Problem Set 2

due: February 3, 2020

1. Consider a one-sector model, with labor endowment \( \bar{L} = 80 \) and capital endowment \( \bar{K} = 100 \). Suppose that the marginal product of capital is represented by \( MP_K = 800 - K \), where \( K \) is the capital input. Determine the rental rate, wage rate, GDP, GNP, and welfare under autarky. [Output is zero if capital input is zero.]

2. Consider a one-sector model, with labor endowment \( \bar{L} = 100 \) and capital endowment \( \bar{K} = 80 \). Suppose that aggregate production function is given by \( Y = L^{0.4}K^{0.6} \), where \( Y \) is the output, \( L \) the labor input, and \( K \) the capital input. The marginal product of labor associated with the production function is \( MP_L = 0.4L^{-0.6}K^{0.6} \), and marginal product of capital is \( MP_K = 0.6L^{0.4}K^{-0.4} \). Determine the rental rate, wage rate, GDP, GNP, and welfare under autarky.

3. Consider again the economy described in question 1. An amount of foreign capital \( k^* = 10 \) comes into the economy. It works and earns the market rental rate, which is remitted out of the country without having to pay any taxes. Determine the new equilibrium (rental rate, wage, and output) and new welfare level. Is the foreign capital beneficial to the local economy?

4. Consider again the economy described in question 2. An amount of foreign capital \( k^* = 10 \) comes into the economy. It works and earns the market rental rate, which is remitted out of the country without having to pay any taxes. Determine the new equilibrium (rental rate, wage, and output) and new welfare level. Is the foreign capital beneficial to the local economy?

5. Consider again the economy described in question 3. Suppose that an exogenous amount of foreign capital \( k^* = 20 \) comes in. Foreign investors earn the market rental rate, but they have to pay a specific income tax of \( t = 50 \) per unit of capital before remitting their earnings out of the country. Determine the new equilibrium (rental rate, wage, and output) and new welfare level. Is the foreign capital beneficial to the local economy?

6. Consider again the economy in exercise 1, with marginal product of capital equal to \( MP_K = 800 - K \), where \( K \) is the capital input. Let this economy be called Home, with a labor endowment of \( \bar{L} = 80 \) and a capital endowment of \( \bar{K} = 100 \). There is another economy called Foreign with the marginal product of capital given by \( MP_K^* = 700 - 2K^* \). Its capital endowment is \( \bar{K}^* = 120 \).
If the Home government wants to use an income tax to maximize its welfare, while the Foreign government remains passive in policy, determine the optimal income rate and the resulting capital movement. Also, determine the resulting GDP, GNP, wage rate, and rental rate of Home.

7. Consider again the economy described in question 2. Suppose that the wage rate, for some reason, is fixed at $\bar{w} = 0.4 > w^a$. Determine the labor employment that supports the wage rate and the resulting unemployment. Find also the rental rate, GDP, GNP, and welfare under the present autarkic equilibrium.

8. Consider again the economy in question 7. Suppose now that foreign capital comes in. Determine the right amount of foreign capital that will induce an additional employment just enough to remove the unemployment. Find also the new rental rate, GDP, GNP, and welfare.