Clawpack Tutorial
Part I

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Conservation Laws Package

www.clawpack.org

(pdf's will be posted and green links can be clicked)

Some collaborators

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David George, USGS CVO
Donna Calhoun, CEA, Paris
Christiane Helzel, Bochum
David Ketcheson, KAUST
Sorin Mitran, UNC

Current students:

Kyle Mandli (PyClaw, GeoClaw, storm surges)
Jonathan Claridge (Implicit AMR)
Grady Lemoine (Cut cells, bone modeling)
Jihwan Kim (multi-layer, submarine landslides)

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Outline

Monday:

- Overview of Clawpack software
- What are hyperbolic problems?
- Finite volume methods
- Riemann problems and Godunov's method
- Downloading and installing
- Running and plotting

Tuesday:

- Specifying boundary conditions
- Riemann solvers
- Limiters

Wednesday:

- Plotting with the Python modules
- Multidimensional, Adaptive mesh refinement

Options for using Clawpack

1. Use IMA computers platinum, carbon, or tan.
   Install from tar file or Subversion: **Instructions**.
2. Install on your own computer.
   Requires some **prerequisites:** Fortran, Python modules.
3. Use the **VirtualClaw** virtual machine.
4. For some applications, use **EagleClaw**
   (Easy Access Graphical Laboratory for Exploring Conservation Laws)

Also perhaps useful:
Class notes on Python, Fortran, version control, etc.

First order hyperbolic PDE in 1 space dimension

Linear: $q_t + Aq_x = 0, \qquad q(x,t) \in \mathbb{R}^m, \ A \in \mathbb{R}^{m \times m}$

Conservation law: $q_t + f(q)_x = 0, \qquad f : \mathbb{R}^m \to \mathbb{R}^m$ (flux)

Quasilinear form: $q_t + f'(q)q_x = 0$

Hyperbolic if A or $f'(q)$ is diagonalizable with real eigenvalues.

Models wave motion or advective transport.

Eigenvalues are wave speeds.

Note: Second order wave equation $p_{tt} = c^2 p_{xx}$ can be written as a first-order system (acoustics).

Some applications where CLAWPACK has been used

- Aerodynamics, supersonic flows
- Seismic waves, tsunamis, flow on the sphere
- Volcanic flows, dusty gas jets, pyroclastic surges
- Ultrasound, lithotripsy, shock wave therapy
- Plasticity, nonlinear elasticity
- Chemotaxis and pattern formation
- Semiconductor modeling
- Multi-fluids, multi-phase flows, bubbly flow
- Combustion, detonation waves
- Astrophysics: binary stars, planetary nebulae, jets,
- Magnetohydrodynamics, plasmas, relativistic flow
- Numerical relativity — gravitational waves, cosmology

Finite differences vs. finite volumes

Finite difference Methods

- Pointwise values $Q_i^n \approx q(x_i, t_n)$
- Approximate derivatives by finite differences
- Assumes smoothness

Finite volume Methods

- Approximate cell averages: $Q_i^n \approx \frac{1}{\Delta x} \int_{x_{i-1/2}}^{x_{i+1/2}} q(x, t_n)\, dx$
- Integral form of conservation law,

$$\frac{\partial}{\partial t} \int_{x_{i-1/2}}^{x_{i+1/2}} q(x,t)\, dx \quad = \quad f(q(x_{i-1/2}, t)) - f(q(x_{i+1/2}, t))$$

leads to conservation law $q_t + f_x = 0$ but also directly to numerical method.

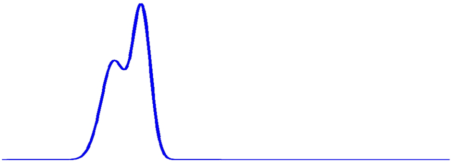
Advection equation

$u =$ constant flow velocity

$q(x,t) =$ tracer concentration, $\quad f(q) = uq$

$\implies \quad q_t + uq_x = 0.$

True solution: $q(x,t) = q(x - ut, 0)$



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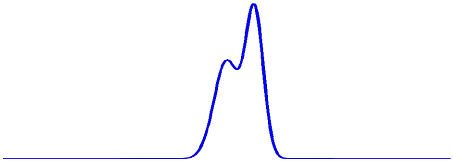

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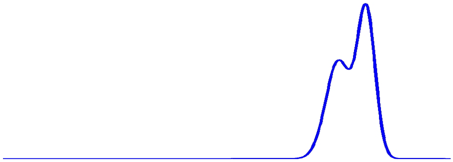


The Riemann problem

The Riemann problem consists of the hyperbolic equation under study together with initial data of the form

$$q(x,0) = \begin{cases} q_l & \text{if } x < 0 \\ q_r & \text{if } x \geq 0 \end{cases}$$

Piecewise constant with a single jump discontinuity from q_l to q_r.

The Riemann problem is fundamental to understanding

- The mathematical theory of hyperbolic problems,
- Godunov-type finite volume methods

Why? Even for nonlinear systems of conservation laws, the Riemann problem can often be solved for general q_l and q_r, and consists of a set of waves propagating at constant speeds.

The Riemann problem for advection

The Riemann problem for the advection equation $q_t + uq_x = 0$ with

$$q(x,0) = \begin{cases} q_l & \text{if } x < 0 \\ q_r & \text{if } x \ge 0 \end{cases}$$

has solution

$$q(x,t) = q(x-ut,0) = \begin{cases} q_l & \text{if } x < ut \\ q_r & \text{if } x \ge ut \end{cases}$$

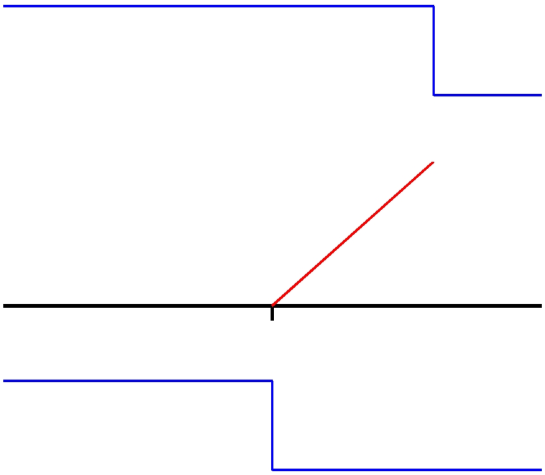
consisting of a single wave of strength $\mathcal{W}^1 = q_r - q_l$ propagating with speed $s^1 = u$.

Riemann solution for advection

$q(x,T)$

x–t plane

$q(x,0)$



Advection examples

- **$CLAW/apps/advection/1d/example1/README.html**
- **Advection in EagleClaw**

Example: Linear acoustics in a 1d tube

$$q = \begin{bmatrix} p \\ u \end{bmatrix} \qquad \begin{aligned} p(x,t) &= \text{pressure perturbation} \\ u(x,t) &= \text{velocity} \end{aligned}$$

Equations:

$$\begin{aligned} p_t + \kappa u_x &= 0 \qquad & \kappa &= \text{bulk modulus} \\ \rho u_t + p_x &= 0 \qquad & \rho &= \text{density} \end{aligned}$$

or

$$\begin{bmatrix} p \\ u \end{bmatrix}_t + \begin{bmatrix} 0 & \kappa \\ 1/\rho & 0 \end{bmatrix} \begin{bmatrix} p \\ u \end{bmatrix}_x = 0.$$

Eigenvalues: $\lambda = \pm c$, where $c = \sqrt{\kappa/\rho}$ = sound speed

Second order form: Can combine equations to obtain

$$p_{tt} = c^2 p_{xx}$$

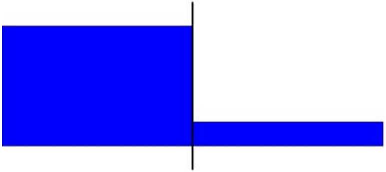
Riemann Problem

Special initial data:

$$q(x,0) = \begin{cases} q_l & \text{if } x < 0 \\ q_r & \text{if } x > 0 \end{cases}$$

Example: Acoustics with bursting diaphram

Pressure:



Acoustic waves propagate with speeds $\pm c$.

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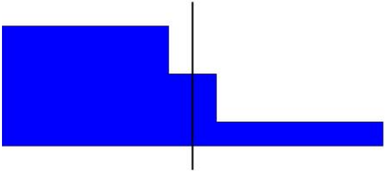

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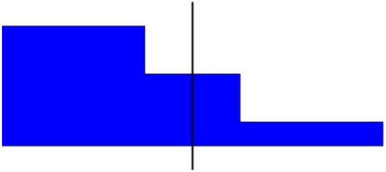

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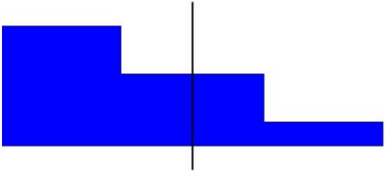

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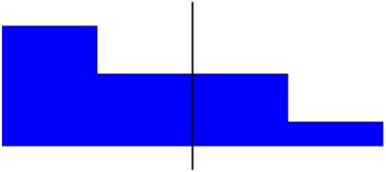

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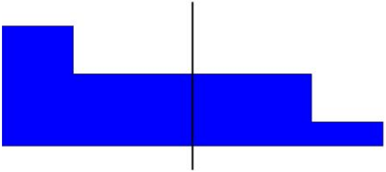

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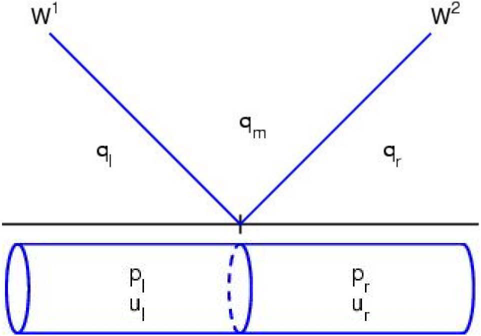
Riemann Problem for acoustics

Waves propagating in x–t space:



Left-going wave $\mathcal{W}^1 = q_m - q_l$ and
right-going wave $\mathcal{W}^2 = q_r - q_m$ are eigenvectors of A.

Acoustics examples

- **$CLAW/apps/acoustics/1d/example2/README.html**
- **Acoustics in EagleClaw**

CLAWPACK — www.clawpack.org

- Open source, 1d, 2d, (3d in V4.3, soon to be ported)
- Originally f77 with Matlab graphics (V4.3).
- Now use Python for user interface, graphics
- Adaptive mesh refinement, GeoClaw.
- Coming: OpenMP and MPI.

User supplies:

- Riemann solver, splitting data into waves and speeds
  (Need not be in conservation form)
- Boundary condition routine to extend data to ghost cells
  Standard `bc1.f` routine includes many standard BC's
- Initial conditions — `qinit.f`
- Source terms — `src1.f`

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