Assignment IV
due Wednesday Feb. 13

Reading: Text Chapter 2, Sections 2.2, 2.3, 2.4  Chapter 3, sections 3.1, 3.2, later 3.4.
Le Bellac 3.2.5  See web links on related material.

1. a) Rewrite McIntyre 2.67 in matrix notation
   b) Rewrite McIntyre 2.69 for the $y$ state instead of $x$ and use ket notation.
   c) Work out $[S_x, S_y]$ in terms of $S_z$.

2. a) Work out the precession frequency $\omega_0$, of the electron and of the proton starting from fundamental constants.
   The $g$ factor for the proton is $2 \times 2.793$ for electrons $g$ is approximately $2 \times 1.001$
   Answer: The proton precession frequency is about 42 MHz/Tesla.
   b) Calculate the “cyclotron” frequency of the electron. This is the frequency of rotation of the charge in a magnetic field. How is it related to the precession frequency found above?

3. A spin $\frac{1}{2} \uparrow$ particle has a magnetic moment $M = \gamma S$. At $t = 0$ the spin is in the $+z$ direction (spin “up”) $|\psi(t = 0)\rangle = |+\rangle$.
   a) $S_x$ is measured at $t = 0$. What results are found and with what probabilities?
   b) Instead of making the measurement of part a) let the system evolve under the influence of a magnetic field $B_0$ which is in the $y$ direction.
      Write the state of the system in the $z$ basis at time $t$.
   c) Is there a time when there is 50% probability for finding the spin “down” ($-z$ direction)? If so, when?
   d) Is there a time when there is 100% probability for finding the spin “down”? If so, when?
   e) Is there a time when there is 100% probability for finding the spin in the $x$ direction? If so, when?
      (these answers can be given in terms of constants like $\omega_0$).

4. The magnetic field is in the $z$ direction. A particle precesses in the $x,y$ plane around the $z$ axis with angular frequency $\omega_0$.
   a) Find $\langle S_x \rangle$ and $\langle S_y \rangle$.
   b) A different approach is to write the state as a linear combination of eigenstates of $S_x$ and from this notice that it precesses. It is sufficient to write the state in the $x$ basis as a function of time and then look at the state at $t = 0$, at the end of a quarter cycle, and at the end of a half cycle. What does the state represent at each of these times?

5. Text Problem 3.5 on page 73.