Summary of PET (SUV) Lecture from Robert Doot, PhD

- Quantitative analysis of FDG uptake is important in tumor imaging, especially for research
- Standard uptake values (SUV) are clinically feasible and require no extra effort
- But SUVs require attention to detail (i.e. weigh pts every time)
- SUV is less precise than more complex quantitative analysis methods
- Protocol standardization improves quantitative precision

Introduction to PET Image Reconstruction

Adam Alessio  
http://faculty.washington.edu/aalessio/  
Nuclear Medicine Lectures  
Imaging Research Laboratory  
Division of Nuclear Medicine  
University of Washington  
Fall 2011  

https://www.rad.washington.edu/researchResearch/groups/irl/

Projection Imaging

'overlay' of all information (non quantitative)

Source \( f(x,y,z) \)  
Detector \( p(x,z) = \int f(x,y,z) \, dy \)

Tomographic Imaging

'Tomo' + 'graphy' = Greek: 'slice' + 'picture'

Source \( f(x,y,z) \)  
Detector orbiting source + detector data for all angles  
True cross-sectional image

Types of imaging systems

Transmission (TX)  
Emission (EM)

... but same mathematics of tomography

From photon detection to data in form of Sinograms

The number of events detected along an (LOR) is proportional to the integral of activity (i.e. FDG concentration) along that line.
**Sinogram Example**

- The sinogram is $P(s, \theta)$ organized as a 2D histogram - Radon Transform of the object

**Data Corrections**

Order of corrections (common application):
- Start with Raw Data: 
  - **Prompt Events** = Trues + Randoms + Scatter
  1. Randoms correction ($Y_r = \text{Prompt-Randoms}$)
  2. Detection efficiency normalization ($Y_n = Y_r \times \text{Norm}$)
  3. Deadtime ($Y_d = Y_n \times \text{Dead}$)
  4. Scatter ($Y_s = Y_d \times \text{Scat}$)
  5. Attenuation ($Y_a = Y_s \times \text{ACF}$) attenuation correction factors
  6. **Image Reconstruction**

**Sinograms reconstructed into images**

**Analytical Methods: Data Formation**

- Point Source - Forward Projection to 3 projections...
  Can get estimate of point source with Back-projection

**Analytical Methods: simple back-projection of point source**

- Instead of getting a point source, end up with:
  - Need to do “Filtered Back-Projection” FBP

**Image & Sinogram Space**

- $1/r = \text{Distance}$
  - Backprojected Image
  - No filtering $\Rightarrow$ Result = true $1/r$
  - To undo this, filter projections with ramp filter before backprojecting…

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Adam Alessio, alessio@u.washington.edu
**Analytical Method: Filtered back-projection**
- Ramp filter then backprojection provides the exact solution in the absence of noise
- Ramp filter accentuates high frequency - Not good for noise

![Filter Comparison](image)


**FBP Characteristics**
- **PROS:**
  - Analytic method ("Inverse Radon transform")
  - FBP is "exact" IF:
    - No noise - ????Which filter is exact??
    - No attenuation
    - Complete, continuously sampled data
    - Uniform spatial resolution
  - Easy to implement
  - Computationally Fast
  - Linear, other properties well understood (2x uptake = 2x intensity in image)
  - Can Adjust filter window to trade off bias vs. variance
- **CONS:**
  - cannot model noise in data,
  - cannot model non-idealities of system, (resolution recovery methods)
  - does not easily work with unusual geometries,
  - cannot include knowledge about the image (like non-negative activity)

**The Reconstruction Problem: An Inverse Problem**

\[ y = Px + n \]

\( x \) is N x 1 image (typically N ~ 128 x 128)
\( y \) is M x 1 data (typically M ~ 280 x 336)
\( P \) is M x N system matrix (provides probability entry \( j \) from \( x \) will be placed in entry \( i \) of \( y \))

- If we could just invert \( P \), we could solve this. But \( P \) is 16,000 x 95,000 entries...Need to solve iteratively.

**Iterative Reconstruction Characteristics**
- **Pros**
  - General results in reduced variance (noise) for a given level of accuracy (bias, resolution, etc.)
  - Reduce or eliminate streaks
  - Incorporate (model) physical effects
    - Counting statistics (noise)
    - Confidence weighting
    - Distance dependent resolution
    - Scatter, attenuation, detector efficiency, deadtime, randoms
    - a priori information (non-negativity, anatomical information, etc.)
- **Cons**
  - Slow (computationally intensive)
  - Non-linear -- hard to analyze
  - A lot of "knobs" to adjust: smoothing parameters, number iterations, etc.
  - Streaks replaced with different noise character (e.g. "blobs")

**Iterative Reconstruction: Basic Components**

1. Description of the form of the image (pixels, voxels, blobs...)
2. System model relating unknown image to each detector measurement: relates image to data (Can include detector response, corrections for attenuation, efficiencies, etc...)
3. Statistical Model describing how each measurement behaves around its mean (Poisson, Gaussian, ...)
4. Objective Function defining the "best" image estimate (Log-likelihood, WLS, MAP...)
5. Method for maximizing the objective function (EM)

Main Point: Lots of options, Not all "EM" algorithms the same

**EM for NM Reconstruction in a Nutshell**

Expectation Maximization is a general method for solving all kinds of statistical estimation problems, in tomography results in easy maximization step

\[
\text{EM: } \begin{cases} 
\text{Back project} & \text{Image}_{\text{new}} = \text{Image}_{\text{old}} \times BP \\
\text{Forward project} & \text{Measured Data} = \text{FP(Image}_{\text{old}}) 
\end{cases}
\]

Forward projection takes image into data space
Back projection takes data into image space

Adam Alessio, aalessio@u.washington.edu
Common Methods

1. **EM** (ML-EM - maximum likelihood method)
2. **OSEM** (Ordered Subset Expectation Maximization)
   - Variant of EM (still Maximum Likelihood) - Pro: Fast | Con: Does not converge
   - Uses subsets of the data to compute each estimate... Subset A then B then...

\[
\text{Image}_{\text{NEW}} = \text{Image}_{\text{OLD}} + \text{BP} \left( \frac{\text{Measured Data}}{\text{FP(Image}_{\text{OLD}})} \right)
\]

OSEM Example

\[
\text{Image}_{\text{NEW}} = \text{Image}_{\text{OLD}} + \text{BP} \left( \frac{\text{Measured Data}}{\text{FP(Image}_{\text{OLD}})} \right)
\]

OSEM “Convergence” (Implicit Smoothing-Stop algorithm before convergence)

\[
\hat{x}_{\text{REGULARIZED}} = \arg \max_x (\text{Plain/Objective}(x) - \text{Regularizer}(x))
\]

Common Methods

3. **Regularized Methods**
   - MAP(maximum a posteriori), PWLS/penalized weighted least squares, GEM (generalized EM)
   - All consist of variation in objective function...
   - Assume have some knowledge about the image before we even get the data (a priori knowledge)
   - PROS: Can enforce noise/resolution properties in final image (don’t need to post-smooth), Can include anatomical information (PET/CT?), Enforce non-negativity, Methods converge to final solution, More accurate model of data
   - CONS: Usually takes longer, has more variables to set and understand (more knobs...), can impose odd noise structures

OSEM Examples (Explicit Smoothing-Post Smooth) 12 subsets, 7 iterations

- No Post Filter
- Post-filter 7.5mm
- Post-filter 15mm

Adam Alessio, aalessio@u.washington.edu
OSEM Examples
(Explicit Smoothing) 12 subsets, 7 iterations

No Post Filter
Post-filter 7.5mm
Post-filter 15mm

<table>
<thead>
<tr>
<th>Noise free</th>
<th>High counts (low noise)</th>
<th>Low counts (high noise)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FWHM 6.79 pixels</td>
<td>FWHM 6.03 pixels</td>
<td>FWHM 4.00 pixels</td>
</tr>
</tbody>
</table>

FBP Examples

Reduce Variance & Increase Bias
(Reduce noise & Increase quantitative error)

Each method can tradeoff Bias and Noise

FBP - can vary cutoff on filter which modifies the ramp filter
OSEM - Can vary number of iterations (stop early) or vary amount of post-recon smoothing

Hypothetical Tradeoff Curves - Which is better: method A or B? method C or D?

FBP - can vary cutoff on filter which modifies the ramp filter
OSEM - Can vary number of iterations (stop early) or vary amount of post-recon smoothing

Hypothetical Tradeoff Curves

Imaging - Fully 3D vs. 2D

- Fully 3D PET data increases sensitivity of scanner (~ 8x)
- Drawbacks:
  - Increased scatter
  - Significant storage and reconstruction computation demands

Fully 3D Reconstruction

- Direct Analytic Approach
  - 3DRP: 3D reprojection (Kinahan and Rogers 1988)
- Iterative Approach
  - Simple conceptual extension: Just need system model that relates voxel to fully 3D data (as opposed to a pixel to 2D data)
  - System model becomes 100-200x larger (big computational challenge. 3D: 1.5 Billion entries to 3D: 1000 Billion entries)
- Rebinning Approach
  - Reduce Fully 3D data to decoupled sets of 2D data, then do normal 2D reconstruction
  - FORE (Fourier Rebinning) most common form
Some Examples:

- FBP, 12 mm
  - Hann, no axial smoothing
- OSEM, 10 mm
  - Gaussian, no axial smoothing

Example of different post filters:

- OSEM, S28, L4, P8, Z6
- OSEM, S28, L4, P8, Z6
- OSEM, S28, L4, P8, Z6

Example of changing number of iterations:

- S28, I1, L4, P8, Z6
- S28, I2, L4, P8, Z6
- S28, I3, L4, P8, Z6
- S28, I4, L4, P8, Z6

Example of Detector Modeling with PET:
Measuring Detector Response (Point Spread Function)

Steps for measuring PSF:
1. 12 Fully-3D acquisitions of Na22 point source at different radial locations
2. Extract radial profile from each
3. Parameterize with discrete cosine transform coefficients
4. Interpolate coefficients to all radial locations to determine unique blurring kernel at each radial location:

\[ P_{PSF}(s_x, s_y) \]

Each radial location has blur in radial direction

Measured profile in black, parameterized profile in red

FDG PET Example:

- No Post Filter
- 3.5mm Post Filter
- 7mm Post Filter

Example of changing number of iterations:

- OSEM+LOR
- OSEM+LOR+PSF
- OSEM+LOR+PSF
- OSEM+LOR+PSF

Coronal view through images reconstructed with OSEM+LOR and proposed spatially varying system model modification.

Observation: With this fixed resolution vs. noise setting, there is enhancement with addition of PSF (~11% higher)

Adam Alessio, aalessio@u.washington.edu
Example of Detector Modeling with PET:

Resolution vs Iteration
Line Source at 5cm from center of FOV

Contrast Recovery vs. “True” Noise across 50 scans

Observation:
- As add sophistication to system model, algorithms tend to converge more slowly
- Inverse problem is more ill-posed when relate voxels to more locations in data space (system model is less-sparse→ harder to invert)

Next Week:
X-ray/CT with Paul Kinahan