Problem Set 3
Due: February 7, 2020

1. Consider a one-sector model, with labor endowment $\bar{L} = 80$ and capital endowment $\bar{K} = 100$. Suppose that the marginal product of capital is represented by $MP_K = 800 - K$, where $K$ is the capital input. [This is the economy described in exercise 1 of problem set 2.] Suppose now that 20 units of domestic capital move to another country and earn a rental rate $r^* = 750$. Determine the new GDP, rental rate, wage rate, and welfare (GNP) of the economy. Referring to the autarkic equilibrium derived in exercise 1 of problem set 2, determine whether the economy has gained from the capital outflow. What is the value of capital outflow surplus?

2. Consider a one-sector model, with labor endowment $\bar{L} = 100$ and capital endowment $\bar{K} = 80$. Suppose that the aggregate production function is given by $\phi = \bar{L}^{0.4}\bar{K}^{0.6}$, where $\phi$ is the output, $\bar{L}$ the labor input, and $\bar{K}$ the capital input. The marginal product of labor associated with the production function is $MP_L = 0.4\bar{L}^{-0.6}\bar{K}^{0.6}$, and marginal product of capital is $MP_K = 0.6\bar{L}^{0.4}\bar{K}^{-0.4}$. [This is the economy described in exercise 2 of problem set 2.] Suppose now that 20 units of domestic capital move to another country and earn a rental rate $r^* = 0.8$. All prices are assumed to be flexible. Determine the new GDP, rental rate, wage rate, and welfare (GNP). Referring to the autarkic equilibrium derived in exercise 2 of problem set 2, determine whether the economy has gained from the capital outflow.

3. Consider again the economy described in question 2. Suppose that the wage rate is downward rigid so that the wage rate stays at the autarkic level as 20 units of capital flow out. Determine the new employment level after the capital outflow. What is the amount of unemployment? What is the unemployment rate (= unemployment/total labor force)? Determine the new GDP, rental rate, wage rate, and welfare (GNP). Does the economy gain from capital outflow?

4. Consider again the economy described in question 1. Denote the amount of domestic capital that flows out by $k$. Let the foreign rental rate be described by the following function: $r^* = 800 - k$. If capital flows freely and costlessly between the economies, determine the amount of domestic capital that goes out eventually. [Prices are flexible.] What is the new welfare of the domestic economy?

5. Suppose that the domestic government can use either an income tax or a quantitative restriction on capital outflow, with all prices flexible and with the foreign
government staying inactive in policy. Determine the optimal income tax rate (in terms of the domestic welfare). What are the resulting welfare, GDP, wage rate, and rental rate? If the government chooses quantitative restriction on the amount of capital outflow? What is the difference between the equilibrium under an optimal income tax and that under a quantitative restriction?