Decay of Radioactivity

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Review of last week: Introduction to Nuclear Physics and Nuclear Decay

Nuclear shell model – “orbitals” for protons and neutrons – and how the energy differences leads to nuclear decay

Differences between isotopes, isobars, isotones, isomers

Know how to diagram basic decays, e.g.

$$^{\text{A}}_{\text{Z}} X \rightarrow _{\text{Z'+1}}^{\text{A}} Y + \beta^{\pm} + \nu + Q$$

Conservation principles: Energy (equivalently, mass); linear momentum; angular momentum (including intrinsic spin); and charge are all conserved in radioactive transitions

Understand where gamma rays come from (shell transitions in the daughter nucleus) and where X-rays come from.

Understand half-life and how to use it
The Math of Radioactive decay

1. Activity, Decay Constant, Units of Activity
2. Exponential Decay Function
3. Determining Decay Factors
4. Activity corrections
5. Parent-Daughter Decay

*Physics in Nuclear Medicine - Chapter 4*
Radioactive decay:

- is a **spontaneous** process
- can **not** be predicted exactly for any single nucleus
- can only be described statistically and probabilistically
  i.e., can only give averages and probabilities

The description of the mathematical aspects of radioactive decay is today's topic.
Activity (A)

• Average number of radioactive decays per unit time (rate)
• Change in number of radioactive nuclei present:
  \[ A = -\frac{dN}{dt} \]
  where \( N \) is the number of nuclei present.
• Activity (\( A \)) decreases with time.
• Measured in Becquerel (Bq):
  \[ 1 \text{ Bq} = 1 \text{ disintegration per second (dps)} \]
• Traditionally measured in Curies (Ci):
  \[ 1 \text{ Ci} = 3.7 \times 10^{10} \text{ Bq} \quad (1 \text{ mCi} = 37 \text{ MBq}) \]
  Traditionally: 1 Ci is the activity of 1g \(^{226}\text{Ra}\).
### Decay Constant ($\lambda$)

- Activity divided by the number of nuclei:
  \[ \lambda = \frac{A(t)}{N(t)} = \frac{\text{Activity}}{\text{Number of Nuclei}} \]

- Constant in time, characteristic of each nuclide
- Directly related to activity: \[ A = \lambda \times N \]
- Has units of (time)$^{-1}$
  
  Example: Tc-99m has $\lambda = 0.1151$ hr$^{-1}$, i.e., 11.5% decay/hr
  Mo-99 has $\lambda = 0.252$ day$^{-1}$, i.e., 25.2% decay/day

- If nuclide has several decay modes, each has its own $\lambda_i$.
  Total decay constant is sum of modes: $\lambda = \lambda_1 + \lambda_2 + \lambda_3 + \cdots$

- Branching ratio = fraction in particular mode: $\text{Br}_i = \lambda_i / \lambda$
  
  Example: $^{18}\text{F}$: 97% $\lambda^+$, 3% Electron Capture (EC)
Question

Example: $N(0) = 1000$ and $\lambda = 0.1 \text{ sec}^{-1}$

What is the activity at time $= 0$ seconds?

What is the number of nuclei left after 2 seconds?
Answer

\[ A(0) = N(0) \lambda \]

\[ A(0) = 1000 \times 0.1 \text{ sec}^{-1} \]

\[ A(0) = 100 \text{ decays / sec} \]
Answer

\[ N(0) = 1000; \quad A(0) = 100 \text{ sec}^{-1} \]
\[ N(1) = 900; \quad A(1) = 90 \text{ sec}^{-1} \]
\[ N(2) = 810; \quad A(2) = 81 \text{ sec}^{-1} \]

Both the number of nuclei and activity decrease by \( \lambda \) with time.
Exponential Decay Equation

\[ N(t) = N(0)e^{-\lambda t} \]

The number remaining nuclei is proportional to the original number.

Mathematical derivation:

\[ \frac{dN}{dt} = -\lambda \cdot N \]

\[ \frac{dN}{dt} = -\lambda * N \Rightarrow \frac{dN}{N} = -\lambda \cdot dt \Rightarrow \int \frac{1}{N} dN = \int -\lambda dt \]

\[ \ln(N_t) - \ln(N_0) = -\lambda t \Rightarrow \ln\left(\frac{N_t}{N_0}\right) = -\lambda t \Rightarrow \frac{N_t}{N_0} = e^{-\lambda t} \Rightarrow N_t = N_0 e^{-\lambda t} \]
Exponential Decay Equation

This also holds for the activity (number of decaying nuclei):

\[ A(t) = A(0) \times e^{-\lambda t} \]

Decay factor: \[ e^{-\lambda t} \]

\[ = \frac{N(t)}{N(0)}: \text{fraction of nuclei remaining} \]

or

\[ = \frac{A(t)}{A(0)}: \text{fraction of activity remaining} \]
Half-life

Half-life: time required to decay a sample to 50% of its initial activity: \( \frac{1}{2} = e^{-\left(\lambda \times T_{1/2}\right)} \)

\[
\ln\left(\frac{1}{2}\right) = -\lambda T_{1/2} \\
-\ln 2 = -\lambda T_{1/2}
\]

\[ T_{1/2} = \frac{\ln 2}{\lambda} \]

\[ \lambda = \frac{\ln 2}{T_{1/2}} \quad (\ln 2 \approx 0.693) \]

Constant in time, characteristic for each nuclide
### Some Clinical Examples

<table>
<thead>
<tr>
<th>Radionuclide</th>
<th>$T_{1/2}$</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flourine 18</td>
<td>110 min</td>
<td>0.0063 min$^{-1}$</td>
</tr>
<tr>
<td>Technetium 99m</td>
<td>6.02 hr</td>
<td>0.1152 hr$^{-1}$</td>
</tr>
<tr>
<td>Iodine 123</td>
<td>13.3 hr</td>
<td>0.0522 hr$^{-1}$</td>
</tr>
<tr>
<td>Molybdenum 99</td>
<td>2.75 d</td>
<td>0.2522 d$^{-1}$</td>
</tr>
<tr>
<td>Iodine 131</td>
<td>8.02 d</td>
<td>0.0864 d$^{-1}$</td>
</tr>
</tbody>
</table>

\[
\lambda \approx \frac{0.693}{T_{1/2}}
\]
Average Lifetime ($\tau$)

Time until decay varies between nuclei from 'very short' to 'very long'.

The average lifetime $\tau$ is characteristic for each nuclide and doesn't change over time:

$$\tau = 1 / \lambda = T_{1/2} / \ln 2 = 1.44 \times T_{1/2}$$

$\tau$ is longer than the half-life by a factor of $1/\ln 2$ ($\sim 1.44$)

Average lifetime important in radiation dosimetry calculations
Determining Decay Factors ($e^{-\lambda t}$)

Table of Decay factors for Tc-99m

<table>
<thead>
<tr>
<th>Minutes</th>
<th>0</th>
<th>15</th>
<th>30</th>
<th>45</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.000</td>
<td>0.972</td>
<td>0.944</td>
<td>0.917</td>
</tr>
<tr>
<td>1</td>
<td>0.891</td>
<td>0.866</td>
<td>0.844</td>
<td>0.817</td>
</tr>
<tr>
<td>3</td>
<td>0.707</td>
<td>0.687</td>
<td>0.667</td>
<td>0.648</td>
</tr>
<tr>
<td>5</td>
<td>0.561</td>
<td>0.545</td>
<td>0.530</td>
<td>0.515</td>
</tr>
<tr>
<td>7</td>
<td>0.445</td>
<td>0.433</td>
<td>0.420</td>
<td>0.408</td>
</tr>
<tr>
<td>9</td>
<td>0.354</td>
<td>0.343</td>
<td>0.334</td>
<td>0.324</td>
</tr>
</tbody>
</table>

Read of DF directly or combine for wider time coverage:
$DF(t_1+t_2+...) = DF(t_1) \times DF(t_2) \times ...$
Other Methods to Determine DF

• Graphical method: DF vs. number of half-lives elapsed

Determine the time as multiples of $T_{1/2}$.

Read the corresponding DF number off the chart.

Example:
Decay factor after 3.5 half lives

DF (3.5) =
Other Methods to Determine DF

- Graphical method: DF vs. number of half-lives elapsed

Determine the time as multiples of $T_{1/2}$.

Read the corresponding DF number off the chart.

Example:

$\text{DF (3.5) = 0.088}$

- Pocket calculator with exponential/logarithm functions
Examples

C-11 has a half-life of 20 minutes. Initial sample has 1000 nuclei.

Q: How many are left after 40 minutes?
   After 80 minutes?
   How many half lives before less than 1 left?
Answer

C-11 has a half-life of 20 minutes. Initial sample has 1000 nuclei.

Q: How many are left after 40 minutes?
   After 80 minutes?
   When is less than 1 left?

A: After 2 half-lives (40 min) \( 1/2^2 = 1/4 \) of the initial activity is left (25%).

After 4 half-lives (80 min) \( 1/16^{th} \) is left (6.25%)

To have less than one left, one needs a DF < 1/1000. This happens after 10 half-lives because \( (1/2)^{10} = 1/1024 \), so after 200 minutes (3 hrs 20 min).
Image Frame Decay Correction

If the acquisition time of an image frame $\Delta t$ is not short compared to the half-life of the used nuclide, then the nuclide is decaying measurably during the acquisition time.

In this situation one needs to adjust the decay factor (i.e., $DF(t)$) to account for decay of activity during $\Delta t$, because $a_0 = DF(t)\Delta t \neq a_d$.

$$DF_{\text{eff}}(t, \Delta t) = DF(t) \cdot \left( \frac{a_d}{a_0} \right) = DF(t) \cdot \left[ \frac{(1 - e^{-x})}{x} \right] \quad x = 0.693 \cdot \Delta t / T_{1/2}$$

Approximations for small $x$:

$$DF_{\text{eff}} \approx \frac{1}{2} \cdot [DF(t) + DF(t + \Delta t)]$$

$$DF_{\text{eff}} \approx DF(t + (\Delta t/2))$$

$$DF_{\text{eff}} \approx DF(t) \cdot [1 - (x/2)]$$
Effective Decay Factor

\[ x = 0.693 \times \Delta t / T_{1/2} \]

Taylor series expansion:  
\[ DF_{\text{eff}}(t, \Delta t) \approx DF(t) \times [1 - (x/2)] \]

\( DF_{\text{eff}} \) is accurate to within 1\% when \( x < 0.25 \)

Midpoint of frame:  
\[ DF_{\text{eff}}(t, \Delta t) \approx DF(t + (\Delta t / 2)) \]

\( DF_{\text{eff}} \) is accurate to within 1\% when \( x < 0.5 \)

Average of DF's:  
\[ DF_{\text{eff}}(t, \Delta t) \approx \frac{[DF(t) + DF(t + \Delta t)]}{2} \]

\( DF_{\text{eff}} \) is accurate to within 1\% when \( x < 0.35 \)
Question

$^{15}$O study – image frame is 30 - 45 sec after injection. How much difference does it make to use or not to use the exact effective DF, and is approximation OK?

The half-live of $^{15}$O is 124 seconds.
Answer - Effective Decay Factor

\( T_{1/2}^{(15\text{O})} = 124 \text{ sec.} \) and \( \Delta t = 15 \text{ sec.} \)

\[
DF_{\text{eff}}(t, \Delta t) = DF(t) \times \frac{(1-\exp(-x))}{x}, \quad x = \ln 2 \times \frac{\Delta t}{T_{1/2}}
\]

\( DF(t) = DF(30\text{sec}) = \exp(-\ln 2 \times t / T_{1/2}) = 0.846 \)

Params: \( x = \ln 2 \times (\Delta t / T_{1/2}) = 0.084, \quad [1-\exp(-x)]/x \approx 0.959 \)

• Exact Solution:
  \[ DF_{\text{eff}} = DF(30 \text{ sec}) \times \frac{[1-\exp(-x)]}{x} = 0.811 \]
  Decay correction factor for this frame: \( 1/0.811 = 1.233 \)

• With approximation: \( DF_{\text{eff}} = DF(t) \times [1 - (x/2)] = 0.810 \)
  ⇒ Approximation is very good in this case.
Specific Activity

- Ratio of total radioisotope activity to total mass of the *element* present
- Measured in Bq/g or Bq/mole
- Useful if radioactive and non-radioactive isotopes of the same element are in mixture (sample *with carrier*)
- Maximum Specific Activity in *carrier-free* samples:

  Carrier Free Specific Activity (CFSA)

  \[
  \text{CFSA} \ [\text{Bq/g}] \approx 4.8 \times 10^{18} / (A \times T_{1/2}) \quad \text{for } T_{1/2} \text{ given in days, } A:\text{atomic weight}
  \]

  \[
  \text{CFSA(Tc-99m)} = 4.8 \times 10^{18} / (99 \times 0.25) = 1.9 \times 10^{17} \text{ Bq/g} \]

  \[
  = 5.3 \times 10^6 \text{ Ci/g}
  \]
Question

What has a higher Carrier Free Specific Activity (CFSA) Carbon-11 or Fluorine-18?
Answer

What has a higher Carrier Free Specific Activity (CFSA) Carbon-11 or Fluorine-18?

\[
\text{CFSA(C-11)} = \frac{4.8 \times 10^{18}}{(11 \times 0.014)} = 3.1 \times 10^{19} \text{ Bq/g} = 8.3 \times 10^8 \text{ Ci/g}
\]

\[
\text{CFSA(F-18)} = \frac{4.8 \times 10^{18}}{(18 \times 0.076)} = 3.5 \times 10^{18} \text{ Bq/g} = 9.45 \times 10^7 \text{ Ci/g}
\]
Decay of Mixed Radionuclide Sample

For unrelated radionuclides in a mixture, the total Activity is the sum of the individual activities:

\[ A_t(t) = A_1(0) * e^{-0.693*t/T_{1/2,1}} + A_2(0) * e^{-0.693*t/T_{1/2,2}} + ... \]

\( A_t(t) \) always eventually follows the slope of the longest half-life nuclide.

This can be used to extrapolate back to the contributors to the mix.
Biologic and Physical Decay

The effective half-life of a radiopharmaceutical with a physical half-life of $T_{1/2, \text{physical}}$ and a biological half-life of $T_{1/2, \text{biological}}$ is:

$$\frac{1}{T_{1/2, \text{eff}}} = \frac{1}{T_{1/2, \text{physical}}} + \frac{1}{T_{1/2, \text{bio}}}$$

or

$$T_{1/2, \text{eff}} = \frac{(T_{1/2, \text{physical}} \times T_{1/2, \text{bio}})}{(T_{1/2, \text{physical}} + T_{1/2, \text{bio}})}$$

or

$$\lambda_{\text{eff}} = \lambda_{\text{physical}} + \lambda_{\text{bio}}$$
Parent-Daughter Decay

Parent-daughter decay is illustrated below:

\[ \begin{align*}
\text{Parent} & : T_{1/2} = T_p \\
\text{Daughter} & : T_{1/2} = T_d
\end{align*} \]

Equation for activity of parent is:

\[ A_p(t) = e^{-\lambda_p t} \]

Equation for activity of daughter is described by the Bateman Equation:

\[ A_d(t) = \left\{ \left[ A_p(0) \frac{\lambda_d}{(\lambda_d - \lambda_p)} \right] \times \left( e^{-\lambda_p t} - e^{-\lambda_d t} \right) \right\} \times B.R. + A_d(0) e^{-\lambda_d t} \]
Secular Equilibrium

Parent-Daughter Decay Special Case:
Half-life of parent very long, decrease of $A_p$ negligible during observation, $\lambda_p \approx 0$.

*Example:* Ra-226 ($T_p \approx 1620$ yr) $\rightarrow$ Rn-222 ($T_d \approx 4.8$ days)

*Bateman Approximation:* $A_d(t) \approx A_p(0) \left(1 - \exp(-\lambda_d t)\right) \times BR$

After one daughter half-life $T_d$:
$A_d(t) \approx \frac{1}{2} A_p(t)$

After 2$\times$T_d: $A_d(t) \approx \frac{3}{4} A_p(t)$

Eventually $A_d \approx A_p$, asymptotically approaches equilibrium

Figure from Physics in Nuclear Medicine (Cherry, Sorenson and Phelps)
Transient Equilibrium

Parent Half-life longer than Daughter’s, but not ‘infinite’

*Example:* $^{99}$Mo ($T_{1/2} = 66\text{hr}$) $\rightarrow$ $^{99m}$Tc ($T_{1/2} = 6\text{ hr}$)

$A_p$ decreases significantly over the observation time, so $\lambda_p \approx 0$.

The daughter activity increases, exceeds that of the parent, reaches a maximum value, then decreases and follows the decay of the parent. Then the *ratio of daughter to parent is constant*: transient equilibrium.

$$A_d/A_p = \left[T_p/(T_p - T_d)\right] \times \text{BR}$$

Figure from Physics in Nuclear Medicine (Cherry, Sorenson and Phelps)
No Equilibrium

If $T_d > T_p$, no equilibrium is possible.

Example: $^{131\text{m}}\text{Te} \ (T_{1/2} = 30\text{hr}) \quad ^{131}\text{I} \ (T_{1/2} = 8 \text{ days})$

Buildup and decay of daughter shows maximum again. When parent distribution is gone, further daughter decay is according to the daughter’s decay equation only.

![Graph showing buildup and decay of $^{131\text{m}}\text{Te}$ and $^{131}\text{I}$ with respect to daughter half-lives.](image)
Summary

Mathematical description of decay only yields probabilities and averages.

Decay equation: $N(t) = N(0) \cdot e^{-\lambda \cdot t} = N(0) \cdot e^{-0.693 \cdot t / T_{1/2}}$

Decay constant: $\lambda = A(t)/N(t)$
Half-life: $T_{1/2} = \ln 2 / \lambda = 0.693 / \lambda$
Average lifetime: $\tau = 1 / \lambda = 1.44 \ T_{1/2}$

Decay factor (DF): $e^{-\lambda \cdot t} = e^{-0.693 \cdot t / T_{1/2}}$

Image frame decay correction - effective decay factors

Specific Activity

Mixed, and parent-daughter decays
## I. Interactions of Charged Particles with Matter

Types of charged particle radiation relevant to Nuclear Medicine

<table>
<thead>
<tr>
<th>Particle</th>
<th>Symbol</th>
<th>Mass (amu)</th>
<th>Charge (relative)</th>
<th>Energy Imparted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron</td>
<td>e-, β-</td>
<td>0.000548</td>
<td>-1</td>
<td>kinetic</td>
</tr>
<tr>
<td>Positron</td>
<td>e+, β+</td>
<td>0.000548</td>
<td>+1</td>
<td>kinetic + $2 \times 0.511\text{MeV}$</td>
</tr>
<tr>
<td>Alpha</td>
<td>$\alpha$</td>
<td>4.0028</td>
<td>+2</td>
<td>kinetic (can be large)</td>
</tr>
</tbody>
</table>
Interactions of **Charged Particles** with Matter

**Description of Passage**

Gradual loss of particle’s energy

Energy transferred to nearby atoms and molecules
Interactions of Charged Particles with Matter

Interaction mechanisms

1. Excitation
2. Ionization
3. Bremsstrahlung
Interactions of **Charged Particles** with Matter

**Excitation**

- Energy is transferred to an orbital electron, but not enough to free it.

- Electron is left in an excited state and energy is dissipated in molecular vibrations or in atomic emission of infrared, visible or UV radiation, etc.

From: Physics in Nuclear Medicine (Cherry, Sorenson and Phelps)
Interactions of **Charged Particles** with Matter

**Ionization**

- Interaction between charged particle and orbital electron
- Energy transferred from passing particle to electron
- When energy is greater than ionization potential, electron is freed (ionization)

- Ionization potential for carbon, nitrogen, and oxygen are in the range of 10-15 eV.

- Ejected electrons energetic enough to cause secondary ionizations are called delta-rays

From: Physics in Nuclear Medicine (Cherry, Sorenson and Phelps)
Interactions of **Charged Particles** with Matter

**Bremsstrahlung**

- Some particles will interact with the nucleus.
- The particle will be deflected by the strong electrical forces exerted on it by the nucleus.
- The particle is rapidly decelerated and loses energy in the “collision”. The energy appears as a photon of electromagnetic radiation.

From: Physics in Nuclear Medicine (Cherry, Sorenson and Phelps)
Interactions of Charged Particles with Matter

Collision versus radiation losses

• Ionization and excitation are collisional losses.

• Bremsstrahlung production is called a radiation loss. Radiation losses increase with increasing particle energy and increasing atomic number of the absorbing material. (X-ray tube anode)

• High-energy electrons ($\beta^-$) in nuclear medicine dissipate most of their energy in collisional losses, which have short primary and secondary path lengths. However, Bremsstrahlung production from betas results in longer-range secondaries (photons) that can be important when shielding large quantities of energetic $\beta$-emitters (e.g., tens of mCi of $^{32}$P).
Interactions of Charged Particles with Matter

Collision versus radiation losses

What is more likely, collision loss or radiative loss?

\[ \% \text{ radiation loss} \approx (ZE_{\beta}^{\text{max}} / 3000) \times 100\% \]

% radiation loss of \( ^{32}\text{P} \) in water

\[ E_{\beta}^{\text{max}} = 1.7 \, \text{MeV} \quad \text{for} \quad ^{32}\text{P} \]

\[ Z_{\text{eff}} = 7.9 \]

% radiation loss = \((7.9 \times 1.7/3000) \times 100\% \approx 0.4\% \)
Interactions of Charged Particles with Matter

Collision versus radiation losses

Is it better to shield $^{32}$P in plastic container or a lead container? Why?
Interactions of Charged Particles with Matter

Collision versus radiation losses

Is it better to shield $^{32}\text{P}$ in plastic container or a lead container? Why?

\[
\text{% radiation loss} \approx \left( \frac{ZE_{\beta}^{\text{max}}}{3000} \right) \times 100\%
\]

\[
\text{% radiation loss water} \approx 0.4\%
\]

\[
\text{% radiation loss lead} \approx 4.6\%
\]
Interactions of Charged Particles with Matter

**Paths of Charged Particles**

**Electron**

- path > range
- range: mm in water

**Alpha**

- path = range
- range: µm in water
Interactions of **Charged Particles** with Matter

### Energy deposition along track

Linear energy transfer (LET): Amount of energy deposited per unit path length (eV/cm)

- Measure of energy deposition density, determines biological consequence of radiation exposure

- “high-LET” radiation (alpha particles, protons) more damaging than “low-LET” radiation (beta particles and ionizing electromagnetic radiation)
Interactions of Charged Particles with Matter

Energy deposition along track

Specific ionization:
Number of ion pairs produced per unit length
(total of both primary and secondary ionization events)

Specific ionization increases as particle slows down. This gives rise to the Bragg ionization peak.

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*Figure 6-7. Specific ionization versus distance traveled for α particles in air. (Adapted from Mladjenovic M: Radioisotope and Radiation Physics. New York, Academic Press, 1973, p 111.)*
II. Interactions of \textbf{photons} with matter

Interaction Mechanisms

1. Coherent (Rayleigh) scattering
2. Photoelectric effect
3. Compton scattering
4. Pair production
Interactions of **Photons** with Matter

**Coherent (or Rayleigh) Scatter**

- Scattering interactions that occur between a photon and an atom as a whole.
- Coherent scattering occurs mainly at energies $<50$ keV (**uncommon in nuclear medicine and often ignored**; accounts for ~5% of x-ray interactions above 70 keV)
- Because of the great mass of an atom very little recoil energy is absorbed by the atom. The photon is therefore deflected with essentially no loss of energy.
- It is the reason the sky is blue and sunsets are red.
An atomic absorption process in which an atom absorbs all the energy of an incident photon.

\[ \mu_{\text{PE}} \propto \rho \frac{Z^{(3-5)}}{E^3} \]

*Z* is atomic number of the material, *E* is energy of the incident photon, and *r* is the density of the material.
Interactions of **Photons** with Matter

**Photoelectric Effect (Absorption)**

Abrupt increase in likelihood of interaction at edges due to increased probability of photoelectric absorption when photon energy just exceeds binding energy.
Collision between a photon and a loosely bound outer shell orbital electron. Interaction looks like a collision between the photon and a “free” electron.
Interactions of *Photons* with Matter

**Compton Scatter**

The probability of Compton scatter is a slowly varying function of energy. It is proportional to the density of the material ($\rho$) but independent of $Z$.

$$\mu_{CS} \propto \rho$$

$\rho$ is the density of the material.

As energy is increased scatter is forward peaked.
Interactions of **Photons** with Matter

**Compton Scatter**

The scattering angle of the photon is determined by the amount of energy transferred in the collision.

\[
E_1 = \frac{E_0}{1 + \frac{E_0}{m_e c^2} (1 - \cos \theta)}
\]

- $E_0$ is original photon energy
- $E_1$ is scattered photon’s energy
- $m_e$ is the electron mass ($m_e c^2 = 511$ keV)
- $\theta$ is the scattering angle ($\theta = 0$ is no scatter)
Pair production occurs when a photon interacts with the electric field of a charged particle. Usually the interaction is with an atomic nucleus but occasionally it is with an electron.

Photon energy is converted into an electron-positron pair and kinetic energy. Initial photon must have an energy of greater than 1.022 MeV (> 2 times rest mass of electron).

Positron will eventually interact with a free electron and produce a pair of 511 keV annihilation photons.
Interactions of **Photons** with Matter

Photoelectric vs Compton fractions
Interactions of **Photons** with Matter

**Attenuation**

When a photon passes through a thickness of absorber material, the probability that it will experience an interaction (i.e., photoelectric, Compton scatter, or pair production) depends on the energy of the photon and on the composition and thickness of the absorber.

**Attenuation ≠ absorption**
Interactions of Photons with Matter

Attenuation

For a narrow beam of monoenergetic photons … the transmission of photons through an absorber is described by an exponential equation:

\[ I(x) = I(0)e^{-\mu x} \]

where

- \( I(0) \) = initial beam intensity,
- \( I(x) \) = beam intensity transmitted through absorber,
- \( x \) = thickness of absorber, and
- \( \mu \) = total linear attenuation coefficient of the absorber at the photon energy of interest (note units: cm\(^{-1}\)).
Interactions of **Photons** with Matter

**Attenuation Coefficients**

There are three basic components to the linear attenuation coefficient:

- $\mu_\tau$ due to the photoelectric effect;
- $\mu_\sigma$ due to Compton scattering; and
- $\mu_\kappa$ due to pair production.

The exponential equation can then be written as:

$$I(x) = I(0)e^{-(\mu_\tau + \mu_\sigma + \mu_\kappa)x}$$

or as

$$I(x) = I(0)e^{-\mu_\tau x}e^{-\mu_\sigma x}e^{-\mu_\kappa x}$$
Interactions of **Photons** with Matter

**Attenuation Coefficient**

Linear attenuation coefficient $\mu_1$
- depends on photon energy
- depends on material composition
- depends on material density
- dimensions are 1/length (e.g., 1/cm, cm$^{-1}$)

Mass attenuation coefficient $\mu_m$

$$\mu_m = \mu_1 / \rho$$  ($\rho$ = density of material yielding $\mu_1$)

- does not depend on material density
- dimensions are length$^2$/mass (e.g., cm$^2$/g)
Interactions of **Photons** with Matter

Mass attenuation coefficients for soft tissue

Bushberg Figure 3-13
Interactions of **Photons** with Matter

Examples of linear attenuation coefficient

**NaI(Tl)**

**BGO**

From: Bicron (Harshaw) Scintillator catalog

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Interactions of **Photons** with Matter

**Narrow beam vs broad beam attenuation**

Without collimation, scattered photons cause artificially high counts to be measured, resulting in smaller measured values for the attenuation coefficients.
Half-value thickness is the amount of material needed to attenuate a photon flux by 1/2 (attenuation factor = 0.5).

\[ 0.5 = \frac{I}{I_0} = e^{-\mu_{HVT}} \]
\[ \mu_{HVT} = -\ln(0.5) \]
\[ HVT = \frac{0.693}{\mu} \]

Tenth value thickness is given by

\[ 0.1 = \frac{I}{I_0} = e^{-\mu_{TVT}} \]
\[ TVT = \frac{2.30}{\mu} \]
### Interactions of Photons with Matter

#### Half and tenth value thicknesses

**Examples**

<table>
<thead>
<tr>
<th>Material (energy)</th>
<th>$\mu$ (cm(^{-1}))</th>
<th>HVT (cm)</th>
<th>TVT (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lead (140 keV)</td>
<td>22.7</td>
<td>0.031</td>
<td>0.10</td>
</tr>
<tr>
<td>Lead (511 keV)</td>
<td>1.7</td>
<td>0.41</td>
<td>1.35</td>
</tr>
<tr>
<td>Water (140 keV)</td>
<td>0.15</td>
<td>4.6</td>
<td>15.4</td>
</tr>
<tr>
<td>Water (511 keV)</td>
<td>0.096</td>
<td>7.2</td>
<td>24.0</td>
</tr>
</tbody>
</table>
A polychromatic beam (multiple energies) (e.g., from x-ray tube, Ga-67, In-111, I-131) has complex attenuation properties.

Since lower energies are attenuated more than higher energies, the higher energy photons are increasingly more prevalent in the attenuated beam.
Interactions of **Photons** with Matter

**Secondary Ionization**

The Photoelectric Effect and Compton Scattering ionize an atom.

Pair production creates two charged particles.

These energetic charged particles in turn ionize more atoms producing many free electrons.

These secondary ionization are the basis for most photon detectors.

- Ionization also leads to breaking molecular bonds → basis of most radiation biological effects.
Example: Attenuation Calculation 1

What fraction of 140 keV photons will escape unscattered from the middle of a 30 cm cylinder?

The photons must travel through 15 cm of water.

\[
\frac{l}{l_0} = e^{-\mu d} = e^{-(0.15/\text{cm})(15\text{cm})} = 0.105 = 10.5\%
\]
Example: Attenuation Calculation 2

What fraction of pairs of 511 keV photons will both escape unscattered from the middle of a 30 cm cylinder?

The photons must each travel through 15 cm of water.

\[
\frac{l}{l_0} = e^{-\mu d} = e^{-(0.096/\text{cm})(15\text{cm})} \times e^{-(0.096/\text{cm})(15\text{cm})} = 5.6\%
\]

Note: fewer 511 keV pairs escape unscattered than a single 140 keV photon.
Example: Attenuation Calculation 3

What thickness of lead is required to attenuate 99% of 511 keV photons?

99% attenuated = 1% surviving
Using the exponential attenuation formula
\[
0.01 = \frac{I}{I_0} = e^{-\mu d} = e^{-\left(1.7/\text{cm}\right)d}
\]
\[
\ln(0.01) = -(0.17/\text{cm})d
\]
\[
d = -\ln(0.01)/(1.7/\text{cm}) = 2.7 \text{ cm}
\]

Alternatively, if the TVT is known (1.35 cm), doubling the TVT results in two consecutive layers which each transmit 1/10 of photons, or a total transmission of 1/100 or 1%. \(2 \times 1.35 \text{ cm} = 2.7 \text{ cm}\).
Example: Buildup Factors
(broad beam versus narrow beam values)

The transmission factor for 511-keV photons in 1 cm of lead was found to be 18% for narrow-beam conditions. Estimate the actual transmission for broad-beam conditions.
Example: Buildup Factors
(broad beam versus narrow beam values)

<table>
<thead>
<tr>
<th>Material</th>
<th>Photon Energy (MeV)</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>7</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>0.1</td>
<td>4.55</td>
<td>11.8</td>
<td>41.3</td>
<td>137</td>
<td>321</td>
<td>938</td>
<td>2170</td>
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<tr>
<td></td>
<td>0.5</td>
<td>2.44</td>
<td>4.88</td>
<td>12.8</td>
<td>32.7</td>
<td>62.9</td>
<td>139</td>
<td>252</td>
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<tr>
<td></td>
<td>1.0</td>
<td>2.08</td>
<td>3.62</td>
<td>7.68</td>
<td>15.8</td>
<td>26.1</td>
<td>47.7</td>
<td>74.0</td>
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<td>2.0</td>
<td>1.83</td>
<td>2.81</td>
<td>4.98</td>
<td>8.65</td>
<td>12.7</td>
<td>20.1</td>
<td>28.0</td>
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<td>4.0</td>
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<td>2.24</td>
<td>3.46</td>
<td>5.30</td>
<td>7.16</td>
<td>10.3</td>
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<td>6.0</td>
<td>1.51</td>
<td>1.97</td>
<td>2.84</td>
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<td>5.37</td>
<td>7.41</td>
<td>9.42</td>
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<td>10.0</td>
<td>1.37</td>
<td>1.68</td>
<td>2.25</td>
<td>3.07</td>
<td>3.86</td>
<td>5.19</td>
<td>6.38</td>
</tr>
<tr>
<td>Lead</td>
<td>0.5</td>
<td>1.24</td>
<td>1.39</td>
<td>1.62</td>
<td>1.88</td>
<td>2.10</td>
<td>2.39</td>
<td>2.64</td>
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<td>2.89</td>
<td>3.51</td>
<td>4.45</td>
<td>5.27</td>
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<td>1.40</td>
<td>1.76</td>
<td>2.52</td>
<td>3.74</td>
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<td>9.08</td>
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<td>1.36</td>
<td>1.67</td>
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<td>5.61</td>
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<td>6.61</td>
<td>13.7</td>
<td>26.6</td>
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<tr>
<td></td>
<td>10.0</td>
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<td>7.37</td>
<td>15.4</td>
<td>50.8</td>
<td>161</td>
</tr>
</tbody>
</table>

*Data taken from Schleien B (ed): The Health Physics and Radiological Health Handbook. Silver Spring, MD, Scinta, 1992. Note that these values apply to the buildup of photon intensity in terms of it related exposure value, a quantity commonly used for radiation protection purposes (see Chapter 22).
The transmission factor for 511-keV photons in 1 cm of lead was found to be 18% for narrow-beam conditions. Estimate the actual transmission for broad-beam conditions.

HVT for lead is 0.4 cm. Therefore, $\mu_l = 0.693/0.4 \text{ cm} = 1.73 \text{ cm}^{-1}$. For 1 cm $\mu_l x = 1.73$. Using values from buildup factor table for lead and linear interpolation:

$$B = 1.24 + 0.73 \times (1.39 - 1.24)$$

$$B = 1.35$$

$$T = B e^{-\mu_l x}$$

$$T = 1.35 \times 18\% = 24\%$$
Main points of:
“Interaction with matter”

Charged particles

- *Short ranged* in tissue
  - ~ µm for alphas (straight path and continual slowing)
  - ~ mm for betas (more sporadic path and rate of energy loss)
- Interactions Types: Excitation, Ionization, Bremsstrahlung
- Linear energy transfer (LET) - nearly continual energy transfer
- Bragg ionization peak - specific ionization peaks as particle slows down

Photons

- Relatively long ranged (range ~cm)
- Local energy deposition - photon deposits much or all of their energy each interaction
- Interactions Types: Rayleigh, Photoelectric, Compton, Pair Production
- Compton - dominant process in tissue-equiv. materials for Nuc. Med. energies
- Beam hardening – preferential absorption of lower E in polychromatic photon beam
- Buildup factors - narrow vs. wide beam attenuation
- Secondary ionization - useful for photon detection