Gene Golub SIAM Summer School 2012

Numerical Methods for Wave Propagation
Finite Volume Methods
Lecture 3

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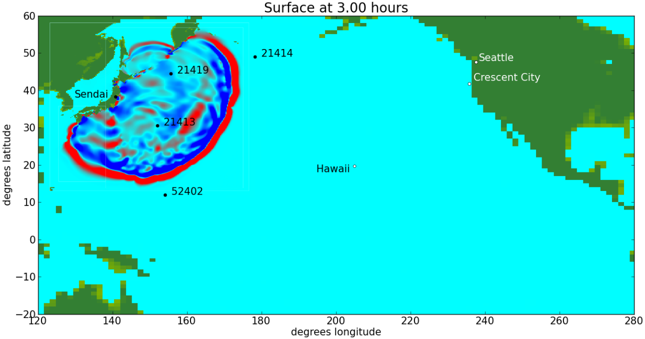
Outline

- Shallow water equations with topography
- Approximate Riemann solvers
- f-wave formulation of wave-propagation method.
- Well-balanced methods to preserve ocean-at-rest.
- Dry state Riemann solvers

Great Tohoku Tsunami, 11 March 2011



Modeling and Simulating Tsunamis with an Eye to Hazard Mitigation, RJL and J. Behrens, SIAM News, May, 2011
http://www.siam.org/news/news.php?id=1882

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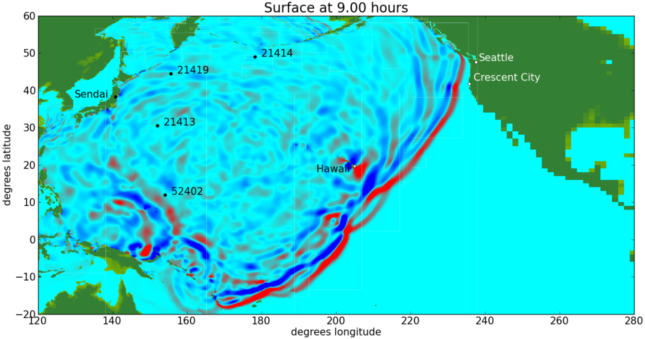

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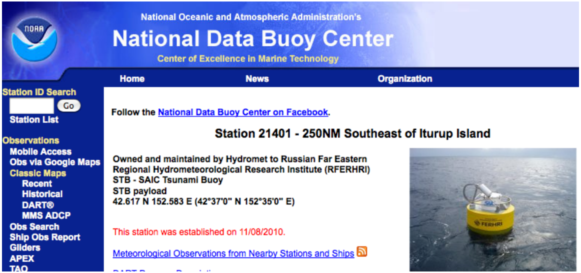
National Oceanic and Atmospheric Administration's
National Data Buoy Center
Center of Excellence in Marine Technology
Home
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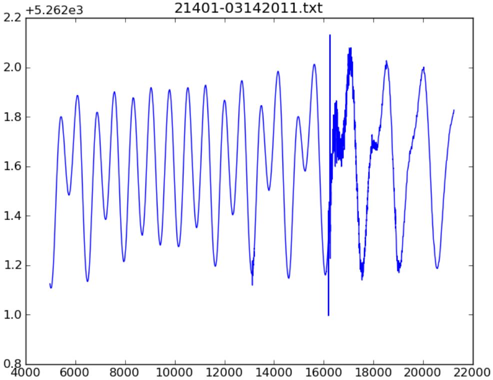
Obs via Google Maps
Classic Maps
Recent
Historical
DART®
MMS ADCP
Obs Search
Ship Obs Report
Gliders
APEX
TAO
Follow the National Data Buoy Center on Facebook.
Station 21401 - 250NM Southeast of Iturup Island
Owned and maintained by Hydromet to Russian Far Eastern Regional Hydrometeorological Research Institute (RFERHRI)
STB - SAIC Tsunami Buoy
STB payload
42.617 N 152.583 E (42°37'0" N 152°35'0" E)
This station was established on 11/08/2010.
Meteorological Observations from Nearby Stations and Ships


21401-03142011.txt
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2.2
2.0
1.8
1.6
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1.2
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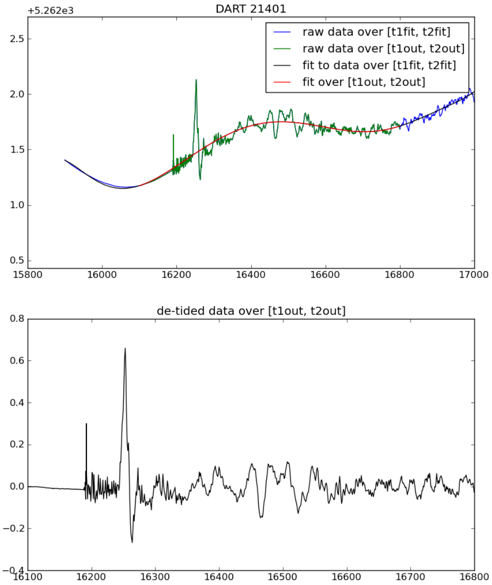
DART 21401
+5.262e3
raw data over [t1fit, t2fit]
raw data over [t1out, t2out]
fit to data over [t1fit, t2fit]
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2.5
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15800
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17000
de-tided data over [t1out, t2out]
0.8
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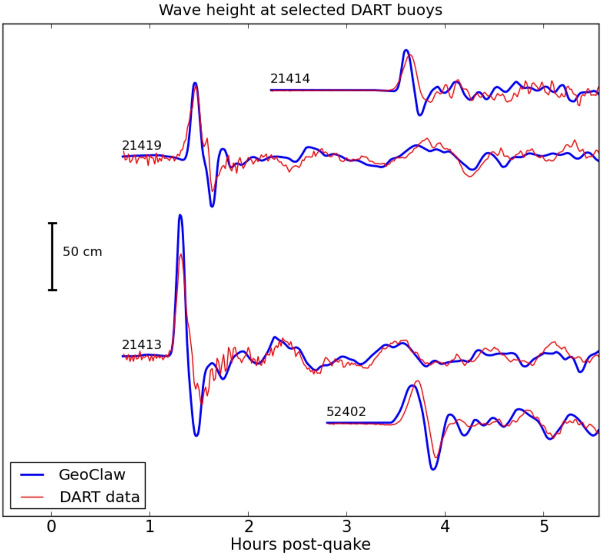

Shallow water equations

$h(x,t) =$ depth
$u(x,t) =$ velocity (depth averaged, varies only with x)

Conservation of mass and momentum hu gives system of two equations.

mass flux $= hu$,
momentum flux $= (hu)u + p$ where $p =$ hydrostatic pressure

$$h_t + (hu)_x = 0$$

$$(hu)_t + \left(hu^2 + \frac{1}{2}gh^2\right)_x = 0$$

Jacobian matrix:

$$f'(q) = \begin{bmatrix} 0 & 1 \\ gh - u^2 & 2u \end{bmatrix}, \qquad \lambda = u \pm \sqrt{gh}.$$

Shallow water equations

Hydrostatic pressure:

Pressure at depth $z > 0$ below the surface is gz from weight of water above.

Depth-averaged pressure is

$$p = \int_0^h gz\,dz = \frac{1}{2}gz^2\Big|_0^h = \frac{1}{2}gh^2.$$

The Riemann problem

Dam break problem for shallow water equations

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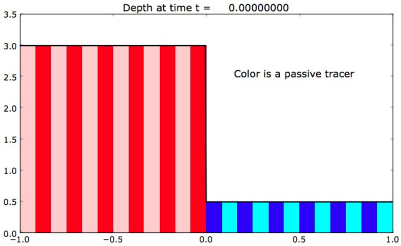

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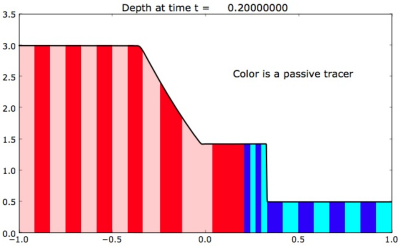

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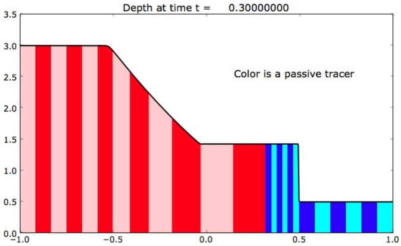

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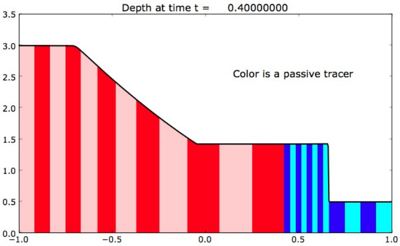

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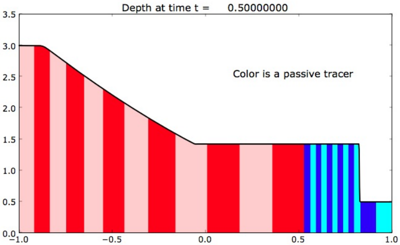
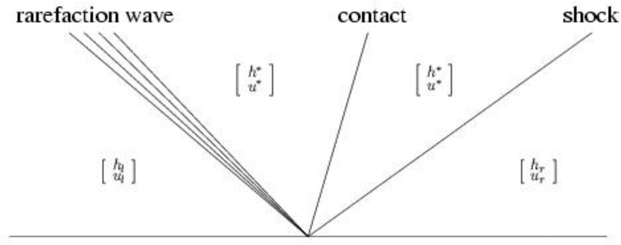


Riemann solution for the SW equations in x-t plane



Similarity solution:
Solution is constant on any ray: $q(x,t) = Q(x/t)$

Riemann solution can be calculated for many problems.
Linear: Eigenvector decomposition. Nonlinear: more difficult.

In practice "approximate Riemann solvers" used numerically.

An isolated shock

If an isolated shock with left and right states q_l and q_r is propagating at speed s

then the Rankine-Hugoniot condition must be satisfied:

$$f(q_r) - f(q_l) = s(q_r - q_l)$$

For a system $q \in \mathbb{R}^m$ this can only hold for certain pairs q_l, q_r:

For a linear system, $f(q_r) - f(q_l) = Aq_r - Aq_l = A(q_r - q_l)$.
So $q_r - q_l$ must be an eigenvector of $f'(q) = A$.

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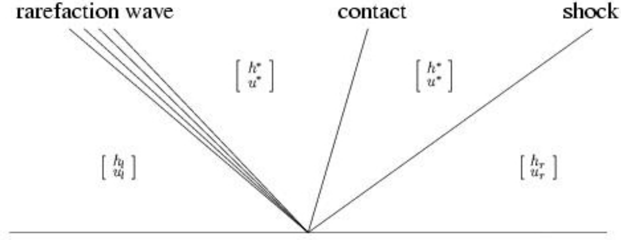
For a nonlinear system, there will be m curves through q_l called the Hugoniot loci.

Riemann solution for the SW equations in x-t plane



Nonlinear Riemann solution: If we know the 1-wave is a rarefaction wave, 2-wave is a shock,
Can find h^* by solving:

$$u_l + 2(\sqrt{gh_l} - \sqrt{gh^*}) = u_r + (h^* - h_r)\sqrt{\frac{g}{2}\left(\frac{1}{h^*} + \frac{1}{h_r}\right)}.$$

Expensive to do at every cell interface!

HLL Solver

Harten – Lax – van Leer (1983): Use only 2 waves with

s^1 =minimum characteristic speed

s^2 =maximum characteristic speed

$$\mathcal{W}^1 = Q^* - Q_\ell, \qquad \mathcal{W}^2 = Q_r - Q^*$$

Conservation implies unique value for middle state Q^*:

$$s^1\mathcal{W}^1 + s^2\mathcal{W}^2 = f(Q_r) - f(Q_\ell)$$

$$\implies Q^* = \frac{f(Q_r) - f(Q_\ell) - s^2 Q_r + s^1 Q_\ell}{s^1 - s^2}.$$

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Choice of speeds:

- Max and min of expected speeds over entire problem,
- Max and min of eigenvalues of $f'(Q_\ell)$ and $f'(Q_r)$.

HLLE Solver

Einfeldt: Choice of speeds for gas dynamics (or shallow water) that guarantees positivity.

Based on characteristic speeds and Roe averages:

$$s^1_{i-1/2} = \min_p(\min(\lambda^p_i, \hat{\lambda}^p_{i-1/2})),$$
$$s^2_{i-1/2} = \max_p(\max(\lambda^p_{i+1}, \hat{\lambda}^p_{i-1/2})).$$

where

λ^p_i is the pth eigenvalue of the Jacobian $f'(Q_i)$,

$\hat{\lambda}^p_{i-1/2}$ is the pth eigenvalue using Roe average $f'(\hat{Q}_{i-1/2})$

Approximate Riemann Solvers

Approximate true Riemann solution by set of waves consisting of finite jumps propagating at constant speeds.

Local linearization:

Replace $q_t + f(q)_x = 0$ by

$$q_t + \hat{A} q_x = 0,$$

where $\hat{A} = \hat{A}(q_l, q_r) \approx f'(q_{ave})$.

Then decompose

$$q_r - q_l = \alpha^1 \hat{r}^1 + \cdots \alpha^m \hat{r}^m$$

to obtain waves $\mathcal{W}^p = \alpha^p \hat{r}^p$ with speeds $s^p = \hat{\lambda}^p$.

Approximate Riemann Solvers

How to use?

One approach: determine $Q^* =$ state along $x/t = 0$,

$$Q^* = Q_{i-1} + \sum_{p:s^p<0} \mathcal{W}^p, \quad F_{i-1/2} = f(Q^*),$$

$$\mathcal{A}^- \Delta Q = F_{i-1/2} - f(Q_{i-1}), \quad \mathcal{A}^+ \Delta Q = f(Q_i) - F_{i-1/2}.$$

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Wave-propagation algorithm uses:

$$\mathcal{A}^-\Delta Q = \sum_{p:s^p<0} s^p\mathcal{W}^p, \qquad \mathcal{A}^+\Delta Q = \sum_{p:s^p>0} s^p\mathcal{W}^p.$$

Conservative only if $\mathcal{A}^-\Delta Q + \mathcal{A}^+\Delta Q = f(Q_i) - f(Q_{i-1})$.

This holds for Roe solver.

Roe Solver

Solve $q_t + \hat{A} q_x = 0$ where $\hat{A}$ satisfies

$$\hat{A}(q_r - q_l) = f(q_r) - f(q_l).$$

Then:

- Good approximation for weak waves (smooth flow)
- Single shock captured exactly:

  $f(q_r) - f(q_l) = s(q_r - q_l) \implies q_r - q_l$ is an eigenvector of $\hat{A}$

- Wave-propagation algorithm is conservative since

  $$\mathcal{A}^- \Delta Q_{i-1/2} + \mathcal{A}^+ \Delta Q_{i-1/2} = \sum s^p_{i-1/2} \mathcal{W}^p_{i-1/2} = A \sum \mathcal{W}^p_{i-1/2}.$$

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Roe average $\hat{A}$ can be determined analytically for many important nonlinear systems (e.g. Euler, shallow water).

Roe solver for Shallow Water

Given $h_l,\ u_l,\ h_r,\ u_r$, define

$$\bar{h} = \frac{h_l + h_r}{2}, \quad \hat{u} = \frac{\sqrt{h_l}u_l + \sqrt{h_r}u_r}{\sqrt{h_l} + \sqrt{hr}}$$

Then

$$\hat{A} = \text{Jacobian matrix evaluated at this average state}$$

satisfies

$$A(q_r - q_l) = f(q_r) - f(q_l).$$

- Roe condition is satisfied,
- Isolated shock modeled well,
- Wave propagation algorithm is conservative,
- High resolution methods obtained using corrections with limited waves.

Roe solver for Shallow Water

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Eigenvalues of $\hat{A} = f'(\hat{q})$ are:

$$\hat{\lambda}^1 = \hat{u} - \hat{c}, \quad \hat{\lambda}^2 = \hat{u} + \hat{c}, \quad \hat{c} = \sqrt{g\bar{h}}.$$

Eigenvectors:

$$\hat{r}^1 = \begin{bmatrix} 1 \\ \hat{u} - \hat{c} \end{bmatrix}, \qquad \hat{r}^2 = \begin{bmatrix} 1 \\ \hat{u} + \hat{c} \end{bmatrix}.$$

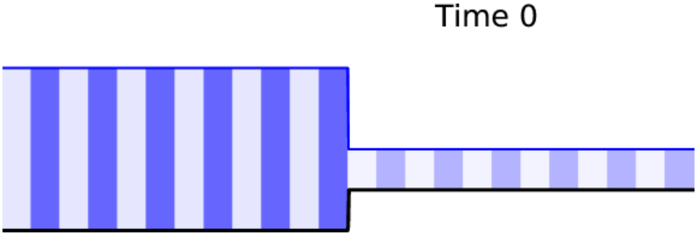
Examples in Clawpack 4.3 to be converted soon!

The Riemann problem over topography

$$h_t + (hu)_x = 0$$

$$(hu)_t + \left(hu^2 + \frac{1}{2}gh^2\right)_x = -ghB_x(x)$$

With piecewise constant $B(x)$, source term is delta function.



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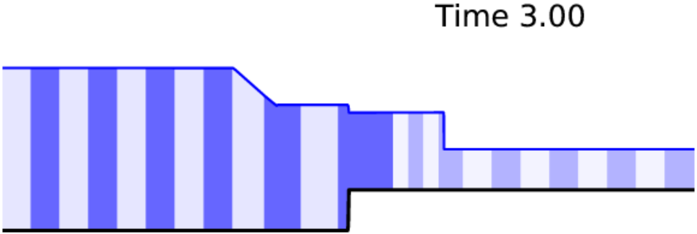

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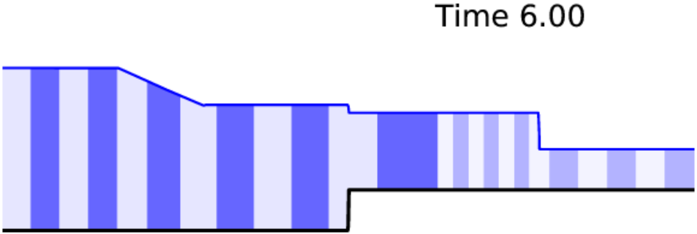
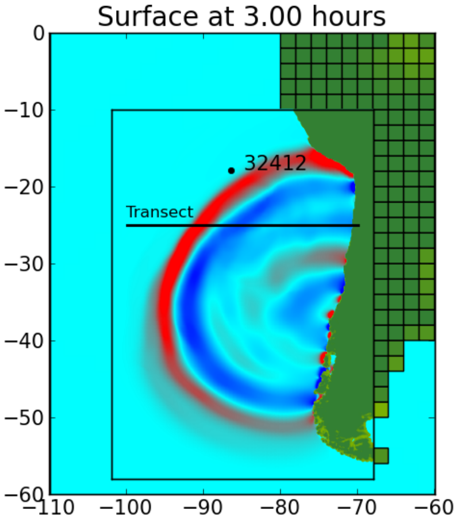
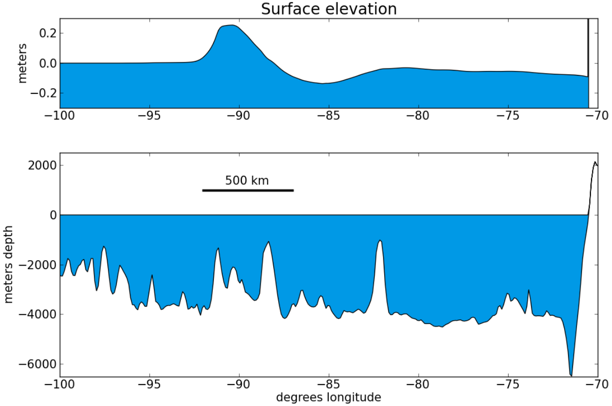


Tsunami from 27 Feb 2010 quake off Chile



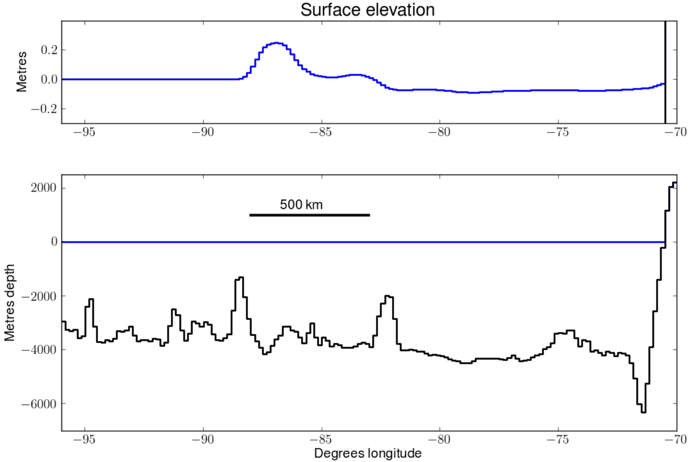
Cross section of Atlantic Ocean & tsunami



Transect of 27 February 2010 tsunami

Bathymetry, depth change by > 1000 m from one cell to next,

Surface elevation changes on order of a few cm.



Source terms and quasi-steady solutions

$$q_t + f(q)_x = \psi(q)$$

Steady-state solution:

$$q_t = 0 \implies f(q)_x = \psi(q)$$

Balance between flux gradient and source.

Quasi-Steady solution:

Small perturbation propagating against steady-state background.

$$q_t \ll f(q)_x \approx \psi(q)$$

Want accurate calculation of perturbation.

Examples:

- Shallow water equations with bottom topography and flat surface
- Stationary atmosphere where pressure gradient balances gravity

Fractional steps for a quasisteady problem

Alternate between solving homogeneous conservation law

$$q_t + f(q)_x = 0 \tag{1}$$

and source term

$$q_t = \psi(q). \tag{2}$$

When $q_t \ll f(q)_x \approx \psi(q)$:

- Solving (1) gives large change in q
- Solving (2) should essentially cancel this change.

Numerical difficulties:

- (1) and (2) are solved by very different methods. Generally will not have proper cancellation.
- Nonlinear limiters are applied to $f(q)_x$ term, not to small-perturbation waves. Large variation in steady state hides structure of waves.

Incorporating source term in f-waves

$q_t + f(q)_x = \psi$ with $f(q)_x \approx \psi$.

Concentrate source at interfaces: $\Psi_{i-1/2}\,\delta(x - x_{i-1/2})$

Split $f(Q_i) - f(Q_{i-1}) - \Delta x \Psi_{i-1/2} = \sum_p \mathcal{Z}^p_{i-1/2}$

Use these waves in wave-propagation algorithm.

Steady state maintained: (Well balanced)

If $\frac{f(Q_i) - f(Q_{i-1})}{\Delta x} = \Psi_{i-1/2}$ then $\mathcal{Z}^p \equiv 0$

Near steady state:

Deviation from steady state is split into waves and limited.

Shallow water equations with bathymetry $B(x)$

$$h_t + (hu)_x = 0$$

$$(hu)_t + \left(hu^2 + \frac{1}{2}gh^2\right)_x = -ghB_x(x)$$

Ocean-at-rest equilibrium:

$$u^e \equiv 0, \qquad h^e(x) + B(x) \equiv \bar{\eta} = \text{sea level.}$$

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Using

$$\Psi_{i-1/2} = -\frac{g}{2}(h_{i-1} + h_i)$$

gives exactly well-balanced method, but only because hydrostatic pressure is quadratic function of h:

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$$\begin{aligned} f(Q_i) - f(Q_{i-1}) - \Psi_{i-1/2}(B_i - B_{i-1}) &= \\ &= \left(\frac{1}{2}gh_i^2 - \frac{1}{2}gh_{i-1}^2\right) + \frac{g}{2}(h_{i-1} + h_i)(B_i - B_{i-1}) \\ &= \frac{g}{2}(h_{i-1} + h_i)((h_i + B_i) - (h_{i-1} + B_{i-1})) \\ &= 0 \quad \text{if } h_i + B_i = h_{i-1} + B_{i-1} = \bar{\eta}. \end{aligned}$$

Flux-based wave decomposition (f-waves)

Choose r^p (e.g. eigenvectors of linearized Jacobian).

Then decompose flux difference:

$$f_r(q_r) - f_l(q_l) = \sum_{p=1}^{m} \beta^p r^p \equiv \sum_{p=1}^{m} \mathcal{Z}^p$$

rather than jump in q:

$$q_r - q_l = \sum_{p=1}^{m} \alpha^p r^p \equiv \sum_{p=1}^{m} \mathcal{W}^p$$

For linear system or Roe solver,

$$\mathcal{Z}^p = s^p \mathcal{W}^p, \qquad s^p = \text{eigenvalue.}$$

Bale, RJL, Mitran, Rossmanith, SISC 2002 [link]

RJL, Pelanti, JCP 2001 [link]

Wave-propagation algorithm using waves

$$
\begin{aligned}
Q_i^{n+1} &= Q_i^n - \frac{\Delta t}{\Delta x}[\mathcal{A}^+\Delta Q_{i-1/2} + \mathcal{A}^-\Delta Q_{i+1/2}] \\
&\quad - \frac{\Delta t}{\Delta x}[\tilde{F}_{i+1/2} - \tilde{F}_{i-1/2}]
\end{aligned}
$$

Standard version: $Q_i - Q_{i-1} = \sum_{p=1}^m \mathcal{W}_{i-1/2}^p$

$$
\begin{aligned}
\mathcal{A}^-\Delta Q_{i+1/2} &= \sum_{p=1}^m (s_{i+1/2}^p)^- \mathcal{W}_{i+1/2}^p \\
\mathcal{A}^+\Delta Q_{i-1/2} &= \sum_{p=1}^m (s_{i-1/2}^p)^+ \mathcal{W}_{i-1/2}^p \\
\tilde{F}_{i-1/2} &= \frac{1}{2}\sum_{p=1}^m |s_{i-1/2}^p| \left(1 - \frac{\Delta t}{\Delta x}|s_{i-1/2}^p|\right) \widetilde{\mathcal{W}}_{i-1/2}^p.
\end{aligned}
$$

Wave-propagation algorithm using f-waves

$$Q_i^{n+1} = Q_i^n - \frac{\Delta t}{\Delta x}[\mathcal{A}^+\Delta Q_{i-1/2} + \mathcal{A}^-\Delta Q_{i+1/2}]$$
$$- \frac{\Delta t}{\Delta x}[\tilde{F}_{i+1/2} - \tilde{F}_{i-1/2}]$$

Using f-waves: $f_i(Q_i) - f_{i-1}(Q_{i-1}) = \sum_{p=1}^m \mathcal{Z}_{i-1/2}^p$

$$\mathcal{A}^-\Delta Q_{i-1/2} = \sum_{p:s_{i-1/2}^p<0} \mathcal{Z}_{i-1/2}^p,$$

$$\mathcal{A}^+\Delta Q_{i-1/2} = \sum_{p:s_{i-1/2}^p>0} \mathcal{Z}_{i-1/2}^p,$$

$$\tilde{F}_{i-1/2} = \frac{1}{2}\sum_{p=1}^m \text{sgn}(s_{i-1/2}^p)\left(1 - \frac{\Delta t}{\Delta x}|s_{i-1/2}^p|\right)\widetilde{\mathcal{Z}}_{i-1/2}^p$$

f-wave approximate Riemann solver

Let $\hat{A}$ be any averaged Jacobian matrix, e.g.
$\hat{A} = f'((q_l + q_r)/2)$.

Use eigenvectors of $\hat{A}$ to do f-wave splitting.

Then $\mathcal{A}^-\Delta Q_{i-1/2} + \mathcal{A}^+\Delta Q_{i+1/2} = f(Q_i) - f(Q_{i-1})$ and so method is conservative.

If $\hat{A} =$ Roe average, then this is equivalent to usual Roe Riemann solver, and $\mathcal{Z}^p = s^p \mathcal{W}^p$.

Clawpack: Use library routine step1fw.f instead of step1.f.

Dry cells and inundation

Use regular grid (e.g. Latitude–Longitude).

Finite volume cells can be wet $(h > 0)$ or dry $(h = 0)$.

Allow state to change dynamically from one step to next.

Need Riemann solver that handles dry states.

Dry cells and inundation

Use regular grid (e.g. Latitude–Longitude).

Finite volume cells can be wet $(h > 0)$ or dry $(h = 0)$.

Allow state to change dynamically from one step to next.

Need Riemann solver that handles dry states.

Use adaptive mesh refinement (AMR) to resolve shoreline.

AMR algorithms have to interact well with wetting/drying.

Also with well-balancing.

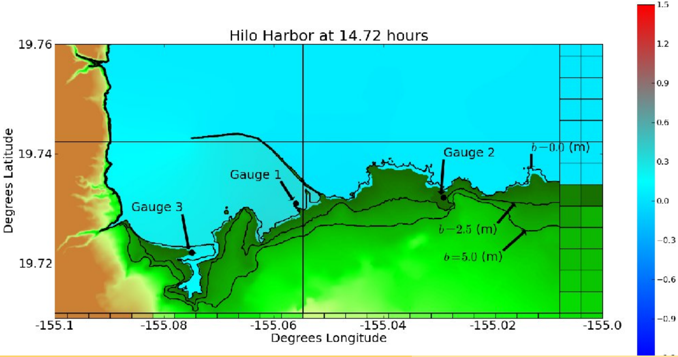
Inundation of Hilo, Hawaii from 27 Feb 2010 event

Resolution $\Delta y \approx 160$ km on Level 1 (covering Pacific Ocean),
$\Delta y \approx 10$m on Level 5 (shown below).

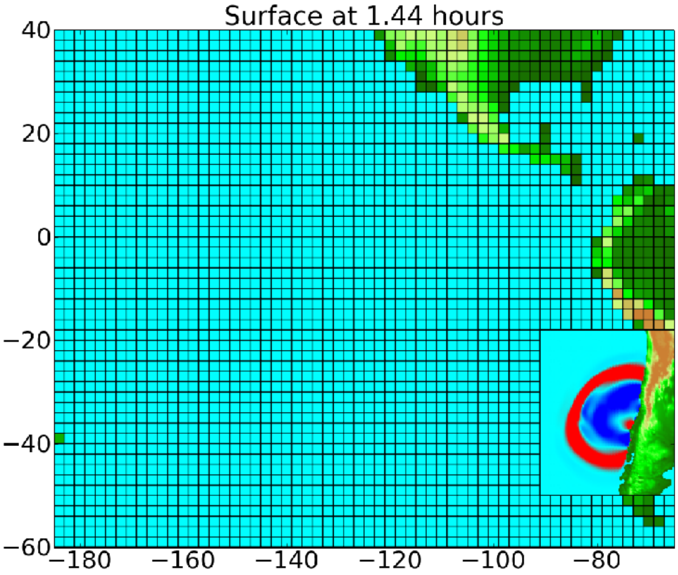
Using 5 levels of refinement with ratios 8, 4, 16, 32.

Total refinement factor: $2^{14} = 16,384$ in each direction.

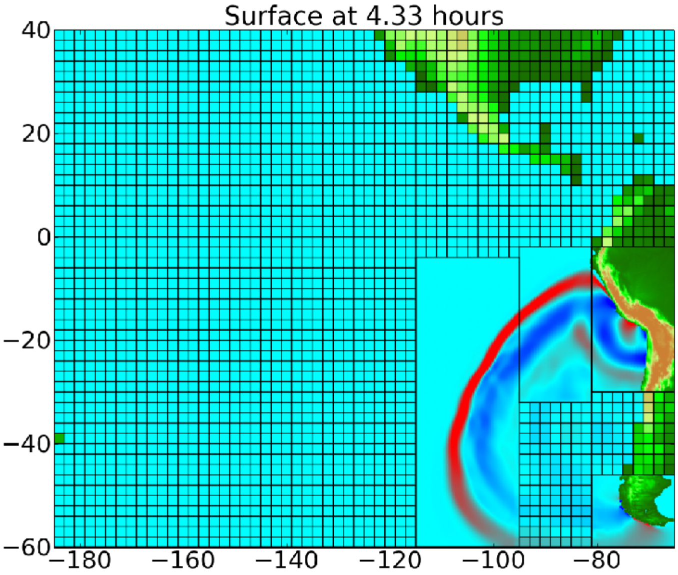
With 15 m displacement at fault:



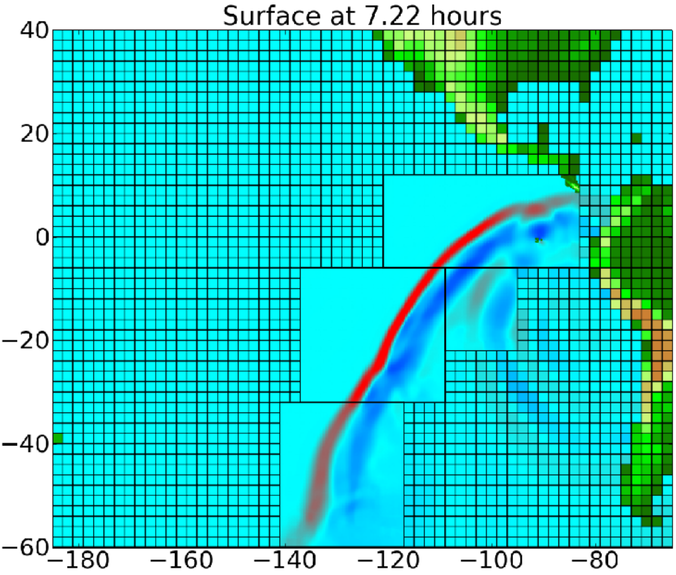
27 February 2010 tsunami



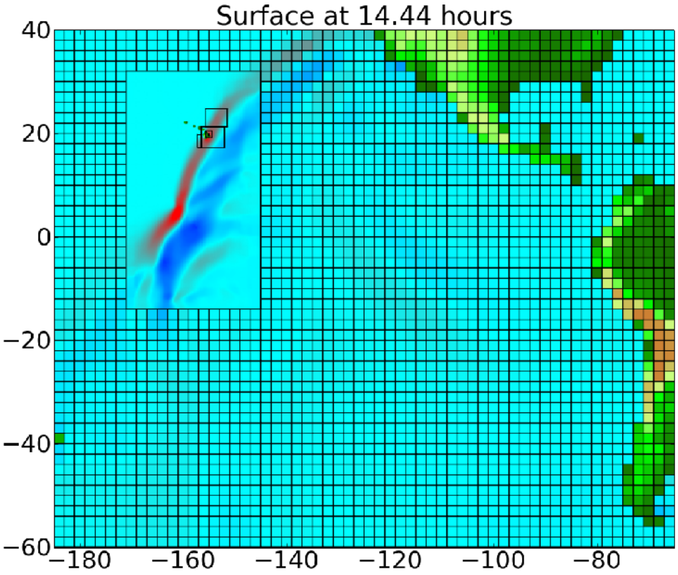
27 February 2010 tsunami



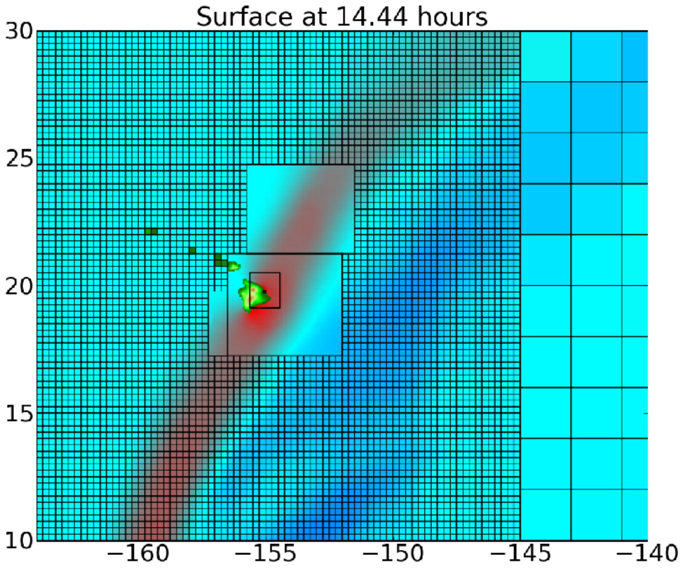
27 February 2010 tsunami



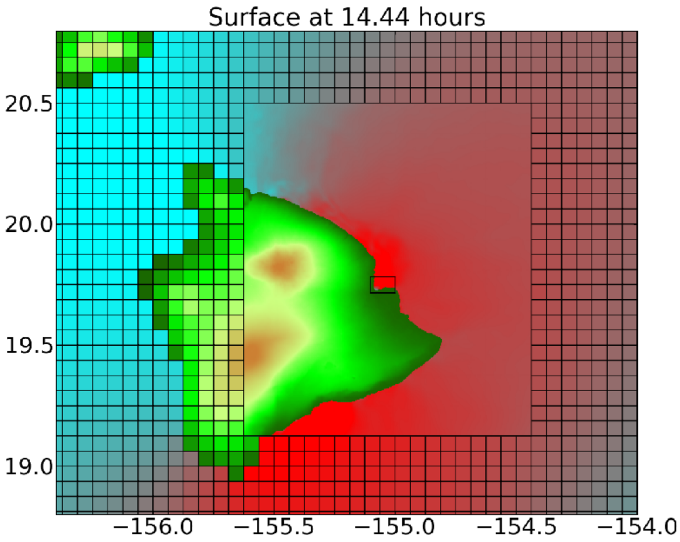
27 February 2010 tsunami



27 February 2010 tsunami



27 February 2010 tsunami



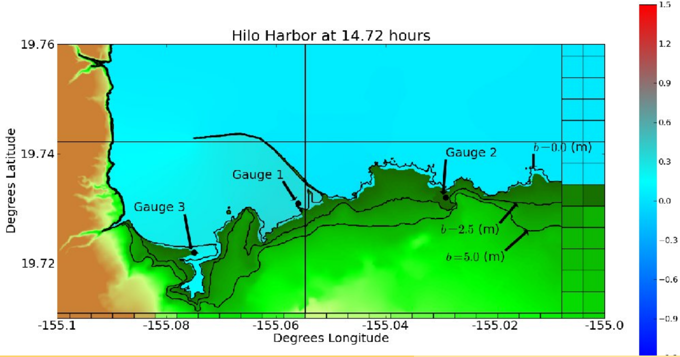
Inundation of Hilo, Hawaii

Using 5 levels of refinement with ratios 8, 4, 16, 32.

Resolution ≈ 160 km on Level 1 and ≈ 10m on Level 5.

Total refinement factor: $2^{14} = 16,384$ in each direction.

With 15 m displacement at fault (27 Feb 2010):



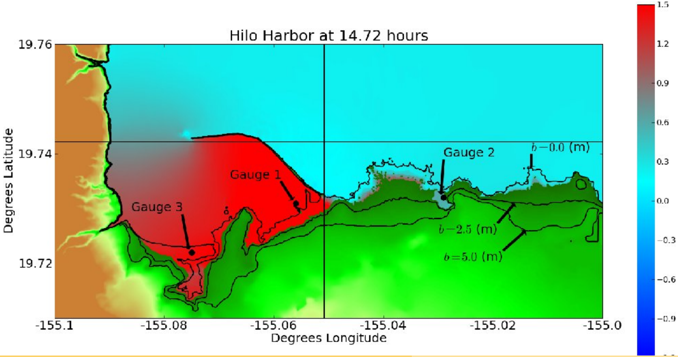
Inundation of Hilo, Hawaii

Using 5 levels of refinement with ratios 8, 4, 16, 32.

Resolution ≈ 160 km on Level 1 and ≈ 10m on Level 5.

Total refinement factor: $2^{14} = 16,384$ in each direction.

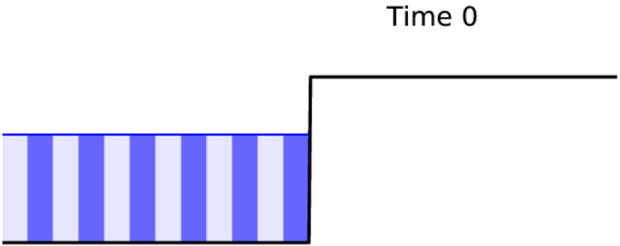
With 90 m displacement at fault (1960?):



The Riemann problem with dry state

$$h_t + (hu)_x = 0$$
$$(hu)_t + \left(hu^2 + \frac{1}{2}gh^2\right)_x = -ghB_x(x)$$

For small velocity $u_\ell > 0$, the shore acts as solid wall:



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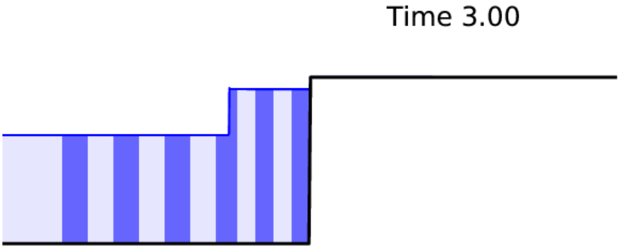

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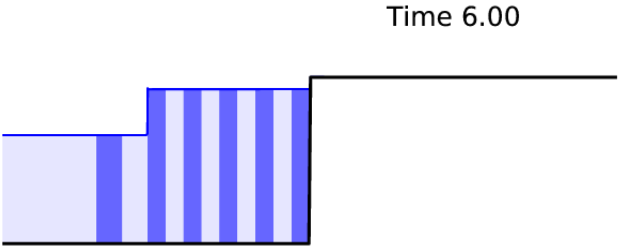

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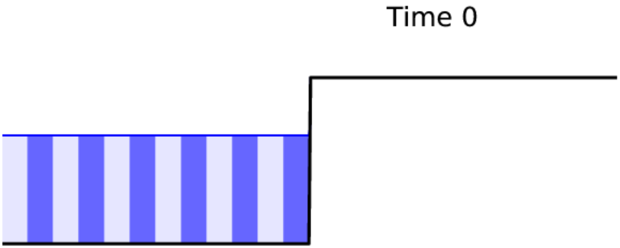

The Riemann problem with dry state

$$h_t + (hu)_x = 0$$

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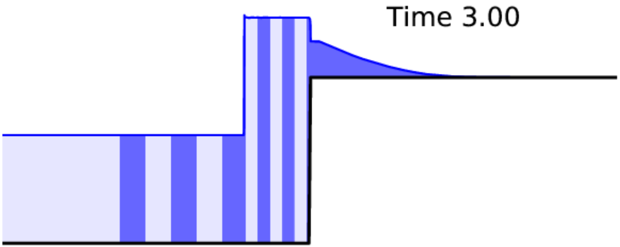

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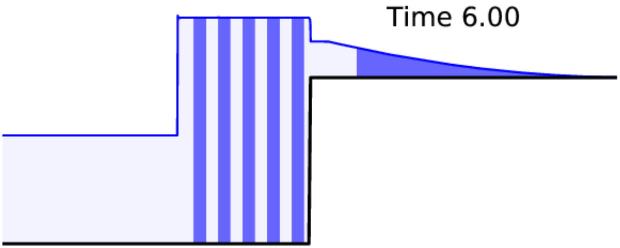

Some references

GeoClaw: **www.clawpack.org/geoclaw**
contains links to the recent paper with references and codes:

Tsunami modeling with adaptively refined finite volume methods, by RJL, D. L. George, M. J. Berger, *Acta Numerica* 2011

The GeoClaw software for depth-averaged flows with adaptive refinement, by M. J. Berger, D. L. George, RJL, and K. M. Mandli, 2011 *Advances in Water Resources*

GeoClaw results for the NTHMP tsunami benchmark problems, with Chamberlain, González, Hirai, Varkovitzky, 2011.