Tutorial suggestions

If you want to experiment with Clawpack, see
www.clawpack.org/g2s3/doc/clawpack-quickstart.html

Determine the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 0 & h_0 \\ g & 0 \end{bmatrix}$$

for the linearized shallow water equations.

2. Solve the Riemann problem for these equations for general data

$$q_\ell = \begin{bmatrix} h_\ell \\ u_\ell \end{bmatrix}, \qquad q_r = \begin{bmatrix} h_r \\ u_r \end{bmatrix}.$$

Find a general expression for the middle state q_m and for the waves $\mathcal{W}^1 = \alpha^1 r^1$ and $\mathcal{W}^2 = \alpha^2 r^2$.

Tutorial suggestions

Take $g = 10$ and $h_0 = 10$ for simplicity.

Experiment with the following Riemann data.

(a) $h_\ell = 1, \ \ h_r = 0, \ \ u_\ell = u_r = 0.$
(Dam break problem.)

(b) $h_\ell = 1, \ \ h_r = 1, \ \ u_\ell = 1, \ \ u_r = -1.$

Colliding streams of water — this Riemann problem can be used for modeling reflecting wall boundary conditions. For any left state $h_\ell, \ u_l$ set $h_r = h_\ell$ and $u_r = -u_\ell.$

Note that in this case the Riemann solution has $u_m = 0$, as would arise for flow into a wall at $x = 0$.

Hints on computing eigenvalues/vectors

Recall that $Ar = \lambda r$ means that $(A - \lambda I)r = 0$.

So $A - \lambda I$ must be a singular matrix, so $det(A - \lambda I) = 0$.

For a 2×2 matrix the determinant is

$$(a_{11} - \lambda)(a_{22} - \lambda) - a_{12}a_{21}$$

Setting this to zero gives a quadratic equation to solve for the two eigenvalues λ^1 and λ^2.

Then examine the singular matrix $(A - \lambda^p I)$ and find a vector r^p satisfying $(A - \lambda I)r^p = 0$. (What linear combination of the columns gives the 0 vector?)