Clawpack Tutorial
Part 2

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Slides posted at
http://www.clawpack.org/links/tutorials
http://faculty.washington.edu/rjl/tutorials

Outline

- Boundary conditions

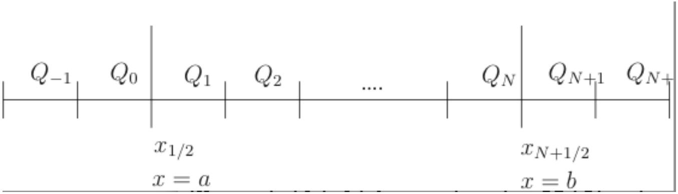
- Python plotting tools
- Specifying plotting parameters
- Options for viewing plots:
  Web pages
  Interactive

- Two space dimensions
  Normal and transverse Riemann solvers

Boundary conditions and ghost cells

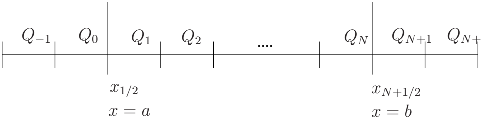
In each time step, the data in cells 1 to $N = \mathtt{mx}$ is used to define ghost cell values in cells outside the physical domain.

The wave-propagation algorithm is then applied on the expanded computational domain, solving Riemann problems at all interfaces.



The data is extended depending on the physical boundary conditons.

Boundary conditions



Periodic:

$$Q^n_{-1} = Q^n_{N-1}, \quad Q^n_0 = Q^n_N, \quad Q^n_{N+1} = Q^n_1, \quad Q^n_{N+2} = Q^n_2$$

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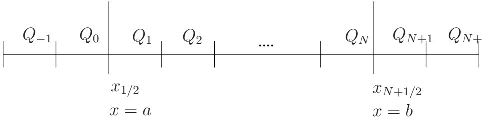

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Extrapolation (outflow):

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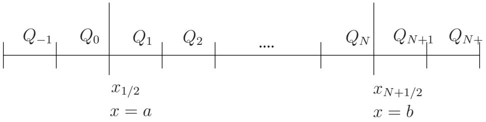

Periodic:

$$Q_{-1}^n = Q_{N-1}^n, \quad Q_0^n = Q_N^n, \quad Q_{N+1}^n = Q_1^n, \quad Q_{N+2}^n = Q_2^n$$

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Solid wall:

For Q_0 : $\quad p_0 = p_1, \quad u_0 = -u_1,$

For Q_{-1} : $\quad p_{-1} = p_2, \quad u_{-1} = -u_2.$

Setting BCs in Clawpack

In `setrun.py`, at each boundary
(`xlower, xupper, ylower, yupper`)
must specify for example:

```
clawdata.mthbc_xlower = 3  # solid wall
clawdata.mthbc_xupper = 1  # extrapolation
```

Set to 2 at both boundaries for periodic.

Setting BCs in Clawpack

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must specify for example:

```
clawdata.mthbc_xlower = 3  # solid wall
clawdata.mthbc_xupper = 1  # extrapolation
```

Set to 2 at both boundaries for periodic.

Set to 0 if you want to impose something special.

In this case need to copy **$CLAW/clawpack/1d/lib/bc1.f** to application directory and modify (along with Makefile).

Extrapolation boundary conditions

If we set $Q_0 = Q_1$ then the Riemann problem at $x_{1/2}$ has zero strength waves:

$$Q_1 - Q_0 = \mathcal{W}^1_{1/2} + \mathcal{W}^2_{1/2}$$

So in particular the incoming wave $\mathcal{W}^2$ has strength 0.

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In 2D or 3D, extrapolation in normal direction is not perfect but works quite well.

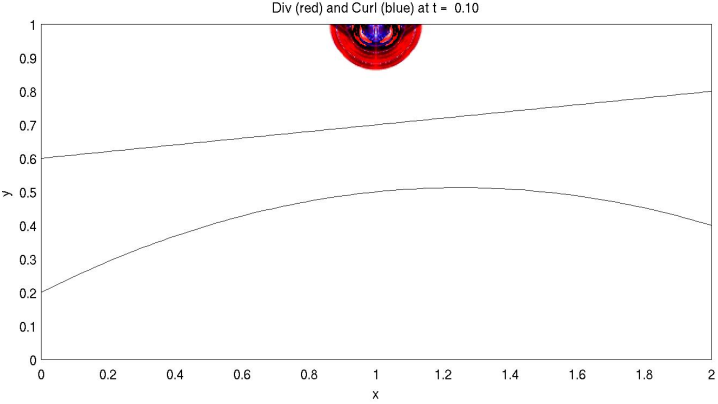
Extrapolation boundary conditions

Examples:

- One-dimensional acoustics:
  $CLAW/book/chap3/acousimple
  $CLAW/apps/acoustics/1d/example2
- Tsunami propagation:
  $CLAW/apps/tsunami/chile2010
- Seismic waves in a half-space on following slides,

Seismic wave in layered medium

Red = div(u) [P-waves], Blue = curl(u) [S-waves]



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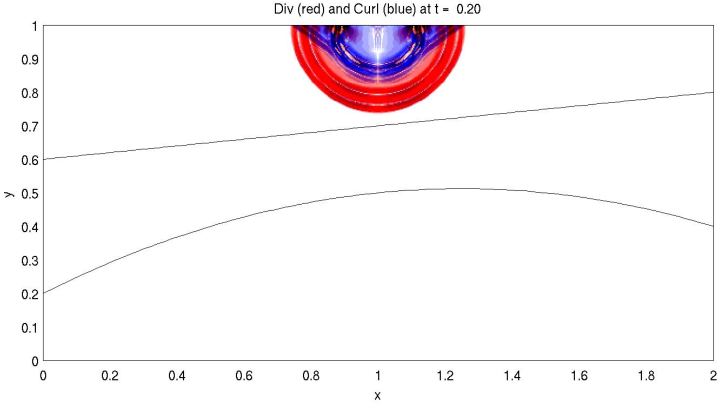

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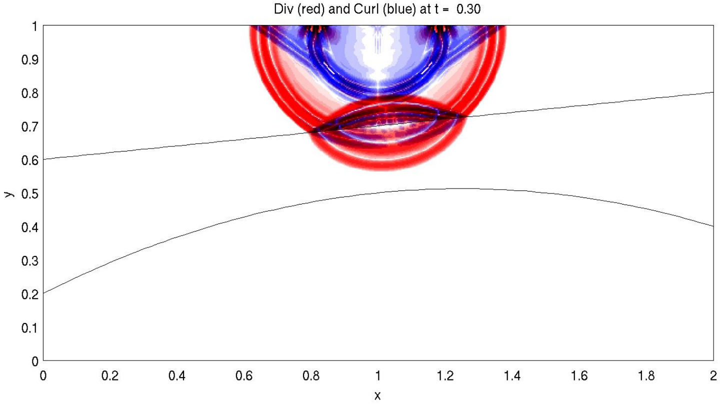

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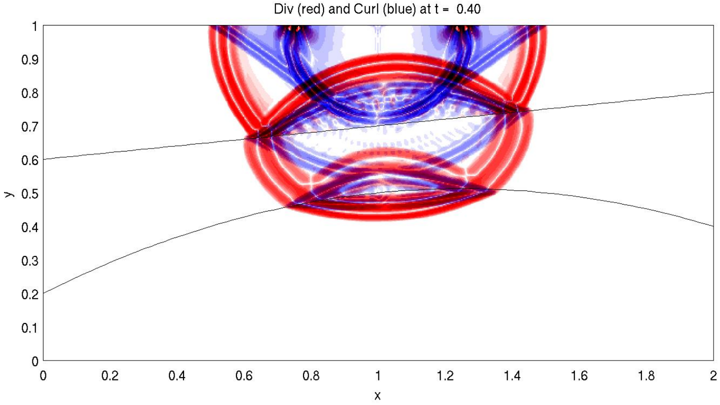

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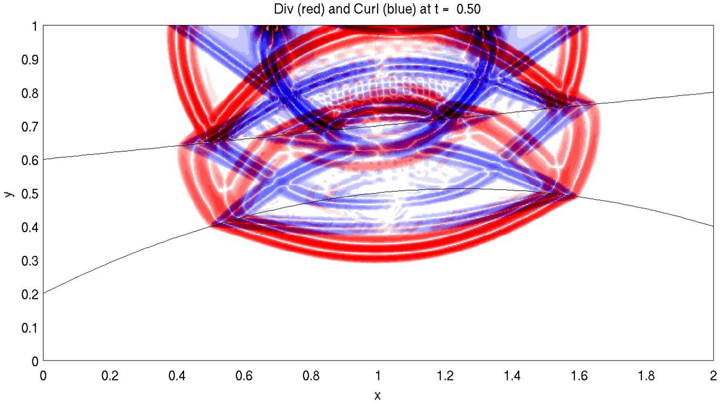

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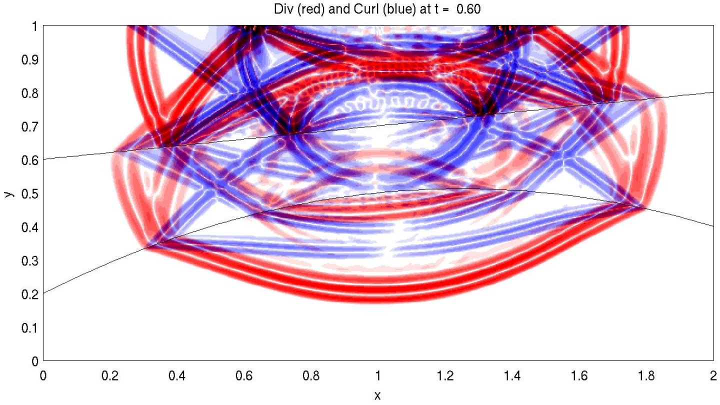

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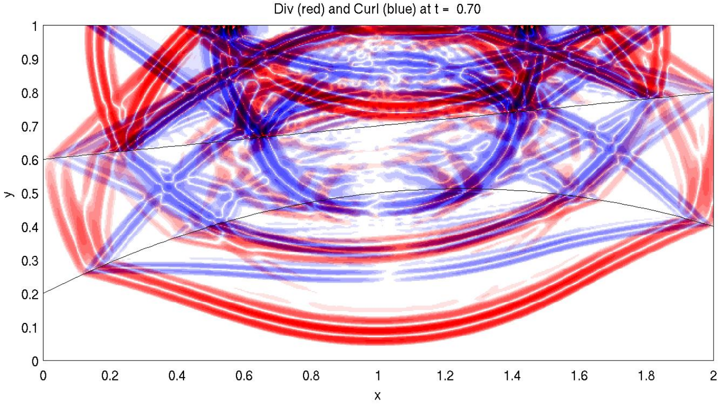

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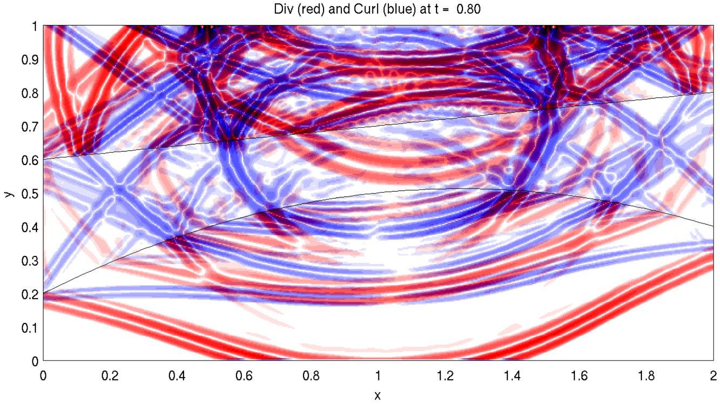

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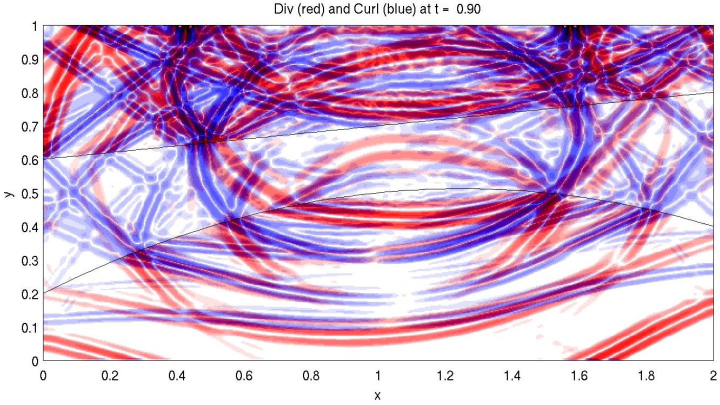

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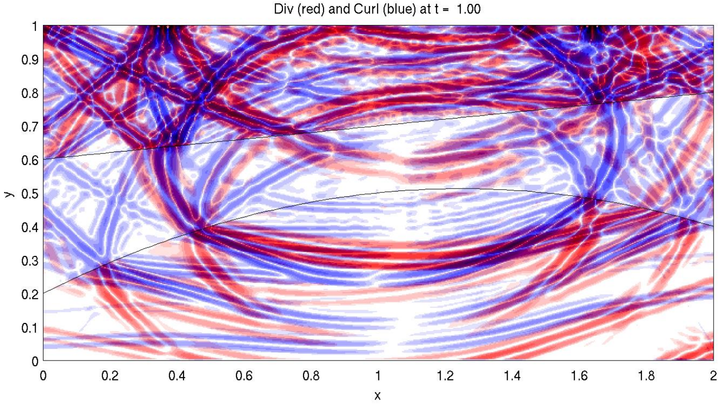

Python plotting tools

Directory `_output` contains files `fort.t000N, fort.q000N` of data at frame N (N'th output time).

`fort.t000N`: Information about this time,
`fort.q000N`: Solution on all grids at this time

There may be many grids at each output time.

Python tools provide a way to specify what plots to produce for each frame:

- One or more figures,
- Each figure has one or more axes,
- Each axes has one or more items,
  (Curve, contour, pcolor, etc.)

`setplot` function for speciying plots

The file `setplot.py` contains a function setplot
Takes an object `plotdata` of class ClawPlotData,
Sets various attributes, and returns the object.

Documentation: **www.clawpack.org/users/setplot.html**

Example: 1 figure with 1 axes showing 1 item:

```
def setplot(plotdata):
    plotfigure = plotdata.new_plotfigure(name,num)
    plotaxes = plotfigure.new_plotaxes(title)
    plotitem = plotaxes.new_plotitem(plot_type)
    # set attributes of these objects
    return plotdata
```

`setplot` function for speciying plots

Example: plot first component of q as blue curve, red circles.

```
plotfigure = plotdata.new_plotfigure('Q', 1)
plotaxes = plotfigure.new_plotaxes('axes1')

plotitem = plotaxes.new_plotitem('1d_plot')
plotitem.plotvar = 0  # Python indexing!
plotitem.plotstyle = '-'
plotitem.color = 'b'  # or [0,0,1] or '#0000ff'

plotitem = plotaxes.new_plotitem('1d_plot')
# plotitem now points to a new object!
plotitem.plotvar = 0
plotitem.plotstyle = 'ro'
```

Plotting examples and documentation

General plotting information:
www.clawpack.org/users/plotting.html

Use of setplot, possible attributes:
www.clawpack.org/users/setplot.html

Examples:

1d: **www.clawpack.org/users/plotexamples.html**

2d: **www.clawpack.org/users/plotexamples2d.html**

FAQ: **www.clawpack.org/users/plotting_faq.html**

Gallery of applications:
www.clawpack.org/users/apps.html

Plotting options

Create a set of webpages showing all plots:

```
$ make .plots
```

Disadvantages:

- May take a while to plot all frames
- Can't zoom in dynamically or explore data

View plots interactively:

```
$ ipyclaw   # alias defined in setenv.bash

In[1]: ip = Iplotclaw()
In[2]: ip.plotloop()
PLOTCLAW>> ?

PLOTCLAW>> q
In[3]: Quit
```

First order hyperbolic PDE in 2 space dimensions

Advection equation: $q_t + uq_x + vq_y = 0$

First-order system: $q_t + Aq_x + Bq_y = 0$

where $q \in \mathbb{R}^m$ and $A, B \in \mathbb{R}^{m \times m}$.

Hyperbolic if $\cos(\theta)A + \sin(\theta)B$ is diagonalizable with real eigenvalues, for all angles θ.

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This is required so that plane-wave data gives a 1d hyperbolic problem:

$$q(x, y, 0) = \breve{q}(x\cos\theta + y\sin\theta)$$

`(\breve q)`

implies contours of q in x–y plane are orthogonal to θ–direction.

Acoustics in 2 dimensions

$$
\begin{aligned}
p_t + K_0(u_x + v_y) &= 0 \\
\rho_0 u_t + p_x &= 0 \\
\rho_0 v_t + p_y &= 0
\end{aligned}
$$

Note: pressure responds to compression or expansion and so p_t is proportional to divergence of velocity.

Second and third equations are $F = ma$.

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Second and third equations are $F = ma$.

Gives hyperbolic system $q_t + Aq_x + Bq_y = 0$ with

$$q = \begin{bmatrix} p \\ u \\ v \end{bmatrix}, \qquad A = \begin{bmatrix} 0 & K_0 & 0 \\ 1/\rho_0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 0 & K_0 \\ 0 & 0 & 0 \\ 1/\rho_0 & 0 & 0 \end{bmatrix}.$$

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Plane waves:

$$A\cos\theta + B\sin\theta = \begin{bmatrix} 0 & K_0\cos\theta & K_0\sin\theta \\ \cos\theta/\rho_0 & 0 & 0 \\ \sin\theta/\rho_0 & 0 & 0 \end{bmatrix}.$$

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Eigenvalues: $\lambda^1 = -c_0, \;\; \lambda^2 = 0, \;\; \lambda^3 = +c_0 \;\; = \sqrt{K_0/\rho_0}$

Independent of angle θ.

Isotropic: sound propagates at same speed in any direction.

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Note: Zero wave speed for “shear wave” with variation only in velocity in direction $(-\sin\theta,\; \cos\theta)$. (Fig 18.1)

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$$
A = \begin{bmatrix} 0 & K_0 & 0 \\ 1/\rho_0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad R^x = \begin{bmatrix} -Z_0 & 0 & Z_0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}
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Solving $q_t + Aq_x = 0$ gives pressure waves in (p, u).
x-variations in v are stationary.

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$$
B = \begin{bmatrix} 0 & 0 & K_0 \\ 0 & 0 & 0 \\ 1/\rho_0 & 0 & 0 \end{bmatrix} \qquad R^y = \begin{bmatrix} -Z_0 & 0 & Z_0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}
$$

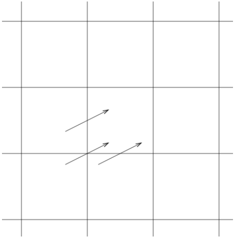
Solving $q_t + Bq_y = 0$ gives pressure waves in (p, v).
y-variations in u are stationary.

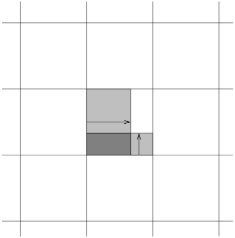
Advection: Donor Cell Upwind

With no correction fluxes, Godunov's method for advection is

Donor Cell Upwind:

$$\begin{aligned} Q_{ij}^{n+1} = Q_{ij} &- \frac{\Delta t}{\Delta x}[u^+(Q_{ij} - Q_{i-1,j}) + u^-(Q_{i+1,j} - Q_{ij})] \\ &- \frac{\Delta t}{\Delta y}[v^+(Q_{ij} - Q_{i,j-1}) + v^-(Q_{i,j+1} - Q_{ij})]. \end{aligned}$$



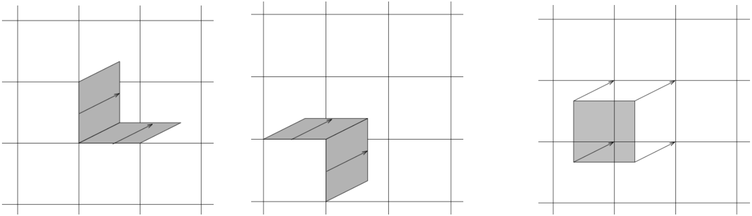


Stable only if $\left|\frac{u\Delta t}{\Delta x}\right| + \left|\frac{v\Delta t}{\Delta y}\right| \le 1.$

Advection: Corner Transport Upwind (CTU)

Correction fluxes can be added to advect waves correctly.

Corner Transport Upwind:



Stable for $\max\left(\left|\frac{u\Delta t}{\Delta x}\right|, \left|\frac{v\Delta t}{\Delta y}\right|\right) \leq 1.$

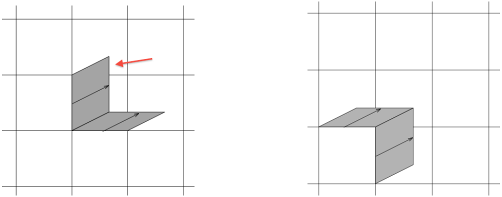
Advection: Corner Transport Upwind (CTU)

Need to transport triangular region from cell (i, j) to $(i, j+1)$:

$$\text{Area } = \frac{1}{2}(u\Delta t)(v\Delta t) \implies \left(\frac{\frac{1}{2}uv(\Delta t)^2}{\Delta x \Delta y}\right)(Q_{ij} - Q_{i-1,j}).$$

Accomplished by correction flux:

$$\tilde{G}_{i,j+1/2} = -\frac{1}{2}\frac{\Delta t}{\Delta x}\, uv(Q_{ij} - Q_{i-1,j})$$



$\frac{\Delta t}{\Delta y}(\tilde{G}_{i,j+1/2} - \tilde{G}_{i,j-1/2})$ gives approximation to $\frac{1}{2}\Delta t^2 uvq_{xy}$.

$\frac{\Delta t}{\Delta x}(\tilde{F}_{i+1/2,j} - \tilde{F}_{i-1/2,j})$ gives similar approximation.

Wave propagation algorithms in 2D

Clawpack requires:

Normal Riemann solver `rpn2.f`
Solves 1d Riemann problem $q_t + Aq_x = 0$
Decomposes $\Delta Q = Q_{ij} - Q_{i-1,j}$ into $\mathcal{A}^+\Delta Q$ and $\mathcal{A}^-\Delta Q$.
For $q_t + Aq_x + Bq_y = 0$, split using eigenvalues, vectors:

$$A = R\Lambda R^{-1} \implies A^- = R\Lambda^- R^{-1}, A^+ = R\Lambda^+ R^{-1}$$

Input parameter `ixy` determines if it's in x or y direction.
In latter case splitting is done using B instead of A.
This is all that's required for dimensional splitting.

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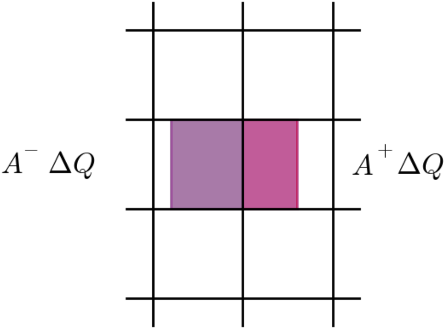
Transverse Riemann solver `rpt2.f`
Decomposes $\mathcal{A}^+\Delta Q$ into $\mathcal{B}^-\mathcal{A}^+\Delta Q$ and $\mathcal{B}^+\mathcal{A}^+\Delta Q$ by splitting this vector into eigenvectors of B.

(Or splits vector into eigenvectors of A if `ixy=2`.)

Wave propagation algorithm for $q_t + Aq_x + Bq_y = 0$

Decompose $A = A^+ + A^-$ and $B = B^+ + B^-$.

For $\Delta Q = Q_{ij} - Q_{i-1,j}$:



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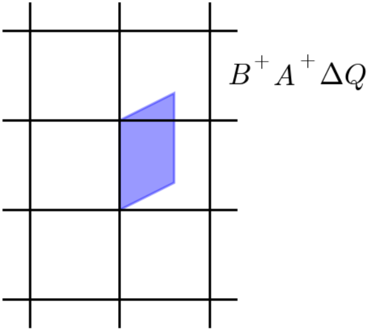

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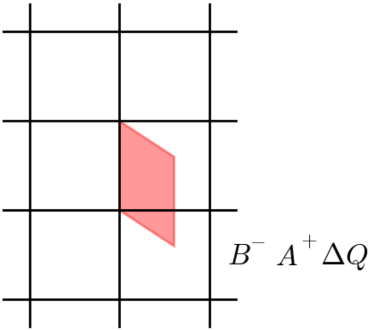

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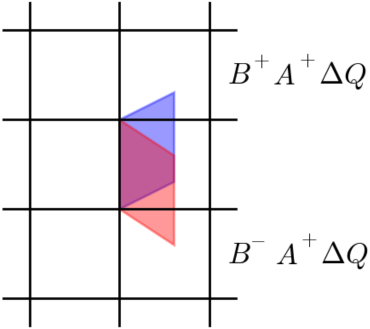

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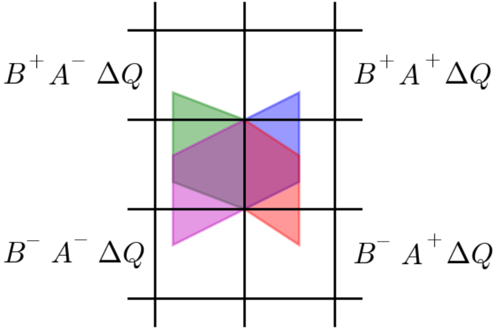
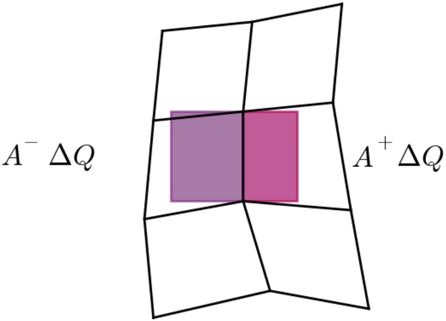


Wave propagation algorithm on a quadrilateral grid



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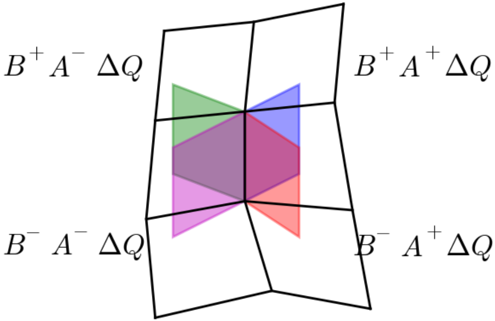

Acoustics in heterogeneous media

$$q_t + A(x,y)q_x + B(x,y)q_y = 0, \qquad q = (p,\ u,\ v)^T,$$

where

$$A = \begin{bmatrix} 0 & K(x,y) & 0 \\ 1/\rho(x,y) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & K(x,y) \\ 0 & 0 & 0 \\ 1/\rho(x,y) & 0 & 0 \end{bmatrix}.$$

Note: Not in conservation form!

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Note: Not in conservation form!

Wave propagation still makes sense. In x-direction:

$$\mathcal{W}^1 = \alpha^1 \begin{bmatrix} -Z_{i-1,j} \\ 1 \\ 0 \end{bmatrix}, \qquad \mathcal{W}^2 = \alpha^2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \qquad \mathcal{W}^3 = \alpha^3 \begin{bmatrix} Z_{ij} \\ 1 \\ 0 \end{bmatrix}.$$

Wave speeds: $s^1_{i-1/2,j} = -c_{i-1,j}, \;\; s^2_{i-1/2,j} = 0, \;\; s^3_{i-1/2,j} = +c_{ij}$.

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Decompose $\Delta Q = (\Delta p,\ \ \Delta u,\ \ \Delta v)^T$:

$$\begin{aligned} \alpha^1_{i-1/2,j} &= (-\Delta Q^1 + Z\Delta Q^2)/(Z_{i-1,j} + Z_{ij}), \\ \alpha^2_{i-1/2,j} &= \Delta Q^3, \\ \alpha^3_{i-1/2,j} &= (\Delta Q^1 + Z_{i-1,j}\Delta Q^2)/(Z_{i-1,j} + Z_{ij}). \end{aligned}$$

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Fluctuations: (Note: $s^1 < 0, \ s^2 = 0, \ s^3 > 0$)

$$\mathcal{A}^-\Delta Q_{i-1/2,j} = s^1_{i-1/2,j}\mathcal{W}^1_{i-1/2,j},$$

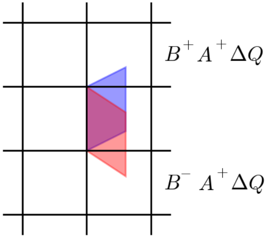
$$\mathcal{A}^+\Delta Q_{i-1/2,j} = s^3_{i-1/2,j}\mathcal{W}^3_{i-1/2,j}.$$

Acoustics in heterogeneous media

Transverse solver: Split right-going fluctuation

$$\mathcal{A}^+\Delta Q_{i-1/2,j} = s^3_{i-1/2,j}\mathcal{W}^3_{i-1/2,j}$$

into up-going and down-going pieces:



Decompose $\mathcal{A}^+\Delta Q_{i-1/2,j}$ into eigenvectors of B. Down-going:

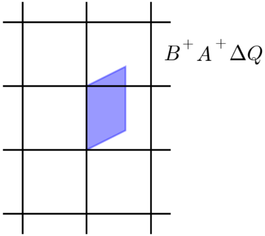
$$\mathcal{A}^+\Delta Q_{i-1/2,j} = \beta^1 \begin{bmatrix} -Z_{i,j-1} \\ 0 \\ 1 \end{bmatrix} + \beta^2 \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} + \beta^3 \begin{bmatrix} Z_{ij} \\ 0 \\ 1 \end{bmatrix},$$

Transverse solver for acoustics

Up-going part: $\mathcal{B}^+\mathcal{A}^+\Delta Q_{i-1/2,j} = c_{i,j+1}\beta^3 r^3$ from

$$\mathcal{A}^+\Delta Q_{i-1/2,j} = \beta^1 \begin{bmatrix} -Z_{ij} \\ 0 \\ 1 \end{bmatrix} + \beta^2 \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} + \beta^3 \begin{bmatrix} Z_{i,j+1} \\ 0 \\ 1 \end{bmatrix},$$

$$\beta^3 = \left((\mathcal{A}^+\Delta Q_{i-1/2,j})^1 + (\mathcal{A}^+\Delta Q_{i-1/2,j})^3 Z_{i,j+1}\right) / (Z_{ij} + Z_{i,j+1}).$$



Transverse Riemann solver in Clawpack

`rpt2` takes vector `asdq` and returns `bmasdq` and `bpasdq` where

`asdq` $= \mathcal{A}^*\Delta Q$ represents either
  $\mathcal{A}^-\Delta Q$ if `imp = 1`, or
  $\mathcal{A}^+\Delta Q$ if `imp = 2`.

Returns $\mathcal{B}^-\mathcal{A}^*\Delta Q$ and $\mathcal{B}^+\mathcal{A}^*\Delta Q$.

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Note: there is also a parameter `ixy`:

`ixy = 1` means normal solve was in x-direction,

`ixy = 2` means normal solve was in y-direction,
In this case `asdq` represents $\mathcal{B}^-\Delta Q$ or $\mathcal{B}^+\Delta Q$ and the routine must return $\mathcal{A}^-\mathcal{B}^*\Delta Q$ and $\mathcal{A}^+\mathcal{B}^*\Delta Q$.