Lecture 5: Image Restoration

BE 244: Biomedical Image Analysis

Enhancement vs. Restoration

- Same goal:
  improve image in some predefined sense
- Image enhancement
  - Subjective process
  - Heuristic procedures
  - Example: contrast stretching
- Image restoration
  - Objective process
  - Criterion for image goodness
  - Example: removal of image blur

Image Degradation

- g(x,y) = H[f(x,y)] + η(x,y)
  - f(x,y): original input image
  - H(): degradation function
  - η(x,y): additive noise
  - g(x,y): degraded output image

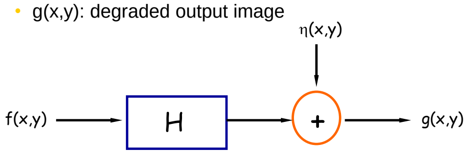
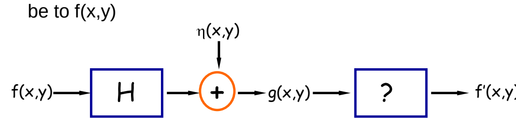


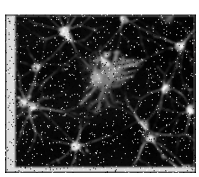
Image Restoration

- Given g(x,y) and some a priori information about H and η(x,y), obtain an estimate f'(x,y) of the original image
- We want the estimate f'(x,y) to be as close as possible to the original input image f(x,y)
- The more we know about H and η the closer f'(x,y) will be to f(x,y)



Noise Probability Density Functions

- Noise is introduced into images during the acquisition and/or transmission processes
- Noise can be correlated or uncorrelated with spatial coordinates



Noise PDFs

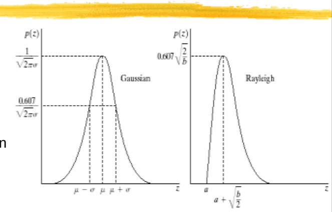
- Gaussian Noise

$$p(z)=\frac{1}{\sqrt{2\pi}\sigma}e^{-(z-\mu)^2/2\sigma^2}$$

  - "Normal" noise distribution
  - Electronic or sensor noise
  - Common noise model – often abused
- Rayleigh Noise

$$p(z)=\begin{cases}\frac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z \le a\\ 0 & \text{for } z < a\end{cases}$$

  - Useful for histogram analysis of images with significant background component (ie skewed)



$$\mu = a+\sqrt{\pi b/4}$$

$$\sigma^2=\frac{b(4-\pi)}{4}$$

Noise PDFs

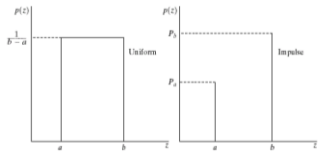
- Uniform Noise

$$p(z) = \begin{cases} \frac{1}{(b-a)} & \text{for } a \le z \le b \\ 0 & \text{otherwise} \end{cases}$$

- Impulse Noise

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$

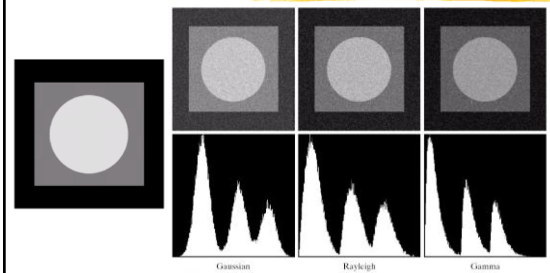
  - "Bipolar", "Salt-and-Pepper", "Shot", or "Spike" noise
  - Impulses can be negative or positive and are typically at saturation levels



$$\mu = \frac{a+b}{2}$$

$$\sigma^2 = \frac{(b-a)^2}{12}$$

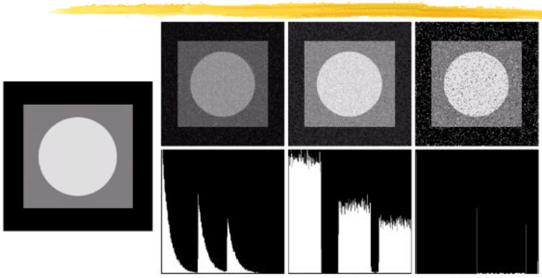
Noise PDFs



a b c
d e f

FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.

Noise PDFs

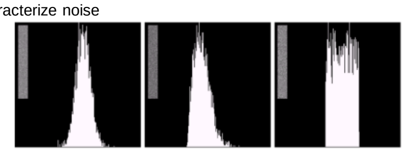


g h i
j k l

FIGURE 5.4 (Continued) Images and histograms resulting from adding exponential, uniform, and impulse noise to the image in Fig. 5.3.

Noise Estimation

- Noise characteristics may be estimated from the Fourier spectrum of an image
  - Periodic (spatially correlated) noise will appear as frequency spikes
- Characteristics may be empirically derived for a given acquistion system by imaging a flat (typically all black) environment
- Regions of interest in existing images may also be used to characterize noise



a b c

FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

Noise reduction

- Mean filters
  - Arithmetic mean filter
  - Geometric mean filter
  - Harmonic mean filter (salt noise, Gaussian noise)
- Order-statistics filters
  - Median filter (salt and pepper noise)
  - Min filter (salt noise)
  - Max filter (pepper noise)
  - Midpoint filter (uniform noise, Gaussian noise)

Image Degradation

- H is linear

  $H[k_1f_1(x,y) + k_2f_2(x,y)] = k_1H[f_1(x,y)] + k_2H[f_2(x,y)]$

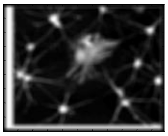
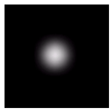
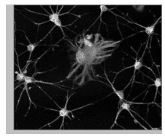
  and position-invariant

  $H[f(x - \alpha, y - \beta)] = g(x - \alpha, y - \beta)$
- Spatial domain

  $g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$
- Frequency domain

  $G(u,v) = H(u,v) F(u,v) + N(u,v)$

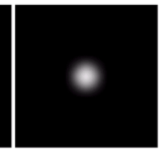
Image Degradation, example

 =  * 

g(x,y) = h(x,y) * f(x,y)

Image Restoration

- Estimation of degradation function H
  - Image observation, $H_s(u,v) = G_s(u,v) / F_s'(u,v)$
  - Experimentation
  - Mathematical modeling

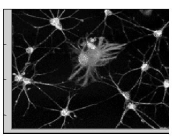
 

- Direct inverse filtering

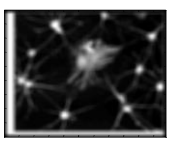
  F'(u,v) = G(u,v) / H(u,v)

  $f'(x,y) = \mathcal{F}^{-1}[F'(u,v)] = \mathcal{F}^{-1}[G(u,v) / H(u,v)]$

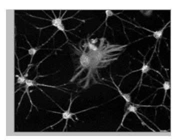
Image Restoration, example



f(x,y)



g(x,y)
f(x,y) blurred with a 7x7 mean filter



f'(x,y)
f(x,y) restored with the inverse filter

Inverse Filtering, problems

- F'(u,v) = G(u,v) / H(u,v)

  F'(u,v) = F(u,v) + N(u,v) / H(u,v)

  F(u,v) = F'(u,v) - N(u,v) / H(u,v)

  N(u,v) = ?
- H(u,v) small -> N(u,v) / H(u,v) large

  may dominate the estimate F'(u,v)

  we need to limit the analysis to frequencies near the origin H(0,0)

Wiener Filter

- For **g = Hf + n**

$$F'(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + S_n(u,v)/S_f(u,v)}\right] G(u,v)$$

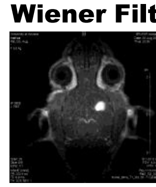
$$S_n(u,v) = |N(u,v)|^2 = \text{noise power spectrum}$$

$$S_f(u,v) = |F(u,v)|^2 = \text{original image power spectrum}$$

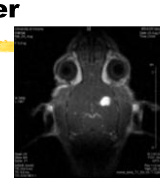
- Also called "least squares filter" because it minimizes

  $\sigma^2 = E\{[f(u,v) - f'(u,v)]\}$
- For $S_\eta(u,v) = 0$ => inverse filter
- For "white noise" $S_n(u,v)$ => constant

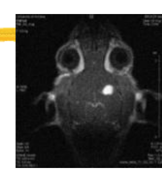
Wiener Filter



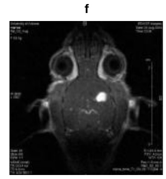
f



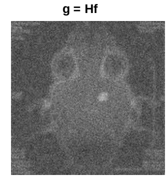
g = Hf



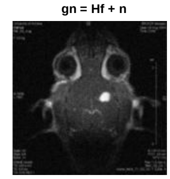
gn = Hf + n



inverse(g)



inverse(gn)



wiener(gn)