Here is one example which shows the relationship between the position and the momentum in a circumstance that is easy to understand. Suppose we have a single slit, and particles are coming from very far away with a certain energy—so that they are all coming essentially horizontally (Fig. 2–2). We are going to concentrate on the vertical components of momentum. All of these particles have a certain horizontal momentum $p_0$, say, in a classical sense. So, in the classical sense, the vertical momentum $p_y$, before the particle goes through the hole, is definitely known. The particle is moving neither up nor down, because it came from a source that is far away—and so the vertical momentum is of course zero.

But now let us suppose that it goes through a hole whose width is $B$. Then after it has come out through the hole, we know the position vertically—the $y$-position—with considerable accuracy—namely $\pm B$.† That is, the uncertainty in position, $\Delta y$, is of order $B$. Now we might also want to say, since we known the momentum is absolutely horizontal, that $\Delta p_y$ is zero; but that is wrong. We once knew the momentum was horizontal, but we do not know it any more. Before the particles passed through the hole, we did not know their vertical positions. Now that we have found the vertical position by having the particle come through the hole, we have lost our information on the vertical momentum!

Why? According to the wave theory, there is a spreading out, or diffraction, of the waves after they go through the slit, just as for light. Therefore there is a certain probability that particles coming out of the slit are not coming exactly straight. The pattern is spread out by the diffraction effect, and the angle of spread, which we can define as the angle of the first minimum, is a measure of the uncertainty in the final angle.

How does the pattern become spread? To say it is spread means that there is some chance for the particle to be moving up or down, that is, to have a component of momentum up or down. We say chance and particle because we can detect this diffraction pattern with a particle counter, and when the counter receives the particle, say at $C$ in Fig. 2–2, it receives the entire particle, so that, in a classical sense, the particle has a vertical momentum, in order to get from the slit up to $C$. 

![Diagram of a slit and particles](image)
So the narrower we make the slit, the wider the pattern gets, and the more is the likelihood that we would find that the particle has sidewise momentum. Thus the uncertainty in the vertical momentum is inversely proportional to the uncertainty of \( y \). In fact, we see that the product of the two is equal to \( p_0 \lambda \). But \( \lambda \) is the wavelength and \( p_0 \) is the momentum, and in accordance with quantum mechanics, the wavelength times the momentum is Planck’s constant \( h \). So we obtain the rule that the uncertainties in the vertical momentum and in the vertical position have a product of the order \( h \): 

\[
\Delta y \Delta p_y \approx h. \tag{2.3}
\]

We cannot prepare a system in which we know the vertical position of a particle and can predict how it will move vertically with greater certainty than given by (2.3). That is, the uncertainty in the vertical momentum must exceed \( h/\Delta y \), where \( \Delta y \) is the uncertainty in our knowledge of the position.

Sometimes people say quantum mechanics is all wrong. When the particle arrived from the left, its vertical momentum was zero. And now that it has gone through the slit, its position is known. Both position and momentum seem to be known with arbitrary accuracy. It is quite true that we can receive a particle, and on reception determine what its position is and what its momentum would have had to have been to have gotten there. That is true, but that is not what the uncertainty relation (2.3) refers to. Equation (2.3) refers to the predictability of a situation, not remarks about the past. It does no good to say “I knew what the momentum was before it went through the slit, and now I know the position,” because now the momentum knowledge is lost. The fact that it went through the slit no longer permits us to predict the vertical momentum. We are talking about a predictive theory, not just measurements after the fact. So we must talk about what we can predict.
Wave model predicts:

- Higher Light Intensity $\rightarrow$ higher electron Kinetic Energy
- Frequency of Light does not affect Kinetic Energy of electrons
- Low Light Intensity $\rightarrow$ electrons are delayed
Observations:

- Higher Light Intensity → No change in electron Kinetic Energy
- Higher Intensity → more electrons ejected
- Higher Light Frequency → Higher electron Kinetic Energy
- Frequency below cutoff → no electrons
- Electrons appear immediately
Photon Model

- A photon has energy $E=hf = \hbar \nu$.
- A photon ejects an electron from well.
- Photon must have higher energy than work function of metal.
- Light intensity is proportional to number of photons.
1. The kinetic energies of the photoelectrons are independent of the light intensity. In other words, a stopping potential (applied voltage) of $-V_0$ is sufficient to stop all photoelectrons, no matter what the light intensity, as shown in Figure 3.11. For a given light intensity there is a maximum photocurrent, which is reached as the applied voltage increases from negative to positive values.

2. The maximum kinetic energy of the photoelectrons, for a given emitting material, depends only on the frequency of the light. In other words, for light of different frequency (Figure 3.12) a different retarding potential $-V_0$ is required to stop the most energetic photoelectrons. The value of $V_0$ depends on the frequency $\nu$ but not on the intensity (see Figure 3.11).

3. The smaller the work function $\phi$ of the emitter material, the smaller is the threshold frequency of the light that can eject photoelectrons. No photoelectrons are produced for frequencies below this threshold frequency, no matter what the intensity. Data similar to Millikan's results (discussed later) are shown in Figure 3.13, where the threshold frequencies $\nu_0$ are measured for three different metals.

4. When the photoelectrons are produced, however, their number is proportional to the intensity of light as shown in Figure 3.14. That is, the maximum photocurrent is proportional to the light intensity.

5. The photoelectrons are emitted almost instantly ($\leq 3 \times 10^{-9}$ s) following illumination of the photocathode, independent of the intensity of the light.
max KE

\text{Photo Energy} = \text{Max KE} + \text{Work Function}

\phi \text{ is work function}

\text{Max KE} = E_x - \phi = hf - \phi

V_{stop}\
\text{Cesium} \quad \text{Potassium} \quad \text{Sodium} \quad \text{Lithium}

f (10^{14} \text{ Hz})

5.0 \quad 5.2 \quad 5.4 \quad 5.6 \quad 5.8 \quad 6.0 \quad 6.2