Available online at www.sciencedirect.com
SCIENCE DIRECT
ELSEVIER
Fisheries Research 77 (2006) 220–225
FISHERIES RESEARCH
www.elsevier.com/locate/fishres

Comparison of Akaike information criterion (AIC) and Bayesian information criterion (BIC) in selection of stock–recruitment relationships

Yanjun Wang *, Qun Liu

Department of Fisheries, Ocean University of China, No. 5 Yushan Road, Qingdao 266003, China

Received 20 May 2004; received in revised form 18 August 2005; accepted 19 August 2005

Abstract

Variations in environmental variables and measurement errors often result in large and heterogeneous variations in fitting fish stock–recruitment (SR) data to a SR statistical model. In this paper, the maximum likelihood method was used to fit the six statistical SR models on six sets of simulated SR data. The best relationships were selected using the Akaike information criterion (AIC) and Bayesian information criterion (BIC) methods, respectively. Which have the advantage of testing the significance of the difference between the functions of different model specifications. The exercises were also conducted on eight sets of real fisheries SR data. The results showed that both AIC and BIC are valid in selecting the most suitable SR relationship. As far as the nested models are concerned, BIC is better than AIC.
© 2005 Elsevier B.V. All rights reserved.

Keywords: Stock–recruitment model; Maximum likelihood; AIC; BIC

1. Introduction

Stock–recruitment (SR) relationship is fundamental for fish stock assessment and management (Ricker, 1975; Hilborn and Walters, 1992). It can be used to answer some important management questions such as whether recruitment can fail owing to over-exploitation (Cushing, 1981). The SR models are often used to derive important management quantities which are used as reference points for managing fish stocks. However, the determination of the SR relationship is perhaps among the most difficult tasks in fisheries. Large variations in recruitment, large measurement errors in spawning stock size (errors-in-variable), and correlations between the present spawning stock size and previous recruitment values (time series bias) can cause large uncertainties (bias and variability) in parameter estimates (Hilborn and Walters, 1992).

Another source of uncertainty in applying the SR relationship is caused by choosing a particular form of SR curves to fit the SR data (model structural error). There are many functional forms of stock–recruitment models in literature (Quinn and Deriso, 1999). Since different SR models can result in quite different estimates of management quantities, thus greatly affecting the fisheries stock management and fish enhancement plans, one needs to select among the different SR relationships even when many of those fit the data almost equally well.

Hiramatsu et al. (1994) present a method for estimating the SR parameters using the maximum likelihood method and selecting the best model using the AIC, and apply this method to several real data sets. AIC has the advantage of testing the significance of the differences between the functions of different model specifications (Akaike, 1973). Sakamoto et al. (1986) describe an alternative to the AIC, called the BIC (Adkison et al., 1996), which is also a tool of selecting the best model.

The purpose of this paper is to test and compare the ability of AIC and BIC in selecting the true SR models by simulated SR data generated by the six SR models. Apart from the simulated data, eight sets of real fisheries SR data are chosen from the published papers. Their SR relationships are estimated

* Corresponding author. Tel.: +86 532 82032605; fax: +86 532 82032605.
E-mail address: wangyj@mail.ouc.edu.cn (Y. Wang).

0165-7836/$ – see front matter © 2005 Elsevier B.V. All rights reserved.
doi:10.1016/j.fishres.2005.08.011

by the maximum likelihood method and the best models are selected using AIC and BIC, respectively.

2. Method and data

2.1. Statistical model

The following six statistical models are defined, including both the linear and the non-linear models:

$$R = \alpha S \tag{1}$$

$$R = \alpha S\,\mathrm{e}^{-\beta S} \tag{2}$$

$$R = \frac{\alpha S}{1 + \beta S} \tag{3}$$

$$R = \alpha S^{\gamma} \tag{4}$$

$$R = \frac{\alpha S}{1 + (\beta S)^{\gamma}} \tag{5}$$

$$R = \alpha S^{\gamma}\,\mathrm{e}^{-\beta S} \tag{6}$$

where R is the recruitment size resulting from parental stock size S, and α, β and γ are the parameters of the SR equations. Model 1 is a linear relation (density-independent) model. Model 2 is the Ricker model (Ricker, 1954), when $\beta = 0$, it becomes the linear relation model. Model 3 is the Beverton–Holt model (Beverton and Holt, 1957), when $\beta = 0$, it also corresponds to the linear relation model. Model 4 is the Cushing model (Cushing, 1971, 1973), when $\gamma = 1$, it corresponds to the linear relation model. Model 5 is the Shepherd model (1982). The Beverton–Holt model is a special case of this general model when $\gamma = 1$. When $\gamma > 1$, the curve is dome-shaped like the Ricker model. And when $\gamma < 1$, the curve increases indefinitely like the Cushing model. Model 6 is the Gamma model. When $\gamma > 0$, the curve is dome-shaped like the Ricker model. When $\beta = 0$, the curve becomes the Cushing model. When γ approaches 0, the curve is similar to that of the Beverton–Holt model (Reish et al., 1985). Each model has from one to three parameters.

We assume R is log-normally distributed around the SR equation. Thus,

$$R = f(S)\,\mathrm{e}^{\varepsilon}$$

where $\varepsilon \sim N\ (0,\ \sigma^2)$. Because the variation in recruitment usually increases with stock size (the data points are more scattered for large stock size in a stock–recruitment scatter plot), the logarithm can transfer the data to the normal distribution and stabilize the variances. In theory, the recruitment process consists of many multiplicative processes from hatching to recruitment, thus, if a log is taken from the recruitment, the log recruitment is the sum of the logs of many processes. On the basis of the central limit theorem, log recruitment is normally distributed (Quinn and Deriso, 1999).

2.2. Simulated data

To evaluate the ability of AIC and BIC to select the best model, we treat the above models as operating models to simulate artificial SR data sets and then fit each of the simulated data sets using the above models, which are then treated as statistical models.

Six sets of artificial SR data are simulated with the true parameters of $\alpha = 10$, $\beta = 0.1$, $\gamma = 2$ using the six models, respectively. White noises of 20% (coefficient of variation) are superimposed on the simulated data. We generate 1000 sets of simulated artificial SR data using each of the six operating models. In total, 6×1000 data sets are simulated. For example, Fig. 1 shows the simulated data sets for one of the simulations.

2.3. Real fisheries data

Eight sets of real fisheries SR data are selected from the published papers. The sources of data used in this paper are shown in Table 1. The selected species include invertebrate, small pelagics, anadromous species, ground fish and large pelagic species. The measures of stock size include quantities such as the number of spawning stock, and the index of spawning stock size. The measures of recruitment are the relative number of recruits (Fig. 2).

2.4. Maximum likelihood method, AIC and BIC

In general, the maximum-likelihood estimators are to be preferred over the least squares because the maximum likelihood is based on careful considerations of how random “errors” arise and how they are distributed. Most reliable fisheries parameter estimations are currently being achieved by using the maximum likelihood method (Hilborn and Walters, 1992; Hilborn and Mangel, 1997). The basic idea behind maximum likelihood is to find the values of the parameters for which the observed data is most likely to occur.

Table 1
The eight sets of real fisheries SR data

Data no.	Populations	Source
1	Chesapeake Bay blue crab (*callinectes sapidus* Linnaeus)	Zhan (1995)
2	Bohai penaeid shrimp (*Penaeus orientalis* Kishinouye)	Deng et al. (1996)
3	Yellow Sea squid (*Loligo japonica* Steenstrup)	Qiu (1986)
4	Pacific sardine (*Sardinops sagax* Jenyns)	Larry and Alec (1995)
5	Peruvian anchovy (*Engraulis ringens* Jenyns)	Cury and Roy (1989)
6	Tillamook Bay chum salmon (*Oncorhynchus keta* Walbaum)	Ricker (1975)
7	Yellowfin tuna (*Thunnus albacares* Bonnaterre)	Wang and Tanaka (1988)
8	Flathead flounder (*Hippoglossoides elassodon* Jordan et Gilbert)	Dyakov (1995)

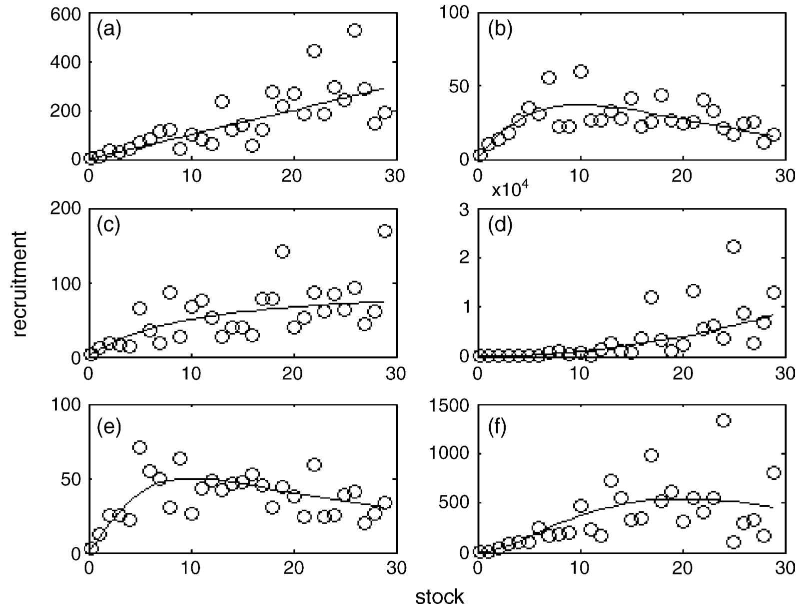

Fig. 1. Scatter plots of the six simulated stock–recruitment data set, the sources of the data are in Table 1. *X*-axis represents stock index and *Y*-axis represents recruitment index. (a) Data set simulated from the linear model. (b) Data set simulated from the Ricker model. (c) Data set simulated from the Beverton–Holt model. (d) Data set simulated from the Cushing model. (e) Data set simulated from the Shepherd model. (f) Data set simulated from the Gamma model.

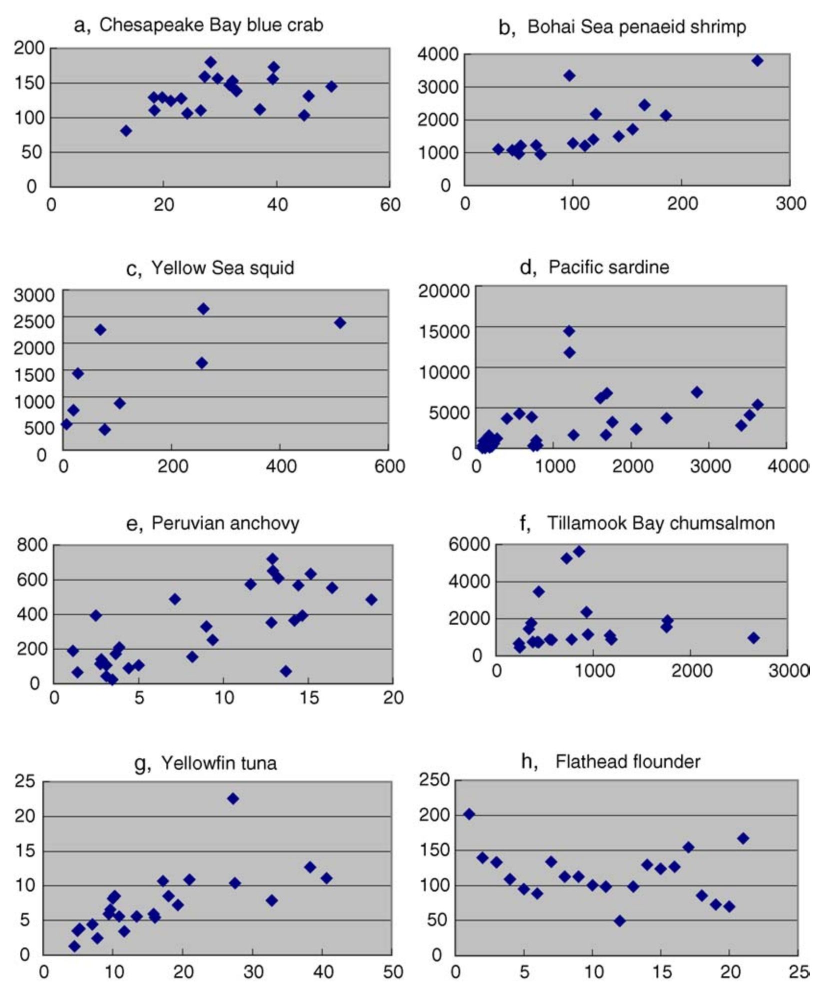

Fig. 2. Scatter plots of the eight real fisheries stock–recruitment data sets. *X*-axis represents stock index and *Y*-axis represents recruitment index.

For the above six models, the likelihood of any set of observation can be represented by the following equation:

$$L(D|\Theta) = \prod_{n}^{i=1} \frac{1}{R_i\sqrt{2\pi}\sigma} \exp\left(-\frac{(\ln(R_i) - \ln(\hat{R}_i))^2}{2\sigma^2}\right)$$

where $L(D|\Theta)$ is the likelihood of the data set D given the parameter vector Θ, Θ denotes (α, β, γ, σ^2) and R_i and $\hat{R}_i$ are the ith observed and predicted recruitment value of the data set D, respectively. Because the reciprocal of the observed recruitment value $\frac{1}{R_i}$, has no effect on the optimization of the model fitting and the model selection, is omitted.

Then, the maximum likelihood estimates of the parameters are obtained using the trust region method in the optimization toolbox of MATLAB6.5. For each SR model, the model is re-written in a linear form. The linear regression parameter estimates obtained for α, β and γ are considered as initial values in the application of the iterative method to estimate the parameters α, β and γ in the non-linear SR models. To obtain the biologically meaningful parameter estimates, all the parameters are bounded as positive values. Then, the best suitable stock–recruitment relationships are selected using AIC and BIC, respectively.

$$\text{AIC} = -2\ \ln(\text{maximum likelihood}) + 2m$$

$$\text{BIC} = -2\ \ln(\text{maximum likelihood}) + m\ \ln(n)$$

where m is the number of the estimated parameters and n is the number of the observations. The model which gives the minimum AIC or BIC is selected as the best model.

Table 2
The average values of the estimated parameters of α, β, γ and AIC and BIC for the six simulated SR data sets which are simulated from the six SR statistical models

Data	Model					
	Linear	Ricker	B–H	Cushing	Shepherd	Gamma
(a) The estimated average α values (true value = 10)						
Linear	10.102	10.636	10.730	10.228	20.122	19.628
Ricker	2.349	10.139	17.447	12.092	8.573	10.049
B–H	4.391	8.304	9.988	9.691	10.595	9.091
Cushing	103.351	103.830	103.729	9.945	104.805	9.813
Shepherd	3.349	11.476	16.913	9.679	10.308	11.449
Gamma	24.228	24.875	24.554	11.870	29.820	9.948
(b) The estimated average β values (true value = 0.1)						
Linear		0.004	0.005		3.409	0.728
Ricker		0.100	0.561		0.114	0.100
B–H		0.044	0.101		0.378	0.032
Cushing		0.000	0.000		0.018	0.011
Shepherd		0.085	0.344		0.106	0.086
Gamma		0.002	0.001		0.125	0.099
(c) The estimated γ average values (true value = 2)						
Linear				0.993	0.397	0.634
Ricker				0.297	2.218	0.997
B–H				0.662	1.160	0.886
Cushing				2.006	29.686	2.078
Shepherd				0.662	2.027	1.011
Gamma				1.306	17.407	1.999
(d) The average AIC values						
Linear	**40.496**	42.087	42.058	41.806	43.651	44.008
Ricker	81.745	**18.375**	37.925	50.452	20.324	19.649
B–H	49.863	29.996	**28.454**	33.578	30.540	30.714
Cushing	102.051	104.422	104.071	**67.401**	105.880	68.678
Shepherd	74.622	25.002	35.351	33.387	**23.330**	26.737
Gamma	71.593	73.291	73.561	67.167	74.122	**54.526**
(e) The average BIC values						
Linear	**41.776**	44.889	44.860	44.608	47.855	48.212
Ricker	83.146	**21.178**	40.727	53.254	24.528	23.852
B–H	51.264	32.799	**31.257**	36.380	34.744	34.918
Cushing	103.452	107.224	106.873	**70.204**	110.084	72.882
Shepherd	76.023	27.804	38.154	36.189	**26.132**	30.941
Gamma	72.994	76.093	76.363	69.969	78.326	**58.730**

The number in bold means the minimum AIC or BIC that is selected as the best model.

3. Results

3.1. Simulated data

Table 2a–c shows the results of the estimated average parameters α, β and γ for the simulated data. Table 2d shows the AIC values for the simulated data. Applying the criterion that the model with the minimum value of AIC is the most suitable model, Table 2d indicates that the AIC method can locate the most suitable model because the model that best fits the data is the model that simulates the data. Table 2e shows the result of the average BIC values. Similar to the result of AIC, for a given set of data, BIC also selects the model that generates the data set.

3.2. Real fisheries data

Table 3a–c shows the results of the estimated average parameters α, β and γ for the real fisheries SR data. Table 3d shows the AIC values for the real fisheries SR data. The linear model is most suitable for the Pacific sardine data (Fig. 2d). The Ricker model is most suitable for the Chesapeake Bay blue crab data (Fig. 2a), Tillamook Bay chum salmon data (Fig. 2f) and Yellowfin tuna (Fig. 2g). The Beverton–Holt model is best suitable for the Flathead flounder data (Fig. 2h). The Cushing model is best suitable for the Yellow Sea squid data (Fig. 2c) and the Peruvian anchovy data (Fig. 2e). The Shepherd model is best suitable for the Bohai penaeid shrimp data (Fig. 2b). Generally speaking, the selection results of BIC are identical with AIC except for the data for the Peruvian anchovy (Fig. 2e). The Cushing model is selected by AIC and the linear model by BIC (Table 3e).

Table 3
The values of the estimated parameters of α, β, γ and the AIC and BIC for the eight real SR data sets

Data	Model					
	Linear	Ricker	B–H	Cushing	Shepherd	Gamma
(a) The estimated α values						
1	4.598	10.338	17.449	54.610	6.790	2.186
2	41.683	90.054	96.447	73.452	61.896	89.993
3	15.839	31.391	89.444	284.644	242.074	256.503
4	2.167	2.648	2.626	3.643	2.443	0.482
5	34.220	43.111	45.357	55.624	225.594	54.818
6	1.923	4.023	6.332	221.887	3.050	0.659
7	0.470	0.622	0.646	0.889	0.536	0.427
8	0.213	0.528	1.465	37.598	0.797	5.563
(b) The estimated β values						
1		0.0269	0.0964		0.0240	0.0500
2		0.1622	0.3200		0.1346	0.2789
3		0.0046	0.0525		0.4694	0.0000
4		0.0002	0.0002		0.0003	0.0005
5		0.0267	0.0396		47.4380	0.0000
6		0.0009	0.0032		0.0009	0.0001
7		0.0170	0.0240		0.0235	0.0302
8		0.0016	0.0112		0.0041	0.0008
(c) The estimated γ values						
1				0.262	2.771	1.669
2				0.541	4.156	1.449
3				0.328	0.740	0.351
4				0.920	3.450	1.320
5				0.744	0.299	0.750
6				0.270	2.335	1.320
7				0.857	0.178	0.669
8				0.755	2.562	1.227
(d) The estimated AIC values						
1	−11.107	**−14.886**	−11.553	−9.961	−14.515	−14.564
2	28.560	22.079	24.255	25.507	**21.562**	22.858
3	27.986	23.780	18.715	**17.877**	19.637	19.559
4	**83.520**	84.453	84.584	85.242	85.956	85.576
5	63.602	64.499	64.386	**63.314**	65.408	65.316
6	49.955	**40.337**	42.088	43.106	42.038	42.128
7	23.769	**20.588**	20.859	21.916	22.270	22.211
8	34.455	13.062	**12.139**	12.724	13.834	13.349
(e) The estimated BIC values						
1	−12.103	**−12.895**	−9.562	−7.969	−11.528	−11.577
2	29.045	23.049	25.225	26.827	**23.016**	24.313
3	28.183	24.174	19.109	**17.921**	20.229	20.150
4	**84.887**	87.188	87.318	87.976	90.057	89.678
5	**64.934**	67.164	67.050	65.978	69.404	69.312
6	49.955	**42.329**	44.079	45.097	45.025	45.115
7	24.904	**22.859**	23.130	24.187	25.677	25.618
8	35.500	15.151	**14.229**	14.813	16.968	16.483

The number in bold means the minimum AIC or BIC that is selected as the best model.

4. Discussion

According to the simulation study (Table 2), we can conclude that AIC and BIC are both robust in selecting the most suitable SR relationships. However, there is a few differences in the selection result of AIC and BIC when applied to real fisheries SR data. Commonly, competing models do not have the same number of parameters. For the Peruvian anchovy data (Fig. 2e), the Cushing model is selected by AIC and the linear model by BIC. In theory, when one of two competing models is a sub-model of the other (i.e. nested), a statistical comparison can be made with an F test, with a likelihood ratio test when a probability distribution of the error structure is specified. If there is not a significant difference between the full model (i.e. complex model) and the reduced model (i.e. simple model), the reduced model is often selected as the best suitable model, i.e. following the parsimonious principle. AIC and BIC can be applied to both non-nested and nested models. Quinn and Deriso (1999) state that the AIC tends to be a conservative criterion because a model with more parameters is often selected, which breaks the parsimonious principle. However, BIC is more likely to result in a parsimonious model, it more seriously penalizes the introduction of additional parameters seriously by adding the term of $m\ln(n)$ in the BIC function, where m is the number of parameters and n is the number of observations.

In order to compare the validity of AIC and BIC, we use the likelihood ratio test method to check the selection results for AIC and BIC for the data set of Peruvian anchovy (Fig. 2e).

Table 4
The AIC, BIC and the likelihood ratio test result for the Peruvian anchovy SR data

Model	Number of parameters	AIC	BIC	Negative log-likelihood
Linear	1	63.602	64.934	30.801
Cushing	2	63.314	65.978	29.657

Table 4 shows the selection results for AIC, BIC and the likelihood ratio test for the Peruvian anchovy data. Twice the difference in the negative log-likelihood between the linear model and the Cushing model is 2.288. The chi-square probability of change in 2.288 with one degree of freedom is greater than 0.1. There is not a significant difference between the Cushing model and the linear model. The reduced model, that is the linear model, is selected as the best model. This result is identical with BIC. Perhaps as far as the nested models are concerned, BIC is better suitable than AIC. Because only limited data sets are used in this paper, further research is needed.

There are a number of methods to select the suitable SR model for a given SR data. Although statistical fit is important for fisheries analyses and the parameters can be given a real world interpretation, the equations should be regarded as an empirical description rather than as an explanation of events. To obtain a theoretical or explanatory statement about the observable world, relationships between stock size and the resulting recruitment should also be based on biological details and life history of the real species. In most cases, statistical fits are identical with biologically based results. For example, The Tillamook Bay chum salmon (Fig. 2f) selected the Ricker model and the Flathead flounder (Fig. 2h) selected the Beverton–Holt model, which confirms the theoretical expectations of the SR models.

For small pelagics, recruitment success can be greatly influenced by physical environment, and probably physical environment effects are much more important than stock effects. Therefore, care must be taken when using AIC and BIC in SR model selection.

Acknowledgments

We would like to thank Dr. Zhenming Su of the University of Michigan and Professor Zhenlin Liang of the Ocean University of China for helping with this work. Comments from reviewer Dr. Steve Cadrin substantially improved the quality of the manuscript. The work is supported by the National Natural Science Foundation of China (30271025).

References

Adkison, M.D., Peterman, R.M., Laointe, M.F., Gillis, D.M., Korman, J., 1996. Alternative models of climatic effects on sockeye salmon, *Oncorhychus nerka*, productivity in Bristol Bay, Alaska, and the Fraser River, British Columbia. Fisheries Oceanogr. 5, 137–152.

Akaike, H., 1973. Information theory and an extension of the maximum likelihood principle. In: Pertaran, B.N., Csaaki, F. (Eds.), International Symposium on Information Theory, 2nd ed. Acadeemiai Kiadi, Budapest, Hugary, pp. 267–281.

Beverton, R.J.H., Holt, S.J., 1957. On the dynamics of exploited fish populations. Fish. Invest. 19, 1–533.

Cury, P., Roy, C., 1989. Optimal environmental window and pelagic fish recruitment success in upwelling areas. Can. J. Fish. Aquat. Sci. 46, 670–680.

Cushing, D.H., 1971. Dependence of recruitment on parent stock in different groups of fishes. J. Cons. Int. Explor. Mer. 33, 340–360.

Cushing, D.H., 1973. Dependence of recruitment on parent stock. J Fish. Res. Board. Can. 30, 1965–1976.

Cushing, D.H., 1981. Fisheries Biology: A Study in Population Dynamics. University of Wisconsin Press, Madison.

Deng, J., Zhu, J., Ren, S., 1996. Study on dynamic of stock recruitment relationship (SRR) of Penaeid Shrimp (*Penaeus chinensis*) in the Bohai sea. J. Fishery Sci. China 3 (2), 20–25 (in Chinese).

Dyakov, Yu.P. 1995. Reproduction of some flounder species from the West Kamchatka Shelf. In Proceeding for the International Symposium on North Pacific Flatfish. Alaska Sea Grant College Program. AK-SG-95-04, pp. 17–33.

Hilborn, R., Mangel, M., 1997. The Ecological Detective: Confronting Models with Data. Princeton University Press, Princeton, New Jersey.

Hilborn, R., Walters, C.J., 1992. Quantitative Fisheries Stock Assessment: Choice, Dynamics and Uncertainty. Chapman & Hall, New York.

Hiramatsu, K., Matsumiya, Y., Kitada, S., 1994. Introduction of suitable stock–recruitment relationship by a comparison of statistical models. Fish. Sci. 60 (4), 411–414.

Larry, D.J., Alec, D.M., 1995. Stock–recruitment models for Pacific sardine (*Sardinops sagax*). Can. J. Fish. Aquat. Sci. 52, 566–577.

Qiu, X., 1986. Studies on the biological characteristics of Squid (*Loligo japonica*) and its resources in the Yellow Sea. Mar. Fisheries Res. 7, 109–119 (in Chinese).

Quinn II, T.J., Deriso, R.B., 1999. Quantitative Fish Dynamics. Oxford University Press, New York.

Reish, R.L., Deriso, R.B., Ruppert, D., Carroll, R.J., 1985. An investigation of the population dynamics of Atlantic menhaden (*Brevoortia tyrannus*). Can. J. Fish. Aquat. Sci. 42, 147–157.

Ricker, W.E., 1954. Stock and recruitment. J. Fish. Res. Bd. Can. 11, 599–623.

Ricker, W.E., 1975. Computation and interpretation of biological statistics of fish populations. Bull. Fish. Res. Board Can., 191.

Sakamoto, Y., Ishiguro, M., Kitagawa, G., 1986. Akaike Information Statistics. KTK Scientific Publishers/D. Reidel Publishing, Tokyo/Dordrecht.

Shepherd, J.G., 1982. A versatile new stock–recruitment relationship for fisheries, and the construction of sustainable yield curves. J. Cons. Int. Explor. Mer. 40, 67–75.

Wang, C., Tanaka, S., 1988. Development of a multi-cohort analysis method and its application to the Indian Ocean yellowfin tuna length composition. Far Seas Fish. Res. Lab. 25, 1–73.

Zhan, B., 1995. Fisheries Resource Assessment. Agriculture Press, Beijing, pp. 212–224 (in Chinese).