Introducing quantum mechanics: One-particle interferences

Valerio Scarani[a)]
Institut de Physique Expérimentale, Ecole Polytechnique Fédérale de Lausanne, CH-1015 Lausanne, Switzerland

Antoine Suarez[b)]
Center for Quantum Philosophy, The Institute for Interdisciplinary Studies, P.O. Box 304, CH-8044 Zurich, Switzerland

(Received 27 August 1997; accepted 26 January 1998)

One-particle quantum interference is presented using probably the simplest setup. This review of experimental facts may be useful as a short self-contained introduction to quantum mechanics, highlighting the dependence of interference on indistinguishability.

I. RICHARD FEYNMAN AND ONE-PARTICLE QUANTUM MECHANICS

In discussing one-particle quantum mechanics (QM), one should gratefully refer to Richard Feynman's didactic effort: He considered the superposition principle (whose simplest nontrivial consequence is precisely one-particle interference) to be "the only mystery" of QM and devoted much time to explaining it. Probably his best achievements in this direction are his three lectures on quantum electrodynamics (QED),[1] in which he describes a way of calculating quantum probabilities without introducing "complicated" mathematical tools, the description of the two-slit experiment, as well as the famous *Gedanken Experimenten* with many variously oriented Stern–Gerlach setups in his lectures for undergraduate physicists.[2]

Feynman had a dual purpose: to introduce people to the formalism of quantum mechanics, and to show the predictive and descriptive power of this theory. For this purpose, he needed some tools and he needed some hours. Note also that the "only mystery" is often mixed with some technicalities one must deal with in the systems he describes. In the two-slit experiment, for instance, readers must have an idea of the ordinary behavior of light, and at the same time they must understand that in QM they are facing a phenomenon whose physical origin is different.[3] Furthermore, the setup ends with a screen, i.e., with a continuum of detectors; while in the experiment we are going to describe, there are only two detectors. The Stern–Gerlach experiment is even more complicated: One needs to know how a classical magnetic moment would behave, and at the end of the explanation, the richness of spin physics may conceal the purely quantum characteristics of the results. This is, of course, also valid for Rauch's experiment.[4]

That's why we find it useful to present one-particle interferences in a simpler way, probably the simplest, using a two-path interferometric device. We are conscious of the fact that one-particle experiments are less astonishing then two-particle experiments;[5] and indeed, the latter must be known to anybody who wants to discuss QM. However, a simple discussion of one-particle experiments is not uninteresting at

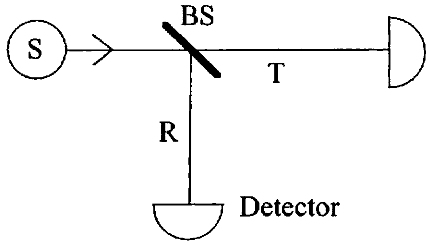

Fig. 1. Calibrating a beam-splitter.

all (as we hope to show), and is of course a necessary prerequisite for a good understanding of two-particle experiments.

II. EXPERIMENTAL FACTS

As a matter of fact, interferometry works for any kind of particle; but since our interferometric setup is built with mirrors and half-mirrors, we choose to discuss the experimental facts using photons: this will avoid the need for the teacher to explain what a mirror for a neutron is. Furthermore, photon energy (and thus, frequency) plays no role in what follows, so that one can imagine working with a laser emitting visible photons. All that matters for the strength of the argument is that *at a given time at most one photon must be traveling in the interferometer*. This condition was technically achieved only in the 70s. Readers interested in a more detailed description (both experimental and theoretical) than the one we are giving may refer to Refs. 6 and 7.

First, we must briefly describe the principle of a half-mirror, hereafter referred to as a beam-splitter (BS). Indeed, this is quite clear (see Fig. 1): If we send many photons toward the BS, and we detect both in the transmission (T) and the reflection (R) directions, we find that half of the photons have been transmitted, half reflected. We write in terms of probabilities:

$$\mathrm{P}(T)=\mathrm{P}(R)=\tfrac{1}{2}. \qquad (1)$$

A very important fact is that *each photon is detected in one and only one detector*; in other words, no "half-photon" can be observed. It is customary to summarize this by saying that the *photon is a particle.*

Consider now, as a second step, the setup of Fig. 2. We have three BSs and four paths (TT, TR, RT, and RR), each path leading to one and only one detector. Classically, *a priori* one can expect two results.

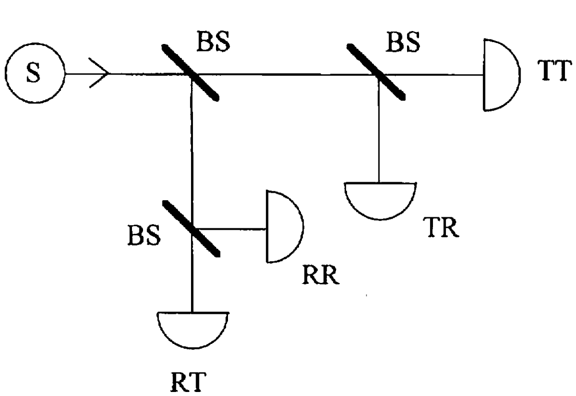

Fig. 2. A classical setup, showing no interferences.

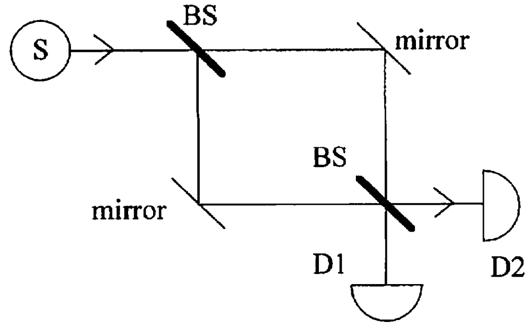

Fig. 3. The quantum interference setup.

(1) The transmission or reflection is determined by an internal parameter of the photon; in this case, a photon which has been transmitted will always be transmitted; i.e., photons can be divided into two ensembles, the "T photons" and the "R photons." In this case $\mathrm{P}(TT)=\mathrm{P}(RR)=\frac{1}{2}$; $\mathrm{P}(TR)=\mathrm{P}(RT)=0$. Such devices exist—they are called "polarizing beam-splitters"—and the internal parameter determining transmission or reflection is the state of polarization of the photon. Readers familiar with QM know that one can build a quantum interferometer with such devices, since superposition of polarization states is allowed. But we don't follow this method.

(2) The second possibility (the one we shall work with) is that transmission and reflection are random (to the extent that this means something, one can assume that everything is always determined by parameters we cannot control. However, this deterministic option will have no influence on what follows). In this case, of course,

$$\mathrm{P}(TT)=\mathrm{P}(RR)=\mathrm{P}(TR)=\mathrm{P}(RT)=\tfrac{1}{4}. \qquad (2)$$

Note that from the mathematical point of view this result is obtained by multiplying probabilities, which means precisely that the "choice" at the second BS is independent of the "choice" at the first BS.

Let's turn now to the setup described in Fig. 3. It will not take long for a classical eye to see that this setup is (or seems to be) a rough version of the previous one: Each photon still undergoes two choices, but we don't distinguish between TT and RR (both end in detector D_1), or between TR and RT (both end in D_2). In this classical approach, the calculation of the probabilities is straightforward: $\mathrm{P}(D_1)=\mathrm{P}(TT)+\mathrm{P}(RR)=\frac{1}{2}=\mathrm{P}(D_2)$. However, this prediction is *wrong*, and the observed probabilities are

$$\mathrm{P}(D_1)=0, \quad \mathrm{P}(D_2)=1, \qquad (3)$$

i.e., all photons end in D_2! Recall that each photon is alone while traveling through the interferometer, and that no "half-photon" can be observed. Moreover, each photon undergoes two impacts at two different BSs, exactly as in the setup described in Fig. 2. But in Fig. 3, two paths lead to the same detector: that is, experiment shows that *the existence of many possible paths leading to detection has an influence on the experimental outcomes.* Since each individual photon behaves as if it had "explored" all the possible paths, people speak of a photon "interfering with itself."

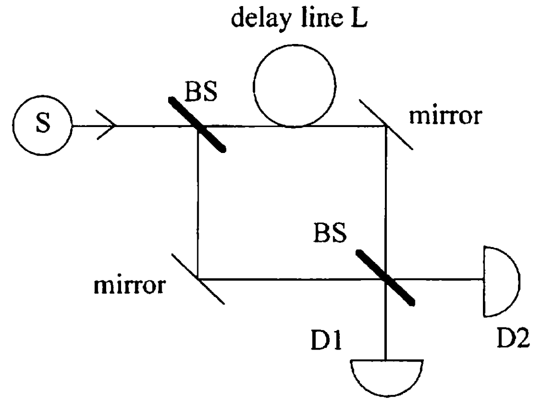

Fig. 4. Generalization of the quantum interference setup, allowing a discussion about distinguishability.

III. TO GO FARTHER

A. Interference fringes and distinguishability

It may be worthwhile to generalize the experiment, as done in Fig. 4. There a delay line (which is usually an optical fiber in the case of photons) has been added, so that one path is longer than the other. The length of the delay line must be compared to the coherence length of the laser. We consider two limiting cases (an appropriate statistical weighting of which accounts for any intermediate case).

(1) If the delay line is short compared to the coherence length, then the result is

$$P(D_1)=\sin^2(L/2), \quad P(D_2)=\cos^2(L/2). \tag{4}$$

Here, L is the optical length $\omega l/c$, with ω the frequency of the photon and l the length of the fiber. Since this section is written for people who know something about QM, we think it useful to sketch the outline of the calculation leading to this result. As a general rule, one must calculate amplitudes of probability, then sum the amplitudes for the interfering paths, take the square modulus of these terms, and finally sum over all the noninterfering possibilities. To apply this general recipe to our case, one must know that: (a) the state in the basis of the eigenstates of a BS is $|\psi\rangle = 1/\sqrt{2}(|T\rangle + i|R\rangle)$; (b) on the path with the delay line, the unitary evolution leads to an extra phase factor e^{iL}. Thus the state vector arriving at the second BS is $|\psi'\rangle=(1/\sqrt{2})(e^{iL}|T\rangle+i|R\rangle)$. Now, applying (a) to the second BS one has

$$|T\rangle=\frac{1}{\sqrt{2}}(|D_1\rangle+i|D_2\rangle), \quad |R\rangle=\frac{1}{\sqrt{2}}(|D_2\rangle+i|D_1\rangle)$$

and consequently at the stage of detection

$$|\psi'\rangle=\frac{e^{iL}-1}{2}|D_1\rangle+i\,\frac{e^{iL}+1}{2}|D_2\rangle$$
$$=ie^{iL/2}[\sin(L/2)|D_1\rangle+\cos(L/2)|D_2\rangle].$$

By squaring the moduli of the probability amplitudes one finds (4) as anticipated.

(2) If the delay line is much longer than the coherence length, then one finds the classical probabilities

$$P(D_1)=P(D_2)=\tfrac{1}{2}. \tag{5}$$

This is the *heart of quantum interference: indistinguishability*. If one can (even in principle) distinguish the path each photon has taken, then one obtains the classical probabilities (no interference). In the setup depicted in Fig. 2, distinguishability is straightforward; in the case of a long delay line, one can in principle distinguish by measuring the arrival time; and in both cases no interference is seen. In other terms, interference arises only when the photon *could have taken* more than one path to give the same final result.

B. Wave–particle duality

Modern QM tends to abandon the concept of "wave–particle duality;" however, in many introductory textbooks these terms are still used; we briefly analyze, in these terms, the experiments described, for those who would try to establish a link between this text and an older approach.

Were a photon a wave, the description of the setup of Fig. 4 would be as follows: The wave is first separated into two partial waves, one of these is dephased, and finally the two are recombined. In the particular case $L=0$ (Fig. 3), the wave is separated and recombined without any modification, so that it continues to propagate in the initial direction. Thus result (3) is immediate evidence if one uses the wave model. But, of course, this model cannot explain why no half-photon can be observed: as usual, we find that neither of the two classical *models* of physical phenomena (particles and waves) alone can describe quantum interferences.

IV. CONCLUSIONS

We have described a *naked* quantum experiment: All that may sound new and astonishing in that what precedes is due to the peculiarities of the quantum world. More precisely, we have produced simple experimental facts leading to the conclusion that the answer to the question "which path has the photon taken?" is not trivial, to the extent that the question itself has meaning.

ACKNOWLEDGMENTS

One of us (VS) must thank two persons: Jean-Paul Fragnière, who teaches Philosophy at Collège St-Michel, Fribourg, asked me to give three lectures on QM to his students; he thus offered me the first "experimental test" for the argument described in this article. My Ph.D. director, Professor Jean-Philippe Ansermet, encouraged me to take some time to write this article, in spite of the fact that we are busy with some very interesting experimental work in NMR.

[a]Corresponding author; electronic mail: valerio.scarani@ipc.dp.epfl.ch
[b]Electronic mail: suarez@leman.ch

[1]R. P. Feynman, *QED—The Strange Theory of Light and Matter* (Princeton U.P., Princeton, 1988).

[2]R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics* (Addison–Wesley, Reading, MA, 1966), Vol. III.

[3]This can clearly be seen in the mathematical approach: In the case of light diffraction, one uses the linearity of Maxwell's equations, i.e., field equations; in the quantum case, one considers superposition states, i.e., the structure of Hilbert space.

[4]For a review, see H. Rauch, "Neutron interferometric tests of quantum mechanics," Helv. Phys. Acta **61**, 589–610 (1988).

[5]Among the many introductory papers that have been written on this subject, the reader may refer to works by N. D. Mermin "Is the moon there when nobody looks? Reality and the quantum theory," Phys. Today 38–47 (April 1985); and "Hidden variables and the two theorems of John Bell," Rev. Mod. Phys. **65**, 803–815 (July 1993). See also: D. M. Green-

berger, M. A. Horne, and A. Zeilinger, "Multiparticle interferometry and the superposition principle," Phys. Today 22–29 (August 1993). An alternative nonlocal description by the two of us has also been published: A. Suarez and V. Scarani, "Does entanglement depend on the timing of the impacts at the beam-splitters?" Phys. Lett. A **232**, 9–14 (1997).

[6]P. Grangier, G. Roger, and A. Aspect, "Experimental Evidence for a Photon Anticorrelation Effects on a Beam Splitter: A New Light on Single-Photon Interferences," Europhys. Lett. **1**, 173–179 (1986).

[7]T. Hellmuth, H. Walther, A. Zajonc, and W. Schleich, "Delayed-choice experiments in quantum interference," Phys. Rev. A **35**, 2532–2541 (1987).