# Economic Analysis of Commuting Service Platform

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**Abstract**

We propose and investigate the concept of commuting service platforms (CSP) that provide commuting services and connect directly commuters (employees) and their worksites. By applying the two-sided market analysis framework, we show under what conditions a CSP may present the two-sidedness. Both the monopoly and duopoly CSPs are then analyzed. We show how the price allocation, i.e., the prices charged to commuters and worksites, can impact the participation and profit of the CSPs. We also add demand constraints to the duopoly model so that the participation rates of worksites and employees are (almost) the same. With demand constraints, the competition between the two CSPs becomes less intense in general. In most parameter settings, both CSPs make medium profits. Only under a few parameter sets will one CSP make high profits while the other CSP struggles with low profits. The analysis results are expected to provide useful insights on how to develop the next generation travel demand management strategies that can leverage the emerging urban mobility services.

## 1 Introduction

Commuting is "recurring travels between home and work or study" (Wikipedia, Accessed: 2018). Accounting for a substantial portion of the total daily trips (e.g., commuting trips are about 1/3 of the total daily trips in the US), commuting trips are important to businesses (employers), local economy, and people's daily life, which also experience the most congestion and related problems, especially in fast growing urban areas. Among all trips, they are probably the easiest to describe: Businesses (employers) lease/purchase space for their activities (often in urban areas), and abide by city ordinance. Employees (in this paper, we focus on the commuting choices of employees, so the terms "employee" and "commuter" are used interchangeably) work at the businesses, meeting the work and arrival times set by employers. Normally, employers make decisions that largely determine employees' work schedule. In response to such schedule, employees (or students if commuting to school is concerned) are commuters who create the commuting traffic (i.e., travel demand) as they decide on mode/vehicle of travel, time of travel, route, and the like. At the same time, public agencies manage/maintain the transportation infrastructure, providing proper capacity (and policies) to serve the demand. Congestion happens when demand exceeds capacity, often at specific periods of time, e.g., the peak periods that are in most cases the commuting periods.

Reducing commuting congestion and related issues have been a long-lasting challenge in transportation. In addition to infrastructure expansion (rare nowadays) and efficient traffic control schemes (such as traffic signal control, routing, etc.), adequate travel demand management (TDM) methods are crucial. TDM focuses on developing relatively longer term planning and coordination strategies to help manage people's time and modes of travel (FHWA, 2012), with the purposes of eliminating certain trips, or switching them to more efficient modes (such as transit), or changing trip starting times (e.g., for peak spreading). TDM has been studied very extensively in the last several decades (Ferguson, 1990) especially for commuting traffic. There are TDM programs in some cities in the US and around the world that developed TDM strategies to reduce congestion by promoting non single-occupancy-vehicle (SOV) travel, among others. One example is the commute trip reduction (CTR) program of the State of Washington (WSDOT, 2009). As discussed above, commuters, their employers, and transportation management agencies are the major players for commuting related decisions. Effective TDM methods should recognize and take advantage of the behavior and interactions of these major players.

In the existing urban transportation system, commuting related decisions by the major players (i.e., commuters, employers, and transportation agencies) are only loosely connected and largely isolated. Businesses (employers) are the major attractor of commuting trips, but have no or little responsibility of managing traffic or congestion; commuters (employees) form the commuting traffic in the transportation system, and to a great extent, have to follow the work schedule established by their employers and thus do not have much flexibility in their commuting schedule (Holguín-Veras *et al.*, 2011); transportation agencies, who provide transportation infrastructure and system capacity, do not have any direct control on travel schedule, demands, etc. Some TDM strategies did recognize this issue and developed policies and programs to keep employers in the loop (such as employer-based transit passes, vanpooling, telecommuting, parking management, etc.) to manage commuting demands by encouraging their employees to switch from SOV travels to more efficient modes or avoid travel at all (Georgia Institute of Technology, 1994; Young, 1992; WSDOT, 2009). Furthermore, many technology companies and government agencies have been implementing flexible work hours or even telecommuting policies so that their employees do not need to follow strict work schedule or go to work every day. However, such strategies are for individual employers/employees and mostly on a voluntary basis, lacking coordinations among different employers and even employees within the same employer. This results in much less significant impact to reduce commuting problems than what they could have achieved, which can be shown clearly by the steady increase in commuting related congestion, e.g., the number of hours wasted in traffic by an average US commuter in urban areas has grown for about 40% in 6 years, from 36 hours in 2009 to 50 hours in 2015 (Inrix, 2015). Therefore, to make TDM strategies more effective, we need mechanisms that can more closely connect/coordinate employers and employees (and also agencies) so that the effect of commuting trips can be directly reflected in their decision makings.

On the other hand, recent technology advances have produced novel mobility modes that have transformed (and will continue to transform) urban transportation. For example, mobile-app based new mobility services have led to the paradigm of mobility as a service (MaaS), the "integration of various forms of transport services into a single mobility service accessible on demand" (ERTICO, 2016). MaaS connects transportation service providers and travelers directly, which includes various forms (Shaheen & Ismail, 2016) such as ridesourcing (e.g., Uber/Lyft), ridesharing, carpooling, carsharing, bikeshring, on-demand shuttle services, among others. By focusing on all types of travels (commuting, entertainment, shopping, etc.), current MaaS only connects travelers with service providers, thus excluding key players of important trips (e.g., employers in commuting trips). As a result, the current form of MaaS may not be effective in solving commuting problems. After all, we have been trying to "nudge" travelers by technologies, incentives, etc. to the point that we probably need other innovative ways as well to collectively solve commuting problems. Meanwhile, there are also employer-sponsored transportation programs (ESTP), in which employers are directly involved with providing commuting services to their employees (Apple Inc. *et al.*, 2012). This is mostly in the form of providing carpool or shuttle services to employees from home to work and vice versa, by either operating the shuttle services directly (e.g., Amazon) or by outsourcing the operations to a third party (e.g., Microsoft). For example, Amazon piloted a shuttle project to bring workers from suburb areas to its Seattle campus in 2016 (Levy, 2016). Microsoft started the Connector program to shuttle employees from adjacent areas to its headquarter in Redmond and offices in Bellevue in 2007. Google and Apple have also implemented similar shuttle services to improve the commuting condition of their employees (Helft, 2007; Dormehl, 2015). More recently, industry innovators are tapping into ESTP by helping design TDM strategies (Luum, 2019) and provide carpool services to co-workers (Scoop, 2019).

While both MaaS and ESTP are rapidly evolving, we see a growing trend of the integration of the two. We believe that integrating MaaS and ESTP to focus on commuting trips, facilitated by proper TDM policies, may provide the needed mechanism to better connect employers and commuters, and as a result providing new ways to solve commuting challenges. In particular, we envision that such integration may produce the so-called employer-based *commuting service platform* (CSP) to provide future urban commuting services. CSP can help match an employer with its potential employees in the long term (called the *planning* level) and provide commuting services on a daily basis (called the *operational* level). We focus on the planning level analysis of CSP in this paper. Employers are motivated to join CSP because (i) they are increasingly aware of the commuting issues and have started to help directly or indirectly their employees' daily commuting (e.g., the above-discussed ESTPs), which has become one of the important strategies for them to recruit/retain the needed talents (Commute Seattle, 2016, 2017; Harrington, 2019); (ii) more companies are supporting sustainability and are becoming more socially responsible (including how to deal with congestion and related issues caused by commuting); (iii) there are pressures from local communities, cities, and even states for companies to take more actions to help resolve commuting issues, e.g., the CTR program in the State of Washington. For employees, it is always in their best interest to find the optimal commuting options that can better balance work and family. Therefore, when selecting employers, possible commuting options and the commuting packages an employer can provide will have an important impact on their decisions. Therefore, CSP can be considered as a platform to connect employers and their potential employees when commuting is concerned, similar to how Amazon connects sellers and potential buyers via its online platform.

With CSP, an employer needs to subscribe for the platform (by paying an annual fee or per "transaction" fee; here a transaction means a commuting service for one of its employees) so that its employees can use the service. Employees will also be charged each time the CSP service is used. For the platform (i.e., CSP), the cost of service will still be the cost of labor and vehicle depreciation, fuel, maybe also the cost of negotiation with employers, etc. However, the source of revenue has new components. Instead of charging every commuter (employee) a fare as traditionally done, CSP has potential revenue sources from both the commuter side and the employer side. That is, CSP can closely connect employers and employees, with proper policies / management strategies from the agencies, which is similar to a two-sided market. Two-sided market is characterized by two distinct sides (e.g., employers and employees on the CSP) who get ultimate benefit by interacting through a common platform (e.g., the CSP in our study) (Rochet & Tirole, 2003). A platform is said to be two-sided if the price allocation but not only the aggregated price of the two sides affects the profit (or participation) (Rochet & Tirole, 2006). Notice here that two-sided market methods have been applied to analyze MaaS where the two sides are service providers and travelers (Zha *et al.*, 2016; Djavadian & Chow, 2017). In a CSP, however, the two sides are employers and employees (commuters), which is markedly different from the analysis for MaaS.

In this paper, we aim to conduct the economics analysis of CSP in the planning level. We are interested in understanding the interactions of employers and employees on CSP, under what conditions a CSP will be a two-sided market, and if so how to apply the two-sided market analysis framework to study the basic interactions of employers and employees on the CSP in a systematic manner. Such analyses can help understand the interactions of key players and how different policies (such as prices charged by CSP for employers and employees) by the platform may lead to different behavior of employers/employees and the resulting system effects, based on which to develop operational level methods of CSP and related TDM strategies. To begin with, we focus on a particular TDM strategy called proximate commute (Mullins, 1999) in this study. Proximate commute allows employees who work for a multi-worksites company/employer (e.g., Starbucks, Key Bank, etc.) to be assigned to worksites closer to their homes, which is beneficial to employees, employers, community, and the environment. Allowing qualified employees voluntarily swapping worksites is one of the ways that a multi-worksites employer could reduce commuting distance for its employees (Mullins, 1995). Here we assume that one or two CSPs are providing the commuting services to all the employees of the employer. We will apply the two-sided market analysis method to understand the interactions of worksites and employees when proximate commute is implemented, and the effect and implications of such interactions/behavior.

For the analysis, we will start with a monopoly (single) CSP that provides one type of commuting services. We will then analyze a duopoly model with two CSPs providing two types of commuting services. For the duopoly model, we are interested in the impact of worksite flexibility on commuting trips, and assume two commuting services provided by the CSPs: non-work-flex (NWF) services for which an employee needs to arrive at the worksite punctually at a particular work starting time (say 9 am in the morning), and work-flex (WF) services for which an employee has more flexibility to arrive at his/her worksite (say from 8 am to 10 am). Different players (commuters, worksites, and the CSPs) may view the two services differently: commuters may like WF due to the flexibility it provides, worksites may prefer NWF since it is easier to manage, while a CSP may prefer WF so that it does not need to send all employees to their worksites simultaneously. Therefore, understanding how the price allocation of the CSPs may influence the choices of the commuters and worksites (i.e., the two sides) regarding the two services (CSPs) will be of paramount importance to devise sensible policies and TDM strategies to encourage the use of one CSP over the other, from the perspective of managing commuting demands and related issues. Besides understanding the economic behaviors of commuters, worksites and CSPs, another concern is how the employees will be matched to the worksites in a two-sided market. For this, we add demand constraints for the duopoly model to ensure that the participation rates of employees and worksites are (almost) the same.

The proximate commute scenarios studied here is a very simplified version of a general CSP. However, we believe that the proposed CSP concept, and the two-sided market based modeling framework and analysis method developed in this paper are the first critical step and a crucial building block to establish and analyze more general CSPs and mobility service platforms for other types of trips in future urban mobility systems. Ultimately, we hope that such analyses can help develop the CSPs and next-generation TDM strategies that can better leverage the emerging systems and technologies.

## 1.1 Literature review

### 1.1.1 TDM and Proximate Commute

Originated from 1970s and 1980s, TDM promotes collaborative efforts from employers, commuters, governments to avoid the costly expansion of the transportation system (Ferguson, 1990). Since then TDM has been guiding the design of transportation and physical infrastructure and encouraging the use of transit, ridesharing, walking, biking, and telework (Mobility-Lab, 2013). There are mainly three groups of TDM strategies to : 1) improve mobility options, such as carsharing services, HOV priority lanes, walking and cycling improvement, public transportation improvement, telecommuting, flexible working hour, etc; 2) apply economic measures such as congestion price, parking regulations, etc; 3) enhance smart growth and land use policies, including transit-oriented development, location-efficient development, etc (Andrea Broaddus, 2009).

In the early 1990s, Southern California launched an employer-based TDM program to improve air quality by trip reduction, which required employers with over 100 employees during peak hours to conduct trip reduction plans (Georgia Institute of Technology, 1994). The cost-effectiveness of the trip reduction plans was studied, among which the commissioned program in an accounting firm resulted in a decrease of 8.4 % in daily vehicle trips (Young, 1992). Commute trip reduction (CTR) is a specific TDM program for commuting trips, aiming to encourage travelers to drive alone less, reduce carbon emissions and keep the busiest commute routes flowing. For example, since the Washington State Legislature passed the CTR Law in 1991 (Kadesh & Roach, 1997), over 1000 worksites and over 530,000 commuters have joined the state CTR program by 2009. The widely adoption of CTR program has resulted in a 9% traffic delay reduction in the Central Puget Sound Region from 2006 to 2009 (WSDOT, 2009).

Proximate commute is a CTR strategy that allows employees of a multi-worksite organization to be assigned to the worksites close to their homes (Mullins, 1995). It aims to reduce commute distances by swapping workers in different worksites or taking commute distances/times into consideration when building a new worksite or recruiting new workers. Key Bank of Washington conducted a demonstration project of proximate commute in 1995. The project lasted for 15 month, during which nearly 500 employees at 30 Key Bank branches in Washington were given the opportunity to voluntarily switch to the branches closer to their homes. 17% of eligible employees enrolled in this program, for whom commute miles reduced by 65%. There was a 33% reduction in the longest commute per Key Bank worksite. The results showed that proximate commute is a low-cost method for reducing employee commute time, distance, expense, which can help increase work force productivity. Employers was also willing to implement proximate commute because their investment can likely be recouped within a year through reduced absenteeism, higher morale and productivity, and other improvements.

Existing TDM strategies and CTR programs have correctly recognized the importance of involving employers, and evaluations have done on the impact of involving employers in TDM to reduce traffic congestion and related issues (Yushimito *et al.*, 2014, 2015). However, those programs and evaluations were often done for individual employers/employees, and lacked coordinations among different employers and employees. More critically, they have not taken the full advantage of emerging mobility options (such as ridesourcing) into consideration. In this paper, we attempts to investigate a specific TDM strategy, i.e., the next-generation proximate commute strategies that are implemented via the CSP.

### 1.1.2 Two-sided market

Two-sided markets are defined as markets where one or several platforms enable interactions between the the two sides and get the two sides "on board" by appropriately charging each side (Rochet & Tirole, 2006). For example, the Uber platform matches drivers and riders, and charges riders while pays wages to drivers ( wages can be considered as negative prices charged to drivers). The two sides choose to join the platform (i.e., consume the services provided by the platform) that makes them better-off. The platform bears the cost of services and charges the two sides to obtain profits. Because of the same-side negative network effects and the cross-side positive network effects, the price allocation but not only the total price of services will affect the participation rates of both sides and the profit of the platform (Rochet & Tirole, 2006). Same-side effects capture the consumer behavior that an agent will usually be worse-off if more agents from the same side join the same platform, whereas cross-side effects exist if an agent from one side benefits from the increasing participation from the other side on a common platform. Depending on the number of platforms and the relations among them, a two-sided market may consist of a single monopoly platform or multiple competitive platforms. When there are competitive platforms, competition among platforms affects the participation and profits on each platform (Armstrong & Wright, 2007). Users from either side could choose to join a single platform, which is referred to as "single-home", or choose to use multiple platforms, which is called "multi-home".

Rochet & Tirole (2003) provided the first comprehensive investigation of the theory of the two-sided market based on a single platform. With an analytical solution of the price allocation for different governance structures, their study unveils how a platform makes profit by courting the two-sides. While illustrated in the context of credit cards, their study provides a benchmark model that is applicable to a wide range of applications of two-sided markets. Armstrong (2006), Armstrong & Wright (2007) analyzed two-sided markets under different degrees of product differentiation on each side of the market. They analyzed the conditions of strong product differentiation on both sides, in which case agents from both sides single-home. The conditions when sellers view the platforms as homogeneous while buyers view them as heterogeneous are also discussed. And in this case buyers still single-home, but sellers choose to multi-home. In the latter case, the platforms compete indirectly for the multi-homing sellers by attracting buyers to join, which is defined as a "competitive bottleneck" equilibria. "Competitive bottleneck" explains the observation that many platforms charge little or nothing to buyers when sellers multi-home.

Two-sided market provides a method to analyze how price structure affects profits and economic efficiency. Take credit cards for an example. Different credit card issuers are platforms; buyers choose to own one (single-home) or multiple (multi-home) types of credit cards for purchase; sellers choose to accept one (single-home) or multiple (multi-home) types of credit cards. A transaction happens on a platform if a buyer purchase from a seller using the credit card issued by the platform. To optimize its profit, the platform needs to decide which side to bear the price burden. This usually leads the platform to make less money on one side, or even subsidize this side, and recoup its cost from the other side. The platform loses profit when subsidizing one side, and this side is regarded as a "loss leader". In the credit card example, buyers are usually the loss leaders and the many promotion programs by credit card issuers (such as points or rebates) are the subsidies.

There are many examples of real-world markets involving two groups of agents interacting via common platforms, which may be characterized as two-sided markets. Examples include: 1) academic publishing; 2) advertising media market; 3) payment systems, such as credit cards; 4) Internet service provider. There are a handful applications of the two-sided market theory in transportation. One example is the matching of drivers and customers in taxi or ridesourcing. By treating the ridesourcing platform as a two-sided market with customers and drivers as the two sides, Zha *et al.* (2016) and Wang *et al.* (2016) studied the matching process with negative same-side externality and the positive cross-side externality. Djavadian & Chow (2017) evaluated an agent-based stochastic day-to-day adjustment process in a two-sided market. A collection of publications in these market is summarized in Table 1.

Table 1: Applications of Two-sided Market

| Field | Platform(s) | Two sides | Findings | Authors |
|---|---|---|---|---|
| **Academic Journals** | Academic journals | authors ; readers | Open access policy makes publications free to readers and charges high publication fees to authors. This policy is good when considering maximizing social welfare, but may harm readers utility, the impact or profit of the journal. | Jeon & Rochet (2010) |
| **Payment card** | credit card, debit card | merchants ;customers | Benchmark model shows that HAC rule not only benefits the multi-card platform but also raises social welfare. However, in the extended model HAC rule may no longer raise social welfare under all parameter settings. | Rochet & Tirole (2008) |
| **Magazine** | Magazine companies | readers; advertisers | Higher demand on the reader side increases advertising rates. Higher demand on the advertiser side reduces the price of magazine to readers. | Kaiser & Wright (2006) |
| **Internet** | Internet Service Provider (ISP) | content providers; broadband users | Network neutrality regulation increases the total surplus under certain parameter ranges for both monopoly and duopoly platforms. | Economides & Tåg (2012) |
| **Flexible mobility services** | The built environment | operators ; travelers | The differences between one-sided and two-sided market. The threshold when the network externalities lead to two-sidedness. | Djavadian & Chow (2017) |
| **Ride-sourcing** | Ride-sourcing services | drivers; passengers | The matching condition/ regulation policy when the first / second best solution holds in monopoly case is found. The study of competing platforms suggests merging of platforms as competition won't lower the price level or improve social welfare | Zha *et al.* (2016) |

Despite the above efforts of two-sided market applications, no study so far has attempted to connect employers and employees by a common platform (e.g., CSP, as proposed in this paper) when commuting trips are considered. Consequently, no study has applied the two-sided market framework to analyze the behavior and interactions of commuters and employers on CSP.

## 2 Preliminaries

### 2.1 Problem Statement

In this paper, we study the proximate commute problem of an employer with multiple worksites when there is a CSP to provide commuting services. There are many examples of such employers in urban areas. For example, as shown in Figure 1, there are 17 Starbucks stores (worksites) in the Seattle downtown area. We will investigate in this paper when proximate commute with CSP will present two-sidedness, and when this happens how the pricing schemes and other mechanisms may impact the participation of the two sides (i.e., worksites and employees) and the profit of the CSP. We will analyze both the monopoly platform and the duopoly platforms in the two-sided markets. For the monopoly platform, there will be only one type of CSP service in the market. For the duopoly platforms, we focus on how work place flexibility may influence employees/employers' choices of the platforms. For this, as discussed above, we assume there are two types of CSPs to provide NWF services and WF services respectively. Through such investigations, we hope to gain deeper understanding of CSPs, how the price allocation of a CSP may impact its scale (i.e., the participation of the two sides) and profit, which can provide useful insight on how to design CSP and next-generation TDM strategies based on emerging technologies and mobility options such as MaaS.

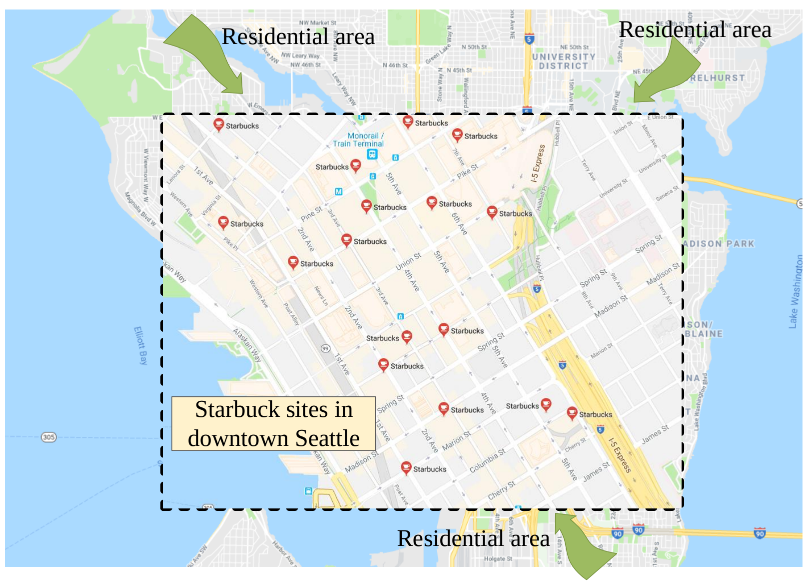



Figure 1: The distribution of Starbucks in downtown Seattle and the commuting trends

We make several assumptions to simplify the real world proximate commute problem:

**(a)** One or multiple CSPs exist to provide commuting services. CSPs charge both worksites and employees for using the service.
**(b)** A commuter can choose which worksite to work for based on his/her own preference.
**(c)** The participation of worksites and employees is not pre-defined in the monopoly model (Section 4) or the duopoly model (Section 5), which is determined by the market equilibrium.
**(d)** We assume that the employees are evenly split among the worksites on the same CSP. In section 5.3, we relax Assumption (c) to add demand constraints to the duopoly model so that the participation of worksites and employees are almost the same, with small (and bounded) deviations. This essentially match the total number of employees with the total number of worksites. In the future, we can further relax this assumption by adding demand constraints to each worksite directly when the location of the worksite is specified in a transportation network.

Assumption (a) ensures that worksites and employees interact on the CSP, and the CSP's price strategy may impact their behavior of using the platform. Under assumption (b), employees are exchangeable among different worksites, which is the key concept of proximate commute. From the two-sided market analysis, we would be able to obtain the proportion of commuters/worksites participating in a platform and the resulting profits. We also start with assumption (c) such that the participation rates of employees and worksites are not pre-specified. We then relax this by adding a demand constraint to relate the participation rates of employees and worksites in assumption (d). In future research, we will specify the home locations of commuters and the locations of worksites; this may allow us to add demand constraints to each worksite directly.

## 2.2 Model Setting

An agent from either side (i.e., worksites or employees) pays a fee to join a CSP. By doing so, the agent gets a fixed benefit. Also, an agent gets better-off when the cross-side network effect increases, and gets worse-off when the same-side network effect increases. An agent chooses the CSP when s/he has higher utility. A CSP sets prices to the two sides to maximize its profit. Here is a list of notations. More specific definitions of those variables/parameters are given in Section 4 and 5.

**Sets:**

- $i$ labels of platforms, $i \in \{N, W\}$; $N, W$ denote NWF CSP, WF CSP, respectively.
- $k$ labels of different groups, $k \in \{B, C\}$; $B, C$ denote worksites, commuters (employees), respectively.

**Variables:**

- $q^k$ (monopoly model) the fraction of group $k$ agents join the CSP.
- $q_i^k$ (duopoly model) the fraction of group $k$ agents single-homing on CSP $i$.
- $Q^k$ (duopoly model) the fraction of group $k$ agents multi-homing on both CSPs. We assume that commuters only single-home, business sites can choose to single-home or multi-home, thus $Q^B \in [0,1]$, $Q^C = 0$.
- $p_i^k$ subscription price of a group $k$ agent on CSP $i$, charged by the platform (static cost), $p_i^k \geq 0$.
- $x^k$ range from $[0,1]$, the location of a group $k$ agent at the unit interval in Hotelling Model.
- $U_i^k$ the utility of a group $k$ agent on CSP $i$.
- $R_i$ the profit of CSP $i$.

Here we use unit measure of group $k$ agents, thus $Q^k + q_N^k + q_W^k = 1$ in the duopoly model.

**Parameters:**

$U_0^k$ the intrinsic benefit of a group $k$ agent when joining a CSP.

$t^k$ the rate of inconvenience cost (i.e., same-side "congestion" effects) of the Hotelling model.

$\beta_i$ the rate of cross-side benefit of commuters on CSP $i$. A commuter obtains benefit $\beta_i q_i^B$ by joining CSP $i$, as she has the potential to choose from $q_i^B$ worksites.

$\alpha_i$ the rate of cross-side benefit of worksites on CSP $i$. A worksite obtains benefit $\alpha_i q_i^C$ by joining CSP $i$, as it has the potential to choose from $q_i^B$ commuters.

$b^k$ the rate of cross-side benefit of group $k$ agents in monopoly model

$f_i^k$ the cost of CSP $i$ to serve group $k$ agents in duopoly model.

$f^k$ the cost of the CSP to serve group $k$ agents in monopoly model.

**Auxiliary parameters:**

$\alpha^+, \alpha^-$ the sum/difference of the cross-side benefit rate of worksites on the two CSPs, respectively, defined for analytical purpose, $\alpha^+ = \alpha_N + \alpha_W$, $\alpha^- = \alpha_N - \alpha_W$

$\beta^+, \beta^-$ the sum/difference of the cross-side benefit rate of commuters on the two CSPs, respectively, defined for analytical purpose, $\beta^+ = \beta_N + \beta_W$, $\beta^- = \beta_N - \beta_W$

$\psi_W^k$ a term of equilibrium prices in Proposition 2, affected by different cross-side/same-side network effect on the two CSPs

## 3 Hotelling model and linear demand specification

To model the inconvenience cost of worksites and employees joining the CSP, we apply the Hotelling model (Hotelling, 1929) that has been applied to many studies in two-sided markets. Economides & Tåg (2012) used Hotelling model for monopoly and duopoly two-sided markets. Rochet & Tirole (2003), Armstrong (2006) and Kaiser & Wright (2006) applied Hotelling model for duopoly two-sided market models. In this study, we apply Hotelling model in both monopoly platform and duopoly platforms. A classic Hotelling model depicts that customers are uniformly distributed on a unit length street, and two stores locate at the two ends ($x = 0$ and $x = 1$) of the street. Hotelling competition assumes that consumers purchase at those stores if and only if the minimum utility of consuming at the two stores is larger than some constant $\bar{U}$ (Fudenberg & Tirole, 1991). Under this assumption, the Nash equilibrium is achieved when the utility of purchasing at the two stores are the same. Denote the Nash equilibrium as $x^*$, $x^* \in [0,1]$. Under equilibrium, consumers located to the left of $x^*$ choose the store at $x = 0$, the rest of consumers choose the store at $x = 1$. Therefore, $x$ also indicates the proportion of customers who choose the store at $x = 0$. The role of Hotelling model in monopoly platform and duopoly platforms are similar. Here we use duopoly platforms as an example, and illustrate how the Hotelling model is applied to represent inconvenience cost.

In duopoly platforms, the NWF CSP ($x = 1$) and the WF CSP ($x = 0$) are located at the two ends of a unit interval, as shown in Figure 2. The demands of worksites and commuters are both specified by Hotelling models. Take worksites as an example, worksites distribute uniformly along the unit interval. $x^B$ denotes the location of a worksite, which also indicates preference of the worksite over the WF CSP (or the NWF CSP): smaller $x^B$ implies the worksite prefers more of the WF CSP (and less of the NWF CSP) . $t^B$ denotes the rate of inconvenience cost of worksites. The worksite located at $x^B$ experiences a inconvenience cost of $t^B x^B$ when joining the WF CSP, or a inconvenience cost of $t^B(1 - x^B)$ when joining the NWF CSP (shown in equation (15) and (16)). Under such setting, the cost term induced by Hotelling model ($t^B x^B$ or $t^B(1 - x^B)$) reflects the same-side "congestion" effects. It means that a worksite will be worse-off if more other worksites join the same CSP. $t^B$ can be understood as the same-side "congestion" effects of the two CSPs. At the same time, $t^B$ also reflects the level of competition between the two CSP. The competition becomes stronger for worksites when $t^B$ decreases (Armstrong & Wright, 2007). This can be explained as follows. For the same level of $x^B$, if we decrease $t^B$, the gap of the inconvenience cost of joining the two platforms is smaller, namely $t^B x^B - t^B(1-x^B)$ is smaller. This indicates that worksites experience less distinct inconvenience cost on the two CSPs when $t^B$ is small, which means the level of competition is high. At the equilibrium, the optimal $x^{B*}$ represents the participation rate of businesses in the WF CSP, and $1-x^{B*}$ is the participation rate of businesses in the NWF CSP.

Same analysis can be applied to the commuter side, which is omitted here. We can also apply the Hotelling model to the monopoly platform, with a similar interpretation as shown in Figure 2 that one end is the CSP and the other end is "not joining the CSP".

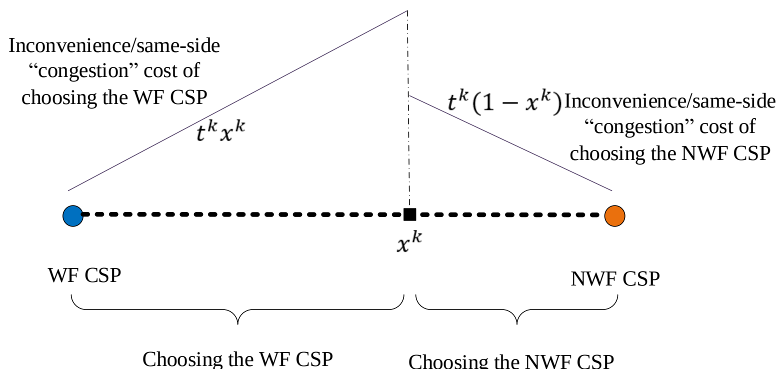



Figure 2: An illustration of Hotelling Model ($k \in \{B, C\}$)

# 4 Monopoly platform

In a monopoly model, agents from the two sides choose to join the CSP or be off the market. The utilities of agents are affected by the same-side and cross-side network effects. In this section, we first discuss two-sidedness of the CSP, then analyze how network effects impact the participation, price strategies, and profits of the CSP.

## 4.1 Two-sidedness in commuting problems

Before introducing the model, we want to answer why the envisioned CSP is a two-sided market, and why the two-side market theory is important in studying the CSP. We first present a benchmark model of the monopoly platform similar to Armstrong (2006) to illustrate the two-sidedness of a market with CSP. Note that in this benchmark model, the same-side congestion effect will not be considered, which will be added later in Section 4.2. As stated above, worksites and commuters are the two sides, denoted as $k \in \{B, C\}$. We assume unit mass for both sides, i.e., $q^B, q^C \in [0,1]$. $q^B$ and $q^C$ are then also the participation rates of the two sides. By joining the CSP, a group $k$ agent incurs a fixed benefit $U_0^k$. In the monopoly model, there is only one CSP sending commuters to worksites. The costs for the CSP to serve the two groups are $f^B and f^C$, respectively. $f^C$ represents the per commuter transportation cost, while $f^B$ represents the negotiation cost of attracting a worksite to join the CSP. The rate of the cross-side network benefits are measured by $b^k$. By joining the CSP, each agent from group $k$ experiences a benefit of $b^k q^l$ under the assumption that s/he values the participation of the other group $l$. This is intuitively understandable. A commuter on the CSP will be better-off if more worksites join the CSP since s/he will have more choices of worksites. Similarly, a worksite on the CSP will also benefit if more commuters join the CSP since this can potentially attract more employees (commuters) to the worksite, which is the desired results when the worksite subscribes for the CSP.

Based on the above discussions, the utility of a group $k$ agent is determined by

$$U^k = U_0^k + b^k q^l - p^k \quad \forall k,l \in \{B,C\} \text{ and } k \neq l \tag{1}$$

Supposing that the participation of group $k$ agents on the CSP can be measured by an increasing function of utility, the participation of group $k$ agents is:

$$q^k = \phi^k(U^k) \tag{2}$$

From equation (1), the price of group $k$ can be expressed as $p^k = U_0^k + b^k q^l - U^k$. Profit of the CSP can be written as,

$$\begin{aligned} R &= (p^B - f^B)\phi^B(U^B) + (p^C - f^C)\phi^C(U^C) \\ &= \left[(U_0^B + b^B\phi^C(U^C) - U^B) - f^B\right]\phi^B(U^B) + \left[(U_0^C + b^C\phi^B(U^B) - U^C) - f^C\right]\phi^C(U^C) \end{aligned} \tag{3}$$

The equilibrium price can be obtained by maximizing the profit of the platform:

$$p^k = f^k - U_0^k - b^l\phi^l(U^l) + \frac{\phi^k(U^k)}{[\phi^k(U^k)]'} \quad \forall k,l \in \{B,C\} \text{ and } k \neq l \tag{4}$$

The profit maximizing prices can be written in the form of Lerner indices and elasticities (Lerner, 1934). As shown in Proposition 1.

**Proposition 1.** *Write*

$$\eta^B(p^B \mid q^C) = \frac{p^B[\phi^B(U^B)]'}{\phi^B(U^B)} = \frac{p^B[\phi^B(U_0^B + b^B q^C - p^B)]'}{\phi^B(U_0^B + b^B q^C - p^B)} \tag{5}$$

$$\eta^B(p^C \mid q^B) = \frac{p^C[\phi^C(U^C)]'}{\phi^C(U^C)} = \frac{p^C[\phi^C(U_0^C + b^C q^B - p^C)]'}{\phi^C(U_0^C + b^C q^B - p^C)} \tag{6}$$

*for a group's price elasticity of demand given the level of participation by another group. Then the optimal prices satisfy*

$$\frac{p^B - (f^B - U_0^B - b^C q^C)}{p^B} = \frac{1}{\eta^B(p^B \mid q^C)}; \quad \frac{p^C - (f^C - U_0^C - b^B q^B)}{p^C} = \frac{1}{\eta^C(p^C \mid q^B)} \tag{7}$$

Based on this benchmark model and Proposition 1, we illustrate two important concepts:

*(i) Loss leader:* under the optimal price structure, it is possible for group $k$ agents to act as the loss leader (to be subsidized), that is, when $p^k < f^k$. From equation (7), this occurs if the group's elasticity of demand $\eta^k(p^k \mid q^l)$ is large and/or the cross-side benefit ($b^l$) enjoyed by group $l$ ($k,l \in \{B,C\}, k \neq l$) is large. On the other hand, when $b^l$ is small and/or $\eta^k(p^k \mid q^l)$ is small, the CSP charges higher price to group $k$ agents. This implies that, if worksites value the number of commuters that join the CSP, and/or commuters' elasticity of demand is high, the CSP will subsidize the commuters to attract more commuters, thus more worksites to join the platform.

*(ii) Two-sidedness:* two-sidedness is conceptually defined from either of the two aspects: 1) "the volume of transaction on the platform depends on the allocation of price between the two sides but not only on the aggregated price level" (Rochet & Tirole, 2006); 2) the decision of each group of agents affects the outcomes of the other group of agents, typically through an externality, i.e. cross-side positive network effects and same-side negative network effects (Rysman, 2009; Caillaud & Jullien, 2003; Armstrong, 2006). Because CSP services transport employees to their worksites, no agent from one group would be willing to join the CSP unless agents from the other group also join the CSP (if no commuter joins the CSP, it's irrational for worksites to join the CSP and vice-versa). As a result, the price of one group is affected by the participation and network effects of

both groups. In the benchmark model, if there is no network effects (either cross-side or same-side) and only the aggregated price level affects the transaction, the participation of the two sides will be the same, $q^B = q^C = \phi(p^B + p^C)$. The model will be reduced to $R = (p^B + p^C - f^B - f^C)\phi(p^B + p^C)$. In the reduced model, if the CSP holds the aggregated price to be constant, its profit will not change even when the price allocation differs. In this case, the model will reduce to one-sided. In the models in section 4.2 and 5, we assume there are network effects in the market. Therefore, participation always changes with price allocation and network effects from both groups. Two-sidedness is important for us to unveil how to incentivize participation on the CSP under different level of network effects.

Now that we are clear about the definition of two-sidedness, we can continue to argue that one-sided logic is not suitable for the proposed CSPs. Wright (2004) listed eight fallacies of using one-sided logic in two-sided markets, most of which are applicable for the CSPs. For example, the prices of one-sided markets reflect the relative costs of products, which means that the price is high for the high-cost product/service, and the price can be low for the low-cost product/service. In our scenario, the CSP offers rides to commuters, thus the cost of serving a commuter can be represented by the transportation cost of sending the commuter from home to a worksite. The cost of serving a worksite is not as obvious. Imagine that a CSP may need to negotiate with worksites to convince them to subscribe for its service, and this process can be regarded as the cost of serving worksites. If the market is one-sided, the price charged to a customer (or a worksite) tends to be no less than the marginal cost of serving the customer (or the worksite). However, this is not true for CSPs. If the worksites value the commuters more than vice-versa, the CSP will reduce the price of commuters to attract more commuters, thus more worksites. Commuters are subsidized, so the price charged to commuters may be lower than the marginal cost. Therefore, one-sided logic cannot explain for the surplus (cross-side benefit) a worksite enjoys when an extra commuter join the same CSP. Furthermore, under the one-sided logic, competition between CSPs reduces prices to the marginal cost, and higher price-cost margin makes a CSP more competitive among the others, which is not accurate for CSP. A CSP in our setting tends to be more competitive when it sets a good price allocation by properly assessing the cross-side benefits and the same-side "congestion" effects between the two sides (see section 5).

Based on our discussion of two-sideness, we can summarize the condition of two-sidedness of the monopoly platform as follows,

**(A0)** $b^k > 0, t^k > 0$, ensures that network effects exist ($b^k$ represents cross-side network effects. Same-side effects $t^k$ was introduced in section 3).

When the CSP extracts profits from both sides in a market with network effects, the participation of one side will be affected by the decisions of both sides, thus two-sidedness holds. Condition **(A0)** is the basic feature of a two-sided market, which is applied to all of the models in this paper. In section 4.2, we add Hotelling Model to the monopoly platform, which can also be understood as same-side negative network effects. In section 5, we analyze a market with two CSPs, each of which satisfies condition **(A0)**.

### 4.2 Monopoly platform with linear demand specification

In this part, we add inconvenience cost to the benchmark model mentioned above. We introduce the Hotelling Model to the utility function to represent the horizontal differentiation between joining vs. not joining the CSP. A group $k$ agent incurs fixed benefit $U_0^k$ when joining CSP. Group $k$ agents are uniformly distributed on a unit interval $[0,1]$ with the WF CSP at $x = 0$. $t^k$ is the rate of inconvenience cost (same-side "congestion" effect) when an agent from group $k$ joins the CSP. A group $k$ agent located at $x^k$ experiences an inconvenience cost (same-side "congestion" effect) of $t^k x^k$ to join the WF CSP, or a cost of $t^k(1-x^k)$ if s/he does not join the NWF CSP. Adding the new utility terms $U_0^k$ and $-t^k x^k$ to equation (1), we obtain the following utility functions:

$$U^C = U_0^C + b^C q^B - t^C x^C - p^C \qquad U^B = U_0^B + b^B q^C - t^B x^B - p^B \tag{8}$$

Here are the conditions that ensure the equilibrium prices of the monopoly model are feasible (Economides & Tåg, 2012) :

**(A1)** Cross-side positive effects are not strong. When same-side negative effects and cross-side positive effects follow the condition $t^B t^C > (b^B + b^C)^2$, the profit function is concave and the equilibrium prices are feasible;
**(A2)** The parameter setting satisfies $4t^B t^C - (b^B + b^C)^2 \geq \max\{(v^C - f^C)(b^B + b^C) + 2t^C(v^B - f^B), (b^B + b^C)(v^B - f^B) + 2t^B(v^C - f^C)\}$, which ensures $q^B, q^C \leq 1$.

Equilibrium exists when worksites or commuters are indifferent between choosing and not choosing the CSP. In the Hotelling model, agents from both sides are uniformly distributed on a unit interval. Notice that we assume unit mass for both sides, thus $x^k = q^k$. The demand of two groups are:

$$q^B = \frac{b^B(U_0^C - p^C) + t^C(U_0^B - p^B)}{t^B t^C - b^B b^C} \qquad q^C = \frac{b^C(U_0^B - p^B) + t^B(U_0^C - p^C)}{t^B t^C - b^B b^C} \tag{9}$$

Substitute equation (9) to profit function, we can write CSP profit as a function of the prices:

$$R = \frac{b^B(U_0^C - p^C) + t^C(U_0^B - p^B)}{t^B t^C - b^B b^C}(p^B - f^B) + \frac{b^C(U_0^B - p^B) + t^B(U_0^C - p^C)}{t^B t^C - b^B b^C}(p^C - f^C) \tag{10}$$

Maximize CSP's profit using the first order condition of equation (10). The equilibrium prices are

$$p^B = \frac{-b^{B^2} f^B - b^{C^2} U_0^B + (2t^B t^C - b^B b^C)(f^B + U_0^B) + t^B(b^B - b^C)(U_0^C - f^C)}{4t^B t^C - (b^B + b^C)^2} \tag{11}$$

$$p^C = \frac{-b^{C^2} f^C - b^{B^2} v^C + (2t^B t^C - b^B b^C)(f^C + U_0^C) + t^C(b^B - b^C)(f^B - U_0^B)}{4t^B t^C - (b^B + b^C)^2} \tag{12}$$

Substitute equation (11) and (12) into equation (9). Demands on the CSP platform are

$$q^B = \frac{(U_0^C - f^C)(b^B + b^C) + 2t^C(U_0^B - f^B)}{4t^B t^C - (b^B + b^C)^2} \quad q^C = \frac{(b^B + b^C)(U_0^B - f^B) + 2t^B(U_0^C - f^C)}{4t^B t^C - (b^B + b^C)^2} \tag{13}$$

Substitute equation (11) and (12) into equation (10), we get the profit at equilibrium price

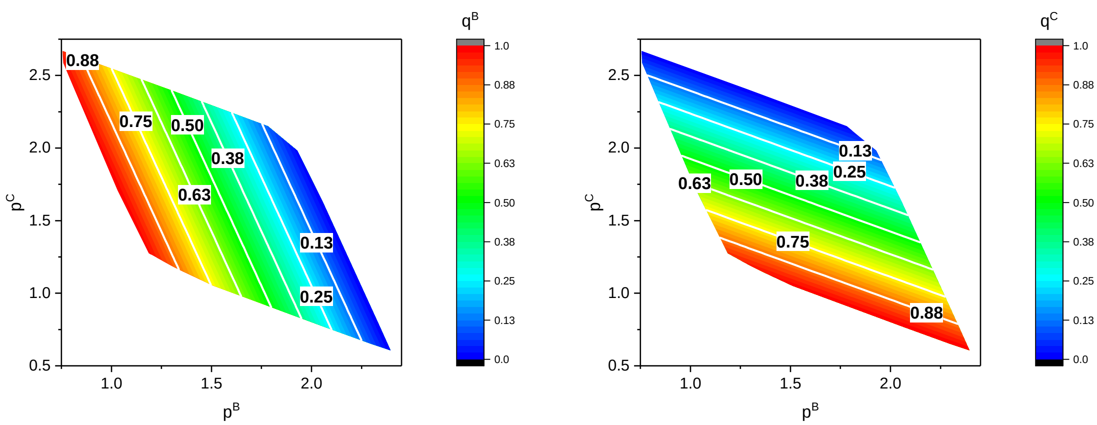
$$R = \frac{(b^B + b^C)(f^B - U_0^B)(f^C - U_0^C) + t^C(f^B - U_0^B)^2 + t^B(f^C - U_0^C)^2}{4t^B t^C - (b^B + b^C)^2} \tag{14}$$

Equation (9) shows the relation between price and participation. When network effects and fix benefits are given, quantity can be expressed as a linear combination of prices, $q^k = f(p^B, p^C)$. This also indicates the key characteristic of a two-sided market, participation of one side of the market is affected by the allocation of prices between the two sides. The equilibrium prices of the two sides are symmetric. It's hard to draw conclusions on the distribution of price allocations based on the equilibrium expressions shown in equation (11) and (12). A comprehensive analysis on how network effects impact the distribution of prices, quantities and profits are presented in the numerical experiments next.

### 4.3 Numerical experiments of monopoly platform

In the numerical experiment, we use the Starbucks stores in downtown Seattle as an example (Figure 1) to explain the participation of the two sides. For this, we assign Starbucks commuters and stores (worksites) as the two sides to the CSP based on their preferences, and maximize the profit of the CSP by selecting the optimal price strategies under different network effects. The baseline parameters for this case study are $U_0^B = 1.9$, $U_0^C = 2.1$, $b^B = 0.5$, $b^C = 0.7$, $t^B = 1.1$, $t^C = 1.5$, $f^B = 0.73$, $f^C = 0.75$. Under this setting, CSP's cost of serving commuters ($f^C$) is larger than that of the worksites ($f^B$). A commuter incurs higher benefit when joining the CSP than a worksite ($U_0^C > U_0^B$) for the approximate commute problem studied here. Commuters value the number of worksites more than worksites value commuters ($b^C > b^B$); commuters dislike the participation of other commuters more than that of worksites ($t^C > t^B$). There are 17 Starbucks in total. The number of employees at each Starbucks may vary depends on the size of the store. Let's say that there are 10 employees at each store, thus 170 commuters in total. Given the parameter setting, there are $17 * q^B$ worksites choosing the CSP, $170 * q^C$ commuters choosing the CSP. We assume that the commuters on the CSP are split evenly among the worksites that choose the CSP (assumption (d)). For the agents that choose the CSP, the number of employees at each worksite is $\frac{170*q^C}{17*q^B} = \frac{10*q^C}{q^B}$. For agents that do not choose the CSP, there are $\frac{10*(1-q^C)}{(1-q^B)}$ employees at each worksite. The number of employees at each worksite is actually determined by the ratio $q^C/q^B$. When implementing our model to a real world problem, we need to add an extra constraint so that this ratio is limited within a threshold ($q^C/q^B \approx 1$), which ensures that there are reasonable number of employees at each worksites. We added this constraint in section 5.3.

#### 4.3.1 Demand-price relation




(a) $q^B$ (b) $q^C$

Figure 3: The change of participation as a function of $(p^B, p^C)$

According to equation (9), given the parameters, demand can be written as a linear function of prices. An example of the demand-price relation is shown in Figure 3 when using the baseline parameters. Because of the cross-side positive network effects, the choices of Starbucks worksites affect the choices of commuters (vice versa). Therefore, if the CSP increases the price of commuters ($p^C$), less commuters will choose the CSP, the participation of Starbucks worksites will decrease as well. Under the current parameter setting, the price of one side has dominant impact on the participation of the same side, and has minor impact on the participation of the other side. For example, the participation of commuters ($q^C$) decreases when the CSP increases the price for the commuters or the Starbucks worksites, which however decreases faster with the price of commuters ($p^C$).

#### 4.3.2 Cross-side positive network effects

In order to explore further the cross-side positive effects, we test $b^B$ and $b^C$ while fixing other parameters to the baseline values. Results in Figure 4 show that when the overall cross-side effect ($b^B + b^C$) increases, the participation ($q^B$ and $q^C$) from both sides increases, the aggregated price ($p^B + p^C$) keeps steady, and the CSP's profit ($R$) increases. Notice that the aggregated price does not change, implying that CSP profit increases as a result of the increased participation from both sides. This tells us that the price allocation, not only the aggregated price, is effective to change the participation and profit of the CSP. This indicates the two-sidedness of the CSP. Without raising the aggregated price, the CSP may make use of the cross-side effects to attract more end-users (both Starbucks stores and employees), thus increasing its profit. The CSP can achieve higher profit when the two sides value the choices of each other (larger cross-side effects). It is common for big companies like Starbucks to provide commuting subsidies to improve employees' satisfaction at work. With CSP, this means that the Starbucks worksites are willing to pay more subscription fee to the CSP. In such case, the CSP can charge the worksites with higher prices and take the commuters side as a loss leader so that more worksites and commutes prefer the CSP over other options.

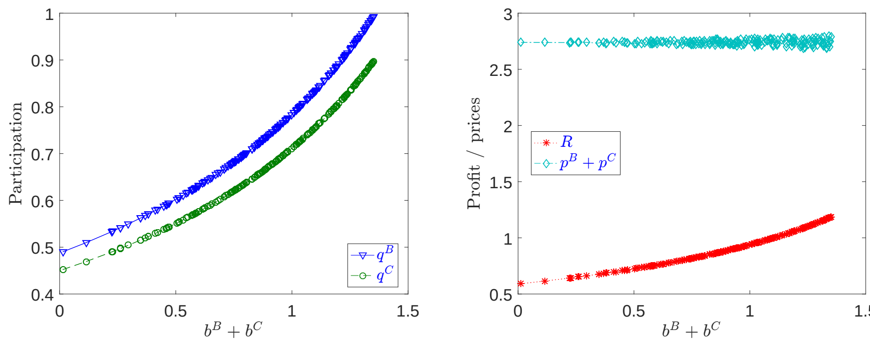



Figure 4: Sensitivity analysis of $b^B + b^C$

Since the equilibrium price structure is affected by both $b^B$ and $b^C$, we test $b^B$ unilaterally in order to capture the change of price structure. In this test, we use the baseline parameters except for $b^B$. Results are shown in Figure 5. When worksites value the number of commuters more ($b^B$ increases), the CSP can make more profit by reducing the price of commuters (thus attracting more participation from both sides) and recouping profit from worksites. By setting lower prices to commuters, more commuters will be willing to join the CSP. Worksites highly value the number of commuters, and as a result they will join CSP even if their price is high.

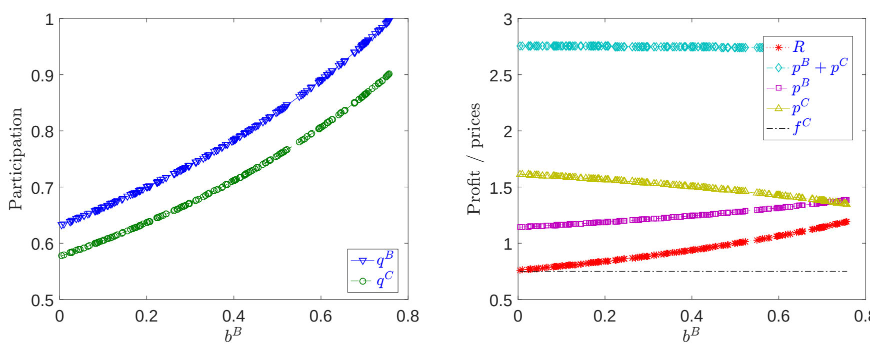



Figure 5: Sensitivity analysis of $b^B$

In Figure 5, increasing cross-side positive effect of worksites reduces the price of commuters. In the monopoly model, all parameters follow the conditions described in **(A1)** and **(A2)**. So under baseline parameter setting, $b^B$ cannot exceeds 0.8. Because of these constraints on the parameters, the CSP starts to set lower prices to commuters but not to the extent of subsidizing. With an attempt to show the "loss leader", we change $t^B, t^C$ to $t^B = 2, t^C = 2.2$, while keeping the other parameters the same as the baseline values. We then unilaterally test $b^B$. Results show that CSP will subsidize commuters when $b^B$ keeps increasing (Figure 6). When $b^B$ exceeds 2.25, the price of commuters $p^C$ falls below cost $f^C = 0.75$, in which case the commuter side is the "loss leader". At the same time, the CSP recoups profit by charging worksites much higher prices. Comparing Figure 5 and Figure 6, we can see that when worksites highly value the number of commuters, CSP has the intention to reduce the price of commuters, whereas the actual "subsidizing" level will be depended on the parameter setting of the model.

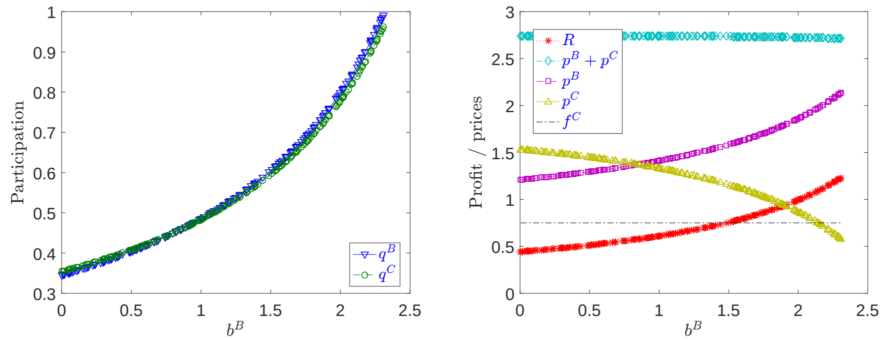
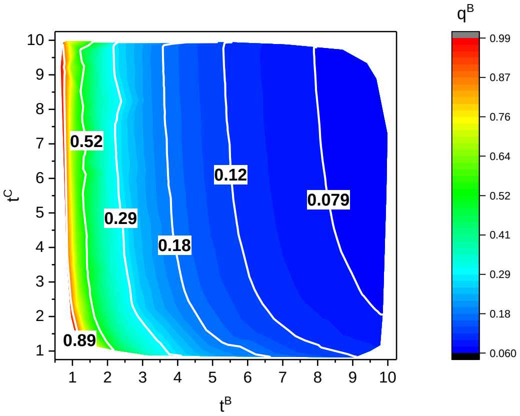


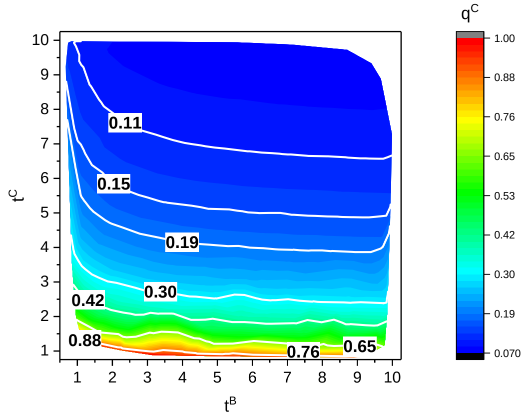

Figure 6: Sensitivity analysis of $b^B$ (when $t^B$ and $t^C$ are adjusted)

#### 4.3.3 Same-side “congestion” effects

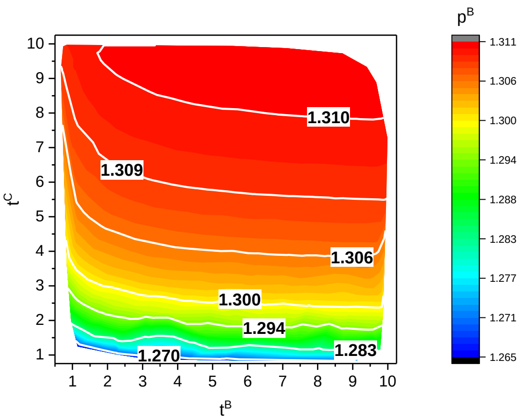
We use the baseline parameters and only change the value of $t^C$ and $t^B$ to investigate the same-side “congestion” effects. The resulting patterns of participation, prices and profits are presented in Figure 7. Intuitively, when $t^B$ becomes larger, a worksite will experience larger disutility if s/he joins CSP given existing worksites on CSP, in which case the participation of worksites is discouraged. The results shown in Figure 7a agree with our intuition. When each worksite is less discouraged by fellow worksites on CSP ($t^B$ is small), the participation of worksites increases. Also notice that, as $t^B$ gets smaller, $t^C$ will have weaker impact on $q^B$. This means that when $t^B$ is small, the number of worksites on the CSP ($q^B$) is mainly affected by the same-side “congestion" effect of worksites ($t^B$). On the other hand, if the same-side “congestion” effect of worksites ($t^B$) is larger, the same-side “congestion” effect of the commuter side ($t^C$) will have a greater impact on the participation of worksites ($q^B$). Thus. when $t^B$ is large, the same-side “congestion" effect of both groups will discourage the participation of worksites. Commuters have similar behaviors, as shown in Figure 7b. Figure 7c and 7d show that the price allocation changes very slowly with the same-side “congestion” effects, with a price variation below 0.05. But it is interesting that the price of worksites is generally more sensitive to the same-side “congestion” effect of commuters. When $t^B$ becomes larger given small $t^C$, the CSP will lose many worksites and a few commuters. The CSP will fail to maintain its profits in such case. Therefore, when same-side “congestion” effect gets too large, more agents from both groups will leave the CSP. Ultimately, $t^B, t^C$ will affect the CSP profits, as shown in Figure 7e. The increase of the same-side “congestion” effects of either side will reduce CSP profits mainly because of the loss of participants. The contour lines in Figure 7e show that for a range of combination of $t^B, t^C$, the CSP can manage to maintain the same profit level (e.g., larger than 0.5). Based on the above analysis, when the participation of commuters and worksites is high for the CSP, it may happen that the commuting services of the CSP may degrade if the number of commuters exceeds the capacity of the service. In such scenario, the potential CSP commuters or Starbucks worksites will experience higher same-side “congestion” effects. This implies that when there are increasing participation of the CSP, the CSP needs to control the same-side effects in order to maintain the profit and service quality.



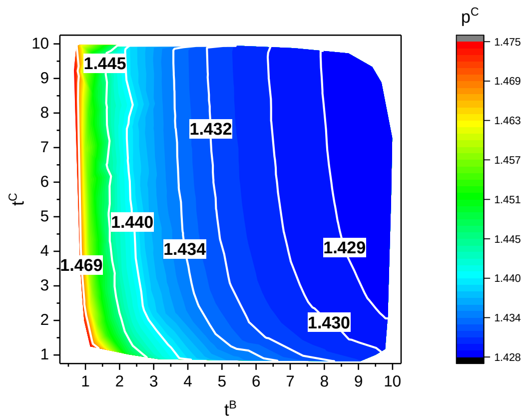

(a) $q^B$



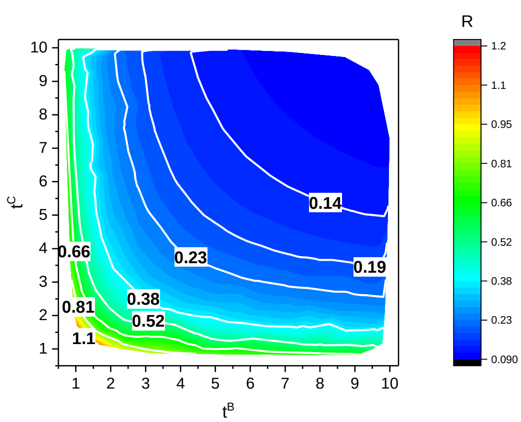

(b) $q^C$




(c) $p^B$




(d) $p^C$




(e) $R$

Figure 7: Sensitivity analysis of $t^B, t^C$

# 5 Duopoly platforms

## 5.1 General Model for duopoly platforms

Practically, it is common to see employees have many options for commuting. In other words, there are usually multiple CSPs in real life commuting services. Thus, a two-sided market with more than one platform is also valuable for understanding the competition among CSPs. Here we investigate the competition in the proposed two-sided market when there are two CSPs. We still consider worksites and commuters as the two sides. The two CSPs are specified as the WF CSP and the NWF CSP, denoted as $i \in \{W, N\}$. We consider two cases: (i) worksites and commuters single-home (i.e., a worksite or a commuter can only join one of the two CSPs) ; (ii) worksites multi-home (i.e., a worksite can join both CSPs), commuters single-home (i.e., a commuter can only join one of the two CSPs). We apply the model similar as Armstrong (2006) to the scenario in this paper. We assume that an agent from group $k$ has an incurred utility $U_0^k$ by joining the market. The cost for platform $i$ to serve a group $k$ agent is $f_i^k$. We consider that each agent from group $C$ (or $C$) values the number of agents from the other group $B$ (or $C$) with whom s/he can interact with. In particular, an agent from group $C$ obtains benefit $\beta_i(q_i^B + Q^B)$ by joining CSP $i$, as s/he will have the potential to interact with $q_i^B + Q^B$ agents from group $B$. $\beta_i$ denotes the cross-side benefit rate of commuters on CSP $i$. On the other hand, an agent from group $B$ also values the number of agents from group $C$. A group $B$ agent obtains cross-side benefit $\alpha_i(q_i^C + Q^C)$ by joining CSP $i$. $\alpha_i$ can be interpreted as the cross-side benefit rate of worksites on CSP $i$. CSP $i$ charges a non-negative subscription price $p_i^k$ to agents from group $k$. We assume that when an agent chooses a platform, s/he will be worse off when more other agents from the same group join the same platform, referred to as same-side negative network effect (same-side "congestion" effect) as shown by the Hotelling model in Section 3. The analysis of the same-side effect for the duopoly model is similar to that for the monopoly model, which is omitted here. The utility of a worksite located at $x^B \in [0, 1]$, when s/he joins WF CSP or NWF CSP, is given by:

$$U_W^B = \underbrace{U_0^B}_{\text{fixed benefit}} - \underbrace{p_W^B}_{\text{subscription price}} - \underbrace{t^B x^B}_{\substack{\text{same-side/inconvenience}\\ \text{costs}}} + \underbrace{\alpha_W(q_W^C + Q^C)}_{\text{cross-side benefits}} \tag{15}$$

$$U_N^B = \underbrace{U_0^B}_{\text{fixed benefit}} - \underbrace{p_N^B}_{\text{subscription price}} - \underbrace{t^B(1 - x^B)}_{\substack{\text{same-side/inconvenience}\\ \text{costs}}} + \underbrace{\alpha_N(q_N^C + Q^C)}_{\text{cross-side benefits}} \tag{16}$$

The utility of a commuter located at $x^C \in [0, 1]$, when s/he joins WF CSP or NWF CSP, is given by:

$$U_W^C = \underbrace{U_0^C}_{\text{fixed benefit}} - \underbrace{p_W^C}_{\text{subscription price}} - \underbrace{t^C x^C}_{\substack{\text{same-side/inconvenience}\\ \text{costs}}} + \underbrace{\beta_W(q_W^B + Q^B)}_{\text{cross-side benefits}} \tag{17}$$

$$U_N^C = \underbrace{U_0^C}_{\text{fixed benefit}} - \underbrace{p_N^C}_{\text{subscription price}} - \underbrace{t^C(1 - x^C)}_{\substack{\text{same-side/inconvenience}\\ \text{costs}}} + \underbrace{\beta_N(q_N^B + Q^B)}_{\text{cross-side benefits}} \tag{18}$$

When a worksite multi-homes, s/he obtains utility:

$$U_{NW}^B = \underbrace{U_0^B}_{\text{fixed benefit}} - \underbrace{(p_W^B + p_N^B)}_{\text{subscription price}} - \underbrace{t^B}_{\substack{\text{same-side/inconvenience}\\ \text{costs}}} + \underbrace{(\alpha_W q_W^C + \alpha_N q_N^C)}_{\text{cross-side benefits}} \tag{19}$$

The profit of platform $i$ is:

$$R_i = \underbrace{(q_i^B + Q^B)(p_i^B - f_i^B)}_{\text{profit collected from worksites}} + \underbrace{(q_i^C + Q^C)(p_i^C - f_i^C)}_{\text{profit collected from commuters}} \qquad \forall i \in \{W, N\} \tag{20}$$

Based on this general Model, we can derive duopoly platforms when both sides singlehome in section 5.2. Section 5.3 adds demand constraints to the model in section 5.2. The multi-home model is presented in Appenix A.

## 5.2 Duopoly platforms when worksites and commuters single-home

The following conditions ensure that agents from both groups are single-homing and the equilibrium price is feasible:

**(B1)** $U_0^B$ and $U_0^C$ are sufficiently high such that all agents wish to subscribe to at least one CSP;
**(B2)** $t^B > \alpha_W q_W^C + \alpha_N q_N^C$ and $t^C > \beta_W q_W^B + \beta_N q_N^B$ : ensures that the incremental utility from single-home to multi-home is always negative, so that no agent multi-homes at any non-negative prices set by the two CSPs;
**(B3)** $4t^B t^C > (\alpha^+ + \beta^+)^2$: ensures that the profits of the CSPs are positive.

**Lemma 1.** *Under condition **(B2)**, no agent multi-homes at any non-negative price set by the two CSPs.*

*Proof.* Single-home case can be further divided into sub-cases: (i) all agents from group $k$ prefer one CSP to another, $q_W^k q_N^k = 0, q_W^k + q_N^k = 1$; (ii) agents from group $k$ choose either CSPs, $q_W^k q_N^k > 0, q_W^k + q_N^k = 1$. To guarantee all agents single-home, we need to show that group $B$ agents with the lowest utility does not want to multi-home. (The proof of group $C$ single-homing can be derived similarly)

Case (i): all group $B$ agents prefer the WF CSP to the NWF CSP, so that $U_W^B > U_N^B$. Here, the agents with the lowest utility is located at $x^k = 1$, and they are most likely to multi-home. Evaluated at $x^B = 1$, the incremental utility of multi-homing with respect to single-homing is $U_{WN}^B - U_W^B|_{x^B=1} = U_0^B - (p_W^B + p_W^C) - t^B + (\alpha_W q_W^C + \alpha_N q_N^C) - [U_0^B - p_W^B - t^B + \alpha_W q_W^C] = -p_N^B + \alpha_N q_N^C$. We assume $U_W^k > U_N^k$, thus $U_W^B|_{x^B=1} > U_N^B|_{x^B=1}$ holds, from which yields $-p_W^B - t^B + \alpha_W q_W^C > -p_N^B + \alpha_N q_N^C$. From condition (A2), we know that $t^B > \max\{\alpha_W q_W^C, \alpha_N q_N^C\}$, so $-p_W^B - t^B + \alpha_W q_W^C$ is negative. Hence the incremental utility of multi-homing for group $B$ is also negative. Similarly, we can prove that the incremental utility of multi-homing for group $C$ is also negative under condition (A2).

Case (ii): agents from group $k$ are most likely to multi-home when they are indifferent between the two CSPs, namely, $U_W^B|_{x^B=x^*} = U_N^B|_{x^B=x^*}$. This is the lowest utility an agent experiences by choosing single-home. Evaluate at location $x^*$, the incremental utility from multi-homing with respect to single-homing is $U_{WN}^B - \frac{1}{2}(U_W^B|_{x^B=x^*} + U_W^B|_{x^B=x^*}) = U_0^B - (p_W^B + p_W^C) - t^B + (\alpha_W q_W^C + \alpha_N q_N^C) - \frac{1}{2}[2U_0^B - p_W^B - p_N^B - t^B + \alpha_W q_W^C + \alpha_N q_N^C] = \frac{1}{2}[-p_W^B - p_N^B - t^B + \alpha_W q_W^C + \alpha_N q_N^C]$, given condition (A2), $t^B > \alpha_W q_W^C + \alpha_N q_N^C$, so the incremental utility is negative. ■

**Proposition 2.** *Under condition (B1)-(B3), an equilibrium exists when all agents single-home. Assuming that the two CSPs charge the same pair of prices, $p_W^B = p_N^B$ and $p_W^C = p_N^C$, half of the agents from each group will join each platform, i.e., $q_W^B = q_W^C = q_N^B = q_N^C = 0.5$. If $f_W^B + t^B + \psi_W^B > \frac{\beta^+}{2}$ and $f_W^C + t^C + \psi_W^C > \frac{\alpha^+}{2}$, the equilibrium prices are:*

$$p_W^B = f_W^B + t^B - \frac{\beta^+}{2} + \psi_W^B \qquad p_W^C = f_W^C + t^C - \frac{\alpha^+}{2} + \psi_W^C \tag{21}$$

*where*

$$\psi_W^B = \frac{2(\beta^+ - \alpha^+)\beta^- t^B + (\beta^{+2} - 4t^B t^C)\alpha^-}{8t^B t^C - 2\alpha^+\beta^+} \qquad \psi_W^C = \frac{2(\alpha^+ - \beta^+)\alpha^- t^C + (\alpha^{+2} - 4t^B t^C)\beta^-}{8t^B t^C - 2\alpha^+\beta^+} \tag{22}$$

*Each CSP makes profit:*

$$R_i = \frac{t^B + t^C - \frac{\beta^+ + \alpha^+}{2} + \psi_W^B + \psi_W^C}{2} > 0 \quad \forall i \in \{W, N\} \tag{23}$$

*Proof.* We know that $q_N^k + q_W^k = 1$. According to Nash equilibrium, an agent experiences the same utility from either CSP, i.e., $U_W^B = U_N^B$ and $U_W^C = U_N^C$. From equation (15)-(18) we get the following relations:

$$U_0^B - p_W^B - t^B x^B + \alpha_W(q_W^C + Q^C) = U_0^B - p_N^B - t^B(1 - x^B) + \alpha_N(q_N^C + Q^C) \tag{24}$$

$$U_0^C - p_W^C - t^C x^C + \beta_W(q_W^B + Q^B) = U_0^C - p_N^C - t^C(1 - x^C) + \beta_N(q_N^B + Q^B) \tag{25}$$

Set $x^k = q_W^k$. We obtain the participation of each group on a CSP. Set $\alpha^+ = \alpha_N + \alpha_W$, $\beta^+ = \beta_N + \beta_W$, $\alpha^- = \alpha_N - \alpha_W$, $\beta^- = \beta_N - \beta_W$, the number of group $k$ agents joining the WF CSP can be written as:

$$q_W^B = \frac{1}{2} + \frac{\alpha^+(p_N^C - p_W^C) + 2t^C(p_N^B - p_W^B)}{4t^B t^C - \alpha^+\beta^+} - \frac{t^C\alpha^- + \frac{\alpha^+\beta^-}{2}}{4t^B t^C - \alpha^+\beta^+} \tag{26}$$

$$q_W^C = \frac{1}{2} + \frac{\beta^+(p_N^B - p_W^B) + 2t^B(p_N^C - p_W^C)}{4t^B t^C - \alpha^+\beta^+} - \frac{t^B\beta^- + \frac{\alpha^-\beta^+}{2}}{4t^B t^C - \alpha^+\beta^+} \tag{27}$$

Condition **(B3)** ensures that $4t^B t^C - \alpha^+ \beta^+ > 0$ and the profit function is concave with respect to prices. Substitute the demand functions, equation (26) and (27), into the profit function (20). Assume $p_W^B = p_N^B$ and $p_W^C = p_N^C$. Based on the first order condition of profit function (20) over prices, we can obtain the optimal prices.

$$\frac{\partial R_i}{\partial p_i^k} = 0 \qquad \forall i \in \{W, N\}, k \in \{B, C\} \tag{28}$$
$$s.t. \qquad p_W^B = p_N^B, p_W^C = p_N^C$$

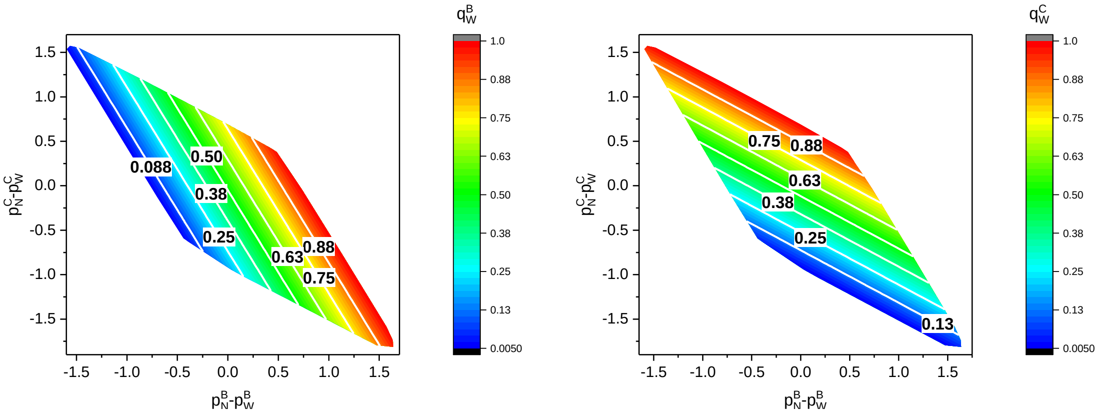
■

Note that we assume $p_W^B = p_N^B$ and $p_W^C = p_N^C$ in Proposition 2 for more concise expressions of equilibrium prices. We do not have such assumptions when generating numerical results in section 5.2.1 and 5.3.1.

### 5.2.1 Numerical experiments for duopoly model (single-homing)

To avoid redundancy, we present the results for the duopoly model selectively. The same-side effects of the duopoly model is similar as the monopoly model, so the numerical experiments for the same-side effects are omitted here. Also, the analysis for the cross-side effects of the two groups are similar, so we only analyze the cross-side effects of worksites. The baseline parameters are set as $\alpha_N = 0.7$, $\alpha_W = 0.6$, $\beta_N = 0.5$, $\beta_W = 0.8$, $t^B = 1.1$, $t^C = 1.2$, $f_W^B = 0.7$, $f_N^B = 0.73$, $f_W^C = 0.73$, $f_N^C = 0.75$. Under this parameter setting, worksites experience more cross-side benefits when joining the NWF CSP than the WF CSP ($\alpha_N > \alpha_W$). In reality, this happens when some worksites set fixed working hours for their employees to achieve certain productivity goals. Commuters obtain more cross-side benefits from the number of worksites on the WF platform ($\beta_W > \beta_N$). This is because commuters tend to value more flexible working hours. Commuters dislike the participation of other commuters more than that of worksites ($t^C > t^B$). For CSPs, the service cost of each commuter ($f_i^C$) is higher than the per-worksite cost ($f_i^B$). The WF CSP spends less than the NWF CSP to serve customers from the same group ($f_W^k < f_N^k$). For the duopoly model, we continue to use the Starbucks example. Since we have not added the transportation network to our model, we assume that employees who choose CSP $i$ are evenly split among the worksites that also choose CSP $i$ (assumption (d)). For the WF CSP subscribers, there are $\frac{170 * q_W^C}{17 * q_W^B} = \frac{10 * q_W^C}{q_W^B}$ employees at each Starbucks worksite. For the NWF CSP subscribers, there are $\frac{10 * q_N^C}{q_N^B} = \frac{10 * (1 - q_W^C)}{(1 - q_W^B)}$ employees at each Starbucks worksite. For the duopoly model, the number of employees at a worksite is also affected by the ratio $q_W^C / q_W^B$. We will first show the analytical results when $q_W^C / q_W^B$ is not constrained in this section, and then show the results when $q_W^C / q_W^B$ is constrained (so that $q_W^C \approx q_W^B$) in section 5.3.

According to equation (26) and (27) , participation (demand) can be expressed as a linear function of $(p_N^C - p_W^C)$ and $(p_N^B - p_W^B)$ when we hold the other parameters constant. Using the baseline parameters, we obtain the demand-price relation as shown in Figure 8. The number of agents from either group on the WF CSP increases when the WF CSP sets lower prices than the NWF CSP. With the current parameters, $p_N^B - p_W^B$ has larger impact than $p_N^C - p_W^C$ on the participation of worksites on the WF CSP ($q_W^B$). Similarly, $p_N^C - p_W^C$ has larger impact than $p_N^B - p_W^B$ on the participation of commuters on the WF CSP ($q_W^C$). This indicates that the participation of group $k$ on CSP $i$ is affected by the prices of both groups on the two CSPs. CSPs should be more strategic to allocate the prices between the two groups in order to be more competitive in the market. When the prices of commuters ($q_N^C$ and $q_W^C$) are constant, the prices of worksites on the two CSPs change so that $p_N^B - p_W^B$ increases, the participation of worksites on the WF CSP ($q_W^B$) increases much faster than that of commuters ($q_W^C$). Different from the monopoly model, the Starbucks worksites and employees now need to choose between two kinds of CSPs. The choices of the two groups affect each other because of the cross-side positive network effects. Figure 8b shows that the participation of commuters is mainly affected by the relative prices for commuters on the two CSPs, and is also affected by the prices of worksites.



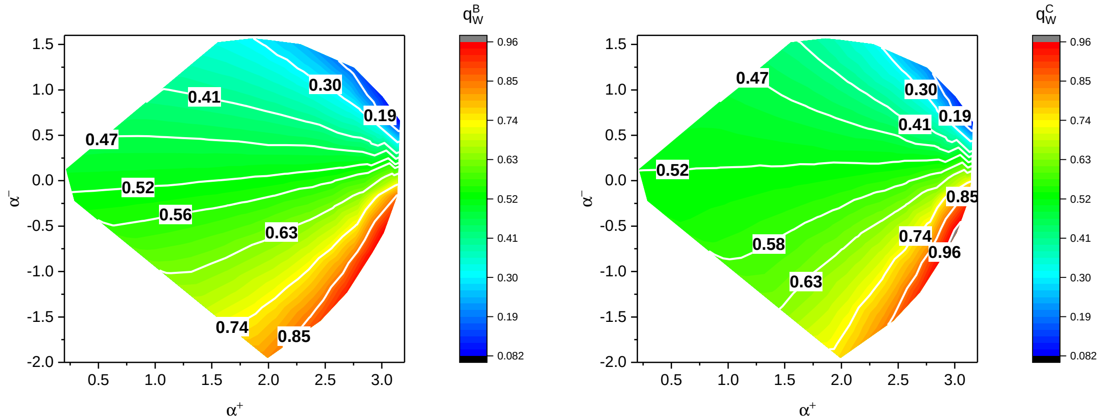

(a) $q_W^B$ (b) $q_W^C$

Figure 8: The change of participation as a function of $(p_N^B - p_W^B, p_N^C - p_W^C)$

$\alpha_N$ and $\alpha_W$ are the cross-side benefit rates of worksites on the two CSPs. We define $\alpha^+$ (positive, $\alpha^+ = \alpha_N + \alpha_W$) representing the overall cross-side benefit rate of worksites from the two CSPs, and $\alpha^-$ (can be negative, $\alpha^- = \alpha_N - \alpha_W$) is the difference of cross-side benefit rate of worksites between the WF CSP and the NWF CSP. When $\alpha^- > 0$, worksites value higher of the commuters on the NWF CSP; when $\alpha^- < 0$, worksites value higher of the commuters on the WF CSP. $\alpha^+, \alpha^-$ are linear combinations of $\alpha_N$ and $\alpha_W$. Here we test the sensitivity of $\alpha^+, \alpha^-$ in order to see how the cross-side benefits of the two CSPs affect participation, prices and CSP profits.

Fixing other parameters as the baseline values, we change the value of $\alpha^+, \alpha^-$ unilaterally. Figure 9a and 9b show that when $\alpha^- \approx 0$, the participation of worksites / commuters does not change much with $\alpha^+$. Under this scenario, worksites/commuters are indifferent toward the two CSPs and the overall level of cross-side benefit rate ($\alpha^+$) marginally affects the participation of worksites/commuters on the WF CSP. When the cross-side benefit rate on the WF CSP exceeds that of the NWF CSP ($\alpha^- < 0$), the WF CSP becomes more attractive to both worksites and commuters. When $\alpha^-$ is negative and fixed, if $\alpha^+$ increases, the WF CSP becomes more attractive to both sides. Note that in this case, $\alpha_N < \alpha_W$ and $\alpha_N$ increases together with $\alpha_W$. Even $\alpha_N$ increases with $\alpha_W$, participation on the WF CSP still keeps increasing (and thus the participation on the NWF CSP is decreasing), showing that $\alpha_W$ becomes the dominant factor to the change of participation ($q_W^B$ and $q_W^C$). The same is true when $\alpha^-$ is larger. Therefore, the larger cross-side benefit is dominant in deciding the participation of the two CSPs from both groups. Similar participation of the two groups indicates that the cross-side benefit effects lead the participation of the two groups to deviate in the same direction for the duopoly model, which is consistent with the findings from the monopoly model. Starbucks worksites tend to subsidize the type of commuting service that is preferred by their employees, which means the worksites value the choices of commuters ($\alpha_W, \alpha_N$). The commuters benefit from the WF CSP because shorter commuting time, more flexible time schedule. Higher employee satisfaction leads to higher work productivity, in which way worksites will also benefit from choosing WF CSP. In this case, the worksites will value the employees on the WF CSP more ($\alpha_W > \alpha_N$), participation on the WF CSP ($q_W^C, q_W^B$) increases.

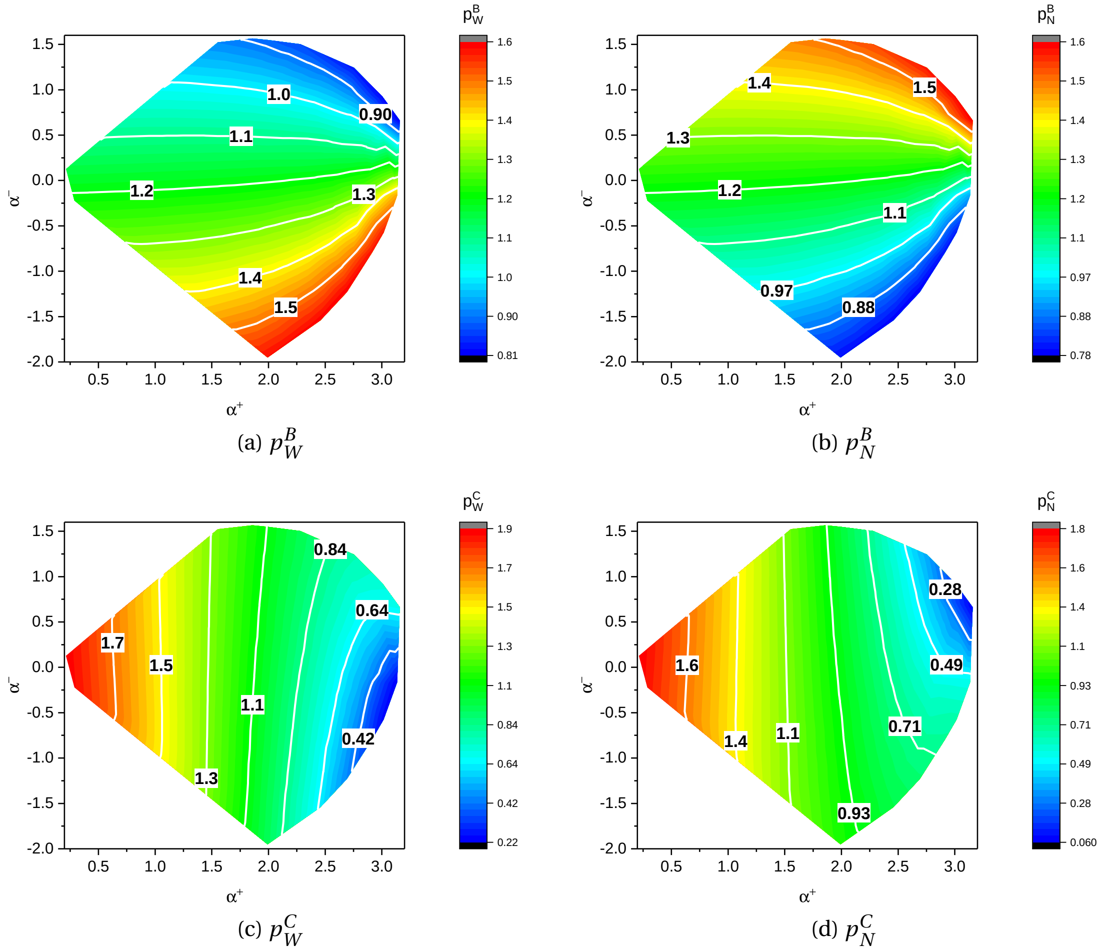
Although we do not specify the number of employees on each worksite, Figure 9a and 9b show similar patterns of participation from the two sides on the WF CSP. The numerical tests show that $q_W^B$ is always close to $q_W^C$, and more particularly $-0.0675 \le q_W^C - q_W^B \le 0.0854$ holds. This means that under equilibrium, the fraction of commuters choosing a CSP is close to the fraction of worksites on the same CSP, indicating there is a relative reasonable range of the number of employees working for each worksite.




(a) $q_W^B$ (b) $q_W^C$

Figure 9: The change of participation with $\alpha^+, \alpha^-$

When we change $\alpha^+, \alpha^-$, each CSP adjusts its price structure to maximize profit. The price structure patterns are shown in Figure 10. We can see that the prices of worksites are mainly affected by $\alpha^-$, while the prices of commuters are mainly affected by $\alpha^+$. By comparing Figure 9a and Figure 10a, we notice that for worksites, prices have the similar pattern as that of participation. The reason is that $\alpha_W$ and $\alpha_N$ are the cross-side benefit of worksites, and the relative value of these two parameters ($\alpha^-$) can be understood as the cross-side benefit discrepancy of the two CSPs from the perspective of worksites. When $\alpha^- < 0$, worksites get higher cross-side benefits on the WF CSP. Knowing that worksites care less about price and care more about the number of commuters on the WF CSP, the WF CSP sets higher price to worksites (bottom-right part of Figure 10a). In the meantime, worksites are less attracted to the number of commuters on the NWF CSP. The NWF CSP tries to set lower price to worksites to encourage participation (bottom-right part of Figure 10b), but the participation is still low (bottom-right part of Figure 9a and 9b, the high participation on the WF CSP also means that the participation on the NWF CSP is low). The case when $\alpha^- > 0$ can be understood in a similar way. From commuters' perspective, the overall cross-side effect of worksites ($\alpha^+$) matters more. Because $\alpha^+$ measures how much the worksites group value commuters. If the worksites value commuters more, meaning $\alpha^+ > \beta^+ = 1.3$, then the CSPs will set lower prices to commuters to attract both groups on board. When $\alpha^+$ is much larger than $\beta^+$, the CSP will even take the commuters group as a loss leader. In section 4.1, we define the threshold of subsidization/loss leader as $p_i^k < f_i^k$. Here is an example when the commuters group is a loss leader in the duopoly model. In Figure 10c, $p_W^C < f_W^C = 0.73$ when $\alpha^+$ exceeds 2.4, in which case commuters group is a loss leader to the WF CSP. Otherwise, if the worksites value commuters less, meaning $\alpha^+ < \beta^+ = 1.3$, CSPs will increase the prices of commuters and set lower prices to worksites. This shows that the aggregated cross-side benefit of worksites ($\alpha^+$) in the duopoly model has similar effect as the cross-side benefit of worksites ($b^B$) in the monopoly model ($\beta^+$ is similar as $b^C$). The price patterns in Figure 10 are consistent with the findings in the monopoly model. For the Starbucks example, if the worksites on the WF CSP value the commuters more ($\alpha^- < 0$, $\alpha_W > \alpha_N$), they are willing to pay higher subscription fee on the WF CSP. The reverse is also true when the worksites on the NWF CSP value the commuters more ($\alpha^- > 0$, $\alpha_N > \alpha_W$). In both cases, the CSP with higher cross-side benefit can set higher prices to the worksites, and offer discount price to the commuters to increase participation.



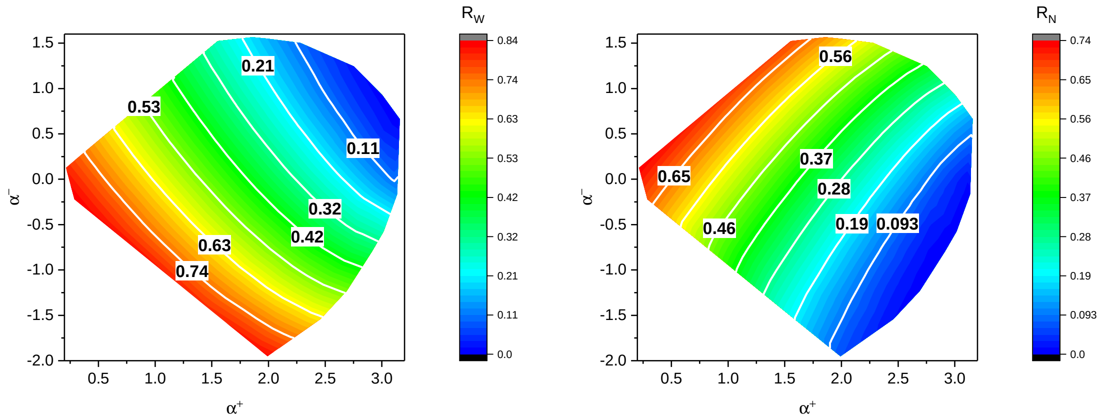

(a) $p_W^B$ (b) $p_N^B$

(c) $p_W^C$ (d) $p_N^C$

Figure 10: The change of prices with $\alpha^+, \alpha^-$

Figure 11 shows how profits on the two CSPs change with $\alpha^+, \alpha^-$. When $\alpha^- < 0$ ($\alpha_W > \alpha_N$), the WF CSP can make higher profit. Similarly, the NWF CSP tends to make higher profit when $\alpha^- > 0$ ($\alpha_N > \alpha_W$). In contrast, large overall cross-side benefit rate ($\alpha^+$) is not always desired by CSPs. When $\alpha^- = const$, profits on both CSPs decrease with $\alpha^+$. Referring to Figure 9 and 10, we can explain why profits decrease with $\alpha^+$. Take the WF CSP as an example, when $\alpha^- > 0$ and $\alpha^+$ is large (the top-right part of Figure 11a and 11b), the WF CSP charges low prices for both groups (the top-right part of Figure 10a and 10c), but few agents from both groups join WF CSP (the top-right part of Figure 9a and 9b ). Thus, the WF CSP gains low profit because of the low participation and low prices. Consider another situation, when $\alpha^- < 0$ and $\alpha^+$ is large (the bottom-right part of Figure 11a and 11b). The WF CSP can attract lots of participants from both groups (the bottom-right part of Figure 9a and 9b), and even takes advantage of high $\alpha_W$ to charge worksites high price (the bottom-right part of Figure 10a), but it still makes low profit. The reason is that, the WF CSP subsidizes commuters too much and fails to recoup enough profits from worksites. From Figure 10c, we can see that the price of commuters decreases with the overall cross-side benefit ($\alpha^+$). The commuters are charged with only 0.42 ($0.42 = p_W^C < f_W^C = 0.73$) when $\alpha^+ \approx 2.7$, which implies more commuters on WF CSP means more profit loss. Similar analysis can also applies to the NWF CSP. At the bottom-right part of Figure 11a and 11b, we notice that the profit of NWF CSP is lower than that of WF CSP. This means that large $\alpha^+$ intensifies the competition between the two CSPs. One of the CSP attracts lots of participants by making use of cross-side network effects and over subsidizing, while the other platform fails to attract consumers even if it sets very low prices. Neither of the CSPs makes good profits in this case. The profit patterns also distinguish the CSPs from the one-sided market. Even the highest participation from both sides (the bottom-right part of Figure 9a, 9b and 11a) cannot ensure high profit on the

WF CSP, although its profit is higher than that of the NWF CSP. The most desirable cases for the WF CSP are on the bottom-left part of Figure 11a, when the CSP charges medium prices to worksites and commuters without taking any of them as loss leaders (the bottom-left part of Figure 10a, 10c), and is also able to have more than 50% of the customer share from the both sides (the bottom-left part of Figure 9a, 9b).




(a) $R_W$ (b) $R_N$

Figure 11: The change of CSP profit with $\alpha^+, \alpha^-$

Similar as the monopoly model, the participation of the two groups are similar when we change the cross-side network effects (Figure 9a, 9b). However, it is still possible that $q^C_W/q^B_W$ deviate from 1 under some parameter settings. In the following section, we will add the demand constraints to the duopoly model.

## 5.3 Duopoly platforms with demand constraints

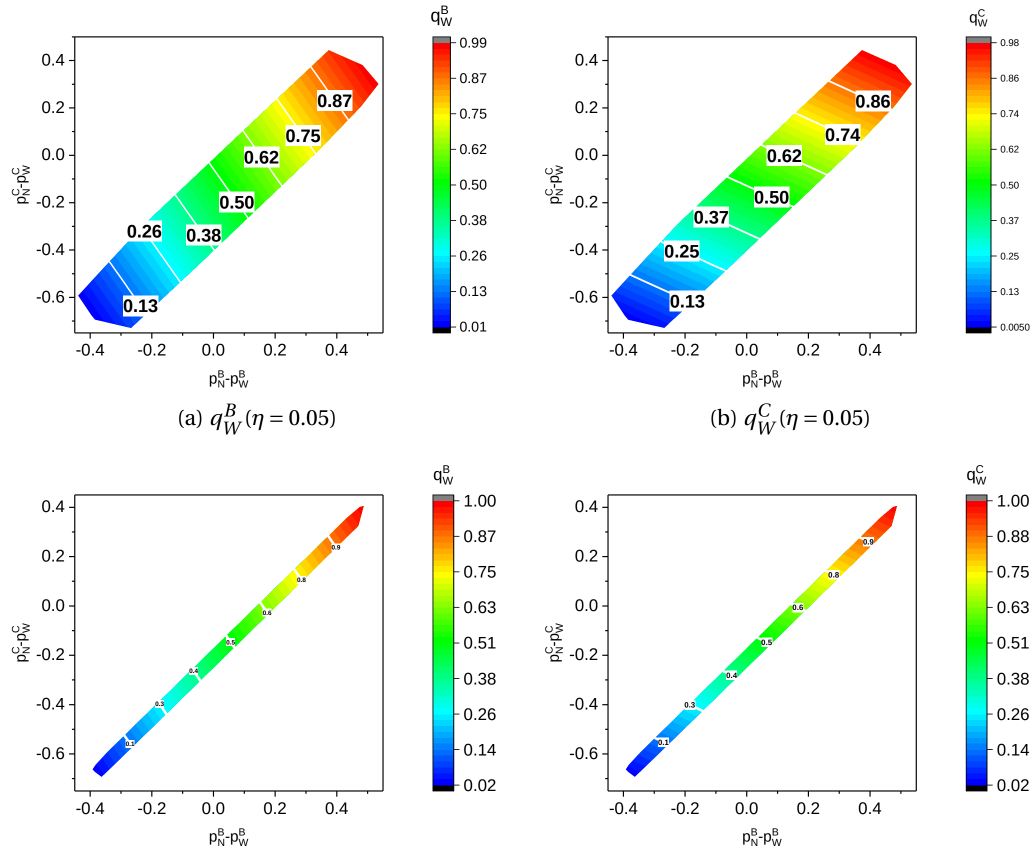
In section 5.2, we assume that each worksite does not specify the number of employees. Practically, a worksite loses profit if there are too many or too few employees. Thus, it is critical to ensure a reasonable range of the number of employees at each worksite. Notice that the results in section 4.2 and 5.2 already show similar participation for worksites and commuters, since the two groups are modeled to benefit from the cross-side effects. In order for the model to be more practical, here we impose stricter demand constraints for worksites. In particular, we assume that employees who choose a CSP are evenly split among the worksites on the same CSP. We can then add the demand constraints to the duopoly model in section 5.2 as:

$$q^C_i = q^B_i \pm \eta \quad i \in \{W, N\} \tag{29}$$

where $\eta$ reflects the demand flexibility of each worksite (either on the WF CSP or the NWF CSP). Take Starbucks as an example, remember there are 17 Starbucks worksites in downtown Seattle. Ideally, each worksite has 10 employees, but this number may change because of the discrepant preferences of CSPs from the two groups. In the proposed demand constraints, $\eta$ . For example, when we set $\eta = 0.05$, $q^B_W = 0.4$, 40% of worksites choose the WF CSP, the percentage of commuters choosing the WF CSP is 35% ~ 45%. Because we assume unit quantity for both worksites and commuters, 60% of worksites choose the NWF CSP, 55% ~ 65% of commuters choose the NWF CSP. It is obvious that the number of employees at each worksite converges to 10 when $\eta \to 0$.

### 5.3.1 Numerical experiments for duopoly platforms with demand constraints

The baseline parameters are the same as the previous duopoly model (section 5.2.1), $\alpha_N = 0.7$, $\alpha_W = 0.6$, $\beta_N = 0.5$, $\beta_W = 0.8$, $t^B = 1.1$, $t^C = 1.2$, $f^B_W = 0.7$, $f^B_N = 0.73$, $f^C_W = 0.73$, $f^C_N = 0.75$. This section shows the results when the demand constraints (i.e., assumption (d)) are added to the duopoly model, and make a comparison with the duopoly model without demand constraints. We only present the results of the duopoly model with demand constraints when $\eta = 0.05$ and $\eta = 0.01$ in the following discussions. For the Starbucks example, this means that $q_W^C / q_W^B = (q_W^B \pm \eta)/q_W^B \approx 1$. So that the number of employees at each worksite is $\frac{170 * q_W^C}{17 * q_W^B} \approx \frac{170}{17} = 10$.




(a) $q_W^B (\eta = 0.05)$

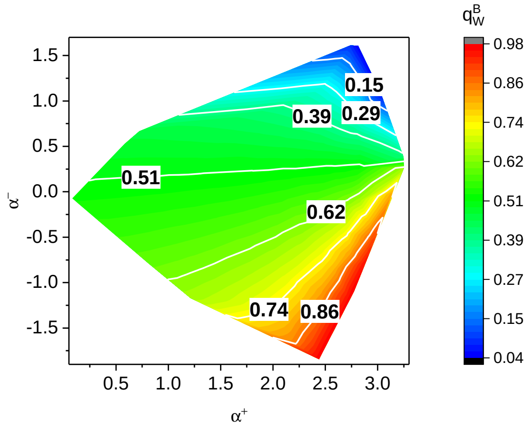
(b) $q_W^C (\eta = 0.05)$

(c) $q_W^B (\eta = 0.01)$

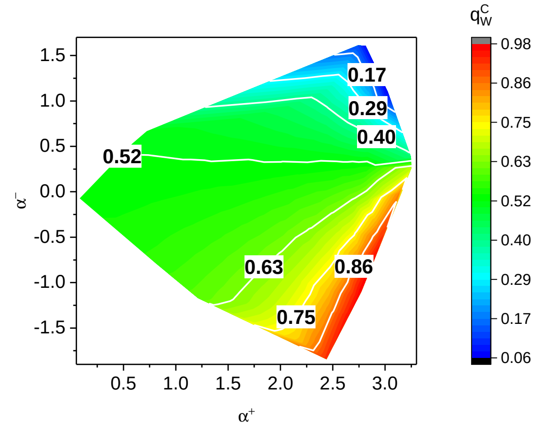
(d) $q_W^C (\eta = 0.01)$

Figure 12: Demand-price relation of the duopoly platforms (with demand constraints)

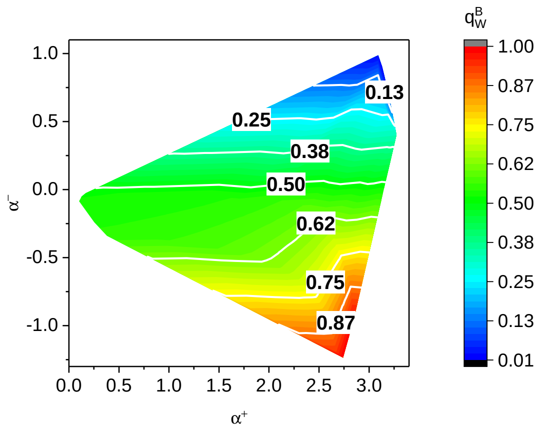
Under Nash equilibrium, the relation between prices and participation is shown in Figure 12. The number of group $k$ agents joining the WF CSP has the same expression as equation (26) and (27) besides the constraints from equation (29). From Figure 12, we can see that the demand constraints are very effective in forcing the participation of worksites and commuters to be similar on the same CSP. The demand constraints filter the data points in the top-left and bottom-right of Figure 8 and reserve the data points along the diagonal line. When we reduce $\eta$ from 0.05 to 0.01, the demand constraints become stricter, the feasible region of the demand-price relation shrinks toward the diagonal line more dramatically. Thus, the proposed demand constraints effectively select price allocations to force the participation of the two sides to be similar on the same CSP. Results shown in Figure 12 are consistent with the results in the duopoly model (Figure 8) and the monopoly model (Figure 3). The participation of worksites on the WF CSP ($q_W^B$) is more sensitive to the relative prices of worksites on the two CSPs ($p_N^B - p_W^B$), and less sensitive to the relative prices of commuters on the two CSPs ($p_N^C - p_W^C$). The participation of worksites on the WF CSP ($q_W^B$) increases when the WF CSP set lower prices to the two groups than the NWF CSP. The same is true for the commuters side.



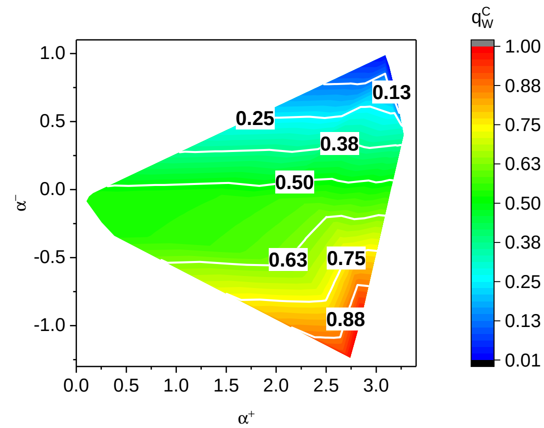

(a) $q_W^B (\eta = 0.05)$




(b) $q_W^C (\eta = 0.05)$




(c) $q_W^B (\eta = 0.01)$

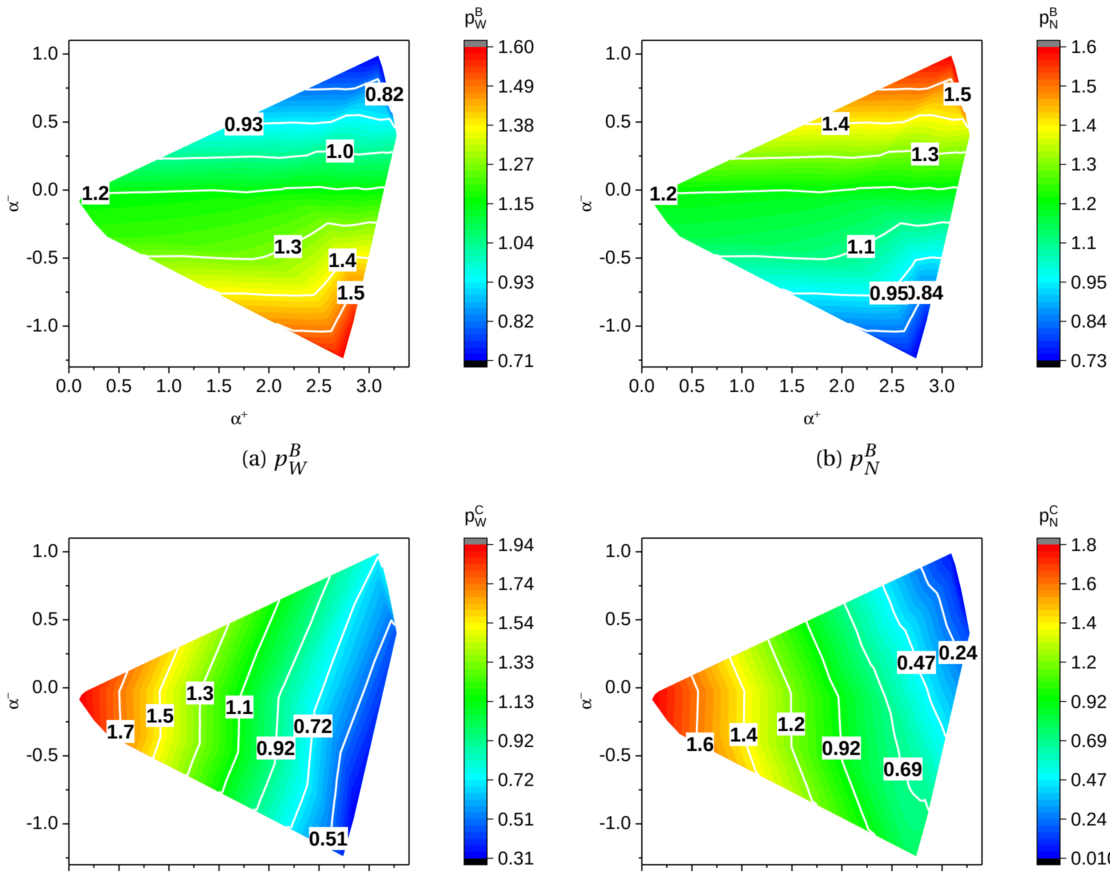



(d) $q_W^C (\eta = 0.01)$

Figure 13: The change of participation and with $\alpha^+, \alpha^-$ $(\eta = 0.05, 0.01)$

We continue to investigate what changes demand constraints bring to the duopoly model regarding the cross-side effects. Generally, the participation patterns shown in Figure 13 are consistent with Figure 9. When $\alpha^- = 0$ ($\alpha_W = \alpha_N$), the participation of worksites on the WF CSP ($q_W^B$) is marginally affected by the aggregated cross-side benefits ($\alpha^+$). When $\alpha^- > 0$ ($\alpha_W < \alpha_N$), the participation of worksites on the WF CSP ($q_W^B$) decreases with $\alpha^+$. When $\alpha^- < 0$ ($\alpha_W > \alpha_N$), the participation of worksites on the WF CSP ($q_W^B$) increases with $\alpha^+$. The participation of commuters on the WF CSP ($q_W^C$) shows similar patterns. It is interesting to see that demand constraints weaken the influence of the aggregated cross-side benefit ($\alpha^+$), and increase the influence of the relative cross-side benefit ($\alpha^-$), and this change is stronger when the demand constraints are stricter. When the demand constriants are less strict, i.e., $\eta = 0.05$, the participation of worksites on the WF CSP ($q_W^B$) changes slightly with $\alpha^+$, especially when $\alpha^+ < 2$. When we add stricter demand constraints to the worksites, i.e., $\eta = 0.01$, the influence of the aggregated cross-side benefits ($\alpha^+$) is further weakened, and only a small range of aggregated cross-side benefits ($2.25 < \alpha^+ < 2.75$) affects the participation. As shown in Figure 13c and 13d, the relative value of cross-side benefits ($\alpha^-$) largely decides the participation of the two sides. However, the participation does change when the absolute value of $\alpha^-$ is large (the top or bottom part of Figure 13c and 13d). In Figure 9, the larger cross-side effect on the two CSPs dominants participation, in Figure 13, the dominant role of the larger cross-side effect is weakened. Therefore, demand constraints control the expansion of the CSP with larger market share. Let's assume that $\alpha^- = const$. With demand constraints, the WF CSP cannot attract more customers when $\alpha_W$ increases (notice $\alpha_N$ is also increasing because $\alpha^- = const$, bottom part of Figure 13c, 13d). In contrast, without demand constraints, the WF CSP can increase its user base when the cross-side benefit on the WF CSP increases, even when $\alpha_N$ increases at the same time (bottom part in Figure

9a, 9b).




(a) $p_W^B$ (b) $p_N^B$ (c) $p_W^C$ (d) $p_N^C$

Figure 14: The change of prices with $\alpha^+, \alpha^-$ $(\eta = 0.01)$

The demand constraints change the price patterns in a similar way, weakening the effects of the aggregated cross-side benefit ($\alpha^+$) and strengthening the effects of the relative cross-side benefit ($\alpha^-$). The prices ($p_W^B$ and $p_N^B$) of worksites shown in Figure 10 are mainly affected by the relative cross-side effects ($\alpha^-$). When $\alpha^- < 0$ ($\alpha_W > \alpha_N$ ), worksites experience higher cross-side benefits from the WF CSP, thus choosing the WF CSP even if the price is high. Under such scenario, the NWF CSP fails to attract worksites even if it sets very low prices. Commuters are almost only affected by the aggregated cross-side benefits ($\alpha^+$) in Figure 10. When we add the demand constraints, the relative cross-side benefit ($\alpha^-$) starts to have larger impact on the prices of commuters ($p_W^C$ and $p_N^C$). When $\alpha^+$ is fixed, the price of commuters on the WF CSP ($p_W^C$) increases slowly with $\alpha^-$ (Figure 14c). With demand constraints, under the same relative cross-side benefit ($\alpha^-$ is fixed and negative), the WF CSP is less likely to take advantage of high $\alpha_W$ to increase participation from either side (bottom part of Figure 13c, 13d). Also, the WF CSP is less likely to take the commuters side as a loss leader and recoup profit by charging worksites with high prices (bottom part of Figure 14a, 14c). This is also true for the NWF CSP. Therefore, the competition between the two CSPs is reduced to some extent due to the demand constraints.

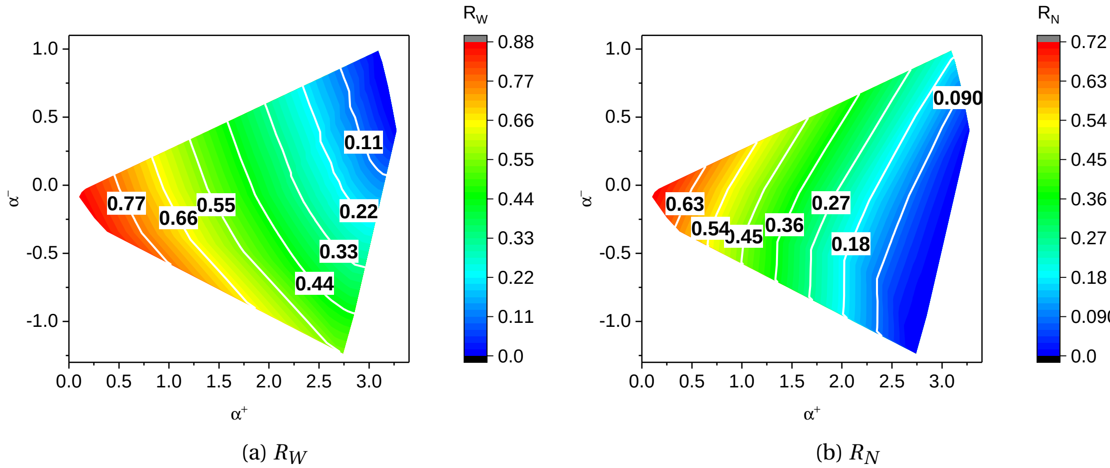



Figure 15: The change of CSP profit with $\alpha^+, \alpha^-$ $(\eta = 0.01)$

The profit patterns in Figure 15 are consistent with Figure 11. The feasible region in Figure 15 shrinks because of the demand constraints. The profit patterns in Figure 15 reserve most of the region with medium profits and cut out a substantial amount of cases when there are extreme profits (either very high or very low). In reality, mostly likely a worksite has demand constraints, under which the competition between two CSPs are milder than the results shown in Figure 11. In Figure 15, under most price strategies, the two CSPs share the market and make good profits. The cases when one CSP makes very high profits while the other CSP struggles with low profits only happens when the cross-side benefits follow certain distribution, i.e., the left edge or the right edge in Figure 15a and 15b. The cases when neither of the CSPs makes decent profit are also reduced. Overall, the demand constraints reduce the competition between the two CSP.

## 6 Discussions and implications

In this paper, we have presented the numerical results of the monopoly model, and the duopoly models with and without demand constraints. Here we summarize the major findings and implications to build CSPs and associated TDM strategies in practice. A summary of the numerical experiments for the three models is shown in Table 2. The baseline parameters used is summarized in Table 3. The results from the duopoly model is consistent with that of the monopoly model. The demand constraints reduce the competition between the WF CSP and the NWF CSP, which helps both CSPs make good profit.

First our analyses have shown the value of considering employers and commuters as the two sides, and the commuting service platform that connects the two. Two-sided markets have unique features compared with one-sided markets: 1) the volume of transactions on the CSP depends on the allocation of price between the two sides but not only on the aggregated price level; 2) the decision of each group of agents affects the outcomes of the other group of agents, typically through an externality, i.e. cross-side positive network effects and same-side negative network effects. Such an analysis framework can help develop CSPs and new TDM strategies.

Table 2: Summary of Major Results

| | Cross-side positive effects | | | Same-side negative effects | | |
|---|---|---|---|---|---|---|
| | Participation | Price | Profit | Participation | Price | Profit |
| Monopoly model | Increasing cross-side network effects from one group raise the participation of both groups. | When the worksites value the commuters more, the CSP takes the commuter group as a loss leader (low price) and recoups profits from worksites (high price). The reverse is also true. | When the cross-side effects of one group or both groups increase, the CSP can achieve higher profit. | Participation decreases with the same-side effects of either group. And the same-side effect from the same group is the dominant factor. | Prices change very slowly with the same-side effects. The price of one group is affected more by the same-side effect on the other group. | The profit decreases with the same-side effects. But the CSP is able to maintain profit under some combinations of the same-side effects. |
| Duopoly model | The larger cross-side effect $(\alpha_W, \alpha_N)$ on the two CSPs dominants participation. When worksites value commuters more on a CSP, the CSP can attract more participation from both groups by allocating prices, which is similar to the monopoly model. | A CSP sets higher price for worksites if the cross-side effect of worksites $(\alpha_W, \alpha_N)$ is higher. If the overall cross-side effect of worksites $(\alpha^+)$ is high, the price of commuters can be low, similar to the results from the monopoly model. | When a CSP attract more than 50% agents from both groups and do not subsidize either groups, it can gain high profits. When one group is regarded as a loss leader, neither of the CSPs can make high profits. | The results are similar to the monopoly model. We choose not to show the numerical experiment. | | |
| Duopoly model with demand constraints | The impact of the relative cross-side effect is amplified, while the impact of the overall cross-side effect is weakened. The dominant role of the larger cross-side effect is weakened. | | The competition between the two CSPs are milder. There are fewer cases for extreme profits (either very high or very low). | Same as the duopoly model | | |

Table 3: Summary of Baseline Parameters

| | $U_0^B$ | $U_0^C$ | $b^B$ | $b^C$ | $t^B$ | $t^C$ | $f^B$ | $f^C$ | $\alpha_N$ | $\alpha_W$ | $\beta_N$ | $\beta_W$ | $f_W^B$ | $f_N^B$ | $f_W^C$ | $f_N^C$ | $\eta$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Monopoly model | 1.9 | 2.1 | 0.5 | 0.7 | 1.1 | 1.5 | 0.73 | 0.75 | — | — | — | — | — | — | — | — | — |
| Duopoly models | — | — | — | — | 1.1 | 1.2 | — | — | 0.7 | 0.6 | 0.5 | 0.8 | 0.7 | 0.73 | 0.73 | 0.75 | — |
| Duopoly model with demand constraints | Same as the duopoly model | | | | | | | | | | | | | | | | 0.05 / 0.01 |

The application of the two-sided market theory on commuting services leads to many interesting findings. In **the monopoly model**, when a CSP changes the price allocation between the two sides, the participation from both sides will be affected. Actually, even if the CSP only changes the price of one side, e.g., worksites, the participation of both sides will change (Figure 3). Given specific parameter settings, the participation of worksites is more sensitive to the price of the same side (worksites), and less sensitive to the price of the other side, i.e., commuters (Figure 3). The cross-side positive network effects bring more benefits to one side if the participation of the other side increases (Figure 4, 5, 6). Thus, increasing cross-side network effects from one side or both sides raise the participation of both sides. Part of the reason is that high cross-side network effects increase the utilities of agents who choose the CSP (Equation (8)), which means more agents prefer the CSP over not joining the market. The other part of the reason is that in such a two-sided market, the CSP can make use of the cross-side benefits and sets lower price to the side that is being highly valued by the other side. When worksites highly value the number of commuters on a CSP, the CSP may reduce the price charged on commuters and recoup its profit from the worksites. Worksites are attracted by the commuters on the CSP and will not be discouraged by the high price. The reverse will be also true if commuters value highly the number of worksites on the CSP. Under specific parameter settings, the CSP is willing to subsidize the commuters to maintain or increase the overall profit (Figure 6). The same-side negative effects discourage an agent to join the CSP when many agents from the same side have already chosen the same CSP (Figure 7). Thus, high level of same-side effects reduce participation and profit on a CSP. However, the CSP can maintain its profit under some combinations of the same-side effects, shown by the contour line in Figure 7e.

**The duopoly model** inherits the main characteristics of the monopoly model, and helps us understand the competition between the two CSPs. The participation of one side is still affected by the price allocation of both sides (Figure 8). The participation of worksites is more sensitive to the relative price of worksites ($p_N^B - p_W^B$), and less sensitive to the relative price of commuters ($p_N^C - p_W^C$). Generally, the cross-side benefits of worksites on the two CSPs ($\alpha_W$ and $\alpha_N$) encourage participation (Figure 9), which is consistent with the monopoly model. However, the actual participation pattern is more complex. When $\alpha^- = 0$ ($\alpha_W = \alpha_N$), the two CSPs have the same level of cross-side benefit for worksites, thus participation remains the same if the aggregated cross-side benefit ($\alpha^+$) changes. The relative higher cross-side benefit has the dominant effects on participation. For example, if $\alpha^- < 0$ and is fixed ($\alpha_W > \alpha_N$), $\alpha_W$ becomes the major factor of participation, thus the participation of worksites on the WF CSP ($q_W^B$) increases with $\alpha^+$ (notice that $\alpha^-$ is fixed and $\alpha^+$ is increasing). The same is true when $\alpha^- > 0$ ($\alpha_W < \alpha_N$). The price of worksites is mainly affected by the relative value of the cross-side benefits ($\alpha^-$; Figure 10a, 10b), while the price of commuters is mainly affected by the aggregated cross-side benefits ($\alpha^+$; Figure 10c, 10d). Remember that high cross-side benefits attracts more worksites on a CSP. Therefore, the CSP with higher cross-side benefits can attract more worksites even if it sets high prices. The WF CSP sets high price to worksites in the right-bottom part of Figure 10a, in which case commuters are subsidized and worksites are still attracted to the WF CSP. The prices of commuters on the two CSPs mainly change with the aggregated cross-side benefit. When $\alpha^+$ is large and $\alpha^- < 0$ ($\alpha_W > \alpha_N$), commuters are subsidized on the WF CSP, which means the CSP sets low price to commuters (right-bottom part of Figure 10c). Because of the competition between the two CSPs, the NWF CSP also need to set lower price to commuters (right-bottom part of Figure 10d). Thus, when either of the CSPs is subsidizing the commuters, the other CSP will also be forced to set lower price to commuters. Notice that the subsidization is mainly caused by the relative cross-side benefit ($\alpha^-$), so the price of commuters is merely affected by $\alpha^-$. Thus, the price of commuters is almost only affected by $\alpha^+$. There exist "bad" competitions where neither of the CSPs makes high profit. For example, in the right-bottom part of Figure 11, the NWF CSP has low profit because of low participation from both sides. Although the WF CSP successfully attracts participation from both sides, its profit is still low (slightly higher than that of the NWF CSP) because of the subsidization to the commuters side.

When **demand constraints** are added to the duopoly CSPs, the demand constraints force the participation of commuters to be similar as that of worksites. As a result, the feasible region of demand-price relation is shrank to the diagonal line (Figure 12). In the same example of the cross-side benefits of worksites ($\alpha^-$ and

$\alpha^+$), the aggregated cross-side benefit ($\alpha^+$) has lower impacts on the participation and prices, while the relative cross-side benefit ($\alpha^-$) has higher impacts (Figure 13, 14), comparing with the duopoly model without demand constraints. In the sensitivity analysis of the cross-side benefits of worksites, the participation of worksites changes less with the aggregated cross-side benefit ($\alpha^+$) in order to be similar as the participation of commuters (Figure 13). Here the participation of commuters directly changes with the cross-side benefits of commuters ($\beta_W, \beta_N$) (accroding to the symmetry of the model). It is also true that the participation of commuters changes with the relative cross-side benefits of worksites ($\alpha^-$) indirectly. As discussed above, if $\alpha_W$ is relatively higher, the commuters on the WF CSP will be subsidized. At the same time, the NWF CSP will also set lower price to commuters to compete with the WF CSP. The commuters are attracted to the CSP where they can get higher utilities. Thus, the participation of commuters is more sensitive to the relative cross-side benefits ($\alpha^-$) and less sensitive to the aggregated cross-side benefits ($\alpha^+$). When we add the demand constraints, the participation from both sides follows this rule (becomes more sensitive to $\alpha^-$ and less sensitive to $\alpha^+$). Similar trends occur to the pattern of prices (Figure 14). Actually, it is more reasonable for the relative cross-side benefit ($\alpha^-$) to have higher impact on the equilibrium results. The relative cross-side benefit ($\alpha^-$) reflects the level of competition between the two CSPs, since the only variables in the experiment are $\alpha^+$ and $\alpha^-$. The patterns of profit also prove that the demand constraints improve the duopoly model (Figure 15). The demand constraints lead to "friendlier" competitions between the two CSPs. The cases when neither of the CSPs makes decent profit are reduced. Both CSPs make reasonable profit under most parameter settings, which means the CSPs can better co-exist in the market.

The above analysis results may help us draw some initial insights on how to **build CSPs** in practice. First, the envisioned CSP can help enhance the collaboration among CSPs, employers, and commuters, which is beneficial to all players in the market: (i) the CSP can manage to obtain profits from both sides; (ii) the employers can out-source the commuting subsidization of their employees to a third party (in our case, the CSPs) conveniently; (iii) the commuters can have more affordable/accessible commuting choices. The profit of a CSP is affected by cross-side positive network effects as well as same-side negative network effects. In practice, cross-side benefits exist when the commuting decisions of individual commuters are related to the subsidization programs of their employers. Therefore, adjusting price strategies according to the cross-side effects is very critical for a CSP to attract participation. For example, if worksites value commuters more, a CSP can set lower price to the commuters and set higher price to the worksites to attract more commuters, thus more worksites to participate. High same-side effects may exist when the participation on a CSP exceeds the maximum number of services the CSP can provide, or the CSP becomes less efficient when the amount of customers increases. Imagine when a CSP hires big vans to pick up employees at their homes and then send them to a worksite. If the number of commuters taking the van increases, the picking up time will increase. Knowing this, some commuters may choose more time efficient ways of commuting and choose not to join the CSP. Thus, when a CSP tries to attract more customers, it is important to develop strategies that can serve the increasing demand efficiently. There are several ways to build practical CSPs. Some existing MaaS, such as ridesourcing companies, can extend the existing platforms to add CSP as a new category of services that are specialized for commuting. For example, a ridesourcing company may have contracts with business owners and assign vehicles to transport commuters to their worksites (each vehicle picks up multiple commuters from the same or different employers) and vice versa. Such ridesourcing vehicles can either send commuters from their homes to the worksites directly, or send them from homes to stations of public transit. To be more effective, CSP needs to enable (real time) communications between commuters and their employers (e.g., managers) in order to resolve commuting related (and work schedule related) issues promptly. This will hopefully help prompt more efficient ridesourcing modes (such as ridesplitting) which are currently under utilized (Li *et al.*, 2019). The integration of employers into current MaaS platforms will also lead to a win-win-win situation: commuters can choose convenient and less expensive mobility services, MaaS companies have access to larger demands (by working with employers directly) that may result in larger profit, and business owners can also benefit from CSPs because their employees have more convenient ways to get to work and the total commuting trips of their companies are causing less congestion in the adjacent areas (and thus better meet regulations on commuting trip reductions).

As the first step in proposing and studying CSP, our analysis may also help develop the next-generation **TDM strategies** to better leverage the emerging MaaS technologies. CSP is able to integrate several kinds of TDM strategies as well as enhance the collaboration of business owners and MaaS. In this paper, we designed CSPs that consider two TDM strategies, proximate commuting and flexible working hours. Benefited from the connections (contracts) with business owners, CSPs have more information about the demand pattern during the peak hours. Hence, CSPs can improve the performance of current TDM strategies and also make it possible for new types of TDM strategies to take place. Currently, the business owners take responsibility for the traffic surge in the adjacent area caused by the commuting trips of their employees, thus many of them have launched programs to reduce single occupant vehicles (Kadesh & Roach, 1997). Such single occupant vehicles can be further reduced through CSPs. Meanwhile, transportation agencies will be able to implement TDM strategies more efficiently with the presence of CSPs. They can increase the usage of certain types of commuting services that are more beneficial to the urban transportation network. For example, agencies can provide incentives to encourage employers to subscribe for the WF CSP since the platform encourages peak spreading (and thus helps reduce peak hour congestion). From our study, this will motivate more commuters (employees) to choose the WF platform. As a result, the actual use of the WF CSP will increase, which could help ease the peak hour congestion. The envisioned CSP will also provide more options to implement TDM strategies. For example, instead of putting forward regulations that directly guide individual commuters/companies to adopting TDM strategies (such as ridesharing and flexible working hours), transportation agencies can implement and increase the impact of TDM strategies by working with CSPs who may then have more (positive) influences on the commuting related decisions of employers and commuters. .

## 7 Concluding Remarks

This paper applied the two-sided market theory to study the demand-price relation and network effects in a market where employers and employees are directly connected by the envisioned *commuting service platforms* (CSP). A benchmark model was proposed to clarify the definition of the two-sidedness and the threshold of subsidization. Models for both the monopoly platform and the duopoly platforms were constructed. The duopoly model was further improved with demand constraints, which ensures that the participation rates of worksites and employees are almost the same.

The analyses presented in this paper allows us to obtain a basic understanding of CSP and the interactions of its major players (employers, employees, and the platform), as well as how such interactions may impact the participation of the platform and its prices and profit, as detailed in Section 6. Such analyses and findings can help gain useful insights on how to build CSPs and how to develop associated TDM strategies in practice .

For future research, we will relax Assumption (d) to add demand constraint (i.e., the number of employees) for each worksite in the Proximate Commute problem. We will then extend the analysis of CSP to more general commuting problems. Furthermore, the CSP proposed here is largely an abstract concept. To implement such platforms in practice, we will need to consider the myriad existing and future mobility services and think about innovative ways to combine them into an integrated CSP to serve practice commuting needs. For example, network level studies based on the economic analysis presented in this paper, i.e., whether there are transfers between different modes, design the optimal number of commuters each vehicle picks up, determine the pick up locations, find the optimal routes for the CSP vehicles, etc. Although the analyses and findings here are helpful, such practical CSP design problem is still challenging and merits further investigations. For this, understanding the behaviors and interactions of the major players (employers, employees, and agencies) with respect to commuting options (e.g., WF and NWF) and how to present them effectively on CSPs are crucial. The investigations will also help develop the next-generation TDM strategies that leverage the proposed CSP and emerging mobility options. Results on these investigations may be reported in subsequent papers.

## 8 Acknowledgments

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# Appendices

## Appendix A Duopoly model when workistes multi-home

Worksites tend to view the CSPs as homogenous. In contrast, commuters tend to view the CSPs as heterogeneous because they often evaluate the level of service of a CSP based on various factors, i.e., how convenient it is to choose a CSP based on a commuter's schedule; travel time of a CSP, which is likely to be different on the WF CSP and the NWF CSP; the comfort level of the CSP services, etc. In section 5.2, we assume that the same-side "congestion" effects are high enough (condition (B2)) so that no agents will multi-home. Here we relax this constraint and allow worksites to multi-home. Here are the conditions for the multi-home model:

**(C1)** $U_0^B = 0$ and $U_0^C$ is high enough so that commuters wish to join at least one of the CSPs
**(C2)** (i) $t^B = 0$, the cost of joining a CSP for worksites is low, so that it is possible for worksites to choose both CSPs; (ii) $t^C > \beta_N q_N^B + \beta_W q_W^B$, the same-side "congestion" effects of commuters are high, ensuring that commuters single-home
**(C3)** $f_W^B < \min\{\frac{\alpha_W}{3}\}$, $f_i^C < \min\{\frac{3\alpha_W}{4}\}$: ensures that the CSPs are willing to serve worksites.

To find the equilibrium in this setting, the consistent demand configurations need to be characterized. We first list all the possible configurations of worksites in a duopoly model, and then show that worksites will always choose Configuration 1 under conditions (C1) ~ (C3). Commuters single-home under condition **(C2)** since **Lemma 1** still applies.

*Configuration 1: worksites multi-home*

Given that $Q^B = 1, q_W^B = q_N^B = 0$, the fraction of commuters joining the WF CSP is determined by the Hotelling Model,

$$q_W^C = \frac{1}{2} + \frac{p_N^C - p_W^C + \beta_W - \beta_N}{2t^C} \tag{30}$$

The number of commuters joining the NWF CSP is $1 - q_W^C$. We assume that when a worksite finds it indifferent to join or not join a CSP, it will join the platform. It is optimal for worksites to mutlti-home when $U_{NW}^B \geq \max\{U_N^B, U_W^B, 0\}$, i.e., $-(p_W^B + p_N^B) - t^B + \alpha_W q_W^C + \alpha_N q_N^C \geq \max\{-p_W^B - t^B x^B + \alpha_W q_W^C, -p_N^B - t^B(1-x^B) + \alpha_N q_N^C, 0\}$, which implies that multi-homing is preferred over single-homing or not join any of the CSPs. The first two inequalities can be written as,

$$p_W^B \leq (\frac{1}{2} + \frac{p_N^C - p_W^C + \beta_W - \beta_N}{2t^C})\alpha_W \tag{31}$$

$$p_N^B \leq (\frac{1}{2} + \frac{p_W^C - p_N^C + \beta_N - \beta_W}{2t^C})\alpha_N \tag{32}$$

Profits of the CSPs are,

$$R_W = p_W^B - f_W^B + (p_W^C - f_W^C)(\frac{1}{2} + \frac{p_N^C - p_W^C + \beta_W - \beta_N}{2t^C}) \tag{33}$$

$$R_N = p_N^B - f_N^B + (p_N^C - f_N^C)(\frac{1}{2} + \frac{p_W^C - p_N^C + \beta_N - \beta_W}{2t^C}) \tag{34}$$

*Configuration 2: worksites single-home on the WF CSP*

The fraction of commuters joining the WF CSP is,

$$q_W^C = \frac{1}{2} + \frac{p_N^C - p_W^C + \beta_W}{2t^C} \tag{35}$$

Worksites choose to single-home on the WF CSP when $U_W^B \geq \max\{U_N^B, U_{NW}^B, 0\}$, i.e., $-p_W^B - t^B x^B + \alpha_W q_W^C \geq \max\{-p_N^B - t^B(1-x^B) + \alpha_N q_N^C, -(p_W^B + p_N^B) - t^B + \alpha_W q_W^C + \alpha_N q_N^C, 0\}$. These inequalities can be written as,

$$p_W^B \leq (\frac{1}{2} + \frac{p_N^C - p_W^C + \beta_W}{2t^C})\alpha_W \tag{36}$$

$$p_N^B \geq (\frac{1}{2} + \frac{p_W^C - p_N^C - \beta_W}{2t^C})\alpha_N \tag{37}$$

Profits of the CSPs are,

$$R_W = p_W^B - f_W^B + (p_W^C - f_W^C)(\frac{1}{2} + \frac{p_N^C - p_W^C + \beta_W}{2t^C}) \tag{38}$$

$$R_N = (p_N^C - f_N^C)(\frac{1}{2} + \frac{p_W^C - p_N^C - \beta_W}{2t^C}) \tag{39}$$

*Configuration 3: worksites single-home on the NWF CSP*

The fraction of commuters on the WF CSP is,

$$q_W^C = \frac{1}{2} + \frac{p_N^C - p_W^C - \beta_N}{2t^C} \tag{40}$$

Worksites choose to single-home on the WF CSP when $U_N^B \geq \max\{U_W^B, U_{NW}^B, 0\}$, i.e., $-p_N^B - t^B(1-x^B) + \alpha_N q_N^C \geq \max\{-p_W^B - t^B x^B + \alpha_W q_W^C, -(p_W^B + p_N^B) - t^B + \alpha_W q_W^C + \alpha_N q_N^C, 0\}$. These inequalities can be written as,

$$p_W^B \geq (\frac{1}{2} + \frac{p_N^C - p_W^C - \beta_N}{2t^C})\alpha_W \tag{41}$$

$$p_N^B \leq (\frac{1}{2} + \frac{p_W^C - p_N^C + \beta_N}{2t^C})\alpha_N \tag{42}$$

Profits of the CSPs are,

$$R_W = (p_W^C - f_W^C)(\frac{1}{2} + \frac{p_N^C - p_W^C - \beta_N}{2t^C}) \tag{43}$$

$$R_N = p_N^B - f_N^B + (p_N^C - f_N^C)(\frac{1}{2} + \frac{p_W^C - p_N^C + \beta_N}{2t^C}) \tag{44}$$

*Configuration 4: worksites join neither of the CSPs*

The proportion of commuters joining the WF CSP is the same as that in Configuration 1. Worksites do not want to join any CSP when $0 \geq \max\{-p_W^B - t^B x^B + \alpha_W q_W^C, -p_N^B - t^B(1-x^B) + \alpha_N q_N^C, -(p_W^B + p_N^B) - t^B + \alpha_W q_W^C + \alpha_N q_N^C, 0\}$. This requires each inequality (31) and (32) be reversed. The CSPs only make profits from the commuter side. There exists price range when some of the configurations overlap. To make the explanation more concise, we assume that the two CSPs set the same prices, i.e., $p_W^B = p_N^B = p^B, p_W^C = p_N^C = p^C$. Configuration 1, 2 and 3 are all consistent when $\max\{(\frac{1}{2} - \frac{\beta_N}{2t^C})\alpha_N, (\frac{1}{2} - \frac{\beta_N}{2t^C})\alpha_W\} \leq p^B \leq \min\{(\frac{1}{2} + \frac{\beta_W - \beta_N}{2t^C})\alpha_W, (\frac{1}{2} + \frac{\beta_N - \beta_W}{2t^C})\alpha_N\}$. Configuration 4 is the reverse of configuration 1, so it is impossible for them to overlap. Configuration 2, 3 and 4 are consistent at the same time when $\max\{(\frac{1}{2} + \frac{\beta_W - \beta_N}{2t^C})\alpha_W, (\frac{1}{2} + \frac{\beta_N - \beta_W}{2t^C})\alpha_N\} \leq p^B \leq \min\{(\frac{1}{2} + \frac{\beta_W}{2t^C})\alpha_W, (\frac{1}{2} + \frac{\beta_N}{2t^C})\alpha_N\}$.

### A.0.1 One-sided cross-side network effects

The analysis is very easy when the cross-side benefits are one-sided. In this section we will discuss about such cases. The simplified network effects will still unveil important insights from the duopoly model. Suppose that $\beta_W = \beta_N = 0$. In this case, the equilibrium is described in the following proposition,

**Proposition 3.** *Let condition **(C1)-(C3)** hold and assume $\beta_W = \beta_N = 0$. Then the equilibrium is unique, CSPs will serve both sides of the market, with worksites multi-home and commuters single-home. The optimal prices for worksites are $p_W^B = (\frac{1}{2} + \frac{f_N^C - f_W^C - \alpha^-}{6t^C})\alpha_W, p_N^B = (\frac{1}{2} + \frac{f_W^C - f_N^C + \alpha^-}{6t^C})\alpha_N$. The equilibrium prices of commuters are depend on the parameter settings of the model. If $\frac{f_N^C}{3} + \frac{2(f_W^C)}{3} + t^C \geq \frac{\alpha_N}{3} + \frac{2\alpha_W}{3}$ and $\frac{2(f_N^C)}{3} + \frac{f_W^C}{3} + t^C \geq \frac{2\alpha_N}{3} + \frac{\alpha_W}{3}$, the optimal prices for commuters are,*

$$p_W^C = \frac{f_N^C - \alpha_N}{3} + \frac{2(f_W^C - \alpha_W)}{3} + t^C \tag{45}$$

$$p_N^C = \frac{2(f_N^C - \alpha_N)}{3} + \frac{f_W^C - \alpha_W}{3} + t^C \tag{46}$$

*CSPs make profits,*

$$R_W = -f_W^B + (\frac{f_N^C - f_W^C - \alpha^-}{6t^C} + \frac{1}{2})(\frac{f_N^C - f_W^C - \alpha^-}{3} + t^C) \tag{47}$$

$$R_N = -f_N^B + (\frac{f_W^C - f_N^C + \alpha^-}{6t^C} + \frac{1}{2})(\frac{f_W^C - f_N^C + \alpha^-}{3} + t^C) \tag{48}$$

*If $\frac{f_N^C}{3} + \frac{2(f_W^C)}{3} + t^C < \frac{\alpha_N}{3} + \frac{2\alpha_W}{3}$ and $\frac{2(f_N^C)}{3} + \frac{f_W^C}{3} + t^C < \frac{2\alpha_N}{3} + \frac{\alpha_W}{3}$ , the equilibrium prices for commuters are $p_W^C = 0, p_N^C = 0$. The profits of the CSPs are,*

$$R_W = -f_W^B + (\frac{f_N^C - f_W^C - \alpha^-}{6t^C} + \frac{1}{2})(\alpha_W - f_W^C) \tag{49}$$

$$R_N = -f_N^B + (\frac{f_W^C - f_N^C + \alpha^-}{6t^C} + \frac{1}{2})(\alpha_N - f_N^C) \tag{50}$$

*Proof.* It takes 2 steps to proof **Proposition 3**. First we show how we derive the equilibrium prices when both CSPs are willing to serve worksites. In the second step, we will explain why each CSP is better off by serving worksites.

Step (i): Suppose both CSPs are willing to serve the worksites, then the equilibrium prices follow the expressions presented in **Proposition 3**.

Since the decisions of commuters are not affected by worksites ($\beta_W = 0, \beta_N = 0$), the participation of commuters takes the form of equation (30). Worksites, knowing the decisions of commuters, choose the WF CSP if $p_W^B \leq q_W^C \alpha_W$, or choose the NWF CSP if $p_N^B \leq q_N^C \alpha_N$, as characterized in inequalities (36) and (42). Worksites' decisionS of joining one CSP is independent of their decisions of joining the other CSP. Thus, CSPs will fully extract the surplus from worksites. In other words, each CSP will set the prices to worksites as high as possible. Set $p_W^B = q_W^C \alpha_W$, yields the profit function of the WF CSP. Similarly, we obtain the profit function of the NWF CSP when setting $p_N^B = q_N^C \alpha_N$.

$$R_W = -f_W^B + (p_W^C + \alpha_W - f_W^C)(\frac{1}{2} + \frac{p_N^C - p_W^C}{2t^C}) \tag{51}$$

$$R_N = -f_N^B + (p_N^C + \alpha_N - f_N^C)(\frac{1}{2} + \frac{p_W^C - p_N^C}{2t^C}) \tag{52}$$

From the perspective of CSPs, the revenue from worksites can be regarded as a reduction to the marginal cost of commuters, from $f_i^C$ to $f_i^C - \alpha_i$. Given that both CSPs serve worksites, the equilibrium is unique. Under condition **(C3)**, the profits of CSPs are non-negative. The profit maximization problems of CSPs are,

$$\frac{\partial R_i}{p_i^C} = 0 \quad \forall i \in \{W, N\}$$

which yield the price structure in **Proposition 3**.

Step (ii): Each CSP is better off when serving worksites.

In this part, we are going to prove that the WF CSP is better off when serving worksites. Similar proof applies to the NWF CSP. First, we study the cases when the equilibrium prices of commuters are positive, i.e., $\frac{f_N^C}{3} + \frac{2(f_W^C)}{3} + t^C \geq \frac{\alpha_N}{3} + \frac{2\alpha_W}{3}$ . Profit functions are given in equation (47) and (48). Suppose to the contrary, the WF CSP stops serving worksites, its profit with price $p_W^C$ is,

$$R_W = (p_W^C - f_W^C)(\frac{1}{2} + \frac{\frac{2(f_N^C - \alpha_N)}{3} + \frac{f_W^C - \alpha_W}{3} + t^C - p_W^C}{2t^C}) \tag{53}$$

which is maximized at $p_W^C = \frac{f_N^C}{3} + \frac{f_W^C}{3} - \frac{\alpha_W}{6} - \frac{\alpha_N}{3} + t^C$. The WF CSP makes positive profits only if the price is larger than cost, i.e., $t^C > \frac{f_W^C - f_N^C}{3} + \frac{\alpha_W}{6} + \frac{\alpha_N}{3}$, in which case it obtains profit,

$$\widetilde{R}_W = \frac{1}{72t^C}(2\alpha_N + \alpha_W - 2f_N^C + 2f_W^C - 6t^C)^2$$

Under condition **(C3)**, $\widetilde{R}_W$ is less than the profit we obtained in **Proposition 3**. The proof is as follows.

From condition **(C3)**, we know that $f_i^C < \frac{3}{4}\alpha_W$, thus, $-\alpha_W^2 + \frac{4}{3}\alpha_W(f_W^C - f_N^C) < 0$, so that,

$$\begin{aligned}
& \frac{1}{72t^C}(2\alpha_N + \alpha_W - 2f_N^C + 2f_W^C - 6t^C)^2 < -f_W^B + (\frac{f_N^C - f_W^C - \alpha^-}{6t^C} + \frac{1}{2})(\frac{f_N^C - f_W^C - \alpha^-}{3} + t^C) \\
\iff \quad & 4\alpha_N\alpha_W + \alpha_W^2 - 4\alpha_W f_W^C - 12\alpha_W t^C < -8\alpha_N\alpha_W + 4\alpha_W^2 + 8\alpha_W f_N^C - 8\alpha_W f_W^C + 24\alpha_W t^C - 72 f_W^B t^C \\
\iff \quad & 12\alpha_W(\alpha_N - 3t^C) < 3\alpha_W - 72 f_W^B t^C + 4\alpha_W(f_W^C - f_N^C) \\
\iff \quad & 4\alpha_W(3t^C - \alpha_N) > -\alpha_W^2 + 24 f_W^B t^C + \frac{4}{3}\alpha_W(f_W^C - f_N^C) \\
\iff \quad & 8\alpha_W t^C > -\alpha_W^2 + 24 f_W^B t^C + \frac{4}{3}\alpha_W(f_W^C - f_N^C) \quad (-\alpha_W^2 + \frac{4}{3}\alpha_W(f_W^C - f_N^C) < 0) \\
\iff \quad & \alpha_W t^C > 3 f_W^B t^C \\
\iff \quad & \frac{1}{3}\alpha_W > f_W^B
\end{aligned}$$

Condition **(C3)** states that $\frac{1}{3}\alpha_W > f_W^B$. So the inequality $\frac{1}{72t^C}(2\alpha_N+\alpha_W-2f_N^C+2f_W^C-6t^C)^2 < -f_W^B+(\frac{f_N^C-f_W^C-\alpha^-}{6t^C}+\frac{1}{2})(\frac{f_N^C-f_W^C-\alpha^-}{3}+t^C)$ holds, which proves that $\widetilde{R}_W < -f_W^B+(\frac{f_N^C-f_W^C-\alpha^-}{6t^C}+\frac{1}{2})(\frac{f_N^C-f_W^C-\alpha^-}{3}+t^C)$. In the same way, we can show that when $\frac{f_N^C}{3}+\frac{2(f_W^C)}{3}+t^C < \frac{\alpha_N}{3}+\frac{2\alpha_W}{3}$, the WF CSP will lose profit if not serving worksites. ■

**Proposition 3** finds the conditions under which an equilibrium exists where worksites multi-home and have their surplus fully extracted. CSPs choose to compete indirectly for commuters instead of competing directly for worksites.