Interactions of Radiation with Matter

William Hunter, PhD

wcjh@uw.edu

Nuclear Medicine Basic Science Lectures
http://www.rad.washington.edu/research/our-research/groups/irl/education/basic-science-resident-lectures

September 20, 2011

Passage of radiation through matter depends on …

1. Type of radiation
   • charged particle (e.g., electron, proton, etc.)
   • electromagnetic (e.g., high energy photon such as x-ray or γ-ray)

2. Energy of radiation (e.g., in keV or MeV)

3. Nature of matter traversed (atomic number & density)

Mathematics description nly yields probabilities and averages

Decay equation: \( N(t) = N(0) \times e^{-\lambda t} = N(0) \times e^{-0.693 \frac{t}{T_{1/2}}} \)

Activity: \( A(t) = -\frac{dN(t)}{dt} = \lambda N(t) \)

Half-life: \( T_{1/2} = \ln(2) / \lambda = 0.693 / \lambda \)

Average lifetime: \( \tau = \frac{1}{\lambda} \)

Decay factor (DF): \( e^{-\lambda t} = e^{-0.693 \frac{t}{T_{1/2}}} \)

Image frame decay correction - effective decay factors

Approximation for short frame times relative to isotope half-life: e.g., \( DF_{eff} \approx \frac{1}{2} [DF(t) + DF(t+Dt)] \)

Specific Activity

\( CFSA \ [Bq/g] = \frac{\text{max SA}}{A \times T_{1/2}} \)

for \( T_{1/2} \) given in days, \( A \): atomic weight

Mixed radionuclide samples

\( N_{\text{total}}(t) \) always eventually follows the slope of the longest half-life nuclide.

Parent-daughter decay rates

1. Secular equilibrium: \( T_{1/2,\text{parent}} >> T_{1/2,\text{daughter}} \) 
   Daughter activity quickly asymptotes to parent's

2. Transient equilibrium: \( T_{1/2,\text{parent}} \geq T_{1/2,\text{daughter}} \)
   Daughter activity slowly asymptotes to parent's

Types of Radiation

Particles ejected from an unstable nucleus:

• Alpha radiation = 2p+2n (same as a He nucleus).
  Usually from a heavy element

• Beta radiation = β⁻ or β⁺ (electron or positron)
  For example: neutron conversion inside the nucleus:
  \( (n \rightarrow p + \beta^- + \nu_e \ldots \) or other permutations)

• Heavy Ions and Neutrons

Electromagnetic energy (photons):

• Gamma radiation = γ
  De-excitation of a nucleus following another nuclear decay

• Other electromagnetic radiation
  All photons from sources other than nuclear decay
  (e.g., atomic electron-orbital transitions)
I. Interactions of Charged Particles with Matter

Types of charged particle radiation relevant to Nuclear Medicine

<table>
<thead>
<tr>
<th>Particle</th>
<th>Symbol</th>
<th>Mass (amu)</th>
<th>Charge (relative)</th>
<th>Energy Imparted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron</td>
<td>e-, β-</td>
<td>0.000548</td>
<td>-1</td>
<td>kinetic</td>
</tr>
<tr>
<td>Positron</td>
<td>e+, β+</td>
<td>0.000548</td>
<td>+1</td>
<td>kinetic + 2 × 0.511MeV</td>
</tr>
<tr>
<td>Alpha</td>
<td>α</td>
<td>4.0028</td>
<td>+2</td>
<td>kinetic (can be large)</td>
</tr>
</tbody>
</table>

Gradual loss of particle’s energy
Energy transferred to nearby atoms and molecules

Interaction mechanisms

1. Excitation
2. Ionization
3. Bremsstrahlung

Excitation

- Energy is transferred to an orbital electron, but not enough to free it.
- Electron is left in an excited state and energy is dissipated in molecular vibrations or in atomic emission of infrared, visible or UV radiation, etc.
Interaction between charged particle and orbital electron
- Energy transferred from passing particle to electron
- When energy is greater than ionization potential, electron is freed (IONIZATION)

- Ionization potential for carbon, nitrogen, and oxygen are in the range of 10-15 eV.
- Ejected electrons energetic enough to cause secondary ionizations are called delta-rays

Bremsstrahlung
- Some particles will interact with the nucleus.
- The particle will be deflected by the strong electrical forces exerted on it by the nucleus.
- The particle is rapidly decelerated and loses energy in the “collision”. The energy appears as a photon of electromagnetic radiation.

- Photons produced by Bremsstrahlung are called X-rays

Ionization and excitation are collisional losses.
- Bremsstrahlung production is called a radiation loss. Radiation losses increase with increasing particle energy and increasing atomic number of the absorbing material. (x-ray tube anode)
- High-energy electrons ($\beta^-$) in nuclear medicine dissipate most of their energy in collisional losses, which have short primary and secondary path lengths. However, Bremsstrahlung production from betas results in longer-range secondaries (photons) that can be important when shielding large quantities of energetic $\beta^-$-emitters (e.g., tens of mCi of $^{32}$P).
Linear energy transfer (LET):
- Amount of energy deposited per unit path length (eV/cm)
- Measure of energy deposition density, determines biological consequence of radiation exposure
- “High-LET” radiation (alpha particles, protons) more damaging than “low-LET” radiation (beta particles and ionizing electromagnetic radiation)

Specific ionization:
- Number of ion pairs produced per unit length (total of both primary and secondary ionization events)
- Specific ionization increases as particle slows down. This gives rise to the Bragg ionization peak.

Interaction Mechanisms
1. Coherent (Rayleigh) scattering
2. Photoelectric effect
3. Compton scattering
4. Pair production
• Scattering interactions that occur between a photon and an atom as a whole.
• Coherent scattering occurs mainly at energies <50 keV (uncommon in nuclear medicine and often ignored; accounts for <5% of x-ray interactions above 70 keV)
• Because of the great mass of an atom very little recoil energy is absorbed by the atom. The photon is therefore deflected with essentially no loss of energy.
• It is the reason the sky is blue and sunsets are red.

**Interactions of Photons with Matter**

**Coherent (or Rayleigh) Scatter**

*Before*  

*After*  

\[ \gamma \]

An atomic absorption process in which an atom absorbs all the energy of an incident photon.

\[ \mu_{\text{PE}} \propto Z^2 \rho \frac{E}{E_f} \]

*From: Physics in Nuclear Medicine (Cherry, Sorenson and Phelps)*

**Interactions of Photons with Matter**

**Photoelectric Effect (Absorption)**

An abrupt increase in likelihood of interaction at edges due to increased probability of photoelectric absorption when photon energy just exceeds binding energy.

\[ \mu_{\text{PE}} \propto Z^2 \rho \frac{E}{E_f} \]

**Compton Scatter**

Collision between a photon and a loosely bound outer shell orbital electron. Interaction looks like a collision between the photon and a “free” electron.
The probability of Compton scatter is a slowly varying function of energy. It is proportional to the density of the material ($\rho$) but independent of $Z$.

$$\mu_{CS} \propto \rho$$

$\rho$ is the density of the material.

As energy is increased scatter is forward peaked.

The scattering angle of the photon is determined by the amount of energy transferred in the collision.

$$E_1 = \frac{E_0}{1 + E_0 m_e c^2 (1 - \cos \theta)}$$

$E_0$ is original photon energy

$E_1$ is scattered photon’s energy

$m_e$ is the electron mass ($m_e c^2 = 511$ keV)

$\theta$ is the scattering angle ($\theta = 0$ is no scatter)

How likely is scattering to occur at different angles?

- low energies: all angles ($\theta$) are equally probable
- high energies: forward angle scatter is favored

The relative frequency of scatter angle for given incident energy is described by Klein-Nishina equation.

Polar plot of Klein-Nishina equation (relative # per solid angle)
Pair production occurs when a photon interacts with the electric field of a charged particle. Usually the interaction is with an atomic nucleus but occasionally it is with an electron.

Photon energy is converted into an electron-positron pair and kinetic energy. Initial photon must have an energy of greater than 1.022 MeV (> 2 times rest mass of electron).

Positron will eventually interact with a free electron and produce a pair of 511 keV annihilation photons.

\[
\begin{align*}
\gamma & \quad \text{Before} \\
\Downarrow & \\
\gamma & \quad \text{After} \\
\end{align*}
\]

When a photon passes through a thickness of absorber material, the probability that it will experience an interaction (i.e., photoelectric, Compton scatter, or pair production) depends on the energy of the photon and on the composition and thickness of the absorber.

\[
I(x) = I(0)e^{-\mu x}
\]

where

- \(I(0)\) = initial beam intensity,
- \(I(x)\) = beam intensity transmitted through absorber
- \(x\) = thickness of absorber, and
- \(\mu\) = total linear attenuation coefficient of the absorber at the photon energy of interest (note units: cm\(^{-1}\)).
There are three basic components to the linear attenuation coefficient:

- $\mu_\tau$ due to the photoelectric effect;
- $\mu_\sigma$ due to Compton scattering; and
- $\mu_\kappa$ due to pair production.

The exponential equation can then be written as:

$$I(x) = I(0) \exp\left[-(\mu_\tau + \mu_\sigma + \mu_\kappa)x\right]$$

or as

$$I(x) = I(0) \exp[-\mu_\tau x] \exp[-\mu_\sigma x] \exp[-\mu_\kappa x]$$

- Linear attenuation coefficient $\mu_i$
  - depends on photon energy
  - depends on material composition
  - depends on material density
  - dimensions are 1/length (e.g., 1/cm, cm$^{-1}$)

- Mass attenuation coefficient $\mu_m$
  - $\mu_m = \mu_i \rho$ ($\rho$ = density of material yielding $\mu_i$)
  - does not depend on material density
  - dimensions are length$^2$/mass (e.g., cm$^2$/g)

Examples of linear attenuation coefficient:

- NaI(Tl)
- BGO
Interactions of Photons with Matter

Narrow beam vs broad beam attenuation

Without collimation, scattered photons cause artificially high counts to be measured, resulting in smaller measured values for the attenuation coefficients.

\[
\frac{I}{I_0} = e^{-\mu \text{HVT}}
\]

\[
\mu_{\text{HVT}} = -\ln(0.5)
\]

\[
\text{HVT} = \frac{0.693}{\mu}
\]

Half-value thickness is the amount of material needed to attenuate a photon flux by 1/2 (attenuation factor = 0.5).

Tenth value thickness is given by

\[
\frac{I}{I_0} = e^{-\mu \text{TVT}}
\]

\[
\mu_{\text{TVT}} = -\ln(0.1)
\]

\[
\text{TVT} = \frac{2.30}{\mu}
\]

Half and tenth value thicknesses

Examples

<table>
<thead>
<tr>
<th>Material (energy)</th>
<th>(\mu) (cm(^{-1}))</th>
<th>HVT (cm)</th>
<th>TVT (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lead (140 keV)</td>
<td>22.7</td>
<td>0.031</td>
<td>0.10</td>
</tr>
<tr>
<td>Lead (511 keV)</td>
<td>1.7</td>
<td>0.41</td>
<td>1.35</td>
</tr>
<tr>
<td>Water (140 keV)</td>
<td>0.15</td>
<td>4.6</td>
<td>15.4</td>
</tr>
<tr>
<td>Water (511 keV)</td>
<td>0.096</td>
<td>7.2</td>
<td>24.0</td>
</tr>
</tbody>
</table>

Beam Hardening

- A polychromatic beam (multiple energies) (e.g., from x-ray tube, Ga-67, In-111, I-131) has complex attenuation properties.

- Since lower energies are attenuated more than higher energies, the higher energy photons are increasingly more prevalent in the attenuated beam.
Secondary Ionization

• The Photoelectric Effect & Compton Scattering ionize an atom.
• Pair production creates two charged particles
• These energetic charged particles in turn ionize more atoms producing many free electrons.
• These secondary ionization are the basis for most photon detectors.
• Ionization also leads to breaking molecular bonds → basis of most radiation biological effects.

Example: Attenuation Calculation 1

What fraction of 140 keV photons will escape unscattered from the middle of a 30 cm cylinder?

The photons must travel through 15 cm of water.

\[ \frac{I}{I_0} = e^{-\mu d} = e^{-(0.15/cm)(15cm)} = 0.105 = 10.5\% \]

Example: Attenuation Calculation 2

What thickness of lead is required to attenuate 99% of 511 keV photons?

99% attenuated = 1% surviving
Using the exponential attenuation formula

\[ 0.01 = \frac{I}{I_0} = e^{-\mu d} = e^{-(1.7/cm)d} \]
\[ \ln(0.01) = -(0.17/cm)d \]
\[ d = -\ln(0.01)/(1.7/cm) = 2.7 \text{ cm} \]

Alternatively, if the TVT is known (1.35 cm), doubling the TVT results in two consecutive layers which each transmit 1/10 of photons, or a total transmission of 1/100 or 1%. 2 * 1.35 cm = 2.7 cm.
Example: Buildup Factors
(broad beam versus narrow beam values)

The transmission factor for 511-keV photons in 1 cm of lead was found to be 18% for narrow-beam conditions. Estimate the actual transmission for broad-beam conditions.

From: Physics in Nuclear Medicine (Cherry, Sorenson and Phelps)

Why are radiation losses (i.e. Bremsstrahlung) more important than collisional losses when shielding large quantities of energetic C-emitters?

a) Radiation losses are more frequent than collisional losses.
b) Photon radiation travels further than beta radiation.
c) Both (a) and (b)
d) Neither (a) and (b)
Main points of this week’s lecture:
“Interaction with matter”

Charged particles
- Short ranged in tissue
  - µm for alphas (straight path and continual slowing)
  - mm for betas (more sporadic path and rate of energy loss)
- Interactions Types: Excitation, Ionization, Bremsstrahlung
- Linear energy transfer (LET) - nearly continual energy transfer
- Bragg ionization peak - specific ionization peaks as particle slows down

Photons
- Relatively long ranged (range ~cm)
- Local energy deposition - photon deposits much or all of their energy each interaction
- Interactions Types: Rayleigh, Photoelectric, Compton, Pair Production
- Compton - dominant process in tissue-equivalent materials for Nucl. Med. energies
- Buildup factors - narrow vs. wide beam attenuation
- Secondary ionization - useful for photon detection

NEXT WEEK: Dr. Lewellen - Radiation Detectors
Suggested reading: Chapter 7 of Cherry, Sorenson, and Phelps

Klein Nishina formula
Polar plot of Klein-Nishina equation
(relative # per solid angle)

For an incident photon of energy $E_i$, the differential cross section is:

$$\frac{d\sigma}{d\Omega} = \pi r_0^2 \left( \frac{E_i}{m_e c^2} \right)^2 \left( P(\theta) + P(E_i, \theta)^{-1} - 1 + \cos^2(\theta) \right)$$

where $r_0$ is the fine structure constant, $\theta$ is the scattering angle, $m_e$ is the mass of an electron, and $P(E_i, \theta)$ is the ratio of photon energy after and before the collision:

$$P(E_i, \theta) = \frac{1}{1 + \frac{E_i}{m_e c^2} \left( 1 - \cos(\theta) \right)}$$

From: Klein, UC Berkeley masters thesis

Fall 2011, Copyright UW Imaging Research Laboratory