General Equilibrium in Vertical Market Structures: Monopoly, Monopsony, Predatory Behavior and the Law

Richard E. Just
University of Maryland

and

Gordon C. Rausser
University of California, Berkeley

Prepared for Presentation
at the

Benefit-Cost Analysis Conference
University of Washington
Seattle, Washington
May 18-19, 2006

Preliminary: Not for quotation without permission of the authors.
Introduction

Recent court decisions have drawn a sharp distinction between “predatory bidding” and “predatory selling.” In the case of “predatory bidding” the literature has drawn a distinction between overbuying and raising rivals’ cost. The former is intended to cause harm to input market competitors ultimately allowing the predatory firm to exercise monopsony power. Raising rivals’ cost is instead intended to raise input cost of the output market competitors and thus allow the predatory firm to exercise market power by raising or maintaining prices. The standard in pure “predatory selling” instances, after many decades of economic and legal debate, was resolved by the United States Supreme Court in *Brooke Group Ltd. v. Brown & Williamson Tobacco Corp.*, 509 US 209 (1993). This ruling found that suppliers in output markets are not predatory unless (1) the prices charged are below the seller’s cost and (2) the seller has a “dangerous probability” of recouping its lost profits once it has driven its competitors from the market. In this paper we analyze whether the standard for liability in “buy-side” or monopsony cases should be the same or as high as the standards for liability in “sell-side” or monopoly cases.

From a theoretical perspective, much of the debate among Kirkwood, Salop and Zerbe is sourced with the distinction between consumer welfare and economic efficiency, the distinction between partial equilibrium and general equilibrium economic welfare analysis, and the distinction between substitutes and complements. In this paper we develop a model that allows us to isolate the implications of each of these distinctions. Moreover, we are able to specify the theoretical standards for predatory conduct and the fundamental forces that dictate violations of such standards. Some of the questions that can be answered by our theoretical formulation
include (a) Under what conditions does overbuying lead to consumer harm? (b) Should predatory buying be required to satisfy the below cost pricing test of Brooke Group? (c) Should allegations of “raising rivals’ cost” also be subject to the same below cost pricing test of Brooke Group?

After specifying the general equilibrium model and the competitive equilibrium benchmark, the first formal analysis evaluates market power in output markets. For this case we prove the proposition that if a concentrated industry has market power only in the output market and related sectors behave competitively, then overbuying in the input market is not profitable. Here the key to monopoly rents is restricting output, not driving up the prices of an input or equivalently overbuying an input. We also show that, under the specified conditions, monopolistic firms achieve greater rents or monopoly profits under general equilibrium than they would achieve under partial equilibrium models. One of the more interesting implications of the general equilibrium lens is that the existing Department of Justice Merger Guidelines can often give inaccurate results in assessing the profitability of a firm raising its prices by 5 or 10 percent.

After setting out three major propositions under monopoly power in the output market, we turn to distortions in the input market focusing on monopsonistic power. Here we find, contrary to the Ninth Circuit ruling in Ross Simmons v Weyerhauser matter, if a concentrated industry does not have the ability to alter its output price through its input buying behavior, then the industry cannot increase its profits by overbuying the input. Instead, under the general equilibrium lens, the traditional monopsony result is obtained where the input market quantity is restricted. Under the same lens we also demonstrate that monopsonistic firms cannot gain as much rent as conventional estimates based on partial equilibrium models would suggest. In essence, a firm has less market power and distorts the price in an input market less once equilibrium adjustments of a related industry are taken into account. We also show that a firm that has the ability to manipulate price by a given amount such as specified by the Department of Justice Merger Guidelines is invalid if done with ordinary or partial equilibrium input supplies.
We also consider the more general case where the vertical structure consists of a single firm or colluding firms that have market power in both their input and output markets. Here we are able to develop seven propositions that turn on elasticities of full equilibrium demand and supplies as well as the degree of substitutability or complementarity in both demand and supply. Finally we present the case of *naked overbuying* as a means of exercising market power.

We emphasize at the outset that all these results are developed in a static model rather than a two-stage model where the firm with market power first drives out its competitors and then exercises greater market power than previously held in a subsequent recoupment stage. Admittedly, some of the relevant legal literature considers the two-stage approach, and some even seems to suggest such a two-stage framework is the only explanation for overbuying. In contrast, we show that such extreme behaviors are possible in a static framework when general equilibrium adjustments are considered. Further, we suggest that such models offer a more practical explanation for the substantive impacts of continued overbuying or other predatory behavior because two-stage models do not explain well why firms do not re-enter markets just as easily as they leave unless other anticompetitive factors are present.

**Equilibrium Analysis of Economic Welfare**

To address these issues, we use the approach advanced by Just, Hueth, and Schmitz (2004, 355-361) for comparison of welfare effects where equilibrium adjustments occur across many markets as well as many types of consumers and producers. This approach permits an analysis of indirect equilibrium adjustments that determine the implications of monopolistic behavior in markets that are interdependent with other markets. Such a framework can explain seemingly extreme monopoly behavior including overbuying even in static models where recoupment periods are not necessary. Before developing specific results pertinent to the market...
structure considered in this paper, we summarize the underlying equilibrium measurement of welfare.

**Assumption 1.** Suppose each of \( J \) utility-maximizing consumers has exogenous income \( m_j \), and is endowed with a nonnegative \( N \)-vector of resources \( r_j \), has monotonically increasing, quasiconcave, and twice differentiable utility \( U_j(c_j) \), where \( c_j \) is a corresponding nonnegative \( N \)-vector of consumption quantities, the budget constraint is \( p(c_j - r_j) = m_j \), and \( p \) is a corresponding \( N \)-vector of prices faced by all consumers and firms in equilibrium.

**Assumption 2.** Suppose each of \( K \) firms maximizes profit \( pq_k \) given an implicit multivariate production function \( f_k(q_k) = 0 \) where \( q_k \) is an \( N \)-vector of netputts (\( q_{kn} > 0 \) for outputs and \( q_{kn} < 0 \) for inputs) where each scalar function in \( f_k \) is monotonically increasing, concave, and twice differentiable in the netput vector, other than for those netputs that have identically zero marginal effects in individual equations (allowing each production process to use a subset of all goods as inputs producing a different subset of all goods as outputs).

**Proposition 1.** Under Assumptions 1 and 2, the aggregate equilibrium welfare effect (sum of compensating or equivalent variations in the case of compensated demands evaluated at ex ante or ex post utility, respectively) of moving from competitive pricing to distorting use of market power in a single market \( n \) is given by

\[
\Delta W = \int \left[ p_n^0(\delta) q_n^s(p_n)dp_n - p_n^d(\delta) q_n^d(p_n)dp_n \right]
\]

where \( q_n^s(\cdot) \) is the aggregate equilibrium quantity supplied of good \( n \), and \( q_n^d(\cdot) \) is the aggregate equilibrium quantity demanded of good \( n \), \( \delta = p_n^d(\delta) - p_n^s(\delta) \) is the effective price distortion introduced in market \( n \), and \( p_n^s(\delta) \) and \( p_n^d(\delta) \) represent the respective marginal cost and marginal benefit of good \( n \) considering all equilibrium adjustments in other markets in response to changes in \( \delta \), \( p_n^0 = p_n^s(0) = p_n^d(0) \).

**Proof:** See Just, Hueth, and Schmitz (2004, 355-361).
Proposition 1 allows an account of equilibrium adjustments that occur throughout an economy in response to the distortion in a single market. Further, the welfare effects (compensating or equivalent variation) of a change in $\delta$ can be measured for individual groups of producers using standard estimates of profit functions and for individual groups of consumers using standard estimates of expenditure or indirect utility functions by evaluation at the initial and subsequent equilibrium price vectors.\(^1\) If other markets are distorted, then this result can be modified accordingly (Just, Hueth, and Schmitz, pp. 361-365) but, in effect, only the case of a single distortion is needed for results in this paper.

The graphical implications of Proposition 1 are presented in Figure 1. With no distortion, equilibrium in market $n$ is described by the intersection of ordinary supply, $\bar{q}^{s}_{n}(p_n,0)$, and ordinary demand, $\bar{q}^{d}_{n}(p_n,0)$, where “0” denotes conditioning on prices associated with the equilibrium prices throughout the economy under no distortions.\(^2\) If the distortion $\delta$ is introduced in market $n$, then after equilibrium adjustments throughout the economy, ordinary supply shifts to $\bar{q}^{s}_{n}(p_n,\delta_0)$ and ordinary demand shifts to $\bar{q}^{d}_{n}(p_n,\delta_0)$, which are conditioned on prices throughout the economy with a specific distortion, $\delta_0$, in market $n$. The effective full equilibrium supply and demand relationships that implicitly include equilibrium adjustments throughout the economy in response to changes in the distortion $\delta$ are $q^{s}_n(\delta)$ and $q^{d}_n(\delta)$, respectively.

With monopoly pricing in market $n$, $p^{d}_{n}(\delta)$ represents the equilibrium market $n$ price, and $\delta = p^{d}_{n} - p^{s}_{n}$ represents the difference in price and full equilibrium marginal revenue, $EMR$. This marginal revenue is not the marginal revenue associated with either the ordinary demand relationship before or after equilibrium adjustments. Rather, by analogy with the simple single-market monopoly problem, it is the marginal revenue associated with the full equilibrium

---

\(^1\) In the case of indirect utility functions, the welfare effects are not measured by the change in the function. Rather, compensating variation, CV, is defined by $V(p^1,m^1) - CV = V(p^0,m^0)$ and equivalent variation, EV, is defined by $V(p^1,m^1) = V(p^0,m^0 + EV)$ where $V$ is the indirect utility function and superscripts 0 and 1 represent initial and subsequent equilibrium conditions.

\(^2\) Throughout this paper, the terms “ordinary supply” and “ordinary demand” are taken to refer to partial equilibrium supplies and demands, respectively, which take as given the behavior of all industries not directly involved in the relevant market.
demand, \( q_n^d(\delta) \), that describes how price responds with equilibrium adjustments throughout the economy in response to changes in the market \( n \) distortion.

With monopsony, \( p_n^s(\delta) \) represents the equilibrium market \( n \) price and \( \delta = p_n^d - p_n^s \) represents the difference in the full equilibrium marginal outlay, \( EMO \), and price. This marginal outlay is not the marginal outlay associated with either the ordinary supply relationship before or after equilibrium adjustments. Rather, by analogy with the simple single-market monopsony problem, it is the marginal outlay associated with the full equilibrium supply, \( q_n^s(\delta) \), that describes how price responds with equilibrium adjustment throughout the economy to changes in the market \( n \) distortion.

The application of this result to the parallel vertical market structure of this paper is illustrated simplistically for the case of perfect substitutes in demand for final products and perfect substitutes in supply of inputs in Figure 2. Suppose in Figure 2(a) that output demand jointly facing two products or industries is \( p(y + z) \) where \( y \) and \( z \) are the quantities sold of each of the products. Suppose also that both products or industries use the same input in production and thus the input supply jointly facing the two industries is \( w(x_y + x_z) \) where \( x_y \) and \( x_z \) are the respective quantities of the input used by the two industries (both the input and output market are represented on the same diagram, assuming for the graphical analysis that the production process transforms the input unit-for-unit into outputs). If the \( y \) industry consists of a single firm whereas industry \( z \) is a competitive industry, we have the dominant-firm-competitive-fringe structure as a special case of Figure 2. Figure 2(b) represents the competitive response of production activity in the \( z \) industry as a function of the difference in the input and output price. Specifically, supply at the origin of Figure 2(b), shown in reverse, is the point at which the corresponding difference in prices in (a) is just high enough that the \( z \) industry would start to produce. Suppose with increasing marginal cost for the \( z \) industry that at output price \( p^0 \) and input price \( w^0 \) the \( z \) industry uses input quantity \( x_z^0 \). The corresponding excess demand, \( ED \), and excess supply, \( ES \), to the \( y \)
industry are shown in Figure 2(a). Note that the two vertical dotted lines in Figure 1(a) sum to the vertical dotted line in Figure 2(b).

To maximize profits, the \( y \) industry can use the input supply and excess supply relationships directly from Figure 2(a) as shown in Figure 2(c). For comparability, the output demand and excess demand relationships from Figure 2(a) must be transformed into input price equivalents by inversely applying the production technology of the \( y \) industry for purposes of determining how much to produce. That is, where \( y = y(x_y) \) is the production function of the \( y \) industry and \( x_y = y^{-1}(y) \) is the associated inverse function, the equivalent input demand \( D^* \) in Figure 2(c) is found by substituting the demand relationship in Figure 2(a) into \( y^{-1}(\cdot) \). The equivalent excess demand, \( ED^* \), in Figure 2(c) is found similarly. Then the \( y \) industry maximizes profit by equating the full equilibrium marginal revenue, \( MR^* \), associated with \( ED^* \), and the full equilibrium marginal outlay, \( MO \), associated with the excess supply.

The core insights in this paper arise because the production technologies for the two industries may not be similar and may not be unit-for-unit technologies. In contrast to the traditional monopoly-monopsony result where market quantities are restricted to increase profits, equilibrium adjustments can cause displacement of the \( z \) industry by the \( y \) industry in the case of overbuying. Moreover, these results are modified when the outputs are not perfect substitutes in demand or the inputs are not purchased from the same market but in related markets.

**The Model**

Based on the general economy model, we are now in a position to evaluate a parallel vertical market structure that exists within the general economy. To abstract from the complications where compensating variation does not coincide with equivalent variation (nor with consumer surplus), consumer demand will be presumed to originate from a representative consumer, and that prices of all goods, other than two related goods of interest, are set by
competitive conditions elsewhere in the economy. As a result, expenditures on other goods can be treated as a composite commodity, \( n \), which we call the numeraire. More concretely, suppose that demand is generated by maximization of a representative consumer utility that is quasilinear in the numeraire, \( u(y, z) + n \), where \( y \) and \( z \) are non-negative consumption quantities of the two goods of interest and standard assumptions imply \( u_y > 0, u_z > 0, u_{yy} < 0, u_{zz} < 0 \), and \( u_{yy} u_{zz} - u^2_{yz} \geq 0 \) where subscripts of \( u \) denote differentiation.\(^3\)

Suppose the consumer’s budget constraint is \( p_y y + p_z z + n = m \) where \( p_y \) and \( p_z \) are prices of the respective goods and \( m \) is income. Substituting the budget constraint, the consumer’s utility maximization problem becomes \( \max_{y, z} u(y, z) + m - p_y y - p_z z \). The resulting first-order conditions yield the consumer demands in implicit form,

\[
\begin{align*}
(1) \quad p_y &= u_y(y, z) \\
(2) \quad p_z &= u_z(y, z).
\end{align*}
\]

Downward sloping demands follow from the concavity conditions, \( u_{yy} < 0 \) and \( u_{zz} < 0 \). The two goods are complements (substitutes) in demand if \( u_{yz} > (\leq) 0 \).

Suppose the two goods, \( y \) and \( z \), each has one major input. For simplicity and clarity, suppose the quantities of any other inputs are fixed. Thus, the respective production technologies can be represented by

\[
\begin{align*}
(3) \quad y &= y(x_y) \\
(4) \quad z &= z(x_z)
\end{align*}
\]

where \( x_y \) and \( x_z \) represent the respective input quantities and standard assumptions imply \( y' > 0, y'' < 0, x' > 0, \) and \( x'' < 0 \), where primes denote differentiation.

---

\(^3\) While quasi-concavity can be assumed for consumer problems, we use the more restrictive assumption that \( u_{yy} u_{zz} - u^2_{yz} \geq 0 \), except where noted below, for symmetry of the mathematical analysis.
Suppose the inputs are related in supply so that the industries or products compete both for inputs and sales of total output. To represent the related nature of supply, suppose the respective inputs are manufactured by another competitive industry with cost function $c(x_y, x_z)$. Thus, input supplies in implicit form follow

\[
\begin{align*}
(5) \quad w_y &= c_y(x_y, x_z) \\
(6) \quad w_z &= c_z(x_y, x_z)
\end{align*}
\]

where $y$ and $z$ subscripts of $c$ represent differentiation with respect to $x_y$ and $x_z$, respectively, and standard assumptions imply $c_y > 0$, $c_z > 0$, $c_{yy} > 0$, $c_{zz} > 0$, and $c_{yy}c_{zz} - c_{yz}^2 \geq 0$, where $c_{yz} > (\text{<}) 0$ if $x_y$ and $x_z$ are substitutes (complements) in supply. For convenience, we also define

\[
\hat{c}_w = \frac{1}{c_{yz}} > (\text{<}) 0 \quad \text{and} \quad \hat{c}_x = -\frac{c_{yz}}{c_{yy}} < (\text{>}) 0
\]

if $x_y$ and $x_z$ are substitutes (complements).

Suppose that the $z$ industry always operates competitively as if composed of many firms. The profit of the $z$ industry is $\pi_z = p_z \cdot z(x_z) - w_zx_z$. The first-order condition for profit maximization requires

\[
(7) \quad w_z = p_z \cdot z'(x_z).
\]

The second-order condition for a maximum is satisfied because $z'' < 0$ and prices are regarded as uninfluenced by the firm’s actions.

Finally, suppose behavior of the $y$ industry is given by

\[
(8) \quad \max_{x_y} \pi_y = p_y \cdot y(x_y) - w_yx_y.
\]

---

4 This industry may represent a hypothetical firm formed by aggregating the behavior of many producers under competitive conditions.

5 For the special case where $c_{yz}c_{zz} - c_{yz}^2 = 0$, which is not normally admitted in standard convexity conditions, we will introduce a concept of perfect substitutes in supply where, in effect, $c(x_y, x_z)$ becomes $c(x_y + x_z)$ and $c(\cdot)$ is a convex function.
Equations (1)-(7) are sufficient to determine the full equilibrium supply and demand relationships facing the $y$ industry. A variety of cases emerge depending on market structure and the potential use of market power by the $y$ industry.

**Competitive Behavior**

If the $y$ industry is composed of many firms that do not collude, then the first-order condition for (8) requires

\[(9) \quad w = p_y y'.\]

As for the $z$ industry, the second-order condition is satisfied because $y'' < 0$ and prices are regarded as uninfluenced by firm actions. This yields the case where $\delta = 0$ in Figure 1.

Focusing on the $y$ industry for given $x_y$, the system composed of (1)-(7) can be reduced to a two equation system that describes the full equilibrium input supply and output demand facing the $y$ industry, viz.,

\[(10) \quad p_y = u_y(y(x_y),z(\hat{c}(w_y,x_y)))\]

\[(11) \quad c_z(x_y,\hat{c}(w_y,x_y)) = u_z(y(x_y),z(\hat{c}(w,x_y)))z'(\hat{c}(w,x_y)).\]

Equations (10) and (11) define implicitly the full equilibrium supply and demand relationships for the $y$ industry. Because (10) and (11) are not in explicit form, comparative static methods can be used to determine

\[(12) \quad \frac{dp_y}{dx_y} = u_{yy}y' + \frac{u_{yz}z'}{c_{yz}} [c_{yy} + \frac{dw_y}{dx_y}]\]

\[(13) \quad \frac{dw_y}{dx_y} = c_{yz} + \frac{(c_{yz} - u_{yz}y'z')c_{yz}}{\pi_z} .\]
where throughout this paper we define for notational simplicity \( \pi_{zz} \equiv u_{zz} z^2 + u_z z'' - c_{zz} < 0 \), which is the marginal effect of \( x_z \) on the first-order condition of the \( z \) industry given demand for \( z \) and supply of \( x_z \). The relationships in (12) and (13) implicitly define the input and output prices for the \( y \) industry as a function of its input level \( x_y \), or equivalently in terms of its output level, \( y = y(x_y) \).

Condition (9) together with (10) and (11) defines the competitive equilibrium output price \( p_y = \overline{p}_y \), input price \( w_y = \overline{w}_y \), and input quantity \( x_y = \overline{x}_y \), where other equilibrium quantities and prices follow from \( \overline{y} = y(\overline{x}_y) \), \( \overline{x}_z = \hat{c}(\overline{w}_y, \overline{x}_y) \), \( \overline{z} = z(\overline{x}_z) \), \( \overline{w}_z = c_z(\overline{x}_y, \overline{x}_z) \), and \( \overline{p}_z = u_z(\overline{y}, \overline{z}) \).

### Market Power Only In the Output Market

The first noncompetitive market structure that can be easily evaluated is the case with market power only in the output market. The \( y \) industry would have market power only in the output market if many other industries or many firms in another industry also use the same input \( x \), effectively rendering input price \( w_y \) unaffected by \( y \) industry activity. But suppose only one competitive industry produces \( x_z \) with a supply represented implicitly by\(^6\)

\[
(6') \quad w_z = c_z(x_z).
\]

\(^6\) Three different approaches can be used for this case. First, the industry that produces \( x_z \) can be considered an independent industry of the one that produces \( x_y \), as suggested directly by equation (6'). Second, if the same industry produces both \( x_y \) and \( x_z \), as maintained thus far, then the two equation system that describes its supplies in (5) and (6) can be replaced by supplies generated from a standard profit function, \( \pi(w_y, w_z) \), where input prices are suppressed because they are regarded as fixed. Thus, supplies in explicit form are \( x_y = \partial \pi / \partial w_y = \tilde{\pi}_y(w_y, w_z) \) and \( x_z = \partial \pi / \partial w_z = \tilde{\pi}_z(w_y, w_z) \). If \( x_y \) faces a perfectly elastic demand because many other industries or firms use it, then only the latter condition is relevant for the \( y \) industry because \( w_y \) is fixed. Further, because \( w_y \) is fixed, the latter condition may be represented simply as a monotonic functional relationship between \( x_y \) and \( w_z \); which for continuity can be represented by (6'). Thus, either of these first two approaches yield the same results as derived in the section. As a third approach, intuition may suggest that if the industry that produces \( x_y \) has a perfectly elastic supply of \( x_z \), then it also has a perfectly elastic supply of its related product \( x_z \). In this case, both equations (5) and (6) are simply dropped from the system that determines equilibrium supply and demand for the \( y \) industry. With this approach, all qualitative results obtained in this section are unaltered. The only quantitative difference is that all \( c_{zz} \) terms vanish.
For this case, $w_y$ is fixed. Accordingly, equation (5) is dropped and equation (6) is replaced by (6') in the system that structures the equilibrium. The system in (1)-(4), (6') and (7) can be reduced to

$$(10') \quad p_y = u_y(y(x_y), z(x_z))$$

$$(11') \quad c_z(x_z) = u_z(y(x_y), z(x_z))z'(x_z).$$

Equations (10') and (11') define implicitly the full equilibrium supply and demand relationships for the $y$ industry in this case.

Because equations (10') and (11') are not in explicit form, once again comparative static methods must be applied to determine properties of the full equilibrium supply and demand facing the $y$ industry,

$$(12') \quad \frac{dp_y}{dx_y} = \frac{(u_{zz}u_{yy} - u_{yz}^2)z'^2 y' - (c_{zz} - u_{zz}z')u_{yy}y'}{\pi_{zz}} = u_{yy}y' + u_{yz}z' \frac{dx_z}{dx_y} = p_{yy}y' < 0$$

$$(13') \quad \frac{dx_z}{dx_y} = -\frac{u_{yz}y'z'}{\pi_{zz}} > (<) 0 \text{ if } u_{yz} > (<) 0$$

where $p_{yy}$ is defined as the slope of the full equilibrium demand for $y$ after equilibrium adjustments in the $z$ sector, which in this case is derived as

$$(14') \quad p_{yy} = u_{yy} + \frac{u_{yz}^2z'^2}{\pi_{zz}} < 0.$$ 

Negativity of $p_{yy}$ is evident from the first right-hand expression of (12').

In this case, the first-order condition for maximizing $\pi_y = p_y \cdot y(x_y) - w_y x_y$ using (12') is

$$(9') \quad w_y = p_{yy}y' + p_yy'.$$

This condition requires the equilibrium output price, $p_y$, to be greater than ordinary marginal cost, $w_y/y'$, for the $y$ industry because $p_{yy}y' < 0$. This case corresponds to Figure 3 where

$$\delta = -p_{yy}y' > 0.$$
Because the input price is fixed, the full equilibrium marginal outlay coincides with the full equilibrium input supply and all partial equilibrium input supplies which are all perfectly elastic.

As is typical of monopoly problems, the second-order condition for this problem involves complicating third derivatives of the utility function. Accordingly, some conditions are possible where the second-order condition fails. However, the second-order condition can clearly be satisfied both for some cases where \( \frac{dx_y}{dx_z} \) is positive and some cases where \( \frac{dx_y}{dx_z} \) is negative. For example, suppose third derivatives of the utility function vanish. Then the second-order condition becomes

\[
p_{xy}(2y'^2 + yy'') - \frac{(2cz_z - 2uz_z' + uz'z')u_{yz}y'y''z''}{\pi_{zz}^2} \frac{dx_z}{dx_y} + p_{xy}y'' < 0.
\]

Because the first and last terms are negative under the plausible assumption that the \( y \) technology is not sharply downward bending, \( 2y'^2 + yy'' > 0 \), this condition can possibly be satisfied in some circumstances when \( \frac{dx_y}{dx_z} \) is positive and in some circumstances when \( \frac{dx_y}{dx_z} \) is negative. In particular, the second-order condition holds when \( 2uz'z'' > 2cz_z + uz'z' \) and \( \frac{dx_y}{dx_z} > 0 \), or when \( 2uz'z'' < 2cz_z + uz'z' \) and \( \frac{dx_y}{dx_z} < 0 \).

Not surprisingly, these results show that if the \( y \) industry does not have the ability to reduce industry \( z \) activity by driving up the price of the input, then the \( y \) industry cannot profitably increase its output price by overbuying the input. The full equilibrium demand for its output is downward sloping in its input quantity or, equivalently, by dividing (12') by \( y' \), in its output quantity. Thus, a rather traditional monopoly result is obtained where the market quantity is restricted.

**Proposition 2.** *With the parallel vertical market structure in (1)-(8), if the concentrated industry has market power only in the output market and the related vertical sector behaves competitively, then overbuying in the input market is not profitable but, rather, output is restricted to increase the output price.*
Even though Proposition 2 is similar to the typical monopoly pricing result, the same equilibrium does not arise if the $y$ industry optimizes its profit in a conventional partial equilibrium sense. To see this, note that the traditional partial equilibrium monopoly pricing rule equates the monopolist’s marginal cost and marginal revenue based on the ordinary output demand as given by (1) where $z$ market activity is taken as given by the monopolist. In this case, the first-order condition for maximizing the monopolist’s profit, $\pi_y^* = p_y \cdot y(x_y^*) - w_{xy}$, requires (9*)

$$w = u_{yy} y' + p_y y'.$$

The only difference in this first-order condition and (9') is that $u_{yy}$ replaces $p_{yy}$. Equation (14') implies that $p_{yy} < u_{yy}$ because the numerator of the right-hand fraction is positive while the denominator is negative. Thus, the full equilibrium demand is steeper than the ordinary demand.

Because $p_{yy}$ negatively exceeds $u_{yy}$, the $y$ market has a larger distortion under general equilibrium (informed) monopoly behavior than under partial equilibrium monopoly behavior. That is, as shown by comparing the respective first-order conditions, the difference in the $y$ industry output price and marginal cost, $p_y - w_y / y'$, is greater under informed monopoly behavior. This implies that monopolistic firms can gain greater monopoly profits than traditional estimates with partial equilibrium models would suggest. The reason is that full equilibrium demands that embody price adjustments in other markets are more inelastic than ordinary demands that hold behavior constant in related markets.

**Proposition 3.** With the parallel vertical market structure in (1)-(8), if the concentrated industry has market power only in the output market and the related vertical sector behaves competitively, then the concentrated industry maximizes profit by introducing a larger monopoly distortion in price than associated with the conventional partial equilibrium monopoly case.

---

7 Throughout this paper, the term “informed monopoly behavior” is defined as monopoly behavior that takes account of equilibrium adjustments that occur in related sectors and the effects of those adjustments on the full equilibrium supply and demand facing the $y$ industry. This is in contrast to partial equilibrium monopoly behavior, which does not.
which is thus a larger price distortion than if there were no related sector. This holds regardless of whether the output of the related sector is a complement or substitute in demand.

Not surprisingly, from (13'), the effect of monopoly behavior by the \( y \) industry, which reduces \( x_y \) from the competitive equilibrium after equilibrium adjustments, is either to reduce \( z \) industry activity (input and output levels) if \( y \) and \( z \) are complements, or to increase \( z \) industry activity if \( y \) and \( z \) are substitutes. In either case, according to (12'), this response makes the full equilibrium demand facing the \( y \) industry less elastic than if no adjustment occurred in the \( z \) industry (as in the partial equilibrium case).

In the case of complements, raising the price of \( y \) by restricting sales reduces the demand and price for \( z \), which increases the ordinary demand for \( y \), thus making the full equilibrium demand facing the \( y \) industry more inelastic than the ordinary demand that holds \( z \) industry activity constant. In the case of substitutes, raising the price of \( y \) by restricting sales increases the demand and price for \( z \), which increases the ordinary demand for \( y \), thus also making the full equilibrium demand facing the \( y \) industry more inelastic than the ordinary demand that holds \( z \) industry activity constant. This is why the \( y \) industry has more market power and distorts the price in the \( y \) market more after equilibrium adjustments of the related industry than partial equilibrium analysis implies.

While the price distortion is greater with informed monopoly, another issue is whether the market quantity with informed monopoly is less than suggested by the conventional partial equilibrium monopoly outcome. Where competitive equilibrium is denoted by overbars, the competitive equilibrium satisfies \( \bar{w} = \bar{p}_y y' \). Subtracting this relationship from (9*) yields

\[
w - \bar{w} = u_{yy} y' + (p_y - \bar{p}_y) y' = 0
\]

where the latter equality follows from perfectly elastic input supply. Thus, \( u_{yy} y + (p_y - \bar{p}_y) = 0 \). Following the partial equilibrium monopoly calculus, which
holds $z$ fixed in (4), \( p_y - \bar{p}_y = -\int_y^\infty u_{yy} dy \). Thus, the partial equilibrium monopoly solution has $y$ satisfying $u_{yy}y - \int_y^\infty u_{yy} dy = 0$. Under linearity ($u_{yy}$ is constant), this condition becomes $u_{yy} y + u_{yy} (y - \bar{y}) = 0$, which yields the familiar result, $y = \bar{y} / 2$.

Comparing the informed equilibrium monopoly case with the competitive equilibrium condition by subtracting (9'), which yields $w - \bar{w} = p_{yy,yy'} + (p_y - \bar{p}_y)y' = 0$ and thus requires $p_{yy,y} + (p_y - \bar{p}_y) = 0$. In this case, $p_y - \bar{p}_y = -\int_y^\infty p_{yy} dy$. Thus, the informed equilibrium monopoly output satisfies $p_{yy} y - \int_y^\infty p_{yy} dy = 0$. Under linearity of demand and production technologies ($u_{yy}, u_{zz}, u_{zz}$ are constants and $z'' = 0$), this condition also yields $y = \bar{y} / 2$. Thus, with linearity, both approaches restrict the market quantity to the same degree while the price distortion is greater under informed equilibrium monopoly behavior. Thus, the deadweight loss is greater under informed equilibrium monopoly behavior.

More generally, these results show that both the conventional partial equilibrium monopoly quantity and the informed monopoly quantity can be greater (less) than half of the competitive market quantity as the corresponding demand is downward (upward) bending. Such analysis also reveals that the full equilibrium demand is more upward bending or less downward bending than the ordinary demand if $p_{yyyy} > u_{yyyy}$, in which case the informed equilibrium monopoly quantity is less than the ordinary monopoly quantity, while the opposite is true if $p_{yyyy} < u_{yyyy}$.

**Proposition 4.** With the parallel vertical market structure in (1)-(8), if the concentrated industry has market power only in the output market and the related vertical sector behaves competitively, then the concentrated industry may restrict the market quantity more or less than

---

8To simplify, when inside an integral, $u_{yy}$ is assumed to vary with $y$ along the path of integration. But, when outside an integral, $u_{yy}$ is assumed to be evaluated at the optimal partial equilibrium monopoly solution.

9To simplify, when inside an integral, $p_{yy}$ is assumed to vary with $y$ along the path of integration. But, when outside an integral, $p_{yy}$ is assumed to be evaluated at the optimal informed monopoly solution.

10Such results depend heavily on third derivatives. For example, one can show that \( \text{sign}(p_{yyyy} - u_{yyyy}) = (-)\text{sign}(u_{yy}) \) if $2u_{yzz}$ is smaller (greater) than both $u_{yy} u_{zz} / u_{zz}$ and $u_{yy}^2 / u_z$.  

---

16
the traditional partial equilibrium monopoly case (or, equivalently, more or less than if there were no related sector).

Certainly in the case of linearity or where the market quantity is smaller with informed monopoly, the deadweight loss will be larger than in the conventional partial equilibrium monopoly case or, equivalently, without a related sector. Also, as in conventional monopoly models, both consumer welfare and overall social efficiency are harmed by monopoly behavior.

Perhaps surprisingly, one of the most interesting implications of the general equilibrium lens is that the ability to exploit a market is increased by having a related sector that generates either a complement or a substitute product. The Department of Justice Guidelines provide a rule for determining the relevant market that depends on the ability of a firm to profit from raising price by 5 percent or 10 percent. Propositions 2 through 4 show that this ability may be possible given equilibrium adjustments in related markets even though it is not present under the ordinary partial equilibrium elasticity of the immediate demand facing the firm. Thus, many more cases may pass the Guidelines rule if equilibrium adjustments in other markets are considered.

**Market Power Only In the Input Market**

A single firm using input \(x_y\) to produce output \(y\) would have market power only in the input market if many firms that do not use input \(x_y\) employ alternative production technologies to produce output \(y\). This might be the case if only one firm, either by patent or trade secret, has a process that uses input \(x_y\) to produce \(y\). In this case, the price \(p_y\) would be unaffected by \(y\) industry activity. For this case, the demand for \(z\) can be represented effectively by\(^{11}\)

\(^{11}\) If only \(p_y\) is fixed, the representative consumer’s optimized utility can be represented by an indirect utility function, \(V(p_y, p_z, m)\). By Roy’s identity, the demands are \(y = -((\partial V / \partial p_y)/(\partial V / \partial m)) \equiv d_y(p_y, p_z, m)\) and \(z = -((\partial V / \partial p_z)/(\partial V / \partial m)) \equiv d_z(p_y, p_z, m)\). If \(y\) is available in perfectly elastic supply because other industries produce it, then only the latter condition is relevant for the single firm that produces \(y\) using \(x_y\). Because \(p_y\) and \(m\) are predetermined, the latter condition can be regarded simply as a monotonic functional relationship between \(z\) and \(p_z\),
(2") \quad p_z = u_z(z).

Accordingly, equation (1) is dropped and equation (2) is replaced by (2") in the system that describes equilibrium. The system in (2''), (3)-(7) can be reduced to

(10'') \quad w_y = c_y(x_y, x_z)

(11'') \quad c_z(x_y, x_z) = u_z(z(x_z))z'(x_z)

which define implicitly the relevant full equilibrium supply and demand relationships.

Comparative static analysis of (10'') and (11'') yield

(12'') \quad \frac{dw_y}{dx_y} = \left(\frac{u_{zz}z^2 + u_z z^2}{\pi_{zz}}\right) c_{yy} = \left(\frac{c_{zz} c_{yy} - c_{yy}^2}{\pi_{zz}}\right) c_{yy} + c_{yz} \frac{dw}{dx_y} = s_{yy} > 0

(13'') \quad \frac{dx_z}{dx_y} = \left(\frac{c_{yz}}{\pi_{zz}}\right) > (\times 0) \text{ as } c_{yz} < (>) 0

where, for later notational simplicity, \( s_{yy} \) is defined as the slope of the full equilibrium supply of \( x_y \) considering equilibrium adjustments in the \( z \) sector, which in this case is

(15'') \quad s_{yy} = c_{yy} + \left(\frac{c_{yz}^2}{\pi_{zz}}\right) > 0.

Positivity of \( s_{yy} \) is evident from the first right-hand side expression of (12'').

In this case, the first-order condition for (8) using (12'') is

(9'') \quad w_y = p_y y' - s_{yy} x_y.

This condition requires that the equilibrium input price, \( w_{ys} \), must be less than the ordinary value marginal product, \( p_y' \), for the \( y \) industry because \( s_{yy} x_y > 0 \). This result is depicted in Figure 4 where

which for continuity we simply represent as \( p_z = u_z(z) \). Note that alternatively, if intuition suggests that the price of output \( z \) should be unaffected by \( y \) industry activity if the price of output \( y \) is unaffected, then the only difference in results in this section is that both equations (1) and (2) are dropped from the system that determines equilibrium supply and demand for the \( y \) industry. Under such circumstances, all qualitative results obtained in this section are unaltered. The only quantitative difference is that all terms involving \( u_{zz} \) vanish and \( u_z \) is simply replaced by \( p_z \).
\[ \delta = s_{yy} x_y > 0. \]

In this case, the full equilibrium marginal revenue coincides with the full equilibrium output demand and all partial equilibrium output demands, which are all perfectly elastic.

Again, the second-order condition involves complicating third derivatives, in this case of the cost function and \( z \) production function. Some local conditions are possible where the second-order condition fails. But ignoring third derivatives, the condition reduces to

\[ p_y y^* - c_{yy} - c_{zz}^2 \frac{y}{\pi_x} + \frac{c_{zz}^2 y}{\pi_x^2} x + \frac{2u_{zz} z}{\pi_x} + u_{zz} z^* \frac{dx_z}{dy} < 0. \]

The first and second terms are negative and the third term is positive. The sign of the fourth term is opposite that of \( u_{zz} \), and the sign of the fifth term is opposite that of \( c_{zz} \). It suffices to note that this condition can possibly be satisfied in some circumstances when \( c_{zz} > 0 \) and in some circumstances when \( c_{zz} < 0 \), although the quantitative possibilities for \( c_{zz} \) are broader when \( u_{zz} \) is smaller negatively or larger positively.

These results show that if the \( y \) industry does not have the ability to alter its output price by indirectly affecting industry \( z \) activity through input buying behavior, then the \( y \) industry cannot increase profits by overbuying the input. Because the full equilibrium supply of its input is upward sloping in its input quantity, a rather traditional monopsony result is obtained where the input market quantity is restricted.

**Proposition 5.** With the parallel vertical market structure in (1)-(8), if the concentrated industry has market power only in the input market and the related vertical sector behaves competitively, then overbuying in the input market is not profitable but, rather, input market purchases are restricted to reduce the input price.

Even though this result is similar to the typical monopsony pricing result, the same equilibrium does not occur if the \( y \) industry optimizes its profit in the conventional partial
equilibrium sense. To see this, note that the traditional partial equilibrium monopsony pricing rule equates the monopsonist’s value marginal product and marginal outlay where the marginal outlay is based on the ordinary input supply as given by (5), which takes activity in the z sector as given. In this case, the first-order condition for maximizing the monopsonist’s profit,

\[ \pi_y = p_y \cdot y(x_y) - w_y x_y, \]

requires

\[ (9**) \quad w_y = p_y y' - c_{yy} x_y. \]

The only difference in this first-order condition and (9") is that \( c_{yy} \) replaces \( s_{yy} \). Equation (15") implies that \( s_{yy} < c_{yy} \) because both the numerator and denominator of the right-hand fraction are positive. Thus, the full equilibrium supply is not as steep as the ordinary supply.

Because \( s_{yy} \) is smaller than \( c_{yy} \), the \( y \) market has a smaller distortion with informed monopsony behavior (accounting for equilibrium adjustments in other markets) than with conventional partial equilibrium monopsony behavior. Accordingly, as shown by comparing the respective first-order conditions, the difference in the marginal revenue product and input price, \( p_y y' - w_y \), is less under informed monopsony behavior. This implies that monopsonistic firms cannot gain as much monopsony profit as conventional estimates based on partial equilibrium models would suggest. The reason is that full equilibrium supplies that embody price adjustments in other markets are more elastic than ordinary supplies that hold behavior constant in related markets.

**Proposition 6.** With the parallel vertical market structure in (1)-(8), if the concentrated industry has market power only in the input market and the related vertical sector behaves competitively, then the concentrated industry maximizes profit by introducing a smaller monopsony distortion in price than associated with the conventional partial equilibrium monopsony case, and thus a smaller price distortion than if there were no related sector. This holds regardless of whether the input of the related sector is a complement or substitute in supply.
Not surprisingly, from (13’’), the effect of monopsony behavior by the \( y \) industry (which reduces \( x_y \) from the competitive equilibrium after equilibrium adjustments) is either to reduce \( z \) industry activity (input and output levels) if \( y \) and \( z \) are complements, or increase \( z \) industry activity if \( y \) and \( z \) are substitutes. In either case, according to (12’’), this response makes the full equilibrium supply of \( x_y \) facing the \( y \) industry more elastic than if no adjustment occurred in the \( z \) industry (i.e., as in the partial equilibrium case).

In the case of complements in supply, reducing the price of \( x_y \) by restricting purchases reduces the supply and increases price \( w_z \) for the \( z \) industry. In turn, the \( z \) industry reduces purchases of \( x_z \), which reduces the ordinary supply of \( x_y \) to the \( y \) industry, thus making the full equilibrium demand facing the \( y \) industry more elastic than the ordinary supply that holds \( z \) industry activity constant. In the case of substitutes, reducing the price of \( x_y \) by restricting purchases increases the supply and reduces price \( w_z \) for the \( z \) industry. As a result, the \( z \) industry increases purchases of \( x_z \), which reduces the ordinary supply of \( x_y \) to the \( y \) industry, thus making the full equilibrium demand facing the \( y \) industry more elastic than the ordinary supply that holds \( z \) industry activity constant. This is why the \( y \) industry has less market power and distorts the price in the \( x_y \) market less considering equilibrium adjustments of the related industry than in the case of partial equilibrium optimization.

While the price distortion is less with informed monopsony, another issue is whether the market quantity with informed monopsony is less than suggested by conventional partial equilibrium monopsony pricing. Where competitive equilibrium is denoted by overbars, the competitive equilibrium satisfies \( \bar{w}_y = \bar{p}_y y' \). Subtracting this relationship from (9**) yields \( w_y - \bar{w}_y = (p_y - \bar{p}_y)y' - c_{yy}x_y = -c_{yy}x_y \) where the latter equality follows from perfectly elastic output demand, which implies \( p_y - \bar{p}_y = 0 \). Following the partial equilibrium monopsony
calculus, which holds $x_z$ fixed in (5), $\bar{w}_y - w_y = \int_{x_z}^{x_y} c_{yy} \, dx_y$. 12 Thus, the partial equilibrium monopsony solution has $x_y$ satisfying $c_{yy} = \int_{x_z}^{x_y} c_{yy} \, dx_y$. Under linearity ($c_{yy}$ is constant), this condition becomes $c_{yy} x_y = (\bar{x}_y - x_y) c_{yy}$, which yields the familiar result, $x_y = \bar{x}_y / 2$.

Now compare to the informed equilibrium monopsony case where subtracting the competitive equilibrium condition from (9") yields $w - \bar{w} = (p_y - \bar{p}_y) y' - s_{yy} x_y = -s_{yy} x_y$, where the latter equality follows from perfectly elastic output demand. In this case, $\bar{w}_y - w_y = \int_{x_z}^{x_y} s_{yy} \, dx_y$. 13 Thus, the informed equilibrium monopsony input satisfies $s_{yy} = \int_{x_z}^{x_y} s_{yy} \, dx_y$. Under linearity of supply and $z$ industry technology ($c_{yy}, c_{zz}, c_{yz}, z'$ are constants and $z'' = 0$), this condition also yields $x_y = \bar{x}_y / 2$. Thus, with linearity, both approaches restrict the market quantity to the same degree while the price distortion is smaller under informed monopsony behavior. Thus, the deadweight loss is smaller under informed monopsony behavior.

More generally, these results show that both the conventional partial equilibrium monopsony quantity and the informed monopsony quantity can be greater (less) than half of the competitive market quantity as the corresponding supply is upward (downward) bending. Such analysis also reveals that the full equilibrium supply is more upward bending or less downward bending than the ordinary supply if $s_{yyy} > c_{yyy}$, in which case the informed monopsony quantity is greater than the conventional partial equilibrium monopsony quantity, while the opposite is true in the converse case.14

**Proposition 7.** With the parallel vertical market structure in (1)-(8), if the concentrated industry has market power only in the input market and the related vertical sector behaves competitively, then the concentrated industry may restrict the market quantity more or less than the

---

12 For simplicity, when inside an integral, $c_{yy}$ is assumed to vary with $x$, along the path of integration. But, when appearing an integral, $c_{yy}$ is assumed to be evaluated at the optimal partial equilibrium monopsony solution.

13 For simplicity, when inside an integral, $s_{yy}$ is assumed to vary with $x_y$ along the path of integration. But, when outside an integral, $s_{yy}$ is assumed to be evaluated at the optimal informed monopsony solution.

14 Such results depend heavily on third derivatives. For example, one can show that $s_{yyy} - c_{yyy} > (\leq) 0$ if $c_{yy} c_{yz} < (\geq) 0$ and $c_{zz} > (\leq) u_{zz} z^2 + u_{zz} z''$. 
conventional partial equilibrium monopsony case (or, equivalently, more or less than if there were no related sector).

Certainly in the case of linearity or where the market quantity is greater with informed monopsony, the deadweight loss will be smaller than in the conventional partial equilibrium monopsony case or, equivalently, without a related sector. In this case, because the price of the $y$ industry output if fixed, whether consumers are better off or worse off depends on the indirect effects on the price of the $z$ industry output. If $c_{yz} > (<) 0$ then $dx_z/dx_y < (>) 0$ implying that industry $z$ output increases (decreases) as the $y$ industry moves from competitive to monopsonistic behavior, which following (2") can only occur if $p_z$ decreases (increases). Thus, consumers gain if the inputs are substitutes ($c_{yz} > 0$) and lose if the inputs are complements ($c_{yz} < 0$) even though social welfare is harmed in either case.

The interesting contrast between this case and the previous one is that the ability to exploit a market is decreased by having a related sector when a firm has market power only in the input market, but is increased from having a related sector when the firm has market power only in the output market. Thus, these results demonstrate that showing a firm has the ability to manipulate price by a given amount, such as specified by the Department of Justice Guidelines, is not valid if done with ordinary input supplies.

Market Power in Both Input and Output Markets

Finally, we consider the more general case where the $y$ industry consists of a single firm or colluding firms that have market power in both their input and output markets. In this case, equilibrium is described by (1)-(7). For the purpose of deriving the core results, we introduce the following definition:
Definition. If \( u_{yz}y'z' > (<) c_{yz} \) then the outputs are more (less) complements in demand than inputs are substitutes in supply (which also includes the case where inputs are complements in supply) if \( u_{yz} > 0 \), or outputs are less (more) substitutes in demand than inputs are complements in supply if \( u_{yz} < 0 \). Similarly, if \( c_{yz} > (<) u_{yz}y'z' \) then inputs are more (less) substitutes in supply than outputs are complements in demand (which also includes the case where outputs are substitutes in demand) if \( c_{yz} > 0 \), or inputs are less (more) complements in supply than outputs are substitutes in demand if \( c_{yz} < 0 \).

The intuition of this definition follows from noting that \( c_{yz} \) is the cross derivative of the cost function of the supplying industry with respect to the two input quantities, while \( u_{yz}y'z' \) is the cross derivative of consumer utility with respect to the two input quantities after substituting the production technologies, \( u(y, z) = u(y(x_y), z(x_z)) \). For simplicity, the relationship of inputs will always refer to supply and the relationship of outputs will always refer to demand.

Again, for given \( x_y \), the system composed of (1)-(7) can be reduced to the two equation system in (10) and (11), which generates (12) and (13), for which further manipulation reveals

\[
\begin{align*}
(12'') \quad & \frac{dp_y}{dx_y} = u_{yy}y' - \frac{(u_{yz}y'z' - c_{yz})u_{yz}z'}{\pi_{zz}} = p_{yy}y' > (<) 0 \text{ as } u_{yy}y'\pi_{zz} - (u_{yz}y'z' - c_{yz})u_{yz}z' < (>) 0 \\
(13'') \quad & \frac{dw_y}{dx_y} = c_{yy} + \frac{(c_{yz} - u_{yz}y'z')c_{yz}}{\pi_{zz}} = s_{yy} > (<) 0 \text{ as } c_{yy}\pi_{zz} + (c_{yz} - u_{yz}y'z')c_{yz} < (>) 0.
\end{align*}
\]

An interesting aspect of these results is that the full equilibrium demand is not necessarily more or less elastic than the ordinary demand. From (12''), \( p_{yy} \) differs from \( u_{yy} \) by

\[
(14'') \quad p_{yy} - u_{yy} = - \frac{(u_{yz}y'z' - c_{yz})u_{yz}z'}{y'\pi_{zz}} > (<) 0 \text{ as } (u_{yz}y'z' - c_{yz})u_{yz}z' > (<) 0.
\]

Proposition 8. With the parallel vertical market structure in (1)-(8), if the concentrated industry has market power in both its input and output markets and the related vertical sector behaves competitively, then the full equilibrium demand relationship facing the concentrated industry is
less (more) elastic than the ordinary demand if outputs are less (more) complements than inputs are substitutes, or outputs are less (more) substitutes than inputs are complements. In particular, the full equilibrium demand relationship is more elastic than the ordinary demand if either both inputs and outputs are substitutes or both are complements.

A further interesting and peculiar nature of the equilibrium relationship in (12") is that the full equilibrium demand facing the $y$ industry is not necessarily downward sloping. In fact, comparing to (14"), as $\pi_{zz}$ approaches zero, the condition for $p_{yy} > 0$ becomes the same as for $p_{yy} > u_{yy}$.

**Proposition 9.** Under the market structure of Proposition 8 where the full equilibrium demand is more elastic than the ordinary demand, the full equilibrium demand becomes upward sloping if the elasticity of ordinary output demand is sufficiently elastic and the effect of $x_z$ on the marginal profit-maximization conditions of the $z$ industry is sufficiently small relative to the relatedness of the sectors.

From (14"), relatedness of sectors in Proposition 9 is measured by the magnitude of 

$$(u_{yz} y'z' - c_{yz}) u_{yz} z'.$$

To examine plausibility of the conditions in Proposition 9 and later results, we will consider extreme but plausible cases of substitution and complementarity. In particular, let the case where $x_y$ and $x_z$ are perfect substitutes in supply be defined by the case where $c(y, x_y, x_z) = c(y + x_z)$. With perfect substitutes in supply, both industries effectively use the same input in their respective production processes. Thus, supply of the input in implicit form becomes

$$w_{yz} = w_2 = w = c'(x_y + x_z)$$

in which case $c_{yy} = c_{zz} = c' > 0$, $c_{yz} = c_{zy} = c^* > 0$, $\hat{c}_w = 1/c^*$, and $\hat{c}_x = -1$. Note that this is the extreme case of positive $c_{yz}$ where the assumption $c_{yy} c_{zz} - c_{yz}^2 \geq 0$ is satisfied with strict equality. Similarly, we will define as perfect complements in supply the case
where $c_{yz}$ is negative and $c_{yy}c_{zz} - c_{yz}^2 \geq 0$ is satisfied with strict equality. For simplicity, we assume in this case that $c_y = c_z = c^* > 0$, $c_{yy} = c_{zz} = -c_{yz} = c^* > 0$.

Similarly, for perfect substitutes in demand we consider the specific case where the utility function takes the form $u(y,z) = u(y + z)$. Thus, demand in implicit form satisfies

$$
p_y = p_z = p = u'(y + z)$$

in which case $u_y = u_z = u' > 0$ and $u_{yy} = u_{zz} = u_{yz} = u'' < 0$. This is the case where $u_{yz} < 0$ and the assumption $u_{yy}u_{zz} - u_{yz}^2 \geq 0$ is satisfied with strict equality. With perfect substitutes in demand, both industries effectively sell into the same market. Conversely, we define perfect complements in demand as the case where $u_{yz} > 0$ and $u_{yy}u_{zz} - u_{yz}^2 \geq 0$ is satisfied with strict equality. More generally, cases of more extreme complementarity are admitted in standard consumer theory, such as fixed proportions consumption, but this case is sufficient to demonstrate plausibility of certain cases, and this more restrictive terminology simplifies subsequent discussion.

To see that the conditions of Proposition 9 are plausible, note that if the $z$ industry technology is linear and both inputs and outputs are perfect complements or both are perfect substitutes then the condition $(u_{yy}u_{zz} - u_{yz}^2)u_{yz}z' < (>) 0$ in (12‴) can be expressed as

$$(u_{yy}u_{zz} - u_{yz}^2)y'z'^2 + u_{yy}y'(u_zz - c_{zz}) + c_{yz}u_{yz}z' = u''(z' - y') < (>) 0$$

as $z' > (>) y'$. The first left-hand term vanishes with perfect substitutes or perfect complements in demand. The third left-hand term is negative if both inputs and outputs are complements or both are substitutes and dominates the second term in the case of perfect complements/substitutes if $z'' = 0$ and $z' > y'$. Thus, the full equilibrium demand is upward sloping if the marginal productivity in the $z$ industry is higher than in the $y$ industry. With these results, Proposition 9 can be restated.

**Proposition 9'**. Under the market structure of Proposition 8 where the full equilibrium demand is more elastic than the ordinary demand, the full equilibrium demand becomes upward sloping if outputs are sufficiently strong substitutes or complements in demand, concavity in the $z$
technology is sufficiently weak, and marginal productivity in the z industry is sufficiently greater than for the y industry.

Similarly, the full equilibrium supply is not necessarily more or less elastic than the ordinary supply. From (13''''), \( s_{yy} \), differs from \( c_{yy} \) by

\[
(15'''') \quad s_{yy} - c_{yy} = \frac{(c_{yz} - u_{yz} y' z') c_{yz}}{\pi_{zz}} > ( <) 0 \quad \text{as} \quad \pi_{zz} \to (>) 0.
\]

**Proposition 10.** With the parallel vertical market structure in (1)-(8), if the concentrated industry has market power in both its input and output markets and the related vertical sector behaves competitively, then the full equilibrium supply relationship facing the concentrated industry is less (more) elastic than the ordinary supply if inputs are more (less) substitutes than outputs are complements, or inputs are more (less) complements than outputs are substitutes. In particular, the full equilibrium supply relationship is more (less) elastic than the ordinary supply if both inputs and outputs are complements (substitutes).

Further, the full equilibrium supply facing the y industry in (13'''') is not necessarily upward sloping. In fact, comparing to (15''''), as \( \pi_{zz} \) approaches zero, the condition for \( s_{yy} < 0 \) becomes the same as for \( s_{yy} < c_{yy} \).

**Proposition 11.** Under the market structure Proposition 10 where the full equilibrium supply is more elastic than the ordinary supply, the full equilibrium supply becomes downward sloping if the elasticity of the ordinary supply is sufficiently high and the effect of \( x_z \) on the marginal profit-maximization conditions of the z industry is sufficiently small relative to the relatedness of the sectors.

From (15''''), relatedness of sectors in Proposition 11 is measured by the magnitude of

\[
(c_{yz} - u_{yz} y' z') c_{yz}.
\]
To see that the conditions of Proposition 11 are plausible, note that if the $z$ technology is linear and either inputs are prefect substitutes while outputs are perfect complements, or inputs are perfect complements while outputs are perfect substitutes, then the condition

\[
c_{yy} \pi_{zz} + (c_{yz} - u_{yz} y'z')c_{yz} < (>) 0 \text{ in (13')}\]

can be expressed as

\[
c_{yy} c_{zz} - c_{yz}^2 + c_{yz} y'z' - c_{yy} (u_{zz} z'' + u_z z') = c'' u' z' (y' - z') < 0 \text{ as } y' > (<) z'.
\]

The first two left-hand terms cancel one another with perfect substitutes or perfect complements in supply and the fourth left-hand term is positive. The third left-hand term is positive if either inputs are perfect substitutes while outputs are perfect complements, or inputs are perfect complements while outputs are perfect substitutes. Further, the third term dominates the fourth term in the case of perfect complements/substitutes if $z'' = 0$ and $y' > z'$. Thus, the full equilibrium supply is downward sloping if the marginal productivity in the $y$ industry is higher than in the $z$ industry.

**Proposition 11'.** *Under the market structure of Proposition 10 where the full equilibrium supply is more elastic than the ordinary supply, the full equilibrium supply becomes downward sloping if inputs are sufficiently strong substitutes or complements in demand, concavity in the $z$ technology is sufficiently weak, and marginal productivity in the $y$ industry is sufficiently greater than for the $z$ industry.*

Propositions 9' and 11' make clear that negative sloping full equilibrium supply cannot occur simultaneously with positively sloping full equilibrium demand because the conditions on marginal productivity comparisons between the two industries are mutually exclusive even with extreme cases of substitution and complementarity. Adding concavity in the $z$ technology only makes the conditions more stringent. In fact, comparing the slopes of the full equilibrium supply and demand from (12'') and (13'') reveals

\[
(17) \quad \left[ \frac{dw_y}{dx_y} - \frac{dp_z}{dx_y} \right] \pi_{zz} = (c_{yy} - u_{yy})u_z z'' - (c_{yy} - u_{yy} y')(c_{zz} - u_{zz} z''') + (c_{yz} - u_{yz} z')(c_{yz} - u_{yz} y' z').
\]
The first two right-hand side terms of (17) are clearly negative. While the final right-hand term can be positive, it can be shown to be dominated by the second right-hand term by examining the relationship for the respective multiplicative components. In particular, suppose consumer utility is expressed as a function of the inputs, \( u^*(x_y, x_z) \equiv u(y(x_y), z(x_z)) \). Then \( u^*(x_y, x_z) \) must be concave in \( x_y \) and \( x_z \) because \( u(y, z), y(x_y), \) and \( z(x_z) \) are all concave. Further, \( c(x_y, x_z) - u^*(x_y, x_z) \) must be convex. Convexity implies

\[
(18) \quad (c_{yy} - u^*_{yy})(c_{zz} - u^*_{zz}) - (c_{yz} - u^*_{yz})^2 > 0.
\]

Without loss of generality, suppose (17) is evaluated at arbitrary values of \( y, z, x_y, \) and \( x_z \) and that the units of measurement for \( y \) and \( z \) are chosen so that \( u^*_{yy} = u^*_{zz} \) and the units of measurement for \( x_y \) and \( x_z \) are chosen so that \( c_{yy} = c_{zz} \) at these arbitrary values.\(^{15}\) Then (18) along with concavity of \( u^*(x_y, x_z) \) and convexity of \( c(x_y, x_z) \) implies \( (c_{zz} - u^*_{zz}) - (c_{yz} - u^*_{yz}) > 0 \), which implies that the multiplicative components of the latter two terms of (17) satisfy

\[
(c_{zz} - u^*_{zz}z^2) - (c_{yz} - u^*_{yz}y'z') > 0.
\]

Alternatively, the function \( c(x_y, x_z) - u^*(x_y, x_z) / y' \) must also be convex where \( y' \) is evaluated at the arbitrary value of \( x_y \) at which (17) is to be evaluated. This convexity implies

\[
(c_{yy} - u^*_{yy} / y') (c_{zz} - u^*_{zz} / y') - (c_{yz} - u^*_{yz} / y')^2 > 0,
\]

which further implies

\[
(c_{yy} - u^*_{yy} / y') - (c_{yz} - u^*_{yz} / y') = (c_{yy} - u^*_{yy}y'^2 / y') - (c_{yz} - u^*_{yz}y'z' / y') > 0.
\]

The latter relationship implies that the first multiplicative components of the latter two terms of (17) satisfy

\[
(c_{yy} - u^*_{yy}y') - (c_{yz} - u^*_{yz}z') > 0.
\]

Whereas the convexity result in (18) is a standard condition necessary for existence of equilibrium in problems involving both consumers and producers, the latter convexity result requires that the difference in the ordinary supply and demand for a good is decreasing faster in its own factor input than in the factor input of the other good. In the parallel vertical market structure of this paper, this result follows from standard convexity and concavity assumptions.

\(^{15}\) For example, this is effectively the same as changing the units of measurement for \( x_z \) to some alternative scale so that quantity is measured by \( \alpha x_z \), in which case \( c_{zz} \) is replaced by \( \alpha^2 c_{zz} \) where \( \alpha \) is chosen so that \( \alpha^2 c_{zz} = c_{yy} \).
Proposition 12. With the parallel vertical market structure in (1)-(8), if the concentrated industry has market power in both its input and output markets, and the related vertical sector behaves competitively, then the full equilibrium supply relationship facing the concentrated industry after transformation by its production technology always intersects the full equilibrium demand relationship from below.

Proposition 12 is worded in terms of the full equilibrium relationships as a function of the output $y$, rather than the input $x_y$. The result is proven above in terms of the input but holds for output as well because the production transformation is monotonic. With Proposition 12, analyzing the sign of $\delta$ is sufficient to determine whether the equilibrium input use of the concentrated sector is larger or smaller than in the competitive equilibrium.

To consider the net effect of these results above, the first-order condition for maximizing $y$ industry profit, $\pi_y = p_y \cdot y(x_y) - w x_y$, is

$$(9'''') \quad w_y = \frac{dp_y}{dx_y} y + p_y y' - \frac{dw_y}{dx_y} x_y.$$ 

Equations (12''') and (13''') imply, in the notation of Figure 1, that

$$(16''') \quad \delta = \frac{dp_y}{dx_y} y + \frac{dw_y}{dx_y} x_y$$

$$\quad = -\left[ u_{yy} y' - \frac{(u_{yz} y' z' - c_{yz}) u_{zz} z'}{\pi_{zz}} \right] y + \left[ c_{yy} y - \frac{(u_{yz} y' z' - c_{yz}) c_{yz}}{\pi_{zz}} \right] x_y \geq (<) 0$$

as $\delta^* \equiv \frac{c_{yz} - u_{yz} y' z'}{c_{zz} - u_{zz} z'^2 - u_{zz} z' c_{yz} x_y - u_{yy} y'} < (> 1$.

The result (16''') shows that the distortion $\delta$ can be either positive or negative. When $\delta$ is positive, as in the cases of either monopoly or monopsony alone, the $y$ industry contracts its production to exercise market power most profitably. However, when $\delta$ is negative, the $y$ industry finds expanding production beyond the competitive equilibrium to increase profits. This is the case of
overbuying the input, i.e., buying more than in the competitive equilibrium. Because the input and output quantities are monotonically related, this also results in selling more \( y \) industry output than in the competitive equilibrium.

To clarify the variety of outcomes that are possible under (16''), we consider a variety of special cases involving either perfect substitutes or perfect complements in input supply and output demand and in the case of linear \( z \) technology. If both inputs and outputs are perfect substitutes or both are perfect complements, then

\[
\delta^* = \frac{c_{yz} - u_{yz} y' z'}{c_{zz} - u_{zz} z'^2 - u_{zz} y z'} c_{yy} x_y - u_{yy} y y' = \frac{c'' - u' y' z'}{c'' - u' z'^2} c_x x_y - u'' y y' > ( < ) 1 \text{ as } (y' - z')(z' - y / x_y) > (<) 0
\]

(in the case of perfect complements the minus signs cancel). Thus, overbuying occurs when \( y' > (<) z' \) and \( y / x_y < (>) z' \). With perfect substitutes, this is the case where both industries use the same input and effectively sell into the same output market using different technologies for production. If the \( z \) technology is concave instead of linear, then the denominator in \( \delta^* \) is increased so the possibility of overbuying (\( \delta < 0 \)) is reduced and the conditions leading to overbuying become more stringent.

**Proposition 13.** With the parallel vertical market structure in (1)-(8), if the concentrated industry has market power in both its input and output markets, the related vertical sector behaves competitively and has a linear technology, and both inputs and outputs are perfect substitutes or both are perfect complements, then overbuying of the input relative to the competitive equilibrium occurs when marginal productivity of the concentrated industry is

\[16 \text{ To show this and a succeeding result, note that } \delta^* = 1 \text{ if } y' = z', \text{ and that replacing } y' \text{ by } z'e \text{ in } \delta^* \text{ and differentiating with respect to } e \text{ reveals that } \partial \delta^* / \partial e > (<) 0 \text{ as } (c_{yz} u_{yy} y - c_{yy} u_{yz} z x_y)(c_{yz} - u_{yz} y z') > (<) 0 \text{ if } z'' = 0. \]

Thus, in the case where both inputs and outputs are perfect substitutes or both are perfect complements, \((c_{yz} u_{yy} y - c_{yy} u_{yz} z x_y)(c_{yz} - u_{yz} y z') = c'' u (y - z' x_y)(c'' x_y - u'' y z') > (<) 0 \text{ as } z' - y / x_y > (<) 0.\]
greater (less) than marginal productivity of the related competitive industry, and average productivity of the concentrated industry is less (greater) than marginal productivity of the related competitive industry. Relaxing linearity of the technology of the related industry further restricts the conditions for overbuying.

If inputs are perfect substitutes while outputs are perfect complements, or if inputs are perfect complements while outputs are perfect substitutes (minus signs cancel between the two cases), then\textsuperscript{17}

\[
\delta^* = \frac{c_{yz} - u_{yz}y'z'}{c_{zz} - u_{zz}z^2} \frac{c_{yx} - u_{yx}y'z'}{c_{yy} - u_{yy}y'y''} = \frac{c'' + u'y'z'c''x_y + u''yz'}{c'' + u''z'z''c''x_y + u''y'y''} \frac{1}{\begin{pmatrix} y' - z' \\ c'' + u''z'y/x_y \end{pmatrix}} < (>) 0.
\]

Thus, overbuying occurs when \( y' > (>) z' \) and \( c'' + u''z'y/x_y < (>) 0 \). With perfect substitutes in supply, this is the case where both industries use the same input. With perfect substitutes in demand, this is the case where both industries effectively sell into the same market. If the \( z \) technology is concave instead of linear then, as before, the denominator in \( \delta^* \) is increased so the possibility of overbuying \( (\delta < 0) \) is reduced and the conditions leading to overbuying become more stringent.

**Proposition 14.** With the parallel vertical market structure in (1)-(8), if the concentrated industry has market power in both its input and output markets, the related vertical sector behaves competitively and has a linear technology, and either inputs are perfect substitutes while outputs are perfect complements, or inputs are perfect complements while outputs are perfect substitutes, then overbuying of the input relative to the competitive equilibrium occurs when marginal productivity of the concentrated industry is greater (less) than marginal productivity of the related competitive industry, and the convexity of the cost function of the input industry is

\textsuperscript{17} In this case, \( (c_{yz} - u_{yz}y'(z'x_y) + c_{yx} - u_{yx}y'z') = c''y + (c''x_y + u''y'y') > (>) 0 \) as \( c''x_y + u''y'y' < (>) 0 \).
less (greater) than the concavity of consumer utility multiplied by the average productivity of the concentrated industry and the marginal productivity of the related competitive industry. Relaxing linearity of the technology of the related industry further restricts the conditions for overbuying.

As for previous cases, the second-order conditions for Propositions 13 and 14 involve complicating third derivatives for the utility and cost functions as well as the $z$ technology. Again, some local conditions are clearly possible where the second-order condition fails, but distinct conditions exist where the second-order condition holds for each of the special cases of Propositions 13 and 14, including cases where $\delta$ is positive and $\delta$ is negative. If third derivatives of the utility and cost functions vanish and the $z$ production technology is linear, then the second-order condition is

$$p_{yy}y' + p_{yy}y'' + \frac{(c_{yy}c_{zz} - c_{yz}^2) + (u_{yy}u_{zz} - u_{yz}^2)(y'^2 + yy^*)z'^2}{\pi_{zz}}$$

$$- \frac{c_{zz}u_{yy}(y'^2 + yy^*) + c_{yz}u_{zz}z'^2 - c_{yz}u_{yz}(2y'z' + x_{y}, y^*)}{\pi_{zz}} < 0$$

(18)

The first three left-hand terms are clearly negative if the $y$ production technology is not too sharply downward bending, $y'^2 + yy^* > 0$. To evaluate the last term, without loss of generality, suppose similar to the approach used above for (17) that (18) is evaluated at arbitrary values of $y$, $z$, $x_y$, and $x_z$ and that the units of measurement for $y$ and $z$ are chosen so that $u_{yy} = u_{zz}$ and the units of measurement for $x_y$ and $x_z$ are chosen so that $c_{yy} = c_{zz}$ at these arbitrary values. Then the numerator of the last term can be written as

$$c_{yy}u_{yy}(y' - z')^2 + (2y'z' + yy^*)(c_{yy}u_{yy} - c_{yz}u_{yz}) + c_{yz}u_{yz}y^*(y - x_{y}z')$$

(19)

where the first term is clearly negative and the second term is negative if the $y$ production technology is not too sharply downward bending, in this case $2y'z' + yy^* > 0$, because $c_{yy}u_{yy} < c_{yz}u_{yz}$ follows from $c_{yy}c_{zz} - c_{yz}^2 > 0$ and $u_{yy}u_{zz} - u_{yz}^2 > 0$. Thus, (18) is negative as long as the last term of (19) does not dominate all other terms. Two practical conditions make the last
term small. First, as the \( y \) technology approaches linearity, the latter term vanishes. Second, the last term vanishes as the average productivity of \( x_y \) in \( y \) approaches the marginal productivity of \( x_z \) in \( z \). Thus, practical cases satisfying the second-order condition can possibly hold for all qualitative combinations of \( u_{yz} \) and \( c_{yz} \). In particular, the second-order condition holds for all cases where both inputs and outputs are substitutes or both are complements if the average productivity of \( x_y \) in \( y \) is less than the marginal productivity of \( x_z \) in \( z \), and for all cases where either inputs are perfect substitutes while outputs are perfect complements, or inputs are perfect complements while outputs are perfect substitutes if the average productivity of \( x_y \) in \( y \) is greater than the marginal productivity of \( x_z \) in \( z \).

**Naked Overbuying as a Means of Exercising Market Power**

Another form of predatory behavior that can be examined in a general equilibrium framework is naked overbuying where the firm with market power buys amounts either of its own input or that of its competitor that are simply discarded. To analyze this case, we consider only buying amounts of the competitors input, which is equivalent to buying additional amounts of its own input in the case of perfect substitutes, and is a more efficient way to influence the market in the case of less-than-perfect substitutes. In this case, equation (6) is replaced by

\[
(6^*) \quad w_z = c_z (x_y, x_z + x_0)
\]

where \( x_0 \) is the amount of the competitors’ input bought and discarded by the firm with market power. For this case, the system composed of (1)-(5), (6*), and (7) can be solved for

\[
(10^*) \quad p_y = u_y (y(x_y), z(\hat{c}(w_y, x_y) - x_0))
\]

\[
(11^*) \quad c_z (x_y, \hat{c}(w_y, x_y)) = u_z (y(x_y), z(\hat{c}(w_y, x_y) - x_0))z'(\hat{c}(w_y, x_y) - x_0),
\]

which define the full equilibrium supply and demand.
Comparative static analysis of (10*) and (11*) yield

\[
\frac{dp_y}{dx_0} = \frac{c_{zz}u_{zz}z' - (u_{zz}z'^2 + u_zz'z)(u_{zz} - u_{yz})z'}{\pi_{zz}}
\]

\[
\frac{dw_y}{dx_0} = \frac{(u_{zz}z'^2 + u_zz'z)c_{yz}}{\pi_{zz}} > (<) 0 \text{ as } c_{yz} > (<) 0
\]

we well as the results as in (12'') and (13'''). Further, writing (6*) as \(w_z = c_z(x_y, \hat{c}(w_y, x_y))\) yields

\[
\frac{dw_z}{dx_0} = \frac{c_{yz}(u_{zz}z'^2 + u_zz'z') - c_{zz}u_{yz}z'z'}{\pi_{zz}} > (<) 0 \text{ as } c_{yz} > (<) 0 \text{ and } u_{yz} > (<) 0
\]

\[
\frac{dw_z}{dx_0} = \frac{u_{zz}z'^2 + u_zz'z}{\pi_{zz}} c_{zz} > 0.
\]

**Proposition 15.** With the parallel vertical market structure in (1)-(5), (6*), and (7)-(8), if the concentrated industry has market power in both its input and output market and the related vertical sector behaves competitively, naked overbuying of the related industry’s input unambiguously causes the related industry’s input price to increase while it causes the industry’s own input price to increase (decrease) if inputs are substitutes (complements). Demand for the industry with market power increases if outputs are perfect substitutes and the related industry’s input supply is not perfectly elastic, and clearly decreases if outputs are not perfect substitutes and input supply to the related industry is perfectly elastic.

To verify the latter claim of Proposition 15, note that the latter numerator term of (12*) vanishes under perfect substitutes \((u_{zz} = u_{yz} = u^*)\), but is positive otherwise \((u_{zz} - u_{yz} < 0)\) after rescaling as discussed above. On the other hand, the former numerator term vanishes when the related industry has perfectly elastic supply \((c_{zz} = 0)\) but is negative with an upward sloping input supply \((c_{zz} > 0)\).

The firm with market power evaluating naked overbuying solves the profit maximization problem given by
(8*) \[ \max_{x_y, x_0 \geq 0} \pi_y = p_y \cdot y(x_y) - w_y x_y - w_z x_0 \]

using (12**'), (13**'), (12*), (13*), (20), and (21). The first-order condition for \( x_y \) again leads to (9**) where (16**) applies if \( x_0 = 0 \), while the first-order condition for \( x_0 \) yields

\[
\frac{dp_y}{dx_0} y - \frac{dw_y}{dx_0} x_y - \frac{dw_z}{dx_0} x_0 - w_z
\]

(22)

\[
\pi_z = \frac{c_{zz} u_{zz} y' - (u_{zz} z'^2 + u_z z') (u_{zz} - u_{yz}) y' z'}{\pi_{zz}} - \frac{(u_{zz} z'^2 + u_z z') (c_{yz} x_y + c_{zz} x_0)}{\pi_{zz}} = w_z.
\]

Because the signs of terms in (12**) and (13**) are unaffected by the addition of \( x_0 \) to the problem when \( x_0 = 0 \), the firm with market power is better off with naked overbuying if and only if the first-order condition for \( x_0 \) is positive when evaluated at \( x_0 = 0 \) and the \( x_y \) that solves the profit maximization problem at \( x_0 = 0 \) (assuming second-order conditions hold). If this first-order condition is negative at this point, then the results without \( x_0 \) in the problem apply because the firm would choose \( x_0 = 0 \) at the boundary condition.

The result in (22) is qualitatively ambiguous. The first right-hand term is positive (after rescaling as discussed above). The second right-hand term can be positive or negative, but evaluated at \( x_0 = 0 \) it is negative (positive) if inputs are substitutes (complements). Of course, the third right-hand term is negative and can dominate if the related industry’s input price is sufficiently high.

**Proposition 16.** With the parallel vertical market structure in (1)-(5), (6*), and (7)-(8), if the concentrated industry has market power in both its input and output market and the related vertical sector behaves competitively, naked overbuying of the related industry’s input tends to be preferred as the related input industry’s input price is lower and both inputs and outputs are weaker substitutes or stronger complements.
Conclusion

This paper has developed a framework to evaluate static explanations for predatory overbuying in input markets and predatory overselling in output markets. The intent is to fully understand predatory behavior that is profitably sustainable. Much can be learned from the comparative static analysis before developing the two-stage predatory formulation where optimality depends on a second-stage recoupment period (at least in the case with related industries).  

While the literature on predatory behavior has drawn a distinction between raising rivals’ costs and predatory overbuying that causes contraction of a related industry, our results show that optimal behavior can involve a combination of the two. In the case of substitutes in a static model, raising rivals’ costs is the means by which contraction of the related industry is achieved. Given the existence of a related competitive industry, a firm with market power in both its input and output markets can be attracted either to (i) overbuy its input as a means of raising rivals’ costs so as to take advantage of opportunities to exploit monopoly power in an expanded output market or (ii) by exercising monopsony power in an expanded input market initially caused by overselling in an output market. In contrast to the Supreme Court ruling in *Brooke Group Ltd. v. Brown & Williamson Tobacco Corp.*, 509 US 209 (1993), these results show that (i) predatory selling in output markets will not necessarily lead to output prices below the seller’s cost because the input market is exploited to reduce input prices relatively more, and that (ii) a second-stage recoupment period after driving competitors from the market is not necessary to make this behavior profitable. Parallel results are apparent for the case of predatory overbuying.

---

18 The conceptual results of this paper apply for various time horizons. Any substantive difference in a two-stage model will depend on having costs of expansion and contraction that differ from one another or that differ between industries. If the costs of expansion and contraction follow standard cost curves over longer time periods (are reversible), then the model of this paper is applicable and two-stage issues are unimportant. So understanding of how two-stage results differ depends on understanding how marginal costs of expansion differ from marginal costs of contraction.

19 Of course, we recognize that much of the literature on predatory overbuying is based on the presumption that overbuying causes firms to exit, as in a two-stage case of recoupment.
Moreover, such action may result in raising prices to consumers, which not only causes loss in overall economic efficiency, but also loss in consumer welfare in particular (thus satisfying the narrower legal definition of efficiency emphasized by Salop). But this loss in consumer welfare may occur either through higher prices for the primary consumer good (in cases of overbuying where $dp_y/dx_y > 0$) or by causing a relatively higher price for a related consumer good (in cases of overbuying where $dp_y/dx_y < 0$).

A further set of results in this paper apply to the case of complements. While apparently not considered in the legal literature defining predatory behavior, overbuying can reduce costs to a related industry in the case of complements, and thus increase the ability to exploit an output market if the related output is also a complement. Likewise, overselling an output in the case of complements can cause increase demand for a related input, which in the case of complementary inputs increases the ability to exploit an input market. The general equilibrium model reveals that the case where both inputs and outputs are complements is virtually identical in effect as the case where both are substitutes. While the case of complements is less common in reality, it seems that any legal standard should by symmetric. In any case, the generality of the results here is broad enough to consider the fundamental case that is the core of the debate between Salop and Zerbe.

With the analytical understanding provided by the framework of this paper, the four-step rule proposed by Salop is shown to relate to a special case. That is, overbuying can be associated with Salop’s first step of artificially inflated input purchasing. However, in the case of complements, this will not lead to injury to competitors according to Salop’s second step. Yet, market power may be achieved in the output market (Salop’s third step), which may cause consumer harm in the output market if outputs are also complements (Salop’s fourth step).

Our results also show that issues in “buy-side” monopsony cases are not simply a mirror image of issues in “sell-side” monopoly cases when related industries are considered. These
issues are understood as mirror images of one another in the conventional partial equilibrium framework. However, once the equilibrium effects of market power are considered, monopoly turns out to be a far more serious concern than monopsony.

The framework of our analysis allows standard estimates of supply, demand, and production technologies to be used to determine the resulting behavior and its deviation from competitive standards. The general equilibrium model is the basis for determining, by standard measures of welfare economics, whether overbuying leads to consumer harm and thus violates the rule of reason under the Sherman Act. In particular, results show in a static model of perpetual predatory overbuying that the purpose of overbuying and consequent raising of rivals’ costs is to more heavily exploit the output market, which necessarily harms consumers. This can happen even if the market output of the subject good do not contract from competitive levels because greater market demand for the subject good is achieved by influencing the related output market through predatory buying of the input.

References


Figure 1. Equilibrium Measurement of the Welfare Effects of Monopolization.
Figure 2. Use of Market Power by One Industry with Parallel Vertical Structures.
Figure 3. Equilibrium Welfare Effects of Monopoly.
Figure 4. Equilibrium Welfare Effects of Monopsony.