1 Photoelectric effect

1.1 Definition

Projection of light onto a clean metal surface causing the ejection of electrons.

1.2 Observations

- The photoelectric effect occurs if and only if the frequency (wavelength) of the light is greater than (shorter than) some value.
- Intensity of the light projected has an effect only on the number of electrons ejected not their maximum Kinetic Energy.
- The “escaping” photoelectrons have a certain maximum Kinetic Energy, dependent on the metal used and the frequency of the projected light.

Hence the equation relating the kinetic energy of the ejected photoelectrons to the energy of the incoming light is given by:

\[ KE_{\text{max}} = E_\gamma - W \]  \hspace{1cm} (1)

where \( E_\gamma \) is the incoming energy and \( W \) is the metal dependent work function.
1.3 Photon model

Using the photon model we shall try to explain the photoelectric effect

1.3.1 Equations of the photon model

The equation relating energy $E$ and frequency $f$, (or angular frequency $\omega$) of light is given by:

$$ E = hf = \hbar \omega $$

and since $\omega = 2\pi f$ then the relationship between $h$ and $\hbar$ is given by:

$$ \hbar = \frac{h}{2\pi} $$

Note: some texts might denote $f$ as $\nu$

1.3.2 Units, Constants and Conventions

- $1\text{eV} = 1.6 \times 10^{-19}\text{J}$
- $h = 6.6 \times 10^{-34}\text{Js}$
- $hc \approx 1240\text{eV} \times \text{nm}$
- $\hbar hc \approx 200\text{eV} \times \text{nm}$

1.3.3 Energy of visible light

Having section 1.3.1 in mind we derive equation (2) into the form:

$$ E = hf = \frac{hc}{\lambda} = \frac{2\pi h c}{\lambda} $$

and using values from 1.3.2 and a $\lambda$ of about 500 nm as typical for visible light we obtain:

$$ E_{\text{visible light}} \approx \frac{1240\text{eV} \times \text{nm}}{500\text{nm}} \approx 2.5\text{eV} $$

1.3.4 Momentum

Recall that, Newtonian Momentum relationship between momentum and energy is given by $E = \frac{p^2}{2m}$ $\Rightarrow$ $p = \sqrt{2mE}$ and including its rest energy is:

$$ E = \frac{p^2}{2m} + mc^2 $$

however, since photons are massless their Energy/Momentum relationship is given by $E = pc$ $\Rightarrow$ $p = \frac{E}{c}$ or in its relativistic form:

$$ E^2 = p^2c^2 + m^2c^4 $$
2 Bohr model of the atom

Bohr represented a model for the atom that could explain the simplest one, Hydrogen. The gist of the theory is the following: what if the proton is at the center, is very massive so it won’t ”wobble”, and has the same magnitude of charge as the electron.

2.1 energy of an orbiting electron

Kinetic energy is derived by:

\[ \sum F = ma \Rightarrow \frac{ke^2}{r^2} = \frac{mv^2}{r} \Rightarrow mv^2 = \frac{ke^2}{r} \Rightarrow KE = \frac{1}{2}mv^2 = \frac{1}{2}\frac{ke^2}{r} \]

The potential energy can be derived from E & M and is given by

\[ PE = -\frac{ke^2}{r} \]

Hence the total energy is

\[ E_{\text{total}} = KE + PE \Rightarrow E_{\text{total}} = \frac{1}{2}\frac{ke^2}{r} - \frac{ke^2}{r} \]

\[ E_{\text{total}} = -\frac{1}{2}\frac{ke^2}{r} \] (7)

2.2 Radius of the orbit

From equation (7) we can also derive the radius of the orbit the following way:

\[ \frac{ke^2}{r^2} = \frac{mv^2}{r} \Rightarrow \frac{1}{r} = \frac{ke^2}{mv^2r^2} \]

since angular momentum is given by \( L = mvr \Rightarrow L^2 = m^2v^2r^2 \) then:

\[ \frac{1}{R} = \frac{ke^2}{L^2}m \] (8)

combining equations 7 & 8 we obtain:

\[ E_{\text{total}} = -\frac{1}{2}\frac{ke^2ke^2m}{L^2} \]

now here comes Bohr’s genius hypothesis, angular momentum is an integer multiple of a universal constant, \( L = nh \) and this turns our equation into:

\[ E_{\text{total}} = -\frac{1}{2}(ke^2)^2 \frac{m}{\hbar^2n^2} \]

or
\[ E_{\text{total}} = -\frac{1}{2} \left( \frac{ke^2}{\hbar c} \right)^2 mc^2 \frac{1}{n^2} \]  \hspace{1cm} (9)

2.2.1 Two notes

The first note is on \( \frac{ke^2}{\hbar c} \) which has the value \( \frac{1}{137} \) and the fact that it is unit-less made many wonder for years about its specialty (i.e. 137 is a prime number) but as it turns out it isn’t really \( \frac{1}{137} \) but rather \( \frac{1}{137.0360} \). As far as our needs for this subject, however, all we need to know are that

\[ \alpha = \frac{ke^2}{\hbar c} \approx \frac{1}{137} \]

and \( \alpha \) is the ”fine structure” constant. The other note about equation 9 is that for an electron \( mc^2 = \frac{1}{2} \text{MeV} = \frac{1}{2} \times 10^6 \text{eV} \) with the above two mentioned we rewrite equation 9 for \( n = 1 \) (ground state) as \( E_{\text{total}} = -\frac{1}{2} \alpha^2 mc^2 \) which for the hydrogen atom has the value of \( E_{\text{total}} = -13.6 \text{eV} \) and from this value we can drive the radius of the orbiting electron:

\[ \frac{1}{r} = \frac{\alpha}{\hbar c} mc^2 \Rightarrow r = \frac{\hbar c}{\alpha mc^2} = \ldots = .05 \text{nm} \]  \hspace{1cm} (10)