Clawpack Tutorial
Part 3 — GeoClaw

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www.clawpack.org/geoclaw

Slides posted at
http://www.clawpack.org/links/tutorials
http://faculty.washington.edu/rjl/tutorials

(green indicates links)

Outline

- GeoClaw overview
- Adaptive Mesh Refinement (AMR)
- Computational dificulties
- Tsunami models
- Data sources
- Case study

Depth-averaged models

Reduce three-dimensional free surface problem to...

Two-dimensional Shallow Water (St. Venant) Equations

$$h_t + (hu)_x + (hv)_y = 0$$

$$(hu)_t + \left(hu^2 + \frac{1}{2}gh^2\right)_x + (huv)_y = -ghB_x(x,y)$$

$$(hv)_t + (huv)_x + \left(hv^2 + \frac{1}{2}gh^2\right)_y = -ghB_y(x,y)$$

where (u, v) are velocities in the horizontal directions (x, y),

$B(x, y) =$ bathymetry (underwater topography),

$\frac{1}{2}gh^2 =$ hydrostatic pressure.

Depth-averaged models

Advantages:

- 2D rather than 3D
  Often critical for realistic geophysical flows
  Vastly different spatial scales, e.g. ocean to harbor
  Need Adaptive Mesh Refinement even in 2D!
- No free surface $\eta(x, y, t)$.

Depth-averaged models

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Possible problems:

- When is this valid?
- What if fluid is not homogeneous, or
  shallow water assumptions don't hold?
- Often still a free boundary in the x-y domain,
  at the shoreline or at the margins of the flow.
- Small perturbations to steady state hard to capture.

GeoClaw Software **www.clawpack.org/geoclaw**

Based on Dave George's thesis work TsunamiClaw.

Currently includes:

- 2d library for depth-averaged flows over topography.
- Handles dry cells where depth $= 0$.
- Well-balanced Riemann solvers for small amplitude waves on ocean at rest.
- Well balancing and dry cells in conjunction with adaptive refinement.
- General tools for dealing with multiple data sets at different resolutions.
- Tools for specifying regions where refinement is desired.
- Python plotting tools.
- Output of time series at gauge locations or on fixed grids.

Collaborators

David George, Mendenhall postdoctoral Fellow at the
USGS Cascades Volcano Observatory (CVO)
Tsunamis, dam break, debris flows

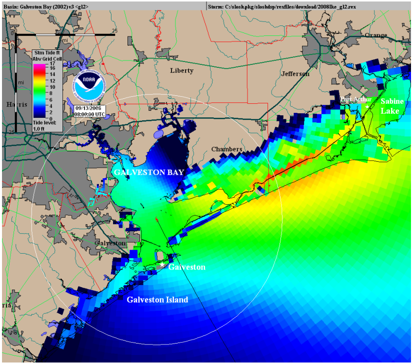
Marsha Berger, Courant Institute, NYU
Adaptive Mesh Refinement (AMR)

Kyle Mandli, UW graduate student
PyClaw, Two-layer shallow water, storm surge

Numerous other students and colleagues

Supported in part by NSF, ONR

Storm Surge Modeling



Source: SLOSH model run, NOAA/NHC -
`www.nhc.noaa.gov/ssurge/index.shtml`

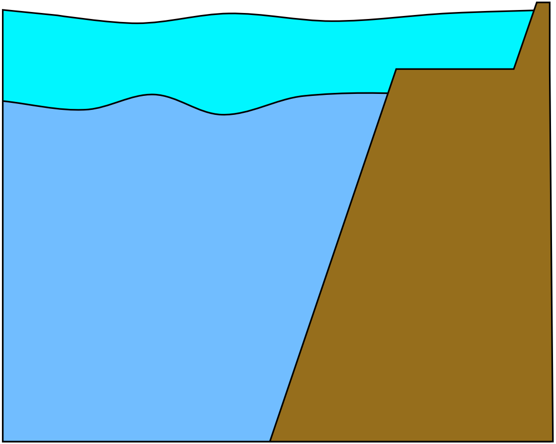
Goal: Given storm track and intensity, predict ocean response

Requirements:

- Handle complex bathymetry
- Inundation at coastline
- Represent multiple scales - ranges from 10 m to 1000 km
- Ensemble prediction capabilities $\rightarrow$ fast and efficient solvers
- Simulate storm surge physics

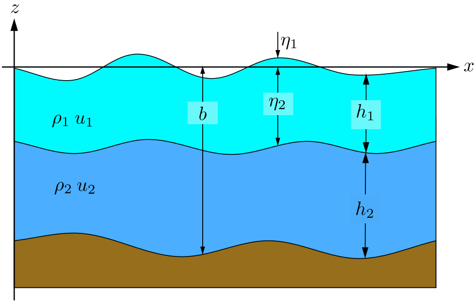
Two-layer model — Kyle Mandli

Top layer goes onto shelf and on-shore.



Bottom layer ends at continental slope.

Multi-Layer Shallow Water Equations



$$(h_1)_t + (h_1 u_1)_x = 0$$

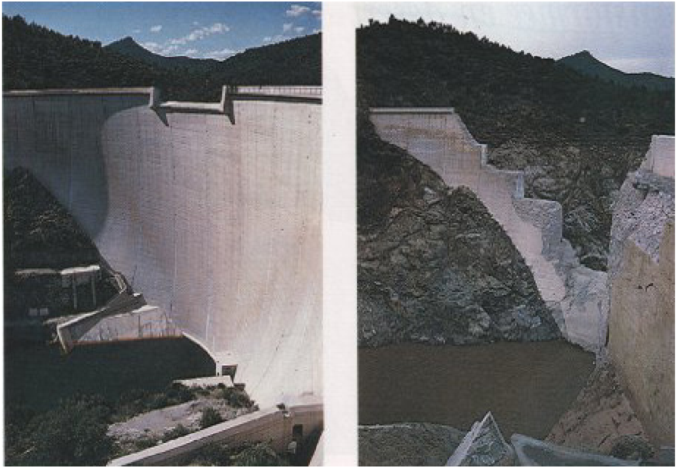
$$(h_1 u_1)_t + (h_1 u_1^2 + 1/2 g h_1^2)_x = \boxed{\mathbf{-g h_1 (h_2)_x}} - g h_1 b_x$$

$$(h_2)_t + (h_2 u_2)_x = 0$$

$$(h_2 u_2)_t + (h_2 u_2^2 + 1/2 g h_2^2)_x = \boxed{\mathbf{-r g h_2 (h_1)_x}} - g h_2 b_x \qquad \mathbf{r = \rho_1/\rho_2}$$

Malpasset Dam Failure

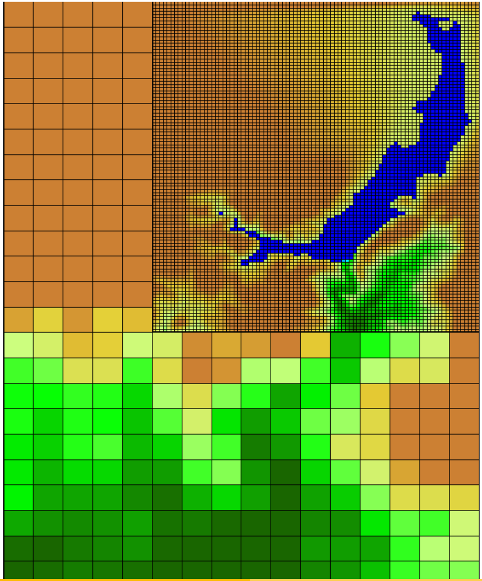
Catastrophic failure in 1959



Malpasset Dam Failure

Modeling work by David George, using GeoClaw

Coarse: 400m cell side, Level 2: 50m, Level 3: 12m, Level 4: 3m



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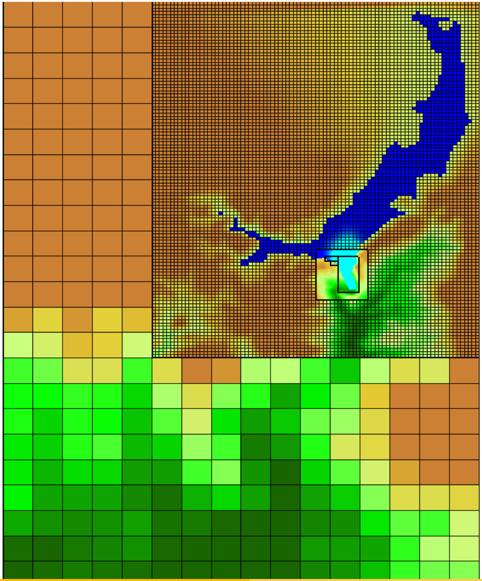

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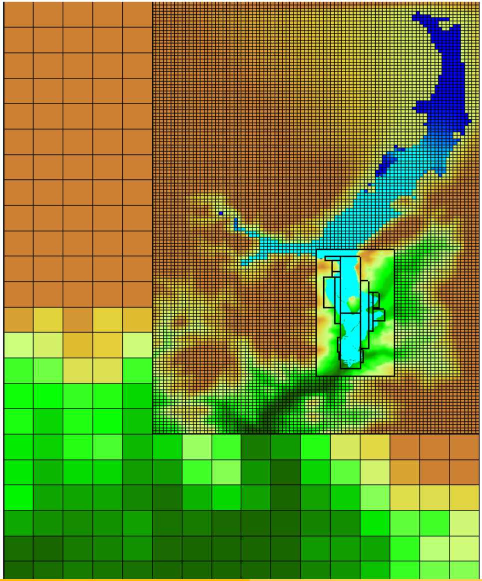

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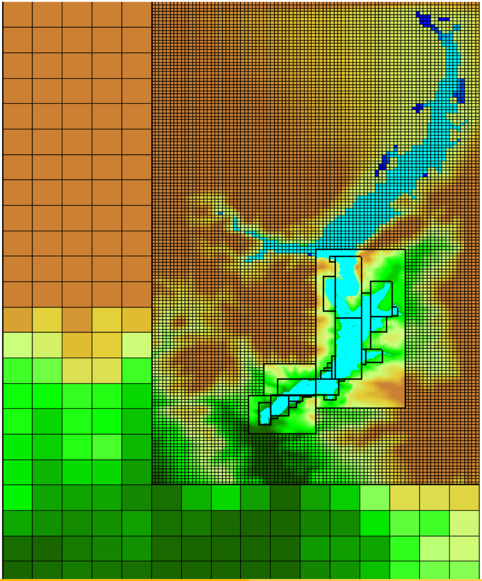

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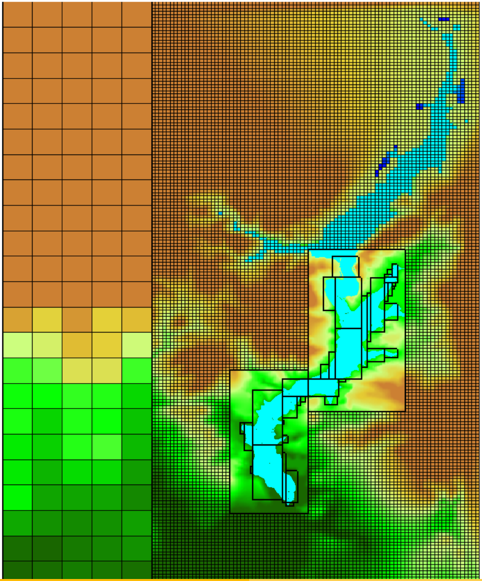

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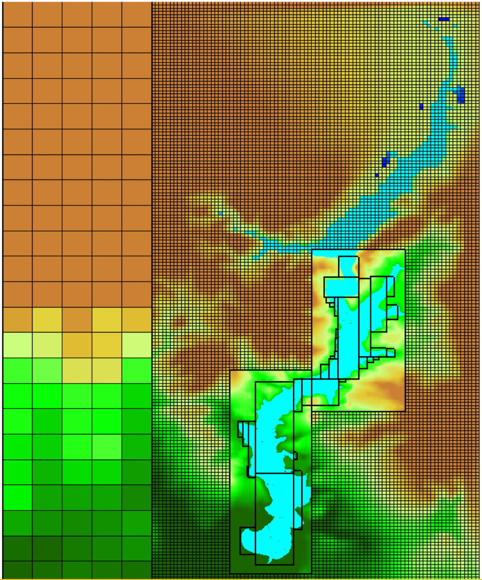

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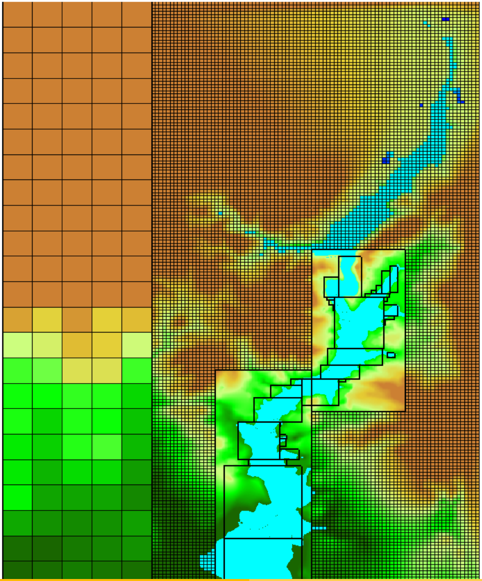
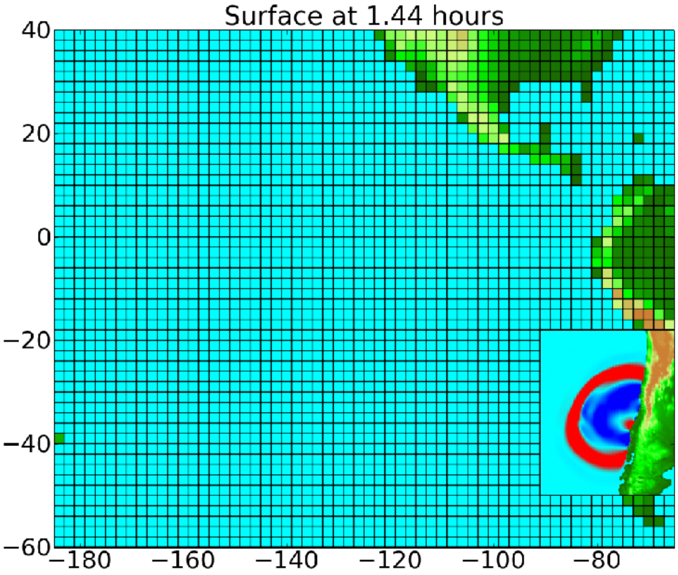


27 February 2010 tsunami



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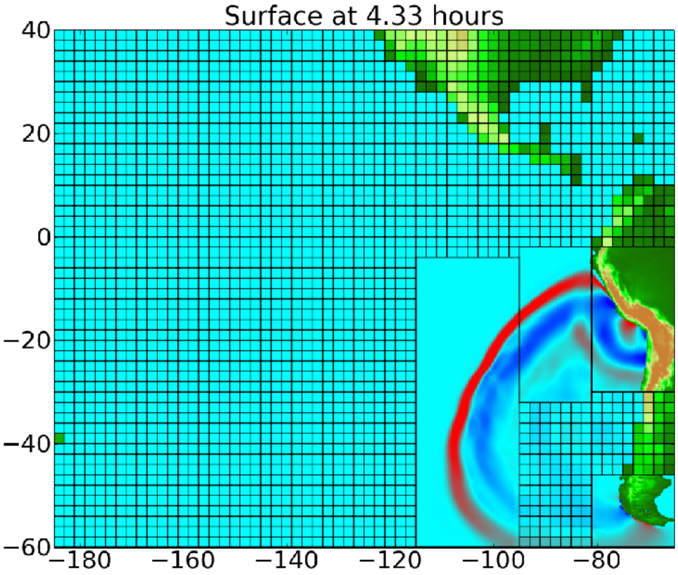

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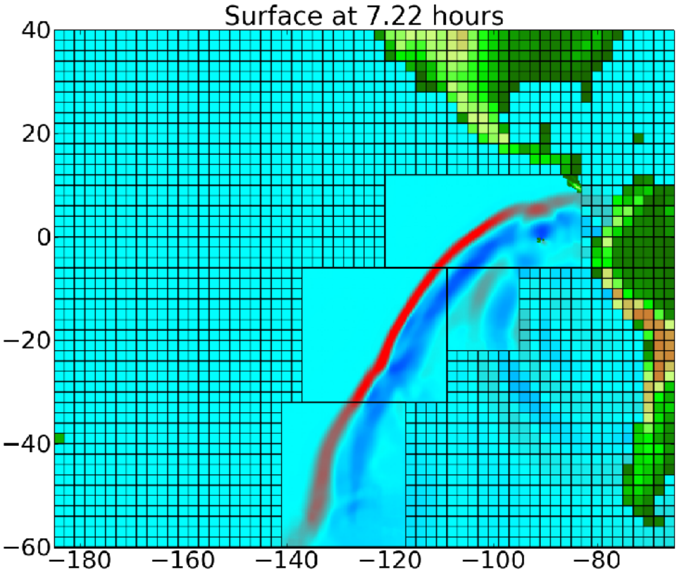

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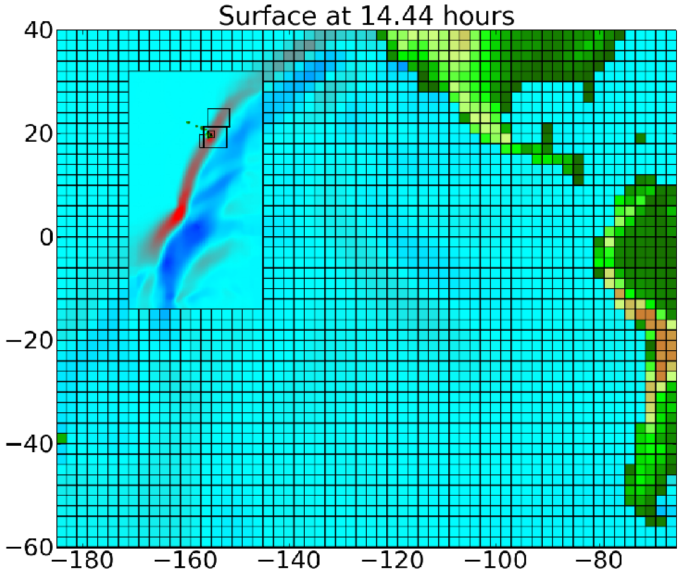

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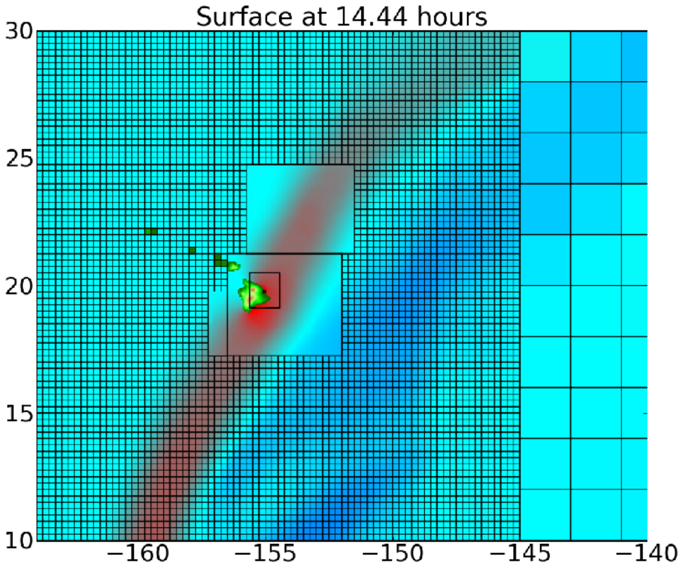

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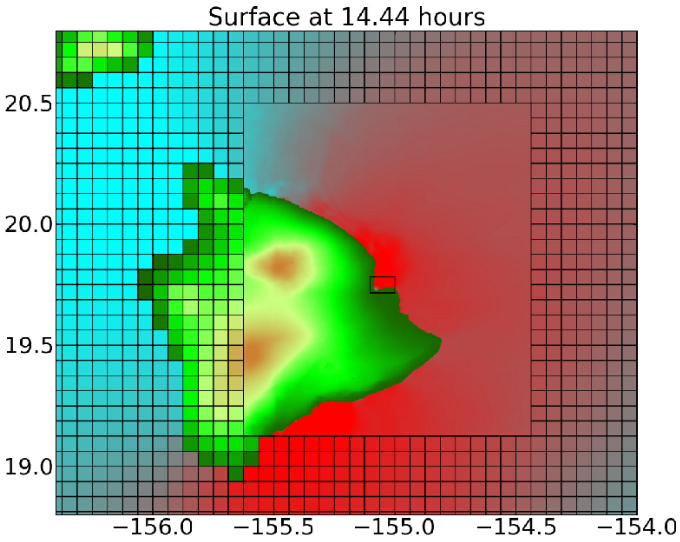


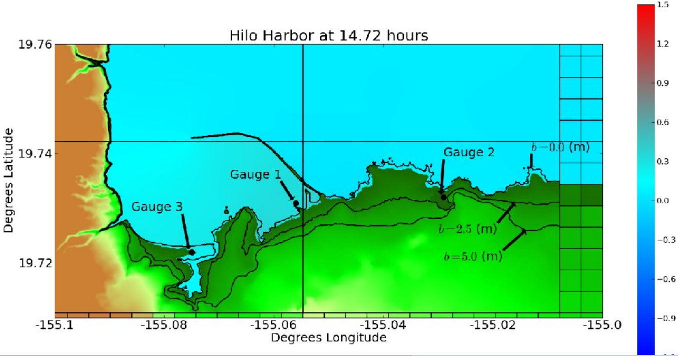
Inundation of Hilo, Hawaii

Using 5 levels of refinement with ratios 8, 4, 16, 32.

Resolution ≈ 160 km on Level 1 and ≈ 10m on Level 5.

Total refinement factor: $2^{14} = 16,384$ in each direction.

With 15 m displacement at fault:



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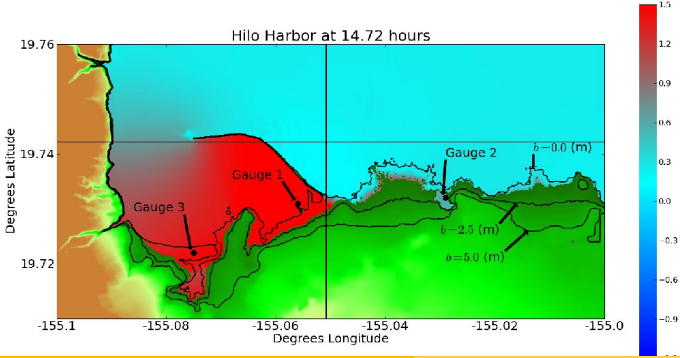
With 90 m displacement at fault:



Adaptive Mesh Refinement (AMR)

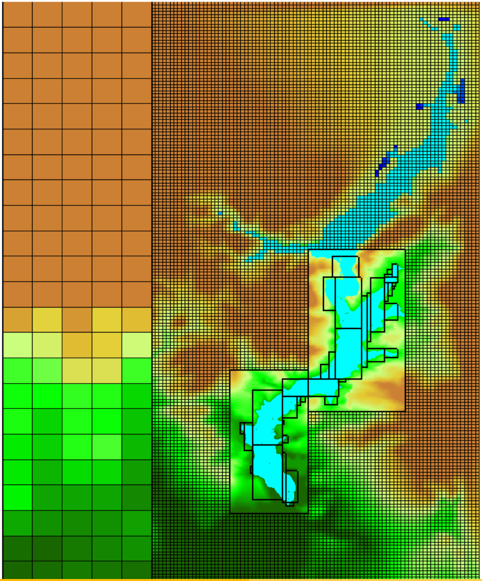
- Cluster grid points where needed
- Automatically adapt to solution
- Refined region moves in time-dependent problem

Basic approaches:

- Cell-by-cell refinement
  Quad-tree or Oct-tree data structure
  Structured or unstructured grid
- Refinement on "rectangular" patches
  Berger-Colella-Oliger style
  (AMRCLAW and CHOMBO-CLAW)

Nested AMR grids

Coarse: 400m cell side, Level 2: 50m, Level 3: 12m, Level 4: 3m

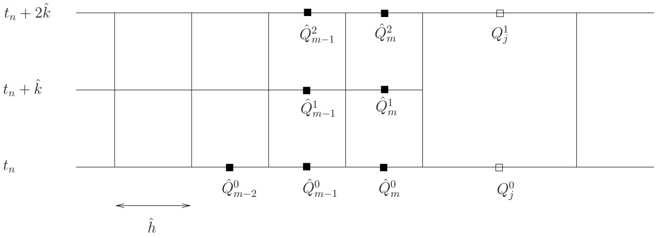


AMR Issues

- Refinement in time as well as space
- Conservation at grid interfaces
- Accuracy at interfaces, Spurious reflections?
- Refinement strategy, error estimation
- Clustering flagged points into rectangular patches

Time stepping algorithm for AMR

- Take 1 time step of length k on coarse grid with spacing h.
- Use space-time interpolation to set ghost cell values on fine grid near interface.
- Take L time steps on fine grid.
  $L =$ refinement ratio, $\hat{h} = h/L$, $\hat{k} = k/L$.
- Replace coarse grid value by average of fine grid values on regions of overlap — better approximation and consistent representations.
- Conservative fix-up near edges.



Flagging Cells for Refinement

Every `kcheck` time-steps at each level (except finest), check all grid cells and flag those needing refinement.

Use one or more of the following flagging criteria:

- Richardson estimation of truncation error.
  Compare result after last two time steps on this grid with one time step on a coarsened grid.
- Estimate spatial gradient of one or more components of solution.
- Check for regions where refinement is user-forced to some level.
- Problem-specific, e.g. near shore for tsunami simulation.
- Other user-supplied criterion set in `flag2refine.f`.

Clustering Flagged Cells for Refinement

Use Berger-Rigoutsos algorithm
[IEEE Trans. Sys. Man & Cyber.] 21(1991), p. 1278]

Clusters flagged points into a set of rectangular patches.

Tradeoff between:

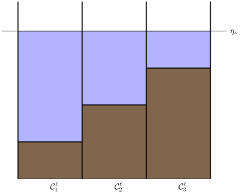
- Many small patches cover flagged points with minimal refinement of unflagged points.
- But.... increases overhead associated with each patch, e.g. boundary values: ghost cell values set by copying or interpolation from other grids,

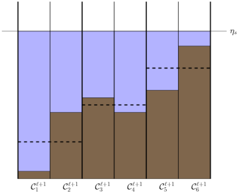
B-G algorithm has cut-off paramter: require that this fraction of refined cells be flagged (usually set to 0.7).

Refinement of topography

Topography should be consistent between different levels.

$$B_1^{\ell} = \frac{1}{2}(B_1^{\ell+1} + B_2^{\ell+1})$$





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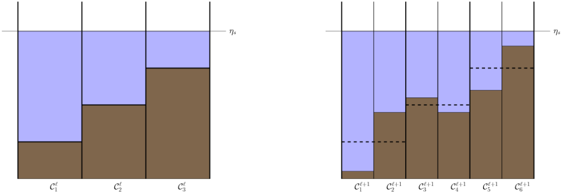
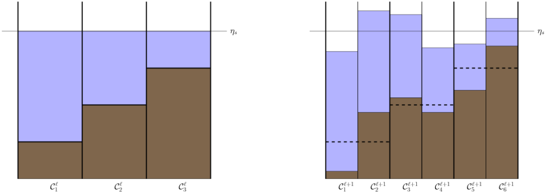
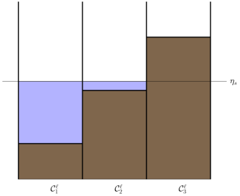


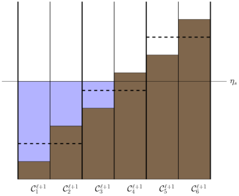
Important to interpolate surface, not depth, as in...



Refinement of topography near shore

Again need to maintain flat surface before wave arrives:





Mass cannot always be conserved!

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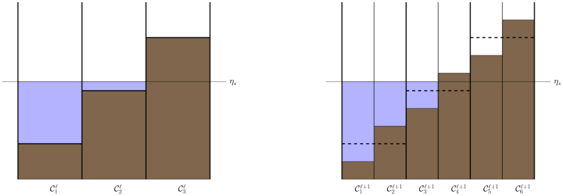

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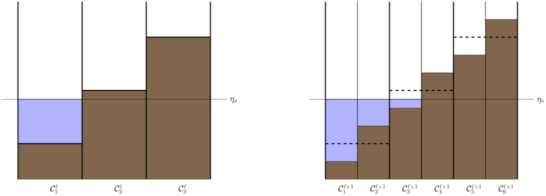
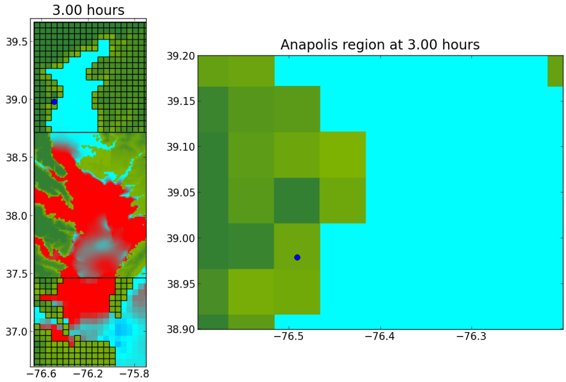


Chesapeake Bay and Anapolis

Cannot conserve mass when refining near shore!



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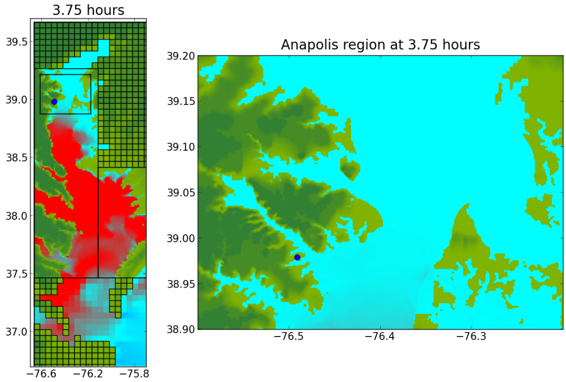
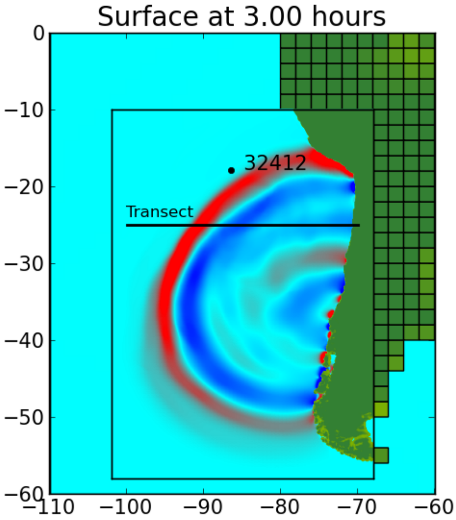


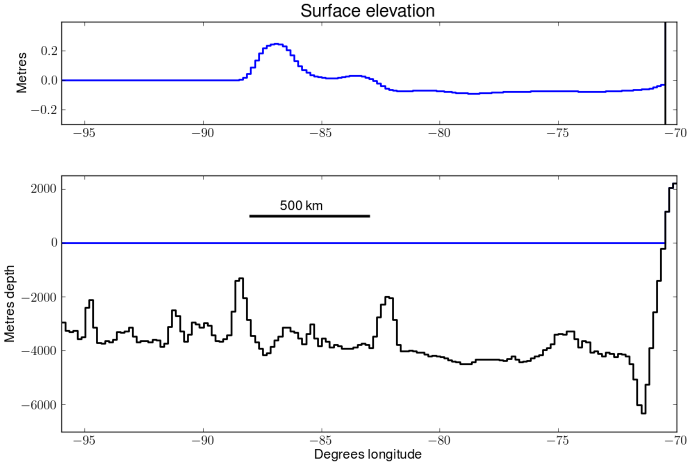
Tsunami from 27 Feb 2010 quake off Chile



Transect of 27 February 2010 tsunami

Bathymetry, depth change by > 1000 m from one cell to next,

Surface elevation changes on order of a few cm.



Incorporating source term in f-waves

$q_t + f(q)_x = \psi$ with $f(q)_x \approx \psi$.

Concentrate source at interfaces: $\Psi_{i-1/2}\,\delta(x - x_{i-1/2})$

Split $f(Q_i) - f(Q_{i-1}) - \Delta x \Psi_{i-1/2} = \sum_p \mathcal{Z}^p_{i-1/2}$

Use these waves in wave-propagation algorithm.

Steady state maintained: (Well balanced)

If $\frac{f(Q_i)-f(Q_{i-1})}{\Delta x} = \Psi_{i-1/2}$ then $\mathcal{Z}^p \equiv 0$

Near steady state:

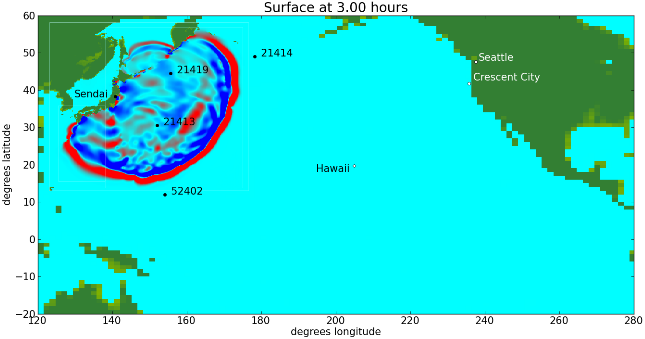
Deviation from steady state is split into waves and limited.

11 March 2011 Great Tohoku Tsunami

Some preliminary results:
www.clawpack.org/links/honshu2011

Wiki / Subversion repository for data and code:
http://kingkong.amath.washington.edu/trac/tsunami-benchmarks/wiki/honshu2011

Great Tohoku Tsunami, 11 March 2011



Modeling and Simulating Tsunamis with an Eye to Hazard Mitigation, RJL and J. Behrens, SIAM News, May, 2011

http://www.siam.org/news/news.php?id=1882

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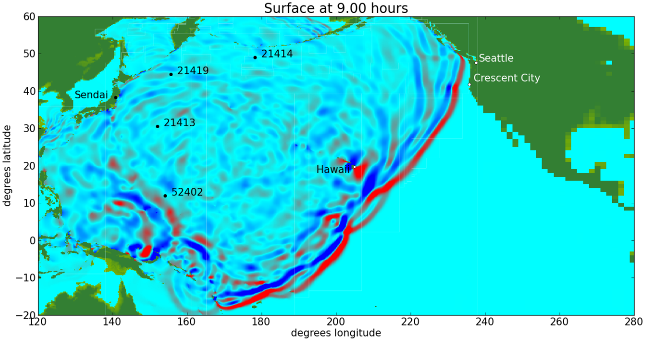

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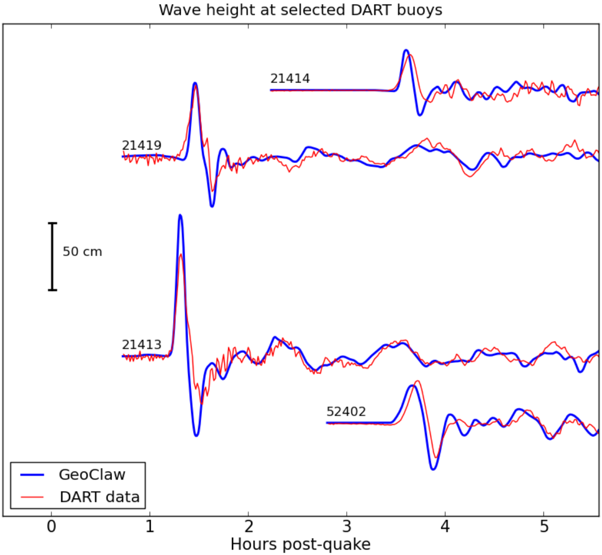


Bathymetry Data

Design-a-Grid website, NOAA National Geophysical Data Center (NGDC)

`http://www.ngdc.noaa.gov/mgg/gdas/gd_designagrid.html`

World oceans to resolution of 1 arc-minute,

Can choose 1, 2, 4, or 10-minute data.

1 minute latitude ≈ 1.8 km.

US coastal regions to 3 arc-seconds ≈ 90 m.

Earthquake source models

Typically many different models proposed.

Example: for Great Tohoku Tsunami,

Currently using Preliminary Version 3 of a UCSB model
Guangfu Shao, Xiangyu Li, Chen Ji, UCSB
Takahiro Maeda, NIED

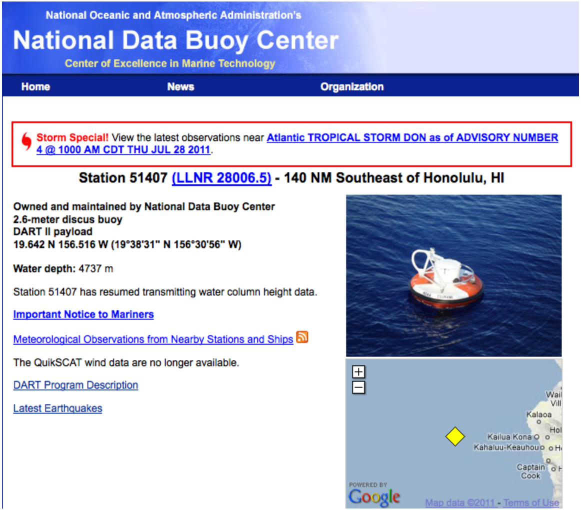
http://www.geol.ucsb.edu/faculty/ji/big_earthquakes/2011/03/0311/Honshu_main.html

190 25×20 km subfaults on a single fault plane

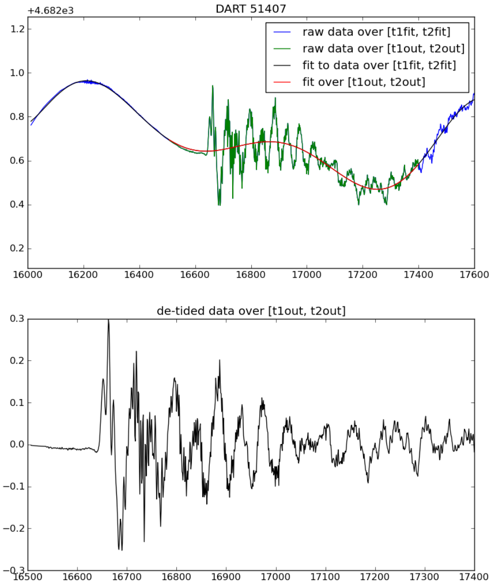
Common strike and dip, different rake and slip, rupture time.

Okada model used to convert into seafloor deformation.

Currently static deformation.
Should include dynamic rupture (≈ 180 sec).



http://www.ndbc.noaa.gov/station_page.php?station=51407



DART 51407
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raw data over [t1fit, t2fit]
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