Evaluating Policy Interventions with General Equilibrium Externalities*

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Abstract

This paper describes a method for quantifying the effects of nonseparable externalities (or public goods) on public policies with important general equilibrium consequences. We calibrate a general equilibrium model with nonseparable air quality benefits in order to measure the excess burden and total net benefits of a new energy tax in the 1995 U.S. economy. With and without the nonseparable air quality responses, our excess burden estimates for a new 5% energy tax (with a pre-existing 40% income tax) differ from 6-27% when air quality is assumed to be twice as substitutable with leisure as the average good. We find similar, but larger differences (11-37%) when the goods are assumed to be strong complements. The magnitude of these effects and the asymmetry across the substitution patterns are due in large part to the character of the general equilibrium feedbacks that the externality produces.

Keywords: excess burden, externalities, general equilibrium.
JEL Classification Codes: H21, H23, Q51

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1 Introduction

This paper describes a method for assessing the general equilibrium implications of nonseparable externalities. We evaluate both the excess burden (or gross cost) and the full welfare effects (net benefits) of policies that alter the level of the externality. Our empirical analysis uses a consistent description of general equilibrium that simultaneously characterizes a market economy and the polluting activities that give rise to air quality externalities. Market data, measures of ambient air quality and emission rates, along with estimates of the incremental willingness to pay to reduce air pollution are used to calibrate this model for the U.S. economy.

When air quality and consumption goods are nonseparable elements of consumer preferences or production technologies, a change in the level of air quality can affect market demands. When these market responses also determine air quality, there is a feedback effect. Pollution affects the level and composition of economic activity which, in turn, affects the level of pollution. These two features — nonseparability and feedbacks — define what we refer to as general equilibrium externalities. In this context, the costs and benefits of a new policy depend on: the direct efficiency costs incurred in newly regulated sectors, the impact of these regulations on the costs of pre-existing distortions in other sectors, and the character of the interaction between the level of the externality and market demands.

There is a substantial theoretical literature on the effect of nonseparable externalities and public goods on excess burden in both the public finance tradition (Boadway and Keen 1993, Kaplow 1996) and in the study of second-best environmental taxation (Schwartz and Repetto 2000, Williams 2002, Williams 2003). These studies uniformly agree that the relationship (substitutes or complements) between the externality (or public good) and the labor market may be an important determinant of the excess burden of a new regulation. They also highlight the need to establish the empirical content of these connections. This is one of the principal objectives of this paper.

There has been far less systematic discussion of the full equilibrium implications of feedbacks between economic activity and external effects. While the basic mechanism is present in several previous studies, authors have tended to focus on the effect of a change in the level of externality on labor supply in welfare calculations without linking the equilibrium level of the externality back to equilibrium consumption and leisure choices in a way that provides any general insight into how these feedback mechanisms function.

The analytical section of the paper expands on previous theoretical work by drawing on the theory of individual markets which exhibit reciprocal externalities (Diamond and Mirrlees 1973, Sandmo 1980, Cornes 1980).[1] This formulation allows us to describe the general equilibrium consequences of feedbacks in terms of compensated demand responses and income effects — quantities for which theory and practice can provide intuition on the magnitude and sign of each component. Furthermore, our numerical simulations are organized to parallel this analytical framework — we solve the model with sets of restrictions that map directly back to the partial effects identified in the analytical discussion.

The specific numerical model that we employ is based on one proposed by Goulder and Williams (2003) to study the excess burden of new energy taxes — we design our amendments to their model so that the analysis replicates the Goulder and Williams results when we make air quality separable. The basic template on which the Goulder and Williams model is built also underlies a number of studies of second-best environmental taxation by Goulder, Williams and co-authors (Bovenberg and Goulder 1997, Goulder, Parry and Burtraw 1997, Goulder, Parry, Williams and Burtraw 1999).[2] It has the virtue of being transparent and well-established, allowing our results to be compared with these benchmarks.

Briefly, we find that, with and without the nonseparable air quality responses, our excess burden estimates for a new 5% energy tax (with a pre-existing 40% income tax) differ from 6-27% when air quality is assumed to be twice as substitutable with leisure as the average good (as measured by the Allen-Uzawa elasticity of substitution.) The range depends upon the magnitude of the assumed marginal willingness to pay for air quality improvements. We find similar, but larger differences (11-37%) when the goods are assumed to be strong complements. The magnitude of these effects and the asymmetry across the substitution patterns are due in large part to the character of the general equilibrium feedbacks that the externality produces. Feedbacks also affect the size of the realized environmental benefits of the energy tax.

Previous empirical studies on the excess burden of environmental taxation have assumed separable environmental benefits because there is little evidence on the sign or magnitude of aggregate substitution patterns between environmental goods and key market goods. There is also a real methodological challenge associated with consistently calibrating nonseparable models, a fact that likely explains why, to

[1] For the most part, this literature has missed this connection to general equilibrium externalities and focused, instead, on describing the conditions under which "anomalous" quantity responses (i.e. upward sloping compensated demand functions) due to reciprocal externalities are possible.

[2] This research is closely related to an extensive set of analytical work on the modeling of tax interaction effects and the double dividend hypothesis. See Fullerton (1997) for a discussion of the importance of normalization rules in describing the effects of taxes and Fullerton and Metcalf (2001) for an overview of the analytical work especially relevant to the evaluation of environmental policy instruments.

our knowledge, no previous study has attempted sensitivity analysis along these lines.

At the same time, there is ample evidence from micro-level, non-market valuation studies that non-separability is part of the "natural structure" (Gorman 1976) of many public policy decisions. We demonstrate that consistent calibration is feasible, and that the general equilibrium implications of these types of external effects can be substantial when the model is calibrated to plausible values for the benefits of air quality improvements.

Section 2 provides a brief review of the logic of general equilibrium measures of welfare change associated with a new tax. We also adapt the Cornes (1980) discussion of reciprocal externalities to show how the feedback between the externality and market demands behaves in a simple, two-commodity example. Section 3 describes the numerical model and a strategy for the consistent calibration of CGE models with nonseparable externalities. Section 4 presents our core numerical findings on the effects of the pollution externality on excess burden and net benefit measurements. We also report on sensitivity analyses with respect to the magnitude of the MWTP for pollution reductions used in calibrating the model and a richer specification of nonseparabilities between air quality, leisure and consumption goods in the final demand system. In section 5 we step back from our calibrated model and consider two questions. First, is the scale of air quality effects on the national economy plausible? Second, how should we resolve the substitution/complementarity relationship which is important to the magnitude of the effects of non-separability and feedbacks? Section 6 summarizes our conclusions.

2 Externalities and General Equilibrium

2.1 The Model

In a representative agent framework, preferences are described by a function including leisure (l), n consumption goods (C_i, $i = 1, \ldots, n$), and air quality (q):

$$U = U(l, C_1, \ldots, C_n, q) \tag{1}$$

The consumers are constrained by their monetary budget:

$$\sum_i P_i(1 + \tau_{C_i})C_i + (1 - \tau_L)l = L + g$$

where P_i is the market price of consumer good i, τ_L is an ad valorem labor income tax, L is labor supplied to the economy and τ_{C_i} are taxes on final consumption. The Harberger (1964) normalization implies that all tax revenue is returned lump sum to consumers:

$$g = \tau_L L + \sum_i (P_i \tau_{C_i} C_i + \sum_j P_j \tau_{I_{ij}} I_{ij})$$

where I_{ij} is the intermediate demand for good j in sector i, and $\tau_{I_{ij}}$ is the associated input tax. The wage is the numeraire price which is normalized to unity. Consumers are also time constrained:

$$l + L = T$$

where T is the consumer total time endowment which the representative agent splits between leisure consumption and labor supply. Goods are produced using other goods and labor as inputs:

$$Y_i = f(I_{iL}, I_{i1}, \ldots, I_{in})$$

where I_{iL} is demand for labor in sector i ($\sum_i I_{iL} = L$). Air quality is a byproduct of production:

$$q = h(Y_1, \ldots, Y_n), \ \frac{\partial h}{\partial Y_i} \leq 0$$

2.2 Equilibrium

Assuming an interior solution, equilibrium requires that prices clear all markets:

$$Y_i - C_i - \sum_j I_{ij} = 0 \tag{2}$$

All profit-maximizing firms operate at zero profits:

$$(1 + \tau_L) - P_i \frac{\partial Y_i}{\partial I_{iL}} = 0 \tag{3}$$

$$P_j(1 + \tau_{I_{ij}}) - P_i \frac{\partial Y_i}{\partial I_{ij}} = 0 \tag{4}$$

Air quality reflects equilibrium output levels:

$$q - h(Y_1, \ldots, Y_n) = 0 \tag{5}$$

And all consumers (represented by our single agent) maximize utility subject to their budget constraints and taking the prevailing air quality conditions as fixed:

$$\begin{aligned} \frac{\partial U(l, C_1, \ldots, C_n; q)}{\partial C_i} - \lambda P_i(1 + \tau_{C_i}) &= 0 & (6) \\ \frac{\partial U(l, C_1, \ldots, C_n; q)}{\partial l} - \lambda(1 - \tau_L) &= 0 & (7) \\ \sum_i P_i(1 + \tau_{C_i})C_i + (1 - \tau_L)l - (L + g) &= 0 & (8) \end{aligned}$$

where λ is the marginal utility of income.

2.3 Welfare Measurement

The marginal change in U as a result of a new tax on a final consumption good, τ_{C_k} is:[3]

$$\frac{dU}{d\tau_{C_k}} = \frac{\partial U}{\partial C_k}\frac{dC_k}{d\tau_{C_k}} + \sum_{i \neq k}^{M} \frac{\partial U}{\partial C_i}\frac{dC_i}{d\tau_{C_k}} + \frac{\partial U}{\partial l}\frac{dl}{d\tau_{C_k}} + \frac{\partial U}{\partial q}\frac{dq}{d\tau_{C_k}} \tag{9}$$

As Harberger demonstrated, the monetized value of the tax change can be expressed in terms of the marginal tax rates. Using the first-order conditions for the firm and consumer optimization problems along with the consumer's time constraint, we have:

$$\frac{1}{\lambda}\frac{dU}{d\tau_{C_k}} = \tau_{C_k}\frac{dC_k}{d\tau_{C_k}} + \tau_{C_i}\sum_{i \neq k}^{M}\frac{dC_i}{d\tau_{C_k}} + \sum_{ij}^{M}\tau_{I_{ij}}\frac{dI_{ij}}{d\tau_{C_k}} - \tau_L\frac{dl}{d\tau_{C_k}} + \frac{1}{\lambda}\frac{\partial U}{\partial q}\frac{dq}{d\tau_{C_k}} \tag{10}$$

When only the labor tax, τ_L, and the new commodity tax, τ_{C_k}, are present, (10) reduces to:

$$\frac{1}{\lambda}\frac{dU}{d\tau_{C_k}} = \tau_{C_k}\frac{dC_k}{d\tau_{C_k}} - \tau_L\frac{dl}{d\tau_{C_k}} + \frac{1}{\lambda}\frac{\partial U}{\partial q}\frac{dq}{d\tau_{C_k}} \tag{11}$$

The first term on the right-hand side of (11) is the direct effect of the new tax on sector k. The second

[3]The empirical exercise described in section 3 calls for a tax on an intermediate good, energy, because we are describing a policy that affects air quality and because we have chosen to build on the results presented in Goulder and Williams (2003). In contrast, the analytical discussion in this section and section 2.4 focuses on the effect of a new tax on final consumption good, C_k, in the name of economy of notation. The algebra, but not the underlying logic, would change if we were to re-write these expressions for a tax on an intermediate good.

term is the tax interaction effect in the labor market — the extent to which the introduction of τ_{C_k} changes the size of the pre-existing labor market distortion. The final term represents the direct environmental benefits due to the effect of the new tax on air quality.

To illustrate how nonseparability of air quality in the preference function affects the costs and benefits of the new tax, it is useful to decompose the total derivatives in (11) into the partial price, income and air quality changes.

$$\frac{dC_i}{d\tau_{C_k}} = \frac{\partial C_i^c}{\partial \tau_{C_k}} + \frac{\partial C_i}{\partial y}\frac{dy}{d\tau_{C_k}} + \frac{\partial C_i^c}{\partial q}\frac{\partial q^c}{\partial \tau_{C_k}} + \frac{\partial C_i^c}{\partial q}\frac{\partial q}{\partial y}\frac{dy}{d\tau_{C_k}} \tag{12}$$

where the superscript "c" indicates compensated demand responses. $\frac{dy}{d\tau_{C_k}}$ is the marginal change in income associated with the policy which (because tax revenue is returned lump sum to consumers) is equal to the total monetized value of the policy change:

$$\frac{dy}{d\tau_{C_k}} = \frac{1}{\lambda}\frac{dU}{d\tau_{C_k}} \tag{13}$$

Using (12) and (13), and an expression comparable to (12) for leisure demand, we can rewrite (11) as:

$$\frac{1}{\lambda}\frac{dU}{d\tau_{C_k}} = \frac{\tau_{C_k}\frac{\partial C_k^c}{\partial \tau_{C_k}} - \tau_L\frac{\partial l^c}{\partial \tau_{C_k}} + \tau_{C_k}\frac{\partial C_k^c}{\partial q}\frac{\partial q^c}{\partial \tau_{C_k}} - \tau_L\frac{\partial l^c}{\partial q}\frac{\partial q^c}{\partial \tau_{C_k}} + \frac{1}{\lambda}\frac{\partial U}{\partial q}\frac{\partial q^c}{\partial \tau_{C_k}}}{1 - \left[\tau_{C_k}\frac{\partial C_k}{\partial y} - \tau_L\frac{\partial l}{\partial y} + \tau_{C_k}\frac{\partial C_k}{\partial q}\frac{\partial q}{\partial y} - \tau_L\frac{\partial l}{\partial q}\frac{\partial q}{\partial y} + \frac{1}{\lambda}\frac{\partial U}{\partial q}\frac{\partial q}{\partial y}\right]} \tag{14}$$

Equation (14) demonstrates that there are both substitution and income effects — direct and indirect — associated with the nonseparability of air quality in our model.

Restricting our attention to the efficiency costs of the new tax, we can write the full excess burden of the policy by dropping the final numerator term from (14).

$$EB = \frac{\overbrace{\tau_{C_k}\frac{\partial C_k^c}{\partial \tau_{C_k}} - \tau_L\frac{\partial l^c}{\partial \tau_{C_k}}}^{Prices} + \overbrace{\tau_{C_k}\frac{\partial C_k^c}{\partial q}\frac{\partial q^c}{\partial \tau_{C_k}} - \tau_L\frac{\partial l^c}{\partial q}\frac{\partial q^c}{\partial \tau_{C_k}}}^{Air\ Quality}}{1 - \left[\tau_{C_k}\frac{\partial C_k}{\partial y} - \tau_L\frac{\partial l}{\partial y} + \tau_{C_k}\frac{\partial C_k}{\partial q}\frac{\partial q}{\partial y} - \tau_L\frac{\partial l}{\partial q}\frac{\partial q}{\partial y} + \frac{1}{\lambda}\frac{\partial U}{\partial q}\frac{\partial q}{\partial y}\right]}d\tau_{C_k} \tag{15}$$

The first two terms in the numerator ($Prices$) of (15) are the substitution effects induced by the price change caused by the tax. If leisure is an average substitute for good k (as most studies of second-best environmental taxation assume), then these price effects increase the cost of the tax.

The latter two terms ($Air\ Quality$) are the substitution effects caused by the change in air quality. The sign of these terms will naturally depend on whether the taxed good and leisure are substitutes or complements for air quality. If air quality is a complement to good k, this relationship will tend to

decrease the cost of the policy, ceteris paribus. On the other hand, if leisure is an air quality complement (better air quality stimulates more trips to the park, for example), this will tend to increase the overall cost of the tax by further exacerbating the under-supply of labor implied by the pre-existing labor tax.

The denominator of (15) also contains both direct income effects from the tax and indirect effects of income on demands through the implied change in air quality. Air quality will, in general, fall with an increase in income ($\frac{\partial q}{\partial y} < 0$) as demand for consumer goods which produce pollution increases. In this model the representative consumer has no mechanism to directly select q. Thus, arguments that the demand for environmental quality increase with income are consistent with our preference specification. The model structure does not offer the consumer an income to act on these demands. Hence, the air-quality-induced income effects will tend to reinforce their substitution counterparts — to diminish the excess burden of the new tax if good k is an air quality complement or if leisure is an air quality substitute.

In the numerical analysis that follows we report measures for discrete energy tax changes of both the full welfare consequences (the line integral associated with (14)) as well as the excess burden (the integral of (15)). Among the possible measures of excess burden, we also distinguish between the general equilibrium excess burden (based on (15)), a general equilibrium measure that ignores the impact of the air quality change based on:

$$EB^G = \frac{\tau_{C_k} \frac{\partial C_k^c}{\partial \tau_{C_k}} - \tau_L \frac{\partial l^c}{\partial \tau_{C_k}}}{1 - \left[\tau_{C_k} \frac{\partial C_k}{\partial y} - \tau_L \frac{\partial l}{\partial y}\right]} \tag{16}$$

and a partial equilibrium measure based on:

$$EB^P = \frac{\tau_{C_k} \frac{\partial C_k^c}{\partial \tau_{C_k}}}{1 - \tau_{C_k} \frac{\partial C_k}{\partial y}} \tag{17}$$

2.4 Reciprocal Externalities and Feedback Effects

What remains is to describe the general equilibrium change in the level of the air quality amenity, $\frac{dq}{d\tau_{C_k}}$. As we noted in the previous section, air quality arises from the production of pollution-intensive goods:

$$q = h(Y_1, \ldots, Y_n)$$

That is, consumption choices lead to changes in the level of q but these responses are not reflected in the first-order conditions underlying the representative agent's demands. Because q is a nonseparable element of the demand system, changes in its level affect market demands which, in turn, feed back to

further influence air quality.

We use the Cornes (1980) logic to illustrate the equilibrium implications of this mechanism. To keep the discussion tractable, we work with a simplified, two-commodity example. Our numerical model allows us to consider more general patterns of interaction.

Consider two consumption goods C_j and C_k. As before, the policy in question involves a tax on C_k. The total effect of a change in τ_{C_k} is given in equation (18).

$$\frac{dC_k}{d\tau_{C_k}} = \frac{\partial C_k}{\partial \tau_{C_k}} + \frac{\partial C_k}{\partial q}\left(\frac{dq_k}{d\tau_{C_k}} + \frac{dq_j}{d\tau_{C_k}}\right) \tag{18}$$

The first term on the right side of (18) represents the direct price response and the second term represents the indirect response in the amenity level induced by the pollution generated by consumption changes of C_k and C_j as a result of the tax change. A similar expression can be derived for C_j as in (19).

$$\frac{dC_j}{d\tau_{C_k}} = \frac{\partial C_j}{\partial \tau_{C_k}} + \frac{\partial C_j}{\partial q}\left(\frac{dq_k}{d\tau_{C_k}} + \frac{dq_j}{d\tau_{C_k}}\right) \tag{19}$$

Assuming a one-to-one response of q to changes in C_k and C_j we can substitute using $\frac{dC_k}{d\tau_{C_k}} = -\frac{dq_k}{d\tau_{C_k}}$ and $\frac{dC_j}{d\tau_{C_k}} = -\frac{dq_j}{d\tau_{C_k}}$. These relationships imply that equation (19) can be expressed in terms of the price effect and the effect of air quality in terms of its underlying connection to demand:

$$\frac{dC_j}{d\tau_{C_k}} = \frac{\frac{\partial C_j}{\partial \tau_{C_k}} - \frac{\partial C_j}{\partial q}\left(\frac{dC_k}{d\tau_{C_k}}\right)}{1 + \frac{\partial C_j}{\partial q}} \tag{20}$$

Substituting (20) into (18) and re-arranging terms we have an illustration of the cascading effects of the externality in equation (21).

$$\frac{dC_k}{d\tau_{C_k}} = \frac{\frac{\partial C_k}{\partial \tau_{C_k}} - \frac{\partial C_k}{\partial q}\frac{\partial C_j}{\partial \tau_{C_k}} / \left(1 + \frac{\partial C_j}{\partial q}\right)}{1 + \frac{\partial C_k}{\partial q} - \frac{\partial C_k}{\partial q}\frac{\partial C_j}{\partial q} / \left(1 + \frac{\partial C_j}{\partial q}\right)} \tag{21}$$

Expression (21) describes the total effect of a change in the price of good k (through the new tax, τ_{C_k}) on the demand for commodity k in terms of: a traditional price (tax) response (first numerator term), a response to the change in the level of the externality due to traditional price responses in other markets (second numerator term), and the implications of the feedback mechanism (parenthesized term in the numerator and all denominator terms) for both market k and the other externality-generating market,

j.[4]

In the absence of strong income effects, we know that first term in the numerator of (21) must be negative. The character of the externality effects (second numerator and all denominator terms collectively) will depend on whether the two market goods and the air quality amenity are substitutes or complements.

The feedback effects may either reinforce (when the denominator and in the parenthesized term in the numerator approach zero) or mitigate (when these terms grow large) the partial equilibrium price and externality responses in the numerator of (21).

In the context of our discussion of excess burden in (15), the cases where we would like to sign the terms in (21) are the taxed good itself (C_k) and leisure. If we simplify the problem so that only sector k is a source of pollution, (21) reduces to:

$$\frac{dC_k}{d\tau_{C_k}} = \frac{\partial C_k}{\partial \tau_{C_k}} / \left(1 + \frac{\partial C_k}{\partial q}\right) \tag{22}$$

If C_k is a substitute to q, $\frac{\partial C_k}{\partial q} < 0$ and the feedback effect will tend to reinforce the negative effect of the tax on demand. This will act to increase the distortionary cost of the tax in this market. The opposite holds true if C_k is a air quality complement. Note, however, that there is a positive cost associated with the tax in this market regardless of the sign of these relationships.

Leisure is affected by the externality generated in sector k, but is not a source of pollution itself. The expression for leisure corresponding to (21) therefore is:

$$\frac{dl}{d\tau_{C_k}} = \frac{\partial l}{\partial \tau_{C_k}} - \frac{\partial l}{\partial q}\frac{\partial C_k}{\partial \tau_{C_k}} / \left(1 + \frac{\partial C_k}{\partial q}\right) \tag{23}$$

If leisure is a substitute for C_k (as we assume throughout the numerical analysis), the partial price response (first term) is positive. If leisure is an air quality substitute, the second term in (23) will be negative and the externality response will offset the positive price response. This will reduce the excess burden due to the labor market distortion. Likewise, costs will be enhanced by the externality effect when leisure and air quality are complements. The magnitude of the externality effects will, again, depend on whether the feedback on the pollution sector is reinforcing or offsetting.

Table 1 summarizes these results, ranking the different feedback regimes from most to least costly

[4]The existence of stable equilibria in a model with feedbacks is a concern. In the present example, stability requires that $-1 < \frac{\partial C_j}{\partial q} < 1$. See Cornes (1980) for a detailed discussion of stability in markets with reciprocal externalities.

for each of the two markets discussed. The ranking assumes constant absolute magnitudes for all of the partial effects.

Table 1: Excess Burden Ranking by Feedback Regime

	Sign of Partial Effects			
	$\frac{\partial C_k}{\partial \tau_{C_k}}$	$\frac{\partial C_k}{\partial q}$	$\frac{\partial l}{\partial \tau_{C_k}}$	$\frac{\partial l}{\partial q}$
	Market k Costs			
	$-$	$-$		
$EB \Uparrow$	$-$	$+$		
	Labor Market Costs			
	$-$	$-$	$+$	$+$
	$-$	$+$	$+$	$+$
	$-$	$+$	$+$	$-$
$EB \Uparrow$	$-$	$-$	$+$	$-$

To evaluate the quantitative importance of these effects in a numerical model we selectively control the differential effects of the interactions in ways that directly parallel our analytical expressions. We compare scenarios in which air quality is a nonseparable element of the demand system (hence $\frac{\partial C_k}{\partial q} \neq 0$ and $\frac{dC_k}{d\tau_{C_k}}$ is as in (21) or (22), for example) with scenarios in which air quality is separable ($\frac{\partial C_k}{\partial q} = 0$ and $\frac{dC_k}{d\tau_{C_k}} = \frac{\partial C_k}{\partial \tau_{C_k}}$) and there are no feedback effects. An important advantage of the numerical analysis is that it allows us to assess the importance of feedback responses to non-marginal policy changes. Even in our simple two commodity example, (21) suggests that large policy changes could result in significantly nonlinear behavior which is outside the scope of the marginal analysis typical of theoretical contributions to this literature.

2.5 Literature

Several authors have noted the potential effects of nonseparabilities GE measures of the benefits of policy action or in other related policy contexts, dating to Deaton's (1981) early discussion of the design of optimal tax structures.[5] What has been missing is a framework to calibrate consistently numerical CGE

[5]He concluded his analysis of the issues in design of optimal taxes observing that "although separability between goods and leisure in all cases simplifies the tax rule, in some cases reducing it to a prescription for uniform taxes, the structure of taxes, whether uniform, repressive or progressive, depends crucially on exactly how separability is formulated" (p. 1259)

Parry (1995) also discusses the role of nonseparability for the welfare losses due to the introduction of pollution taxes with pre-existing labor taxes. His analytical treatment cannot deal with the general equilibrium effects and assumes substitution relationships between the pollution good and leisure are more likely than complementarity.

models using the available estimates of the incremental willingness to pay for non-market environmental amenities.

Especially notable in this early work are the papers by Boadway and Keen (1993) and Williams's analysis of nonseparabilities as sources for health (2003) and productivity (2002) effects. Boadway and Keen's primary focus is on how preference heterogeneity and self selection affect the relationship between public good provision rules and optimal taxation. Their results provide a conceptual foundation for Kaplow's (1996) demonstration that income tax structures can be modified to finance public goods in a way that avoids the distortionary effects of taxation. His framework used the Boadway-Keen insight and adjusts income taxes at each income level to offset (or compensate) for the benefits from the public good increments financed by a change to the tax system.

In our context what is important about Boadway and Keen's analysis is their discussion of the role of separabilities. They consider several special cases noting that public good provision rules (and thus measures of the excess burden or benefits from intervention) would be affected by the structure of non-separabilities. For example, if public goods are separable but leisure and market goods are not and preferences are identical, then the substitution or complementarity relationships between market goods and leisure affect whether one over (or under) provides public goods with optimal income taxes relative to the conventional Samuelson rule. While informative, their analyses do not offer direct insights for the nonseparable case. Their analyses are comparative in that they consider the effects of heterogeneity and self selection with optimal income taxes relative to the Samuelson rule. They are unable to draw conclusions about the relative levels of provision of public goods and thus the quantitative extent of the welfare gains or losses even in the special cases they consider.

Several papers have considered the implications of specific nonseparabilities in the context of analyzing tax interactions. In Schwartz and Repetto (2000) for example, the authors describe environmental quality as a nonseparable argument in preferences and suggest that the effects of environmental services could magnify or reduce the tax interaction effect depending on the relationship between these services and leisure. Their discussion misses the distinction in the nature of feedback effects for consumption goods versus leisure as illustrated in our equations (21)-(23). Williams introduces the environment in preferences as a separable argument and assumes that it influences health expenditures and labor productivity in his 2002 and 2003 studies. The former is treated as an exogenous change in income and the latter is introduced as a perfect substitute for leisure. As Espinosa and Smith (1995) demonstrated, introducing environmental services as a perfect substitute for a market good will simplify the calibration

of CGE models because the willingness to pay for reductions in pollution will exactly equal the savings in expenditure on the substitute. Thus, all of these proposals introduce specific simplifications to the potential cascading effects of feedbacks illustrated by our equation (21). Judging the importance of these interactions requires a general formulation of a numerical model that allows us to compare alternative restricted cases. To date, a general strategy for model calibration with nonseparable externalities has precluded this type of analysis. We address this issue below.

3 Quantitative Assessment

Our numerical model is designed to permit direct comparison with the Goulder and Williams (GW) assessment of the excess burden of energy taxes in the 1995 U.S. economy. The model describes preferences as a nested, constant elasticity of substitution (NCES) function with leisure in one nest and all consumption goods in another. Leisure and consumption goods are linked as substitutes or complements to air quality in two different ways through the representative agent's preference function — one in which leisure and air quality are nonseparable, and another in which leisure, air quality, and one of the consumption goods all combine nonseparably. Equation (24) describes the first of these specifications.

$$\begin{aligned} U(l, C_1, ..., C_n, q) &= \left[\alpha\left(\gamma\left(\frac{l}{l_0}\right)^{\xi} + (1-\gamma)\left(\frac{q}{q_0}\right)^{\xi}\right)^{\rho/\xi}\right. \\ &\quad \left. + (1-\alpha)\left(\sum_{i=1}^{n}\delta_i\left(\frac{C_i}{C_{0i}}\right)^{\psi}\right)^{\rho/\psi}\right]^{1/\rho} \end{aligned} \tag{24}$$

$(1-\alpha)$ can be interpreted as the value share of the $1, \ldots, n$ consumption goods in full income in equilibrium. l_0 and C_{0i} are benchmark levels of the choice variables, and δ_i are the benchmark value shares of consumption goods within the consumption nest. ψ and ρ are substitution elasticity parameters for the consumption nest and the overall utility function, respectively. q and q_0 refer to current and benchmark air quality levels, γ is the benchmark value share of leisure in the new leisure-air quality nest, and ξ is the elasticity parameter for the new nest.

The calibrated values of both α and γ determine the value of air quality in the benchmark equilibrium. For example, because γ can be interpreted as a value share of leisure within the nest, it is defined as:

$$\gamma = \frac{(1-\tau_L)l_0}{(1-\tau_L)l_0 + MWTP_0 q_0}$$

where $MWTP_0$ represents one of the estimates of marginal willingness to pay for air quality we develop in section 3.1.

Within the structure described in (24), both leisure and air quality are weakly separable from consumption goods — a change in the level of either of these commodities does not affect the marginal rate of substitution between the goods in the consumption good nest. In the numerical simulations we vary the parameter ξ, which establishes the elasticity of substitution between leisure and air quality given the other calibration points in the model (described in detail in section 3.1.)

The second preference specification selects one consumption good — consumer services (CSV) — to enter the leisure bundle as part of a subnest with air quality. In this specification, the expression $(1-\gamma)\left(\frac{q}{q_0}\right)^{\xi}$ from (24) is replaced by a third level sub-function

$$(1-\gamma)\left(\nu\left(\frac{q}{q_0}\right)^{\kappa}+(1-\nu)\left(\frac{C_{CSV}}{C_{CSV0}}\right)^{\kappa}\right)^{\xi/\kappa}$$

and the consumption good bundle no longer includes consumer services

$$\left(\sum_{i=1,\neq CSV}^{n}\delta_i\left(\frac{C_i}{C_{0i}}\right)^{\psi}\right)^{\rho/\psi}$$

Air quality is inversely proportional to the aggregate level of pollution produced by the various sectors of the economy:

$$q=\left(\sum_i \beta_i Y_i\right)^{-1} \tag{25}$$

where β_i is the exogenous pollution coefficient for sector i.[6]

Firm production technologies follow a simple CES form:

$$Y_i=\left[\sum_j \theta_{ij}\left(\frac{I_{ij}}{I_{0ij}}\right)^{\zeta}+\theta_{iL}\left(\frac{I_{iL}}{I_{0iL}}\right)^{\zeta}\right]^{1/\zeta}$$

[6]Using a representative agent model to describe an externality requires we treat air quality as quasi-fixed from the consumer's perspective. Partial equilibrium models used to estimate individual willingness to pay for improvements in amenities often assume people can adjust to differential levels of amenities. The hedonic model, for example, relies on households recognizing site specific differences in amenities in adjusting their selections of homes in response to them. In a representative agent model treating pollution as an amenity requires that we ignore the general equilibrium effects associated with households changing locations. This practice parallels the treatement of the lump sum return of tax revenue.

3.1 Calibration

With the exception of the air quality treatment, our calibration of the model conforms to the procedure presented in Goulder and Williams (2003). Specifically,

- We use the same benchmark dataset to calibrate the parameters which describe the benchmark demand for final goods ($\alpha,\ \gamma,\ \nu,\ \delta$), intermediate ($\theta_{ij}$), and intermediate labor demands (θ_{iL}) using data on the value of these demands in the 1995 U.S. economy reported in the *Survey of Current Business*. The calibration of α, γ and ν also relies on the air quality MWTP and quantity estimates from Smith and Huang (1995) and the U.S. EPA, respectively.

- Benchmark leisure demand (l_0) and the elasticity of substitution between the leisure-air quality bundle and the consumption good bundle (ρ) are calibrated to imply compensated labor supply elasticity of 0.25 and an uncompensated elasticity of 0.05.

- The substitution parameter in the bundle of consumption goods (ψ) in the utility function is chosen to imply an elasticity of substitution between these goods of 0.85.

- The substitution parameter in all production functions, ζ, is chosen to match an energy demand elasticity of 0.9.

- ξ (and κ) are chosen to imply an elasticity of substitution, σ_{l-q}, between leisure and air quality (and consumer services) that is half (complements), the same as (separable), or twice (substitutes) the rate of substitution between either of these goods and other consumption goods (σ_u). Specifically, $\sigma_{l-q} = \sigma_u/2$, $\sigma_{l-q} = \sigma_u$, or $\sigma_{l-q} = 2\sigma_u$ where $\xi = \kappa = 1 - 1/\sigma_{l-q}$ and $\rho = 1 - 1/\sigma_u$.

The full details of the calibration are reported in the appendix.

Producing a consistent calibration of CGE demand responses when nonseparable externalities enter the model presents a challenge because changes in externality level are linked to the rest of the final demand vector through changes in the sectoral activity levels in the general equilibrium model.

For example, in the case of the compensated labor supply elasticity we seek to match the model response to the assumed compensated labor supply elasticity (η_L^C) defined in equation (26).

$$\eta_L^C = \frac{\partial L^C}{\partial w}\frac{w_0}{L_0} \tag{26}$$

where L^C represents the consumer's compensated labor supply. Labor supply is a function of price and the amount of amenities in the nonseparable case. The level of amenities, in turn, responds to changes in relative price because the consumption goods affecting those amenities change. This is the feedback mechanism. When there are feedback in the model, the relevant calibration point becomes:

$$\eta_L^C = \left[\frac{\partial L^C}{\partial w} + \frac{\partial L^C}{\partial q}\frac{dq}{dw}\right]\frac{w_0}{L_0} \tag{27}$$

Calibration to match market and non-market (i.e. total emissions and the associated air quality) requires calculating the derivatives in (27) using the general description of the economy to compute the correct air quality responses. To illustrate what is involved, consider a general statement of the optimization problem that must be solved to consistently calibrate a model with nonseparable externalities.

Let $F(x;\tau) = 0$ represent the equilibrium system equations, where $x = (U, C, L, l, Y, I, P, w, q, \lambda, g)$ is a stacked vector of the model variables, and the individual equations that make up $F(.)$ correspond to equations (2)-(8). τ is the vector of benchmark tax rates across all sectors of the economy. To calculate uncompensated price experiments we must produce a version of the model with exogenous tax income:

$$F^U(x^U;\tau, g) = 0$$

where $x^U = (U, C, L, l, Y, I, P, w, q, \lambda)$. In compensated price experiments, we replace the budget balance condition in (8) with a constant utility condition:

$$\bar{U} - U(C, l, q) = 0 \tag{28}$$

where

$$F^C(x;\tau) = 0$$

is defined by equations (2)-(7) and (28).

In this context, the derivatives required to calibrate the elasticities can be calculated using a numerical differencing technique in which tax rate perturbations provide the basis for the marginal price change:

$$\frac{\partial x_k}{\partial \tau_i} \approx \frac{x_k(\tau + \delta^i) - x_k(\tau)}{\delta},$$

where

$$\delta^i = \begin{pmatrix} \delta_1^i \\ \vdots \\ \delta_N^i \end{pmatrix} \qquad \text{with} \quad \delta_j^i = \begin{cases} \delta & i = j \\ 0 & i \neq j. \end{cases}$$

The full set of equations that characterize the calibration problem in the current model are:

$$\begin{aligned} F^U(x^U; \tau, g) &= 0 \\ F^U(x^U; \tau + \delta^i, g) &= 0 \\ F^C(x^C; \tau) &= 0 \\ F^C(x^C; \tau + \delta^i) &= 0 \\ \frac{x_k^U(\tau + \delta^i) - x_k^U(\tau)}{\delta} \frac{P_{0i}}{x_{0k}} &= \eta_{ki}^U \\ \frac{x_k^C(\tau + \delta^i) - x_k^C(\tau)}{\delta} \frac{P_{0i}}{x_{0k}} &= \eta_{ki}^C \end{aligned} \tag{29}$$

We use this technique to calibrate the energy and labor responses in the model as these are the markets that are responsible for generating the excess burden from the new energy tax. In principle, however, all model features which depend upon the realization of equilibrium externality levels could be accommodated by this approach.

3.2 Air Quality and Non-market Value Data

Data on 1995 levels of PM_{10} particulate matter concentrations in the United States are taken from the United States Environmental Protection Agency (1996) and measures of the marginal willingness to pay (MWTP) for reductions in PM_{10} concentrations come from Smith and Huang (1995). Table 3 shows benchmark PM_{10} emission levels and intensities by sector. Sectoral intensities are computed as the level of physical emissions attributable to the sector divided by the sector's value of output. To derive the virtual price of air quality, we convert emissions to ambient concentrations with a simple conversion factor and then to air quality (based on equation (25)).

Using the estimates for PM_{10} emissions by sector, we constructed the emission coefficients per unit of output (measured in dollars). In addition, the aggregate annual PM_{10} emissions in millions of short tons (42) together with the 1995 annual arithmetic mean of PM_{10} (25, in micrograms per cubic meter) were used to convert emissions factors to measures the ambient concentration. We use the EPA rate to convert

estimates of the MWTP to reduce particulate matter (measured as TSP) into the MWTP for PM_{10}.

The process begins with annualized MWTP from Smith and Huang (1995) in 1982-84 dollars and converts it to 1995 dollars:

$$\underbrace{MWTP}_{1982\ \$/TSP} * \underbrace{1.524}_{1995:1982\ CPI} * \underbrace{\frac{1}{.55}}_{PM_{10}/TSP} * \underbrace{\frac{25}{42}}_{conc./mil.\ tons} * \underbrace{10^8}_{households} = \underbrace{AMWTP}_{1995\ \$/mil.\ tons\ PM_{10}}$$

Three sets of estimates are developed: the middle estimate from the Smith and Huang meta analyses of hedonic estimates (i.e. adjusting for the characteristics of the hedonic studies used to develop these measures); the average of the unadjusted hedonic estimates included in the Smith and Huang study; and a "break even" value for the MWTP which implies that air quality benefits just offset the efficiency costs of a 5% energy tax when air quality is assumed to be separable from the rest of the model. The mapping of the values from in Smith and Huang to the implied aggregate MWTP measures in our study are shown in table 2.

Table 2: Aggregate MWTP Estimates for PM_{10} Reductions

	Hedonic Estimates	Implied AMWTP	\$ AQ as % of GDP
Middle Estimate	\$5.01	\$8.26x10^8	0.79
Average Estimate	\$9.76	\$16.10x10^8	1.53
Break-Even Policy	\$18.08	\$29.83x10^8	2.80

NOTE — Hedonic Estimates — Middle and Average Estimates from Smith and Huang (1995) meta-analysis, 1982-84 \$ per TSP converted to annualized term using a 30 year mortgage at 8% interest; Implied AMWTP — sum of the MWTP for all households (N) in the economy, 1995 dollars per million short tons PM_{10}; Break-Even Policy — the implied 1982-84 \$ per TSP from our numerical model

At the benchmark equilibrium with the base case estimate for the MWTP for air quality, the virtual expenditures for air quality are 1.5% of virtual GDP (i.e. market income plus the expenditures required to purchase air quality at the exogenously specified virtual price).

Based on the EPA emission factors, the transportation sector is responsible almost seventy percent of total PM_{10} emissions, followed by agriculture at just over twenty percent. Natural sources make up another five percent, while utilities, energy production, and manufacturing together account for the remaining five percent. The intensity values in table 3 correspond to the calibration of the β terms in

Table 3: PM_{10} Benchmark Emission Levels and Intensities by Sector

	LEVEL OF EMISSIONS	% TOTAL EMISSIONS	INTENSITY
TRANSPORTATION	29.5	69.2	20.7
AGRICULTURE	9.1	21.4	1.7
MANUFACTURING	1.0	2.2	0.0
ENERGY	0.7	1.5	0.1
UTILITIES	0.3	0.6	0.2
NATURAL	2.2	5.1	

NOTE — LEVEL — Millions of Short Tons PM_{10}; INTENSITY — Short Tons PM_{10} per 1995 U.S. $ of Output x 10^5.

(25). Transportation is by far the most PM_{10}-intensive sector, with a $100,000 increase in the annual value of this sector leading to a 20.7 short-ton increase in PM_{10}. Agriculture is roughly one tenth as intensive as transportation, and the remaining industries are less than a tenth as intensive as agriculture.

Table 4: Final Consumption by % of Income and % Energy Inputs

	% INCOME	% ENERGY DEMAND
CONSUMER MANUFACTURES	34.2	0.4
CONSUMER SERVICES	20.7	0.0
LEISURE	20.0	0.0
FOOD AND ALCOHOL	18.7	0.0
TRANSPORTATION	3.2	35.3
UTILITIES	3.2	39.6

NOTE — % INCOME — exclusive of virtual value of air quality; % ENERGY — value of energy inputs to row sector as a percentage of total energy demand.

Table 4 shows the shares of the final consumption goods in total income (including the value of leisure) and the value of energy used in the production of each of these final consumption goods as a percentage of the total value of energy in the economy. Both measures provide a gauge of the magnitude of the effects that a new energy tax would have in the economy represented by the numerical model.[7]

[7]The energy figures in table 4 include only the value of energy that is a *direct* input to the production process. The effect of the tax will also depend on the indirect value of energy through the use of intermediate goods in production that are themselves made with energy.

4 Simulation Results

4.1 Excess Burden

Table 5 describes the implications of feedback effects for the excess burden produced by 2.5%, 5% and 10% energy taxes. These results are based on the specification of the representative agent's utility function in (24) and a benchmark value of air quality which is calibrated using the average estimate of MWTP from the Smith-Huang hedonic meta-analysis.

Table 5: Excess Burden Effects of AQ Feedbacks, MWTP=$9.76

Tax Level	Substitution Pattern	EB	EB^G	EB^P
2.5%	*Complement*	1.49	1.21	0.14
2.5%	*Separable*	1.21	1.21	0.14
2.5%	*Substitute*	1.06	1.21	0.14
5.0%	*Complement*	3.16	2.63	0.53
5.0%	*Separable*	2.63	2.63	0.53
5.0%	*Substitute*	2.33	2.63	0.53
10.0%	*Complement*	7.01	5.98	1.98
10.0%	*Separable*	5.98	5.98	1.98
10.0%	*Substitute*	5.39	5.98	1.98

NOTE — EB — gross economic cost (billions 1995 US $); EB^P — PE approximation (billions 1995 US $); EB^G — GE approximation which ignores AQ responses (billions 1995 US $).

For each level of the energy tax applied, the table describes model calculations under the three different air-quality separability assumptions described in section 3.1 — where leisure and air quality are complements, separable, or substitutes, relative to the degree of substitution between leisure and other goods in final consumption. These scenarios are indicated in the second column of the table.

The final three columns of the table describe the discrete-change analogs of the incremental measures of excess burden defined in section 2.3. EB is the full, general equilibrium cost of the energy tax, inclusive of the substitution patterns implied by the air quality feedbacks. EB^G is the general equilibrium cost ignoring the effects of air quality. In other words, it captures changes in distortions across all markets of the economy, but takes into account only those effects caused by the price changes implied by the energy tax. Finally, EB^P is a simple, partial equilibrium measure of excess burden (analogous to the

Harberger triangle approximation). It captures only the price effect of the energy tax and only as it influences the degree of distortion in the energy market itself.

For the 5% tax, the full excess burden of the tax is $2.63 billion in 1995 dollars when air quality is separable from other demands and, as a consequence, it has no influence on market demands. Because there are no feedbacks, there is no difference between the full excess burden and EB^G, the cost measure which ignores the effects of equilibrium changes in air quality. Both of the general equilibrium cost measures are substantially larger than EB^P. The difference between these measures reflects the impact of the tax interaction effect in the labor market. In fact, the most of the efficiency cost of the policy — $ 2.1 billion — is due to the fact that it exacerbates the pre-existing labor market distortion. These scenarios — where air quality is separable — provide our link to the existing literature; they replicate the energy tax simulations found in Goulder and Williams (2003).

When leisure and air quality are strong complements, the full excess burden of the policy increases to $3.16 billion for the 5% tax — a $530 million increase. The energy tax leads to an improvement in air quality. Because air quality and leisure are complements, air quality improvements lead to increased demand for leisure, which forces labor supply to contract more strongly than it otherwise would. Similarly, when leisure and air quality are substitutes, excess burden is smaller ($2.33 billion) than in the case where the externality is a separable element of the model.

In all of our simulations, the magnitude of the air quality effect is smaller in the substitutes case than in the complements case. Following the discussion in section 2.4, this is because the feedback effect is reinforcing in the case of complements and offsetting in the case of substitutes. For example, when leisure and air quality are complements, the air quality improvement leads to increased demand for leisure. This causes labor supply to contract. But labor is a primary factor (the only primary factor in our empirical model) used in the production of pollution-generating goods. Hence, the labor supply decrease leads to further air quality improvements. This process continues until the system reaches a fixed point.

The pattern of costs we have just described holds at all of the tax levels that we simulated. Naturally, the excess burden of the energy tax increases at higher tax rates. The effects of nonseparable air quality do as well. However, whereas the total cost of the policy more than doubles with a doubling of the tax rate, the effects of nonseparability are slightly less than double for the same tax rate increase.

4.2 Net Benefits

Table 6 reports on the full welfare implications for the same energy tax simulations. The column labeled EB is, as before, defined as the full costs of the policy. EV reports equivalent variation for the policy, so it includes both the efficiency costs covered by EB as well as the value of the air quality improvements. Just as equation (14) defines the full welfare implications for a marginal change in the tax, EV describes the analogous measure for a non-marginal change (i.e. the line integral of (14)). The difference between EV and EB, Δ, is the direct air quality benefits of the policy (analogous to the final term in (11)). In this case, it is for a non-marginal policy change.

Table 6: Net Benefit Effects of AQ Feedbacks, MWTP=$9.76

Tax Level	Substitution Pattern	EV	EB	Δ
2.5%	*Complement*	0.76	1.49	0.73
2.5%	*Separable*	0.49	1.21	0.72
2.5%	*Substitute*	0.34	1.06	0.72
5.0%	*Complement*	1.73	3.16	1.44
5.0%	*Separable*	1.20	2.63	1.43
5.0%	*Substitute*	0.90	2.33	1.42
10.0%	*Complement*	4.17	7.01	2.84
10.0%	*Separable*	3.12	5.98	2.85
10.0%	*Substitute*	2.54	5.39	2.85

NOTE — EV — equivalent variation measure of total welfare effect of the tax (billions 1995 US $); EB — gross economic cost (billions 1995 US $).

The efficiency costs of the energy taxes *exceed* the air quality benefits in all of the scenarios presented in table 6. Based on this comparison, one might be tempted to conclude that a policymaker should reject all of these policies based on a benefit-cost criterion. It is important to recognize, however, that the purpose of these experiments is to illustrate a framework for studying the effects of nonseparable externalities on the introduction of a new tax. The tax was not intended as a policy to reduce pollution. It should also be acknowledged that our empirical model is based on a single measure of the benefits of air quality improvements. In particular, the benefits measure focuses on the the value of PM_{10} reductions that are capitalized in housing values. It does not consider whether other pollutants that would be reduced by an energy tax. By some accounts it might be regarded as an incomplete measure of the

benefits of reducing PM_{10}.[8]

Considering, again, the 5% tax scenarios, we see that EV is larger when air quality and leisure are complements ($1.73 billion) than when air quality is separable ($1.20 billion) and smaller in the substitutes case. These results largely reflect the impact of the substitution pattern on the excess burden of the policy that we discussed in the previous section. At the 5% tax level, the value of the air quality benefits (Δ) is $1.43 billion.

The benefits of the policy increase almost linearly with the increases in the tax rate, with the value approximately doubling for a doubling of the tax rate. In these simulations, therefore, the feedback mechanism operates almost entirely on the excess burden of the policy through the relationship between air quality and the labor market.

4.3 Sensitivity Analysis

Table 7 considers the impact of the 5% energy tax, but varies the calibration of the model in two other dimensions — the benchmark MWTP for air quality and the preference structure which describes final demand. The first column of the table indicates the value of the air quality amenity that is assumed in the benchmark equilibrium for each simulation as a percentage of GDP. The final five columns repeat the measure of costs and benefits present in tables 5 and 6 for these alternative simulations. The top half of the table maintains the preference structure used throughout the rest of the analysis, while the bottom half considers the possibility of nonseparability between leisure, air quality and consumer services. In this case, leisure and air quality are no longer average substitutes for services.

In the top half of table 7, the simulations at the 1.53% calibration value for the aggregate measure of the shadow value of air quality correspond to the 5% energy tax results presented in the previous two sections. When the benchmark value of air quality is smaller (0.79%), the influence of the nonseparability is also smaller so that excess burden increases less with complementarity and is diminished less by substitution. These effects seem to diminish at a diminishing rate, however, reflecting the importance of considering how non-marginal changes alter conclusions about general equilibrium welfare costs.

Naturally, the benefits of the air quality improvement are also diminished when the model is calibrated to a smaller benchmark MWTP, and in an approximately linear fashion, suggesting once again that the

[8] To keep the model simple we focus exclusively on the benefits of PM_{10}-based reductions, so our analysis does not capture the substantial benefits of coincident reductions in other pollutants (ozone, nitrogen oxide, and sulphur dioxide, for example) that one would attribute to reduced fossil fuel use. We also based the MWTP on hedonic property value studies, so we will also fail to capture the value of the environmental improvements to the extent that they are not capitalized in property values. Finally, the energy taxes are not constructed to represent least-cost abatement policies.

Table 7: EV and EB by MWTP and Preference Structure

$AQ/GDP (%)	Substitution Pattern	EV	EB	Δ	EB^G	EB^P
		Simple l-q Bundle				
0.79%	*Complement*	2.17	2.91	0.74	2.80	0.54
0.79%	*Separable*	1.88	2.62	0.74	2.80	0.53
0.79%	*Substitute*	1.72	2.47	0.74	2.80	0.53
1.53%	*Complement*	1.73	3.16	1.44	2.80	0.54
1.53%	*Separable*	1.20	2.63	1.43	2.80	0.53
1.53%	*Substitute*	0.90	2.33	1.42	2.80	0.53
2.80%	*Complement*	0.87	3.55	2.67	2.80	0.54
2.80%	*Separable*	0.00	2.63	2.63	2.80	0.53
2.80%	*Substitute*	-0.50	2.10	2.60	2.79	0.53
		Extended l-q-CSV Bundle				
0.79%	*Complement*	2.30	3.20	0.90	2.67	0.40
0.79%	*Separable*	1.64	2.51	0.87	2.66	0.39
0.79%	*Substitute*	1.12	1.96	0.84	2.66	0.38
1.53%	*Complement*	1.69	3.29	1.61	2.67	0.40
1.53%	*Separable*	0.98	2.52	1.53	2.66	0.39
1.53%	*Substitute*	0.44	1.92	1.48	2.65	0.38
2.80%	*Complement*	0.60	3.45	2.85	2.68	0.40
2.80%	*Separable*	-0.18	2.52	2.70	2.66	0.39
2.80%	*Substitute*	-0.74	1.84	2.58	2.65	0.38

NOTE — EV — equivalent variation measure of total welfare effect of the tax (billions 1995 US $); EB — gross economic cost (billions 1995 US $); $\Delta = EB - EV$; EB^P — PE approximation (billions 1995 US $); EB^G — GE approximation which ignores AQ responses (billions 1995 US $).

influence of feedback effects on the value of benefits are small for these calibrations.

The final set of simulations in the top half of the table (2.80%) is calibrated such that the 5% energy tax represents a break-even policy when air quality is separable in the model — the net benefits of the new energy tax (EV) are zero and air quality gains just offset efficiency costs. These scenarios are meant to demonstrate the importance of nonseparability for a policy where the costs and benefits are of similar magnitude.

Making leisure and air quality complements in this case increases the excess burden of the policy from $2.63 billion to $3.55 billion — a difference of more than 35%. It reduces costs by roughly 20% in the substitutes case. The value of the benefits due to the air quality effects of the tax is also altered, although

substantially less so, varying by \$70 million (2.7%) across the extremes in specifying the substitution possibilities.

While the costs and benefits of the policies move together across the substitution scenarios, the differences to the costs dominate so that the break-even policy becomes a beneficial policy in the substitutes case and a harmful one in the complements case.

The bottom half of table 7 reports on the effects of introducing nonseparability between leisure, air quality and consumer services (CSV). Along with leisure, services would seem to be the natural place to explore the effects of nonseparability with externalities. When all three are relative substitutes the scenario could be interpreted as one in which the health impacts of improved air quality — services then might represent medical services which are a substitute for a better environment or higher labor productivity.[9] In the complements case, services would represent that services that are combined with the environmental good and leisure to produce welfare from outdoor recreation.

Starting with the separable air quality case at the middle MWTP estimate (1.53%), we see that both EV and EB are somewhat smaller than in the same simulations with the simple preference structure. This is because consumer services are no longer an average substitute for leisure – our separable scenario implies that services are a stronger-than-average substitute for leisure than other consumption goods. The energy tax has the effect of raising demand for services because it is not an energy-intensive good. (This is shown in table 4.) Because services and leisure are relatively strong substitutes, higher demand for services leads to substitution out of leisure, which tends to diminish the size of the pre-existing distortion in the labor market.

When air quality is nonseparable and all three goods are strong complements to each other, the excess burden of the policy goes up by \$770 million to \$3.29 billion. This result stems from the relationship between air quality and services to leisure. Both air quality and services lead to complementary increases in leisure, which exacerbates the labor market distortion. As before, the substitution case mirrors the complements but with smaller magnitude effects.

The effects of the feedback mechanism also lead to the larger variations across the substitution patterns. In particular, the reinforcing feedback onto the air quality improvements in the complements case that we discussed earlier is even stronger as air quality increases lead to subsequent increases in demand for services. The effect is to take inputs away from the production of pollution-intensive production.

[9]These examples parallel the situations considered in Williams' papers (health (2003) and productivity (2002)). However, our analysis does not restrict the effects to be affine transformations of labor time. His analysis does. As a result, our numerical results distinguish pure substitution and income effects of changes in air quality.

When we relax the assumption in the simple preference structure that leisure and all consumption goods are average substitutes, the feedback effects are quite noticeable in their effect on value of the environmental benefits. This is true even in the case where the shadow value of air quality in the benchmark represents only 0.79% of GDP. Air quality benefits are larger in the complements case and smaller in the substitutes case, because the reinforcing feedback leads to general equilibrium increases in air quality above the direct effect of the energy tax on pollution-intensive demands.

While the feedbacks effects are, in general, larger with the extended preference structure, their effect does seem to be somewhat attenuated as the benchmark MWTP for air quality is increased. Once again, this result highlights the importance of a framework that allows consideration of non-marginal policy changes. Considering the excess burden results across the two preference structures, the differences are large for both the 0.79% and 1.53% MWTP calibrations, but relatively similar at the break-even calibration of 2.80%.[10]

5 Discussion

When we construct a full income concept that includes market goods and non-market environmental services, priced at their marginal willingness to pay for current quality levels, it seems reasonable to ask what is a plausible relative size for the "environmental sector"? What is its share of aggregate virtual income (i.e. market and non-market)? We don't know the answer to this question. The widely cited and controversial Costanza, d'Arge, de Groot, Farber, Grasso, Hannon, Limburg, Naeem, O'Neill, Paruelo, Raskin, Sutton and van den Belt (1997) paper estimated that the annual willingness to pay for all ecosystems would be a multiple (greater than two) of aggregate income. Viewed by many economists as an uninformative exercise, this study implicitly posed our question. All evaluations of the economic values of environmental resources have been constructed for benefit-cost analyses of policies that are intended to change the "amounts" of these resources.

At each level of economic activity, environmental resources provide consumers life-support, amenity, and recreational services. Boyd and Banzhaf (2006) have recently advocated describing ecosystem services as "... components of nature directly enjoyed, consumed, or used to yield human well being" (p. 8).[11] Ecosystem services in their framework do not correspond to functions or processes that ecosystems

[10]Note that the "break-even" policy does not, in fact, break even for the separable version of the model with the extended preference structure because of the deviation of consumer services from average substitution with leisure — benefits are higher and costs are lower as a result and the policy is actual a beneficial one in this model.

[11]The term ecosystem generally refers to sets of interrelated resources — a fishery and its habitat, the atmospheric system and

support. They are also not measured by the activities people undertake using them (e.g. recreation). For our purposes what is important in their discussion is that these types of services contribute to human welfare. As a result, when the concept of income is expanded to reflect this contribution, we must acknowledge that for any selected baseline, the measures used for their "amounts" together with what is selected for their marginal values will yield a share of the generalized income concept. We have limited intuition to evaluate this fraction. That is why we reported our implied size of the air quality sector as a fraction of GDP in Table 7. In all cases it is likely too small. Estimates of the economic value of a twenty percent improvement in air quality in Southern California as a share of income (see Sieg, Smith, Banzhaf and Walsh (2004)) are about this size.

Murhpy and Topel (2005) confront a related question when considering a counterfactual that involves the annual value of the longevity improvements in the U.S. due to advances in medical science. Their estimates amount to one half the average GDP over this period. If we performed a comparable exercise for the services of environmental resources, we would "run out" of actual income to make payments defined based on virtual income. This was the Costanza et al. problem.

The issue is not unique to environmental services. The same type of challenge arises in calibrating a nonseparable contribution for leisure in preferences. We don't know what to assume about the endowment of an individual's time that is available for labor and leisure. Two decades ago, Fullerton, Henderson and Shoven (1984) illustrated that even when we use existing estimates of the labor supply elasticity an assumption must be made about this endowment. Moreover, there was a difference of 49% in their estimate of welfare gain for tax reform due to changing the endowment from 70 to 50 hours.

Our second question is related. If we are to use a comparable logic to that of Fullerton et al. (1984) — relying on estimates of the elasticity of marginal value for specific resources as well as measures of the amounts of the amenity service involved we are still left with more assumptions to be made. In this case, a key issue concerns how large should the substitution or complementarity between amenities and leisure be?

Previous theoretical work has tended to define these relationships based on the polar cases — perfect complements or substitutes. Using these assumptions would, naturally, magnify or contract the importance of nonseparability in the sorts of calculations we have presented here. Beyond this common sense response, the question is difficult to answer because estimating these relationships has not been a re-

linked terrestrial environment. For our purpose, ecosystems are sets of environmental resources where these interrelationships are explicitly taken into account. The logic of Boyd-Banzhaf definition applies with equal force to service concepts for individual environmental resources.

search priority. Most of the existing revealed preference estimates of the marginal willingness to pay for environmental amenities are either incapable of identifying preference parameters with this level of resolution (i.e. this is generally the situation for hedonic models) or use maintained preference restrictions to estimate the marginal willingness to pay that generally preclude deriving an answer. That is, a relationship is assumed in order to derive an estimate for the marginal value (i.e. weak complementarity is one example, see Smith and Banzhaf (2004)).

Thus, our analysis suggests that empirical work focusing on the measurement of these elasticities would be especially important to progress in integrating the non-market sector into the national accounts.

6 Implications

Over twenty years ago, Sandmo (1980) concluded his analysis of reciprocal externalities by noting that any judgment on whether feedbacks in this case were mere curiosities or consequential should be based on real applications. It is in this spirit that we have described a technique for welfare measurement in the presence of general equilibrium externalities. Conventional practice in measuring the general equilibrium effects of new taxes on the excess burden (Goulder and Williams 2003) or the welfare costs of policy interventions (Hazilla and Kopp 1990) has argued that the effects of environmental amenities can be treated as separable. Our energy tax scenarios suggest that maintaining this assumption can be important source of error — due to both the direct influence of nonseparability and through the feedback effects that are implied by general equilibrium.

Lastly, applications of our modeling strategy are not limited to environmental problems. Many non-market services should probably be treated as making nonseparable contributions to consumer's preferences or firm's technologies. Moreover, the mechanisms responsible for providing these services often resemble externalities. As we noted, a recent example where these concerns arise is Murhpy and Topel's (2005) evaluation of the aggregate benefits of medical progress in enhancing longevity. These authors note that increased longevity between 1970 and 2000 would be worth (in annual willingness to pay terms) about 3.2 trillion dollars. Their analysis assumes that health makes a separable contribution to a discounted inter-temporal preference function. At this scale of impact, about half of annual GDP over the period, the issues of nonseparability and feedbacks that we have raised for environmental regulations are certainly relevant to policy modeling of these changes of this magnitude.

Appendices

A Elements of the Numerical Model

Table 8 lists the dimensions of the economic model. The model describes a general equilibrium in sectors of the economy and primary factors.

Table 8: Elements of the Model

	Primary Factors
LAB	Labor
	Intermediate Sectors
ENE	Energy
SVC	Services
AGR	Agriculture
MNF	Manufactures
	Final Consumption Sectors
FDA	Food and Alcohol
CSV	Consumer Services
CMN	Consumer Manufactures
TRN	Transportation
UTL	Utilities

Benchmark data on quantities, prices, and elasticities provide the calibration point for the production and utility functions that describe the economy.

Key assumptions and notation:

- The model is identical to that used in Goulder and Williams (2003) except in the form of the utility function. Whenever possible we maintain the same calibration as Goulder and Williams (2003).
- All goods are produced via constant elasticity of substitution (CES) production functions. This implies constant returns to scale technology in all sectors.
- The representative agent's welfare is produced through the consumption of consumer goods, leisure, and air quality, subject to time endowment and income constraints. The utility function is a nested CES function.

Table 9: Intermediate Production Benchmark Values

	Energy	Services	Agriculture	Manufactures
Energy	253,800.3	35,748.4	12,135.2	83,751.8
Services	55,608.3	1,182,177.2	48,378.1	753,981.8
Agriculture	174.6	109,776.9	353,617.4	32,591.6
Manufactures	108,723.6	537,487.8	58,516.9	2,017,510.8
Labor	79,221.2	2,239,303.1	55,472.4	1,143,765.5
TOTAL	497,528.0	4,104,493.4	528,120.0	4,031,601.6

SOURCE – Reproduced from Table B2 in Goulder and Williams (2003).

NOTE – All figures in millions of US 1995 $.

Table 10: Final Consumption Production Benchmark Values

	Food & Alcohol	Consumer Services	Consumer Manufactures	Transport.	Utilities
Energy	297.6	34.6	5,571.4	50,320.6	55,868.1
Services	480,375.7	835,116.3	571,872.7	92,237.5	84,745.9
Agriculture	24,721.9	105.5	7,131.1	0.5	0.5
Manufactures	315,431.3	75,867.5	917,510.0	0.5	553.2
TOTAL	820,826.4	911,123.9	1,502,085.1	142,559.1	141,167.7

SOURCE – Reproduced from Table B3 in Goulder and Williams (2003).

NOTE – All figures in millions of US 1995 $.

Table 11: Model Notation and Parameter Values

	Sets	
C	Final Consumption Goods	{FDA,CSV,CMN,TRN,UTL}
I	Intermediate Goods	{ENE, SVC, AGR, MNF}
	Parameters	
τ_{ene}	Per unit tax rate on energy sector	{0.025, 0.05, 0.1}
τ_L	Ad valorem labor tax rate	0.4
T	Aggregate time endowment	$\sim \eta_L^U = 0.05$, $\eta_L^C = 0.25$
ζ	Substitution between inputs in intermediate and final sectors	$\sim \epsilon_{\text{ENE}} = 0.9$
ρ	Substitution between leisure and other consumption	$\sim \eta_L^U = 0.05$, $\eta_L^C = 0.25$
ψ	Substitution between consumer goods in consumption nest	$\sim \sigma_c = 0.85$
ξ	Substitution between leisure and air quality (Pref. #1)	$\sim \sigma_{l-q} = \frac{1}{2}\sigma_u$, $\sigma_{l-q} = 2\sigma_u$
κ	Substitution between consumer services and air quality (Pref. #2)	$\sim \sigma_{csv} = \sigma_{l-q}$

$\sim$ reads "calibrated to imply".
η_L^U and η_L^C denote the uncompensated and compensated labor supply elasticities, respectively.
ϵ_{ENE} denotes the own-price demand elasticity of energy.
σ_u denotes the substitution elasticity between leisure bundle and consumption bundle in the utility function.
σ_c denotes the substitution elasticity between final consumption goods, C_i in the utility function.
σ_{l-q} denotes the substitution elasticity between leisure and air quality in the utility function.
σ_{csv} denotes the substitution elasticity between consumer services and air quality in the utility function.

B Non-Market Sector

The estimates for air pollution are derived from the U.S. Environmental Protection Agency report *National Air Quality and Emission Trends Report for 1995*. Tables A6 and A7 report estimates for emissions of PM_{10} by sector from 1986 to 1995. We selected the reports for 1995 and aggregated the reported thousands of short tons of particulate matter as indicated in table 12.

Table 12: Definitions for Mapping Emission Rates to GW Sectors

ENERGY	Fuel combustion for electric utilities (including coal, oil, gas, and internal combustion), industrial (including coal, oil, gas, and internal combustion), and residential wood.
MANUFACTURING	Chemical and allied products manufacturing, metal processing, petroleum and related industries, other industrial processes, solvent utilization, storage and transport, and waste disposal and recycling.
TRANSPORTATION	On-road vehicles and non-road sources.
AGRICULTURE	Agriculture and forestry.
NATURAL SOURCES	Combustion — wildfires, managed burning and other; Fugitive dust and wind erosion.

C Preferences and Technologies

The following figures give a graphical description of the various production technologies in the model. The top level in each figure represents the output, while all subsequent levels of the tree structure describe the nesting structure of the inputs in the nested constant elasticity of substitution production functions. The substitution patterns for each nest are listed in italics at each node of the tree.

Figure 1: Nesting in Household Consumption (Specification #1)

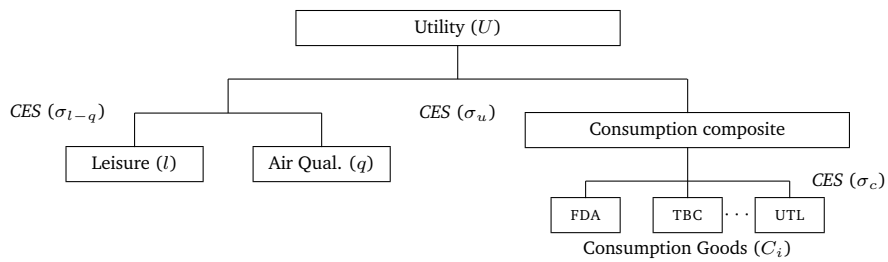


Figure 2: Nesting in Household Consumption (Specification #2)

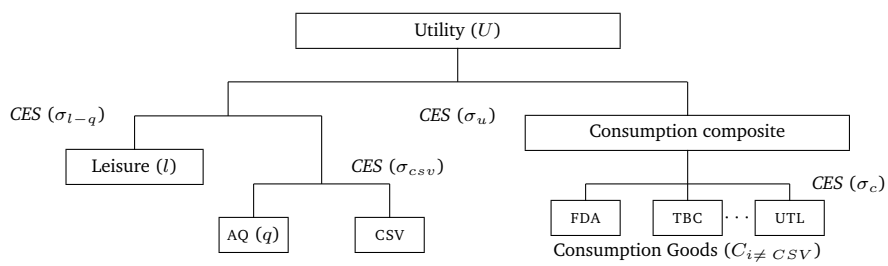


Figure 3: Intermediate Goods

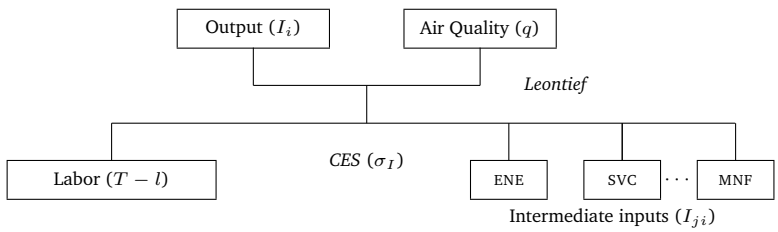
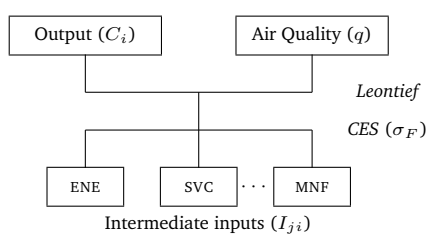


Figure 4: Final Goods



References

Boadway, Robin and Michael Keen, "Public Goods, Self-Selection and Optimal Income Taxation," *International Economic Review*, 1993, *34* (3), 463–478.

Bovenberg, A. Lans and Lawrence H. Goulder, "Optimal Environmental Taxation in the Presence of Other Taxes: General-Equilibrium Analyses," *American Economic Review*, 1997, *84*, 1085–89.

Boyd, James and H. Spencer Banzhaf, "What are Ecosystem Services? The Need for Standardized Environmental Accounting Units," Discussion Paper 06-02, Resources for the Future January 2006.

Cornes, Richard, "External Effects: An Alternative Formulation," *European Economic Review*, 1980, *14*, 307–321.

Costanza, Robert, Ralph d'Arge, Rudolf de Groot, Stephen Farber, Monica Grasso, Bruce Hannon, Karin Limburg, Shahid Naeem, Robert V. O'Neill, Jose Paruelo, Robert G. Raskin, Paul Sutton, and Marjan van den Belt, "The value of the world's ecosystem services and natural capital," *Nature*, May 1997, *387*, 253–260.

Deaton, Angus, "Optimal Taxes and the Structure of Preferences," *Econometrica*, 1981, *49* (5), 1245–1260.

Diamond, Peter A. and James A. Mirrlees, "Aggregate Production with Consumption Externalities," *Quarterly Journal of Economics*, February 1973, *87*, 1–24.

Espinosa, J. Andrés and V. Kerry Smith, "Measuring the Environmental Consequences of Trade Policy: A Nonmarket CGE Analysis," *American Journal of Agricultural Economics*, August 1995, *77* (3), 772–7.

Fullerton, Don, "Environmental Levies and Distortionary Taxation: Comment," *American Economic Review*, 1997, *87*, 245–51.

____ **and Gilbert E. Metcalf**, "Environmental controls, scarcity rents, and pre-existing distortions," *Journal of Public Economics*, 2001, *80* (2), 249–267.

____ **, Yolanda K. Henderson, and John B. Shoven**, "A Comparison of Methodologies in Empirical General Equilibrium Models of Taxation," in H.E. Scarf and J.B. Shoven, eds., *Applied General Equilibrium Analyses*, Cambridge: Cambridge University Press, 1984.

Gorman, W. M., "Tricks with Utility Functions," in M. J. Artes and A. R. Nobay, eds., *Essays in Economic Analysis*, Cambridge: Cambridge University Press, 1976.

Goulder, Lawrence H. and Roberton C. Williams III, "The Substantial Bias from Ignoring General Equilibrium Effects in Estimating Excess Burden, and a Practical Solution," *Journal of Political Economy*, 2003, *111* (4), 898–927.

____ **, Ian W.H. Parry, and Dallas Burtraw**, "Revenue-Raising Vs. Other Approaches to Environmental Protection: The Critical Significance of Pre-existing Tax Distortions," *RAND Journal of Economics*, 1997, *28* (4), 708–731.

____ **,** ____ **, Roberton C. Williams III, and Dallas Burtraw**, "The Cost-Effectiveness of Alternative Instruments for Environmental Protection in a Second-Best Setting," *Journal of Public Economics*, 1999, *72*, 329–60.

Harberger, Arnold C., "The Measurement of Waste," *American Economic Review, Papers and Proceedings*, 1964, *54*, 58–76.

Hazilla, Michael and Raymond J. Kopp, "Social Cost of Environmental Quality Regulations: A General Equilibrium Analysis," *Journal of Political Economy*, August 1990, *98*, 853–873.

Kaplow, Louis, "The Optimal Supply of Public Goods and the Distortionary Cost of Taxation," *National Tax Journal*, 1996, *49* (4), 513–533.

Murhpy, Kevin M. and Robert H. Topel, "The Value of Health and Longevity," Working Paper 11405, National Bureau of Economic Research June 2005.

Sandmo, Agnar, "Anomaly and Stability in the Theory of Externalities," *Quarterly Journal of Economics*, June 1980, *94*, 799–808.

Schwartz, Jesse and Robert Repetto, "Nonseparable Utility and the Double Dividend Debate: Reconsidering the Tax-Interaction Effect," *Environmental and Resource Economics*, 2000, *15* (2), 149–57.

Sieg, Holger, V. Kerry Smith, H. Spencer Banzhaf, and Randall P. Walsh, "Estimating the General Equilibrium Benefits of Large Changes in Spatially Delineated Public Goods," *International Economic Review*, In press November 2004.

Smith, V. Kerry and H. Spencer Banzhaf, "A Diagrammatic Exposition of Weak Complementarity and the Willig Condition," *American Journal of Agricultural Economics*, May 2004, *86*, 455–466.

____ **and Ju-Chin Huang**, "Can Markets Value Air Quality? A Meta-Analysis of Hedonic Property Value Models," *Journal of Political Economy*, 1995, *103*, 209–227.

United States Environmental Protection Agency, *National Air Quality and Emissions Trends Report, 1995* 1996. EPA 454/R-96-05.

Williams, Roberton C., III, "Environmental Tax Interactions When Pollution Affects Health or Productivity," *Journal of Environmental Economics and Management*, 2002, *44*, 261–270.

____ , "Health Effects and Optimal Environmental Taxes," *Journal of Public Economics*, 2003, *87*, 323–335.