

**P-value**

In hypothesis testing, we normally select a prior to conducting the test (which determines the rejection regions) and the conclusions are stated in terms of rejecting or not rejecting  $H_0$ .

An alternative approach is to report the extent to which the test statistic disagrees with  $H_0$  and the reader must decide whether to reject or accept  $H_0$ .

**p-value**

- Measure of disagreement
- Observed significance level of the test
- Probability of observing a test statistic that is at least as contradictory to  $H_0$ . as the calculated test statistic

How to decide whether to reject  $H_0$

1. Select a
2. If p-value < a, reject  $H_0$

**Example 1:** Below is a portion of the EXCEL output used in the lecture notes from Week 3. Interpret the p-values for the coefficients of the independent variables.

	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>
Intercept	11.13845	3.994519	2.788433	0.011011
M	0.310379	0.137684	2.254285	0.034984
A	-0.21824	0.095736	-2.27958	0.033189
C	-0.11753	0.682142	-0.17229	0.864855

**Example 2:** A hypothetical study is set up to study the issue of failed child support payments. A sample of 25 divorced men who failed to make at least one child support payment in the last four years is taken. The following independent variables are considered for this study:

$M_i$  = the number of months the  $i^{\text{th}}$  man was unemployed in the last four years

$A_i$  = the age in years of the  $i^{\text{th}}$  man

$C_i$  = the number of children the  $i^{\text{th}}$  man has fathered

The dependent variable is  $P_i$  = the number of monthly child support payments, the  $i^{\text{th}}$  man missed in the last four years.

Below is the SPSS output for this example (see Example 3 - Week 3 Lecture Notes). Identify the main characteristics of the output below.

## Descriptives

Descriptive Statistics

	N	Minimum	Maximum	Sum	Mean	Std. Deviation	Variance
Payments Missed	25	2	17	206	8.24	3.951	15.607
Months Unemployed	25	2	24	251	10.04	5.549	30.790
Age	25	19	45	661	26.44	7.394	54.673
Number of Children	25	1	4	52	2.08	.909	.827
Valid N (listwise)	25						

## Regression

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.778 <sup>a</sup>	.606	.550	2.651

a Predictors: (Constant), Number of Children, Age, Months Unemployed

**ANOVA(b)**

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	226.993	3	75.664	10.768	.000 <sup>e</sup>
	Residual	147.567	21	7.027		
	Total	374.560	24			

a Predictors: (Constant), Number of Children, Age, Months Unemployed

b Dependent Variable: Payments Missed

**Coefficients(a)**

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	11.138	3.995		2.788	.011	2.831	19.4
	Months Unemployed	.310	.138	.436	2.254	.035	.024	.5
	Age	-.218	.096	-.408	-2.280	.033	-.417	-.0
	Number of Children	-.118	.682	-.027	-.172	.865	-1.536	1.3

a Dependent Variable: Payments Missed

## Interpreting Results--Reprise

### A. Comments about hypotheses and hypothesis testing

#### i. A hypothesis states the relationship between two variables.

A hypothesis needs to clearly state a prediction about a relationship and the unit of analysis for this relationship.

#### ii. Hypothesis testing is about statistical significance, not theoretical validity.

There are variables whose relationship is statistically significant but have no theoretical basis for being related. Likewise, there are theoretical relationships that may not turn out to be statistically significant (due to sample size or limited to variability in X in the data set) but should be included in your model for completeness. All of this takes judgment.

#### iii. T-tests do not test importance of the coefficient.

The magnitude of the coefficient (that is, the slope  $\hat{b}_1$ ,  $\hat{b}_2$ , etc) is a separate issue from the magnitude of the t-statistic (which tells you how confident to be about the coefficient estimate).

#### iv. T-tests are intended for use on samples.

Use a t-test with relatively small samples, not with data for the entire population. With very large samples you can reject almost any null hypothesis.

$t = \frac{\hat{b}_1}{SE_{\hat{b}_1}}$  and as sample size increases, SE diminishes and t approaches infinity, but

this doesn't mean that the true  $\beta$  is not really zero, just that it is a large number of SEs away from zero (but as SEs become very small of this starts to be less important).

## B. Testing hypotheses about multiple coefficients: F-tests

T-tests can only test hypothesis about an individual regression coefficient. So, we use an F-test instead.

### i. F-test #1 (global F-test): Does this model have significant explanatory power?

Is  $R^2$  significant? If we have  $k$  independent variables (predictors) do these explain  $Y$  (the outcome) better than the mean?

**Step 1:** Set up hypotheses: (are all the coefficients really equal to zero?)

$$\begin{aligned} H_0: \quad & \beta_1 = \beta_2 = \beta_3 = \dots \beta_k = 0 \\ & \text{(so } Y_i = \beta_0 + \varepsilon_i \text{, since all other terms are zero under} \\ & \text{ } H_0 \text{, in other words } Y_i = \bar{Y} + e_i) \\ H_a: \quad & H_0 \text{ is untrue (at least one } \beta_i \neq 0) \end{aligned}$$

**Step 2:** Test Statistic

$$F = \frac{MS_{\text{regression}}}{MS_{\text{error}}}$$

$$F = \frac{\frac{SSR}{k}}{\frac{SSE}{n-k-1}} = \frac{\frac{\sum (\hat{Y}_i - \bar{Y})^2}{k}}{\frac{\sum e_i^2}{n-k-1}} = \frac{\frac{R^2}{k}}{\frac{1-R^2}{n-k-1}}$$

**Step 3:**  $F_c$  (the critical F)

can be found on the F-table with  $k$  df for the numerator and  $n-k-1$  degrees of freedom for the denominator

**Step 4:** Set decision rule: if  $F \geq F_c$  then reject  $H_0$   
therefore, at least one coefficient is not zero

F is the ratio of the explained sum of squares to the residual sum of squares adjusted for the number of independent variables ( $k$ ) and the number of observations and the sample ( $n$ ).

The F-test is really testing whether the fit of the equation is providing significantly better predictions of  $Y$  than the mean would.

The F-statistic is a function of  $R^2$ . As  $R^2$  goes up, F goes up. The F-test tests the statistical significance of  $R^2$ .

**Example 3** Using the results given in Example 2, does the model have significant explanatory power? Conduct an F test at the 5% level of significance.

ii. **F-test #2: Do a subset of predictors add to the explanatory power of the model?**

Would a model with fewer explanatory variables be just as good?

First, we create an equation with a set of constraints or restrictions that are put on the regression equation. So, the regression equation is written as if the null hypothesis were true. For this test, that a subset of coefficients are equal to zero.

Second, we compare the fit of this constrained equation to an equation that is unconstrained (or unrestricted) -- where the subset of coefficients are allowed to vary from zero.

One equation must be a subset of the other in order to compare them.

**Hypothesis: (are a subset of coefficients really equal to zero?)**

Restricted equation (null model) $Y = \mathbf{b}_0 + \mathbf{b}_1 X_1 + \mathbf{e}$	$H_0: \beta_1 = \dots = \beta_M = 0$ (all M coefficients are zero)
Unrestricted equation $Y = \mathbf{b}_0 + \mathbf{b}_1 X_1 + \dots + \mathbf{b}_M X_M + \mathbf{e}$	$H_a$ : All M coefficients are not zero.

**Step 1:**  $H_0: \beta_1 \dots = \beta_M = 0$  (all M coefficients are zero)  
 $H_a$ : All M coefficients are not zero

**Step 2:** Calculate Test Statistic

Can be expressed in terms of RSS or  $R^2$ :

$$F_{(M, n-k-1)} = \frac{\frac{RSS_R - RSS_{UR}}{M}}{\frac{RSS_{UR}}{n-k-1}} \qquad F_{(M, n-k-1)} = \frac{\frac{R_{UR}^2 - R_R^2}{M}}{\frac{1 - R_{UR}^2}{n-k-1}}$$

M is the number of constraints on the equation.

M of the k variables are omitted (set equal to 0 under the  $H_0$ ).

$RSS_R$  is the residual sum of squares (unexplained sum of squares) under the restricted or constrained equation and  $RSS_R \geq RSS_{UR}$

You need to calculate the  $RSS_R$  and the  $RSS_{UR}$ . First run the regression with all the independent variables included and get  $RSS_{UR}$  (this is the original regression model). Then run the regression without the M variables (this assumes that they have all been set to zero and gives a new regression model) and get the  $RSS_R$ . The two equations must have the same number of cases.

**Step 3:** Find the critical value (same as above)

**Step 4:** Decision rule: if  $F_{(M, n-k-1)} \geq F_c$  then reject  $H_0$

Rejecting the null means that not all of the coefficients are zero.

### Computer Directions for F-tests

**i. F-test #1: Does this model have significant explanatory power?**

To test the explanatory power of the whole set of explanatory variables, as compared to just using the overall mean of the outcome variable, use the F-statistic and the p-value printed by SPSS or Excel under "ANOVA." If this p-value is less than 0.05, you can reject the null hypothesis (which is that all of the variables have coefficients of zero) at the 5% significance level. If the p-value is less than 0.01 you can reject at the 1% significance level.

ii. **F-test #2: Do a subset of predictors add to the explanatory power of the model?**

To test if, as a group, a subset of explanatory variables have significant explanatory power, you must first run one regression that includes all explanatory variables and then a second regression with only the subset you want to test.

In **Excel**, you'll just have to run the regressions twice, once with all the regressors and once with only a subset. Use the  $R^2$  from each model to compare them as outlined in the lecture notes.

In **SPSS** there is a shortcut. Click on ANALYSIS, REGRESSION, LINEAR and designate your DEPENDENT VARIABLE. Put in all the variables (both those you are testing and those you are not testing) as INDEPENDENT VARIABLES. Now, you'll see BLOCK 1 of 1; click on NEXT to move to the next "block" of explanatory variables. Put in the list of independent variables that you want to test (your subset of variables). Click on METHOD and change this to REMOVE. This will cause SPSS to run two regression equations when you click run. The first will include all of your variables and the second run a regression without the variables you specified. Use the  $R^2$  or residual sum of squares under "ANOVA" to do the F-test as illustrated in the lecture notes.

### **Predictions – using the Regression Model**

If the F and t tests indicate the model has significant explanatory power, we use the regression model to make predictions.

**Example 4:** Use the regression model from Example 2 to predict the number of monthly child support payments missed by a 42 year old father of three children who has been unemployed for 6 months.



## Exam I