JOURNAL OF RESEARCH IN SCIENCE TEACHING VOL. 36, NO. 7, PP. 806–836 (1999)

Dynamic Mental Models in Learning Science: The Importance of Constructing Derivational Linkages among Models

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Received 17 February 1997; revised 3 August 1998; accepted 15 September 1998

Abstract: We present a theory of learning in science based on students deriving conceptual linkages among multiple models which represent physical phenomena at different levels of abstraction. The models vary in the primitive objects and interactions they incorporate and in the reasoning processes that are used in running them. Students derive linkages among models by running a model (embodied in an interactive computer simulation) and reflecting on its emergent behaviors. The emergent properties they identify in turn become the primitive elements of the more abstract, derived model. We describe and illustrate derivational links among three models for basic electricity: a particle model, an aggregate model, and an algebraic model. We then present results of an instructional experiment in which we compared high school students who were exposed to these model derivations with those who were not. In all other respects, both groups of students received identical instruction. The results demonstrate the importance of enabling students to construct derivational linkages among models, both with respect to their understanding of circuit theory and their ability to solve qualitative and quantitative circuit problems. © 1999 John Wiley & Sons, Inc. J Res Sci Teach 36: 806–836, 1999

This article is concerned with how students come to understand abstract scientific models and use them in explaining physical phenomena and solving problems. In a series of studies, we have been evaluating a theory of the development of such understanding which focuses on the linkages among models at different levels in an abstraction hierarchy (Frederiksen & White, 1992; White, 1993a; White & Frederiksen, 1990; White, Frederiksen, & Spoehr, 1993). Lower-level models typically are based on a finer grain of analysis, they focus on more concrete phenomena involving objects and their interactions, and they employ causal reasoning about object interactions (cf. White, 1993a). An example is an electrostatic model of the interaction of charged particles. Higher-level models focus on more abstract representations of variables and

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Contract grant sponsor: Educational Testing Service

 CCC 0022-4308/99/070806-31

relationships among them and they may employ causal reasoning (as in a model of charge flow in which local pressure differences determine rates of flow) or constraint-based reasoning (as in circuit laws that represent algebraically constraints on voltages and currents within an electric circuit). Developing conceptual links from lower-level to higher-level models provides students with a mechanism to explain higher-level model interactions and behaviors. We have found that providing such conceptual linkages improves students' performance in solving problems using the higher-level models. For instance, in our earlier research on learning basic electricity, we found that providing explanations of current flow in terms of the behavior of electrically charged particles helps students to understand the concept of voltage and enables them to apply it in reasoning about electrical circuits using the circuit laws (White et al., 1993).

While our prior research indicates that learning multiple conceptual models improves students' performance in reasoning using the more abstract, higher-level models, our understanding of the cognitive basis of this effect remains incomplete. One possibility is that merely grounding the processes and constraints of abstract models through explanations based upon less abstract models is sufficient. However, another possibility stresses the dynamic nature of lower-level models and the need for students to see how the abstract relations of the high-level models arise as emergent properties of a lower-level model's behavior as it is run. The operation of the lower-level model can be seen as providing a mechanism that explains the basic operating principles of the higher-level model. For instance, by running a lower-level model such as a local pressure-flow model within a variety of circuit configurations, a model-based derivation for high-level electrical circuit laws such as Ohm's law and Kirchhoff's laws is produced. Our purposes in this article are to analyze what goes into a model-based derivation of this sort and describe an experimental investigation of the importance of students making such derivations as they learn electrical circuit theory.

We begin by describing our theory of model linkages. We then illustrate its application by examining linkages among three models for electrical circuits, a particle model, an aggregate model, and an algebraic model. Next, we present results of an instructional experiment in which we vary students' access to model-based derivations while controlling their exposure to the three electricity models. To illustrate the processes students use for linking models, we examine excerpts from protocols of students' reasoning as they learn and use multiple models. We conclude by presenting some theoretical and instructional implications of our work.

Theoretical Overview

The theory of model linkage is based on an analysis of pairwise relations between models at lower and higher levels. Consider model pairs as made up of a source model and a derived model, as illustrated in Figure 1. A source model is a precursor for the derived model if its emergent behaviors provide the primitive objects, interactions, and control processes of the derived model. Our hypothesis is that instruction should be based on and explicitly demonstrate such linkages among models. To facilitate this, several conditions must be met: (a) The source model must be understandable in its own right, which means that it must employ representations and forms of reasoning that are accessible to students, such as reasoning about discrete objects and events based on local mechanisms involving causal interactions (Frederiksen & White, 1992; White, 1993a). (b) Students must learn the source model and be able to run the model in situations used for generating the derived model's properties. (c) The derived model must be conceptually linked to the source model, which means that the primitive objects, interactions, and control processes of the derived model are defined in terms of emergent behaviors of the source model and are demonstrated to students as it is run. Such model derivations are analogous to

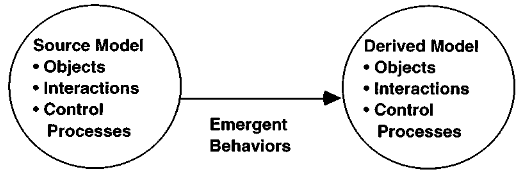


Figure 1. Derivational mapping.

mathematical proofs, with the steps in running the source model corresponding to steps of a proof. However, instead of representing the steps in a derivation using symbolic notation, model-based derivations employ graphic representations (or mental envisionments) of the model at each step in the derivation. Such model-based derivations allow students to reason from the behavior of a lower-level model to establish fundamental processes operating in a more abstract, higher-level model (e.g., Kepler's law can be derived by running a finite-state causal model based on Newtonian physics) (White, 1993b).

Other researchers have used successive models in teaching physics concepts. For example, Clement and colleagues (Brown & Clement, 1989; Clement, 1993; Clement, Brown, & Zietsman, 1989) employed "bridging analogies" to help students understand various difficult concepts in mechanics. One instructional sequence, designed to help students understand the Normal force, first makes an analogy between a spring pushing up on an overlying book and a table pushing up on an overlying book. Although students will agree that the spring does exert an upward force on the book, they do not believe that the table exerts such a force; moreover, they do not recognize the validity of the analogy between the spring and the table. For this analogical model of a table as a spring to be accepted, it must be augmented or "bridged." The next analogical model in the sequence is between the table and a thin flexible board held up by two sawhorses. Students will accept this analogy more readily, and many of them will agree that the flexible board pushes upward on a book resting on it. By introducing this and other bridging analogies, Clement successfully helped students understand that a rigid body such as a table is actually springy and thus capable of exerting a normal force. Microscopic models of springy interatomic bonds within the table are then introduced to explain the ultimate origin of the table's springiness.

Our theory of model derivations is distinct from this and other sets of model sequences in that our models are derivationally linked to one another. The emergent properties of one model become the basis for interactions in the next model. In the bridging analogies work and in other work with multiple analogies (e.g., Gentner & Gentner, 1983), the analogical models share some characteristics such as springiness, but the properties of one do not lead to the next. For instance, the model of the book resting on a spring does not generate behaviors that form the basis of the book on the flexible board model: They are both models of one object pushing up on another because of a springiness mechanism. In fact, the goal of the instructional sequence is for students to recognize the similarity (if not identity) of all the analogical models in the sequence. In contrast, the goal of our sequence is for students to use one model to generate the fundamental behaviors for the next model.

Students' Difficulties in Understanding Electricity

In this article, we develop this theory and its implications for learning within the context of students' understanding of elementary d.c. electrical circuits. Understanding the behavior of electrical circuits—as opposed to learning how to use equations to solve standard circuit problems—is notoriously difficult for students (Cohen, Eylon, & Ganiel, 1983; Fredette, 1981; Steinberg, 1993a). Understanding circuits containing batteries, wires, and light bulbs (or resistors) requires students to reason about electrical potentials at different locations within a circuit and the flow of electrical charge caused by differences in these electrical potentials. For instance, in the circuit shown at the left in Figure 2, students should reason that when a battery voltage is applied to the two resistors in series, it is divided across them in proportion to their resistances. Students also need to be able to envision how changes in the conductivity of any circuit component will have nonlocal effects, causing changes in electric potentials (voltages) throughout the circuit which in turn produce changes in current flows in different circuit branches. Thus, increasing the resistance of R_1 in the circuit will decrease the voltage across R_2, and consequently, the current through R_2. This voltage-centered model is the expert or target form of reasoning that students need to develop (White & Frederiksen, 1990).

A number of reasons have been offered for why this model is so difficult for students to understand, some of them conceptual and others instructional: First, students appear to have strong preconceptions about the flow of material in systems such as this. For example, they tend to regard current as emanating from the battery (Cohen et al., 1983), and their reasoning typically is based upon the local actions of the current as it encounters various obstacles (resistances) around the circuit. For example, for the left-hand circuit in Figure 2, students typically reason that when the current coming out of the battery encounters the first resistor, R_2, it is slowed down, and when that current reaches resistor R_1, it is further slowed down. The current through R_2 is therefore less than that coming out of the battery, and the current through R_1 is even smaller than that coming out of R_2. We have termed this form of reasoning the "current as agent" model (Frederiksen & White, 1992). This type of reasoning has also been called "sequential reasoning" (Dupin & Johsua, 1987; Härtel, 1982) to emphasize that changes which occur downstream from the battery are seen as having no influence on things happening upstream. Such preconceptions about agency and local causality in circuit functioning may impede development of a voltage-centered model.

Second, unaware of the difficulties with their models, students do not hesitate to invoke causal models to explain circuit behavior, even when those models are not taught within their electricity curriculum. Students tend to engage in such causal reasoning when they are asked to solve qualitative circuit problems (for example, when they are asked which branch of a parallel circuit has the most current flowing through it), or when they are dealing with dynamic situations (such as when they are asked to predict a circuit's behavior after a switch has been closed or a light bulb has burned out). In the right-hand circuit in Figure 2, for instance, students will often say that the current always takes the path of least resistance, going through the light bulb L_2 when the switch is open and through the switch when it is closed. Their use of mechanistic reasoning in these situations shows that students do have ideas about a causal model of current

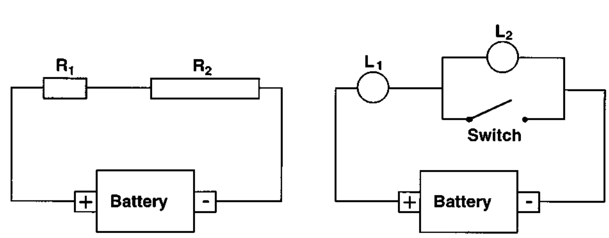


Figure 2. Circuits requiring students to reason about electrical potentials (voltages).

flow in a circuit which they use when they feel it is appropriate. However, they do not have the knowledge of electric potentials needed to develop a more accurate model on their own. Thus, students' tendency to invoke causal models is likely to lead to incorrect views of circuit causality, unless, of course, the curriculum were to provide alternative causal models to take their place.

Unfortunately, electricity curricula seldom provide a physical mechanism for explaining circuit principles such as Ohm's law or how current is propagated within a circuit. High school textbooks, for instance, typically rely on analogies (usually the water analogy) rather than on building a physical model of electricity. However, the water analogy has been shown to be ineffective for teaching electricity (Duit, 1991; Dupin & Johsua, 1989; Tenney & Gentner, 1984). An alternative analogy to this is Steinberg's (1993b) air pressure analogy used within his CASTLE curriculum, in which the changes in voltages (electrical pressure) and current are made the subject of inquiry through the use of circuits containing large capacitors.

Third, much of introductory instruction in electrical circuits is centered on the solution of quantitative circuit problems. The type of reasoning that is involved in such problems is constraint-based reasoning, in the form of manipulating algebraic equations. The problem is that students are not shown how the quantitative circuit theory is related conceptually to a causal model of what is happening in the circuit. Instead, abstract concepts such as voltage are often simply left as variables in an equation (e.g., V in Ohm's law). The student is left with the notion that physical systems can be understood only through mathematical equations, and that these equations cannot be understood in terms of any underlying physical mechanism. As a result, students' natural tendency to reason causally about circuits seems to disappear when they learn to solve quantitative circuit problem (Frederiksen & White, 1992).

But what do students learn in solving quantitative problems? Rather than considering what is happening within a circuit, students learn to use the circuit diagram and problem statement as cues to access formulas that they think fit the problem, to manipulate them algebraically to solve for the required result, and then to plug given quantities into the equations to calculate an answer (Larkin & Chabay, 1989). In other words, what they learn has little to do with developing an understanding of the forces at work in a circuit's functioning.

The Models of Electricity

The Particle Model. It is interesting that the same textbooks that fail to present any form of causal reasoning for d.c. circuits in their units on circuit electricity often do present causal explanations for why electrical charge moves in their unit on electrostatics. In fact, these units are often very effective. It was this form of model that we adopted as the least abstract model in the sequence of models we present to students. Our earlier protocol and instructional studies (Frederiksen & White, 1992; White et al., 1993) showed that students have no difficulty in learning to reason qualitatively about electrons repelling one another and moving within a conductive medium. This type of situation is illustrated in Figure 3, which shows some screen shots of the behavior of the computer simulation we use in our research. Pictured are two connected, conductive areas (they could be slices of a resistor), with a set of negatively charged particles (electrons) initially distributed randomly across the right-hand area. As the simulation runs, the charged particles repel one another with a force that is inversely proportional to the square of the distance between them (Coulomb's law). As time passes, the particles spread out over the two connected areas, so as to keep as far away from each other as possible. This spreading out is an emergent behavior of a system containing charged particles that repel one another and are

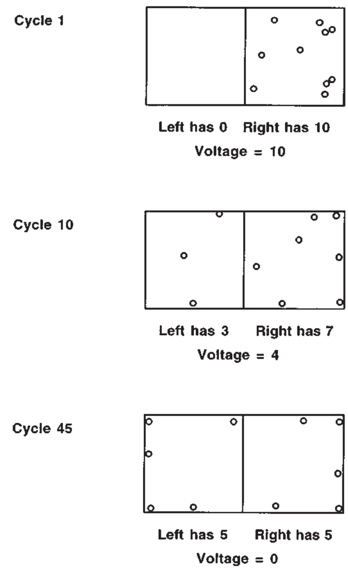


Figure 3. The particle model at three points in the simulation.

free to move on a conductive surface. Students can characterize such emergent system behaviors in terms of simple descriptive laws. Examples of two such simple laws are:

- Electrons will move from areas that are more crowded to areas that are less crowded.
- When two areas have the same densities of particles, there will be no more net movement from one area to the other.

These two laws qualitatively describe a local version of Ohm's law (which we call the "flow equation"): namely, that the number of particles moving from one area of the resistor to the other in a given time interval is proportional to the difference in the density of particles between the two connected areas (Current flow = Constant × Difference in charges).

The Aggregate Model. This simple principle abstracted from the behavior of charged particles can become the basis for linking students' causal understanding of electrostatics to a causal, voltage-based understanding of electrical circuits. To accomplish this, we introduce a simpler but more abstract model for reasoning about electrostatics at the level of granularity expressed in the two laws shown above. In this model, called the aggregate model, students reason about differences in amounts of charge in adjacent areas of a circuit and how these lead to flows of charge which can be calculated using the flow equation. There are two instructional reasons for introducing such a model: First, it is simpler for students to reason qualitatively—as well as quantitatively—with the flow equation than it is for them to simulate mentally the behavior of individual particles moving on a conductive surface. The flow equation involves only simple proportional reasoning rather than visualizing the two-dimensional distribution of particles, which can become extremely difficult when the circuits are complex.[2] Second, and perhaps more important, the flow equation encourages students to think at a higher level of abstraction—about scalar properties of charge (electric potential and current flow), and this should prepare them for later making the transition to purely symbolic, algebraic representations.

For these reasons, we have adopted the flow equation as the basis for a computer simulation of the transient behavior of electrical circuits. This is the discrete-state model we have called the aggregate model (White et al., 1993). The primitive objects in this model are small, equally sized slices of resistors that are connected end to end. In Figure 4, two such resistive slices are connected. In addition to quantizing space in this way, we also quantize time as a sequence of discrete intervals which are modeled as cycles of the simulation. Rather than basing the model on individual charged particles, the model is based on aggregate electrical charge that can flow from one slice of a resistor to another. The rule governing the amount of flow in any time interval is the simple flow principle we abstracted from the behavior of particles. Stated as a difference equation, it is $I = K \cdot \Delta V$, where I is the flow of particles in a unit of time (current), ΔV is the difference in charge (voltage) of the two adjacent areas, and K is a constant determined by the resistance of the conductive material and the length of the time interval. In illustrating the simulation's behavior, the graphic representations are chosen to be consistent with this aggregate flow perspective (they do not require reasoning about particles) and with the desire to focus students' attention on scalar representations of electric potential and current. We represent the charge within each unit area by a bar graph which shows the amount of excess or deficiency in charge in that area (zero represents a balance of positive and negative charge). Current flows are represented by arrows drawn over the connections between unit areas, and their length is proportional to the flow rate. In the simulations, K is taken to be 20%, so that after the two slices are connected, 20% of the difference in charges is transported in each time interval (Figure 4). The charge flow continues until the two connected slices have equal charges.

The causal reasoning of the aggregate model can be applied to whole circuits as well, such as the circuit containing a battery and a resistor shown in Figure 5. The resistor is composed of

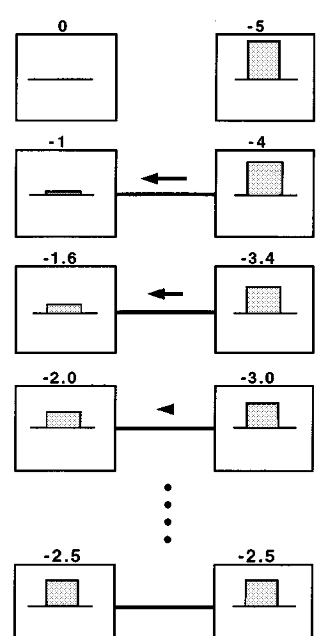


Figure 4. Successive cycles of the aggregate model for two connected resistive slices.

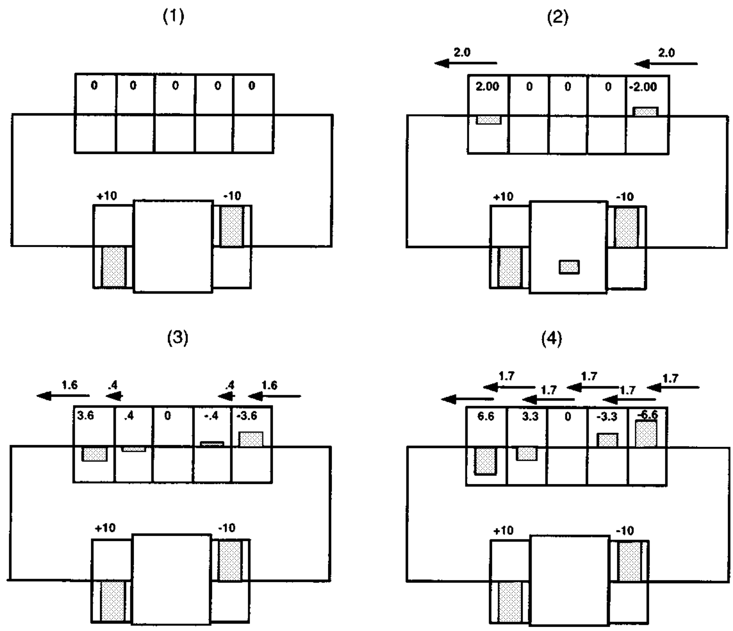


Figure 5. Successive cycles of the aggregate model for a complete circuit. Panels 1–3 show the first three cycles, and Panel 4 shows the steady state (there were many iterations between Panels 3 and 4).

five unit slices of material connected end to end. The battery is modeled as a device that tries to maintain a constant difference between the charges on its terminals. Thus, if charges on its terminals change during the simulation owing to current flow through the circuit, the battery reacts by internally moving charge between its terminals so as to restore the voltage difference between them. Within the circuit, charge flows between any two connected resistor slices or between a battery terminal and an adjacent resistor slice are governed by the flow equation. To run the simulation, one has to calculate the charge flows for all connected slices of material within the circuit at each time step. Thus, on each cycle of the simulation, one first finds the electrical charge differences (voltages) between all adjacent pairs of slices and then uses the flow equation to find the amount of charge that will move from one slice to the other because of these charge differences. Finally, for all slices, one combines the amounts of charge that flow into and out of each slice to find the new net charge on that slice (charge is always conserved as it moves from one slice to another). The visual display is updated to show the new net charges and the flows that led to them on that cycle of the simulation. Over time, the local charges and current flows throughout the circuit keep changing until the circuit reaches a steady state, at which point they have reached stable values.

The dynamic nature of the simulation makes the transition to a steady state salient for students. In our experiments, students run the simulation and watch its behavior change over successive time intervals of the simulation. In each interval, they see small amounts of charge transferred between adjacent regions within the circuit, and these local flows alter the charges on those regions, which in turn determines the flows of charge within the next time interval. This basic process runs until the charges stabilize—that is, until no single slice of any circuit component gains or loses charge.

As students work with such simulations, an accompanying printed workbook draws their attention to a number of emergent behaviors of the system as it runs. For example, in the simulation shown in Figure 4, students notice that when the system settles down to a final steady state, the charges no longer change from one cycle of the simulation to another, and there is no longer a current flow. However, when they are shown the simulation for a complete circuit containing a battery (such as the one shown in Figure 5), they find (much to their surprise) that at the steady state, charge continues to flow with equal amounts of charge flowing into and out of each slice (i.e., there is a dynamic equilibrium). By running this aggregate model for a sequence of circuits of increasing complexity, students are introduced to the steady-state circuit conditions that can in turn be described by the set of quantitative circuit laws. These circuit conditions consist of a set of quantitative constraints on circuit variables (voltage, current, and resistance) that result when the aggregate model reaches its steady state.

The Algebraic Model. In addition to the particle and aggregate models, students develop the algebraic model, which incorporates the standard quantitative circuit laws for direct current circuits. These are algebraic expressions giving constraints on the values of *V, I,* and *R* in circuits that have reached a steady state (i.e., for which these values are no longer changing). Constraint equations include Ohm's law in its three forms ($I = V/R$, $V = I \times R$, and $R = V/I$), formulas for resistances in series ($R_T = R_1 + R_2 + ... + R_n$) and in parallel ($1/R_T = 1/R_1 + 1/R_2 + ... + 1/R_n$), formulas for voltage dividers (voltages across resistors in a series circuit are proportional to their resistances), Kirchhoff's voltage law (voltages across resistors in a circuit loop containing a battery sum to the battery voltage), and Kirchhoff's current law (the currents leaving a node equal the currents entering a node). These are applied first to simple circuits containing a battery and a resistor, then to series circuits, next to parallel circuits, and finally to hybrid series-parallel circuits. In each circuit context, students solve standard circuit problems in which some values of *V, I,* or *R* are given, and these are used to solve for unknown voltages or currents by applying the circuit equations.

Derivational Linkages among the Models

In this section, we present the derivational linkages among the three electricity models we have described. First, as shown in Table 1, each of the three models can be characterized in terms of its primitive elements, which include its elementary objects and their properties, the elementary interactions among objects, and its control processes. For example, for the particle model, the elementary objects include mobile, charged particles, and conductive regions which may be connected. Interactions include those among particles (electrical repulsion) and between particles and edges of regions (bouncing). The control process for running the model is one of iteration over time. Second, as shown in Table 2, each model has characteristic emergent behaviors, which include emergent system properties and concepts that are derived from running the model. For example, iterating the particle model over time yields emergent system properties such as the flow principle, and emergent concepts such as that of aggregate charge and the notion of a steady state. Similar analyses of the primitive elements and emergent behaviors are also given for the aggregate model and the algebraic model.

The correspondences between primitive elements and emergent behaviors in alternative pairs of models are presented in Tables 3 and 4. For example, Table 3 shows how elementary objects and interactions of the aggregate model are derived from the emergent concepts and processes of the particle model. Note that these two models share many common elementary

Table 1
Comparisons among three models of electricity: Primitive elements of the models

	Particle Model	Aggregate Model	Algebraic Model
Elementary objects and properties	• Mobile particles with a unit electric charge • Conductive areas with fixed size and resistance • Connections (permeable boundaries between areas)	• Aggregate charge within a unit area • Resistive areas with a fixed size that hold charge • Batteries that act to keep a fixed difference in charge • Connections (conductive paths) between areas	• Electric potential (V) at any point within a circuit • Resistors which vary in their length (resistance R) • Batteries having a fixed voltage rating • Connections (conductive paths) between components
Elementary interactions	• Electrostatic force between charged particles • Bouncing due to normal force from interaction of particles with edges	• Voltage difference causes flow of charge between connected areas ($I = \mathrm{K} \cdot \Delta V$) • Batteries restore charge difference between terminals when changes occur	• Algebraic relations among variables representing electric charges, resistance, and current (e.g., Ohm's law, $I = V/R$)
Control processes	• Reasoning about changes in particles' motions over successive intervals of time	• Reasoning about how charges and currents change by iterating difference equations over time	• Reasoning about steady states using circuit laws and algebraic, constraint-based reasoning

Table 2
Comparisons among three models of electricity: Emergent behaviors of the models

	Particle Model	Aggregate Model	Algebraic Model
• Emergent system properties	• Flow principle • Differences in amounts of charge in connected areas causes a flow of charge proportional to that difference • Flow leads to new charge distributions	• Constraints on circuit variables when a steady-state occurs • Current flow through a resistor depends on its length (resistance) • Current is constant within resistors/series circuits	• Algebraic derivation of further constraints which apply to particular circuits (e.g., voltage dividers, parallel resistors)
Emergent concepts	• Scalar representing aggregate charge within a conductive area (electric potential) • Scalar representing the rate of charge flow (current) • Distinction between transient and steady-states of a circuit	• Circuit components • Resistors are composed of connected areas • Switches are compsed of areas that can be either connected or unconnected • Transient/steady-state distinction • Dynamic equilibrium	• Circuit concepts • Equivalent resistance (R_T) for resistors connected in series and in parallel) • Procedures for applying algebraic constraints in solving problems

Table 3
Relations between particle model and aggregate model

Particle Model		Aggregate Model
Direct mapping of elementary objects		
Conductive areas	→	Restrictive areas
Connections as permeable boundaries	→	Connections as conductive paths
Direct mapping of control processes		
Iteration over time	→	Iteration over time
Mapping from emergent concepts to objects/properties		
Set of particles within a unit area	→	Aggregate charge within a unit area
Scalar representation of number of charged particles	→	Amount of charge or voltage (V)
Scalar representation of flow of particles	→	Charge flow (I)
Mapping of emergent properties to elementary interactions		
Flow principle (flow is proportional to charge difference)	→	Flow equation for local currents ($I = K \cdot \Delta V$)

objects (e.g., unit resistive areas), and their control processes are the same (causal events iterated over time). However, the flow equation ($I = K \cdot \Delta V$) governing the aggregate model represents an emergent behavior of the particle model. Table 4 shows the correspondences between the algebraic model and the aggregate model. While these two models also share many elementary objects, in this case the rules used in reasoning with the algebraic model are emergent properties of running the aggregate model—namely, that voltages and currents have constant values in the steady state, and that there is a consistent set of constraints on these values. For instance, in the steady state, the currents within a simple series circuit (a resistor connected across a battery) are seen to be everywhere the same, and Ohm's law gives the constraints on the relationship between voltage across and current through the resistor.

Table 4
Relations between aggregate model and algebraic model

Aggregate Model		Algebraic Model
Direct mapping of elementary objects		
Aggregate charge within a unit area	→	Electrical charge or potential (V)
Batteries with fixed voltage rating	→	Batteries with fixed voltage
Connections as conductive paths	→	Connections as conductive paths
Mapping from emergent concepts to objects/properties		
Circuit components as sets of connected areas/nodes (resistors, switches)	→	Circuit components (resistors/switches)
Total resistance depends on number of areas connected	→	Resistor has fixed resistance (R)
Mapping of emergent properties to elementary interactions		
Constraints on steady-state system (e.g., there is a constant current flow through a resistor that depends on its resistance and the voltage across it)	→	Algebraic relations among variables representing electric charges, resistance, and current (e.g., Ohm's law)

Our theory is that in developing an understanding of a physical domain using multiple models, students must see how the primitive elements of one model can be derived from analysis of the behavior of other, lower-level models applied to the same situations (e.g., the same electric circuits). This should be particularly critical in learning the algebraic model, in which students encounter difficult concepts such as the notion of a dynamic steady state and in which systems of multiple constraints replace more familiar concepts such as the notion of causal interactions. Our hypothesis is that it is not enough for students simply to be given an explanation of the linkages between models. Rather, they need to carry out a derivation in which they see how the elementary principles of one model lead to those of another. Specifically, by running the aggregate model, students should see how the origins of the steady state circuit conditions can be derived from a more fundamental, causal model when it is iteratively applied to simulate the changes that occur in a circuit over time.

The purpose of the present study, therefore, is to establish experimentally the importance of presenting students with such iterative derivations of the steady state circuit relations. The crucial experiment to be carried out is to vary students' access to this iterative derivational process while presenting in the clearest way possible every other aspect of the linked models to all students. We must be particularly careful to include for all students the representation of voltages as arising from the charges present at different locations within a circuit, and the causal principle that electrical current is produced by differences in voltages between connected locations. We must also unpack these concepts for all of our students in terms of a mapping to the particle model. Finally, we must visually represent and develop for all students a clear understanding of the final configuration of steady-state voltages and currents in different circuit types. For example, they should see that in series circuits the voltage drop across every pair of adjacent resistor slices is the same, and that voltage drops across larger resistors will be greater because they are made up of more such slices. In short, we want to compare groups of students who differ only in their exposure to the iterative derivation of the steady-state circuit characteristics using the simple causal principle of the aggregate model, i.e., that local voltage differences cause local flows of current.

Method

Participants

The participants were 32 high school students in Grade 10 or 11, who had completed a science course other than physics (usually chemistry). The instruction was given in a laboratory setting in which students followed a written workbook and used computer simulations to learn the model sequence. The students worked for approximately 2 h/day for 2 weeks and were videotaped during the instructional sessions. The students were divided into two instructional groups of 16 students: the transient group and the steady-state group. Although both groups learned the same model sequence, only students in the transient group were provided with computer simulations of the transient state; the steady-state group used simulations that merely jumped from one steady state to another. Within each instructional group, students worked in pairs to run the computer simulations and solve workbook problems. However, all assessments were carried out individually. All students were given an algebra pretest which was included as a covariate in the statistical analyses. The students were also given tests of their knowledge of basic electrical circuits and circuit concepts. The students were matched across groups on the basis of their pretest scores for algebra, their knowledge of circuits, and their grade in school.

Following instruction, all students were also given a set of postinstruction assessments which included a series of paper and pencil tests and a clinical interview.

Instructional Materials and Design

The instructional materials consisted of computer simulations and printed workbooks which presented three different models of circuit behavior: (a) the particle model, (b) the aggregate model, and (c) the algebraic model. By varying the use made of the aggregate model simulation, we created the two instructional conditions: the transient condition and the steady-state condition. All students, regardless of condition, were provided with the aggregate model to provide a link between the particle model and the algebraic model. However, students in the transient condition used the aggregate model to simulate dynamically the changes in charges and current flows over time as the circuit settled into its steady state. Students in the steady-state condition were given the same aggregate model, but they were presented with only the starting condition of the circuit (showing the charges on the battery terminals) and its final, ending condition (showing charges and current flows when it had reached the steady state). In all other respects, the representations, explanations, and sequence of circuits included in the curriculum were the same for the two groups. The two alternate instructional protocols are illustrated in Figure 6.

Particle Model. Both groups of students began instruction spending 2–3 h interacting with a computer simulation based on the particle model. This computer simulation is a simple program written in QuickBasic which runs on a Macintosh computer. It shows the behavior of charged particles (electrons) on a conductive sheet of metal that repel one another owing to a Coulomb interaction (Figure 3). The conductive sheet is divided into areas of equal size, and a counter displays the number of electrons within the left and right areas. Another counter shows the number of particles that pass from the right to the left side (that is, cross the midline) in each time interval. A record of these counts is also kept in a data table. In a series of experiments using the computer simulation, students are given (in experiments at Level 1 of the curriculum)

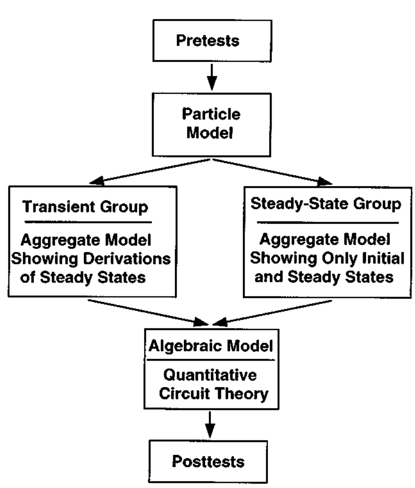


Figure 6. The curriculum sequence for the transient and steady-state groups.

two conductive areas with low resistance, one of which initially contains charged particles, and (in Level 2 of the curriculum) two conductive areas with high resistance, again with one initially containing charged particles. In working with the simulation, students read from a workbook which explained the purpose of the present experiment they were doing, asked them to make predictions, had them run the simulation, and gave them explicit questions to answer about the results of the simulation. These questions were designed to draw out rules or generalizations about the behavior and emergent properties of the simulation in each experiment. An example of a rule is that the net movement of electrons is from the side with the most electrons to the side with the least. Students were encouraged to think of the number of electrons in a unit area as determining the net charge for that area. They were also shown that as the simulation ran, the number of electrons moving (flowing) from one side to the other depended on the difference in amount of charge on the two sides.

Aggregate Model (Transient Condition). Next, students in the transient condition spent approximately 8 h interacting with a computer simulation based on the aggregate model. The computer simulation was that used in the QUEST program, which was implemented on a Xerox 1109 LISP Machine (White & Frederiksen, 1990; White et al., 1993). As previously described, this computer simulation illustrated the transient changes in charges and current flows within a circuit that occur whenever there is a change in circuit conditions, such as when a battery is attached to a circuit or a switch is closed. Students could run the simulation in either step mode or continuous mode. In step mode, the computer simulation ran only one iteration at a time and generated, whenever the student requested it, an explanation of the charge transfers that had just occurred for any slice of a resistor. In the workbooks, students were asked to run the first few steps of the simulation for each circuit in step mode and then to switch to continuous mode to see the final outcome generated by the model. The explanations were given via a Dectalk computer speech synthesis device and were also presented in a window on the computer screen.

The students carried out a series of experiments using the computer simulation which were organized within the curriculum into six instructional levels representing circuits of increasing complexity: Level 1 (charges and conductors) contained circuits having two or three connected areas that had negligible resistance (nodes), and in these problems charges were averaged across the nodes within a single iteration. Level 2 (resistance) contained circuits having two or three connected areas that had appreciable resistance so that the movement of charges took place over multiple iteration (Figure 4). Level 3 (circuits with a battery) contained circuits having a battery, a resistor, and (in some circuits) a switch, some of which were open (incomplete circuits) and others of which were closed (complete circuits) (Figure 5). Level 4 (series circuits) contained circuits having a battery, a switch, and two or more resistors connected in series. Level 5 (parallel circuits) contained circuits having a battery, a switch, and two resistors connected in parallel. Level 6 (series-parallel circuits) contained circuits having a battery, a switch, and three resistors, two of which were connected in parallel. In working with the simulation, students again followed a workbook which explained the purpose of the experiment they were carrying out, asked them to make predictions, had them run the simulation, and gave them explicit questions to answer about the results of the simulation. The questions were designed to draw out rules about the behavior of the simulation in each experiment. For example, in Level 2 (resistance), students developed a rule stating that the amount of current flow between two connected slices at each time step depends on the differences in the voltages of these two connected slices. The workbooks also drew their attention to the steady-state relations between voltages and currents in the circuit. For example, in Level 4 (series circuits), a simple series circuit was presented containing a battery and two resistors (similar to that at the left of Figure 2). The students' attention was drawn to the final voltage differences between adjacent slices of the resistors (which were all the same) and, consequently, to how the differences in the voltage drops across the two resistors are proportional to their length or resistance.

Aggregate Model (Steady-State Condition). In our control condition, the students were also presented with the aggregate model and a workbook which drew out the same qualitative principles used by that model for understanding the relationship between current and voltage in the final steady state of the circuit. The workbook also developed the same causal mechanism, that charge differences cause a current to flow. However, the instruction differed from that of the transient condition in that students did not see the transient states of the simulation and only saw the initial and final states. We used the same computer simulation based on the aggregate model, but set it so that the computer screen was not updated until the steady-state condition was reached. Thus, the instructional time was the same as that for the transient condition. In addition, step mode was not available and computer-generated explanations were accessible only for the initial and final states of the circuit. Thus, students were shown only the initial and final relationships among voltages and current in circuits, but were not given the derivations of those final relationships. The simulation used the same bar graphs and arrows to depict charges and current flows, but only for showing the initial and final states of the circuit.

Algebraic Model. Regardless of whether a group was shown the transient states of the aggregate model, both groups were given an identical set of workbooks for learning the algebraic model which incorporates quantitative circuit theory. Revisiting each of the circuits the students had already encountered in working with Levels 3–6 using the computer simulation, the quantitative workbook developed the quantitative circuit laws as more precise characterizations of the qualitative relations between resistance, voltage, and current which the aggregate model workbook had developed earlier. The algebraic model includes Ohm's law, the formulas for the resistance of resistors connected in series and in parallel and for currents and voltages in series and parallel circuits, as well as the procedures for using these formulas to derive voltages and currents for simple circuits containing a single resistor, series circuits, parallel circuits, and hybrid (series-parallel) circuits. The circuit laws were developed in the context of applying them in solving problems. Students were also taught strategies for applying the circuit laws as they solved circuit problems.

Results

Students' Initial Knowledge of Electricity

To gain an idea of the extent of students' knowledge of electricity prior to instruction, we begin by looking at the written responses of students on the Circuit Concepts Test, which was given as a pretest and posttest. In the Circuit Concepts Test, students were asked to write brief explanations of electricity terms or processes. In their responses, few students showed knowledge of electric forces and their role in an electric circuit. For example, in answer to the question, "What causes electrical charge to flow?" only 24% mentioned voltage or an electrical force as driving current. Nor did the students understand the need for a closed circuit. When they were asked, “Do you need a closed circuit for electrical charge to flow?” only 16% recognized the importance of there being a complete circuit. However, there were two other items for which relatively higher percentages of students were able to respond correctly. One of these asked for a definition of charge flow (34% of the students wrote a correct answer) and the other asked for a definition of resistance (54% of students were correct). Thus, prior to instruction few students showed an understanding of voltage, its role within a circuit, and its implications for why a closed circuit is needed. However, a larger proportion of students appeared to have an idea of what charge flow represents and of the effect of resistance on charge flow.

While their pretest knowledge of circuit electricity was minimal, students’ responses to items on another pretest, the Particle Model Test, suggest that they began instruction with a reasonable knowledge of electrons as charged particles which repel one another. When they were asked in one test item, “What does it mean for an object to be negatively charged?” 49% gave a response that it had “too many electrons for its number of protons.” On another item which asked, “Do similarly charged particles attract or repel one another?” 85% of the students gave a correct response. However, on items which probed their understanding of conductivity and resistance, students were less clear. For instance, when they were asked, “What causes electrons to move within a conductor?” only 25% gave responses we scored as correct. On an item which asked, “Why do electrons move more slowly in a resistor than in a conductor?” only 17% gave correct explanations. These data suggest that students entered the instructional experiment with a better knowledge of the behavior of electrically charged particles than they did of the operation of electric circuits, but that their understanding of the mechanisms of conductance and resistance was not well developed.

Overall Effects of Instruction

To evaluate the overall effects of instruction, we present an analysis of changes in performance for the set of assessments that were given as both pretests and posttests. These include the following: (a) The Particle Model Test focused on students’ knowledge of electrostatics by asking questions which require them to write a short answer. Questions include, for example: “What does it mean for an object to be positively or negatively charged or neutral?” “Do similarly charged particles attract or repel one another?” “What causes electrons to move within a conductor?” and “Why do electrons move more slowly in a resistor than in a conductor?” (b) The Circuit Behavior Test assessed students’ understanding of circuits by asking simple questions about circuits containing a battery, a switch, and one or more light bulbs. For example, in one item students were shown a diagram of a circuit in which a switch and a light bulb were connected in parallel across a battery. For the cases in which the switch was either open or closed, they were asked which elements in the circuit would or would not have voltages across them or currents through them, and also whether the light bulb would be off or on. (c) In the Circuit Concepts Test, students were asked to write brief explanations of electricity terms or processes. Examples of questions are: “What causes electrical charge to flow?” and “Do you need a closed circuit for electrical charge to flow?” In the pretests, we intentionally used questions that we thought even novices had a chance of answering correctly. In the posttests, we repeated those questions and added more challenging ones. In the pre/posttest comparisons, however, we include only results from questions which were identical across the two test forms.

The results of these three assessments are shown in Figure 7. It is clear that the students made significant gains on all three assessments, indicating that they developed a knowledge of both electrostatics and the behavior of circuits. There were no significant differences between the two experimental groups in their scores for these assessments. Both groups improved in their

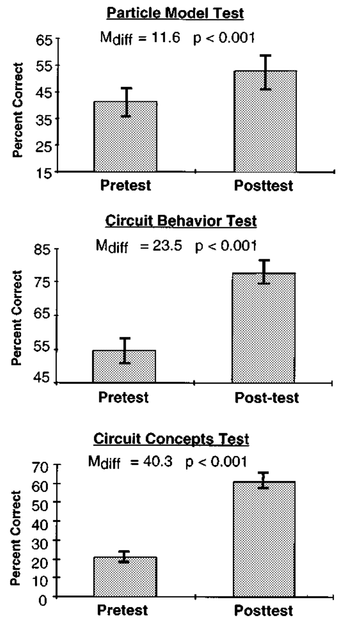


Figure 7. Pretest verus posttest results for students in the two groups combined.

understanding of the particle model, $F(1, 30) = 18.6$, $p < .001$, in their understanding of circuit concepts, $F(1, 30) = 101.7$, $p < .001$, and in their ability to predict voltages, currents, and the state of a light bulb (on or off) in a variety of circuit contexts presented in the Circuit Behavior Test, $F(1, 30) = 80.1$, $p < .001$. The effect sizes, measured in standard deviation units, are large; they are, respectively, 1.1, 2.5, and 2.5 σ.

Effects of Instructional Treatments

We now turn to the analyses of differences between the transient and steady-state groups on the posttests. The posttests included all the pretest questions as well as additional, more difficult problems. Problems in the posttests addressed students' understanding of all three models developed in the curriculum, and thus were designed to reveal differences in knowledge gained by the two groups. The results for the two treatment groups on each of the posttests are presented in Figures 8–11. For each posttest, a sample item is also shown.

The Particle Model Assessment. Our first assessment has to do with the students' understanding of the particle model. The assessment includes problems which directly test their knowledge of the particle model, such as how well they can predict and draw the final distribution of a set of electrons for two connected sections of conductive material, along with short answer questions which were described previously (Figure 8). The questions and problems were chosen to test their understanding of the electrostatics model in which electrons continue to move until they are equally distributed across the two connected sections of conductive material. Our hypothesis was that there would be no difference between the groups for this assessment, since each group was given the same particle model instruction prior to their exposure to

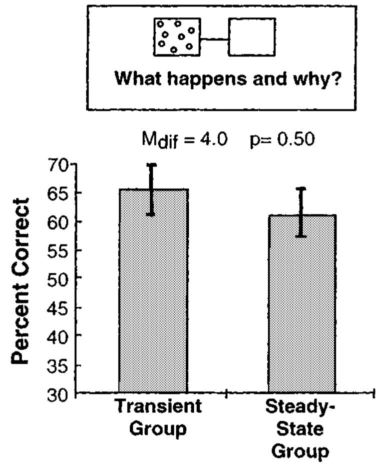


Figure 8. Comparison of transient and steady-state groups on the particle model assessment.

the portion of the curriculum that differed for the two groups. This turned out to be the case. There was no significant difference in the two groups' posttest scores on the particle test, $t(29) = .69$, $p = .50$, two-tailed. Both groups appeared to have mastered the particle model to the same degree.

The form of understanding students developed can be illustrated by the responses of a typical student to the assessment problems. For instance, when asked, "If we connect the two unconnected nodes shown below (Figure 8), what would happen to the concentration of electrons in each node?" the student wrote, "The concentration in the left node would go down, the concentration in the right node would go up. They want to be as far away from each other as they can." When the same student was then asked, "Why is there no 'net flow' between the nodes after many cycles?" she answered, "Because for every e^- which moves from left to right, another moves from right to left." After completing a problem involving resistive materials, she was

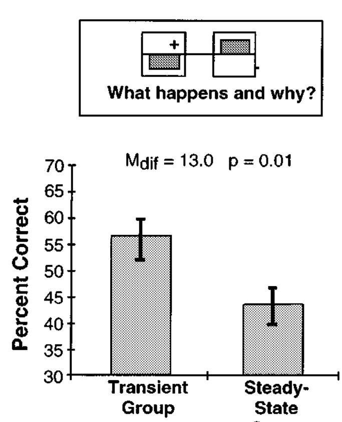


Figure 9. Comparison of transient and steady-state groups on the aggregate model assessment.

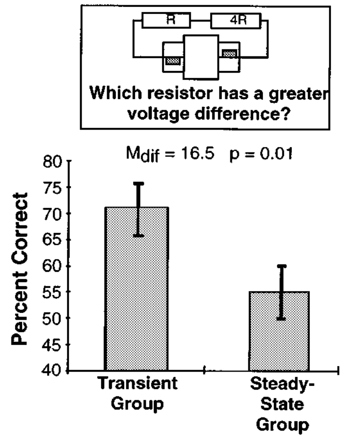


Figure 10. Comparison of transient and steady-state groups on the qualitative reasoning assessment.

asked, "Will it take the same amount of time for the electrons in this case to reach a steady state, as was the case in Problem 9 (where the connected nodes were not resistive)?" In response, she wrote, "It will take longer, because the e^- are trying to flow in resistive materials." Students such as this have a good understanding of the connection between the mutual repulsions among particles and their tendency as a group to drift apart until they reach equal numbers in each area. It is also interesting that students often invoke a notion of agency in describing electrons' behavior, as this student did when she said, "they want to be as far away from each other as possible."

Aggregate Model Assessment. Our second assessment addresses differences in the two groups' understanding of the aggregate model. In the assessment, students were given configurations of two or three connected areas with various charges (including no charge) on them and were asked to predict the final amounts of charge on each section after they were connected. In

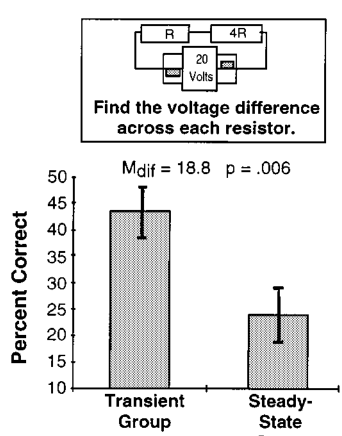


Figure 11. Comparison of transient and steady-state groups on the quantitative reasoning assessment.

the test items, charges were depicted using the bar graph representation. The items also asked students to explain their responses. This assessment may be regarded as addressing the core of the treatment difference. Students in the transient group learned about the aggregate model by seeing it actually run through the transient state until it reached the steady state. These students could directly observe the local effects of charge differences on current flow, since these processes were dynamically represented during the running of the simulation. Students in the steady-state group also learned that charges spread out until each connected section of the resistor has an equal share of the initial charge. However, since they were shown only the starting and ending states of the simulation, their knowledge of the dynamic process could only be gained by imagining the possible behavior of the simulation during the transition from its initial to its final state. One way they could do this was by envisioning how the particle model would behave if it were being run for the same situation. In fact, this was strongly encouraged by presenting both a particle model and an aggregate model simulation (in that order) for the initial situations explored in Levels 1 and 2 of the curriculum (as in Figures 3 and 4). However, envisioning the behavior of the particle model becomes much more difficult when the complexity of circuits is increased (e.g., when they involve both positively and negatively charged areas, or when the circuits contain many components). Our hypothesis was therefore that the students in the transient group would be better able to predict steady-state distributions of charges and current flow because their rules for describing the steady state had been directly supported by experience with the causal, transient-state mechanism.

The results bearing on this hypothesis are presented in Figure 9. Students in the transient group significantly outperformed those in the steady-state group on questions in which they had to envision the current flow and resulting charge distributions when two electrically charged areas were connected, $t(29) = 2.56$, $p = .01$, one-tailed, which represents an effect size of .9 σ. Note that while these questions asked students to describe the final (steady) state in each situation, they also asked students to justify their answers, and these explanations also contributed to their score. Using the aggregate model simulation dynamically clearly helped students see how differences in the charge distributions lead to current flows which in turn alter the charge distributions. Moreover, the students saw how over time this cyclical process leads to a steady state. The next issue was to test whether understanding this process helped students to develop a better understanding of circuit behavior and of the role that voltage differences play in causing charge to flow.

Qualitative Reasoning Assessment. Since the transient group students developed a better understanding of the aggregate model, it was our expectation that these students should also be better able to solve qualitative circuit problems than students in the steady-state group. In solving qualitative circuit problems, students were asked to predict the relative magnitudes of voltages and currents in circuits after they reached a steady state. For example, in these problems they needed to think about the relative magnitudes of current and voltage in different parts of a single circuit (see Figure 2 for an example). We should note that during instruction, both groups of students were given the same set of the workbooks that drew out a set of qualitative rules for predicting voltage differences and currents when a circuit has reached the steady state. The only difference was that students in the transient group were shown the process by which, starting with an initial configuration of voltages, the iterative application of the flow equation leads to a final state of the circuit in which voltages and currents follow those laws. The transient group students therefore should have been more able to see the causal relationship between voltage differences and current flow and to apply these ideas in interpreting the qualitative circuit laws.

They also should have had a better conception of the distinction between the transient and steady states of a circuit, and of the notion of constraints (circuit laws) as a means of describing the unchanging features of the steady-state world. We therefore predicted that students in the transient group would better understand and remember the qualitative, constraint rules needed to solve relative magnitudes problems.

The results are shown in Figure 10. Students in the transient group significantly outperformed those in the dteady-state group on problems which required qualitative reasoning about the magnitudes of voltages and currents in circuits, t(29) = 2.40, p = .01, one-tailed, which represents an effect size of .9 σ. This result supports our hypothesis that a deeper understanding of the aggregate model, of the importance and role of voltage, and of the notion of a steady state will lead to better performance in reasoning qualitatively about the relative magnitudes of voltages and currents in different circuits or branches of a single circuit. Students had a better understanding of the qualitative, steady-state relationships when they saw how those relations emerged from causal processes.

Quantitative Reasoning Assessment. Finally, we predicted that students in the transient group would outperform those in the steady-state group in solving quantitative circuit problems. With their greater understanding of the relationship of voltage difference and current flow, the transient group students should have developed a more principled approach to solving quantitative circuit problems. They should have viewed the equations as a coherent extension of their qualitative reasoning, rather than as simply a set of tricks for solving problems. More specifically, their better understanding of the qualitative relationship that voltage differences cause charge flows should have allowed them to map the *V* in Ohm's law to a fundamental causal principle in circuit operation, and thus enabled them to more meaningfully apply the equation. Results bearing on this hypothesis are given in Figure 11. There was a significant difference between groups on quantitative problems in which students had to calculate voltages and currents for various circuits, $t(29) = 2.69$, $p = .006$, one-tailed, which represents an effect size of 2.7 σ. These data clearly indicate that learning about the transient state helped students make sense of the formulas, because they saw the origins of those equations in the simulation of transient circuit behavior. When the formulas have meaning for students, the students are better able to apply them in a principled way when solving problems.

In addition, we found evidence that the provision of model-based derivations of the steady-state circuit characteristics is particularly effective in enabling students to solve the problems that are usually the most difficult—those which involve reasoning about voltages. We analyzed the effects of the two instructional treatments separately on students' solutions of quantitative problems involving voltage and current. For voltage problems, the means for the transient and steady-state groups were 48% and 25% correct, respectively, with a difference of 24%, $t(29) = 2.33$, $p = .01$, one-tailed. For current problems, the means for the transient and steady-state groups were 37% and 22% correct, respectively, with a difference of 15%, $t(29) = 1.93$, $p = .03$, one-tailed. We conclude that providing derivations of the steady-state circuit conditions helps students develop a clearer understanding of voltage and how it is the fundamental concept needed in analyzing and solving circuit problems.

Gender Effects. For each of our analyses, we carried out analyses of effects due to gender. There were no significant effects of gender in any of the analyses, nor were there any interactions of gender with treatment. Male and female students appeared to profit equally from the curriculum.

Discussion

Taken together, the results support our general hypothesis about the utility of enabling students to derive conceptual linkages among models which represent physical phenomena at increasing levels of abstraction. In particular, our results demonstrate that such a model progression can create a bridge between students' understanding of electrostatics and electrodynamics. Prior to instruction, our students showed little understanding of voltage and its role within a circuit, and they showed little understanding of why one needs a closed circuit for a current to flow. At the same time, however, they began with an intuitive understanding of the particle model, the concept of charge flow, and resistance to flow. Experience carrying out experiments using the aggregate model illustrated how dynamic steady states emerge from transient electrostatic interactions. Working with this model enabled students to parse the evolution from transient to steady-state circuit behavior into a sequence of cause–effect events. These events were themselves understood as the emergent behaviors of the less abstract, more mechanistic particle model which was presented to students at the beginning of the instruction. This model progression enabled students to see how circuit laws describe the emergent behavior of a system that is behaving according to more fundamental physical laws. Moreover, understanding this progression had an impact on their ability to solve both qualitative and quantitative circuit problems. These findings raise several important issues for further analysis.

How Does Seeing the Transient States of a Circuit Promote the Learning and Use of the Aggregate Model?

Reasoning with the aggregate model provides a simple way for thinking about the effects of voltage differences on the flow of charge, and links this idea to the more complex problem of envisioning the behavior of charged particles as depicted in the particle model. Dynamic simulations illustrate the changes in current flow that occur as voltage differences vary, and thus create a clearer appreciation of the causal linkage between voltage differences and current. For these reasons, we believe that students in the transient group focused more on voltage difference as the important variable in the ggregate model, whereas students in the steady-state group focused more on current in making predictions and explanations of charge behavior, and they did not understand current or voltage well. We can illustrate this with some typical responses of students in each group to a question from the aggregate model assessment presented in Figure 12. One student in the transient group responded by saying, "Because it shows that they (the two resistor sections) have the same amount of negative charge and have no voltage difference." In contrast, a student in the steady-state group responded, "Because at a steady state there is no more electrical current flowing." Although both answers and justifications are correct, they differ in their focus. The transient group student described the lack of voltage difference in the final state to justify her answer, while the steady-state group student concentrated on the lack of current.[3] These responses were typical of the two groups. Experience in observing the transient states of a circuit appeared to facilitate students' understanding of the causal agent in the aggregate model: namely, voltage difference.

What Is the Process by Which Seeing the Transient States of a Circuit Promotes the Learning and use of the Qualitative Circuit Laws?

We illustrate this process with excerpts from a transcript of a pair of students in the transient group who were working with the aggregate model computer simulation and trying to an-

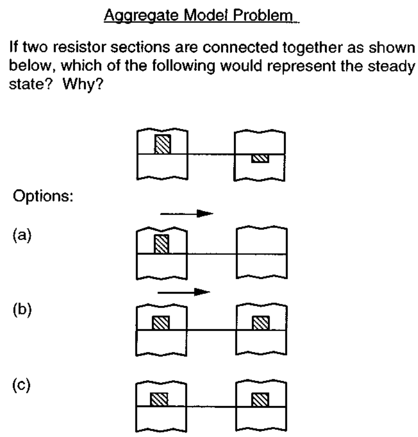


Figure 12. Problem from the aggregate model assessment.

swer questions in the workbook. They were at the crucial point in the curriculum where they were about to encounter for the first time a dynamic equilibrium, reached when a complete circuit containing a battery and a resistor reaches a steady state. The circuit they were working on is shown in Figure 13, which shows the initial state of the circuit presented in the workbook along with the question they were answering at the time of the protocol. In making their pre-

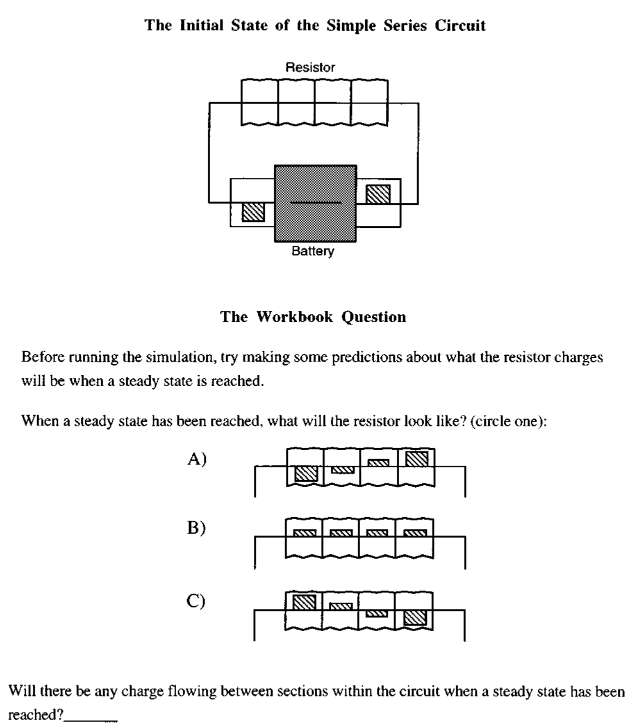


Figure 13. Simple resistor circuit and workbook question that students were discussing in the protocol.

diction, they chose Option B and were about to run the simulation in continuous mode (in continuous mode, the simulation will continue running until a steady state is reached).

Jerry: All right, continuous mode.
David: Wait, wait, wait. Will there be any charge flowing between sections within the circuit when a steady state has been reached?
Jerry: Um, no.
David: I'm gonna say no. I don't think there would be. Unless we're completely wrong.

Here they have made their prediction that no current will flow in the steady state. The students ran the computer simulation in continuous mode. The final state of the circuit they saw on the screen is shown in Figure 14.

Jerry: Go, go, go, go.
David: S—, go back, go back. Oh, it's gonna . . .
Jerry: Oh no.
David: It's gonna be like A [Option A of Figure 10].
Jerry: A. Oh! Why is this? No, no, no.
David: It still has a long way to go; look how much is flowing.
Jerry: Oh, it's looking good.
David: Please . . .
Jerry: That's going down. It's going too slowly, 0.7.
David: It's going down. I don't understand why it's going down.

They were rooting for the computer simulation to behave as they predicted and were agitated that it did not. Further on, when the computer had reached its steady state, they continued:

David: Oh, no.
Jerry: This makes absolutely no sense.
David: Oh, yes it does!
Jerry: Oooh, why?

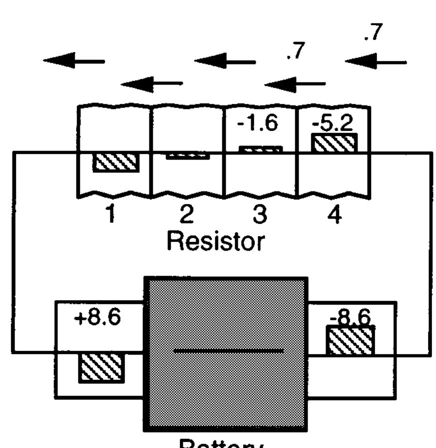


Figure 14. Final state of the computer simulation for the workbook problem discussed in the protocol. Values shown in the computer simulation are those referred to by the students.

David: Why? Because the battery has reached it. See the battery's equal now? [Jerry: Uhhmm.] So nothing is gonna go anywhere.
Jerry: But there's not—
David: But that doesn't change the fact that these two are different. [Points to negative terminal of battery and Section 4 of the resistor.]
Jerry: It makes no sense.

After noting that the positive and negative battery voltage were the same, they recognized that the charges within the circuit were not all the same. A few moments later, they continued:

Jerry: Do you still see? There's still charge flowing.
David: So how can it be steady state? If there's—
Jerry: I think I get it.
David: 'Cause there's no *change* in the charge of any section.

They were concerned that charge was still flowing in the steady state. Then, David realized that it was the charges that were no longer changing. In what follows, Jerry had an insight and made some calculations using the flow equation (the numbers 8.6 and 5.2 referred to are charges in the circuit, and 0.7 is the value of the current flow) (see Figure 14):

Jerry: Oh, I got it! I got it. Watch this.
David: Well, tell me.
Jerry This is really cool, 8.6.
[David cracks a joke.]
Jerry: 8.6 minus 5.2 equals 3.4 divided by 5 equals 0.7.
David: So? So what are you telling me?
Jerry: This [points to number on the negative battery terminal] minus this [points to number on section 4] divided by 5 is 0.7. This [points to Section 4] minus this [points to Section 3] divided by 5 is 0.7.
David: Are you sure? Try it. Try 5.2 minus—
Jerry: This [points to Section 4] minus this [points to Section 3] divided by 5 is 0.7. It's cool.

Jerry saw that there were currents in the steady state because there were differences in charges between adjacent sections of the resistor. He also saw that currents were the same because the voltage differences were all the same. He checked this by calculating currents for several places in the circuit using the flow equation ($I = (1/5) \times \Delta V$). David and Jerry went on to summarize their findings:

David: Well, well, I know that one fifth is gonna be flowing because that's the rule from the resistor, that they've given us about the resistors. But what does that tell us?
Jerry: That tells us that charge is still flowing at . . . uh. This is what I thought it would be, all moving the same in relation to one another.
David: Right.

While they derived the result that currents were everywhere the same, they saw another problem: Why were the charges on the sections of the resistor no longer changing?

Jerry: So, flow is coming [in]—this is the only one I don't understand.
David: But the flow is coming [in] at the same time that it's balancing out. I see. 'Cause the charge is just shifting.

Jerry: Yeah. [David: It's not—] And this is coming, this [points to the negative terminal] is staying the same because this [points to the electrolyte of the battery] is giving it electrons, but this—
David: This tells us right here. [He reads the computer message.] "A steady state has been reached because there's no *change* in the charge of any section." That doesn't mean the charge isn't flowing, it just means there's no change in the charge.
Jerry: That's why you've been hearing that so much and we don't get it. Oh!
David: Thank you, Mr. Computer.
Jerry: That's funny. It's like, we've heard that like a hundred times and we don't get it.
David: We haven't really listened to it.

They realized that on each time step, the flows into and out of a slice balance, leaving the amount of charge unchanged. Later, prompted by a question in the workbook, they tried to write down why they made the wrong prediction:

David: Okay, first of all, why did we think that they [the charges] would be all equal?
Jerry: Because, um, in a closed circuit, in a charged—
David: In a closed circuit, or . . . ?
Jerry: In static electricity, that's what it is. We're used to dealing with static electricity. We have a node that has a 2 charge on it, and you stick it in a circuit, you know what I mean?
David: Just stick it in, chhhk.
Jerry: Yeah, and so. Don't you know, like, like in static electricity that, that—that the charge has to be equally distributed? [David: No, I never heard–] That's static, that's static electricity.
David: Static equilibrium.
Jerry: No, that—yeah, when they're all equal. But this is [David: Is dynamic equilibrium.] dynamic equilibrium. Static electricity this is not. This is what I really thought—this is what real electricity is.

They completed the bridge from electrostatics to electrodynamics. Still later, they pinned down the reason why, if the circuit were open, charge would not flow. Again, they used the flow equation of the aggregate model in solving this problem:

Jerry: I don't know. See, if they were like B [points to Option B] [see Figure 13], and this charge was flowing through [traces circular flow] [David: Yeah.], then they wouldn't stay like that.
David: But you . . .
Jerry: See. If 0.7 went from here (Section 4) to here (Section 3), they'd be 1.4 apart and they wouldn't be equal like B anymore.
David: Oh, you know what?
Jerry: What?
David: If they're all equal, electricity couldn't flow, because you'd have to take one fifth of zero.
Jerry: Ah, that's right, that's right!

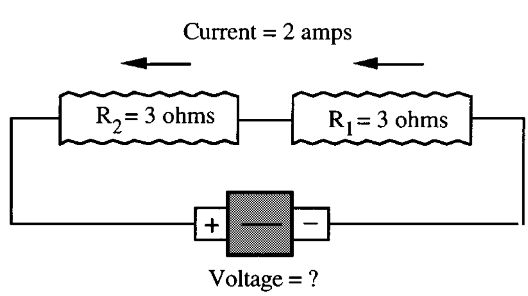
In this session, they traced through the basis of the model derivation, carrying out many calculations in the process. Through this model-based derivation, they were able to link the aggregate model to the dynamic equilibrium of the complete circuit when it reached the steady state. While not all of the students were as articulate as David and Jerry, this protocol increases our confidence that the instructional materials we provide engage students in a process of developing the types of mental models we have sought to have them develop along with their interrelationships using the derivational process.

Why Does Learning Using Dynamic, Model-Based Derivations of the States of a Circuit Have an Impact on Solving Quantitative Circuit Problems?

We have seen that the transient state simulation helps students make the connection between electrostatic interactions and steady-state circuit behavior. The transient state runs on the electrostatic principle that voltage differences cause charge flows. When applied to a circuit context, this principle leads to a dynamic steady state in which the battery's constant voltage drives a steady current through the circuit. By understanding the origins of this dynamic steady state, students are better prepared to understand the steady-state relations among circuit variables. Thus, students' experience with the dynamic simulation should affect their learning to use formulas in solving quantitative problems. When the quantitative circuit formulas were presented in the workbooks in the context of solving circuit problems, they should have made more sense to students in the transient group than to those in the steady-state group. Since the equations had greater meaning, these students should have made fewer errors in choosing and applying formulas during problem solving. Finally, if they were using formulas with understanding, their use of formulas should have been more innovative and less dependent on following the particular problem-solving procedures shown to them in the workbooks.

To illustrate these differences in students' use of the circuit equations, consider the quantitative problem shown in Figure 15. This voltage divider problem was presented in the instructional workbooks. Students had just finished reading and working through a similar problem which explained how to solve such problems by the following method, which we shall refer to as Method 1:

1. First, find the total resistance using $R_T = R_1 + R_2$.
2. Then, find the battery voltage using $V_T = I \times R_T$ (Ohm's law).
3. Next, find the proportions of the total resistance for each resistor in the series circuit using Proportion $R_1 = R_1/R_T$, and Proportion $R_2 = R_2/R_T$.
4. Finally, using those proportions, find the voltage across each resistor using $V_1 = V_T \times (R_1/R_T)$ and $V_2 = V_T \times (R_2/R_T)$.



Given the circuit above, find the voltage difference across each resistor. (Hint: First you have to find the battery's voltage) (Please show all work)

Figure 15. Quantitative workbook problem involving series resistors.

In the workbook problem, the students were asked on their own to find the voltage difference across each resistor in the series circuit. Recall that during instruction, students worked together in pairs, and that there were eight pairs of students in each treatment group.

First, we consider the pairs of students in the transient group. Of these, all eight solved the workbook problem correctly. However, what is more interesting is that three of the eight pairs (37%) used a method other than Method 1 for solving the problem, which we shall call Method 2:

1. First, find the total resistance using $R_T = R_1 + R_1$.
2. Then, find the battery voltage using $V_T = I \times R_T$ (Ohm's law).
3. Finally, apply Ohm's law to find the voltage across each resistor using $V_1 = I \times R_1$ and $V_2 = I \times R_2$.

This method is based on the fact that having been given the current, they can calculate the voltage across each resistor directly with Ohm's law without having to calculate the proportions of total resistance called for in Method 1. One of these pairs of students actually solved the problem using both methods. We take this as evidence that students in the transient group show flexibility in using the circuit equations and do not always rely on following the problem-solving procedure that has been demonstrated for them in the workbook.

Second, we consider the pairs of students in the steady-state group. Of these, two failed to successfully solve the problem, even though they had just been taken through the solution to a similar voltage divider problem in their workbooks. When we look at their methods of solution, all eight attempted to use or successfully used Method 1, the method they had just been shown. It is instructive to look in more detail at the attempted solutions of the two pairs who were unsuccessful in solving the problem. They both ran into the same difficulty. They present, next to one another, two sets of calculations:

R_1 = 3 Ohms/6 Ohms	$V = I \times R$
R_1 = ½ Ohm	$V = 2 \times 6$
R_2 = 3 Ohms/6 Ohms	$V = 12$
R_2 = ½ Ohm	

On the left, they calculated the proportion of the total resistance for each resistor but mislabeled it as the resistance (and given it units of Ohms). Thus, they did not clearly understand that these are proportions and that they are scaleless quantities. Second, they correctly calculated the total voltage using Ohm's law (and implicitly calculated the total resistance of six), but they failed to apply the proportions they calculated to the total voltage to complete the problem. They could find the proportions of total resistance for each resistor, but did not go on to use these proportions to actually find the required voltages. It appears that they either lost sight of the important variable, voltage difference, or they did not understand what the formulas actually meant.

In either case, these examples support our view that by seeing the emergence of the steady state from the transient state, the transient group students made more sense of the steady-state circuit equations, and this allowed them to develop sensible solutions to problems rather than follow by rote the procedures they were taught. This also enabled them to understand some fairly difficult computational tricks in circuit theory, such as the notion of finding proportions of one quantity (resistance) and using those proportions to calculate another (voltage drops). In contrast, while steady-state students may have got some of the calculations correct, in the process they mistook the meaning of quantities they calculated and lost track of how they should have been combined to find the quantities they were after.

Summary and Conclusions

In conclusion, our findings support our general theory about the importance of enabling students to derive conceptual linkages among a sequence of models which represent physical phenomena at increasing levels of abstraction. In particular, our findings demonstrate how enabling students to construct derivational linkages among models enables them to understand the origins of circuit theory and, consequently, to apply the circuit laws in solving qualitative and quantitative problems. We present evidence that learning how to represent and reason about the transient behavior of circuits helps students learn the steady-state circuit relationships and effectively bridges the gap between electrostatics and electrodynamics. In our approach, students learned to envision circuit behavior as a sequence of cause–effect events which obey fundamental laws of physics, and they saw how the steady-state circuit laws emerge from such iterative processes. This understanding led to improved performance in solving both qualitative and quantitative problems about current and voltage. A further benefit is that students developed a general understanding of the alternate forms that models of a single physical system may take, each focusing on different objects and interactions as elementary units of analysis, and each employing a different type of reasoning process. They also learned how models representing alternative perspectives on a physical system can nonetheless be coherently linked. Finally, they learned the nature of the mappings among models through the characterization of emergent properties of lower-level models in models at a higher level in an abstraction hierarchy.

Taken together, this theory and supporting empirical results argue for carefully considering the derivational relations among models that students acquire in instruction and for designing instruction so that students will create a coherent linkage among such models. These model derivations can serve as the “conceptual proofs” of science education, and can play a central role in making science more accessible and meaningful to students as well as in enabling them to acquire powerful, coherent problem-solving expertise.

This research was supported by the Educational Testing Service.

Notes

[1] Only advanced college textbooks introduce dynamic processes. At that level, they are presented in mathematical form using differential equations, making them difficult to understand for students who are not well versed in the calculus.

[2] This is in contrast to Sherwood and Chabay (1991), who advocated bridging between electrostatics and electrodynamics using a detailed model of charge distributions at the particle level.

[3] We note that in a closed circuit, the steady state includes a constant current. This is incompatible with the steady-state student’s conception of the steady state as one in which no current is flowing. Such an incorrect view may explain why these students have difficulty connecting static electrical phenomena with dynamic circuit behavior.

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