1. Vorticity
   
   See DPO chapter 8.6.

2. Sverdrup balance

   The Ekman layer transports drive circulation in the ocean interior through the Sverdrup transport mechanism. The Ekman layer is the thin frictional layer at the top of the ocean, driven directly by the wind. The whole interior ocean is treated as inviscid, that is without friction. (See study questions for why you can do this.)

   Since the wind varies, for instance with latitude (westerlies and trades), the Ekman transport at right angles to the wind also varies. This means there are convergences and divergences in the (horizontal) Ekman transport. Where there is convergence, there must be vertical velocity downward, escaping from the Ekman layer. Where there is divergence, there must be vertical velocity upward, feeding the Ekman transport. We think of these downward and upward velocities as "squashing" or "stretching" the water column beneath the Ekman layer (i.e. most of the ocean depth).

   What is the effect of stretching or squashing? The ocean is a rotating fluid (rotating because of the earth). Therefore it has lots of angular momentum. Stretching and squashing act on the angular momentum, like a spinning skater pulling in or spreading out her arms, and hence spinning faster or more slowly. (conservation of angular momentum, involving the rotation rate and the moment of inertia, which has to do with how tall/thin or short/fat the spinning body is).

   The only important component of the angular momentum (vorticity) of the ocean water columns is the local vertical component. This is because the water layers involved in the general circulation are very thin and vertically stratified compared with their horizontal extent, so that the horizontal velocities (parallel to surface of earth) are much stronger than the vertical velocities (order cm/sec versus $10^{-4}$ cm/sec). The angular momentum (vorticity) has two separate and important components - one is due to local "vorticity" in the flow itself (strong eddying) and the other is due to just the rotation of the earth, which gives everything on the earth angular momentum. These two pieces are called the relative vorticity and the planetary vorticity.
In the general circulation, relative vorticity is not important over most of the ocean, with the only notable exceptions being in the very strong western boundary currents and in the east-west equatorial currents. The planetary vorticity (local vertical component) is largest and positive (in the sense of the right-hand rule) at the north pole. It is largest and negative at the south pole. It is zero on the equator. Therefore the important component of the angular momentum increases northward, from large/negative at the south pole through 0 at the equator to large/positive at the north pole.

When the frictional Ekman layer exports water downward (very small downward vertical velocity), at the top of the ocean, it squashes the water columns, which must then spin more slowly. This can be accomplished by either spinning more slowly in the location of the water column ("inducing negative relative vorticity"), or moving to another latitude where the local rotation rate (parallel to the local vertical) is slower - this would be equatorward. If on the other hand there is Ekman divergence, then the water columns are "stretched", and the column must spin faster, either in place or by moving to higher latitude. Since the general circulation is very large scale and not filled with ever-increasing vortices, the response is to move in latitude rather than to spin up locally. This response to Ekman pumping or suction is called the "Sverdrup flow". and the flow associated with the response is called "Sverdrup transport". Sverdrup transport is equatorward in subtropical regions of Ekman pumping and poleward in subpolar regions of Ekman suction.

While the direction of Sverdrup flow (equatorward or poleward) is dictated by the Ekman pumping, the flow itself is geostrophic. That is, an equatorward flow means that the sea surface is high in the west and low in the east and vice versa. This is an important point - the force balance is still between pressure gradient force and Coriolis force. The Ekman pumping is a very small and the changes induced in the angular momentum (vorticity) are very weak. These set up the pressure gradient that drives the geostrophic flow. Thus the small terms in the momentum equation, that we neglect in geostrophic balance, are important when thinking about the very slow evolution of the flow or just how the forcing by the Ekman pumping (hence frictional stress of the wind on the sea surface) are communicated to the ocean.

3. Western boundary currents.

The net Sverdrup transport added up across the ocean (along a parallel of latitude) must be returned in a narrow flow somewhere. The transport of this narrow flow is the same and in the opposite direction to the transport across the whole ocean width; therefore the velocities are much larger than in the ocean interior.

The return flow must ALWAYS be on the western side, regardless of northern or southern hemisphere. Why? Consider the subtropical gyres, where Sverdrup transport is equatorward. The wind through friction is putting "negative" vorticity into the subtropical ocean, through downward Ekman pumping that send the water columns towards lower rotation, e.g. towards the equator. For a steady state, which the general circulation is, this vorticity must be removed somewhere. It is removed through friction in a narrow boundary current. Why "narrow"?
Because friction acts best on high of velocity, which you would get in a narrow flow, and because the return flow has to be narrow enough to escape forcing by the Ekman pumping. A frictional boundary current has zero velocity along the boundary, and strong velocity offshore. If the boundary current for the northern hemisphere subtropical gyre is on the west, then the relative vorticity of the boundary current is positive (paddlewheel sense). Thus the frictional boundary current puts positive vorticity into the ocean, which balances the negative vorticity put in by the Ekman pumping. This exact argument works for the low pressure regions as well (subpolar gyre, with northward Sverdrup transport and southward western boundary current). It also works in the southern hemisphere.

The resulting strong, narrow western boundary currents returning all of the interior transport back to the latitude where it started. The Gulf Stream, Kuroshio, Oyashio, Labrador Current, etc etc, presented at the beginning of this lecture, are examples of such wind-driven circulation western boundary currents. Because of various other factors, including just inertia, the western boundary currents are generally stronger than is required to simply return the interior flow. This results in an overshoot of the western boundary current at the latitude where it should end if it were very frictional. Thus the Kuroshio and Gulf Stream, for instance, separate and flow far out to sea before finally dying out.

Study questions:

1. What are the common elements of the surface circulation found in the N. Pacific and N. Atlantic?

2. Name the major surface western boundary currents in these two oceans, indicate which way they flow and what gyre circulation they are a part of. What are typical maximum velocities and dynamic height differences for these boundary currents?

3. What horizontal and vertical structure is typical of the subtropical gyre?

4. How deep are the western boundary currents?

5. How deep is the typical wind-driven gyre, excluding the western boundary current?

6. Name the three contributing factors to potential vorticity.


8. Why is the return flow for the Sverdrup interior flow on the western boundary? Explain using potential vorticity arguments.

9. What is the difference between the unit "Sverdrup" and the term "Sverdrup balance" or "Sverdrup interior"?
10. If the Ekman transport southward across 20N in the Atlantic is 5 Sv and the Sverdrup transport southward across the same latitude is 50 Sv, what is the transport of the western boundary current?

11. In which parts of the ocean is it currently assumed that the Sverdrup balance is approximately correct? Where is it clearly incorrect?

Study calculations
1. If the Kuroshio Extension velocity is 10 cm/sec averaged from top to bottom, and the current width is 100km, assuming a depth of 5 km, what is its volume transport? Express transport in Sverdrups. What is its approximate mass transport assuming a constant density?

2. If the Ekman transport at 30N in the Atlantic is -2 Sv southward and the Ekman transport at 20N is -5 Sv southward, what is the average vertical velocity at the base of the Ekman layers between 20N and 30N? Assume that the Atlantic is 5000 km wide.

3. Using the vertical velocity calculated for (2), calculate the southward Sverdrup interior transport at the mid-point (25N).

1 Sv = 1x10^6 m^3/sec.

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