

Coordinate Systems

This chapter discusses coordinate systems and the transformation between different coordinate systems. This chapter:

- analyzes the world2ndc matrix we have worked with in all of the tutorials;
- explains the need for world and normalized device coordinate systems;
- describes and experiments with the world coordinate window;
- derives all essential coordinate transformation operators and demonstrates how to implement these operators.

After this chapter we should:

- understand the difference between modeling design space and device drawing space;
- understand the need for transforming mouse click positions to the world coordinate space.

In addition, with respect to hands-on programming, we should:

- be able to program \mathbf{M}_{w2n} operators to support any user-specified world coordinate window;
- be able to utilize graphics API matrix processors in supporting necessary coordinate transformation operations;
- be able to transform mouse clicks to the world coordinate space.

10.1 Understanding Tutorial 3.1

Recall that in the first tutorial of Chapter 3, we began by measuring and discussing how to draw two squares with the following measurements:

$$\text{LargeSquare}_{\text{wc}} = \begin{cases} V_a = & (160, 122), \\ V_b = & (80, 122), \\ V_c = & (80, 42), \\ V_d = & (160, 42), \end{cases}$$
 (10.1)

wc. Abbreviation for world coordinate. We will study the details of this coordinate system in Section 10.3.

and

SmallSquare_{wc} =
$$\begin{cases} V_a = & (160, 122), \\ V_e = & (210, 122), \\ V_f = & (210, 172), \\ V_g = & (160, 172). \end{cases}$$
(10.2)

Tutorial 3.1 displayed these squares where each millimeter was represented with a pixel on the application window.

Tutorial 10.1. The world2ndc (w2n) Transform

- **Goal.** Understand the w2n transformation operator we have encountered in all of the tutorials.
- **Approach.** Analyze the simplest tutorials we have worked with and study the effect of the operator.

Figure 10.1 is a screenshot of running Tutorial 10.1. This tutorial is identical to Tutorial 3.1 except that the UI drawing area is 200 pixels \times 160 pixels. We can verify that vertices V_e , V_f , and V_g are outside of the UI drawing area and are not visible. Recall that before we developed the UWBGL_D3D_Lib support, drawing was performed during the GrfxWindowD3D::0nPaint() function call, as shown in Listing 10.1. We are now equipped with sufficient knowledge to completely understand this very first graphics API tutorial. In particular, we are interested in understanding the mysterious Step 2 of setting and computing the parameters for the coordinate systems. Based on the knowledge learned from the previous chapters, we understand that in Step 2, we first initialize all three of the D3D matrix processors (WORLD, VIEW, and PROJECTION) to the identity matrix. We then compute the w2n matrix by concatenating a translation and a scaling matrix. Finally, we load the w2n matrix into the VIEW matrix processor. In fact, a similar code

Tutorial 10.1.
Project Name
D3D_ViewTransform0



Figure 10.1. Tutorial 10.1: Re-implement Tutorial 3.1 with 200×160 drawing area.

```
void CGrfxWindowD3D::OnPaint()
                                                                                Source file.
                                                                                GrfxWindowD3D.cpp file in
     Step 1: select graphics hardware render buffer.
                                                                                the GrfxWindow folder of
                                                                                     D3D_ViewTransform0
                                                                                project.
    Step 2: initialize selected hardware and set the coordinate system parameters.
    // Initialize D3D matrix processors setting all three
    // matrix processors to identity matrix
    m_pD3DDevice->SetTransform(D3DTS_WORLD, &identity);
    m_pD3DDevice->SetTransform(D3DTS_VIEW, &identity);
    m_pD3DDevice->SetTransform(D3DTS_PR0JECTION, &identity);
    D3DXMATRIX world2ndc;
                                             // Coordinate Transformation operator
    D3DXVECTOR3 scale(2.0f/width, 2.0f/height, 1.0f); // Parameters for the operator
    D3DXVECTOR3 translate(-1.0f, -1.0f, 0.0);
    // compute the coordinate transformation operator
    D3DXMatrixTransformation(&world2ndc, scale, translate);
    // Load the operator into the D3D VIEW matrix
    m_pD3DDevice->SetTransform(D3DTS_VIEW, &world2ndc);
    Step 3: clear drawing buffer and draw two squares.
    v[0].m_point = D3DXVECTOR3(160,122,0);
    v[1].m_point = D3DXVECTOR3(80,122,0);
    m_pD3DDevice->DrawPrimitiveUP(D3DPT_TRIANGLEFAN, 2, (CONST void *)v, ...);
```

Listing 10.1. CGrfxWndD3D::**OnPaint()** (Tutorial 10.1)

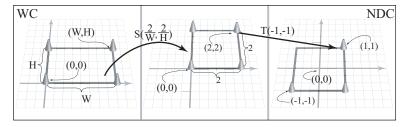


Figure 10.2. The operations of w2n.

fragment exists in every tutorial we have worked with. We perform these operations each time we initialize the graphics API for redraw: in the earlier tutorials when servicing the Redraw/Paint events (in the OnPaint() function); and in the later tutorials, when redrawing the WindowHandler UI drawing area (in the DrawGraphics() function). Figure 10.2 shows the details of the transformation operation that is implemented by the w2n matrix:

- Scale factor. Recall that the width and height parameters are the dimensions of the application window. In Listing 10.1, the scaling factors are defined to be $s_x = \frac{2.0}{\text{width}}$ and $s_y = \frac{2.0}{\text{height}}$. The diagram on the left of Figure 10.2 shows that this scaling factors will shrink a width \times height rectangle into a 2 \times 2 rectangle. For example, in this case we have defined the dimension of the application window to be 200 pixels \times 160 pixels.
- **Displacement.** The displacements of the translation operator are constants for all cases: $t_x = -1$ and $t_y = -1$. The diagram on the right of Figure 10.2 shows how this translation operator moves the 2×2 rectangle resulting from the scaling operation.
- w2n **matrix.** We will refer to this matrix (or the transformation operator) as \mathbf{M}_{w2n} . We see that this is a concatenated operator with a scaling followed by a translation, or

$$\mathbf{M}_{w2n} = \mathbf{ST} = \mathbf{S} \left(\frac{2}{200}, \frac{2}{160} \right) \mathbf{T}(-1, -1).$$
 (10.3)

Notice that \mathbf{M}_{w2n} is loaded into the VIEW matrix processor (or \mathbf{M}_V) of the D3D rendering context (RC). In the case of D3D, we know that all vertices \mathbf{V}_i will be transformed by

$$\mathbf{V}_{it} = \mathbf{V}_i \; \mathbf{M}_W \; \mathbf{M}_V \; \mathbf{M}_P.$$

In this case, both the WORLD (\mathbf{M}_W) and the PROJECTION (\mathbf{M}_P) matrices are initialized to the identity matrix. For this reason, the vertices of the squares will only be transformed by \mathbf{M}_{w2n} in the VIEW matrix processor. For example, $V_a = (160, 122)$ will become V_{at} :

$$\mathbf{V}_{at} = \mathbf{V}_{a} \, \mathbf{M}_{w2n}$$

$$= \begin{bmatrix} 160 \ 122 \end{bmatrix} \mathbf{S}(\frac{2}{200}, \frac{2}{160}) \mathbf{T}(-1, -1)$$

$$= \begin{bmatrix} \frac{160 \times 2}{200} - 1 & \frac{160 \times 2}{150} - 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.6 \ 0.525 \end{bmatrix}.$$

When applying \mathbf{M}_{w2n} to all of the vertices, we get

$$LargeSquare_{ndc} = \begin{cases} V_{at} = & (0.6, & 0.525), \\ V_{bt} = & (-0.2, & 0.525), \\ V_{ct} = & (-0.2, & -0.475), \\ V_{dt} = & (0.5, & -0.475), \end{cases}$$
(10.4)

and

$$SmallSquare_{ndc} = \begin{cases} V_{at} = & (0.6, 0.525), \\ V_{et} = & (1.1, 0.525), \\ V_{ft} = & (1.1, 1.15), \\ V_{\varrho t} = & (0.6, 1.15). \end{cases}$$
(10.5)

Here we see that Direct3D actually transforms the input vertices into much smaller numbers. It is interesting that we defined the vertices of the squares according to LargeSquare_{wc} (Equation (10.1)) and SmallSquare_{wc} (Equation (10.2)), only to define the transform operator \mathbf{M}_{n2w} (Equation (10.3)) to ensure that the D3D API transforms these input vertices to LargeSquare_{ndc} (Equation (10.4)) and SmallSquare_{ndc} (Equation (10.5)). Based on these observations, it is logical to conclude the following.

• The effects of \mathbf{M}_{n2w} . If we define the input vertices according to LargeSquare_{ndc} and SmallSquare_{ndc}, then there would be no need for the \mathbf{M}_{w2n} operator of Equation (10.3).

Tutorial 10.2. Drawing without the w2n Transform

- Goal. Verify the effects of the w2n matrix.
- **Approach.** Draw the two squares defined by the vertices of LargeSquare_{ndc} and SmallSquare_{ndc} with identity in the D3D VIEW matrix processor.

Figure 10.3 is a screenshot of running Tutorial 10.2. We observe the output to be identical to that of Tutorial 10.1. However, the drawing routines of these two tutorials are different in significant ways. From Listing 10.2, we observe the following differences.

• **Step 2.** We do not compute the w2n matrix. Instead, we initialize all three matrix processors to identity and proceed to drawing the squares.

ndc. Abbreviation for *normalized device coordinate*. We will study the details of this coordinate system in Section 10.2.

Tutorial 10.2.
Project Name
D3D_ViewTransform1

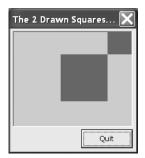


Figure 10.3. Tutorial 10.2.

Source file.

project.

GrfxWindowD3D.cpp file in the *GrfxWindow* folder of the D3D_ViewTransform1

```
void CGrfxWindowD3D::OnPaint()

:

Step 2: initialize selected hardware and set the coordinate system parameters.

// Initialize D3D matrix processors setting all three
// matrix processors to identity matrix
m_pD3DDevice->SetTransform(D3DTS_WORLD, &identity);
m_pD3DDevice->SetTransform(D3DTS_VIEW, &identity);
m_pD3DDevice->SetTransform(D3DTS_PROJECTION, &identity);
```

Step 3: clear drawing buffer and draw two squares.

```
 \vdots \\ // \textit{Vertices of the LargeSquare}_{ndc} \\ v[0].\texttt{m_point} = \texttt{D3DXVECTOR3}(0.60f, 0.525f, 0); // \textit{V}_{at} \\ v[1].\texttt{m_point} = \texttt{D3DXVECTOR3}(-0.2f, 0.525f, 0); // \textit{V}_{bt} \\ v[2].\texttt{m_point} = \texttt{D3DXVECTOR3}(-0.2f, -0.475f, 0); // \textit{V}_{ct} \\ v[3].\texttt{m_point} = \texttt{D3DXVECTOR3}(0.60f, -0.475f, 0); // \textit{V}_{dt} \\ \texttt{m_pD3DDevice->DrawPrimitiveUP}(\texttt{D3DPT_TRIANGLEFAN}, 2, (\texttt{CONST void *}) \texttt{v}, \ldots); \\ // \textit{Vertices of the SmallSquare}_{ndc} \\ v[0].\texttt{m_point} = \texttt{D3DXVECTOR3}(0.6f, 0.525f, 0); // \textit{V}_{at} \\ v[1].\texttt{m_point} = \texttt{D3DXVECTOR3}(1.1f, 0.525f, 0); // \textit{V}_{et} \\ v[2].\texttt{m_point} = \texttt{D3DXVECTOR3}(1.1f, 1.15f, 0); // \textit{V}_{ft} \\ v[3].\texttt{m_point} = \texttt{D3DXVECTOR3}(0.6f, 1.15f, 0); // \textit{V}_{gt} \\ \texttt{m_pD3DDevice->DrawPrimitiveUP}(\texttt{D3DPT_TRIANGLEFAN}, 2, (\texttt{CONST void *}) \texttt{v}, \ldots); \\ \vdots \\ \vdots \\ \end{cases}
```

Listing 10.2. CGrfxWndD3D::**OnPaint()** (Tutorial 10.2)

• **Step 3.** The two squares are specified by vertices defined by LargeSquare_{ndc} (Equations (10.4)) and SmallSquare_{ndc} (Equations (10.5)).

In this tutorial, we verify our observations that we can reproduce the effects of the \mathbf{M}_{w2n} operator of Equation (10.3). Based on the results of this tutorial, we further observe the following.

• When all three matrix processors in the D3D RC are initialized to be the identity matrix, only vertices between the range of ± 1.0 are displayed.

Tutorial 10.3. Verify the ± 1.0 Drawing Area

- Goal. Verify that the application window displays all vertices inside the range of ± 1.0 .
- **Approach.** With all matrix processors set to the identity matrix, draw a circle with center located at the origin ((0,0)) and radius of 1.0.

Figure 10.4 is a screenshot of running Tutorial 10.3. In this case, the output UI drawing area is defined to be 200 pixels \times 200 pixels. Once again, we initialize all the matrix processors of the D3D API to identity and proceed to draw a unit circle located at the origin. Recall that we approximate a circle with a triangle fan where vertices of the triangles are located on the circumference of the circle. We observe that the circle perfectly fits within the application drawing area. This tutorial verifies that the reason we need the \mathbf{M}_{w2n} transform is that the D3D graphics API automatically transforms all vertices from within the range of

$$\begin{cases} -1.0 \le x \le 1.0, \\ -1.0 \le y \le 1.0, \end{cases}$$

to the entire application drawing area. In computer graphics, we refer to this square area covered by ± 1 as the normalized space, or normalized device coordinate (NDC).

Tutorials 10.4 and 10.5. Experimenting with the NDC

- Goal. Understand that the entire NDC is mapped onto the application drawing area, regardless of the dimensions of the application window.
- Approach. Draw the unit circle onto application draw areas with drastically different dimensions and observe the results.

To further understand the transformation performed internally (and automatically) by D3D, in Tutorials 10.4 and 10.5 we define the UI drawing areas to be 100 pixels \times 200 pixels and 200 pixels \times 100 pixels, respectively. In both tutorials, the drawing routines are identical to that of Tutorial 10.3, where the same unit circle with center located at the origin is drawn in each case. Figures 10.5 and 10.6 are screenshots of running Tutorials 10.4 and 10.5. It is interesting that in both cases, just as in the case of Tutorial 10.3, the unit circles fit perfectly within the bounds of the application windows. Of course, in this case, because the application windows are rectangular, the circles are squashed into corresponding ellipses.

Tutorial 10.3. Project Name D3D_ViewTransform2

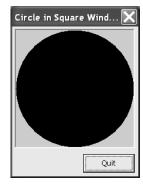


Figure 10.4. Tutorial 10.3: Drawing a circle of radius 1.0 and center at (0,0) with D3D

Tutorial 10.4.

Project Name D3D_ViewTransform3

Tutorial 10.5.Project Name

D3D_ViewTransform4

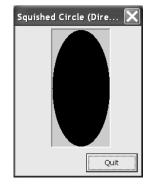


Figure 10.5. Tutorial 10.4: Drawing the same circle onto a 100×200 window.

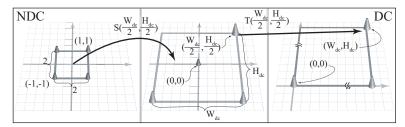


Figure 10.7. D3D's M_{n2d} operator.

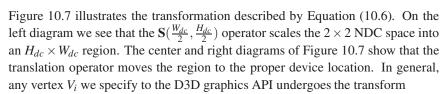
The M_{n2d} **transform.** From our discussions, we observe that the D3D API must be performing

$$\mathbf{M}_{n2d} = \mathbf{S}\left(\frac{W_{dc}}{2}, \frac{H_{dc}}{2}\right) \mathbf{T}\left(\frac{W_{dc}}{2}, \frac{H_{dc}}{2}\right)$$
(10.6)

on all vertices, where

 W_{dc} = Width of drawing area (on device),

 H_{dc} = Height of drawing area (on device).



$$\mathbf{V}_{dc} = \mathbf{V}_i \; \mathbf{M}_W \; \mathbf{M}_V \; \mathbf{M}_P \; \mathbf{M}_{n2d},$$

where \mathbf{M}_W , \mathbf{M}_V , and \mathbf{M}_P are the WORLD, VIEW, and PROJECTION matrix processors of the D3D RC and \mathbf{V}_{dc} is the vertex on the UI drawing area. A very important lesson we have learned so far is that whereas the matrix processors (\mathbf{M}_W , \mathbf{M}_V , and \mathbf{M}_P) are under our program's control, \mathbf{M}_{n2d} will be applied internally by the D3D graphics API automatically and is not under our program's control. Another important observation is that the graphics API (D3D) knows what the underlying display device resolution is (width/height) and computes \mathbf{M}_{n2d} accordingly.

We see the that the \mathbf{M}_{w2n} operator we construct in Step 2 of Listing 10.1 for Tutorial 10.1 (and for every single tutorial we have worked with so far) is to complement the \mathbf{M}_{n2d} transform (Equation (10.6)) that D3D performs automatically. An obvious question is, "Why would D3D automatically perform the \mathbf{M}_{n2d} operation?" To answer this question, we must first understand coordinate systems.

Linearity of affine transformation. Before we leave this section, notice that we analyzed the \mathbf{M}_{w2n} operator from Equation (10.3) based on transforming the

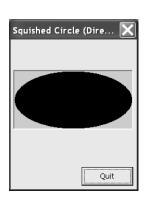


Figure 10.6. Tutorial 10.5: Drawing the circle onto a 200×100 window.

(0,0) to (200,160) rectangular area to the area within the ± 1 range (NDC) (Figure 10.2). This same operator also proportionally transforms the two squares inside the rectangular area where the transformed squares are proportionally located inside the NDC space. This is an example of the linear property of affine transformation: if the transform operator works for a rectangle, then all geometric contents inside the rectangle will also transform appropriately. For this reason, when deriving coordinate transformations, we only need to consider the operator that transforms the enclosing rectangular region of interest.

10.2 Device and Normalized Coordinate Systems

In Section 3.1 when we wanted to describe vertex positions of the squares, we borrowed concepts from the Cartesian coordinate system with the horizontal and vertical axes and units on the axis. From the discussion in the previous section, we see two examples of applying the concepts associated with the Cartesian coordinate system.

- 1. **Device coordinate (DC).** When we draw and refer to positions on the application window, implicitly, we assume a coordinate system. We assume that the origin is located at the lower-left corner of the window, with units being pixels. Note that the DC is a variable coordinate system, where it can be changed even during the lifetime of an application (e.g., by resizing the application window size). The DC has dimension width (W_{dc}) by height (H_{dc}) . The application's drawing area is the DC space.
- 2. Normalized (device) coordinate (NDC). With center at the origin and x/y ranges between -1 and +1, the NDC defines a 2×2 square area. This is the internal coordinate system of the D3D graphics API. We have seen that, as programmers working with D3D, we are responsible for programming the matrix processors such that all vertices are transformed into the NDC (i.e., the \mathbf{M}_{w2n} operator). In turn, D3D will automatically transform vertices from NDC to DC when processing the vertices (i.e., the \mathbf{M}_{n2d}). The NDC never changes.

Although the internal NDC representation causes extra complexity and processing, the NDC representation is also very important for the following reasons.

 Consistency and flexibility. A well-defined constant coordinate system is important for the internal implementation of the D3D API. As programmers of the API, such a well-known coordinate system provides a fixed reference Coordinate system and space. Coordinate systems and coordinate spaces are used interchangeably in this book. For example, device coordinate system and device coordinate space are both referred to as the DC.

NDC and OpenGL. For similar reasons as discussed here, the OpenGL API also defines the NDC as its internal coordinate system. The OpenGL API also performs the exact same \mathbf{M}_{n2d} operator (as defined by Equation (10.6)) on every input vertices.

as the rest of the application changes: as the DC window size changes, we can continue to communicate to the D3D API based on the NDC. In this way, our solution can be completely independent of the size of the UI drawing area. Because our solution is designed with reference to the NDC, our program can run in the same way with a 200×200 or a 500×500 UI window.

• Convenience. With the strategically chosen center (origin) and the coordinate ranges (of ±1), it is straightforward to transform the NDC square to other rectangular regions. For example, we have already seen in Equation (10.6) that it takes simple scaling and translation operations to transform the NDC to DC. In general, it is convenient to transform NDC to any coordinate space, for example, to the coordinate space defined for paper on printouts.

However, for humans, the ± 1 range of NDC is not always intuitive and often inconvenient to work with. For example, it is not straightforward to design a geometric face (e.g., Figure 9.21) where all vertex information must be constraint to between -1 and 1. To compensate for this rigid constraint, we introduce the *world coordinate* space for our programs to work in.

10.3 The World Coordinate System

When designing the geometric face of Figure 9.21, the implicit unit of measurement was the pixel. The implementation of Tutorial 9.9 conformed to this assumption where, for example, the drawing area is exactly 300 pixels \times 300 pixels. It would seem that if we want to display this face design on an application window with a different dimension, we would have to re-measure and re-define all vertex positions. However, if we examine the implementation of Tutorial 9.9 more closely, in the CDrawOnlyHandler::DrawGraphics() function, the w2n matrix is the \mathbf{M}_{w2n} operator:

$$\mathbf{M}_{w2n} = \mathbf{S}\left(\frac{2}{width}, \frac{2}{height}\right) \mathbf{T}(-1, -1),$$

where

$$\begin{cases} width = 300 pixels, \\ height = 300 pixels. \end{cases}$$

```
void CDrawOnlyHandler::DrawGraphics()
  float width = 300.0f, height=300.0f;
    :
    // Construct the matrix that will transform from the world bounds
    // to NDC
    D3DXVECTOR3 scale(2.0f/width, 2.0f/height, 1.0f);
    D3DXVECTOR3 translate(-1.0f, -1.0f, 0.0);
    D3DXMatrixTransformation(&world2ndc, ... &scale, ... &translate);
    m_pD3DDevice->SetTransform(D3DTS_VIEW, &world2ndc);
    :
```

Listing 10.3. The DrawGraphics() function of Tutorial 9.9.

Figure 10.8 illustrates the transformation that takes place. Note the following.

- Step 1. The \mathbf{M}_{w2n} operator transforms all vertices from our *design space* into the NDC. In this case, the design and measurements are based on a drawing area of width = 300 and height = 300. In the implementation of Tutorial 9.9, the values for the width and height are fixed based on the width and height of the application window. However, notice that the input of the \mathbf{M}_{w2n} transformation is our design space and that the output is the NDC space. In fact, neither the input nor the output is related to the DC dimension!
- Step 2. The \mathbf{M}_{n2d} operator is internal to the D3D graphics API. This operator transforms all vertices from the NDC to the final application drawing area, or the device coordinate (DC). Notice that this transformation, \mathbf{M}_{n2d} ,

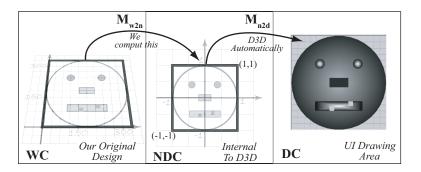


Figure 10.8. Transformation for drawing of the face in Figure 9.21.

Source file.

DrawOnlyHandler.cpp file in the *WindowHandler* folder of the D3D_XformList project. is governed by the dimensions of the application window and is independent of the values we used for the \mathbf{M}_{w2n} operator.

These observations indicate that, in fact, our 300×300 design space need not be related to the application window size. Indeed, the D3D internal NDC space helped separate \mathbf{M}_{w2n} and \mathbf{M}_{n2d} into independent operators, where

Design space
$$\xrightarrow{\mathbf{M}_{w2n}}$$
 NDC $\xrightarrow{\mathbf{M}_{n2d}}$ DC.

As application programmers, we only need to be concerned with the \mathbf{M}_{w2n} operator, which is independent of the DC!

10.3.1 Design Space versus Drawing Area

Tutorial 10.6. DC-Independent Design Space

- Goal. Verify that the 300 × 300 design space of Figure 9.21 is indeed independent of the dimensions of the UI drawing area.
- Approach. Change the dimension of the UI drawing area to 500 × 500 and observe the output.

Figure 10.9 is a screenshot of running Tutorial 10.6. Although the UI front end of this tutorial appears to be significantly different from that of Tutorial 9.9, these two tutorials have identical back-end implementations! The only significant difference is that in this case, the UI drawing area is 500×500 pixels. From the output of this tutorial, we see a larger but the same geometric face as the one observed in Tutorial 9.9. In this tutorial, we display a 300×300 design space in a 500×500 drawing area; we have verified that our design space is indeed independent of the device dimensions.

In computer graphics, we refer to the 300×300 coordinate space where we designed the original face the world coordinate space or the world coordinate system (WC). The world refers to the fact that the geometric objects within this space are the world that we would like to draw onto the output display area. We observe that as long as we correctly construct the \mathbf{M}_{w2n} operator (or the WC-to-NDC operator), we can select to work in any convenient WC space.

10.3.2 Working with a Convenient WC Space

It is important to select a design space or the WC such that it is convenient to specify our geometric objects. For example, in anticipation of specifying the full-

Tutorial 10.6. Project Name:

D3D_XformListCoordSpace Library Support: UWB_MFC_Lib1 UWB_D3D_Lib9



Figure 10.9. Running Tutorial 10.6.

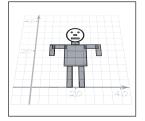


Figure 10.10. A full-figure geometric person.

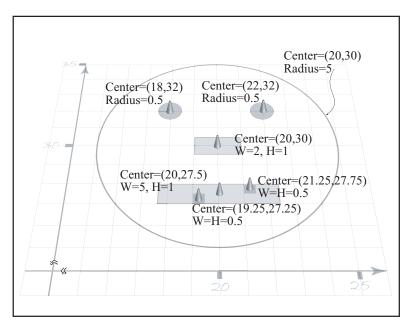


Figure 10.11. WC window of the face from Figure 10.10.

figure geometric person of Figure 10.10, we can choose a more convenient WC space to specify the face of Figure 9.21. Figure 10.11 shows a Window into the WC to illustrate the details of the new geometric face. In this case, the rectangular WC window is bounded by

WC window =
$$\begin{cases} 15 \le x \le 25 \\ 25 \le y \le 35 \end{cases} = \begin{cases} \text{center } = (cx_{wc}, cy_{wc}) = (20, 30), \\ \text{width } = W_{wc} = 10, \\ \text{height } = H_{wc} = 10. \end{cases}$$

If we want to display the content of this WC window, we must construct an appropriate \mathbf{M}_{w2n} operator. As we saw in Figure 10.8, as programmers of the D3D graphics API, our goal is to construct the \mathbf{M}_{w2n} operator to transform the WC window into D3D's internal coordinate system, i.e., the NDC. In turn, D3D will automatically transform the content of the NDC to the drawing area on the application window.

Figure 10.12 illustrates one way to construct the \mathbf{M}_{w2n} operator, where we first move the center of the region to the origin and then scale the region into a 2×2 area, or

$$\mathbf{M}_{w2n} = \mathbf{T}(-20, -30)\mathbf{S}\left(\frac{2}{10}, \frac{2}{10}\right).$$
 (10.7)

Linearity of affine transformation. Recall that to transform geometric contents between rectangular regions, we can concentrate on constructing the operator that transforms between the rectangles that surround the regions. The contents inside the rectangles will transform proportionally.

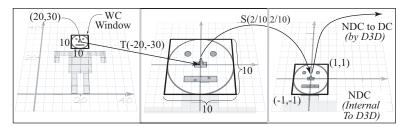


Figure 10.12. M_{w2n} : world-to-NDC transform for the face of Figure 10.11.

Tutoral 10.7. Working with a Convenient WC Space (Figure 10.12)

- **Goal.** Understand and work with a world coordinate (WC) window that is *not* centered on the origin.
- **Approach.** Program the transformation of Figure 10.12 and observe the output.

Figure 10.13 is a screenshot of running Tutorial 10.7. We observe that the output of this tutorial is identical to that of Tutorial 10.6. However, this tutorial implements the geometric face as defined by Figure 10.11 (in CModel.cpp). The DrawGraphics() function shown in Listing 10.4 defines the \mathbf{M}_{w2n} transform operator of Equation 10.7. We see that although the world coordinate spaces and the actual geometric values specified in this tutorial are very different from that of Tutorial 10.6, the output from the two tutorials are identical to each other. This tutorial verifies that we can define the WC window to support a convenient WC space for designing our world and that the WC space is independent of the dimensions of the UI drawing device.

```
void CDrawOnlyHandler::DrawGraphics() \vdots\\ // Translate\ the\ center\ of\ WC\ to\ the\ origin\\ D3DXMatrixTranslation(&toPlace, -20.0f, -30.0f, 0.0);\\ // Scale\ to\ 2x2\\ D3DXMatrixScaling(&toSize,\ 2.0f/10.0f,\ 2.0f/10.0f,\ 1.0);\\ //\ build\ \mathbf{M}_{w2n}\ operator\ of\ Equation\ 10.7\\ D3DXMatrixMultiply(&world2ndc,\ &toPlace,\ &toSize);\\ //\ Load\ to\ the\ VIEW\ matrix\ (M_V)\\ {\tt m_pD3DDevice->SetTransform(D3DTS_VIEW,\ &world2ndc);}
```

Listing 10.4. The DrawGraphics() function of Tutorial 10.7.

Tutorial 10.7.

Project Name:
D3D_ChangedCoordinate
Library Support:
UWB_MFC_Lib1
UWB_D3D_Lib9



Figure 10.13. Tutorial 10.7.

Source file.

DrawOnlyHandler.cpp file in the WindowHandler folder of the D3D_ChangedCoordinates project.

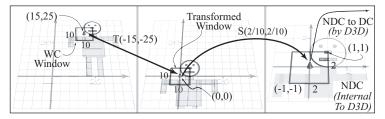


Figure 10.14. Location of WC window as defined by Equations (10.8) and (10.9).

Notice that there are no physical boundaries around the WC window. As programmers, we choose the WC window and program the \mathbf{M}_{w2n} operator accordingly. For example, in Tutorial 10.7, we could easily choose to select a different WC window:

WC window
$$\begin{cases} \text{center} = (15,25), \\ \text{width} = height = 10, \end{cases}$$
 (10.8)

and define the \mathbf{M}_{w2n} to be

$$\mathbf{M}_{w2n} = \mathbf{T}(-15, -25)\mathbf{S}\left(\frac{2}{10}, \frac{2}{10}\right).$$
 (10.9)

Tutorial 10.8. Translating the WC Window (Figure 10.14)

- Goal. Verify our understanding that we can move the WC window in the WC system to show a different part of the WC system.
- **Approach.** Program Equation (10.9) to verify our understanding.

Figure 10.15 is a screenshot of running Tutorial 10.8. The implementation of this tutorial is identical to that of Tutorial 10.7, except we program the \mathbf{M}_{w2n} according Equation (10.9) as shown in Listing 10.5. The image output of the the program does reflect our predictions from Figure 10.14. In this case, we can see the greyish outline of the rest of the geometric person of Figure 10.10. If we examine the class definition in Listing 10.6, at label A we observe that the CModel class defines the geometries for the entire human of Figure 10.10 in the m_figure PrimitiveList object. At label B, we see that the CModel::Draw() function draws all the geometries that are defined in the m_figure object. However, from the output of this tutorial in Figure 10.15, we see that only the geometries inside the WC window are displayed in the UI drawing area. From this tutorial, we see that the graphics API clips away all the geometries outside of the NDC ± 1 range.

Tutorial 10.8.

Project Name:
D3D_TranslateWC
Library Support:
UWB_MFC_Lib1
UWB_D3D_Lib9

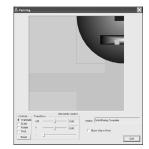


Figure 10.15. Running Tutorial 10.8.

Clipping. Graphics APIs only process and display geometries inside the ± 1 range of the NDC space. Geometries outside of this range are clipped and/or otherwise guaranteed to not show up in the UI drawing area.

Source file.

DrawOnlyHandler.cpp file in the *WindowHandler* folder of the D3D_TranslateWC project.

```
void CDrawOnlyHandler::DrawGraphics() \vdots\\ // \textit{Translate the center of WC to the origin}\\ D3DXMatrixTranslation(&toPlace, -15.0f, -25.0f, 0.0);\\ // \textit{Scale to 2x2}\\ D3DXMatrixScaling(&toSize, 2.0f/10.0f, 2.0f/10.0f, 1.0);\\ // \textit{build } \mathbf{M}_{w2n} \textit{ operator of Equation } 10.9\\ D3DXMatrixMultiply(&world2ndc, &toPlace, &toSize);\\ // \textit{Load to the VIEW matrix } (M_V)\\ \mathbf{m}_{p}D3DDevice->SetTransform(D3DTS_VIEW, &world2ndc);} \end{aligned}
```

Listing 10.5. The DrawGraphics() function of Tutorial 10.8.

CModel::CModel() { // CModel class constructor

```
// geometry for the torso
Source file. Model.cpp file
in the Model folder of the
                            UWB_PrimitiveRectangle* pTorso = new UWB_PrimitiveRectangle();
D3D_TranslateWC project.
                            pTorso->SetCorners( vec3(15,11,0), vec3(25,24,0) );
                            // geometry for the left leg
                            UWB_PrimitiveRectangle* pLeftLeg = new UWB_PrimitiveRectangle();
                             // defines the entire geometry for the simple person of Figure 10.10
                             // m_figure is a PrimitiveList
                             m_figure.Append(pTorso); m_figure.Append(pLeftLeg);
                             // append all defined goemetry into the m_figure PrimitiveList
                       void CModel::DrawModel() // CModel draw function
                             // Set up the WORLD matrix
                             m_xform.SetUpModelStack( m_DrawHelper );
                             // draws the entire goemetric human of Figure 10.10
                             m_figure.Draw( lod, m_DrawHelper );
```

Listing 10.6. The CModel class of Tutorial 10.8.

10.4 The World Coordinate Window

In order to properly support WC design space, in our programs we must identify a rectangular region of interest, or the *window* inside the WC:

WC window =
$$\begin{cases} \text{center} &= (cx_{wc}, cy_{wc}), \\ \text{width} &= W_{wc}, \\ \text{height} &= H_{wc}. \end{cases}$$

We must then construct a corresponding \mathbf{M}_{w2n} to transform this region to the NDC space. In general, we transform the center of the region to the origin $(T(-cx_{wc}, -cy_{wc}))$ and then scale the width and height of the region to the NDC 2×2 square $(S(\frac{2}{W_{wc}}, \frac{2}{H_{wc}}))$:

$$\mathbf{M}_{w2n} = \mathbf{T}(-cx_{wc}, -cy_{wc})\mathbf{S}(\frac{2}{W_{wc}}, \frac{2}{H_{wc}}).$$
(10.10)

10.4.1 Vertices in Different Coordinate Spaces

As programmers working with graphics APIs, we specify vertices in the WC, or V_{wc} . As illustrated in Figure 10.16, this vertex undergoes different transforms until it reaches the DC, or V_{dc} , before being displayed on the output drawing area:

$$\mathbf{V}_{wc} \stackrel{\mathbf{M}_{w2n}}{\longrightarrow} \mathbf{V}_{ndc} \stackrel{\mathbf{M}_{n2d}}{\longrightarrow} \mathbf{V}_{dc},$$

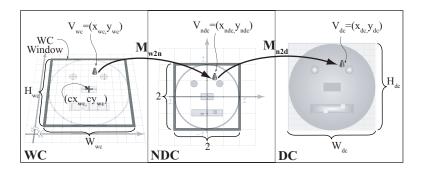


Figure 10.16. Transformation of a vertex between different coordinate systems.

where

$$\mathbf{V}_{wc} = \begin{bmatrix} x_{wc} & y_{wc} \end{bmatrix},
\mathbf{V}_{ndc} = \mathbf{V}_{wc} \mathbf{M}_{w2n},
\mathbf{V}_{dc} = \mathbf{V}_{ndc} \mathbf{M}_{n2d},$$

or

$$\mathbf{V}_{dc} = \mathbf{V}_{ndc} \, \mathbf{M}_{n2d} \\ = \mathbf{V}_{wc} \, \mathbf{M}_{w2n} \, \mathbf{M}_{n2d}.$$

If we let

$$\mathbf{M}_{w2d} = \mathbf{M}_{w2n} \,\mathbf{M}_{n2d},\tag{10.11}$$

then

$$\mathbf{V}_{dc} = \mathbf{V}_{wc} \,\mathbf{M}_{w2d}.\tag{10.12}$$

Recall that \mathbf{M}_{n2d} is defined by Equation (10.6) and that \mathbf{M}_{w2n} is defined by Equation (10.10) (re-listing the two equations):

$$\mathbf{M}_{n2d} = \mathbf{S}(\frac{W_{dc}}{2}, \frac{H_{dc}}{2})\mathbf{T}(\frac{W_{dc}}{2}, \frac{H_{dc}}{2}),$$
(10.6)
$$\mathbf{M}_{w2n} = \mathbf{T}(-cx_{wc}, -cy_{wc})\mathbf{S}(\frac{2}{W_{ve}}, \frac{2}{H_{ver}}).$$
(10.10)

In this way,

$$\begin{aligned} \mathbf{V}_{dc} &= \mathbf{V}_{wc} \ \mathbf{M}_{w2d} \\ &= \mathbf{V}_{wc} \ \mathbf{M}_{w2n} \ \mathbf{M}_{n2d} \\ &= \mathbf{V}_{wc} \ \mathbf{T}(-cx_{wc}, -cy_{wc}) \ \mathbf{S}(\frac{2}{W_{wc}}, \frac{2}{H_{wc}}) \ \mathbf{S}(\frac{W_{dc}}{2}, \frac{H_{dc}}{2}) \ \mathbf{T}(\frac{W_{dc}}{2}, \frac{H_{dc}}{2}), \end{aligned}$$

or

$$\mathbf{V}_{dc} = \mathbf{V}_{wc} \, \mathbf{T}(-cx_{wc}, -cy_{wc}) \, \mathbf{S}(\frac{W_{dc}}{W_{wc}}, \frac{H_{dc}}{H_{wc}}) \, \mathbf{T}(\frac{W_{dc}}{2}, \frac{H_{dc}}{2}). \tag{10.13}$$

Comparing Equations (10.12) and (10.13), we see that

$$\mathbf{M}_{w2d} = \mathbf{T}(-cx_{wc}, -cy_{wc})\mathbf{S}(\frac{W_{dc}}{W_{wc}}, \frac{H_{dc}}{H_{wc}})\mathbf{T}(\frac{W_{dc}}{2}, \frac{H_{dc}}{2}). \tag{10.14}$$

Equation (10.14) says that the operator \mathbf{M}_{w2d} , which transforms from the WC (x_{wc}, y_{wc}) to the DC (x_{dc}, y_{dc}) , does the following.

- Move. The center of the WC window to the origin with $(\mathbf{T}(-cx_{wc}, -cy_{wc}))$. The result of this transform is a $W_{wc} \times H_{wc}$ rectangle centered at the origin.
- Scale. With the WC window width of W_{wc} , the scaling factor $\frac{W_{dc}}{W_{wc}}$ changes the width to W_{dc} . In a similar fashion, the height becomes H_{dc} . After the scaling operator, we have a $W_{dc} \times H_{dc}$ rectangle centered at the origin.

Consecutive scaling operators. Two consecutive scaling operators:

$$S(s_{x1}, s_{v1})S(s_{x2}, s_{v2})$$

have the same effect as scaling once by the combined effect of the scaling factors:

$$S(s_{x1}s_{x2}, s_{y1}s_{y2}).$$

• Move. The rectangle centered at the origin has half its width/height on either side of the y/x axis. The translation of $\mathbf{T}(\frac{W_{dc}}{2}, \frac{H_{dc}}{2})$ moves the lower-left corner of the rectangle to the origin, with the upper-right corner located at (W_{dc}, H_{dc}) . This is the definition of the DC space.

If we expand the operators in Equation (10.13), then, to transform a point (x_{wc}, y_{wc}) from our design space (WC) to a point (x_{dc}, y_{dc}) on the device drawing area (DC),

$$x_{dc} = ((x_{wc} - cx_{wc}) \frac{W_{dc}}{W_{wc}}) + \frac{W_{dc}}{2},$$

$$y_{dc} = ((y_{wc} - cy_{wc}) \frac{H_{dc}}{H_{wc}}) + \frac{H_{dc}}{2},$$
(10.15)

where

Device drawing area
$$\left\{ \begin{array}{ll} \text{width} & = W_{dc}, \\ \text{height} & = H_{dc}, \end{array} \right.$$

and

WC window =
$$\begin{cases} \text{center} &= (cx_{wc}, cy_{wc}), \\ \text{width} &= W_{wc}, \\ \text{height} &= H_{wc}. \end{cases}$$

From Equation (10.15), we see that when the size of the device drawing remains constant (i.e., W_{dc} and H_{dc} do not change), then the transformation from WC to DC is governed by the parameters of the WC window as follows.

- 1. **Center** (cx_{wc}, cy_{wc}) . Defines the location of the WC window. Intuitively, by changing the center we are moving the WC window and thus should observe different rectangular regions in the WC system, or panning of the view. Tutorial 10.10 will examine panning in detail.
- 2. **Dimension** (W_{wc} H_{wc}). Defines the size of the WC window. Intuitively, by changing the dimension, we increase/decrease the rectangular region to be displayed. With a fixed-size UI drawing device, increasing the size of the WC window means showing a larger amount of the WC system in the fixed-size DC drawing area, or a zooming-out effect. With the same logic, decreasing the size of the WC window creates a zooming-in effect. Tutorial 10.11 will examine zooming in detail.
- 3. **Ratio of scaling factors** ($\frac{W_{dc}}{W_{wc}}$ **versus** $\frac{H_{dc}}{H_{wc}}$). We scale the width of the WC window by $\frac{W_{dc}}{W_{wc}}$ and the height by $\frac{H_{dc}}{H_{wc}}$. When these two scaling factors are different, the proportion of the results in DC space will also be different from that of the original WC space. For example, a square will be transformed into a rectangle. Section 10.4.4 will examine this effect in detail.

We remind ourselves that Equation (10.15), or the \mathbf{M}_{w2d} operator, is the *net* transformation that would be applied to vertices specified in WC space. From Equation (10.11), we observe that, as programmers, we are only responsible for \mathbf{M}_{w2n} , or half of the \mathbf{M}_{w2d} operator. The other half of the operator, \mathbf{M}_{n2d} , is computed automatically by the graphics API. In this case, we analyze Equation (10.15) and the \mathbf{M}_{w2d} operator to understand the details of the images generated in the UI drawing device. We will examine the effects of changing the WC window based on tutorial implementations. Let us first extend our library to support working with the WC window.

UWBGL_D3D_Lib10

This library extends from Lib9 by extending the UWB_WindowHandler class to support the definition of the WC window and programming of the \mathbf{M}_{w2n} operator. Recall that the WindowHandler object is defined to abstract a view/controller pair. For the WindowHandler object to properly display different regions of the

```
class UWB_WindowHandler : public UWB_IWindowHandler {
     // Set/Get methods for accessing the m_WCWindow object
A: | virtual void SetWCWindow(const UWB_BoundingBox &window);
     virtual const UWB_BoundingBox* GetWCWindow();
     // Set/Get methods for the drawing device (via m_hAttachedWindow)
B: | virtual void SetDeviceSize(int width, int height);
     virtual void GetDeviceSize(int &width, int &height);
     // wcPt -> dcPt (Equation 10.13)
C: | virtual void WorldToDevice(float wcX wcY, int &dcX &dcY);
     // dcPt -> wcPt (Equation 10.19)
     virtual void DeviceToWorld(int dcX dcY, float &wcX &wcY);
     // Drawing the WCWindow
D: | virtual void DrawWCWindow(UWB_DrawHelper&);
     // Compute \mathbf{M}_{w2n} and load to API matrix processor
E: | virtual void LoadW2NDCXform() = 0;
   protected:
     HWND m_hAttachedWindow; // The UI drawing device (a MFC Window)
     UWB_BoundingBox m_WCWindow; // Window for the World Coordinate
```

Listing 10.7. The WindowHandler class of UWBGL_D3D_Lib10.

Change summary. See p. 522 for a summary of changes to the library.

Source file.

uwbgl_WindowHandler3.h file in the *Common Files/WindowHandler* subfolder of the UWBGL_D3D_Lib10 project.

```
void UWB_WindowHandler::WorldToDevice(float wcx, float wcy, int &dcx, int &dcy) const {
    GetDeviceSize(dcW, dcH); // dcW/dcH are the width/height of the drawing device
    center = m_WCWindow.GetCenter(); // center of the WCWindow
    // Equation 10.13
    dcx = (m_WCWindow.Width() / 2.0f) + ((dcW/m_WCWindow.Width()) * (-center.x + wcx));
    dcy = (m_WCWindow.Height()/ 2.0f) + ((dcH/m_WCWindow.Height()) * (-center.y + wcy));
void UWB_WindowHandler::DeviceToWorld(int dcx, int dcy, float &wcx, float &wcy) const
    GetDeviceSize(dcW, dcH); // dcW/dcH are the width/height of the drawing device
    vec3 center = m_WCWindow.GetCenter(); // center of the WCWindow
    // Equation 10.19
                                                                                       Source file.
    wcx = center.x + ((m_WCWindow.Width()/dcW) * (dcx - (dcW/2.0f)));
                                                                                       uwbgl_WindowHandler3.cpp
    wcy = center.y + ((m_WCWindow.Height()/dcH) * (dcy - (dcH/2.0f)));
                                                                                       file in the Common Files/Win-
                                                                                       dowHandler subfolder of the
Listing 10.8. The WindowHandler transform and draw functions.
                                                                                       UWBGL_D3D_Lib10 project.
class UWBD3D_WindowHandler : public UWB_WindowHandler {
    virtual void LoadW2NDCXform() const; // Computes and loads VIEW matrix with M_{w2n}
void UWBD3D_WindowHandler::LoadW2NDCXform() const
                                                                                       Source file.
    // center of the m_WCWindow (cx_{wc}, cv_{wc})
                                                                                       uwbgl_D3DWindowHandler4.h/cpp
    vec3 center = m_WCWindow.GetCenter();
                                                                                       files in the D3D Files/ Win-
    // \mathbf{T}(-cx_{wc}, -cy_{wc})
                                                                                       dowHandler folder of the
    D3DXMatrixTranslation(&toPlace, -center.x, -center.y);
                                                                                       UWBGL_D3D_Lib10 project.
    // \mathbf{S}(\frac{2}{W_{\text{true}}}, \frac{2}{H_{\text{true}}})
    D3DXMatrixScaling(&toSize, 2.0f/m_WCWindow.Width(), 2.0f/m_WCWindow.Height());
    // \mathbf{M}_{w2n} = \mathbf{T}(-cx_{wc}, -cy_{wc})\mathbf{S}(\frac{2}{W_{wc}}, \frac{2}{H_{wc}})
    D3DXMatrixMultiply(&world2ndc, &toPlace, &toSize);
    // \mathbf{M}_V \leftarrow \mathbf{M}_{w^2n}
    m_pD3DDevice->SetTransform(D3DTS_VIEW, &world2ndc);
```

Listing 10.9. The LoadW2NDCXform() functions.

model, it must support the WC window and the associated transformations. From Listing 10.7, we see that, at label F, we define a UWB_BoundingBox object to represent the WC window. The get/set access functions for the m_WCWindow are defined at label A. Because the UI drawing area (drawing device) is supported by the MFC GUI API (m_hAttachedWindow), the device get/set functions at label B are implemented by interacting with the m_hAttachedWindow object. The two functions at label C implements the WC-to-DC transformations (more details to follow). The drawing function at label D allows us to visualize the WC window as a wire-framed rectangle. The function at label E should compute the \mathbf{M}_{w2n} matrix and load the graphics API with this matrix. Note that this function is a pure virtual function; since WindowHandler is a graphics API-independent class, we do not know how to implement this function. This function will be implemented by the D3DWindowHandler class. Listing 10.8 shows the implementations of the transformation functions. We see that in both cases, the functions are faithful implementations of equations we have derived. We will derive Equation (10.19) in Section 10.5 and discuss how to work with these functions to handle mouse inputs. Listing 10.9 shows the implementation of the LoadW2NDCXform() function in the D3D_WindowHandler class. From the listing, we observe step-by-step implementation of Equation (10.10). The computed \mathbf{M}_{w2n} operator is loaded into the VIEW (\mathbf{M}_V) matrix processor of the D3D API.

Tutorial 10.9. Working with UWBGL_D3D_Lib10

- Goal. Demonstrate how to work with the new UWBGL_D3D_Lib10 library.
- Approach. Re-implement Tutorial 10.8 based on the new library to understand how to work with the new functions.

Figure 10.17 is a screenshot of running Tutorial 10.9. This tutorial is identical to Tutorial 10.8 except that the new implementation takes advantage of the new WC window functionality provided by the UWBGL_D3D_Lib10 library. Listing 10.10 shows that, in the CTutorialDlg::0nInitDialog() function, at label A we initialize the view/controller pair WC window (m_view) as part of the application state initialization process. When redrawing the view/controller pair in the DrawGraphics() function, at label B we call the LoadW2NDCXform() function to compute and load the \mathbf{M}_{w2n} operator into the D3D VIEW matrix processor.

Project Name: D3D_WCSupport Library Support: UWB_MFC_Lib1

UWB_D3D_Lib10

Tutorial 10.9.



Figure 10.17. Running Tutorial 10.9.

```
BOOL CTutorialDlg::OnInitDialog()
    if(!m_view.Initialize(*this, IDC_PLACEHOLDER)) // m_view is a DrawOnlyHandler (D3DWindowHandler)
         return FALSE;
    UWB_BoundingBox wcWindow(vec3(10, 20), vec3(20, 30)); // initialize the wcWindow
    // set the m_WCWindow of WindowHandler object
    m_view.SetWCWindow( wcWindow );
A:
                                                                               Source file.
                                                                               TutorialDlg.cpp file in the
void CDrawOnlyHandler::DrawGraphics()
                                                                               Source Files folder of the
                                                                               D3D_WCSupport project.
    BeginDraw();
B: LoadW2NDCXform(); // compute and load the \mathbf{M}_{w2n} to the VIEW matrix processor
    m_pD3DDevice->Clear( : ); // Clears the device for drawing
    theApp.GetModel().DrawModel(); // Tells the Model to draw itself
    EndDrawAndShow();
```

Listing 10.10. Working with UWBGL_D3D_Lib10 (Tutorial 10.9).

10.4.2 WC Window Position: Panning

Recall that Equation (10.15) defines the transformation of a point from WC space (x_{wc}, y_{wc}) to DC space (x_{dc}, y_{dc}) :

$$x_{dc} = ((x_{wc} - cx_{wc}) \frac{W_{dc}}{W_{wc}}) + \frac{W_{dc}}{2}, y_{dc} = ((y_{wc} - cy_{wc}) \frac{H_{dc}}{H_{wc}}) + \frac{H_{dc}}{2},$$
(10.15)

where $W_{dc} \times H_{dc}$ is the drawing device dimension and

WC window =
$$\begin{cases} center = (cx_{wc}, cy_{wc}), \\ width = W_{wc}, \\ height = H_{wc}. \end{cases}$$

In Tutorial 10.8, we observed that by changing the WC center position (cx_{wc}, cy_{wc}) , we can show different regions of the model defined in the WC system. Based on our discussions, we can predict that a continuous changing of WC window position would create an effect of panning through the WC system.

Tutorial 10.10.

Project Name: D3D_Panning Library Support: UWB_MFC_Lib1 UWB_D3D_Lib10



Figure 10.18. Running Tutorial 10.10.

Tutorial 10.10. Moving the WC Window Center ((cx_{wc}, cy_{wc}))

- Goal. Verify that the effect of changing the center position of a WC window does indeed correspond to continuous displaying of different regions in the WC system.
- **Approach.** Allow the user to interactively change the center position of a WC window and examine the results.

Figure 10.18 is a screenshot of running Tutorial 10.10. In this tutorial, the large (main) view displays a larger region of the WC system (the entire geometric person), while the small view only displays part of the WC system visible in the main view. The red wire-framed rectangle in the main view represents the WC window of the small view. The two slider bars on the lower-right of the application window control the center position $((cx_{wc}, cy_{wc}))$ of the WC window for the small view. By changing these two slider bars, we can observe the red rectangle in the main view and the image showing in the small view pan across the main view. This tutorial verifies that changing the WC window position creates the panning effect. Figure 10.19 illustrates that the implementation of Tutorial 10.10 involves two different types of CWindowHandler objects. The top-center rectangle represents the CDrawOnlyHandler of the small view, whereas the bottomcenter rectangle is the CMainHandler of the main view. As we can see, the CMainHandler maintains a reference to the small view. This reference provides the small view WC window information for the main view to draw the wireframed red rectangle. With the UWBGL_D3D_Lib10 support, both of the Handler objects have an instance of UWB_BoundingBox representing their corresponding WC window. As in the case of Listing 10.10, during DrawGraphics(), each

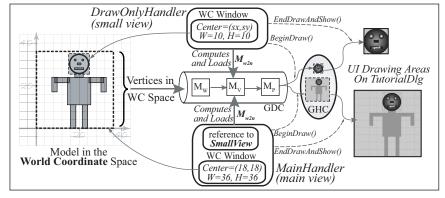


Figure 10.19. Implementation of Tutorial 10.10.

```
class CDrawOnlyHandler : public UWBD3D_WindowHandler {
                                                                                Source file.
                                                                                DrawOnlyHandler.h file in
                                                                                the WindowHandler folder of
    UWBMFC_UIWindow m_window; // the UI drawing area (device)
                                                                                the D3D_Panning project.
};
class CMainHandler : public CDrawOnlyHandler {
    void SetLinkedHandler(CDrawOnlyHandler* pHandler) { m_pLinkedHandler = pHandler; }
  protected:
    CDrawOnlyHandler* m_pLinkedHandler; // reference to the small view
};
Listing 10.11. The WindowHandler classes of Tutorial 10.10.
 class CTutorialDlg : public CDialog {
                                                                                Source file.
                                                                                TutorialDlg.h/cpp file in
                                                                                the Source Files folder of the
A:
     CMainHandler
                       m_main_view; // the main view
                                                                                D3D_Panning project.
     CDrawOnlyHandler m_small_view; // the small view
};
BOOL CTutorialDlg::OnInitDialog() {
     if( !m_main_view.Initialize(*this, IDC_PLACEHOLDER) ) return FALSE;
     if( !m_small_view.Initialize(*this, IDC_PLACEHOLDER3) ) return FALSE;
B: | m_small_view.SetWCWindow(theApp.GetModel().GetWorldBounds());
     m_main_view.SetWCWindow( UWB_BoundingBox( vec3(0,0,0), vec3(36,36,0) ) );
C:
     m_main_view.SetLinkedHandler( &m_small_view );
Listing 10.12. The CTutorialDlg classes of Tutorial 10.10.
```

Handler object calls the LoadW2NDCXform() function to compute and load the D3D RC M_V matrix with their corresponding M_{w2n} operator before drawing. The left side of Figure 10.19 shows that the model is defined in the WC space. When the model draws, it sends all the vertices of all the geometries to the D3D RC.

Source file.

DrawAndMouseHandler.cpp file in the WindowHandler folder of the D3D_Panning project.

```
void CMainHandler::DrawGraphics()
    :
    BeginDraw();
    :
    LoadW2NDCXform(); // compute M<sub>w2n</sub> and load to M<sub>V</sub>
    :
    m_pD3DDevice->Clear(:); // clears the UI drawing area (device) theApp.GetModel().DrawModel(); // model sends all geometries
    :
    // draws the red wire-frame rectangle

B: m_pLinkedHandler->DrawWCWindow(helper);
    EndDrawAndShow();
```

Listing 10.13. The CMainHandler::DrawGraphics() function of Tutorial 10.10.

As we have learned, the \mathbf{M}_{w2n} operator ensures that only the region defined by the corresponding WC window is transformed into the ± 1 NDC space where the rest of the geometries will be clipped away by the graphics API. The right side of Figure 10.19 reminds us that that each Handler object is responsible for initializing the swapchain links with BeqinDraw() and flushing the hardware buffer to the UI draw area with the EndDrawAndShow() function calls. Listing 10.11 shows the definition of the CDrawOnlyHandler and CMainHandler classes of Tutorial 10.10. We see that indeed both of the classes subclass from the UWBD3D_WindowHandler class and that the CMainHandler has a reference to the small view (m pLinkedHandler). Referring to Listing 10.12, at label A we see that the main application window (CTutorialDlg) has an instance of each of the Handler objects representing the main and the small views. At label B, the WC window for each of the views is initialized. At label C, the main view gets a reference to the small view. Listing 10.13 shows the DrawGraphics() function of MainHandler calling LoadW2NDCXForm() to compute and load the corresponding \mathbf{M}_{w2n} operator at label A (before drawing the model). At label B, the WC window of the m_pLinkedHandler (small view) is drawn (as the red wire-frame rectangle) in the main view.

With this structure, the two slider bars are connected to the small view's WC window center position. When the user adjusts the slider bars, the changes are immediately updated to the m_small_view's m_WCWindow. In the subsequent redraw, the updated WC window computes an appropriate \mathbf{M}_{w2n} operator that corresponds to the user input.

10.4.3 WC Window Dimension: Zooming

If we double the WC window size of the small view from Tutorial 10.10 such that

WC window
$$\begin{cases} \text{center} = (15,25), \\ W_{wc} = H_{wc} = 20, \end{cases}$$

then the WC window covers a larger region in the WC system. From Equation (10.15):

$$x_{dc} = ((x_{wc} - cx_{wc}) \frac{W_{dc}}{W_{wc}}) + \frac{W_{dc}}{2}, y_{dc} = ((y_{wc} - cy_{wc}) \frac{H_{dc}}{H_{wc}}) + \frac{H_{dc}}{2},$$
(10.15)

we see that as W_{wc} and H_{wc} are the denominators in the scale factors, doubling these effectively halves the scaling factors. This implies that we should expect the elements on the DC display to decrease in size. As illustrated in Figure 10.20, we see that if we increase the WC window size, more elements in the WC are displayed in the output display area. Correspondingly, each individual element does indeed appear to be smaller on the output display. We see that increasing the WC window size results in the zooming-out sensation. For the same reasons, we can expect a zooming-in effect when we decrease the WC window size.

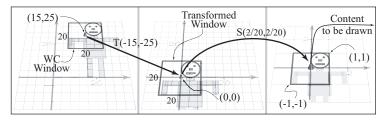


Figure 10.20. Double the WC window size.

Tutorial 10.11. Scaling the WC Window (W_{wc}, H_{wc})

- Goal. Verify zooming effect in relation to the WC window dimensions.
- Approach. Extend Tutorial 10.10 and allow user to adjust the width and height of the small view WC window.

Figure 10.21 is a screenshot of running Tutorial 10.11. This tutorial extends Tutorial 10.10 with two more slider bars at the lower right of the application window. The two new sliders are connected directly to the width (W_{wc}) and height (H_{wc}) of

Tutorial 10.11. Project Name: D3D_Zooming Library Support:

UWB_MFC_Lib1

UWB_D3D_Lib10

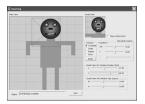


Figure 10.21. Running Tutorial 10.11.

the m_small_view's m_WCWindow. Based on the previous discussions, we understand that as the user adjust the slider bars, the resulting changes will be reflected in the subsequent \mathbf{M}_{w2n} (Equation (10.10)) operator that is loaded into the \mathbf{M}_V matrix. In this tutorial, we can interactively adjust and observe the expected zooming effects. In addition, we observe the following.

- Changing W_{wc} without corresponding changes to H_{wc} (or changing H_{wc} without corresponding changes to W_{wc}) creates an annoying squeezing effect. We will examine this closely in the next section.
- The zooming effect appears to be defined with respect to the center of the WC window. That is, as we zooming in, it appears we are getting closer to the center of the WC window. This is a direct result of our scaling the width and height of the WC window with respect to the center of the WC window. It is left as an exercise for the reader to derive implementations to support zooming with respect to some other position in the WC window.

10.4.4 WC Window Width-to-Height Ratio: Aspect Ratio

Recall that the WC-to-DC transformation is governed by Equation (10.14), the \mathbf{M}_{w2d} operator, or

$$\mathbf{M}_{w2d} = \mathbf{T}(-cx_{wc}, -cy_{wc}) \mathbf{S}(\frac{W_{dc}}{W_{wc}}, \frac{H_{dc}}{H_{wc}}) \mathbf{T}(\frac{W_{dc}}{2}, \frac{H_{dc}}{2}). \quad (10.14)$$

We observe that the middle scaling operator controls the zoom factor. When the scaling factors in the x- and y-directions are different, then the transformation from WC to DC will involve resizing objects in x- and y-directions by different amounts. In the cases of Tutorials10.4 and 10.5, we worked in the NDC space directly with a WC window size of

$$\begin{cases} W_{wc} = 2 \text{ with } -1 \le x \le 1, \\ H_{wc} = 2 \text{ with } -1 \le y \le 1. \end{cases}$$

For both tutorials,

$$W_{dc} \neq H_{dc}$$
.

This means that the factors of the scaling operator in Equation (10.14) are different in the x- and y-directions. For this reason, in both cases we observed the squashed circles. In order to maintain proportional shapes from WC to DC, the x and y scaling factors in the \mathbf{M}_{w2d} operator must be the same, or

$$\frac{W_{dc}}{W_{wc}} = \frac{H_{dc}}{H_{wc}}.$$

Collecting WC and DC terms on either sides of the equation,

$$\frac{W_{dc}}{H_{dc}} = \frac{W_{wc}}{H_{wc}}. (10.16)$$

We define aspect ratio to be:

aspect ratio
$$=\frac{\text{width}}{\text{height}}$$
.

Equation (10.16) says that to maintain proportional shapes when transforming from WC to DC, the aspect ratio of the WC window must be the same as that of the device drawing area.

Tutorial 10.12. Experimenting with Aspect Ratios

- Goal. Understand WC window and DC aspect ratios and verify the artifacts when the ratios do not match.
- **Approach.** Extend Tutorial 10.11 and allow interactive changing of UI drawing dimension.

Figure 10.22 is a screenshot of running Tutorial 10.12. This tutorial extends Tutorial 10.11 by including two more slider bars at the lower right of the application window. These two new slider bars control the device dimension of the small-view UI drawing area, or W_{dc} and H_{dc} . Recall that the other four slider bars control the small-view WC window location (cx_{wc} and cy_{wc}) and dimension (W_{wc} and H_{wc} sliders). From Equation (10.16) we understand that the image displayed in the small view will be distorted if/when we adjust the sliders bars such that

$$\frac{W_{dc}}{W_{wc}} \neq \frac{H_{dc}}{H_{wc}}.$$

We note that in general these four numbers can be controlled by the following.

• Our application. As the programmer of the application, we can design our application to allow user control of these values. For example, our application could allow the user to zoom in the world by allowing the WC window to change size, or our application could allow the user to increase/descrease the UI drawing area. In these cases, in order to maintain object proportions, it is important that our application presents a coherent and meaningful user interface. For example, we should implement zooming functionality by providing the user with one single zoom factor. The single zoom factor would ensure that the ratio $\frac{W_{dc}}{W_{wc}}$ remains constant and thus avoid the situation where WC and DC aspect ratios are different.

Tutorial 10.12.

Project Name:
AspectRatio
Library Support:
UWB_MFC_Lib1
UWB_D3D_Lib10

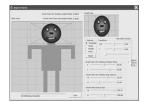


Figure 10.22. Running Tutorial 10.12.

- Window manager. Because our application runs in a shared windowing environment, it is always possible that the environment may change the size of the UI drawing area and inform our application about the changes. In this case, if the DC aspect ratio is changed, then in order to maintain proper proportions, we must change the WC window dimension accordingly. For example, initially our application may have a WC window size of 10×10 and a DC displaying area of 200×200 . If for some reason, the window manager decides to change the DC drawing area to 200×100 , we must update the WC window to match the 0.5 DC aspect ratio. In this case, we can:
 - increase the WC window size to 10 × 20 and show more of the WC space; or
 - decrease the WC window size to 5×10 and show less of the WC space.

The first option guarantees that we will show at least the original WC window, whereas the second option guarantees that we will show at most the original WC window. The choice between the above two options is a policy decision: neither choice is more correct. We should program our application to support the behavior specified by the user.

10.4.5 Summary

The world coordinate (WC) system is introduced to dissociate our model design space from the dimensions of the UI drawing area. As we have seen, this is advantageous for the following reasons.

- With the WC, we can define any coordinate system that is convenient for designing the geometric models and not be concerned with the dimensions of the eventual output drawing area.
- With the WC *window*, we can control the parameters (center, width, and height) to select the exact regions of the WC space to be displayed in the UI drawing area.

To properly support the WC system, the graphics API introduces the normalized device coordinate (NDC) system, a coordinate system bounded by ± 1 . Our job as the graphics API programmer is to compute the \mathbf{M}_{w2n} (Equation (10.10)) operator for each of the WC window regions we wish to display. The graphics API will

automatically clip away all geometries outside of the ± 1 range and will automatically compute the \mathbf{M}_{n2d} (Equation (10.6)) operator for drawing the geometries to the output device. In our library design, the \mathbf{M}_{w2n} operator is computed by the WindowHandler class and loaded into the VIEW matrix processor (M_V) of the D3D RC. Finally, we have seen that when defining the WC window, we must ensure that the WC window aspect ratio is the same as that of the UI drawing device. Otherwise, disproportional scaling will occur, resulting in images that appear squeezed.

10.5 Inverse Transformation

So far, we have concentrated on learning the output of geometric elements. To build interactive applications, we must interact with the user. In particular, we must understand how the added window coordinate system affects a user's picking or selecting an on-screen object by clicking the mouse buttons (in DC space).

In the Tutorial 5.6 implementation of the ball-shooting program, we compared mouse click positions with circles in the AllWorldBalls collection to locate the selected one. Notice that this simple operation involves a comparison across two distinct coordinate systems.

- Mouse-click position (pt_{dc}). This is a point on the display device (that the user's mouse clicked on). In the case of MFC, this position is returned to us in the hardware coordinate space. As described in Chapter 2 (Tutorial 2.4), we *flipped* the y-axis by subtracting the y-value from the height of the device and thereby converting the point into the device coordinate space with the origin at the lower-left corner. In all of the following discussions, we assume that the hardware-to-device transform has already been performed and we will work with points in DC space, pt_{dc} .
- AllWorldBalls collection. This is the set of all geometries in the world.
 By definition, these geometries are defined in the world coordinate (WC) space.

In the case of Tutorial 5.6, although we did not distinguish between these two coordinate systems, the selection operation functioned correctly because we have carefully defined the WC window to coincide exactly with the DC space. In general, we must transform input mouse positions to the WC space before working with them. For example, in the Tutorial 10.7 implementation of displaying the

Hardware coordinate. Recall that the MFC API returns mouse click positions in the hardware coordinate system where the top-left is the origin with *y*-axis incrementing downward and *x*-axis increase rightward.

Hardware-to-device transform. This transformation is performed by the HardwareToDevice() function defined in the WindowHandler class. geometric face of Figure 10.11, we know

DC coordinate:
$$\begin{cases} 0 \le x_{dc} \le 300, \\ 0 \le y_{dc} \le 300, \end{cases}$$

whereas

WC window:
$$\begin{cases} 15 \le x_{wc} \le 25, \\ 25 \le y_{wc} \le 35. \end{cases}$$

In this case, the user's mouse clicks (pt_{dc}) will return points in DC space with range [0,300]. Clearly, this is very different from where the features of the face are defined in the WC space. In this case, we must perform the inverse of the \mathbf{M}_{w2d} operator to transform pt_{dc} into a point in WC space. If we express

$$pt_{dc} = (x_{dc}, y_{dc})$$

as a vector

$$\mathbf{V}_{dc} = \begin{bmatrix} x_{dc} & y_{dc} \end{bmatrix},$$

then we must compute V_{wc} , where

$$\mathbf{V}_{wc} = \mathbf{V}_{dc} \, \mathbf{M}_{w2d}^{-1}$$

From Equation (10.14), recall that \mathbf{M}_{w2d} is

$$\mathbf{M}_{w2d} = \mathbf{T}(-cx_{wc}, -cy_{wc})\mathbf{S}(\frac{W_{dc}}{W_{wc}}, \frac{H_{dc}}{H_{wc}})\mathbf{T}(\frac{W_{dc}}{2}, \frac{H_{dc}}{2}). (10.14)$$

From the discussion in Section 9.2, we know that the inverse of a concatenated operator is simply the inverse of each element concatenated in the reverse order:

$$\mathbf{M}_{w2d}^{-1} = \mathbf{M}_{d2w} = \mathbf{T}^{-1}(\frac{W_{dc}}{2}, \frac{H_{dc}}{2})\mathbf{S}^{-1}(\frac{W_{dc}}{W_{wc}}, \frac{H_{dc}}{H_{wc}})\mathbf{T}^{-1}(-cx_{wc}, -cy_{wc}),$$

which is

$$\mathbf{M}_{d2w} = \mathbf{T}(-\frac{W_{dc}}{2}, -\frac{H_{dc}}{2})\mathbf{S}(\frac{W_{wc}}{W_{dc}}, \frac{H_{wc}}{H_{dc}})\mathbf{T}(cx_{wc}, cy_{wc})$$
(10.17)

or

$$\mathbf{V}_{wc} = \mathbf{V}_{dc} \mathbf{T} \left(-\frac{W_{dc}}{2}, -\frac{H_{dc}}{2} \right) \mathbf{S} \left(\frac{W_{wc}}{W_{dc}}, \frac{H_{wc}}{H_{dc}} \right) \mathbf{T} \left(cx_{wc}, cy_{wc} \right). \tag{10.18}$$

Equation (10.18) says that to transform a point (x_{dc}, y_{dc}) from the device drawing area (DC) to a point (x_{wc}, y_{wc}) in our design space (WC):

$$x_{wc} = ((x_{dc} - \frac{W_{dc}}{2})\frac{W_{wc}}{W_{dc}}) + cx_{wc},$$

$$y_{wc} = ((y_{dc} - \frac{H_{dc}}{2})\frac{H_{wc}}{H_{dc}}) + cy_{wc},$$
(10.19)

where

Device drawing area
$$\left\{ \begin{array}{ll} \text{width} & = W_{dc}, \\ \text{height} & = H_{dc}, \end{array} \right.$$

Inverse transforms.

$$\mathbf{T}^{-1}(t_x, t_y) = \mathbf{T}(-t_x, -t_y)$$

$$\mathbf{S}^{-1}(s_x, s_y) = \mathbf{S}(\frac{1}{s_x}, \frac{1}{s_y})$$

```
void UWB_WindowHandler::DeviceToWorld(int dcx, int dcy, float &wcx, float &wcy) const

:

A: GetDeviceSize(dcW, dcH);  // Get the UI drawing device dimension
    vec3 center = m_WCWindow.GetCenter(); // Center position of the WC Window
    float wcW = m_WCWindow.Width();  // Width of the WC Window
    float wcH = m_WCWindow.Height();  // Height of the WC Window

B: wcx = center.x + ((wcW/dcW) * (dcx - (dcW/2.0f))); // implement Equation 10.17
    wcy = center.y + ((wcH/dcH) * (dcy - (dcH/2.0f))); // by computing Equation 10.19
```

Listing 10.14. The WindowHandler::DeviceToWorld() implementation.

and

WC window =
$$\begin{cases} center = (cx_{wc}, cy_{wc}), \\ width = W_{wc}, \\ height = H_{wc}. \end{cases}$$

Listing 10.14 shows the implementation of Equation (10.19) in the DeviceTo World() transform function of the UWB_WindowHandler class in D3D_UWB_Lib10. At label A, we obtain the dimension of the drawing device and the parameters for the WC window. Label B is a direct implementation of Equation (10.19). In general, we must transform all user mouse-click positions by Equation (10.17) before comparing with points in the WC space.

Tutorials 10.13 and 10.14. Mouse Positions and Inverse Transforms

- **Goal.** Verify mouse positions and the need for device-to-world transformations.
- **Approach.** Extend Tutorial 10.12 to support moving of the small-view WC window by dragging with the left mouse button in the main view.

Figure 10.23 is a screenshot of running Tutorials 10.13 and 10.14. In these tutorials, left mouse button drag in the main view defines the center position for the WC window of the small view. For example, from Figure 10.11 we know that the nose position of the geometric person is (20,30) in WC space. Now, if the user left mouse button clicks at the nose position in the main view, our application will react by moving the center of the small-view WC window to (20,30). If the user left mouse button drags towards the right-eye position, (22,32), our application will track the position by moving the center of the small-view WC window. Notice

Source file.

uwbgl_WindowHandler3.cpp file in the Common Files/ WindowHandler subfolder of the UWBGL_D3D_Lib10 project.

Tutorial 10.13.

Project Name:
D3D_BadMousePan
Library Support:
UWB_MFC_Lib1
UWB_D3D_Lib10

Tutorial 10.14.

Project Name: D3D_MousePan Library Support: UWB_MFC_Lib1 UWB_D3D_Lib10

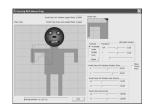


Figure 10.23. Running Tutorial 10.13 and 10.14.

Source file.

DrawAndMouseHandler.cpp
file in the WindowHandler folder of the
D3D_BadMousePan project.

Listing 10.15. The MainHandler mouse event service routines of Tutorial 10.13.

that in our discussion, these are in WC coordinate units, while we know that the mouse positions are in device coordinates. Tutorial 10.13 shows us the results of not transforming points to the WC space and using the mouse positions in the DC space directly. Listing 10.15 shows the mouse button click (OnMouseButton) and mouse move (OnMouseMove) service routines of Tutorial 10.13. In both service routines, at label A we transform the mouse click position from hardware to device coordinate, and then use the DC position to compute the center position for the small-view WC window. For this reason, in Tutorial 10.13, if we click around the top region of the main view, the corresponding DC points will have values around 300, and thus the small-view WC window will be moved to corresponding positions. Because nothing is defined in the 300 range in the WC space, nothing will show up in the small-view drawing area. Recall from Figure 10.10 that the geometric person is defined within the range of [0,40]. This means if we left mouse button click/drag around the lower-left region of the main view, limiting our DC position to within the range of [0,40], we will see the small-view WC window panning around the geometric person. Clearly we must transform the DC positions to WC before computing the WC window position. Tutorial 10.14 extends from Tutorial 10.13 with the simple inclusion of the DeviceToWorld() function

calls. Listing 10.16 shows the mouse event source routines of Tutorial 10.14. As we can see at label A, the DeviceToWorld() function is called (for both mouse button and mouse move event service routines). By running Tutorial 10.14, we can verify that this tutorial functions as expected.

```
// Mouse button services
void CMainHandler::OnMouseButton(bool down, unsigned int nFlags, int hwX, int hwY)
    HardwareToDevice(hwX, hwY, deviceX, deviceY); // hwXY -> deviceXY
A: DeviceToWorld(deviceX, deviceY, wcX, wcY); // deviceXY -> wcXY
    if(nFlags & MK_LBUTTON) // Left Mouse Button down
        // Compute new center of small view WC Window based on wcXY
        ComputeBoundPosition(wcX, wcY);
// Mouse move services
void CMainHandler::OnMouseMove(unsigned int nFlags, int hwX, int hwY)
    HardwareToDevice(hwX, hwY, deviceX, deviceY); // hwXY -> deviceXY
    DeviceToWorld(deviceX, deviceY, wcX, wcY); // deviceXY -> wcXY
A:
    if(nFlags & MK_LBUTTON) // Left Mouse Button down
        // Compute new center of small view WC Window based on wcXY
        ComputeBoundPosition(wcX, wcY);
```

Listing 10.16. The MainHandler mouse event service routines of Tutorial 10.14.

Source file.

DrawAndMouseHandler.cpp file in the WindowHandler folder of the D3D_MousePan project.