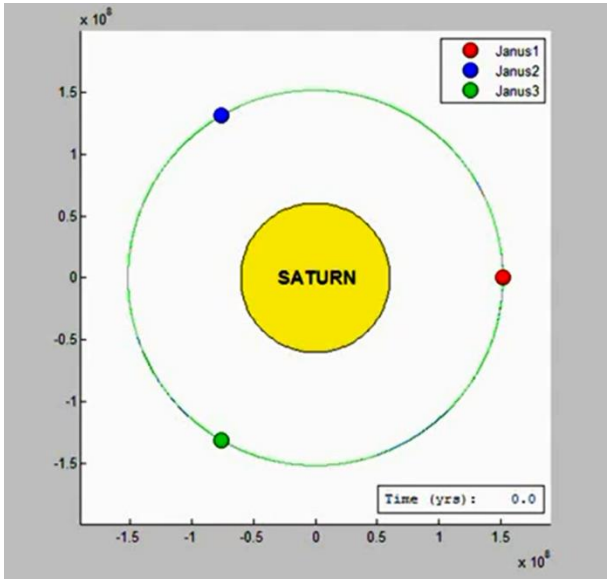
Janus × 2



The Experiments:

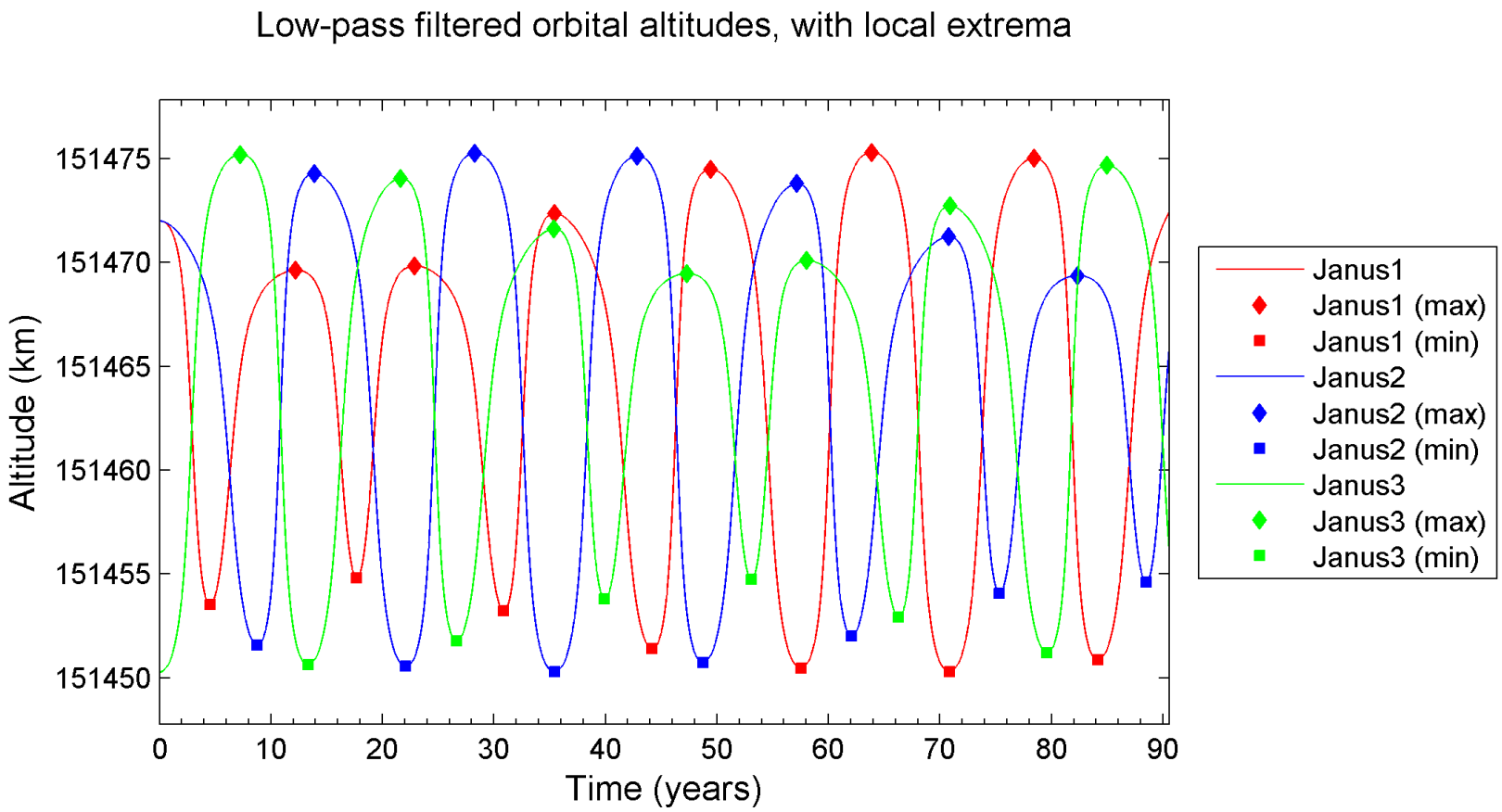
Janus × 3

- Goal: Examine the stability of a system of 3 identical moons, initialized to Janus' upper & lower orbits
- Initial conditions:
  - Janus1 & Janus2 in standard low orbit
  - Janus3 in standard high orbit
  - Angular spacing: $\frac{2\pi}{3}$ radians
- Results:
  - Momentum exchange dynamics get more complex, due to influence of 3rd moon from a distance
  - Orbital height extrema take on a range of values, but within strict bounds
  - Eccentricities consistently near zero
  - Still very stable



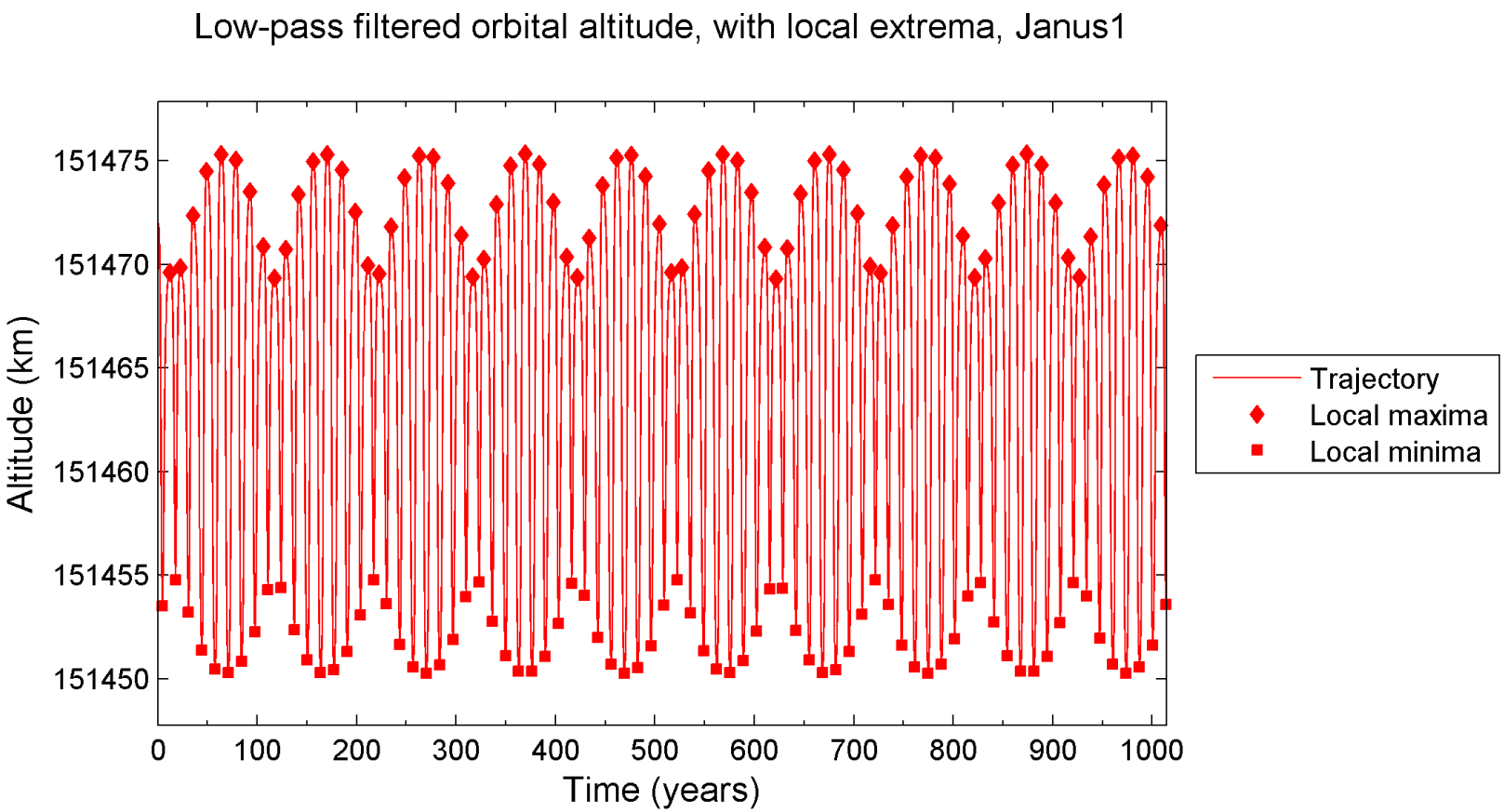
The Experiments:

Janus × 3



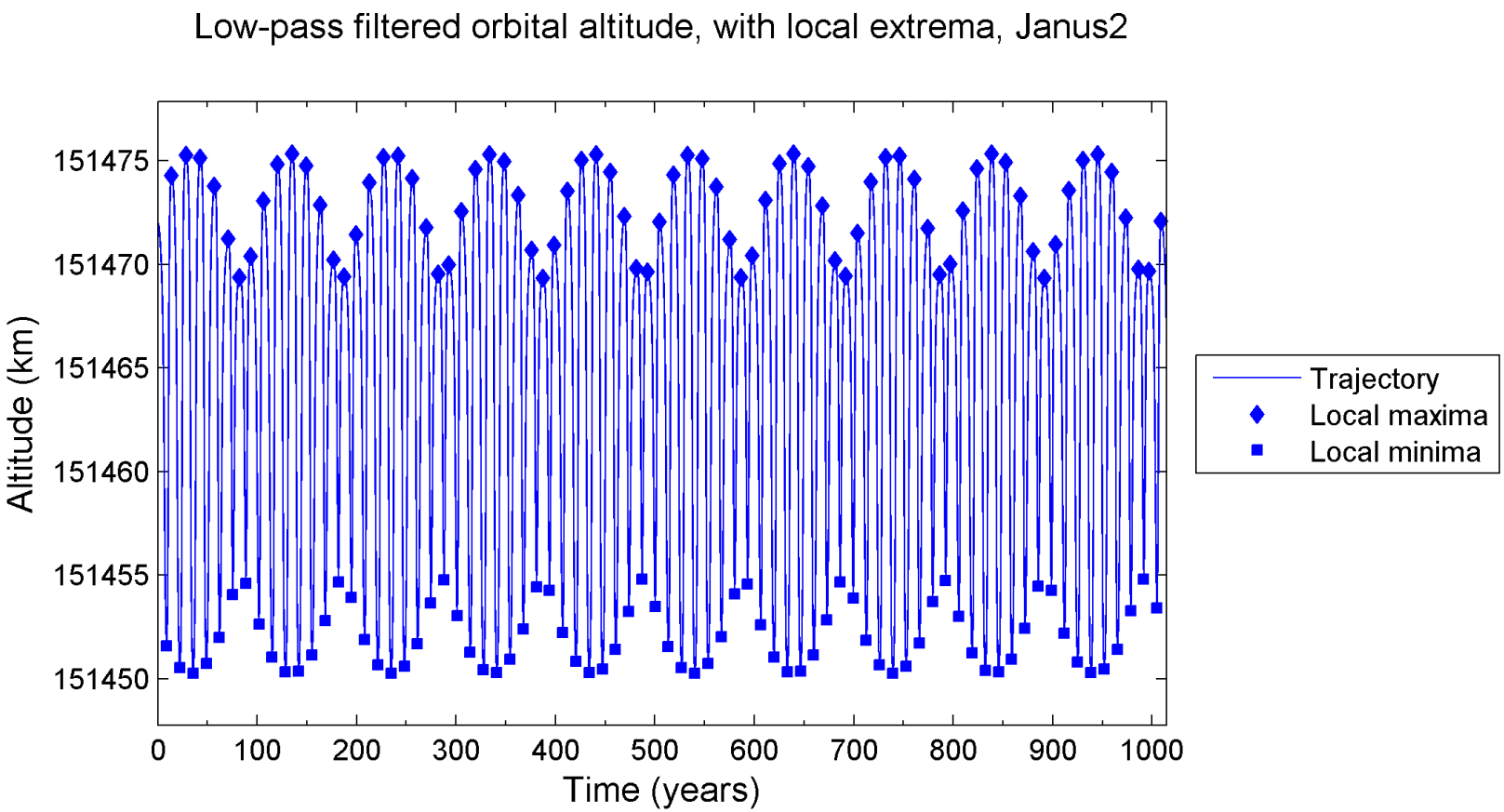
The Experiments:

Janus × 3



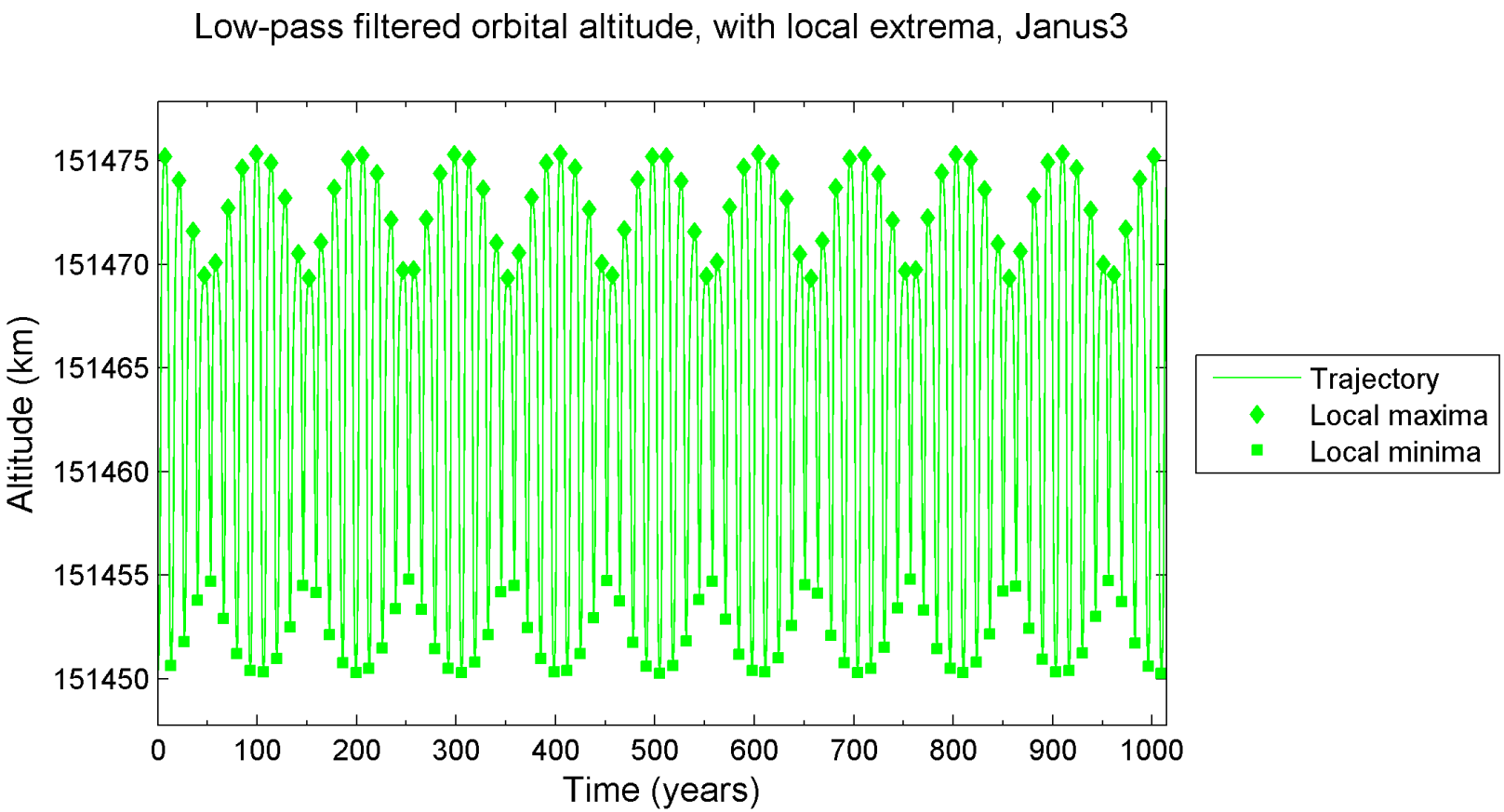
The Experiments:

Janus × 3



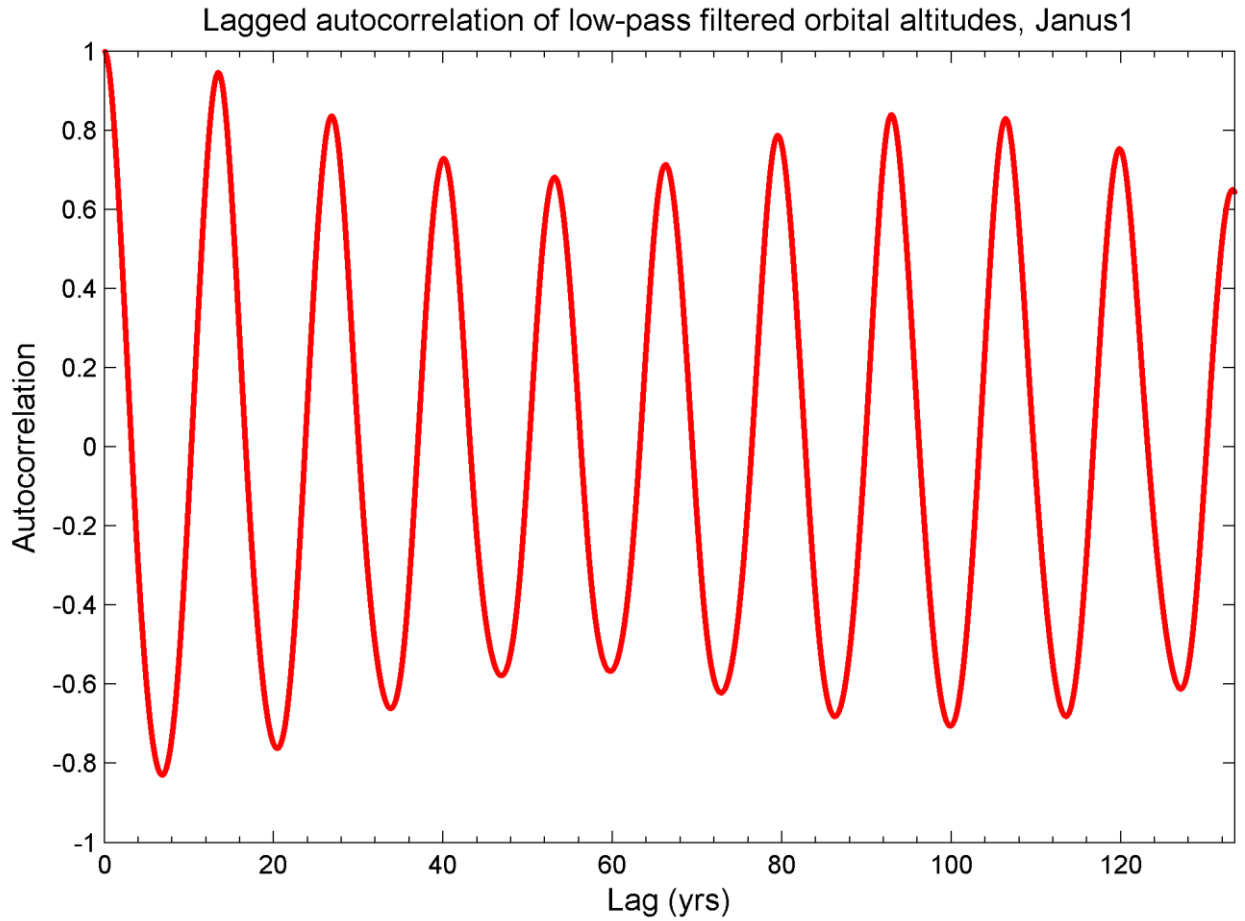
The Experiments:

Janus × 3



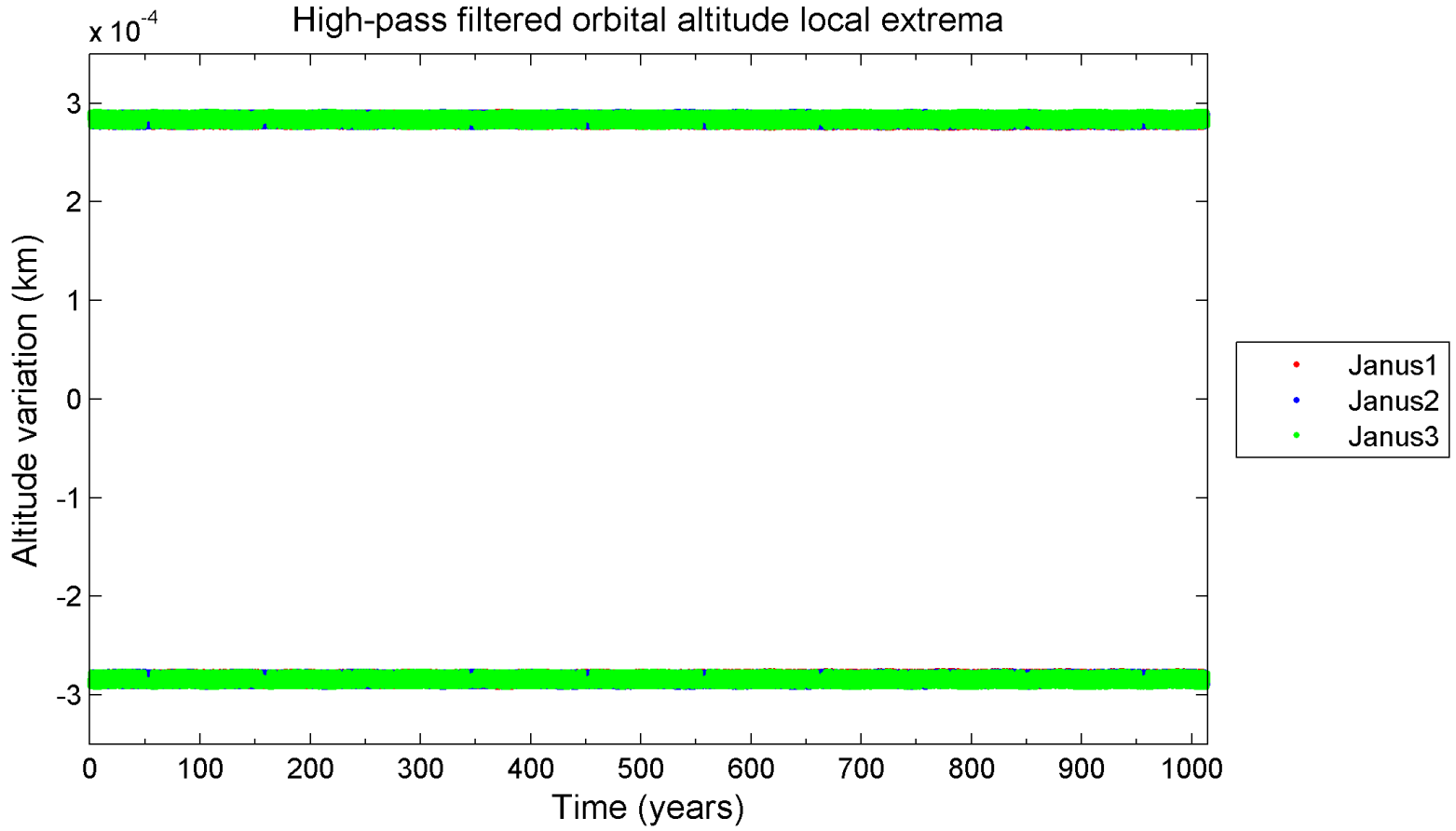
The Experiments:

Janus × 3



The Experiments:

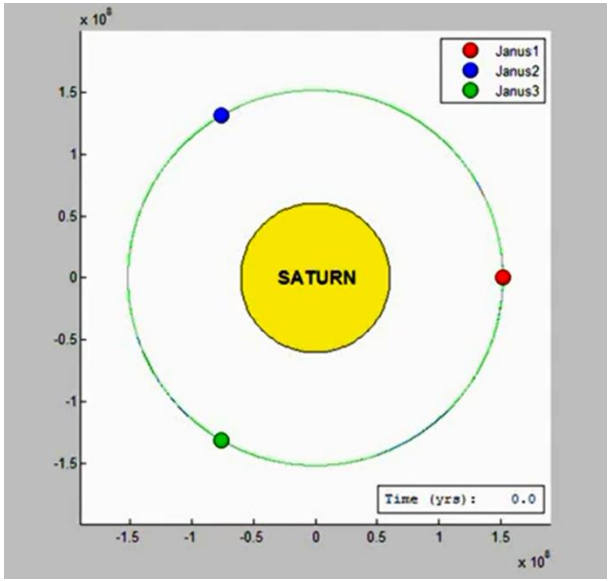
Janus × 3



The Experiments:

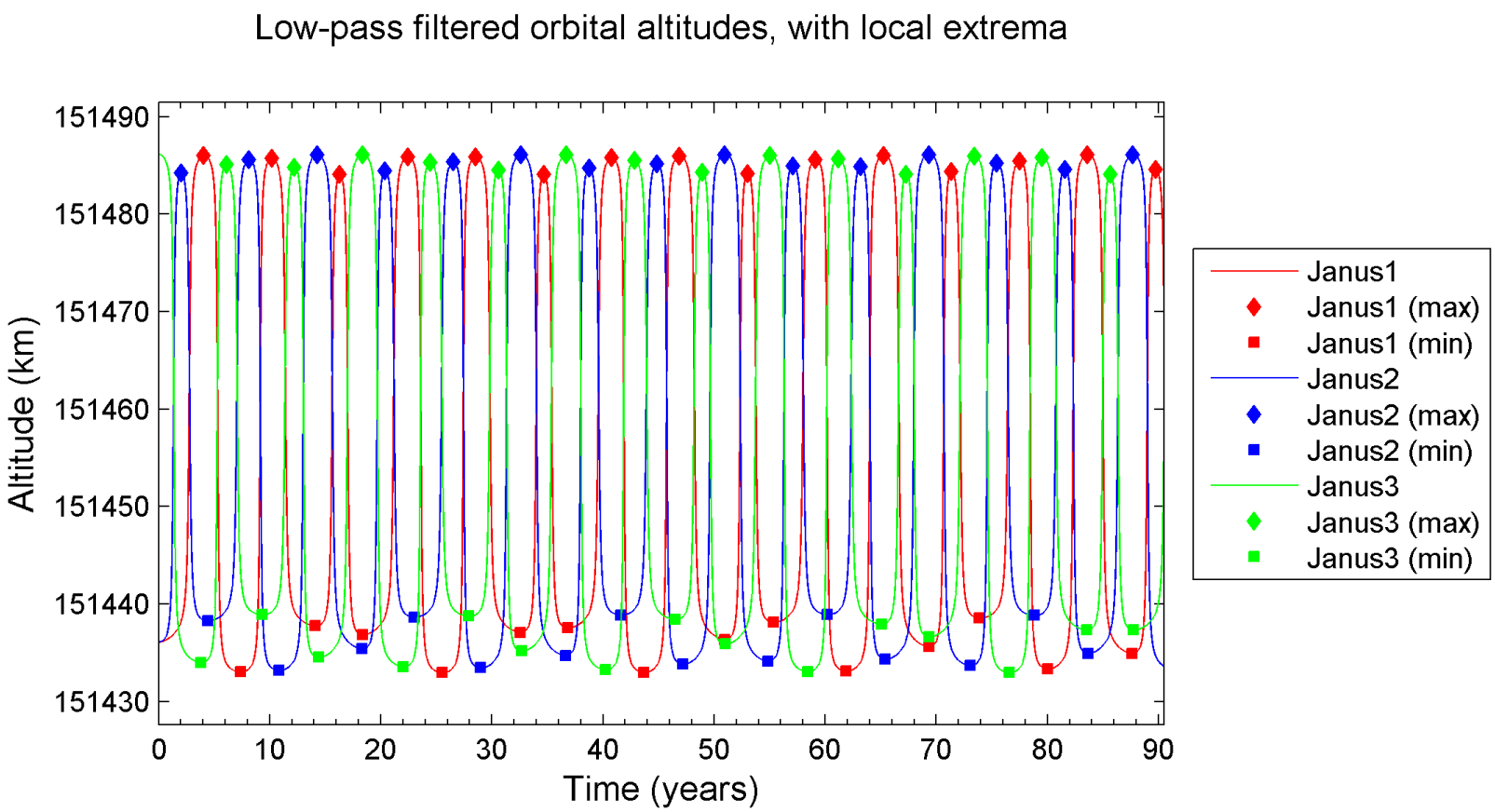
Janus × 3, Δr = 50 km

- Goal: Explore how the dynamics and stability of the *Janus × 3* system changes when the initial altitude difference is raised from 31.7 km to 50
- Initial conditions:
  - Janus1 & Janus2 in mean orbit - 25 km
  - Janus3 in mean orbit + 25 km
  - Angular spacing: $\frac{2\pi}{3}$ radians
- Results:
  - Similarly complex orbital dynamics, but with more frequent orbit swaps and different resonant cycles
  - Orbital heights still vary within strict bounds, similar to before
  - Eccentricities still flat, near zero
  - Still very stable



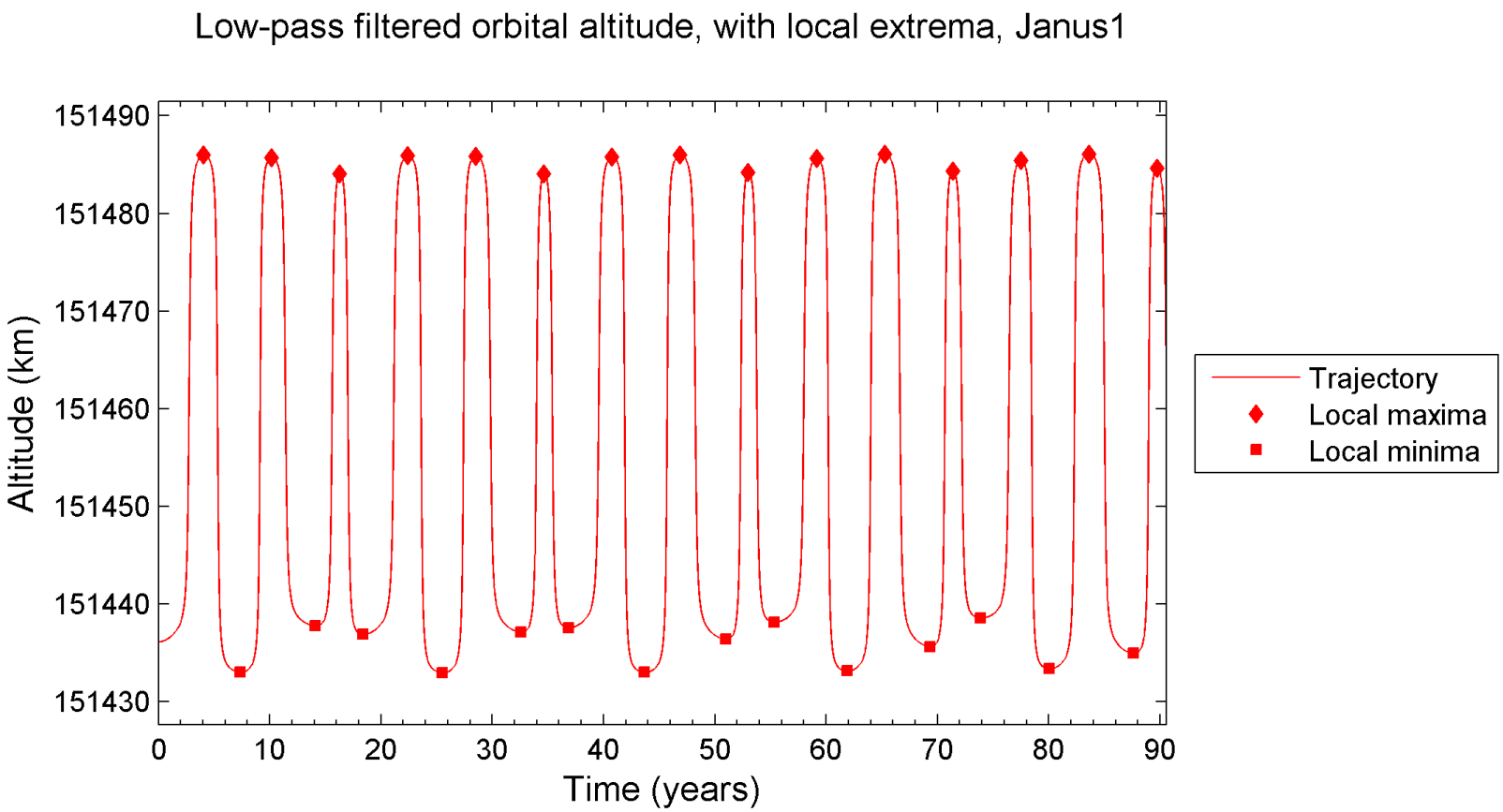
The Experiments:

Janus × 3, Δr = 50 km



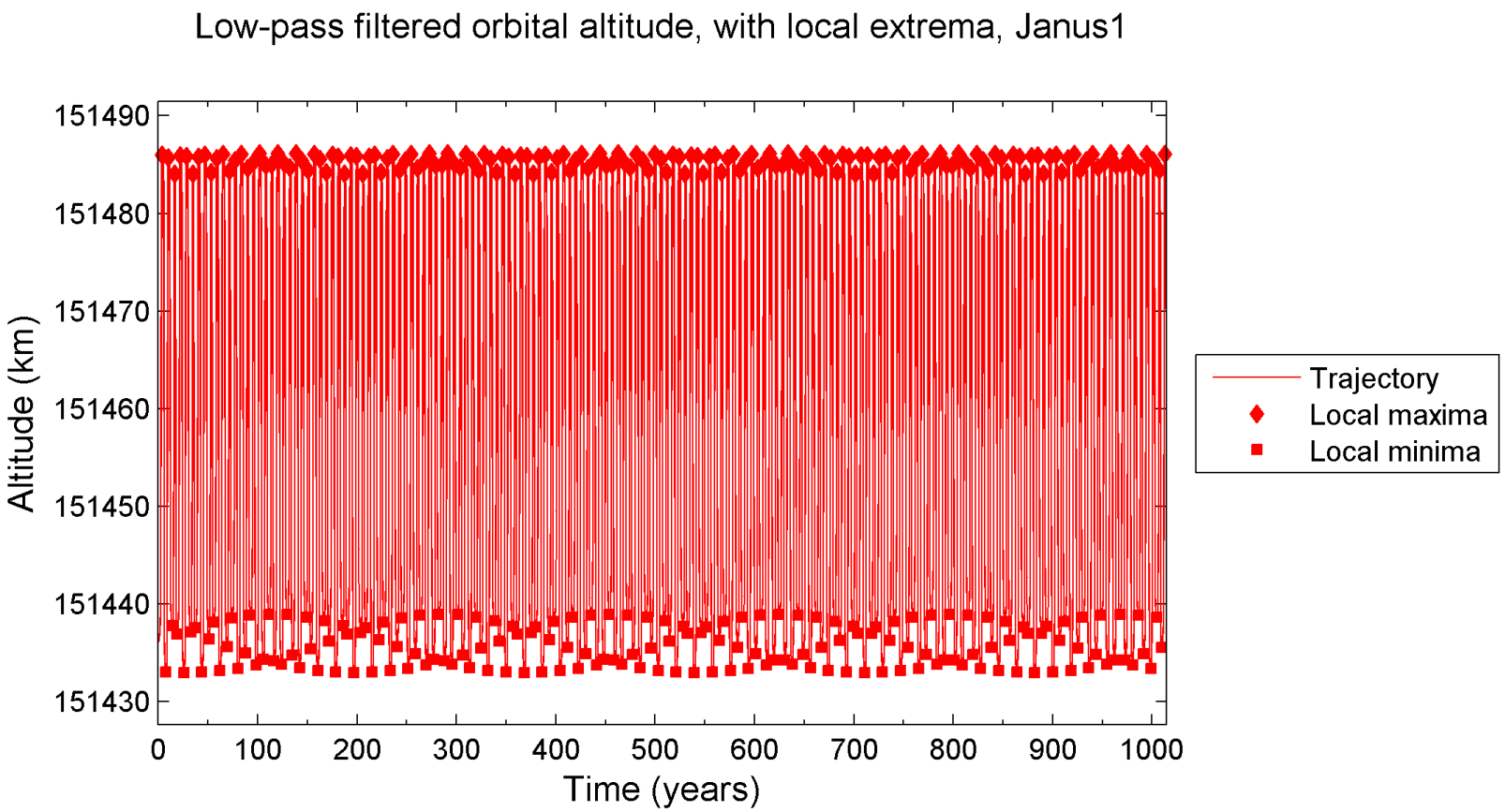
The Experiments:

Janus × 3, Δr = 50 km



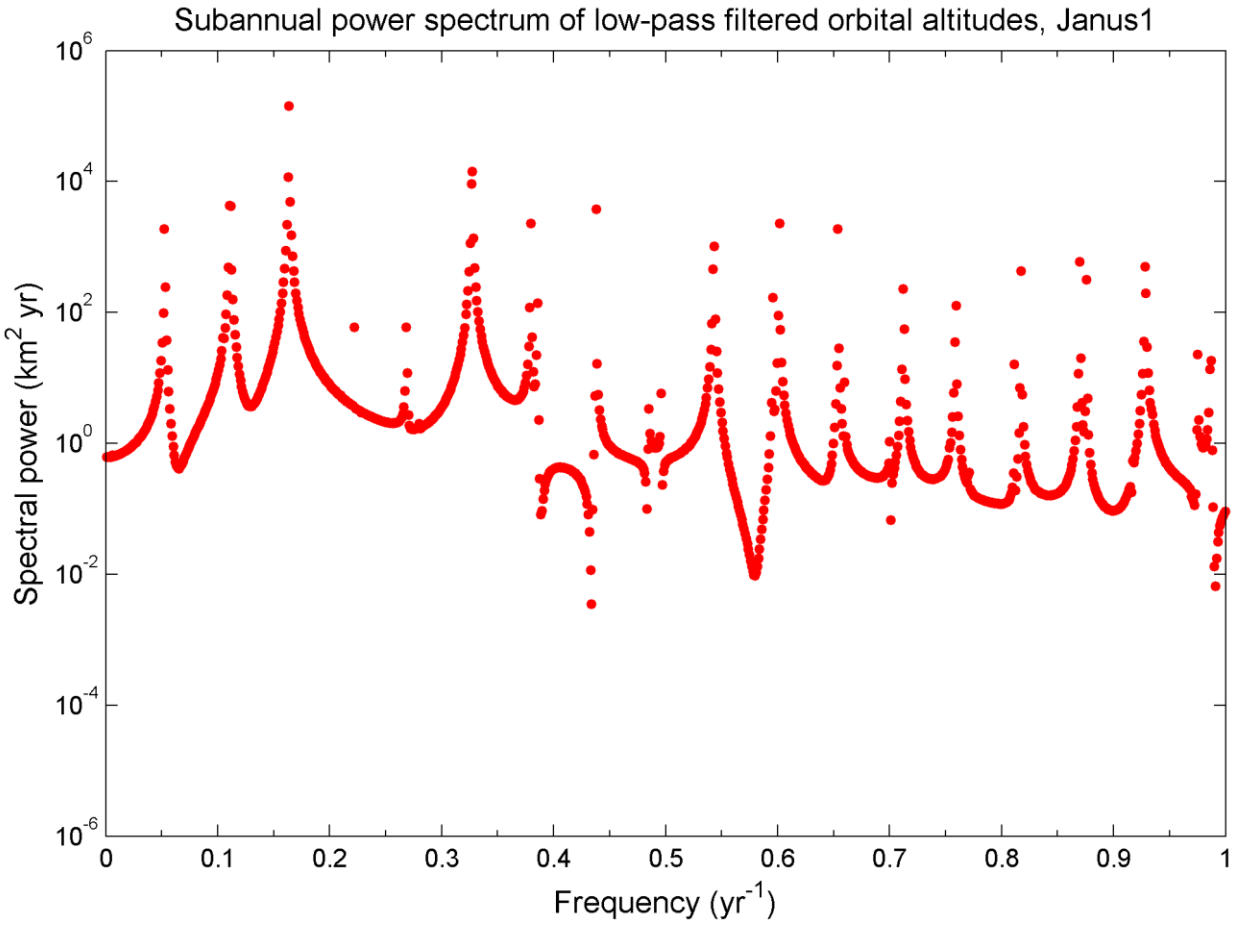
The Experiments:

Janus × 3, Δr = 50 km



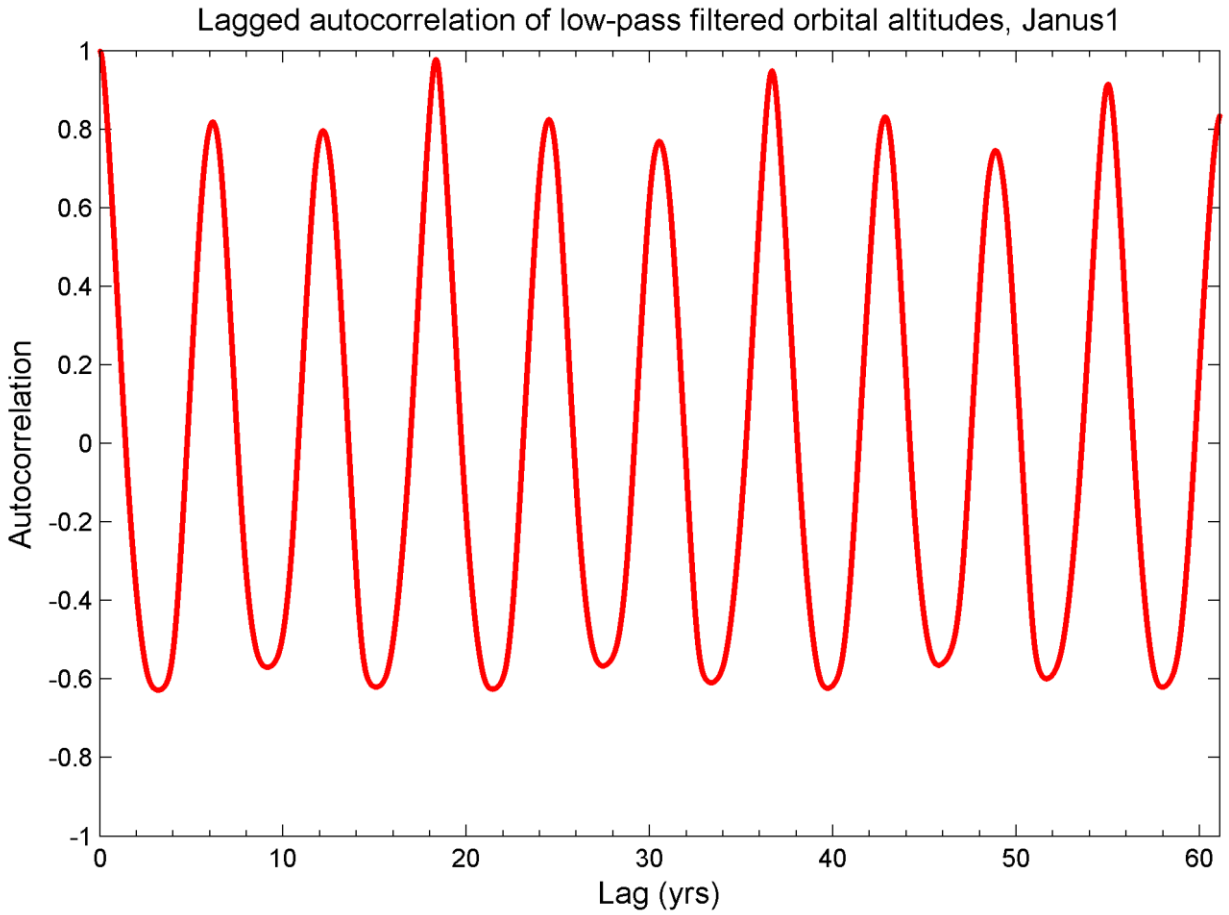
The Experiments:

Janus × 3, Δr = 50 km



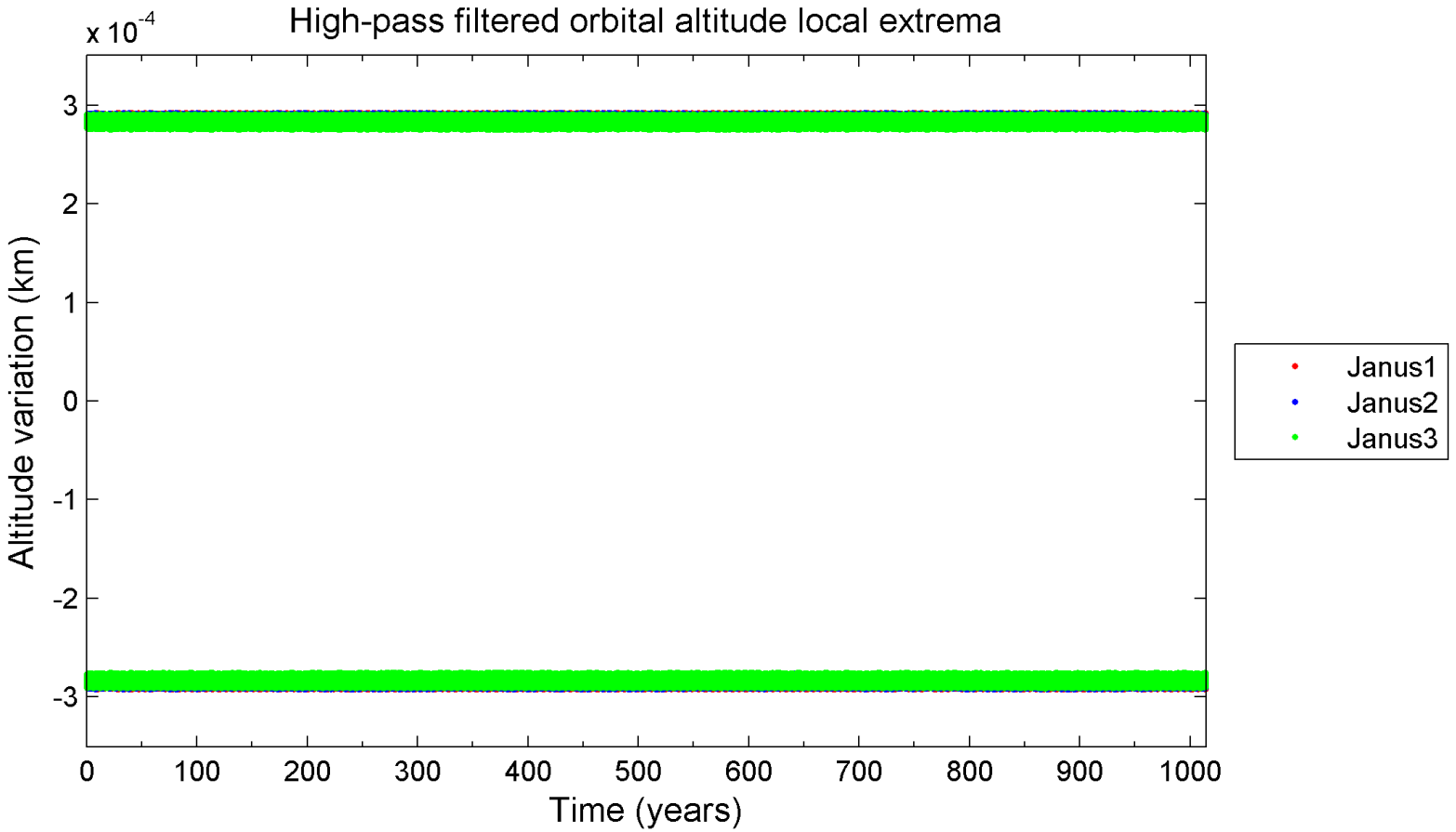
The Experiments:

Janus × 3, Δr = 50 km



The Experiments:

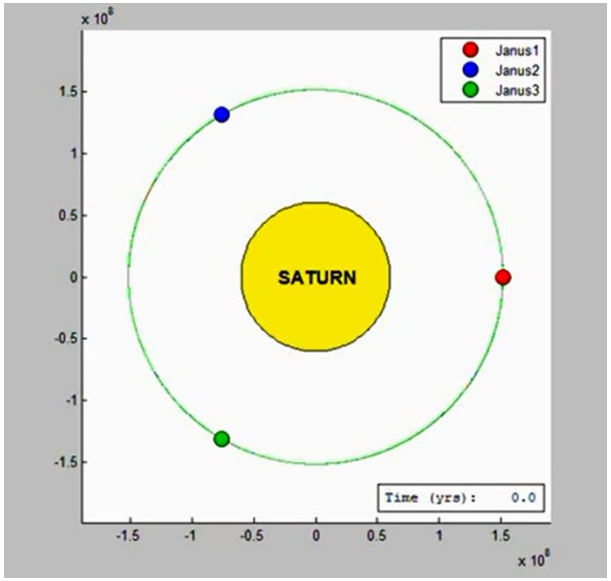
Janus × 3, Δr = 50 km



The Experiments:

Janus × 3, Δr = 100 km

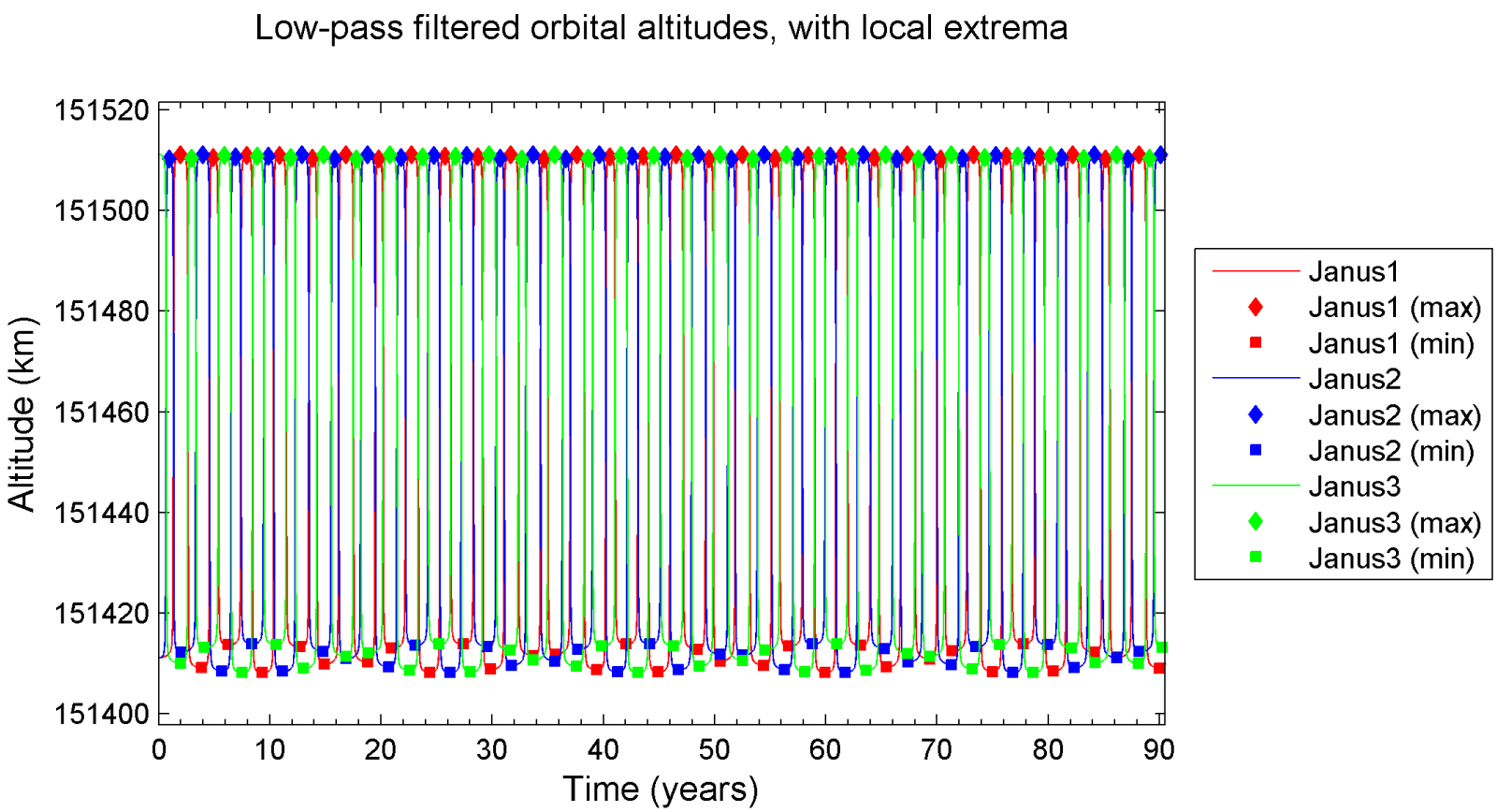
- Goal: Continue to explore the *Janus × 3* system, doubling the initial altitude difference this time to 100 km



- Initial conditions:
  - Janus1 & Janus2 in mean orbit - 50 km
  - Janus3 in mean orbit + 50 km
  - Angular spacing: $\frac{2\pi}{3}$ radians
- Results:
  - Orbit swap cycles continue to accelerate
  - Orbital heights still vary within strict bounds, similar to before
  - Eccentricities just as low, but barely starting to creep in different directions
  - Still very stable, but with tiny seeds of instability

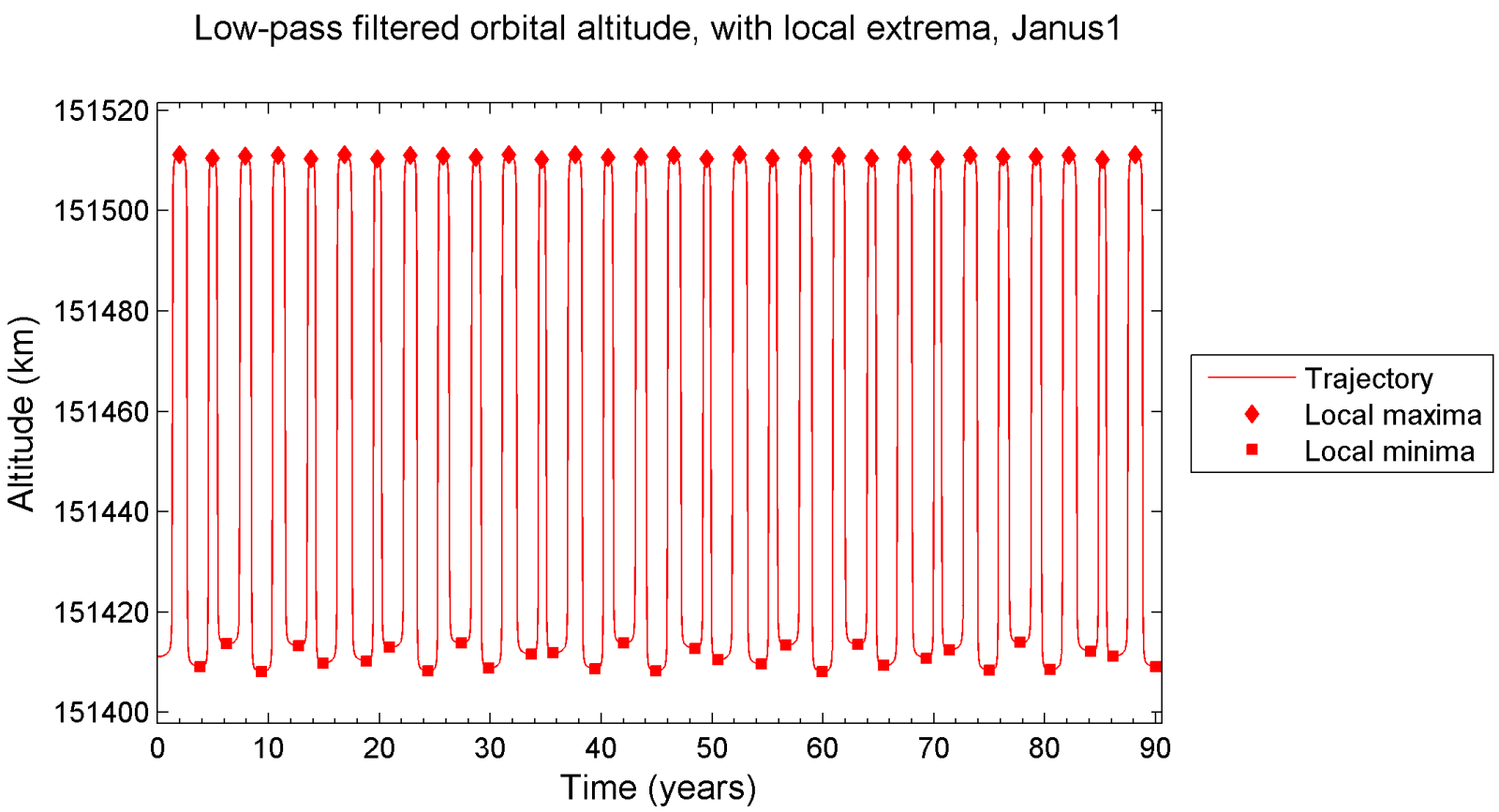
The Experiments:

Janus × 3, Δr = 100 km



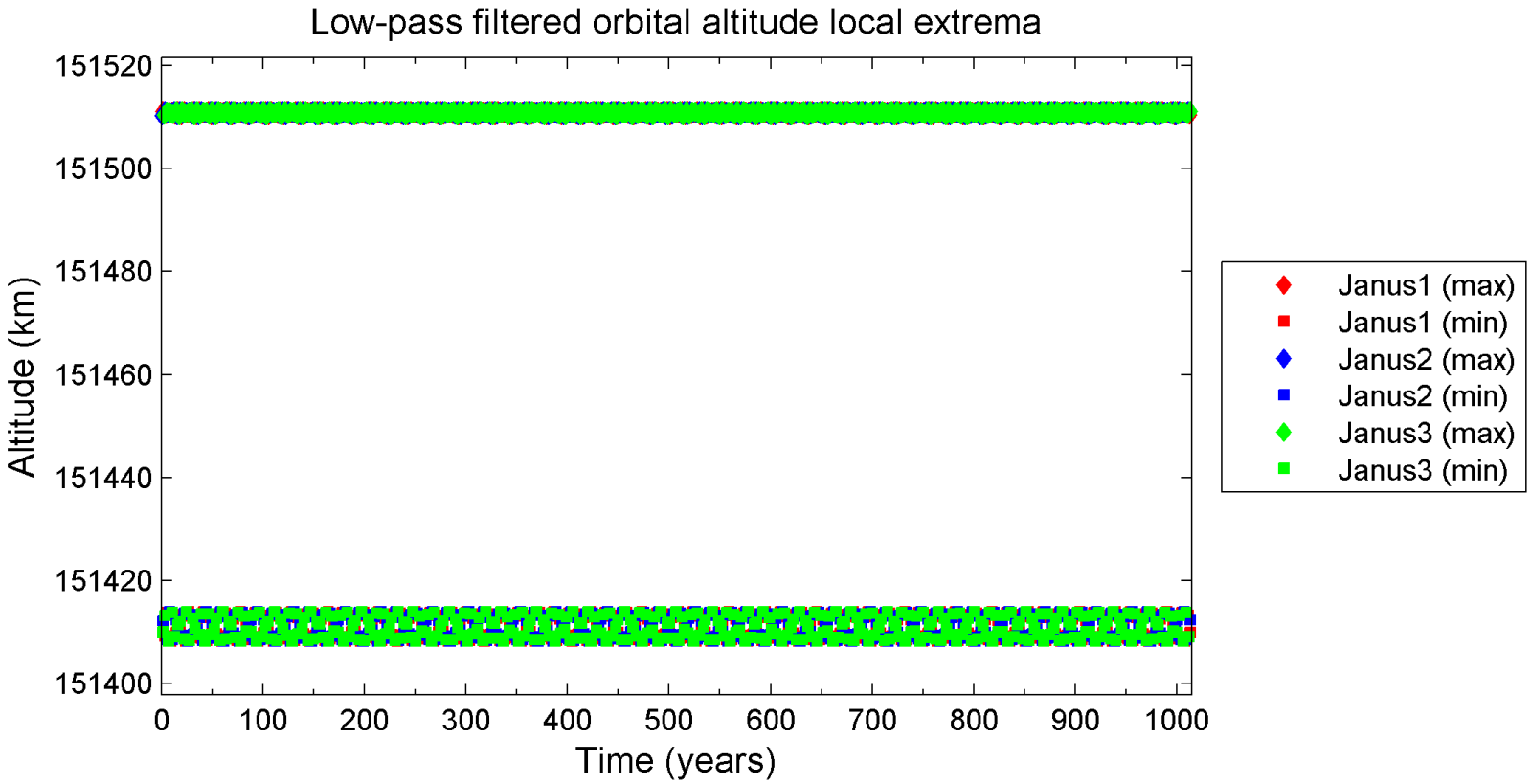
The Experiments:

Janus × 3, Δr = 100 km



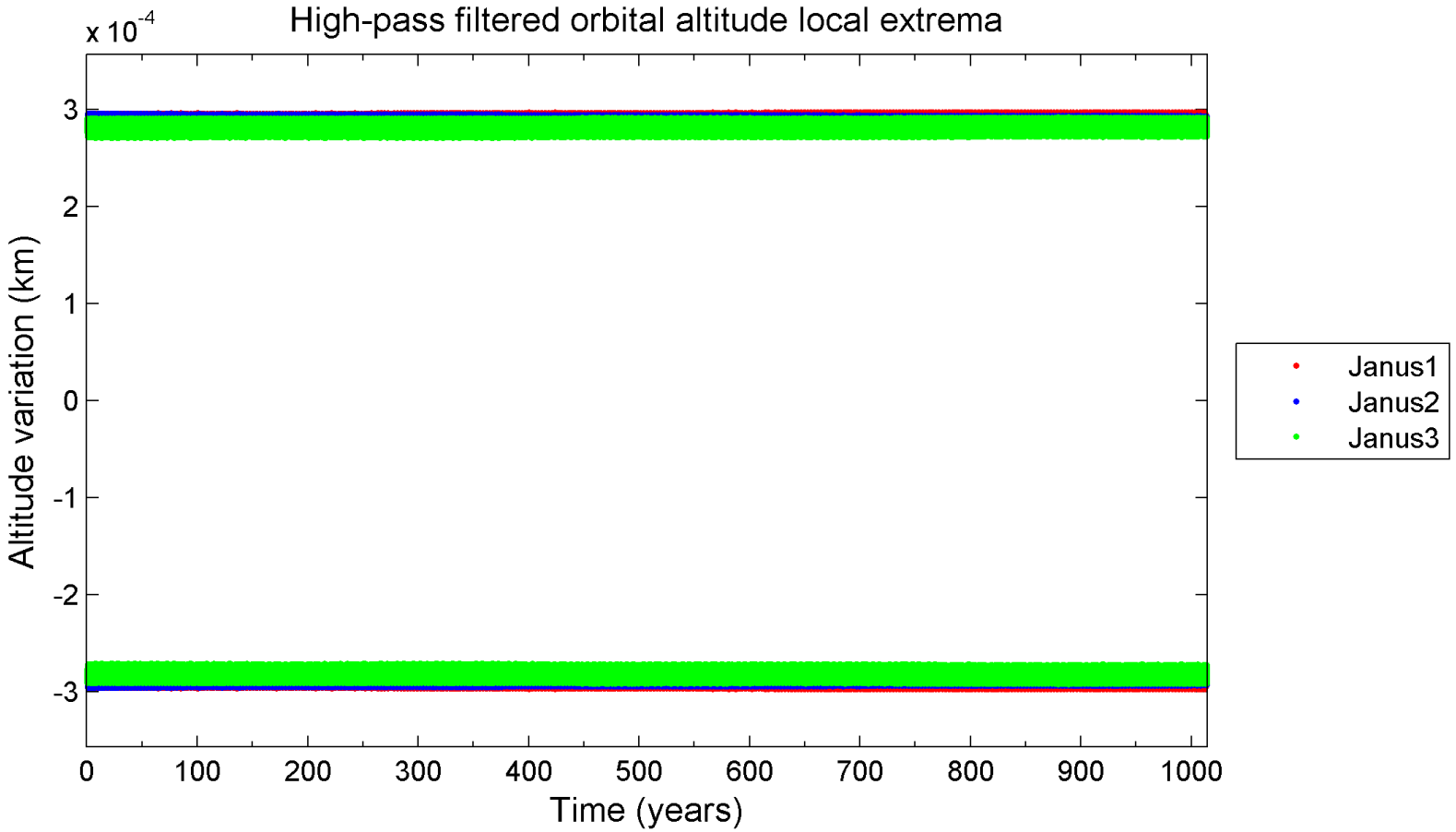
The Experiments:

Janus × 3, Δr = 100 km



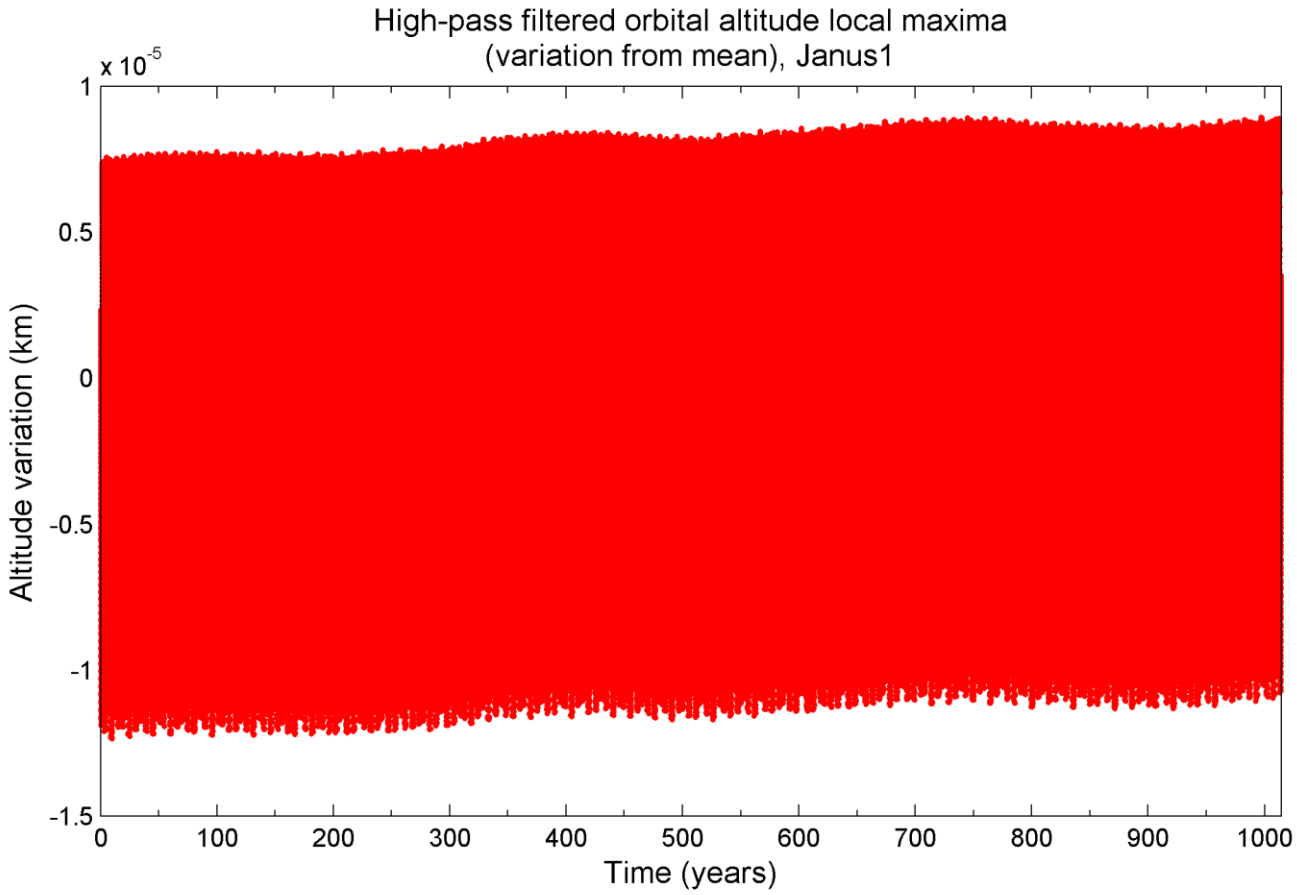
The Experiments:

Janus × 3, Δr = 100 km



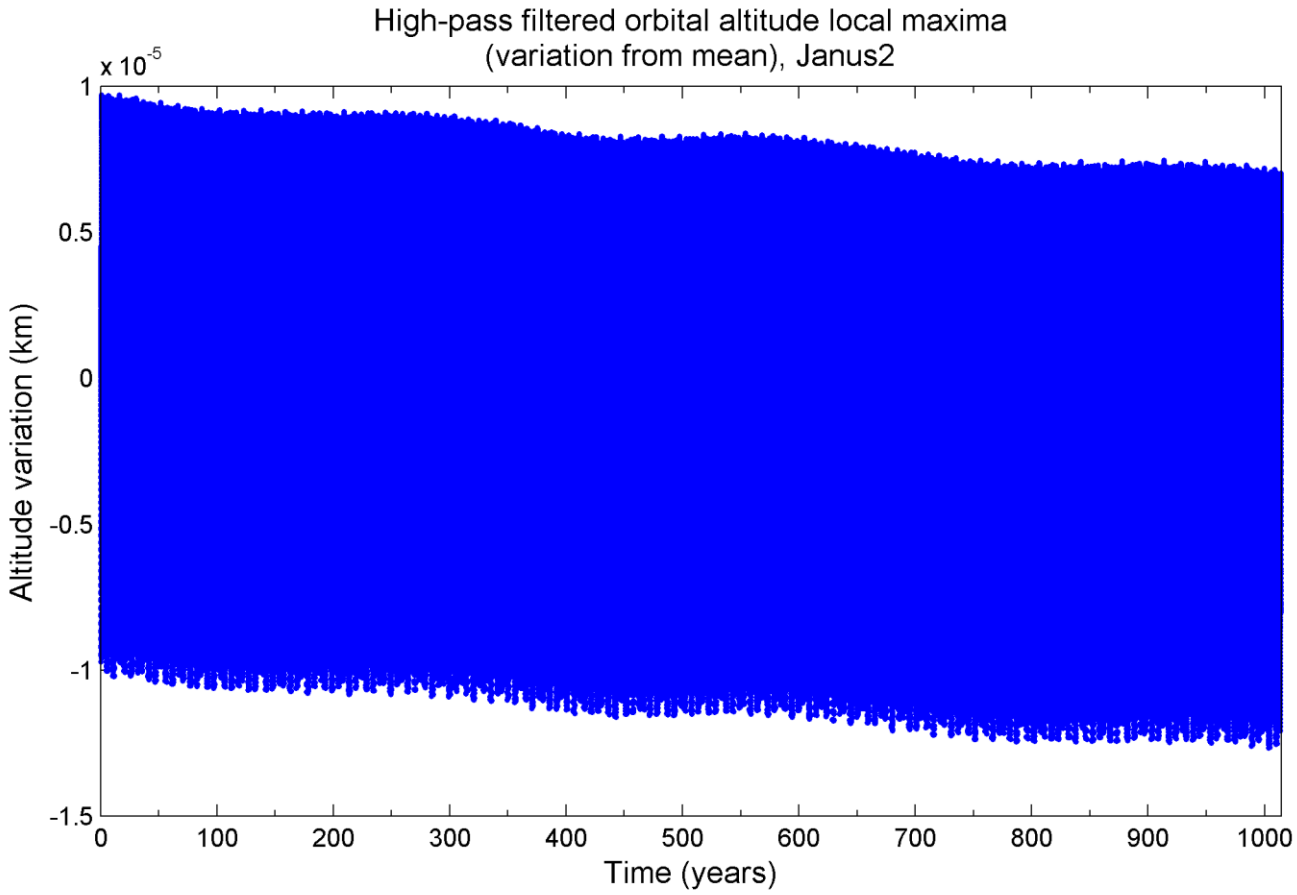
The Experiments:

Janus × 3, Δr = 100 km



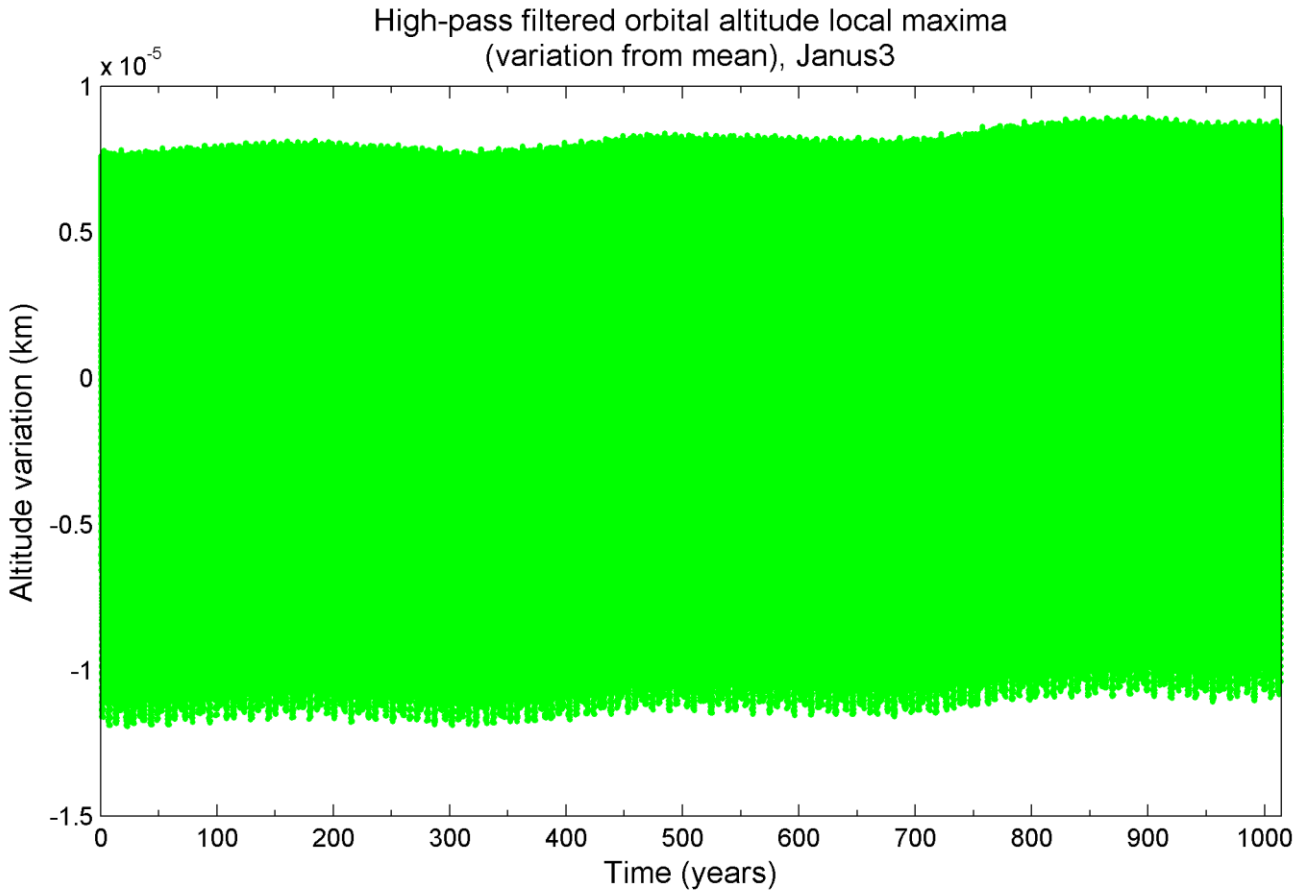
The Experiments:

Janus × 3, Δr = 100 km



The Experiments:

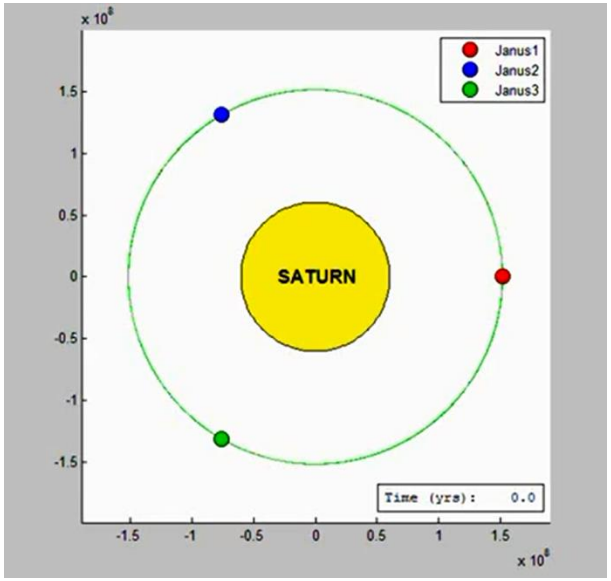
Janus × 3, Δr = 100 km



The Experiments:

Janus × 3, Δr = 150 km

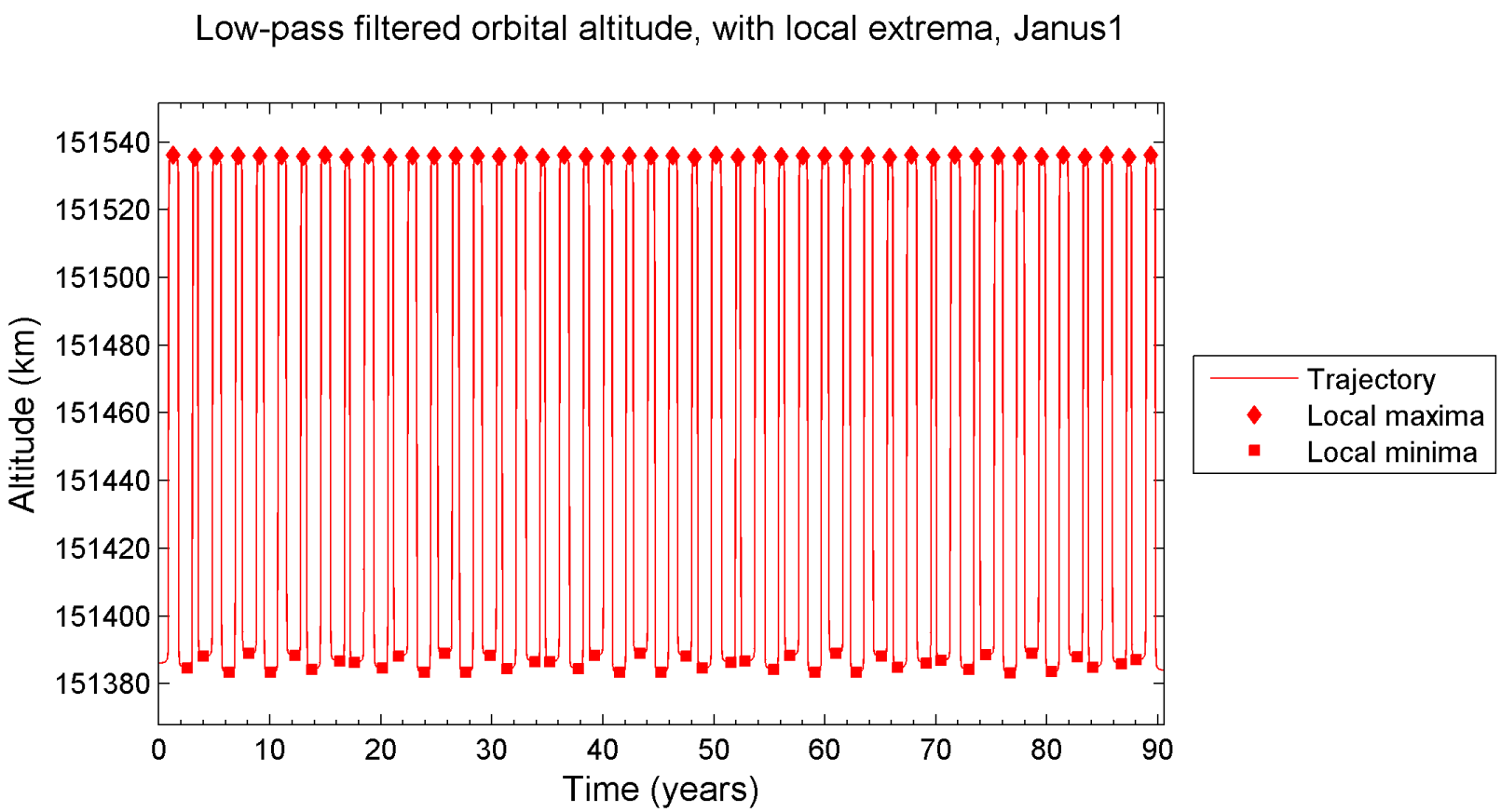
- Goal: Continue to explore the *Janus × 3* system, increasing the initial altitude difference 150 km



- Initial conditions:
  - Janus1 & Janus2 in mean orbit - 75 km
  - Janus3 in mean orbit + 75 km
  - Angular spacing: $\frac{2\pi}{3}$ radians
- Results:
  - Orbit swap cycles continue to accelerate
  - Orbital heights still vary within strict bounds, tighter on the maxima side
  - Eccentricities are suddenly getting periodically lively, though still bounded
  - Still reasonably stable

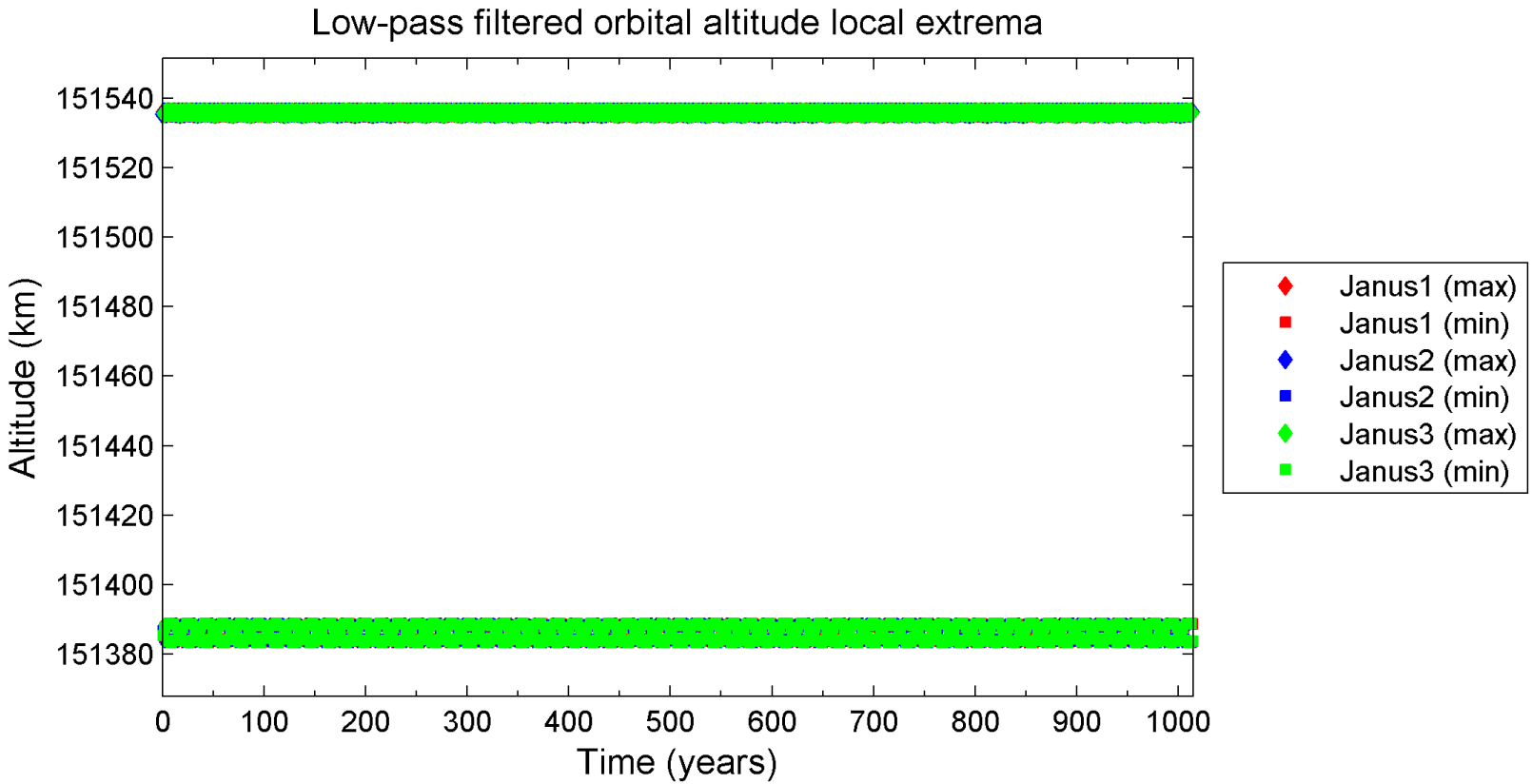
The Experiments:

Janus × 3, Δr = 150 km



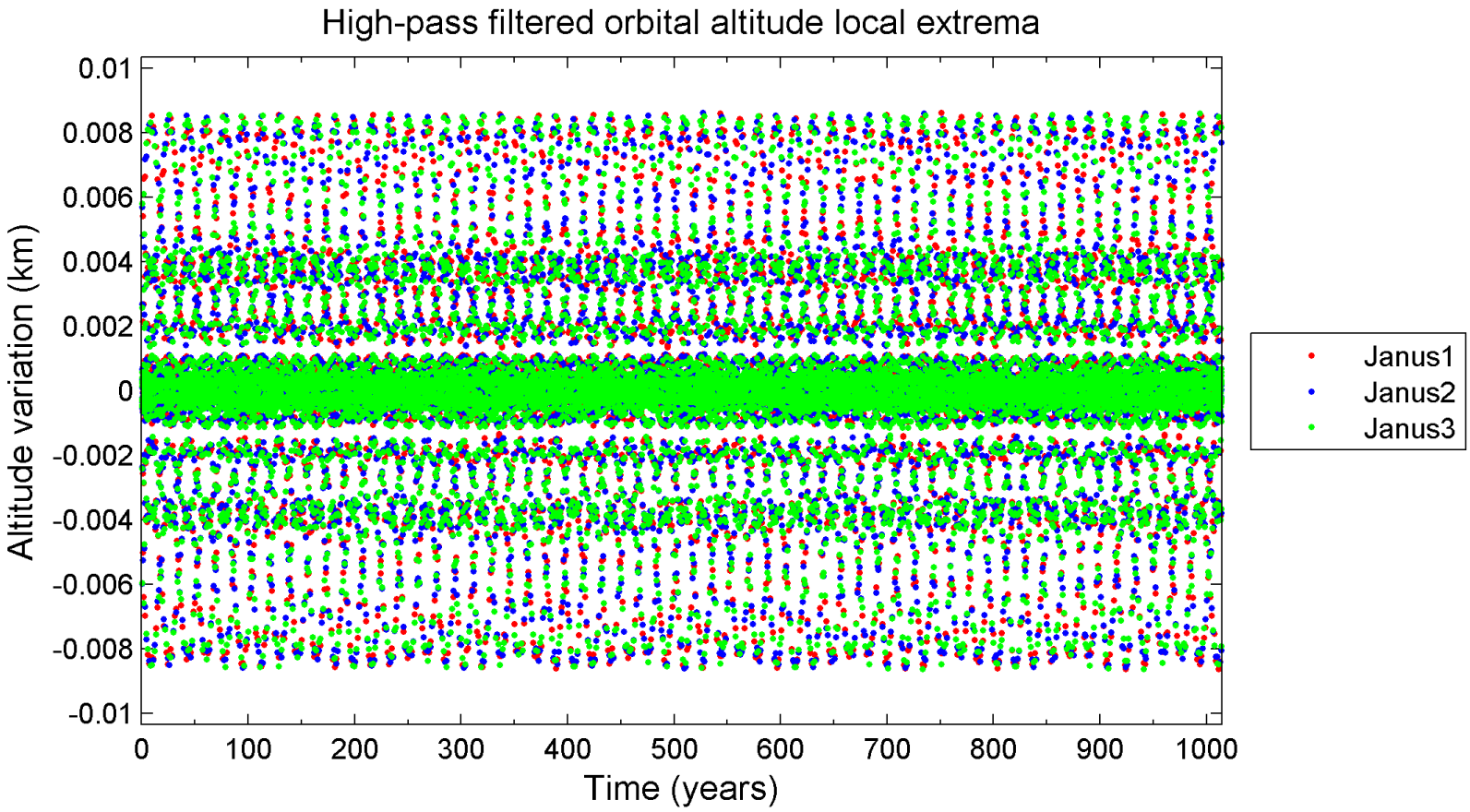
The Experiments:

Janus × 3, Δr = 150 km



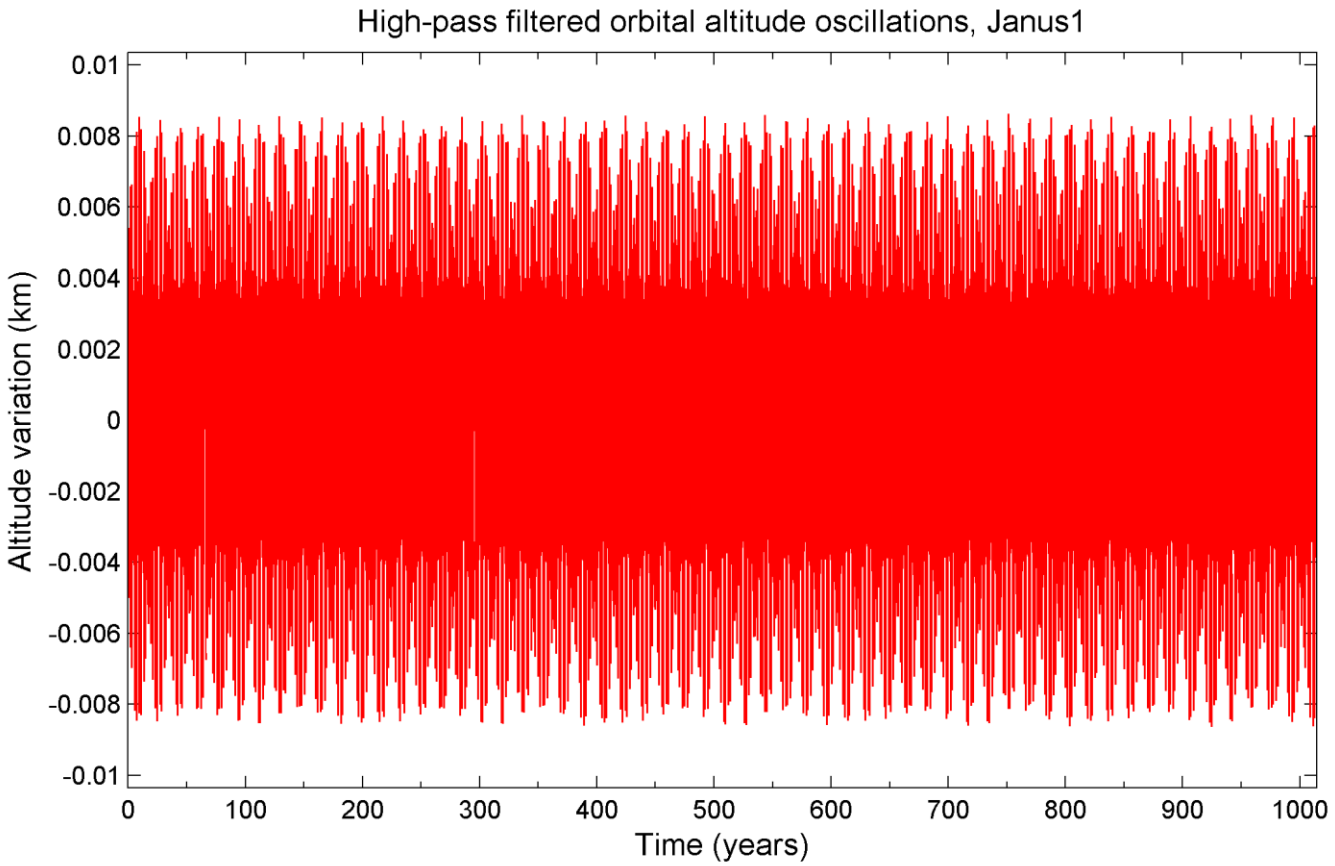
The Experiments:

Janus × 3, Δr = 150 km



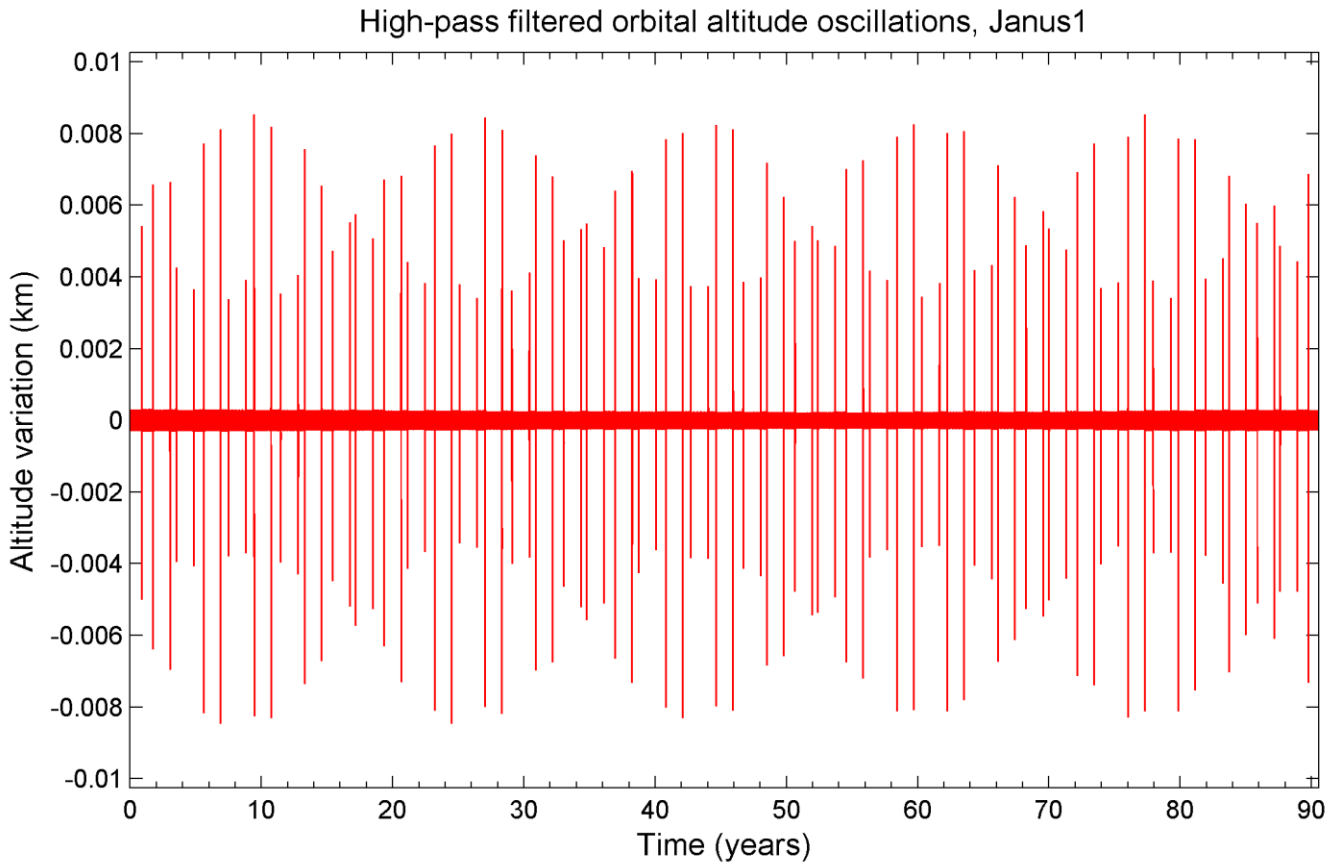
The Experiments:

Janus × 3, Δr = 150 km



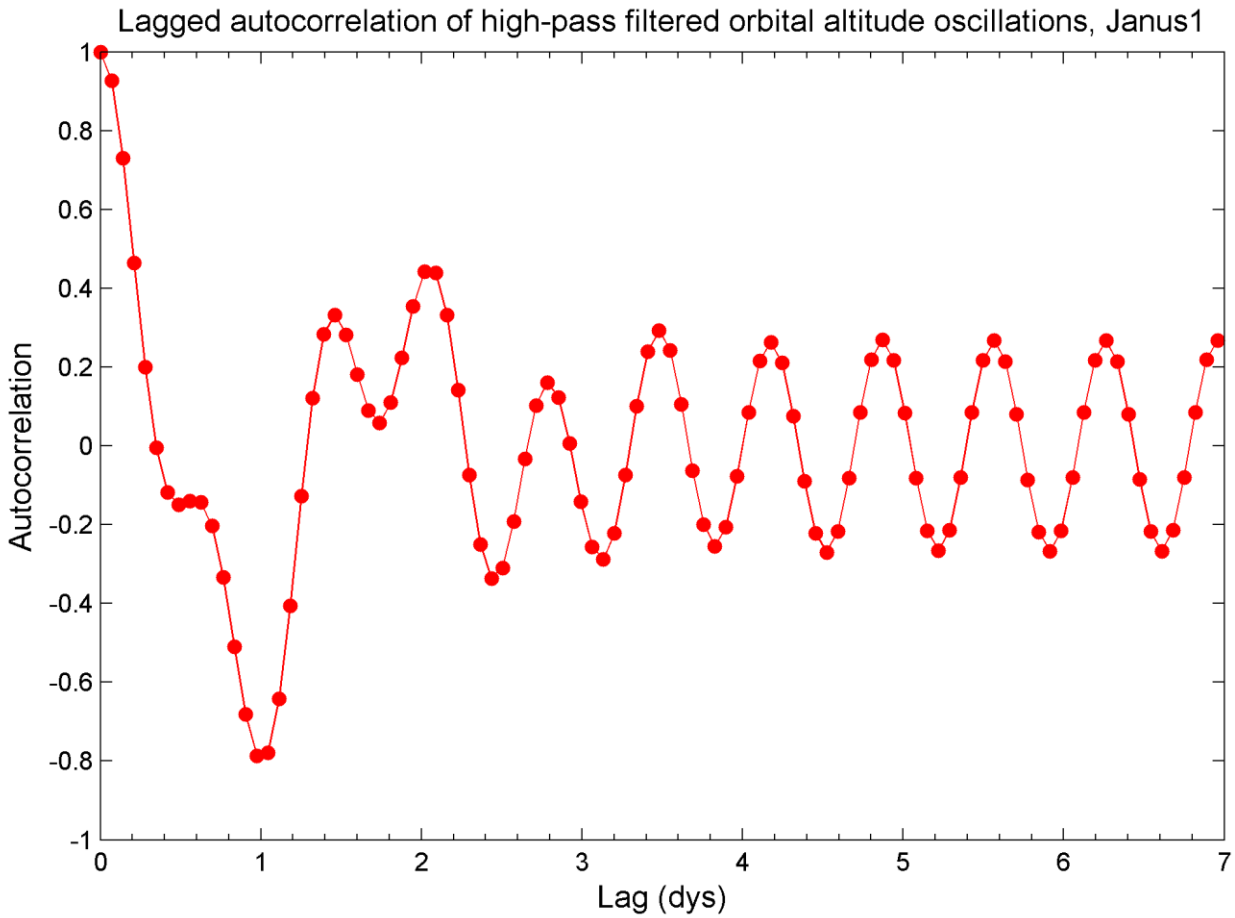
The Experiments:

Janus × 3, Δr = 150 km



The Experiments:

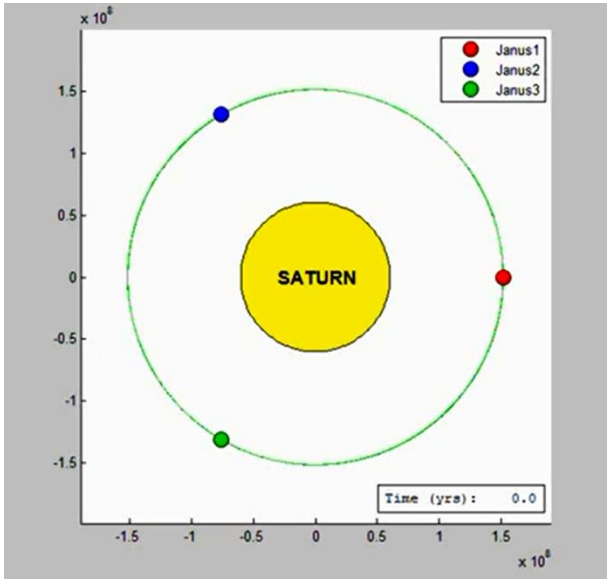
Janus × 3, Δr = 150 km



The Experiments:

Janus × 3, Δr = 200 km

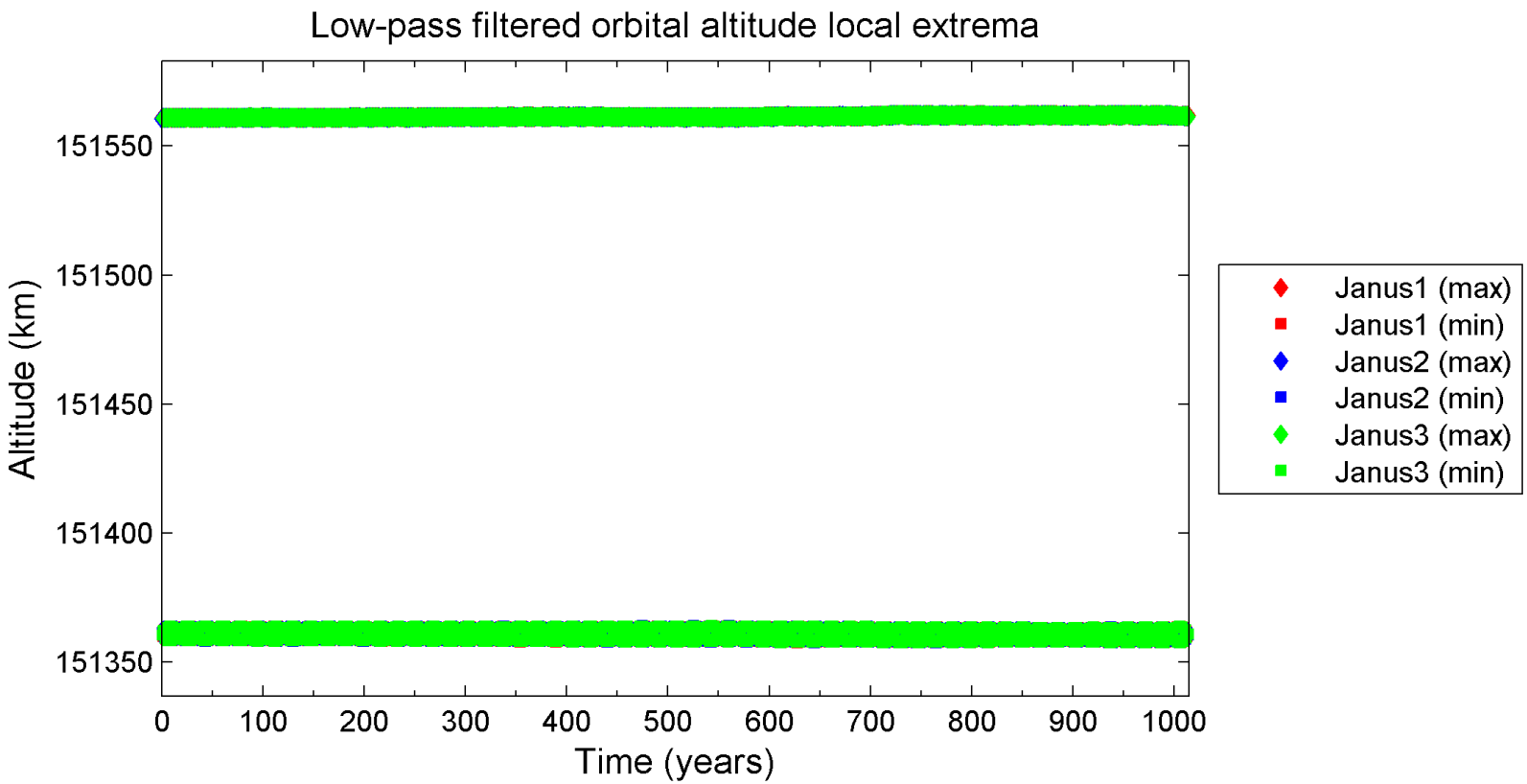
- Goal: Continue to test the limits of the *Janus × 3* system, increasing the initial altitude difference to 200 km



- Initial conditions:
  - Janus1 & Janus2 in mean orbit - 100 km
  - Janus3 in mean orbit + 100 km
  - Angular spacing: $\frac{2\pi}{3}$ radians
- Results:
  - Orbit swap cycle down to 1.4 years now
  - High & low orbit ranges now beginning to slowly diverge
  - Eccentricities have grown substantial now, trending upward over time
  - Dubious stability, likely to degrade over additional millennia

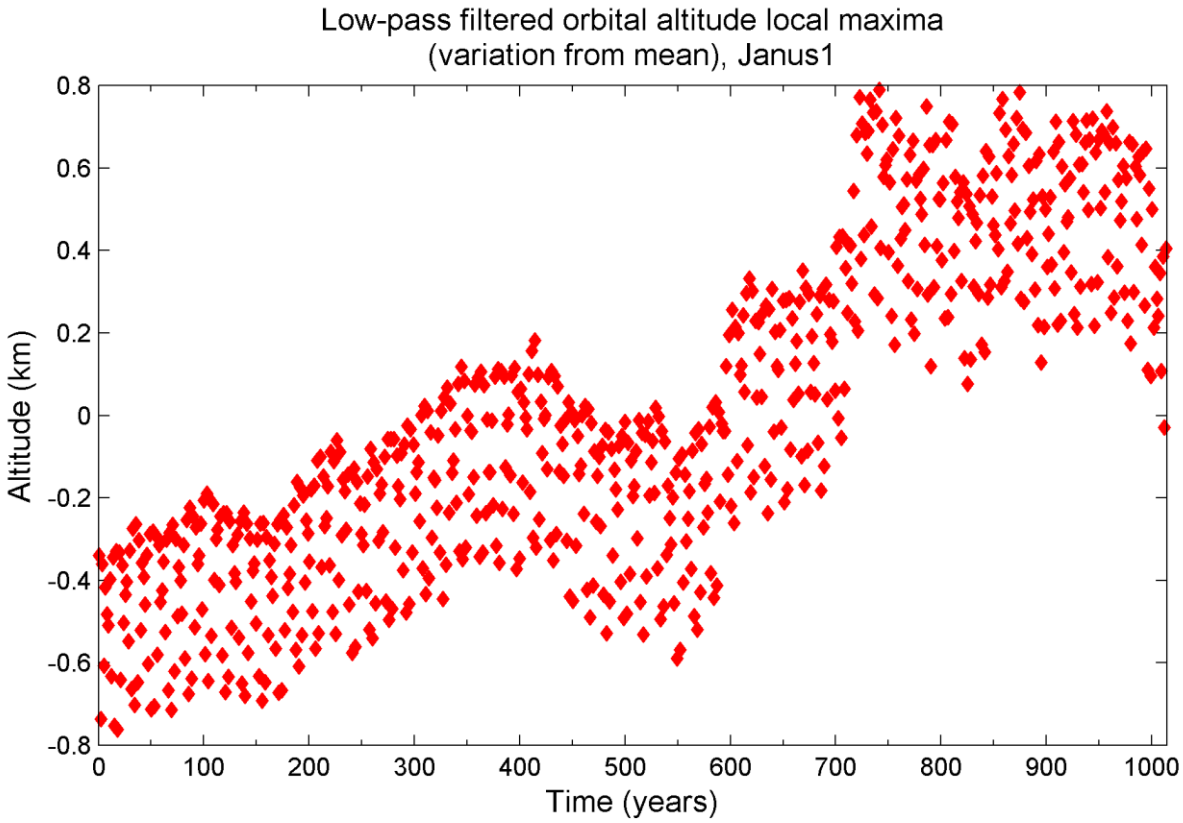
The Experiments:

Janus × 3, Δr = 200 km



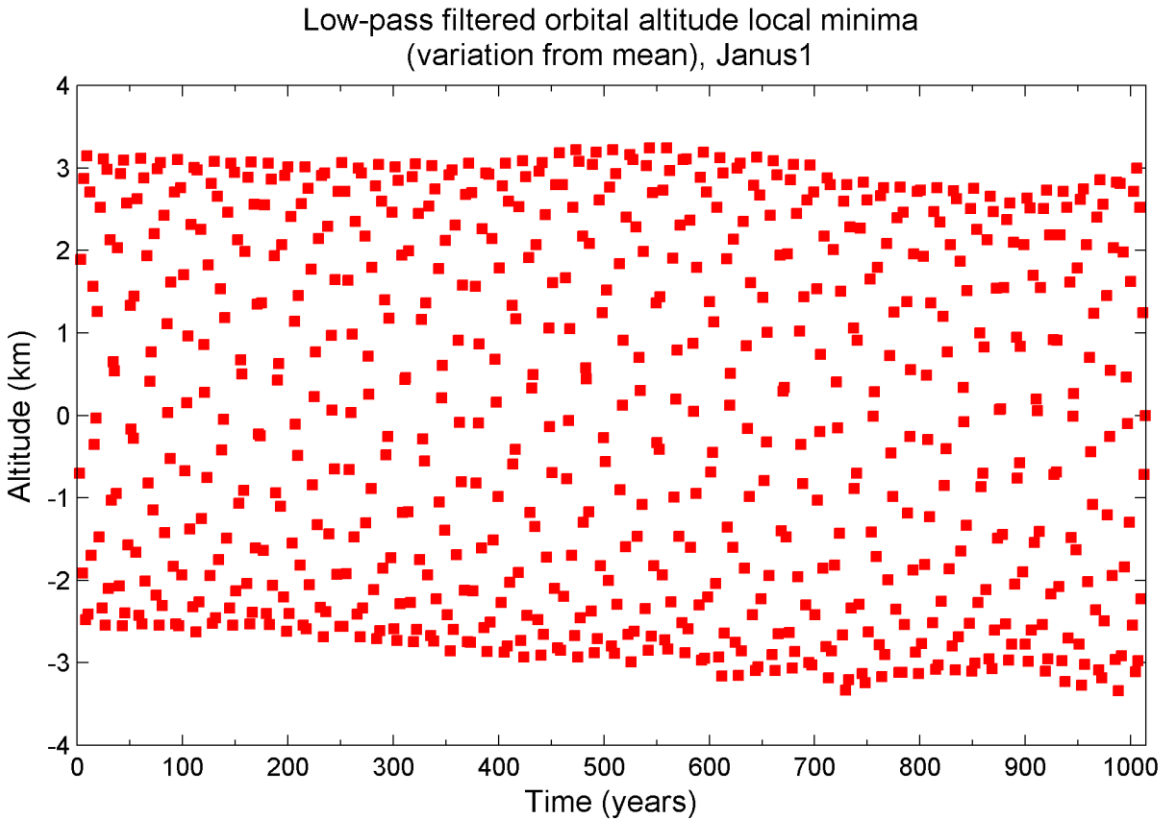
The Experiments:

Janus × 3, Δr = 200 km



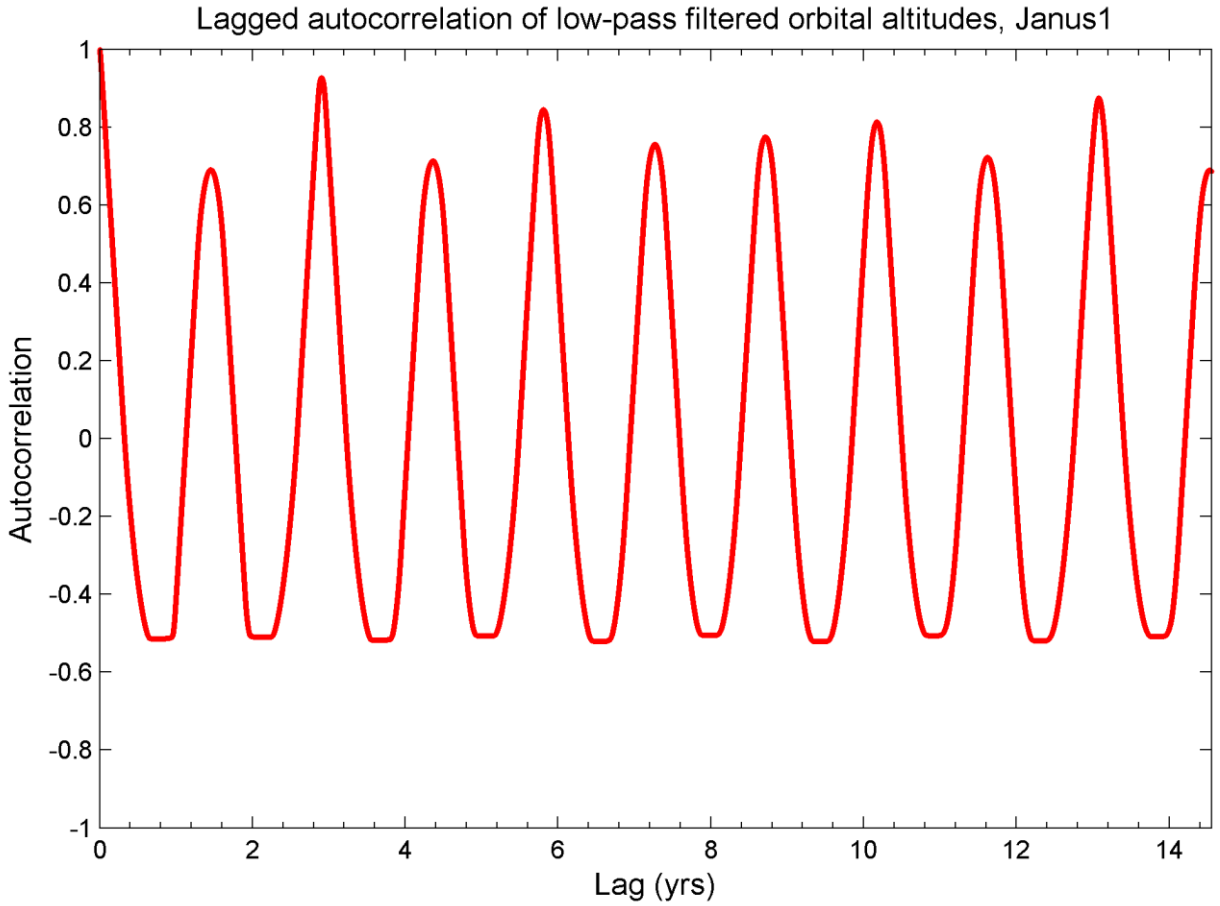
The Experiments:

Janus × 3, Δr = 200 km



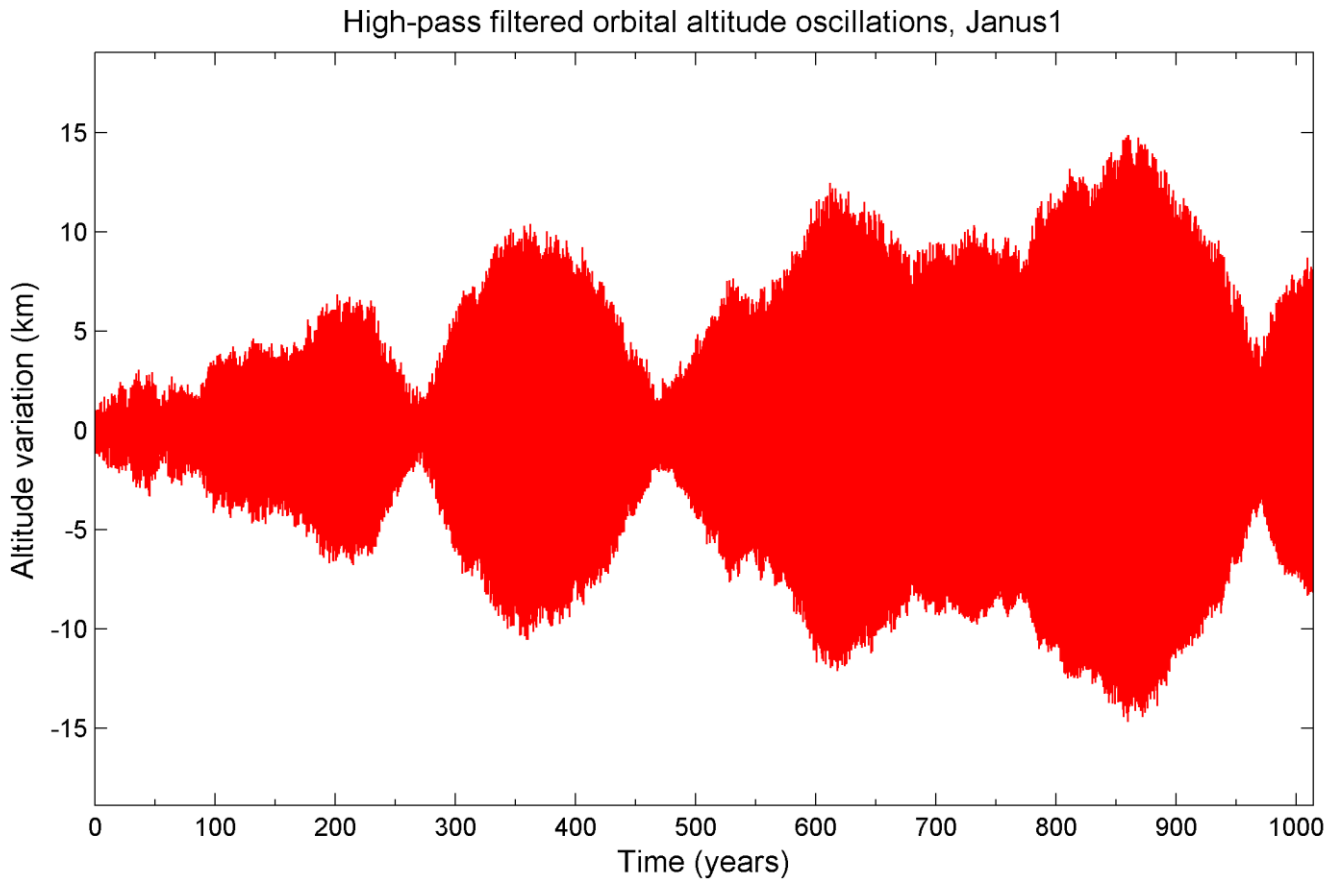
The Experiments:

Janus × 3, Δr = 200 km



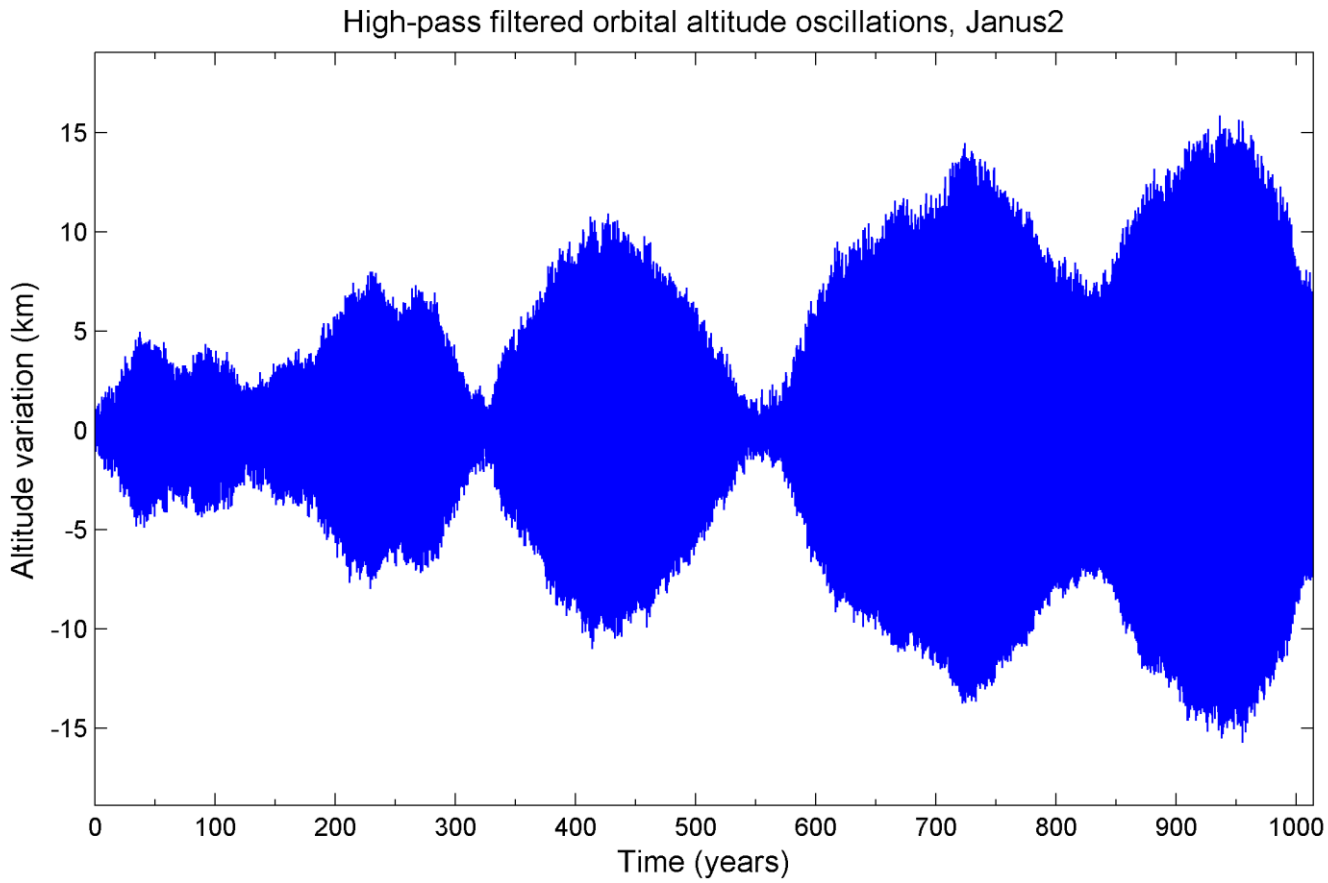
The Experiments:

Janus × 3, Δr = 200 km



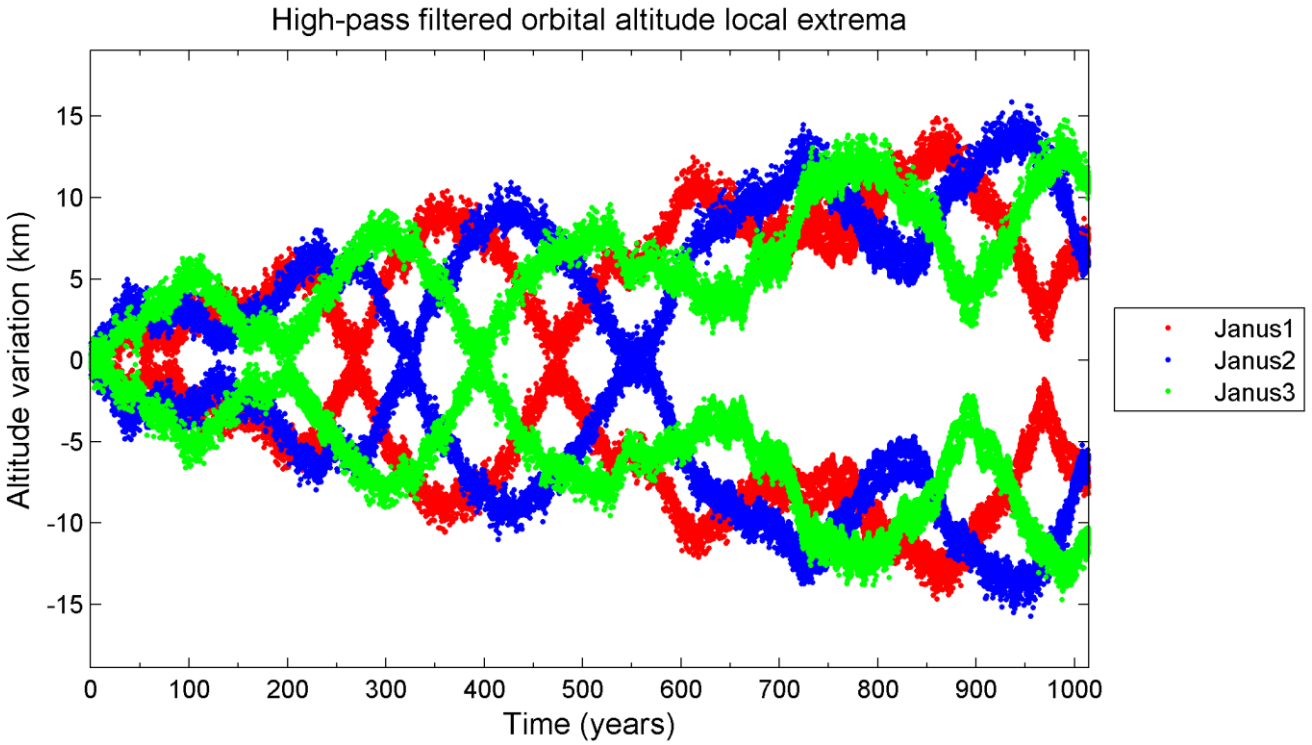
The Experiments:

Janus × 3, Δr = 200 km



The Experiments:

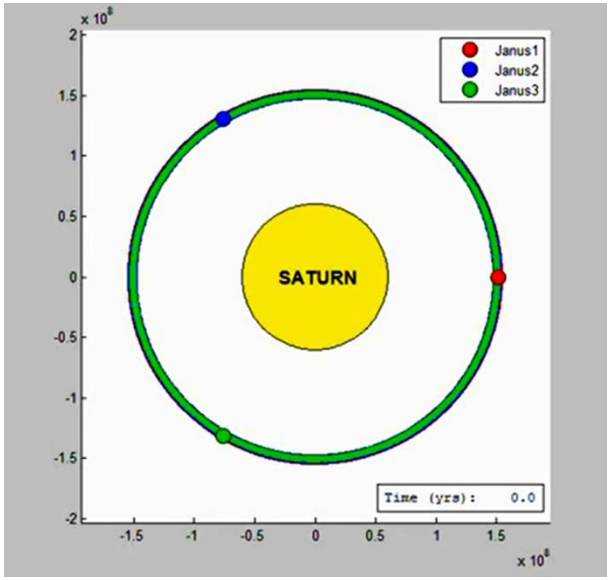
Janus × 3, Δr = 200 km



The Experiments:

Janus × 3, Δr = 225 km

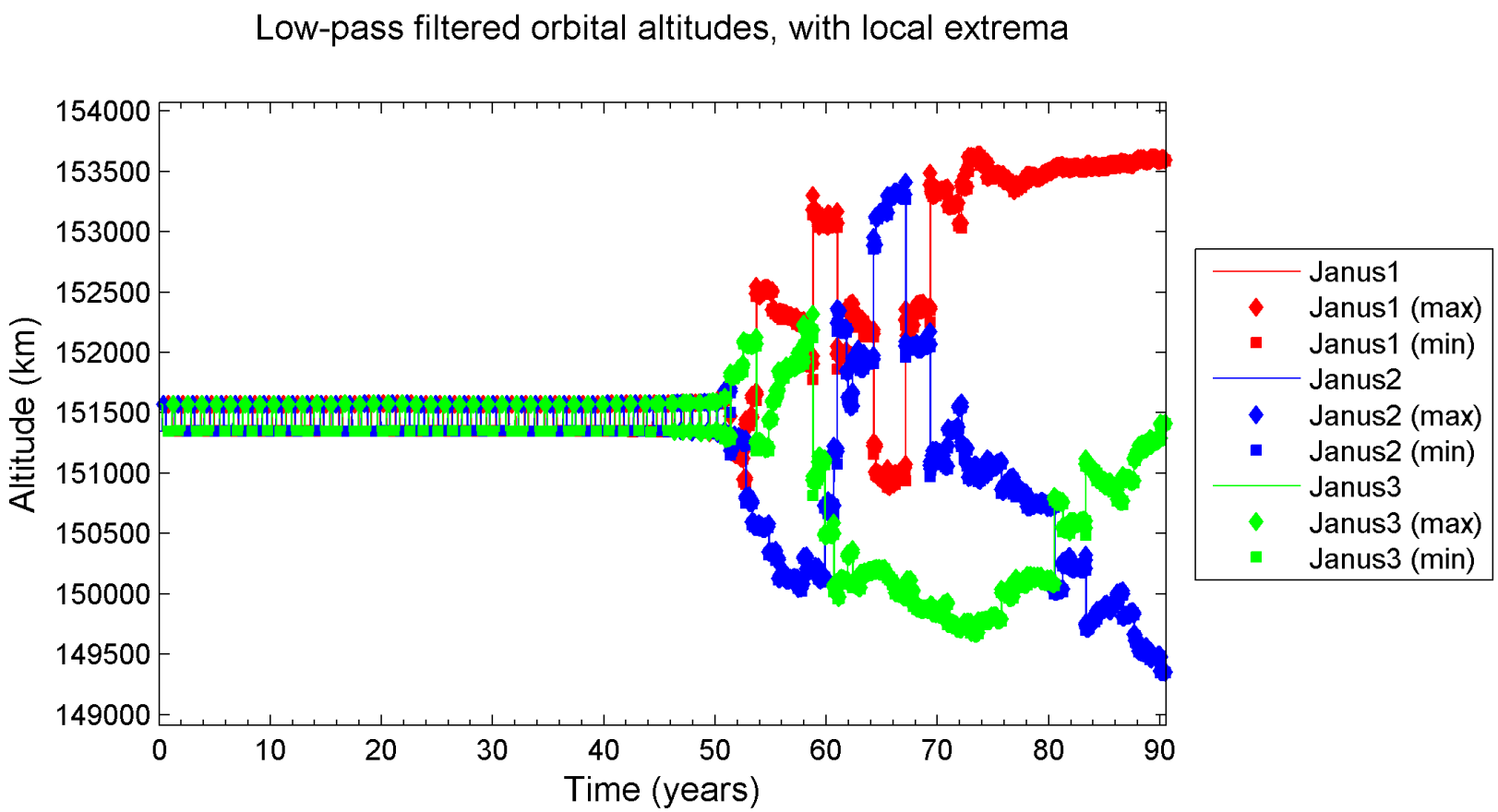
- Goal: Tiptoe into instability in the *Janus × 3* system, increasing the initial altitude difference to 225 km



- Initial conditions:
  - Janus1, Janus2 in mean orbit - 112.5 km
  - Janus3 in mean orbit + 112.5 km
  - Angular spacing: $\frac{2\pi}{3}$ radians
- Results:
  - After first 50 years, satellites escape into separate orbital levels, only fully swapping positions on rare and irregular occasions thereafter
  - After breakup, orbital heights vary by thousands of km
  - Eccentricities blow up concurrently with the orbital breakup, reaching thousands of km as well
  - Highly unstable

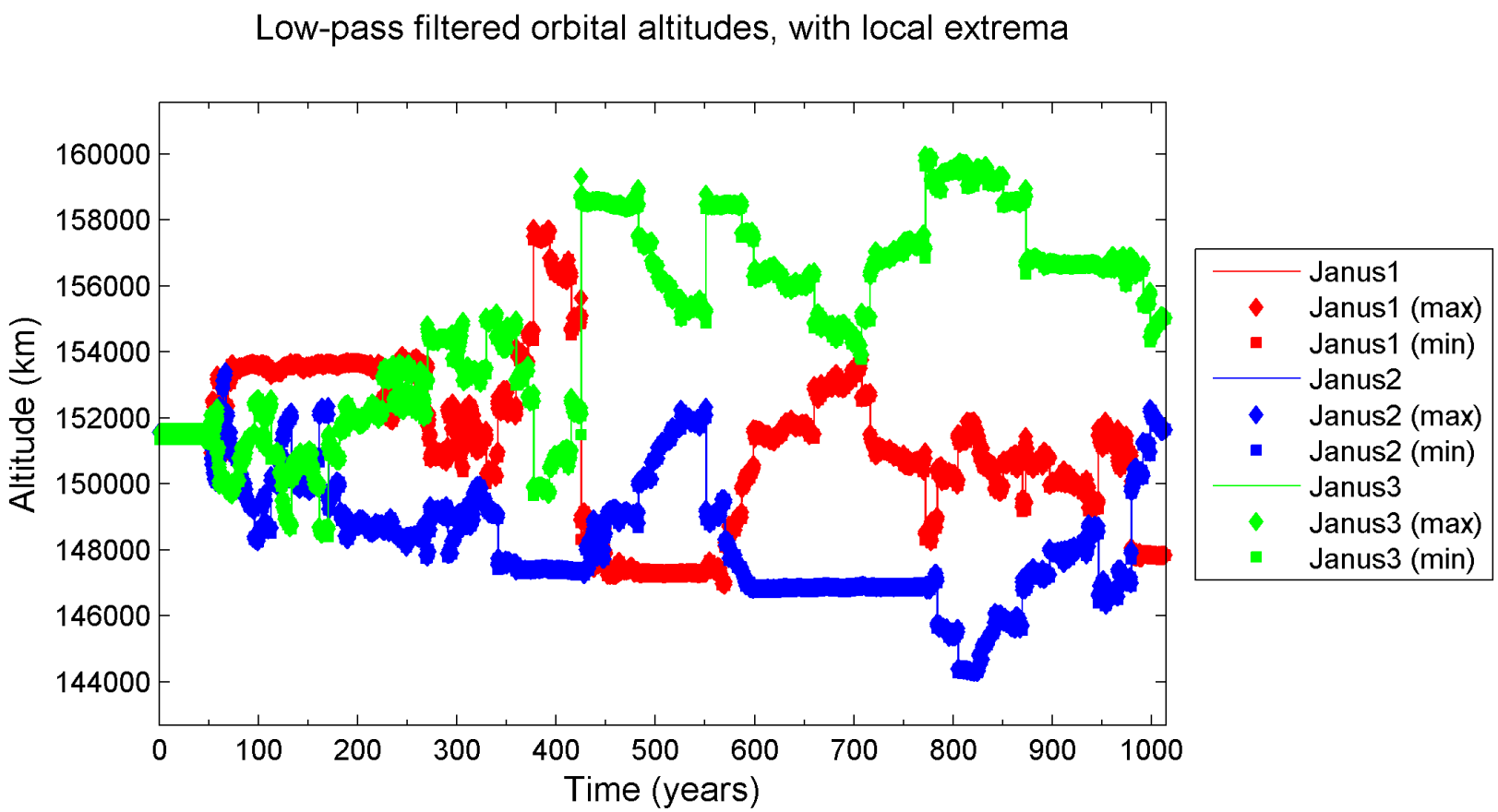
The Experiments:

Janus × 3, Δr = 225 km



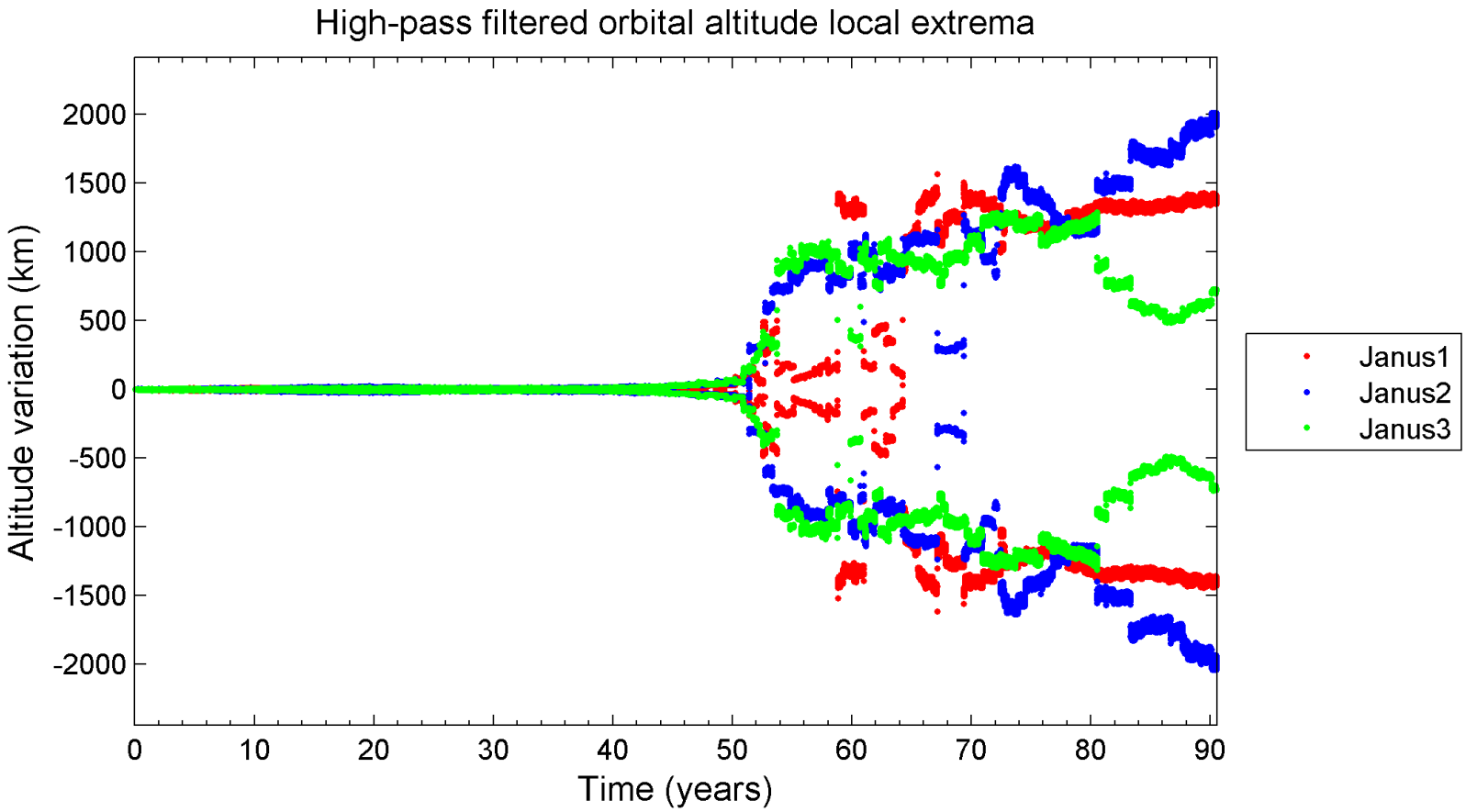
The Experiments:

Janus × 3, Δr = 225 km



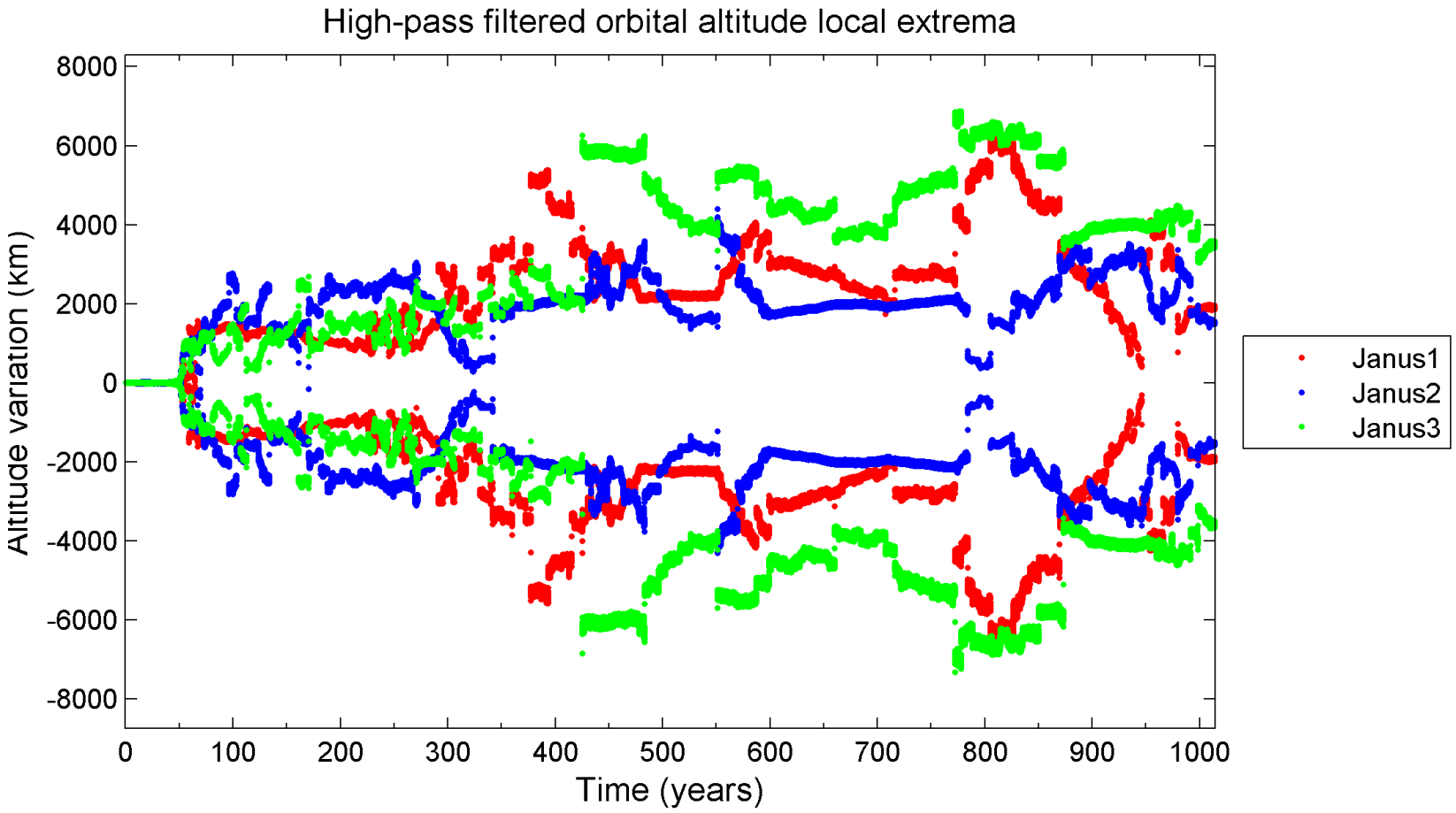
The Experiments:

Janus × 3, Δr = 225 km



The Experiments:

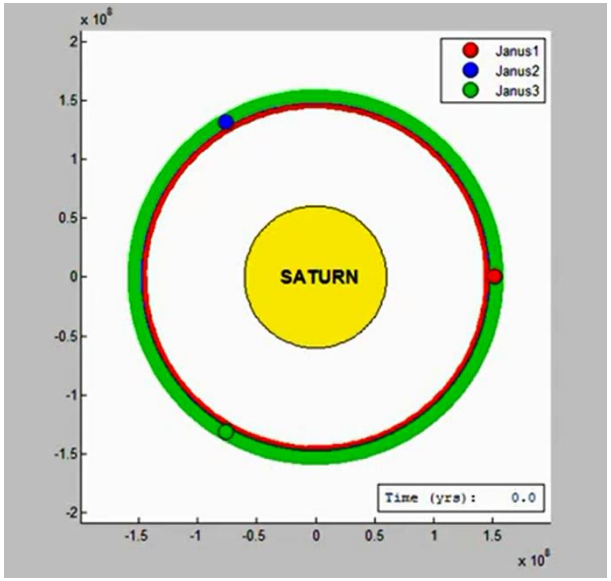
Janus × 3, Δr = 225 km



The Experiments:

Janus × 3, Δr = 250 km

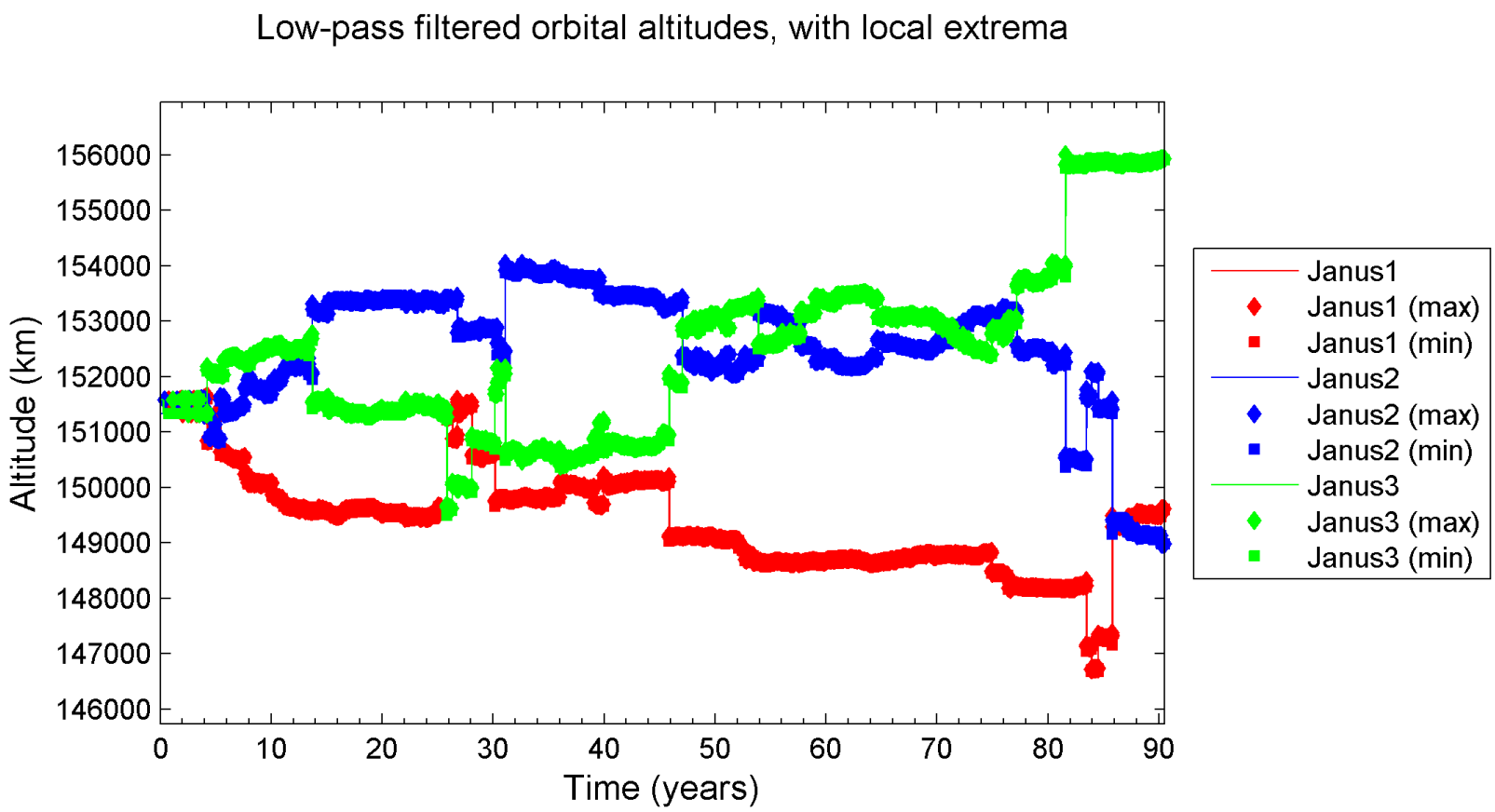
- Goal: Continue to stretch the *Janus × 3* system to the limit, increasing the initial altitude difference to 250 km



- Initial conditions:
  - Janus1, Janus2 in mean orbit - 125 km
  - Janus3 in mean orbit + 125 km
  - Angular spacing: $\frac{2\pi}{3}$ radians
- Results:
  - Orbits diverge almost immediately (~ 4 yrs)
  - Similar erratic divergence pattern to the previous case
  - Eccentricities blow up as well
  - Highly unstable – could not integrate to 1000 years because solver crashed (could not meet tolerance)

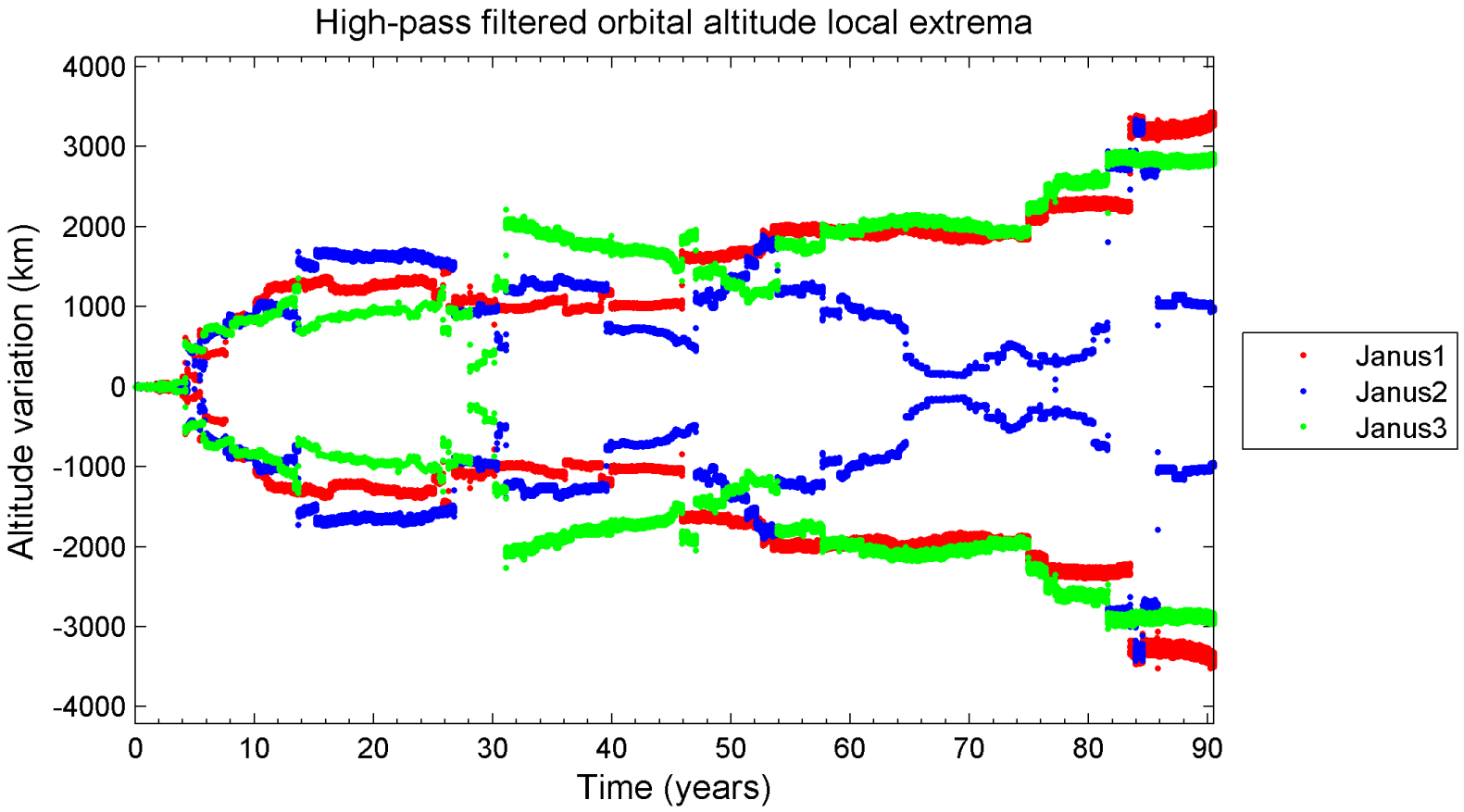
The Experiments:

Janus × 3, Δr = 250 km



The Experiments:

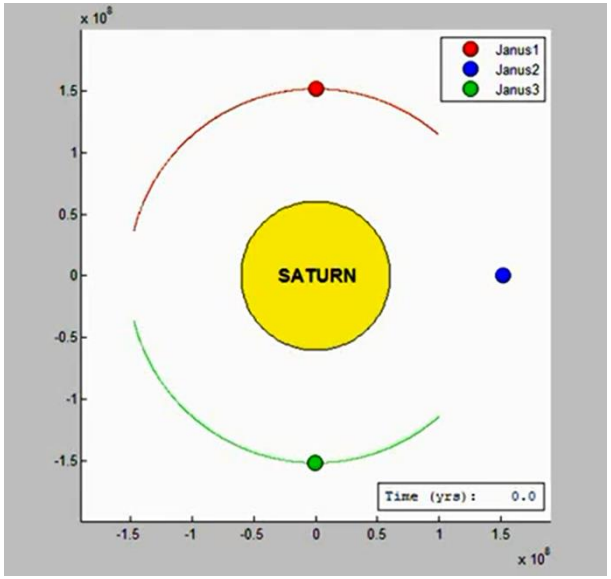
Janus × 3, Δr = 250 km



The Experiments:

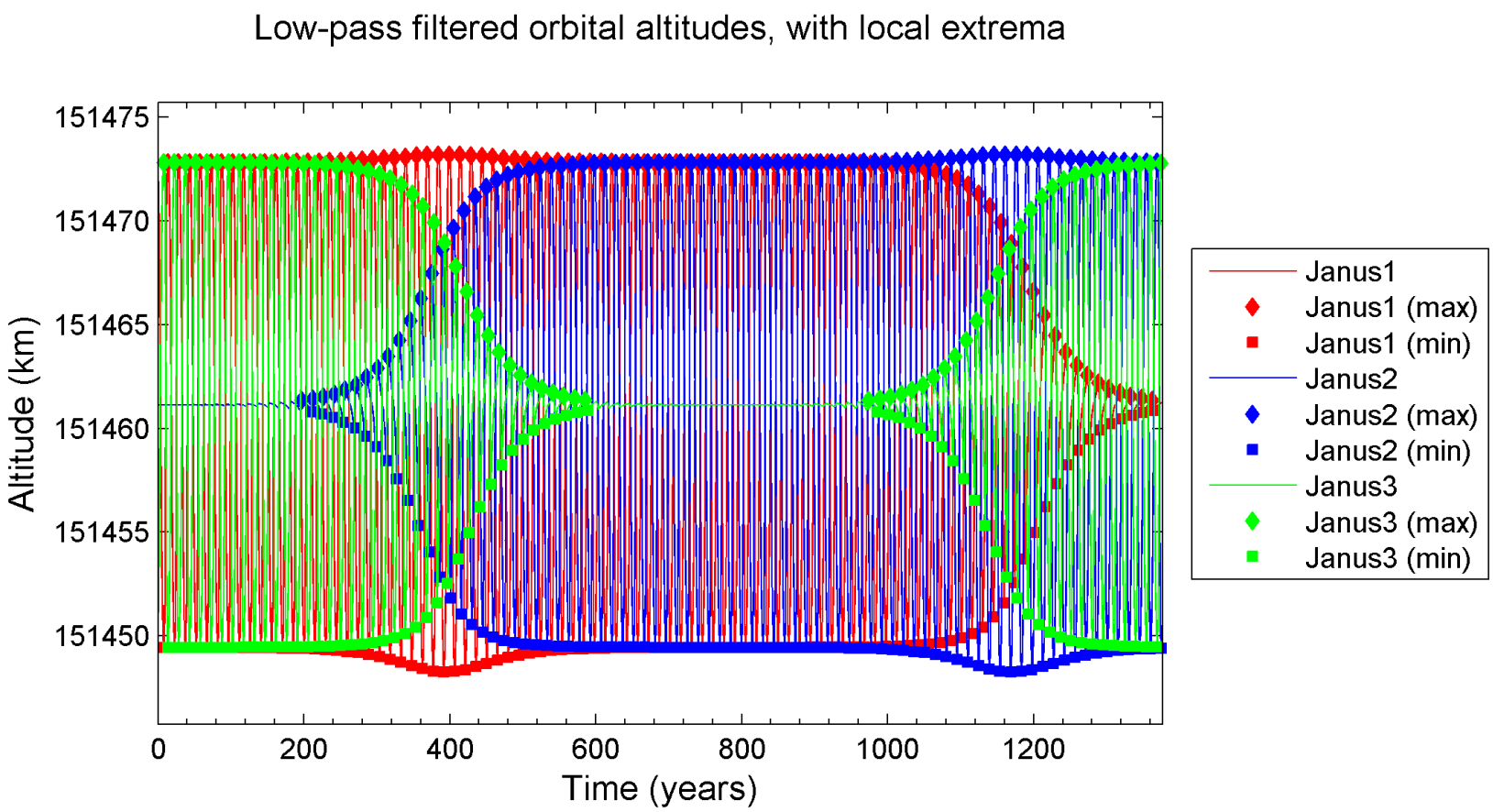
Janus × 3, symmetric orbits

- Goal: See if symmetry & stability can be preserved when Janus × 3 are initialized in a pattern designed for symmetry
- Initial conditions:
  - Janus1 in standard upper orbit, $\theta = \frac{\pi}{2}$
  - Janus2 in mean orbit, $\theta = 0$
  - Janus3 in standard lower orbit, $\theta = -\frac{\pi}{2}$
- Results:
  - Symmetry keeps Janus2 stationary in corotating frame, at first
  - Once Janus2 starts seriously oscillating, Janus3 dampens and takes a turn being the stationary one, then Janus 1
  - Eccentricities remain near zero, although growing slightly over time
  - System remains quite stable over the 1000-year time frame



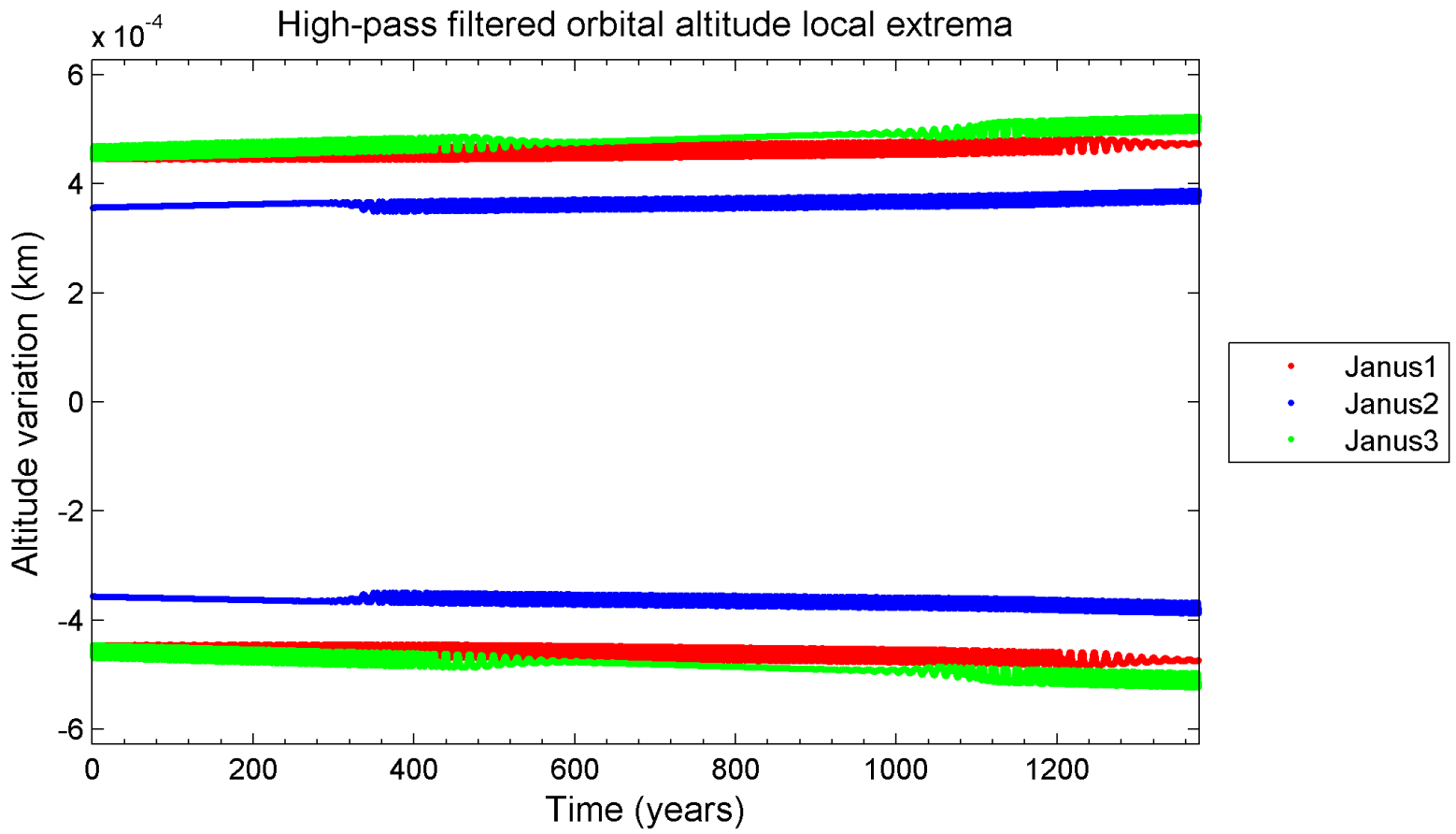
The Experiments:

Janus × 3, symmetric orbits



The Experiments:

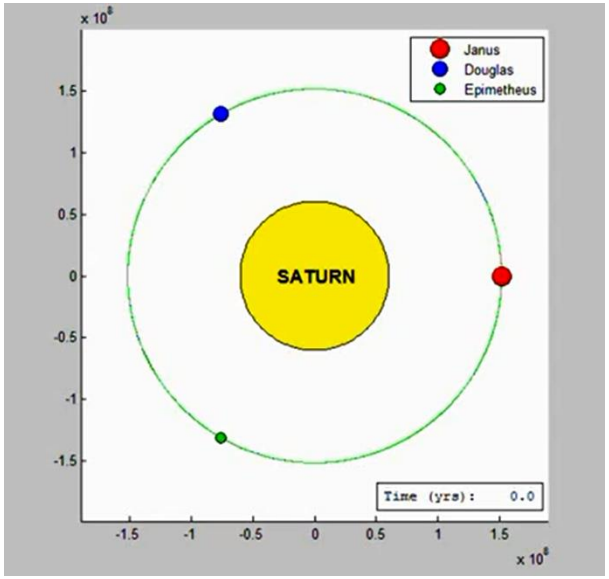
Janus × 3, symmetric orbits



The Experiments:

Janus, Epimetheus & Douglas

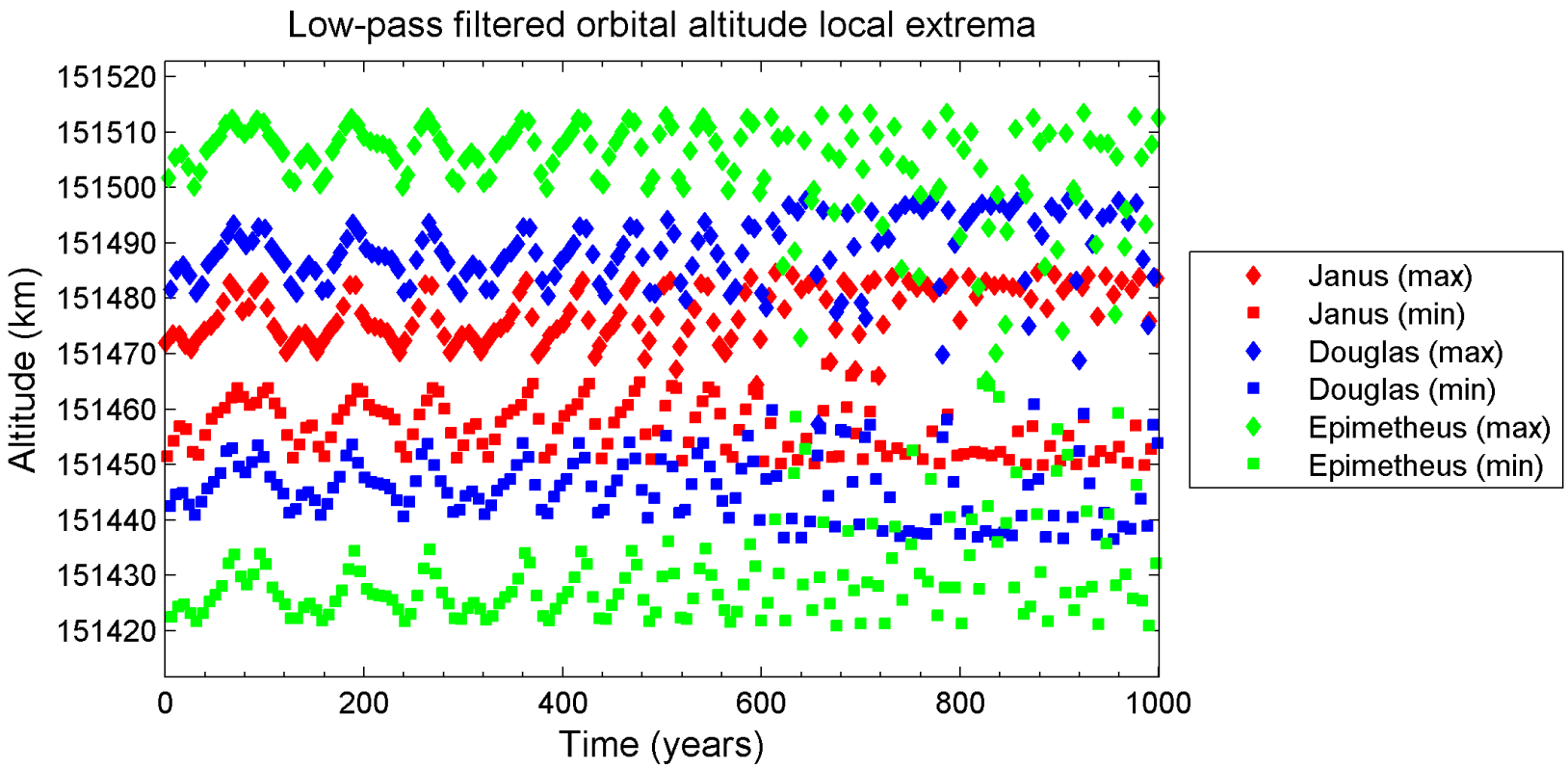
- Goal: Explore dynamics & stability when a 3rd moon of intermediate mass is injected into the Janus-Epimetheus system
- Initial conditions:
  - Janus in standard upper orbit
  - Epimetheus in standard lower orbit
  - Douglas in an upper orbit commensurate with its mass
  - Angular spacing: $\frac{2\pi}{3}$ radians



- Results:
  - Differing masses produce interesting momentum-exchange curves
  - System evolves through variety of different orbit-swap patterns
  - Eccentricities evolve differently over time, but remain very low (near zero)
  - System remains stable over 1000-year span

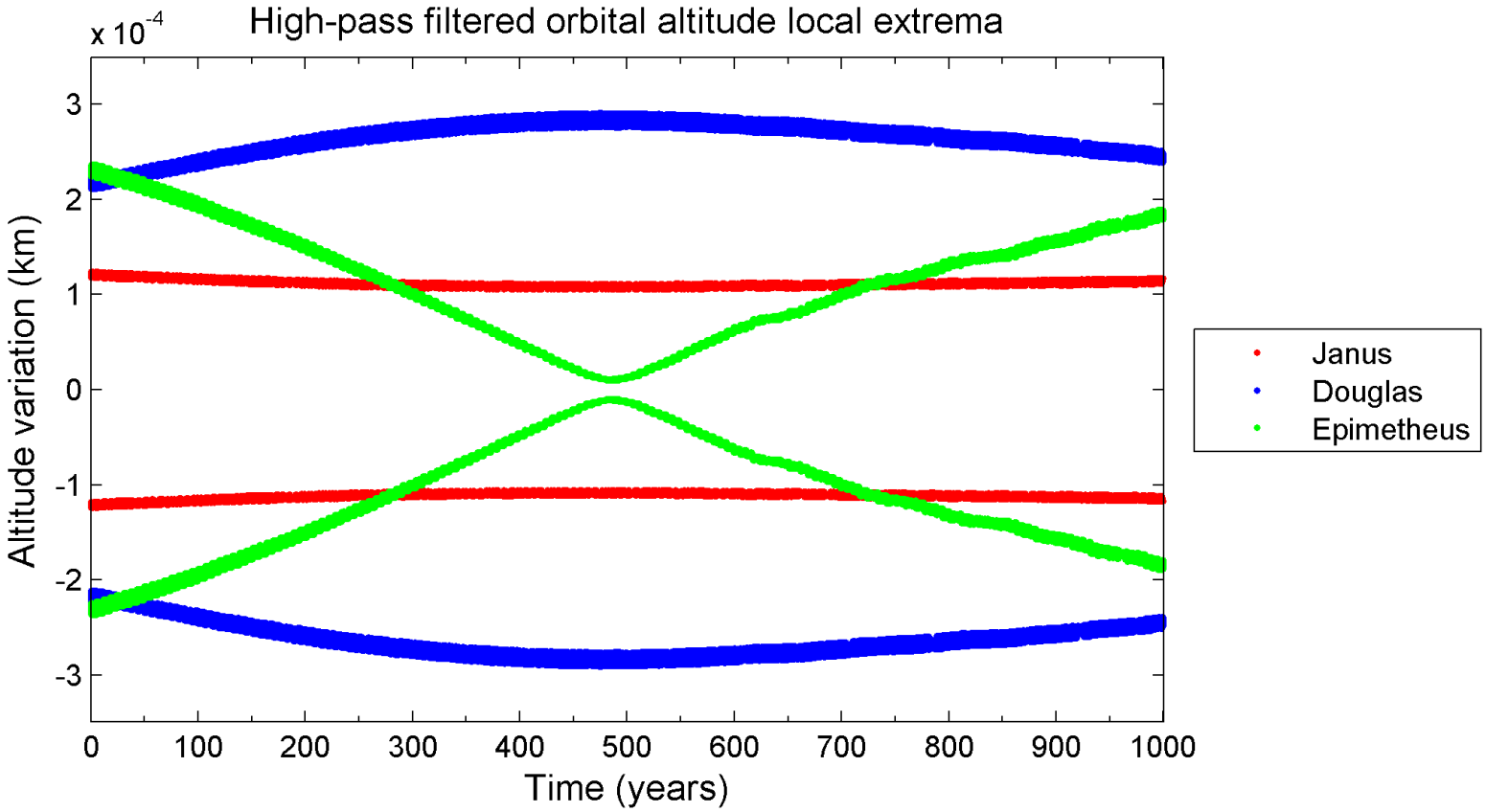
The Experiments:

Janus, Epimetheus & Douglas



The Experiments:

Janus, Epimetheus & Douglas

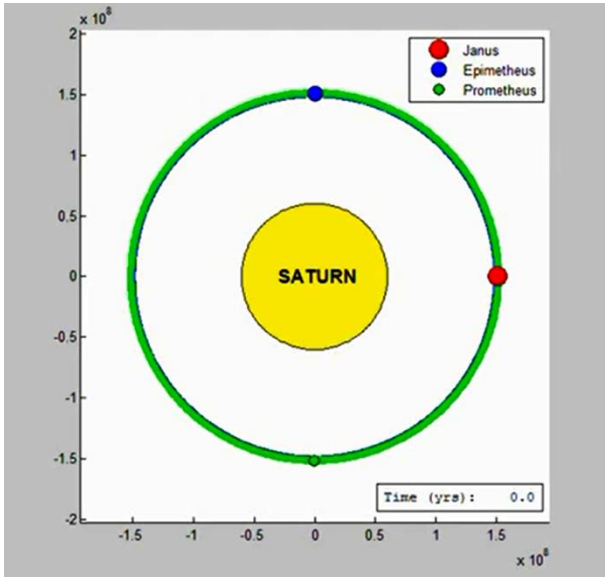


The Experiments:

Janus, Epimetheus & Prometheus

- Goal: Explore dynamics & stability when a 3rd moon significantly smaller than Epimetheus is injected into the Janus-Epimetheus system

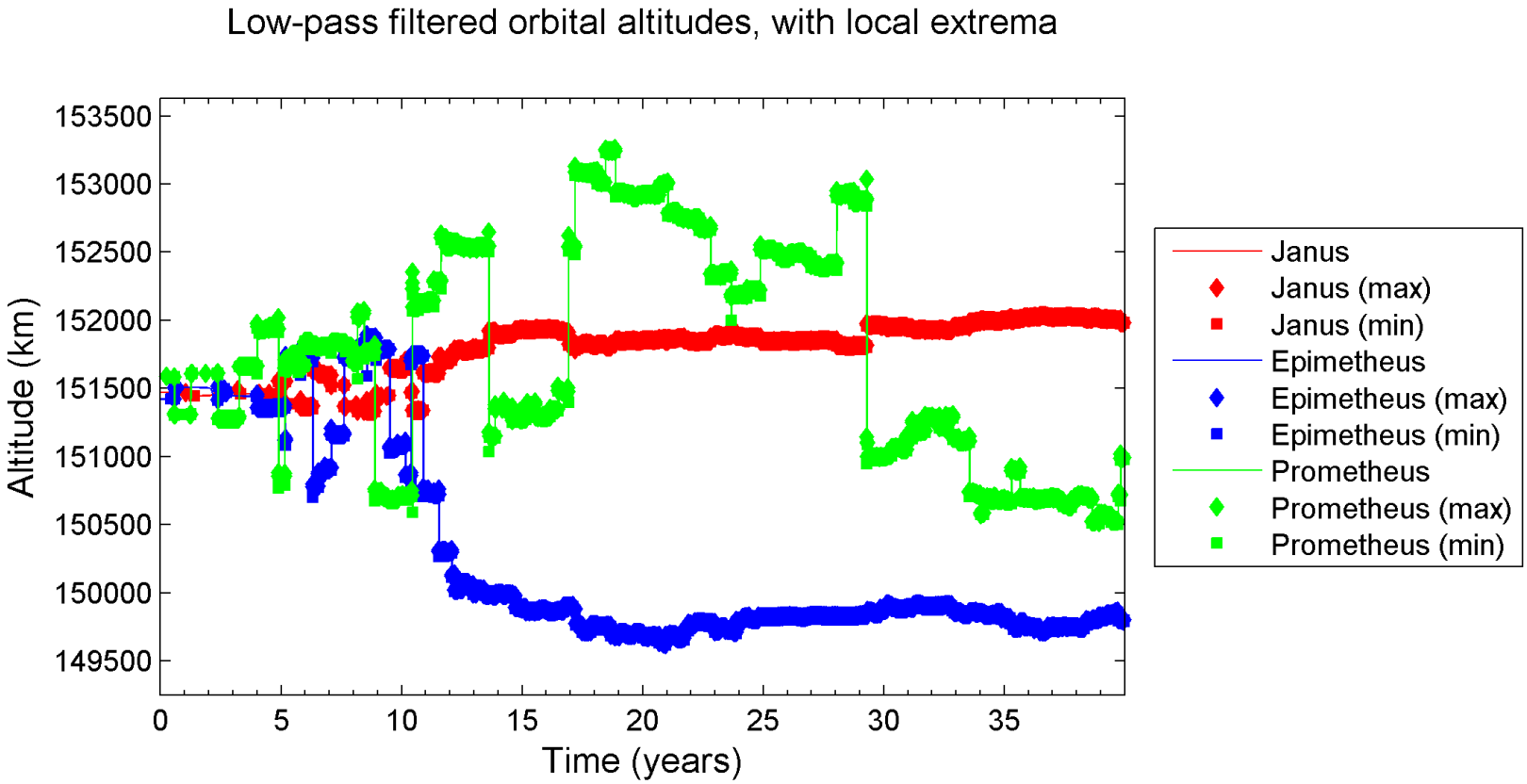
- Initial conditions:
  - Janus in standard upper orbit
  - Epimetheus in standard lower orbit
  - Prometheus in an upper orbit commensurate with its mass
  - Angular spacing: $\frac{\pi}{2}$ radians



- Results:
  - Prometheus breaks out of orbital swap pattern very quickly (~ 4 yrs)
  - After ~ 11 years, all bodies have diverged, and rarely swap orbits again
  - Eccentricities blow up along with the orbital divergence
  - Highly unstable – could not integrate to 1000 years because solver crashed (could not meet tolerance)

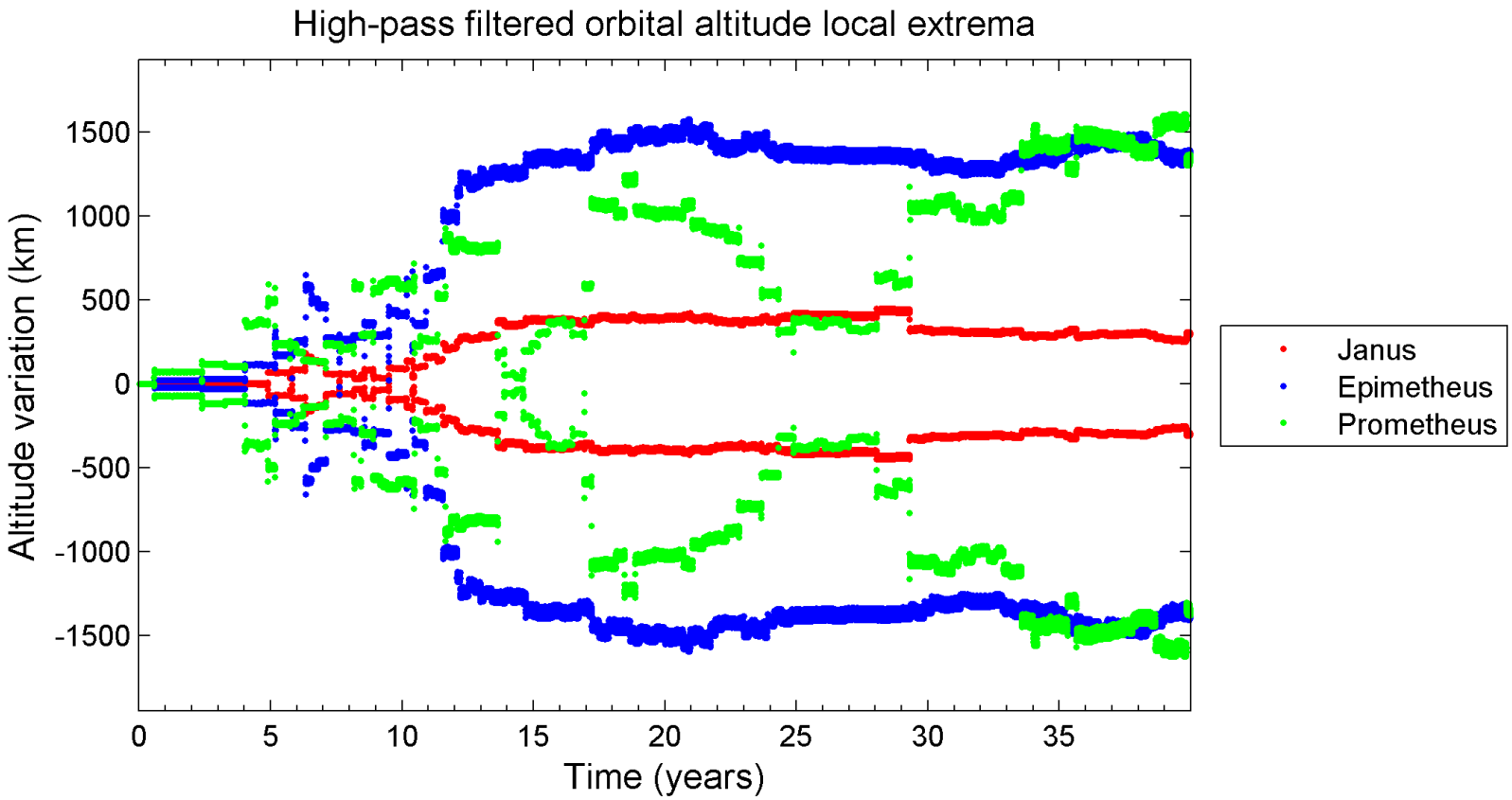
The Experiments:

Janus, Epimetheus & Prometheus



The Experiments:

Janus, Epimetheus & Prometheus



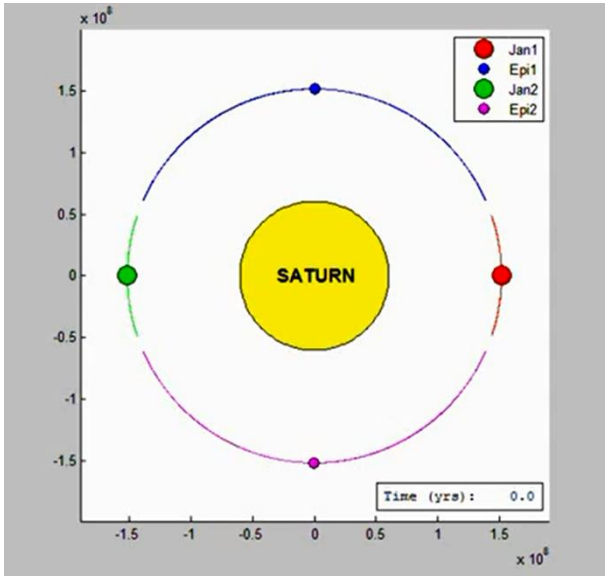
The Experiments:

Janus & Epimetheus, × 2

- Goal: See if symmetry & stability are preserved in a system with 2 Janus-sized moons & 2 Epimetheus-sized ones, set up for radial symmetry

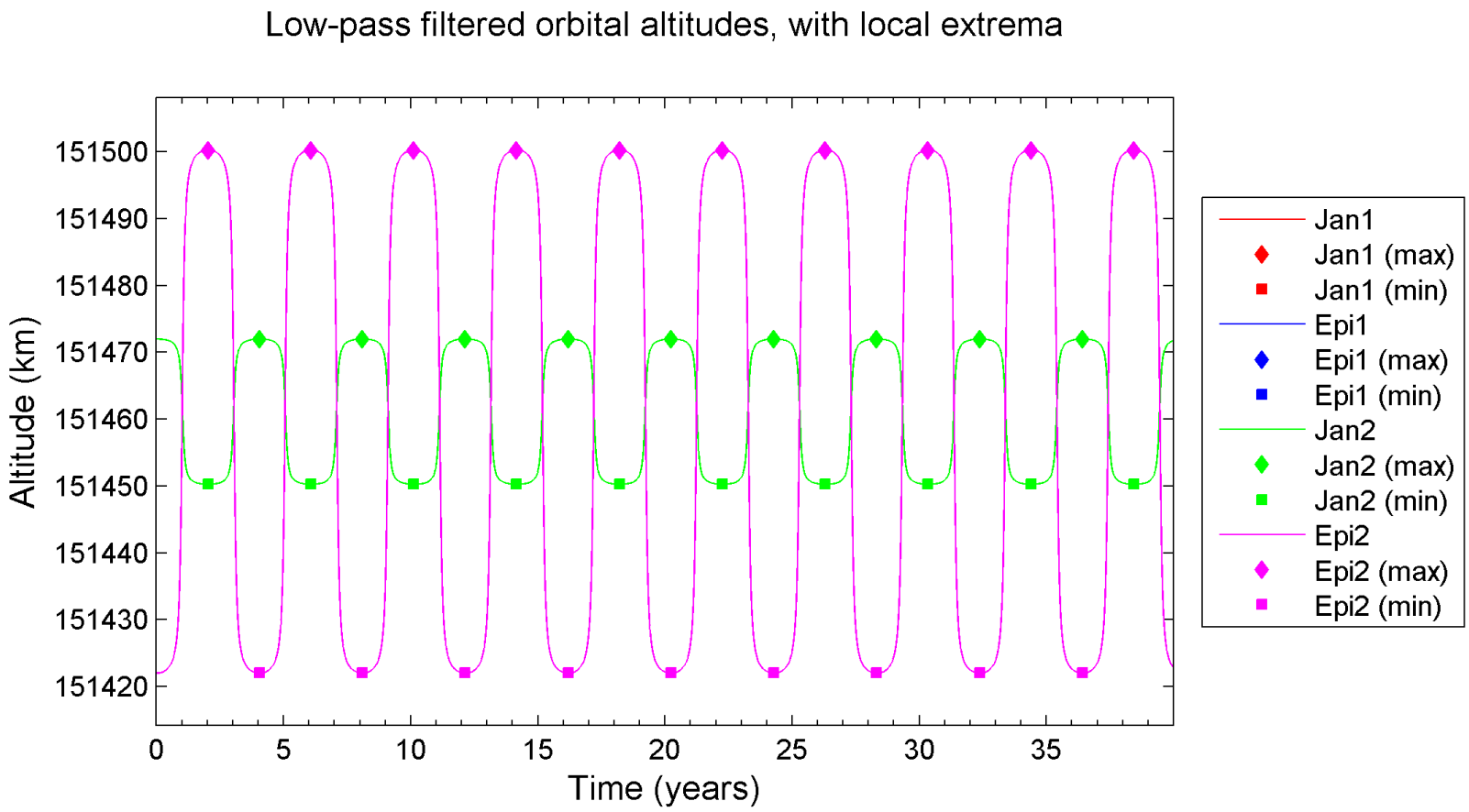
- Initial conditions:
  - Janus 1 & 2 in standard upper orbits
  - Epimetheus 1 & 2 in standard lower orbits, alternating with the Januses
  - Angular spacing: $\frac{\pi}{2}$ radians

- Results:
  - Radial symmetry is well preserved
  - Orbit swap patterns remain perfectly consistent & symmetric
  - Eccentricities stay flat, near zero
  - System is exceedingly stable



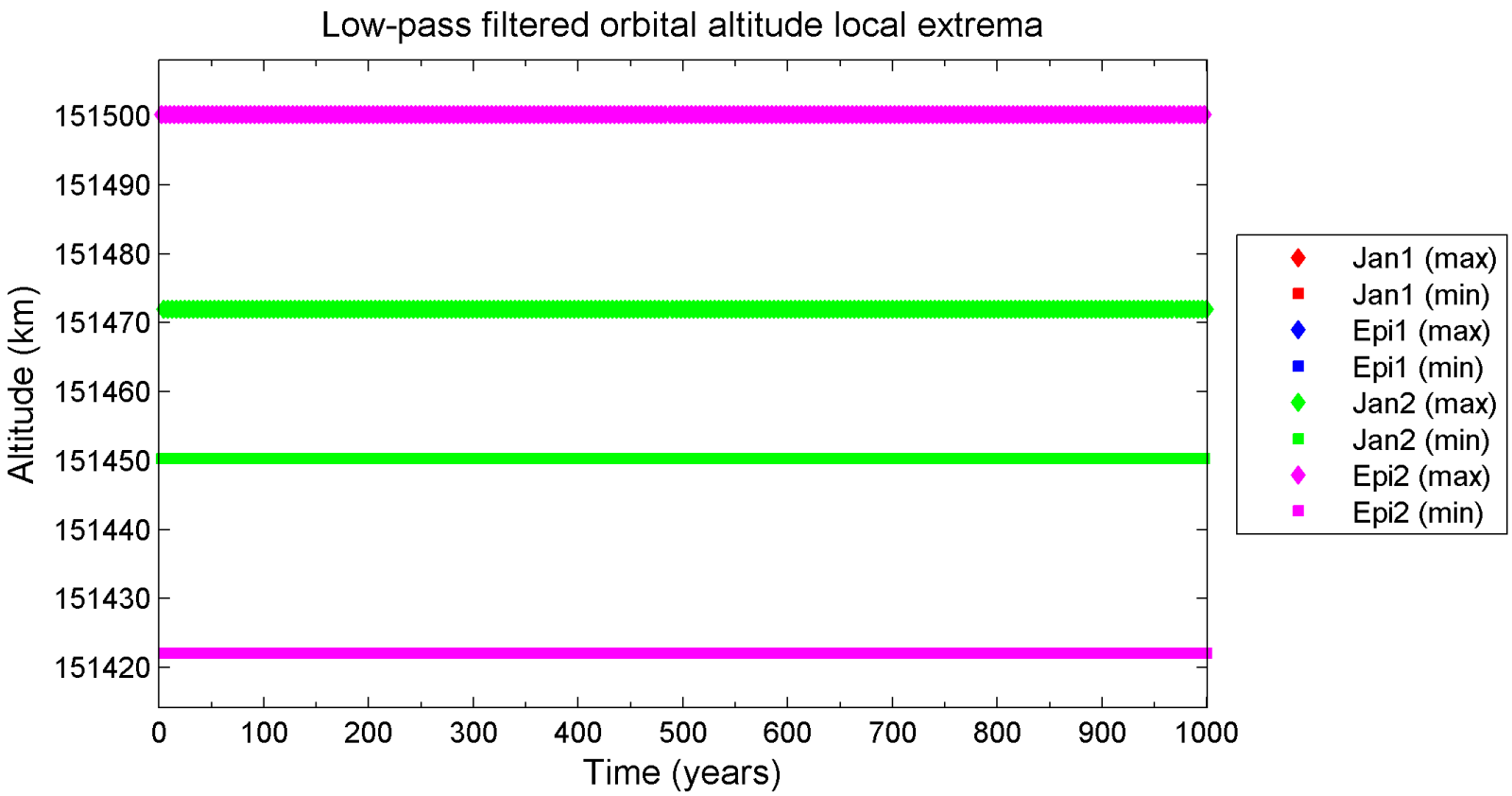
The Experiments:

Janus & Epimetheus, × 2



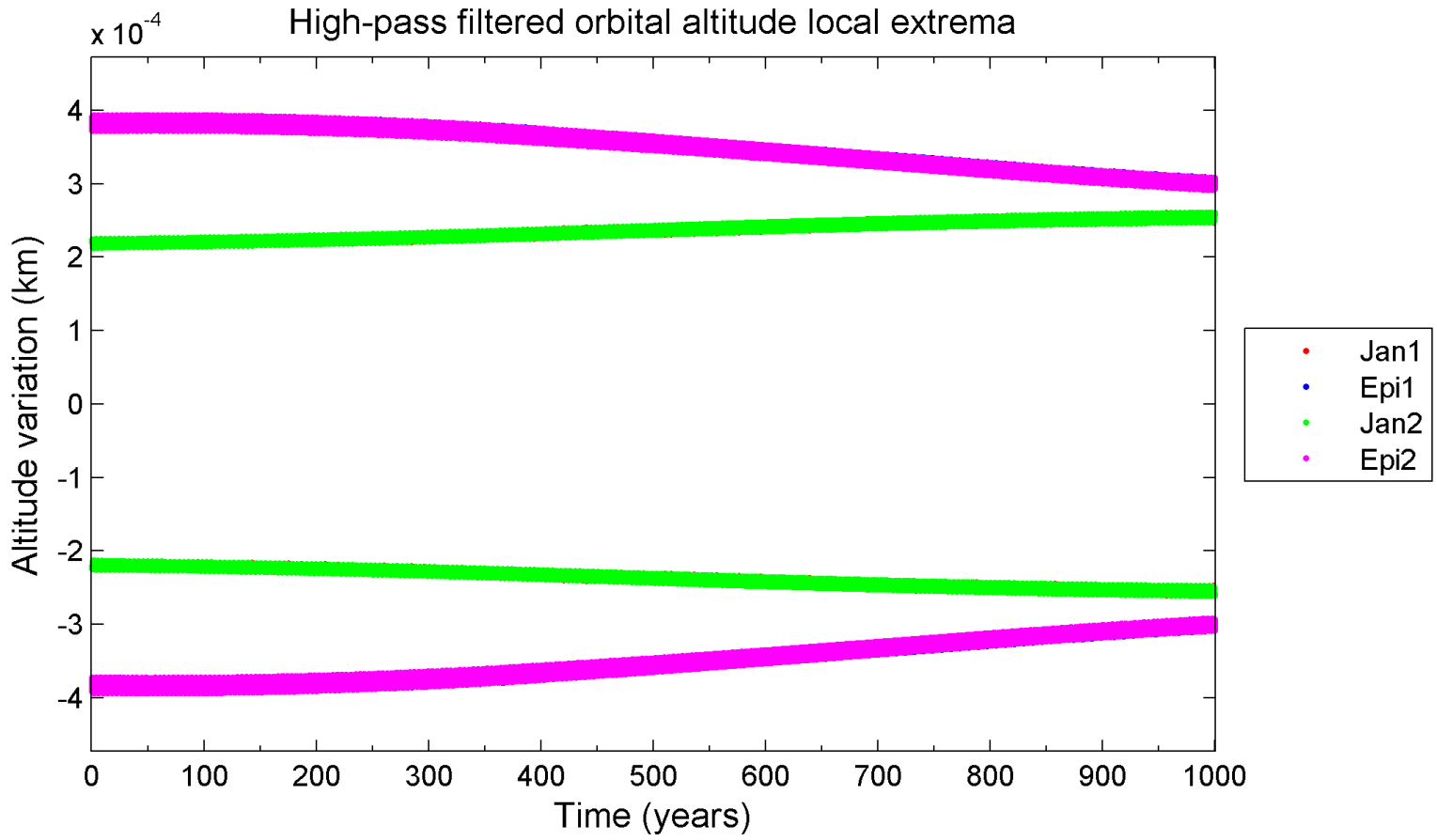
The Experiments:

Janus & Epimetheus, × 2



The Experiments:

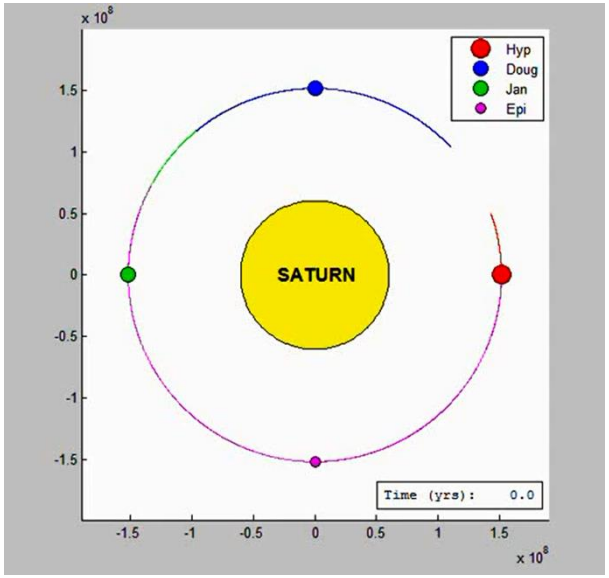
Janus & Epimetheus, × 2



The Experiments:
Janus, Epimetheus & 2 Friends

- Goal: Explore dynamics & stability of a 4-satellite co-orbital system: Janus, Epimetheus, Hyperion & Douglas

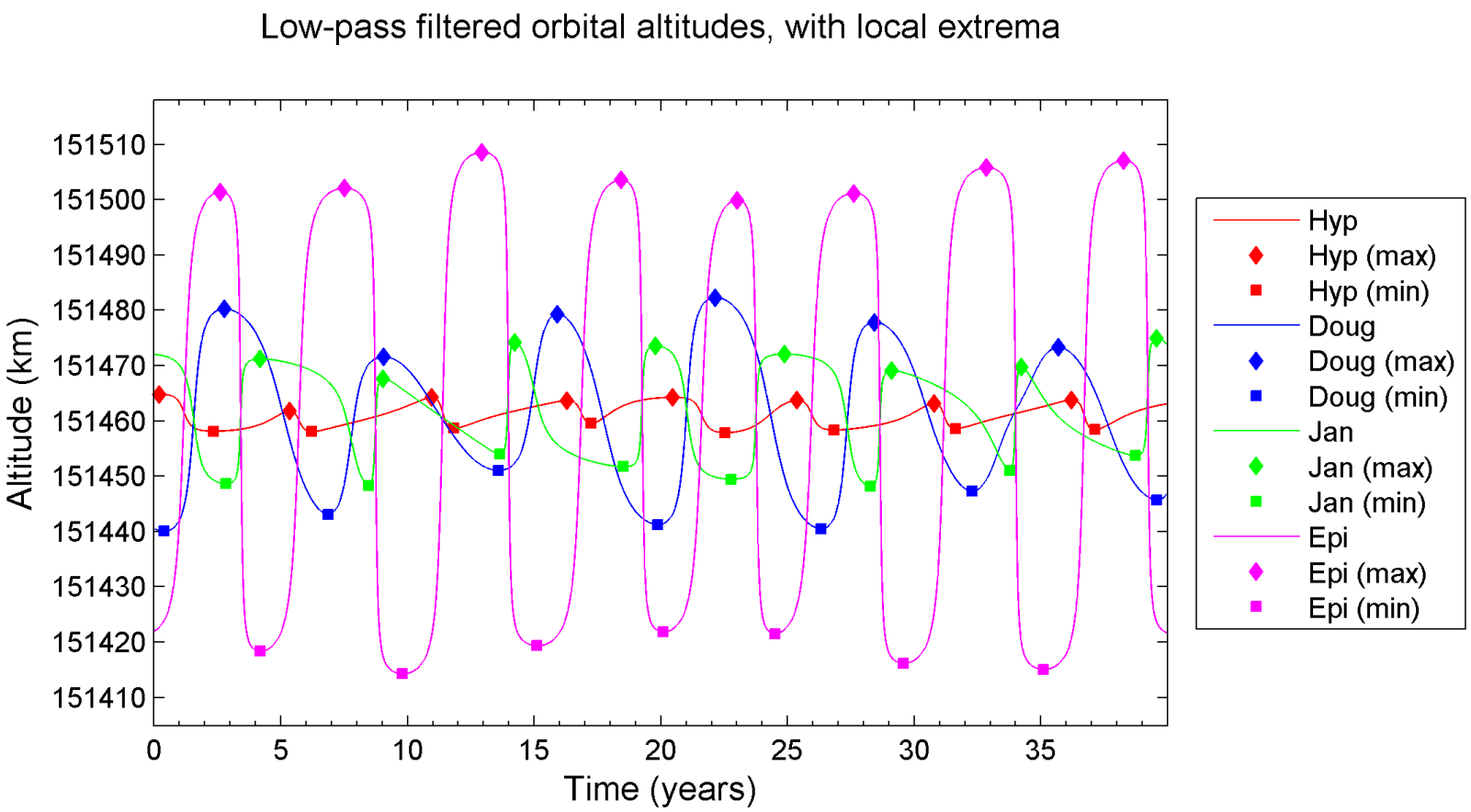
- Initial conditions:
  - Janus & Epimetheus in standard orbits
  - Hyperion (larger than Janus) & Douglas (intermediate mass between J & E) in orbits commensurate with their masses
  - Angular spacing: $\frac{\pi}{2}$ radians



- Results:
  - Differing masses produce interesting momentum-exchange curves
  - System evolves through variety of different orbit-swap patterns
  - Eccentricities evolve differently over time, but remain very low
  - System remains stable over 1000-year span

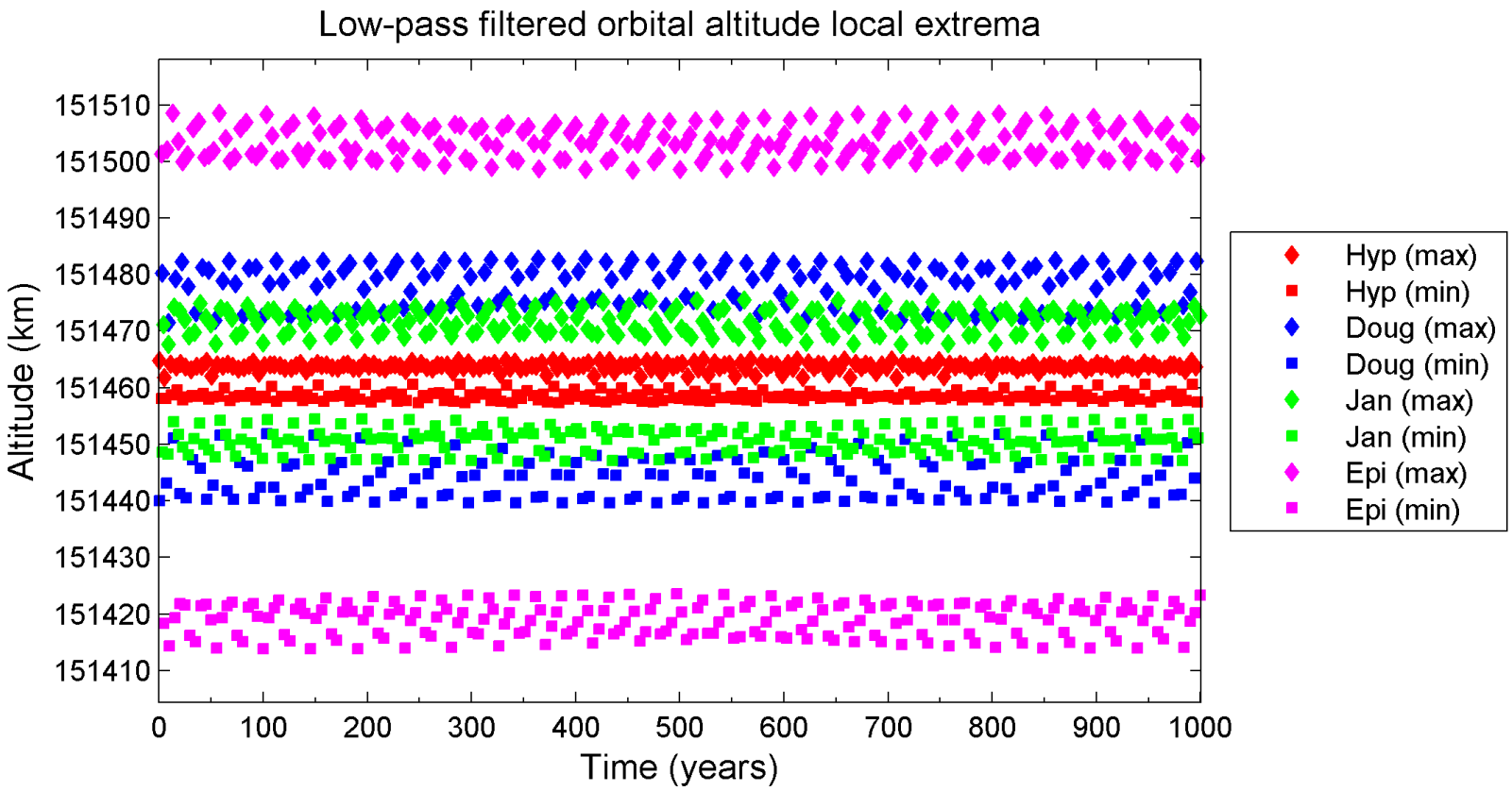
The Experiments:

Janus, Epimetheus & 2 Friends



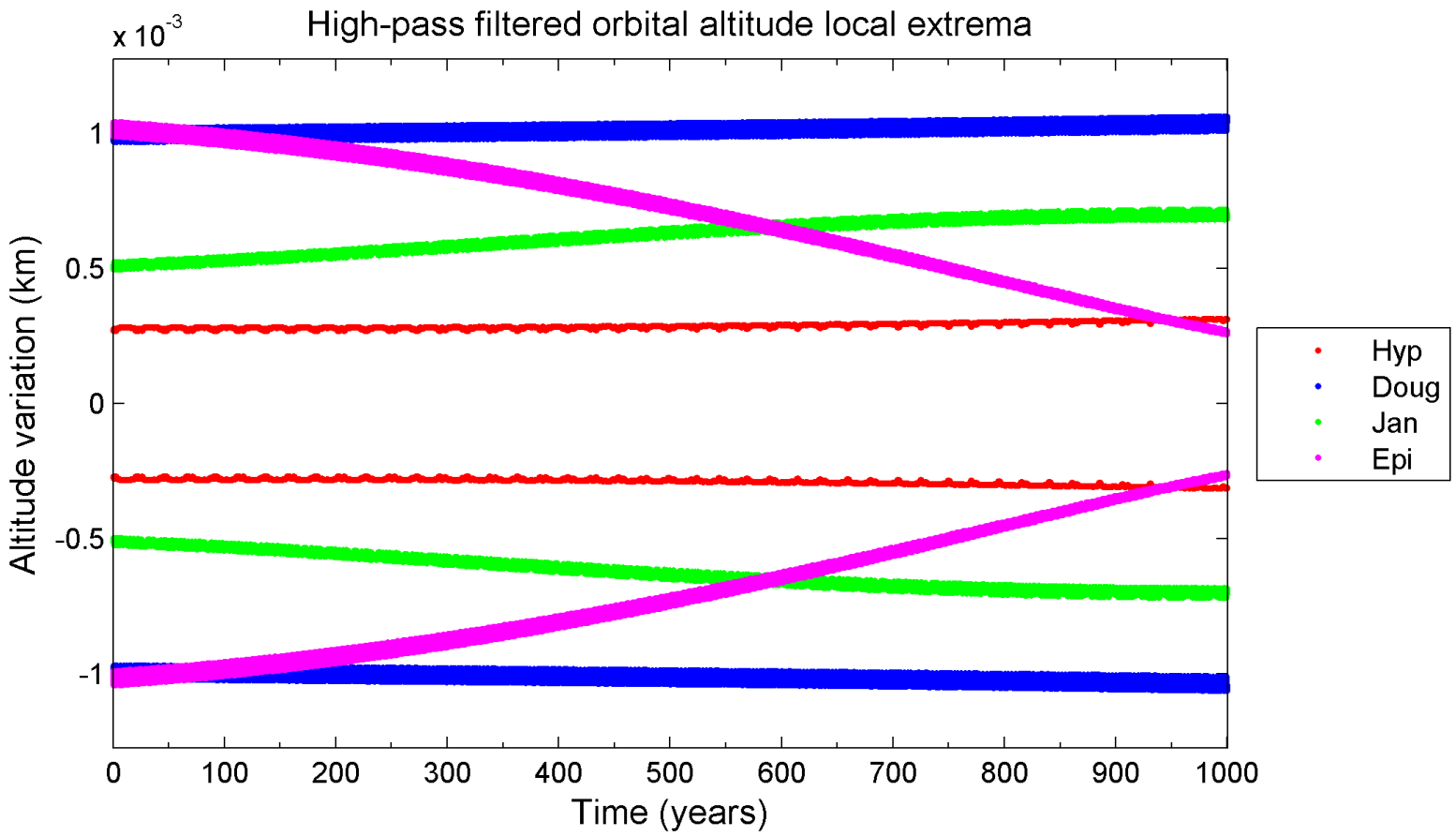
The Experiments:

Janus, Epimetheus & 2 Friends



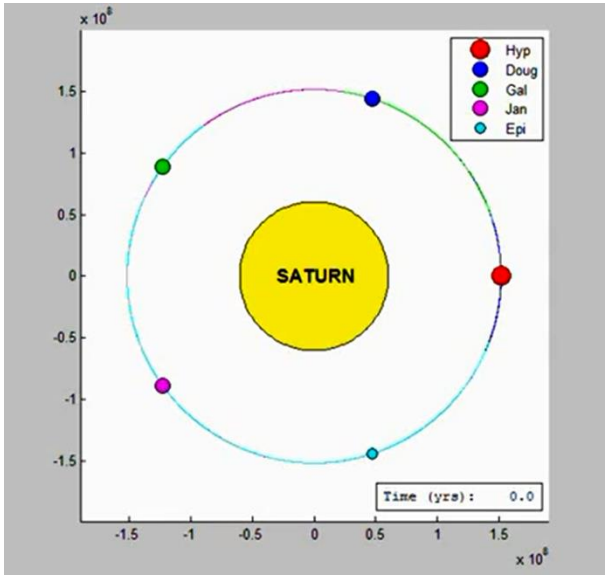
The Experiments:

Janus, Epimetheus & 2 Friends



The Experiments:
Janus, Epimetheus & 3 Friends

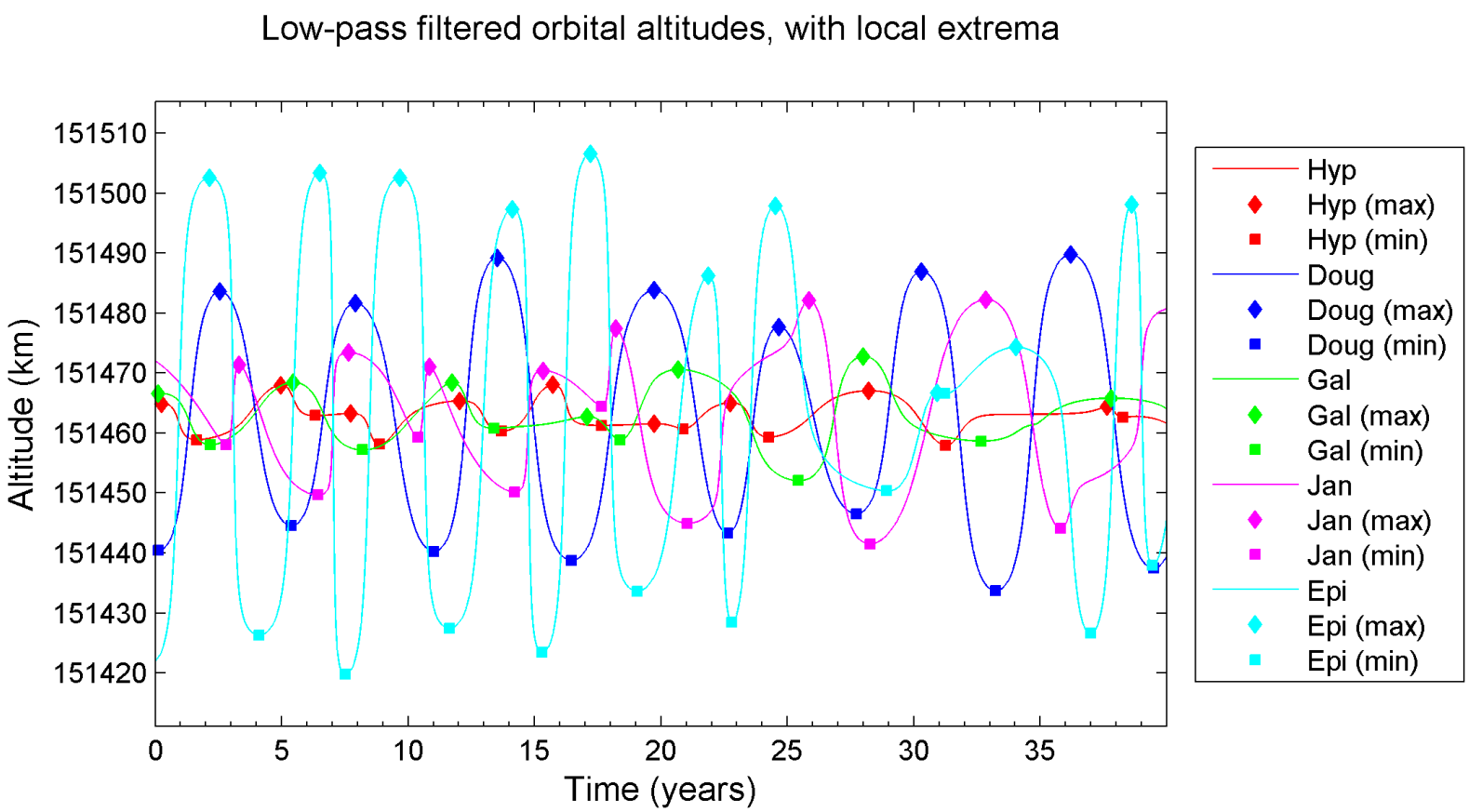
- Goal: Explore dynamics & stability of a 5-satellite co-orbital system: Janus, Epimetheus, Hyperion, Galatea & Douglas



- Initial conditions:
  - Janus & Epimetheus in standard orbits
  - Hyperion & Galatea (larger than Janus) & Douglas (intermediate mass betw. J & E) in orbits commensurate with their masses
  - Angular spacing: $\frac{2\pi}{5}$ radians
- Results (40-year integration):
  - Differing masses produce interesting momentum-exchange curves
  - Orbit swap pattern varies over 40-year time frame
  - Eccentricities different for each body, but remain consistently low
  - System remains stable over this short integration span

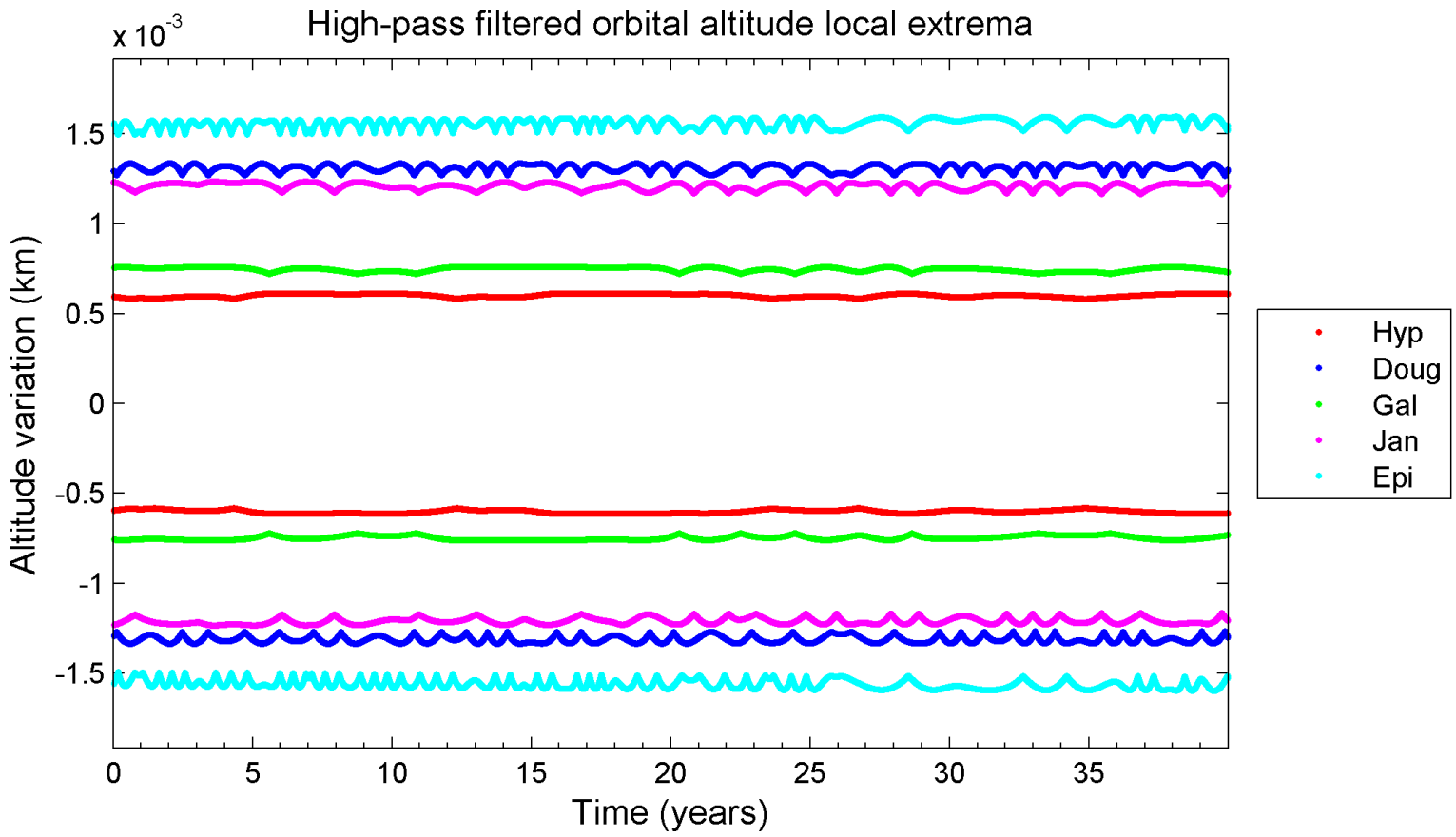
The Experiments:

Janus, Epimetheus & 3 Friends



The Experiments:

Janus, Epimetheus & 3 Friends



Conclusions

- Co-orbital systems of 3 or more moons can remain stable over time, provided the range of orbits occupied starts out sufficiently narrow (relative to the masses of the moons)
- When the range of orbits is wide enough to threaten instability, orbital eccentricities begin to rise prior to orbital breakout
- Radial symmetries can be preserved outright, at least over 1000 years
- Axial symmetries can migrate through the system over time, to be progressively centered on different satellites in turn
- Celestial mechanics animations are cool

Next Steps

- Run selected stable simulations much longer (on faster hardware), to see if stability & symmetries hold up over longer time frames
- Run series of simulations with finer granularity changes in Δr, to determine the actual nature of the functional relationship between Δr and time of orbital breakaway, for a given set of satellite masses
- Write up findings & publish

References

Malinsky S. 2011. Janus and Epimetheus: Momentum Trade in Orbit Around Saturn [presentation]. MS Symposium; Winter 2011; Seattle, WA. Seattle (WA): University of Washington, Department of Applied Mathematics.

NASA Cassini Solstice Mission [Internet]. c2004-2013 [cited 2013 Feb 18]. Available from: http://saturn.jpl.nasa.gov

Thomas PC. 2010. Sizes, shapes, and derived properties of the saturnian satellites after the Cassini nominal mission. Icarus 208 (1): 395–401.

van Deukeren H. 2005. JanusEpimetheus.gif [diagram]. Wikimedia Commons [Internet]. c2005 [cited 2013 Feb 18]. Available from: http://commons.wikimedia.org/wiki/File:JanusEpimetheus.gif

Jrkenti. 2009. Epimetheus-Janus Orbit.png [diagram]. Wikipedia [Internet]. c2009 [cited 2013 Feb 18]. Available from: http://en.wikipedia.org/wiki/File:Epimetheus-Janus_Orbit.png

References

Yoder CF, Colombo G, Synnott SP, Yoder KA. 1983. Theory of Motion of Saturn's Coorbiting Satellites. Icarus 53: 431-443.

Namouni F. 1999. Secular Interactions of Coorbiting Objects. Icarus 137, 293-314.