OFFICE OF EDUCATIONAL ASSESSMENT
Improving Learning Through Assessment

OEA Report 06-06

# Mathematics Placement Test (MPT) Alignment with Washington State College Readiness Mathematics Standards (CRMS)

*Jon Peterson*
*October 2006*

## INTRODUCTION

This report provides a basic description of the Math Placement Tests (MPT) currently in use in five of the six public baccalaureate institutions in Washington State. These tests are developed by math faculty at participating institutions and reflect the college-level curricula common to those institutions. The recently developed College Readiness Mathematics Standards (CRMS) are also described, along with an analysis of the alignment of the MPT to those standards carried out by external agency, Achieve, Inc. In light of limitations of the tests as identified by Achieve, Inc., further analyses were carried out by the University of Washington Office of Educational Assessment and are reported here.

## MATH PLACEMENT TESTS (MPT)

In 1984, Washington State public baccalaureate institutions adopted common mathematics placement tests to assist entering students, together with their academic advisers, in selecting first-year mathematics courses for which they are best prepared. Tests are developed by a collaboration of math faculty from participating institutions.[1] While institutions use the same tests, they do not share placement score cutoffs; these are determined by faculty at each institution in accordance with local curricula. Statewide test administration is coordinated by the Academic Placement Testing Program (APTP), housed in the University of Washington Office of Educational Assessment.

Two versions of the test enable placement of students with varying levels of academic preparation:

- The Intermediate Math Placement Test (I-MPT) is composed of 35 multiple-choice (five alternative) items covering Elementary Algebra, Intermediate Algebra, and Precalculus I, and is intended for students who have less than three or four years of high school math.
- The Advanced Math Placement Test (A-MPT) consists of 30 multiple-choice (five alternative) items covering Intermediate Algebra and Precalculus I and II, and is directed toward students who have taken at least three or four years of high school math, including a precalculus, calculus, or math analysis course.

[1] These are: Central Washington University, Eastern Washington University, University of Washington, Washington State University, and Western Washington University.

At any given time, two parallel forms of each version are in use. The primary form (currently Form F) is used for all regular administrations, while the secondary form (currently Form G) is used for retests. Every three to four years, a new parallel form is created to replace the current secondary form. The existing secondary form replaces the primary form, and that form is retired.

Because the purpose of the tests is to predict how well students will do in college-level mathematics courses, test items have been created according to the content areas considered to be requisite to success in those courses by the faculty who teach them. The number of items for each content area is listed in Table 1, below.

**Table 1. Content areas covered by Mathematics Placement Tests**

| Content Area | Intermediate Test | Advanced Test |
|---|---|---|
| Basic manipulation | 7 | |
| Distance | 2 | |
| Exponents and roots | 3 | |
| Factoring | 2 | |
| Inequalities | 1 | |
| Proportions | 3 | 2 |
| Linear graphs and functions | 6 | 3 |
| Functional notation | 3 | 2 |
| Graph interpretation | 4 | 4 |
| Quadratic graphs and functions | 4 | 2 |
| Equations | | 4 |
| Exponents and logs | | 4 |
| Absolute value | | 2 |
| Simplifying | | 3 |
| Trigonometry | | 4 |
| Total | 35 | 30 |

## COLLEGE READINESS MATHEMATICS STANDARDS (CRMS)

The Washington State Transition Mathematics Projects (TMP) is a consortium of K-12 schools, community and technical colleges, and baccalaureate institutions in Washington State. Over the past two years, this group has developed the College Readiness Mathematics Standards (CRMS) to "define the core knowledge and skills expected of students entering college-level mathematics courses" in Washington State.[2] The purpose of this work is to assist students to progress successfully from high school math to college-level math.

The standards developed by TMP fall into two general categories, Process and Content, as shown in Table 2, below. Each of the standards is comprised of several components, for which TMP identified specific learning indicators. For instance, the *Functions* standard is comprised of six components (e.g., "recognize functional relationships presented in words, tables, graphs,

[2] See http://www.transitionmathproject.org/standards.asp.

and symbols") and 34 learning indicators (e.g., "determine and interpret the meaning of rates of change, intercepts, zeros, extrema, and trends"). As can be seen from the Table, the standards represent much broader content areas than those to which the MPT items were written, consistent with the broader purpose of the TMP.

**Table 2. Definition of College Readiness Mathematics Standards**

| Standard | | TMP Definition |
|---|---|---|
| Process | Reasoning/Problem Solving | The student uses logical reasoning and mathematical knowledge to define and solve problems. |
| | Communication | The student can interpret and present mathematical knowledge in both mathematical and everyday language. |
| | Connections | The student makes connections between ideas in mathematics, to other subject areas, and to real-world situations. |
| Content | Number Sense | The student accurately describes and applies concepts and procedures related to real numbers. |
| | Geometry | The student makes hypotheses, models situations, draws conclusions, and supports claims using geometric concepts and procedures. |
| | Statistics and Probability | The student accurately describes and applies concepts and procedures from probability and statistics to analyze data. |
| | Algebra | The student accurately describes and applies concepts and procedures from Algebra. |
| | Functions | The student accurately describes and applies function concepts and procedures to understand mathematical relationships. |

## ALIGNMENT ANALYSIS BY ACHIEVE, INC.

Achieve, Inc., is a national, non-profit organization created to improve the quality of secondary and post-secondary education by assisting states in aligning their curricula with international standards of college- and work-readiness. Because of the congruence between this work and the goals of TMP, the latter group commissioned Achieve to analyze and report on the alignment of the Washington Assessment of Student Learning (WASL) tests with the newly created CRMS. At the request of the UW Office of Educational Assessment, TMP subsequently extended this request to include a similar analysis of the I-MPT and A-MPT. The structure and content of these tests were created many years prior to CRMS and with a fairly restrictive end (e.g., placement into specific courses) in mind. Nevertheless, we felt it would be useful to learn the extent to which the content coverage matched the broader statement of college-readiness embodied in the CRMS.

The results of the Achieve analysis were reported in June 2006.[3] In carrying out their study, Achieve mapped the 65 individual I-MPT and A-MPT items (Form F) to CRM Standards as shown in Tables 3 and 4, respectively. Although any given item may reflect learning in more than one standard, Achieve identified the single standard to which the item seemed most directly related. Additionally, while some items were mapped to specific learning indicators, others were mapped only to the broader component due to characteristics either of the item or the CRMS.

[3] See http://www.achieve.org/node/756.

As shown in the Tables, Achieve identified only three items as relating to Process standards (1 to *Reasoning/Problem Solving*, 2 to *Communication*, and 0 to *Connections*). Additionally, the remaining 62 items did not map equally to all Content standards; the majority (72%) mapped either to *Algebra* or *Functions*. Although 48% of the components and learning indicators in the CRMS align to these two standards, the I-MPT and A-MPT would seemingly over-represent these two content standards, as noted by Achieve. The authors concluded that the math placement tests do not fully represent all of the areas of capability represented by the standards and, in particular, that the tests address Content but not Process standards. Achieve recommended that the I-MPT and A-MPT be revised taking the CRMS into consideration, particularly with respect to the addition of items relating to Process standards.

## REVIEW AND REANALYSIS BY OEA

The finding that the math placement tests do not provide complete coverage of the CRMS was neither surprising nor disconcerting. Not only were the structure and content of the I-MPT and A-MPT determined years before the CRMS were developed, but the purpose of the test is much more restrictive than that of the CRMS. The MPT are not designed to provide a comprehensive assessment of student readiness for all forms of college-level math, but to place entering college students into specific first-year mathematics courses. That being said, we still felt that there would be value in an accurate mapping of MPT items to the CRMS in that it might suggest modifications to enhance the predictive value of the tests.

As noted above, the strategy adopted by Achieve in mapping MPT items to the CRMS relied on one-to-one connections between items and standards. This approach does not recognize that individual items may call on students to demonstrate multiple capabilities relating to more than a single standard. We believed this to be a significant limitation in approach and so determined to carry out a second mapping of math placement test items to the CRMS.

We began our analysis by reviewing the original content areas for which MPT items were created, as listed in Table 1. It has been several years since these areas have been reviewed, and we discovered that some areas were not defined consistently across the I-MPT and A-MPT. For example, while items involving absolute value equations are included on both tests, only the A-MPT has an *Absolute Value* category. For the purpose of the alignment analysis, and with an eye toward improving the consistency across the tests, we outlined several possible changes to the definition of content areas as shown in Table 5. The redefined content areas then were used as the basis for the alignment analyses, summarized in Tables 6 and 7.

The most observable difference between the results of the Achieve mapping and our subsequent reanalysis results from our decision to map each item to multiple standards, where appropriate. Although we may be interested for some purposes in the *primary* alignments of each item, this is not the case when the focus of our interest is in the overall coverage of the tests. In addition, there is overlap among the standards themselves. TMP states that "although the standards are necessarily presented as separate and distinct areas, they are clearly inter-connected, building upon each other." In fact, the OEA mapping disagreed with that of Achieve on only a single item (I-MPT item #26), though we mapped approximately half of all items onto multiple

standards (15 of 35 on the I-MPT, and 17 of 30 on the A-MPT). The primary result of this approach is that many standards Achieve identified as under-represented by the MPT seem less problematic in light of the OEA mapping. For instance, while Achieve mapped only six of the 65 items to the *Geometry* standard, OEA mapped ten items.

Our analysis did reveal one standard for which coverage was markedly uneven. All of the sixteen items that mapped to the *Probably/Statistics* standard related to a single learning indicator ("use and interpret pie charts, bar graphs, histograms, box-and-whisker plots, scatter plots, stem and leaf, and line graphs"). This may be appropriate, given the content of first-year mathematics courses, or it may be that a broader understanding of this content area is required. If this is the case, additional items should be created to cover the remaining three components of the *Probability/Statistics* standard.

With respect to over-emphasis given to Content versus Process standards, our response is mixed. Although we mapped a larger number of items to the Process standards than did Achieve, our analysis agreed that the coverage was lower than that of the Content standards. Of the 25 items mapped to the *Communication* standard, all mapped to only two learning indicators ("summarize and interpret many different types of graphs" and "create symbolic representations for situations described in everyday language"), and two of the three components associated with this standard are not addressed. Additionally, our analysis agreed with that of Achieve in mapping no items to the *Connections* standard, while we disagreed with Achieve's sole mapping onto the *Reasoning/Problem Solving* standard.

Where we may disagree with Achieve, is in the judgment that the MPT should give the same coverage to Process standards as to Content. Because the tests are in paper-pencil and multiple-choice format (which does not allow students to justify their answers or show their work) it may be unrealistic to expect them to adequately assess Process standards. It also may be unnecessary. While the Process standards are part of the broader outcomes expected of graduating high school students, unless it can be demonstrated that they improve the quality of placement into college-level courses, the lack of coverage in the MPT is not only not problematic, but appropriate.

## CONCLUDING REMARKS AND NEXT STEPS

Consideration of the alignment of the MPT with the CRMS is particularly timely given the current plans to introduce a new form of the placement tests. It is reassuring to read Achieve's praise of the tests' relatively good coverage of the Content standards, strong content and performance centrality, and sufficiently varying level of demand across items in each test. And while neither the I-MPT nor the A-MPT can be argued to comprehensively assess the CRMS, it should be reiterated that these tests are principally designed to place entering college students into first-year mathematics courses for which they are best prepared, rather than to provide a comprehensive assessment of their K-12 mathematics education. Nevertheless, the revised mapping of the MPT items to the CRMS should be included in a deliberate discussion of the structure and content of the next form or forms of the placement tests.

**Table 3. ACHIEVE Mapping of Math Placement Items by Current OEA Subscale and College Readiness Mathematics Standard**

| INTERMEDIATE TEST | College Readiness Mathematics Standards | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Process Standards | | | Content Standards | | | | |
| Current OEA Subscale | Reasoning / Problem-Solving (0 items) | Communication (2 items) | Connections (0 items) | Number Sense (5 items) | Geometry (2 items) | Probability / Statistics (1 item) | Algebra (9 items) | Functions (16 items) |
| Basic Manipulation (7) | | 7 | | 6 | | | 1, 2, 3, 4 | 5 |
| Distance (2) | | | | 17 | 18 | | | |
| Exponents & Roots (3) | | | | 10 | | | 9, 11 | |
| Factoring (2) | | | | | | | 12, 13 | |
| Functional Notation (3) | | | | | | | | 29, 30, 31 |
| Graph Interpretation (4) | | | | | | 35 | | 32, 33, 34 |
| Inequalities (1) | | | | | | | 8 | |
| Linear Graphs & Functions (6) | | | | | | | | 19, 20, 21, 22, 23, 24 |
| Proportions (3) | | | | 15, 16 | 14 | | | |
| Quadratic Graphs & Functions (4) | | 28 | | | | | | 25, 26, 27 |

**Table 4. ACHIEVE Mapping of Math Placement Items by Current OEA Subscale and College Readiness Mathematics Standard**

| ADVANCED TEST | College Readiness Mathematics Standards | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Process Standards | | | Content Standards | | | | |
| Current OEA Subscale | Reasoning / Problem-Solving (1 item) | Communication (0 items) | Connections (0 items) | Number Sense (2 items) | Geometry (4 items) | Probability / Statistics (2 items) | Algebra (13 items) | Functions (8 items) |
| Absolute Value (2) | | | | | | | 29, 30 | |
| Equations (4) | 26 | | | | | | 21, 22 | 20 |
| Exponents & Logs (4) | | | | | | | 14, 15, 18, 19 | |
| Functional Notation (2) | | | | | | | 27, 28 | |
| Graph Interpretation (4) | | | | | | 6 | | 7, 8, 9 |
| Linear Functions (3) | | | | 4 | | 5 | | 3 |
| Proportions (2) | | | | 13 | 12 | | | |
| Quadratic Functions (2) | | | | | | | 10 | 11 |
| Simplifying (3) | | | | | 23 | | 24, 25 | |
| Trigonometry (4) | | | | | 1, 2 | | | 16, 17 |

**Table 5. Proposed Changes to OEA Math Placement Test Subscales**

| OEA Subscale | Intermediate Test | Advanced Test | "Definition"/Examples | Comments |
|---|---|---|---|---|
| *Absolute Value*[4] | Added from advanced | Retained | Understanding of the concept of absolute value (e.g., solving absolute value equations). | Unsure how to handle absolute value *inequalities.* |
| **Basic Univariate Operations**[5] | Completely new | Completely new | Able to solve simple univariate equations. | This new subscale is primarily composed by items from the old "basic manipulations" and "equations" subscales. |
| **Basic Bivariate Operations** | Completely new | Completely new | Able to solve systems of simple equations in two variables. | This new subscale is primarily composed by items from the old "basic manipulations" and "equations" subscales. |
| ~~Basic Manipulations~~[6] | Removed | N/A | This subscale seemed to be a hodgepodge of basic algebraic operations involving one and two variables, as well as simplifying, proportions, constructing linear functions, and computing the perimeter of rectangular objects. | Recommend removal. |
| ~~Distance~~ | Removed | N/A | This subscale was a strange combination of a fairly simple absolute value item and a rather complicated story problem requiring geometric knowledge of the Pythagorean theorem. | Recommend removal. |
| ~~Equations~~ | N/A | Removed | This subscale seemed to be a hodgepodge of basic algebraic operations involving one and two variables. | Recommend removal. |
| Exponents | Could be isolated | Could be isolated | Able to combine and simplify expressions involving rational exponents (e.g., simplify products and quotients of single-term expressions with rational exponents). | Items pertaining to exponents are included on both tests. Could be expanded to "Exponents, Logs, & Roots." |
| Roots | Could be isolated | Could be isolated | Able to combine and simplify expressions involving roots. | Items pertaining to roots are only on the intermediate test. Could be expanded to "Exponents, Logs, & Roots." |
| Logs | Could be isolated | Could be isolated | Able to simplify logarithmic expressions and solve exponential equations through the use of the natural log. | Items pertaining to logs are only on the advanced test. Could be expanded to "Exponents, Logs, & Roots." |
| ~~Exponents & Logs~~ | N/A | Removed | Combination of definitions from above. | Recommend removal. |

[4] All *italicized* subscales are those that currently exist for one test but not the other (e.g., currently there is an "Absolute Value" subscale only on the Advanced test). We recommend that all italicized subscales be added to both tests.

[5] All **bolded** subscales are completely new subscales we recommend be added to both tests.

[6] All ~~struck-through~~ subscales are ones we recommend removing from both tests.

| OEA Subscale | Intermediate Test | Advanced Test | "Definition"/Examples | Comments |
|---|---|---|---|---|
| ~~Exponents & Roots~~ | Removed | N/A | Combination of definitions from above. | Recommend removal. |
| *Factoring* | Retained | Added from intermediate | Able to factor quadratic polynomials. | |
| Functional Notation | Retained | Retained | Able to understand, interpret, and carry out computations involving functional notation. | |
| Geometry | Completely new | Completely new | Able to use geometric properties and relationships of two-dimensional figures to carry out calculations such as area, distance, and distance (e.g., apply the Pythagorean theorem, understand the idea of "congruent parts of congruent triangles"). | This new subscale is primarily composed by items from the old "Basic Manipulations," "Distance," and "Exponents and Logs" subscales, as well as the retained "Factoring" and "Linear Graphs and Functions" subscales. |
| Graph Interpretation | Retained | Retained | Able to interpret graphs for functions that are not explicitly linear or quadratic. | Unsure how to handle the interpretation of graphs that appear to be linear or quadratic. |
| *Inequalities* | Retained | Added from intermediate | Understanding how to manipulate inequalities (e.g., solving linear equalities in one variable). | Unsure how to handle absolute value inequalities. |
| Linear Graphs & Functions | Retained | "Graphs" added | Able to model real-world situations using linear functions as well as understand the algebraic features of a linear function and the features of its graph and/or tabular representation (e.g., determine and interpret the meaning of slope, intercepts, zeros, and extreme). | Some items on both tests involve interpreting graphs of linear functions. |
| Proportions | Retained | Retained | Able to complete multi-step computations using the order of operations and the properties of operations (e.g., commutative property) in situations involving combinations of real numbers. | |
| Quadratic Graphs & Functions | Retained | "Graphs" added | Able to model real-world situations using quadratic functions as well as understand the algebraic features of a quadratic function and the features of its graph (e.g., determine and interpret the meaning of rates of change, intercepts, zeros, and extreme). Able to apply the quadratic formula. | Some items on both tests involve interpreting graphs of quadratic functions. |
| *Simplifying* | Added from advanced | Retained | Able to simplify rational expressions resulting in an equivalent expression. | Could be subsumed by new basic operations subscales |
| *Trigonometry* | Added from advanced | Retained | Able to recognize, analyze, and interpret trigonometric functions (e.g., apply the basic right-triangle trigonometric relationships of sine, cosine, and tangent to solve problems). | Items pertaining to trigonometry are only featured on the advanced test. |

**Table 6. OEA Mapping of Math Placement Items by New OEA Subscales and College Readiness Mathematics Standard**

| INTERMEDIATE TEST | College Readiness Mathematics Standards | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Process Standards | | | Content Standards | | | | |
| Current OEA Subscale | Reasoning / Problem-Solving (0 items) | Communication (13 items) | Connections (0 items) | Number Sense (5 items) | Geometry (5 items) | Probability / Statistics (9 items) | Algebra (15 items) | Functions (18 items) |
| *Absolute Value (1)* | | | | 17 | | | | |
| Basic Univariate Operations (2) | | | | | | | 1, 2 | |
| Basic Bivariate Operations (1) | | | | | | | 4 | |
| ~~Basic Manipulations~~ (7) | | 5, 7 | | 6 | 5, 7 | | 1, 2, 3, 4, 5, 7 | 5, 7 |
| ~~Distance~~ (2) | | 18 | | 17 | 18 | | 18 | 18 |
| Exponents (2) | | | | 10 | | | 10, 11 | |
| Roots (1) | | | | | | | 9 | |
| Logs (0) | | | | | | | | |
| ~~Exponents & Roots~~ (3) | | | | 10 | | | 9, 10, 11 | |
| Factoring (2) | | | | | | | 12, 13 | |
| Functional Notation (3) | | 31 | | | | 31 | | 29, 30, 31 |
| Geometry (4) | | 5, 7, 18 | | | 5, 7, 14, 18 | | 5, 7, 14, 18 | 5, 7, 18 |
| Graph Interpretation (3) | | 32, 33, 35 | | | | 32, 33, 35 | | 32, 33 |
| Inequalities (1) | | | | | | | 8 | |
| Linear Graphs & Functions (7) | | 20, 21, 24, 34 | | | | 20, 21, 24, 34 | | 19, 20, 21, 22, 23, 24, 34 |
| Proportions (3) | | | | 6, 15, 16 | | | | |
| Quadratic Graphs & Functions (4) | | 27, 28 | | | 28 | 27 | 28 | 25, 26, 27 |
| *Simplifying (1)* | | | | | | | 3 | |
| *Trigonometry (0)* | | | | | | | | |

**Table 7. OEA Mapping of Math Placement Items by New OEA Subscales and College Readiness Mathematics Standard**

| ADVANCED TEST | College Readiness Mathematics Standards | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Process Standards | | | Content Standards | | | | |
| Current OEA Subscale | Reasoning / Problem-Solving (0 items) | Communication (12 items) | Connections (0 items) | Number Sense (3 items) | Geometry (5 items) | Probability / Statistics (7 items) | Algebra (19 items) | Functions (14 items) |
| Absolute Value (1) | | | | | | | 30 | |
| Basic Univariate Operations (2) | | | | | | | 21, 22 | |
| Basic Bivariate Operations (2) | | 20, 26 | | | | | 26 | 20, 26 |
| ~~Equations~~ (4) | | 20, 26 | | | | | 21, 22, 26 | 20, 26 |
| Exponents (2) | | | | 15 | | | 14, 15 | |
| Roots (0) | | | | | | | | |
| Logs (2) | | | | | | | 18, 19 | |
| ~~Exponents & Logs~~ (4) | | | | 15 | | | 14, 15, 18, 19 | |
| *Factoring (1)* | | | | | | | 25 | |
| Functional Notation (2) | | | | | | | 27, 28 | 27, 28 |
| *Geometry (2)* | | 23 | | | 12, 23 | | 12, 23 | 23 |
| Graph Interpretation (3) | | 6, 8, 9 | | | | 6, 8, 9 | | 8, 9 |
| *Inequalities (1)* | | | | | | | 29 | |
| Linear *Graphs and* Functions (3) | | 3, 4, 5 | | 4 | 5 | 3, 5 | 4 | 3, 4 |
| Proportions (1) | | 13 | | 13 | | | 13 | 13 |
| Quadratic *Graphs and* Functions (3) | | 7 | | | | 7 | 10, 11 | 7, 11 |
| Simplifying (1) | | | | | | | 24 | |
| Trigonometry (4) | | 16 | | | 1, 2 | 16 | | 16, 17 |