

**Image Enhancement - Spatial Domain**

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BE 244: Medical Image Processing and Analysis

January 28, 2009

**BE244 - Lecture Outline - January 28, 2009**

Image Enhancement - Spatial Domain:

- Basics Spatial filtering
  - Neighborhood operations
  - Spatial convolution
  - Border Issues
- Mean, Median Spatial Filters
  - Calculation Examples
- Other order statistic filter: midpoint filter
- First Second-order derivatives (review)
- Laplacian
- Unsharp Masking

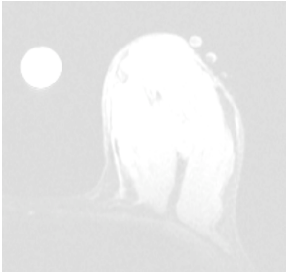
Image Segmentation – Thresholding Basics

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**Introduction to Image Enhancement...**

Water phantom

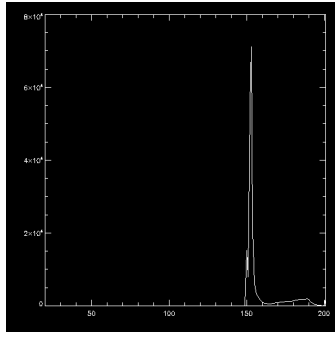
Breast MR  
Normal Volunteer  
T1-weighted image  
Axial pre-contrast slice.



Where do we start ? ....

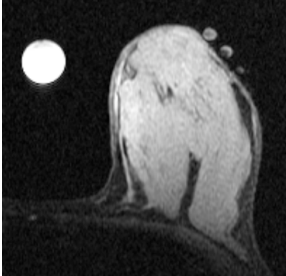
Histogram of image from last slide:

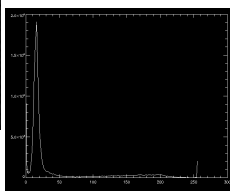
Comments?



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**Final result with simple contrast stretching ...**





**Contrast Mapping**

An image has a max value of 12000. Its minimum is -3200.  
Display it using values 0 - 255.  
Give equation needed to rescale it appropriately.

$$y = [(x - \text{min}) / (\text{max} - \text{min})] * 255$$

Or,

$$y = [(x + 3200) / 15200] * 255$$

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### Spatial Filtering: Graphical illustration

Spatial filtering of image  $f(x,y)$  using neighborhood mask  $h(x,y)$ :

Mask:  
 $h(x,y) = 3 \times 3$  filter  
 with weights as shown

**Spatial convolution:**  

$$g(x,y) = A_0f(x-1,y-1) + A_1f(x,y-1) + A_2f(x+1,y-1) + A_3f(x-1,y) + A_4f(x,y) + A_5f(x+1,y) + A_6f(x-1,y+1) + A_7f(x,y+1) + A_8f(x+1,y+1)$$

Corresponds to multiplication of each pixel under mask with corresponding filter weight, and replacement of this value at the point of coordinates  $(x,y)$ .

### Spatial Filtering: Summary

Neighborhood process, scanning all image  
 ⇒ computationally intensive.

Mask size or shape: any, but arbitrary shapes will play role in result (i.e. Math. Morphology).

Typically: small square/rectangular 2D array, odd number of elements (eg 3x3, 5x5 neighborhoods) to ease programming, centered at pixel being filtered.

### Smoothing Spatial Filter: Mean Filter

- Used for removal (or reduction) of small (irrelevant) details in image
- Every pixel replaced by average of its neighbors
- “Low-pass filter”: removes high spatial frequencies
  - reduces noise in image
  - best for removing gaussian noise

but.....: blurs image  
 reduces sharp edges

Example: *Vertical edge*

0	0	9	9	9
0	0	9	9	9
0	0	9	18	9
0	0	9	9	9
0	0	9	9	9

Filtered using a 3x3 mean filter

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

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### Mean Spatial Filter (1)

Example: *Vertical edge*

0	0	9	9	9
0	0	9	9	9
0	0	9	18	9
0	0	9	9	9
0	0	9	9	9

Filtered using a 3x3 mean filter:

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

### Mean Spatial Filter (2)

Example: *Vertical edge*

0	0	9	9	9
0	0	9	9	9
0	0	9	18	9
0	0	9	9	9
0	0	9	9	9

Filtered using a 3x3 mean filter:

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

0	0	9	9	9
0	3	7	10	9
0	3	7	10	9
0	3	7	10	9
0	0	9	9	9

Issues with borders....

### Spatial Filtering: Border Issues

Issues at borders using  $H \times H$  masks on  $N \times M$  image:

- First  $(H-1)/2$  rows and columns not filtered
- Last  $(H-1)/2$  rows and columns not filtered

Example with 3x3 mask:  
 1st and last rows+columns of original image are not filtered.

-	-	-	-	-
-	X	X	X	-
-	X	X	X	-
-	X	X	X	-
-	-	-	-	-

⇒ 2 main approaches

### Spatial Filtering: Border Issues (2)

2 main approaches to deal with border issues when spatial filtering:

1. **Leave all borders unfiltered (easiest)**
  - Common solution
  - Most information is located at center of image, not border....
2. **Replicate bordering pixels**, before and after image, to define and use HxH neighbors around all pixels of the image
  - More computing complexity

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### Spatial Filtering: Border Issues (3)

2. **Replicate bordering pixels**, before and after image, to define and use HxH neighbors around all pixels of the image

0	0	0	9	9	9	9
0	0	0	9	9	9	9
0	0	0	9	9	9	9
0	0	0	9	18	9	9
0	0	0	9	9	9	9
0	0	0	9	9	9	9
0	0	0	9	9	9	9

Final filtered image:

0	3	6	9	9
0	3	7	10	10
0	3	7	10	10
0	3	7	10	10
0	3	6	9	9

Mean filter effect:

- unsuccessful at removing outlier pixel (18)
- reduced height of outlier but increased its width
- sharp vertical edge changed to gradually sloped edge

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### Mean Spatial Filter: Examples

### Other Example Averaging Filter

Original Image

0	0	8	8	8
0	0	8	8	8
0	0	8	16	8
0	0	8	8	8
0	0	8	8	8

5-point weighted averaging mask

0	1/8	0
1/8	1/2	1/8
0	1/8	0

Spatial averaging of original image using this 5-point weighted averaging mask ? (homework due Feb. 4)

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### Median Filter

**Important Properties:**

1. Nonlinear spatial filter  
(  $\text{median}(Im1) + \text{median}(Im2) \neq \text{median}(Im1+Im2)$  )
2. Good at removing **outlier type noise** (salt and pepper)
3. Better job than mean filter at **preserving edges** within image
4. Its calculation does NOT use convolution operation
5. Mask (HxH) simply defines which pixels are included in calculation
6. Arrange pixel values in window in **increasing (or decreasing) order**
7. Median is the **middle value if H is odd**, and is average of 2 center values if H is even
8. Poor performance when number of noise pixels in window is greater than half the number of pixels in the window.

*Example:*  $f(x) = \{2, 3, 8, 4, 2\}$ , filtered with chosen window = 3x1  
 Output of median filter is  $\text{med}(0)=2$  (boundary value),  $\text{med}(1)=\text{median}\{2, 3, 8\}=3$ ,  $\text{med}(2)=\text{median}\{3, 4, 8\}=4$ ,  $\text{med}(3)=\text{median}\{2, 4, 8\}=4$ ,  $\text{med}(4)=2$  (boundary value).  
 Median filtered sequence =  $\{2, 3, 4, 4, 2\}$

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### Median Filter: Examples

Original image

5	5	9	9	9
5	5	9	9	9
5	5	9	18	9
5	1	9	9	9
5	5	9	9	9

⇒ Median filter using 3x3 window ⇒ Output image ?

Original image

0	0	0	0	0
0	0	0	0	0
5	5	5	5	5
0	0	0	0	0
0	0	0	0	0

⇒ Median filter using 3x3 window ⇒ Output image ?

Comments ?

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### Median Filter: Examples

original

Note the dispersion of water phantom after median (75x75) filtering:  
What is the size (~pixels of circle diameter) of the water phantom?

### Other order statistic filter: Midpoint filter

**Definition:**  
Average of Maximum and Minimum graylevels of the order set of pixels involved in the filter operation.

- Blurs image similarly to mean filter, but it is the best filter to remove uniform type noise from an image.
- Also performs poorly with images containing salt and pepper noise.

Example: original image

0	0	9	9	9
0	0	9	9	9
0	0	9	9	9
0	0	9	9	9
0	0	9	9	9

➔ Find 3x3 midpoint filtered image, and compare with 3x3 mean filtered image

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### Basic Definitions: first / second-order derivatives

**Important definitions:** book pages 125-127, figure 3.38

**Derivatives of a digital function:** defined in terms of differences

*Derivatives will have different behaviors in constant areas, and in areas with step or ramp discontinuities.*

First-order derivative for one-dimensional function  $f(x)$ :  
 $\partial f / \partial x = f(x+1) - f(x)$   
 ➔ Produce thicker edges in the image

Second-order derivative:  
 $\partial^2 f / \partial x^2 = f(x+1) + f(x-1) - 2f(x)$   
 ➔ Stronger response to fine details, produce double response at step changes in gray level.

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### Basic Definitions: Unsharp Masking

**Goal:** Sharpen images by subtracting a blurred version of an image from the image itself.

**Result:** Enhances high frequencies present in the image.

$$f_{\text{high-pass}}(x,y) = f_{\text{orig}}(x,y) - f_{\text{blurred}}(x,y)$$

$$f_{\text{unsharp}}(x,y) = f_{\text{orig}}(x,y) + \lambda f_{\text{high-pass}}(x,y)$$

**Homework:** Program unsharp masking operation, explain choice of parameters.

Unsharp Masking Operations (Jain, Fundamentals of Digital Imaging, 1989)

### Summary Enhancement Techniques

median, 3x3

Shift, t=0.75

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### Crooks Algorithm - Shifting Histogram

Original Image

CF Method: Downshift,  $f = 0.25$ , kernel = 3

CF Method: Upshift,  $f = 0.25$ , kernel = 3

Original Image

CF Method: Downshift,  $f = 0.99$ , kernel = 3

CF Method: Upshift,  $f = 0.99$ , kernel = 3

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### Image Segmentation - Basics

Goal: Divide the image into REGIONS

Two classes of region definition techniques:

Algorithms for construction of regions

Algorithms for contour extraction

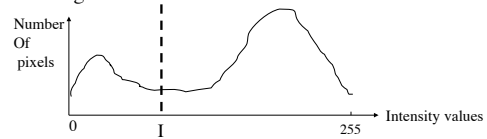
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### Thresholding

#### 1. Definitions

**Histogram**: Calculation of number of pixels at each intensity value in the image

↓  
Displays peaks and valleys corresponding to subpopulations of the image



Choosing intensity level (I) allows thresholding effect = defines 2 subpopulations in the resulting image.

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Binary image: Object (foreground, value 1) in background (value 0)

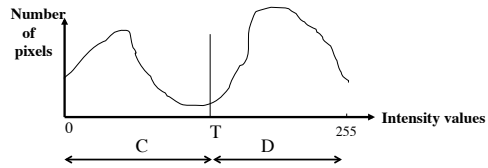
→ histogram shape?



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#### Histogram of gray level image:

Case 1: Image has a bimodal histogram. Define a threshold located in between maxima (the two subpopulations in the image) and separating intensity levels into two groups: C, D.



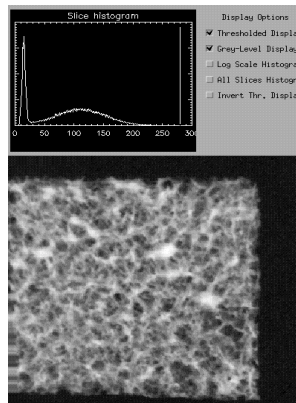
Result of thresholding (using threshold value T): Binary image.

All points in group C (with intensity values <T) take intensity value 0, all points in group D (intensities >=T) take the value 1.

Example: Xray of bovine bone Specimen A

Trabecular bone: bright values

→ Explain histogram



### Thresholding Example 1

Synthetic grid  
(vert=80, horiz=180)

?? Which thresholds were chosen in images 2 and 3?

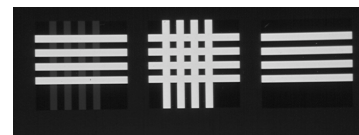


Image 1 (original) Image 2 Image 3

### Thresholding Example 2

Scanned grid

?? Average gray value of circular object ?

Histogram of gray level image:  
 Case 2: Histogram NOT bimodal.

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→ Necessary to define a rule in order to select an acceptable threshold for context of study

#### Global Threshold

Which threshold is acceptable ?

Originals

threshold 1

threshold 2

? ? ?

#### Technique A: fixed threshold

2) quantification of ROI

(threshold = 35% of area under gray values histogram)

Binary mask of ROI

#### Technique B: Experimental Threshold

m1 or m2: average minimum values in image  
 m3: average maximum values

Threshold = 35%(m3-m1)+m1

**Global Threshold:** choice of threshold depends on the intensity values of the image

**Local Threshold:** choice of threshold is made depending on intensity values in local neighborhood calculated at each pixel

**Homework for Feb 4 (descriptive only):**

**Find 2 images (online, book, magazine etc. – please cite sources):**

- one which could be well segmented using a global threshold (explain what would be segmented and how)
- second one presents an histogram which is NOT bimodal, and therefore would require some rule(s) to extract object/segment, please explain. Descriptive only, be creative !

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#### Example: Histogram of Bovine Bone specimen B

**Rules:** Define threshold at

- X% of area under histogram
- Mean histogram Value
- ....

### Thresholding: properties

- No spatial coherence/rule for detected objects: if image is noisy, thresholding can lead to over-segmented binary image
- Histogram provides information on intensity subpopulations, but does not provide spatial information about objects

12	14	15	15	16	16	16
12	1	13	3	12	11	12
7	2	3	4	5	2	13
2	2	1	1	6	1	14
12	2	2	7	7	9	15
13	14	23	13	8	9	11
19	14	21	12	12	20	11

↔

Both images  
have the same  
histogram

8	9	15	1	2	16	16
12	1	13	3	12	11	2
7	12	20	4	19	2	13
14	16	23	15	6	1	2
12	7	2	7	21	9	15
13	14	1	13	12	14	11
5	14	2	12	12	3	11

- Any inhomogeneity due to image modality will bias results
- The number of classes (subpopulations in the image) is generally less than 5.


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### Thresholding techniques:

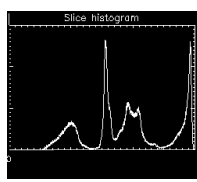
- Well adapted on simple images (non complex content)
- Large objects with good contrast with regard to background

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### Thresholding examples

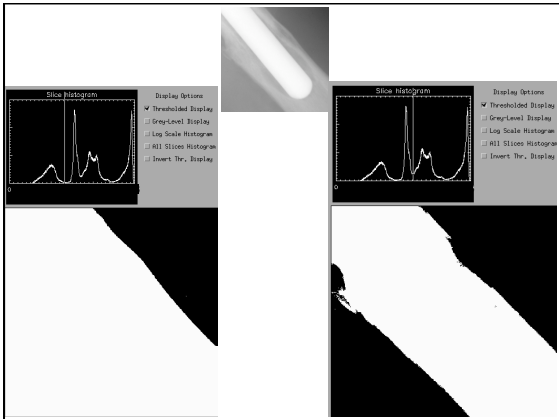


Original image  
Femoral prosthesis  
(detail)

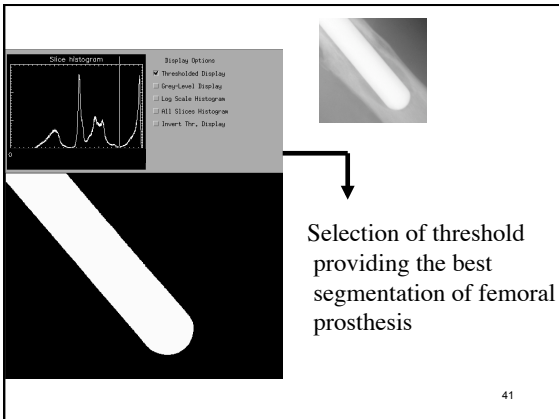


Histogram of original image

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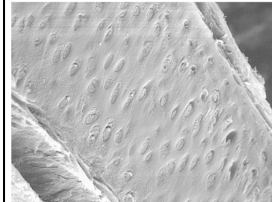



Selection of threshold  
providing the best  
segmentation of femoral  
prosthesis

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### Definition of cartilage patterns from optical microscopy images

Simple global threshold

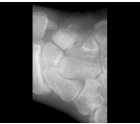
Provides markers of rounded  
structures for future quantification

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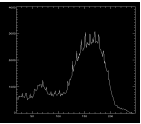
### Optimum Thresholding

\* Assumes image histogram contains 2 predominant peaks \*

**GOAL:** Automatically find valley between the two peaks and threshold the image



Carpal Bones  
(Xray)



**Solution:**

- ⇒ Smooth the histogram to reduce small variations
- ⇒ Use 1D mean filter for smoothing histogram (i.e. 31x1 pixels, size determined depending on content of image)

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### Optimum Thresholding: computation

**Input:** Gray level image, size NxM pixels  
256 gray level values

1. Compute Histogram of image,  $h(i)$ ,  $i$ : intensity levels
2. Smooths histogram curve  $h$  using  $T \times 1$  pixels filter,
3. Locate valley between two peaks, use that level for thresholding
4. Create binary image

**Output:** Binary image → (homework for Feb 4)

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Image A

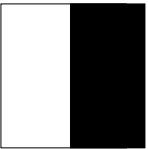
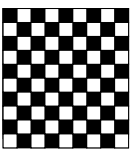


Image B



1. Images A and B: same size, same gray levels
2. Shape of their histograms?
3. Apply 3x3 smoothing mask
4. Are histograms identical after blurring?

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### Answer:

- Number of boundary points between black and white regions is much larger in Image B than in Image A.
- After smoothing, the boundary points will have a different value from black or white, so the two histograms will be different.

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### Histogram Features

**Goal:** Use some statistical parameters obtained directly from the histogram

**Mean**  $\bar{Y} = \sum_{i=1}^N Y_i / N$

**Median:** Estimated from a histogram by finding the smallest number such that the area under the histogram to the left of that number is 50%

**Skewness**  $= \frac{\sum_{i=1}^N (Y_i - \bar{Y})^3}{(N-1)s^3}$       **Measure of symmetry**  
 ("skewed right" distribution = tail on right side)

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### Histogram Features

**Goal:** Use some statistical parameters obtained directly from the histogram

**Mean**  $\bar{Y} = \sum_{i=1}^N Y_i / N$

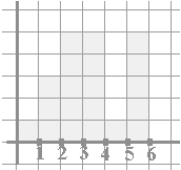
**Kurtosis**  $= \frac{\sum_{i=1}^N (Y_i - \bar{Y})^4}{(N-1)s^4}$       **Measure of data: peaked (high kurtosis)/flat (low kurtosis)**

**Variance**  $s^2 = \sum_{i=1}^N (Y_i - \bar{Y})^2 / (N-1)$

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**Example:**



**3 MODES:**  
3, 4, and 6 (values that appear most often)

**MEAN:**  
The arithmetic mean of a set of values is a sum of all values, divided by their number  
 $(1+2+2+2+3+3+3+3+3+4+4+4+4+5+6+6+6+6)/20 = 3.85$

**MEDIAN:** the middle piece of data, after data sorted from smallest to largest: 1, 2, 2, 2, 3, 3, 3, 3, 3, 4, 4, 4, 4, 4, 5, 6, 6, 6, 6, 6.  
 There is an even number of values, so middle (or median) is between the first and second 4. Since they are identical, the median is four, but if they were different, (i.e. if the median was between a 3 and a 4), we would do  $(3+4)/2=3.5$ .

<http://www.shodor.org/interactivate/discussions/sd1.html>

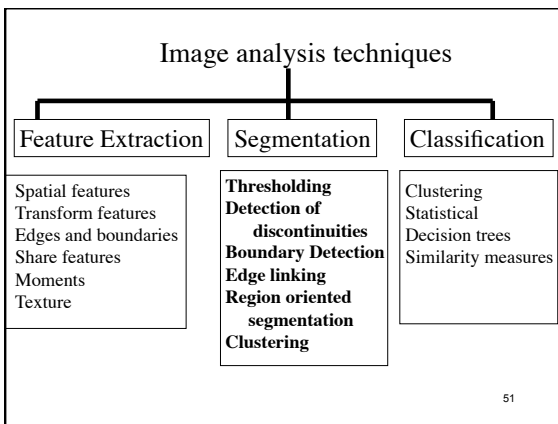
**Alternative Measures of Location**  
 A few of the more common alternative location measures are:

1. **Mid-Mean** - computes a mean using the data between the 25th and 75th percentiles.
2. **Trimmed Mean** - similar to the mid-mean except different percentile values are used. A common choice is to trim 5% of the points in both the lower and upper tails, i.e., calculate the mean for data between the 5th and 95th percentiles.
3. **Winsorized Mean** - similar to the trimmed mean. However, instead of trimming the points, they are set to the lowest (or highest) value. For example, all data below the 5th percentile are set equal to the value of the 5th percentile and all data greater than the 95th percentile are set equal to the 95th percentile.

**The first 3 alternative location estimators above have the advantage of the median: they are not affected by extremes in tails.** However, they generate estimates that are closer to the mean for data that are normal (or nearly so).

4. **Mid-range** =  $(\text{smallest} + \text{largest})/2$ . Not robust (based on two most extreme points). (use typically restricted to situations in which behavior at extreme points is relevant.)

From : <http://www.itl.nist.gov/div898/handbook/eda/section3/eda351.htm>



**Next Session: Monday February 2**

**Image Segmentation:**

Review Thresholding: Read Book pp 595-600  
 Edge & Contour Detection: Read book pp567-585