Summary of Week 4B, Phys. 123A, Spring 2006


Review of Capacitor: \( E = \sigma / \varepsilon_0 \) where \( \sigma \) is the charge density on plates = \( Q/A, Q=AE\varepsilon_0 \) If \( V \) is potential difference between plates then \( V = Ed \) where \( d \) is plate separation.

The new Maxwell term relates magnetic field to a changing electric field
This is analogous to an electric field induced by a changing magnetic field.
So one of Maxwell’s equations becomes
[Integral over closed loop] \( (B \ ds) = \mu_0 i + \mu_0 \varepsilon_0 \ d\Phi_E/dt \) where \( \Phi_E \) is the electric flux.
So a changing electric field produces a magnetic field just like a current does. Example of charging a capacitor; as the current flow the charge on plates and the electric field between the plates increases.

Convert Maxwell’s equations (with no charges or currents) to differential form by integrating around a narrow rectangular loop.
We find \( dE_y/dx = -dB_z/dt \) and another similar equation.
Derive the Wave equation from Maxwell equations by taking one more derivative.
\( d^2E_y/dt^2 = 1/(\mu_0\varepsilon_0) \ d^2E_y/dx^2 \) and a similar wave equation for \( B \).
The speed is \( 1/\sqrt{\mu_0\varepsilon_0} = c \), the speed of light = \( 3 \times 10^8 \) m/s
Plane traveling wave solutions are of the form \( E_y = E_{y0} \sin(kx - \omega t) \)
The solutions for \( E \) and \( B \) are constrained (related) since they have to obey the Maxwell equations.
Conclusions:

- \(E\) is perpendicular to \(B\)
- both are perpendicular to the direction of propagation.
- \(E\) and \(B\) are in phase
- The vector cross product \(\mathbf{E} \times \mathbf{B}\) is in the direction of propagation
- Magnitude are related by \(B_0 = \frac{E_0}{c}\)

Energy transport by plane wave

Review of energy stored in a capacitor
The energy per unit volume is \(\frac{1}{2} \varepsilon_0 E^2\)
The energy per unit volume stored in a magnetic field (in a solenoid) is \(\frac{1}{2} \frac{1}{\mu_0} B^2\)

The energy density has a term due to \(E\) and a term due to \(B\) which can be combined.
The time-averaged energy density (energy per unit volume) is \(\frac{1}{2} \varepsilon_0 E_0^2\)
The intensity is (energy per unit area per second) (watts/m²)
\(\frac{1}{2} \frac{1}{(\mu_0 c)} E_0^2\)
This uses the result: the average value of \(\sin^2 \omega t = \frac{1}{2}\)
The important result is that the intensity is proportional to the square of the electric field.

Polarization. The electric field is transverse in electromagnetic radiation. If it is confined to one transverse plane then the radiation (light) is said to be polarized.
If the electric field has random transverse polarization directions then the light is said to be unpolarized.
Materials exist which transmit only one plane of polarization ("Polaroid").
**The component of the electric field vector along the "pass" direction of the Polaroid is transmitted** and the perpendicular component is stopped for an ideal Polaroid sheet.
If \(E\) is at angle \(\theta\), with respect to then transmission axis (pass direction) then the component of \(E\) along the pass direction is transmitted. So the transmitted intensity is proportional to \(\cos^2 \theta\).
In the case of unpolarized light 50% of the intensity is transmitted.
Circular polarization: Electric fields in perpendicular directions are out of phase by \(\pi/2\).