Electromagnetic Wave Theory

Wei-Chih Wang
National Tsinghua University
Department of Power Mechanical Engineering

Week 4

- Lecture Notes (EM wave theory)
  http://courses.washington.edu/me557/sensors/week2.pdf
- Reading Materials:

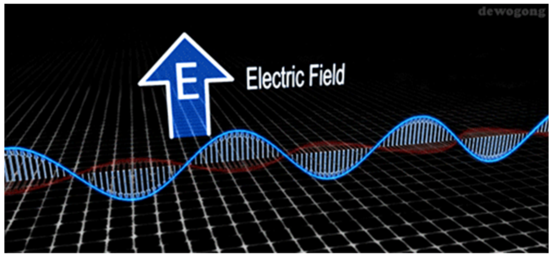
  http://courses.washington.edu/me557/reading/

  - All materials in Week 3 and 4 Electromagnetic-Wave Approach
- **No class This Thursday!!!**
- Homework #1 due Monday week 6 (need more time let me know!)
- Arrange time to do part 1 of Lab 1. Make sure you do a report on Lab 1 after you have completed all the experiment.
- Final Presentation 12/26 1-3PM

Objectives

- Introduction of EM wave
- Mathematics (vector, time harmonic function, complex vector and phasor)
- Maxwell's Equations

$$E(z,t) = \hat{x}E_o \cos(\omega t - kz)$$



This Week

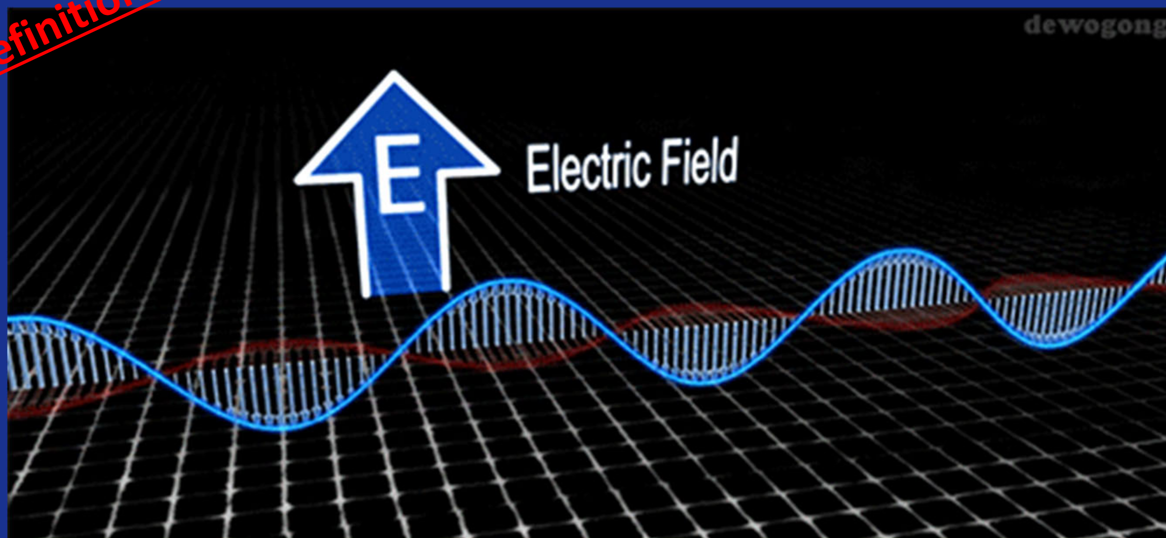
- Introduction of EM wave
- Mathematics (vector, time harmonic function, complex vector and phasor)

Difference between Ray and Wave Optics

- Ray:
- Assume ray as a particle
- gives direction, location (e.g. ray vector)
- Travel in straight path
- Can’t show how much energy is transferring in reflection, refraction. Can’t show interference, diffraction, field and power distribution (beam profile), etc.
- Wave:
- Travel like a wave, has a wavelength, a speed and a frequency
- Show energy exchange between E and B when propagating
- Shows interference diffraction, and field and power distribution (complex vector)

Wave propagation

dictionary definition



Electromagnetic wave by definition:

A **self-propagating** transverse oscillating wave of electromagnetic energy that is radiated by an accelerating or oscillating electric charge and propagates through a vacuum or a material medium as a periodic disturbance of the electromagnetic field at a frequency within the electromagnetic spectrum.

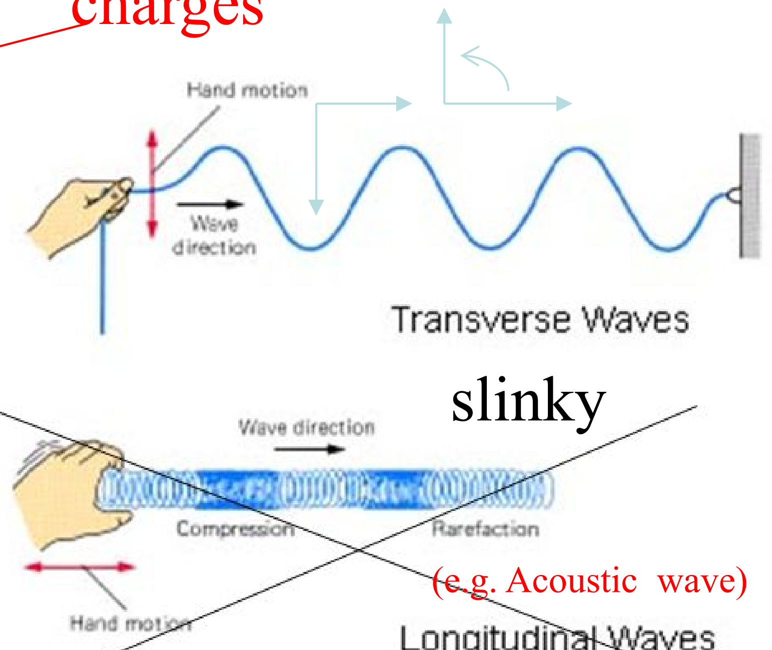
What is a transverse wave? What does the word "transverse" mean?

What is transverse wave and longitudinal waves?

* Transverse waves are waves in which the particles vibrate in an up and down motion.

charges

* "Transverse" means "moving across".
* An example is; a wave moving on a rope.



slinky

(e.g. Acoustic wave)

Rotational force (torque) created by cross product of the axial arm of rope and input up and down force

Demo

- Have student holding hands and move then move their arms up and down to show transverse wave motion
- Then have them lean against each other and then move arms back and forth to show longitudinal wave motion

Traverse and Longitudinal Wave

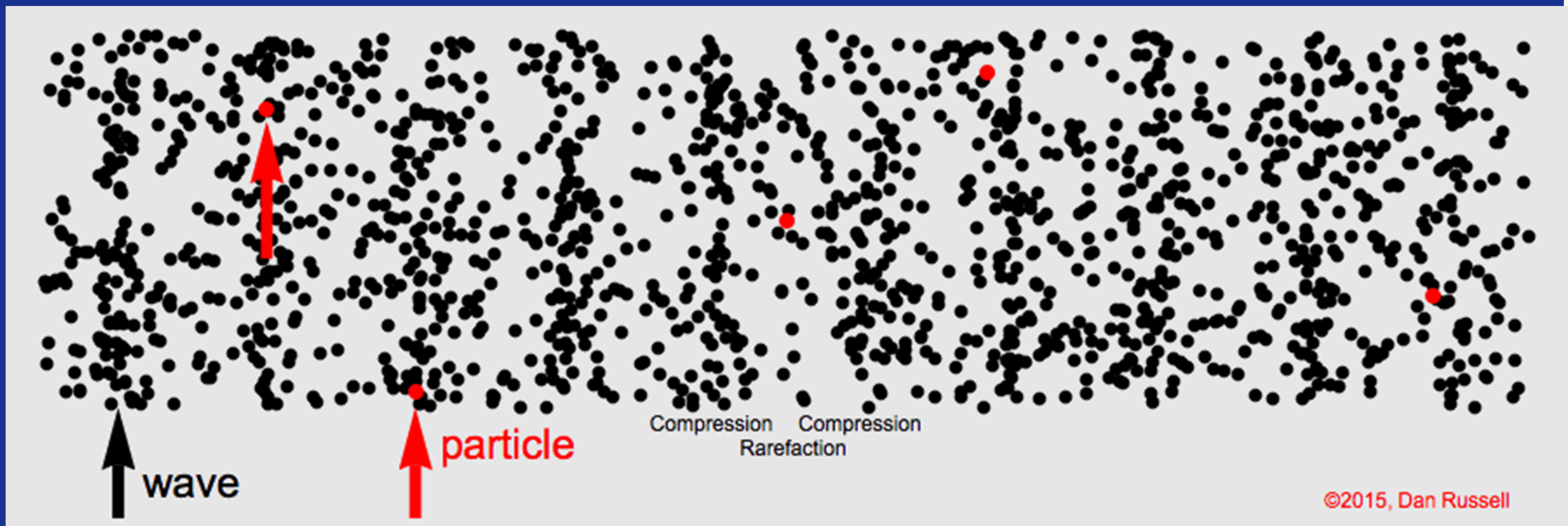
Transverse Wave

- Group velocity (net) is moving forward horizontally, phase velocity (localize particle) oscillates up and down

Transverse Wave

- Wave in Stadium first started at UW Husky Stadium back in 1981

Longitudinal Wave



Group velocity (net) is moving forward horizontally, phase velocity (localize particle) oscillates also horizontally

EM wave Explanation (wave theory)

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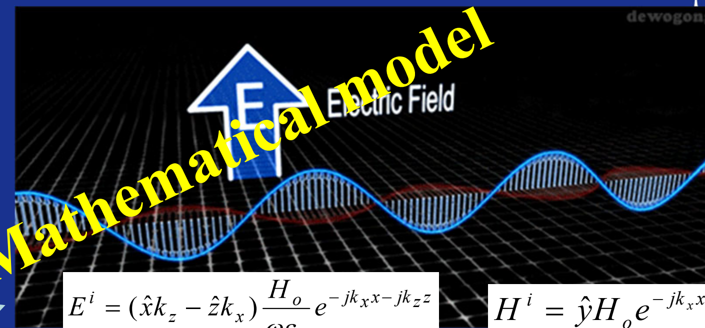
Need electric charges

- Energy absorb causes electrons in atom oscillates and regenerate a new EM wave
- Each electron consists of electric and magnetic components. They are perpendicular to the direction of propagation or direction where energy is transferring.
- While these vibrations occur for only a very short time, they delay the motion of the wave through the medium ~ c/n.

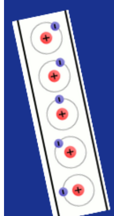
Mathematical model



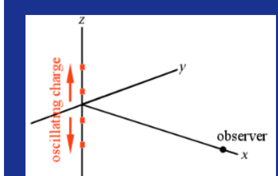
$$E^i = (\hat{x}k_z - \hat{z}k_x)\frac{H_o}{\omega\varepsilon_1}e^{-jk_x x - jk_z z}$$

$$H^i = \hat{y}H_o e^{-jk_x x - jk_z z}$$

E ad B are perpendicular to propagation (energy transfer) direction



Sorry about the animation (not showing oscillation)



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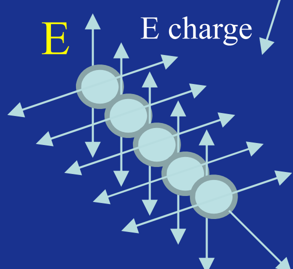
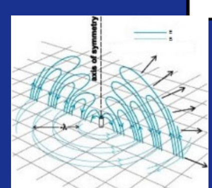
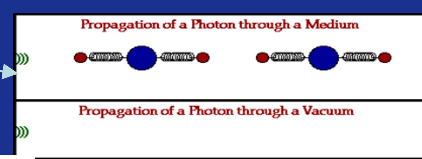
E charge

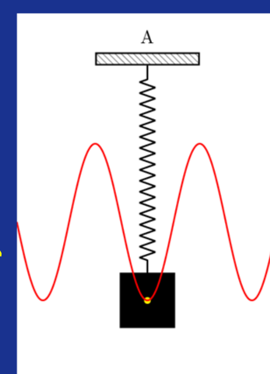




Wave generated by moving charge



$$n = \frac{\sqrt{\mu\epsilon}}{\sqrt{\mu_o\epsilon_o}} = \sqrt{\mu_r\epsilon_r}$$



Mechanical System equivalent (Mass spring system (assume no loss)

DEPARTMENT OF MECHANICAL ENGINEERING
UNIVERSITY OF WASHINGTON

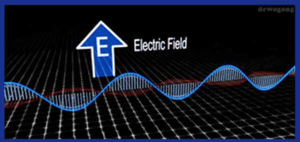
Note

- **Next few pages are basic explanation of light in terms wave and quantum theory and possible explanation for different frequency radiation generation**

Light as EM Wave (Historic perspective)

In 1845, Michael Faraday discovered that the plane of polarization of linearly polarized light is rotated when the light rays travel along the magnetic field direction in the presence of a transparent dielectric, an effect now known as Faraday rotation. T**This was the first evidence that light was related to electromagnetism. In 1846 he speculated that light might be some form of disturbance propagating along magnetic field lines. Faraday proposed in 1847 that light was a high-frequency electromagnetic vibration, which could propagate even in the absence of a medium such as the ether.**

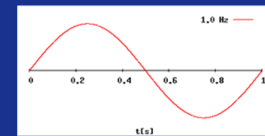
Faraday's work inspired James Clerk Maxwell to study electromagnetic radiation and light. Maxwell discovered that self-propagating electromagnetic waves would travel through space at a constant speed, which happened to be equal to the previously measured speed of light. From this, Maxwell concluded that light was a form of electromagnetic radiation: he first stated this result in 1862 in On Physical Lines of Force. In 1873, he published A Treatise on Electricity and Magnetism, which contained a full mathematical description of the behavior of electric and magnetic fields, still known as **Maxwell's equations**.

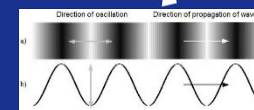


In the quantum theory, photons are seen as **wave packets of the waves described in the classical theory of Maxwell**. The quantum theory was needed to explain effects even with visual light that Maxwell's classical theory could not (such as **spectral lines**).

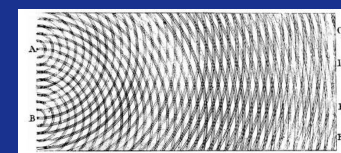
Wave Model

Subsequent many discovery based on wave theory

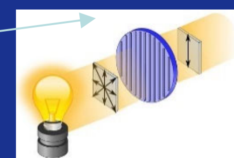




- A wave has a wavelength, a speed and a frequency.
- Grimaldi- observe diffraction of white light through small aperture quote, "light is a fluid that exhibits wave-like motion." (1665)
- Huygen- propose first wave model explaining reflection and refraction(1678)
- Young- perform first interference experiment could only be explained by wave. (1801)
- Malus- observed polarization of light. (1802)
- Fresnel- gives satisfactory explanation of refraction and equation for calculating diffraction from various types of aperture (1816)
- Oersted- discover of current (1820)
- Faraday- magnetic field induces electromotive force (1830)
- Maxwell- Maxwell equation, wave equation, speed of EM wave (1830)
- Hertz- carried out experiment which produce and detect EM wave of frequencies smaller than those of light and law of reflection which can create a standing wave.



Ripple tank interference



Wave Model

Ether theory- wave disturbance by electrical charge

When light was studied in the 1700s by Isaac Newton, who believed that light was made up of particles, he theorized that that there must be some invisible "Aetherial Medium" that light travels through that causes it to diffract. Later scientists such as Fresnel, Poisson, and Maxwell also used this ether idea to explain how electromagnetic waves propagated. These scientists believed that the universe was filled with a stationary substance called "luminiferous ether" that allowed light to propagate from stars at great distances from us. They believed that light traveled at a fixed speed relative to this stationary ether, and because the earth is not a stationary object they postulated that we should be traveling through this ether, and that the speed of **light should be slower when measured in the direction the earth is moving.** In 1887 the famous Michelson-Morley experiment was performed which used an interferometer to measure the very small differences in optical path length that this theory predicted depending on the direction they are measured in. However, much to everyone's dismay, they measured no difference in optical path length no matter how the interferometer was oriented. More experiments to come also seemed to suggest that the expected effects of the earth moving through ether were not there.

Finally, in 1905, Albert Einstein published a paper which explained how many of the phenomena that scientists used ether to explain could be accounted for using ideas that he later would solidify in his famous work on special relativity. However, the debate about ether did not immediately die down. **Over the last century as technologies have improved there have been many more experiments which have shown with progressively better precision that there is no phase shift due to varying optical path lengths in different directions.**

Wave Model

Is wave theory sufficient to explain everything? No

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There **is not a satisfactory explanation that makes it easier to understand how light, and electromagnetic waves propagate without a medium.**

The mathematical concept of fields explains how the forces involved in electrodynamics extend into empty space, however, it does not really explain why it is the way that it is. If you want to understand how electromagnetic fields work, I would say the best way to get this understanding is to study electrodynamics and solve problems until you get a feel for how things work. You have to get used to the idea of invisible field lines extending out from charged objects and wrapping around moving currents.

To answer this question, we need to address a number of **assumptions within it**: Some of the wave definition of how EM radiation and photon generated and prorogate

1. EM (electromagnetic) waves **are self reinforcing**. A changing electric field induces a magnetic field. Meanwhile, a changing magnetic field induces an electric field. When the electric field "falls", it creates a magnetic field. At the E field's zeroth point, the magnetic field is at its peak. Similarly for the magnetic field. That is, each one creates the other, like a see-saw. In addition, the fields induced are at right angles to their changes, the result of which is a beam of EM radiation traveling along through the universe (until it intersects with something). **So, EM waves don't need any other particle to support them, a wave feeds off itself.**
2. **A**n electron carries a negative charge (the proton carries a positive charge). **These particles have an electric charge property. Because of this, they can interact with EM waves and affect them. However, their presence is not required for the wave to propagate** (see first assumption).
3. **W**hat we call a vacuum (as in vacuum of space) **is really not a vacuum**. It is filled with a **sea of particles, real and virtual.** Furthermore, it is **layered with many fields; think of these fields as fluids (that's what I'll call them). There's the electric fluid, the magnetic fluid, the weak fluid, the strong, the quark fluid, and space-time.**
4. **A photon (or EM wave) is actually a wave moving through the electromagnetic fluid - just like a water wave moves through the ocean (water fluid).** So, the vacuum is not empty, **EM waves are self reinforcing**, and **electrons carry electric charge but are not the creators of the electric field (although the bend it and the *photons are the force carriers of that field*).**

So, what is light? What are photons? What are radio waves? Regardless of the total energy transported, all are special aspects of electromagnetic waves that differ in their wavelength. They remain invisible unless they interact with matter.

No Real Answer

Is wave theory sufficient to explain everything? No

- Only electric charges can impede or absorb an EM wave, by being caused to move. However, the absence of any electric charges, so the EM disturbance spreads through a vacuum without limit under the assumption that EM (electromagnetic) waves are self reinforcing. (wave theory)
- If you want a deeper "why" answer (for many of the assumption in wave theory), we do not have one. We know by observation and experiment how the universe behaves, and that it can be described by quite simple mathematics. We don't know why things are the way we find them.

Light as EM Wave (quantum)

recap what quantum theory say about photon and EM wave.

A photon is another way of looking at an electromagnetic wave and it is a quantum-mechanical particle. However, **the photon has zero mass** and **no electric charge.**

The relationship between the photon momentum, wavelength and frequency is:

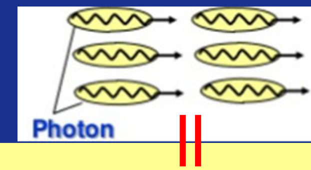
$p = h/\lambda$, where p is the momentum and h being the Planck constant.

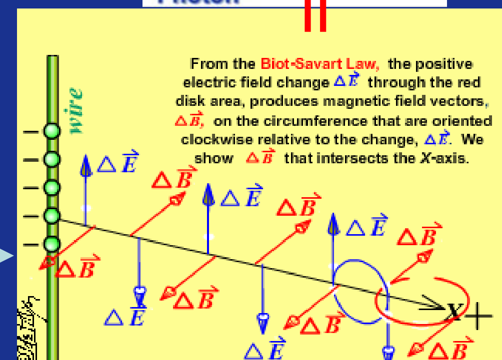
The momentum of a photon (zero mass) is given by $p = E/c$, and the wavelength by $\lambda = c/f$, where c is the speed of light in vacuum.

--> $E = hc/\lambda = hf$ Just a discontinuity about frequency (sometimes f in $\lambda = c/f$ and sometimes v in $E = hv$).

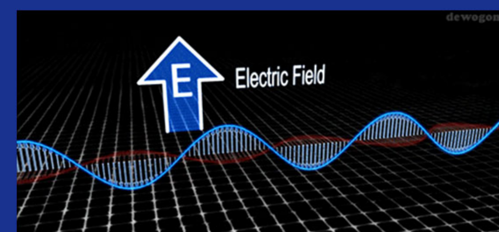
Light as photon and EM Wave

Electromagnetic radiation can be described in terms of a stream of **mass-less charge-less particles,** called photons, each traveling in a wave-like pattern at the speed of light. Each photon contains a certain amount of energy. The different types of radiation are defined by the amount of energy found in the photons (Eg= hC/λ). (quantum)





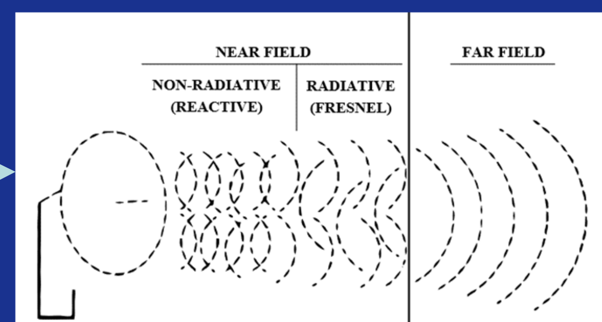
Photons are particles **forming the electromagnetic field and they are also waves.** Their de Broglie wavelength is the same that the one associated to their wavelength of the electromagnetic field. And **the electromagnetic wave propagates at the velocity of light as the photons contained within the electromagnetic field.** The movement produces oscillating electric and magnetic fields, which travel at right angles to each other in a bundle of light energy called a photon



In **homogeneous, isotropic media**, the oscillations of the two fields (E and B) are perpendicular to each other and perpendicular to the direction of energy and wave propagation, forming a **transverse wave**.

$$E(z,t) = \hat{x}E_o \cos(\omega t - kz)$$

Electromagnetic radiation is associated with those EM waves that are free to propagate themselves ("radiate") **without** the continuing influence of the moving charges that produced them, because they have achieved sufficient distance from those charges. Thus, EMR is sometimes referred to as the **far field**



String Theory

Is quantum theory enough to explain everything? No so there is string theory and mutual energy flow theory

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A photon can travel many light years through empty space and after that trip it can still be detected by a human eye.

An electromagnetic wave requires a powerful transmitter and huge receiving antenna antennas to bridge the distance. between two planets.

The path of a photon (EM wave) **is not affected** by electric charges. The electric field is certainly influenced by nearby electric charges. **So the usual statement that photons are electromagnetic waves must be false.**

Where quantum theory fails

Photons are not waves and their carrier is not the electromagnetic field.

According to the Hilbert Book Model **photons can be represented by strings of equidistant one-dimensional shock fronts that travel in our living space**. These strings feature a fixed emission duration that does not depend on the frequency of the photon.

mutual energy flow theorem

The superposition of the retarded wave and the advanced wave created 2 self-energy flow corresponding to the retarded wave and the advanced wave and 1 mutual energy flow which is inner product of the retarded wave and the advanced wave.

The photon is the mutual energy flow. It can be prove that many mutual energy flow can build a macroscopic wave which satisfy Maxwell equations, if the absorbers have uniformly distributed on the infinite big sphere.

The self-energy flows do not carry energy, there are two time-reversal waves cancel the self-energy flows. Hence in microscopic view there 4 waves: the retarded wave, advanced wave and 2 time reversal wave corresponding to the retarded wave and the advanced wave. The photon is built from these 4 waves. Many photon together a macroscopic retarded wave can be built.

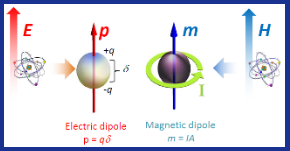
Low and high frequency EM Wave (how wave form)

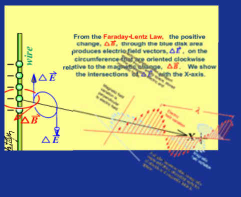
My theory based om wave and quantum, but I combine them so you can see how they might be related.. Please remember EM wave theory is for low frequency and quantum is for higher frequency

Low frequency

- EM radiation produced by accelerating charge (e.g. RF radiation-real ~ low frequency EM radiation)- dipole EM theory

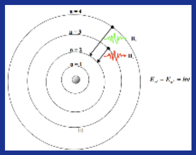
Maxwell didn't and couldn't explain this because he didn't know quantum theory. Photon and EM wave theory are interchangeable in explaining how light or EM wave are generated and propagating

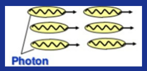




high frequency (but we don't have a charge in photon)

- Photon produced by electrons dropping to lower energy levels (e.g. joule heat-loss through e- with atom or energy release from dying electrons still possible with radiation from higher frequency EM Wave) – quantum theory





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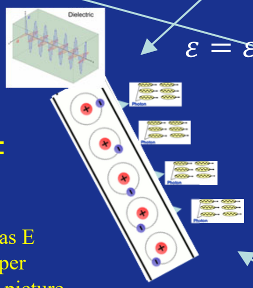
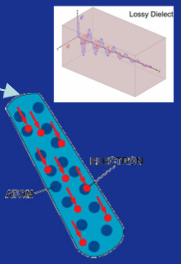
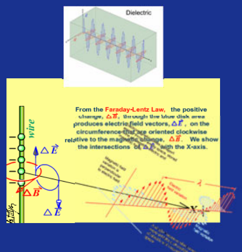


$\varepsilon = \varepsilon' - j\sigma/\omega$

+





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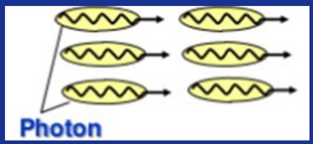
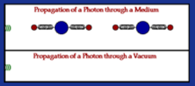
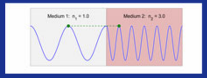


$$\frac{c_o}{c} = \frac{1/\sqrt{\mu_o \varepsilon_o}}{1/\sqrt{\mu_r \mu_o \varepsilon_r \varepsilon_o}} = \sqrt{\varepsilon_r} = n$$

The dropping of energy also involved electron which also has E and H which oscillates like the charge oscillation in the copper wire so the higher frequency radiation still behave light the picture shown here

Pump energy in to create conduction (CREATE CURRENT) and when e' dies, energy releases (PHOTON)



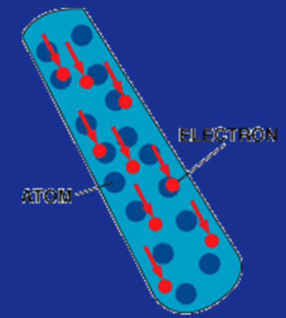


- Produce differently and detect differently but they are fundamentally the same EM wave or photons with different energy levels.

Joule Heat

Joule heating is caused by interactions between charge carriers (usually electrons) and the body of the conductor (usually atomic ions).

A voltage difference between two points of a conductor creates an electric field that accelerates charge carriers in the direction of the electric field, giving them kinetic energy. When the charged particles collide with ions in the conductor, the particles are scattered; their direction of motion becomes random rather than aligned with the electric field, which constitutes thermal motion. Thus, energy from the electrical field is converted into thermal energy



Week 5

- Lecture Notes (EM wave theory)
  http://courses.washington.edu/me557/sensors/week2.pdf
- Reading Materials:

Please read all materials in Week 5 in

http://courses.washington.edu/me557/reading/

And also following notes in Week 5:

- hand written lecture notes on Maxwell's Equation
- hand written lecture notes on dereivation of Wave equation
- Homework #1 due next week (if you need more time just let me know)
- Please start working on first problem in HW #2
- **Final presentation Dec. 26 1:20 to 3:10**

Recap

Low and high frequency EM Wave (how wave form)

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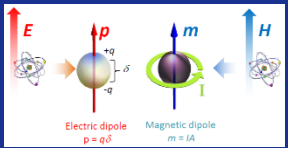

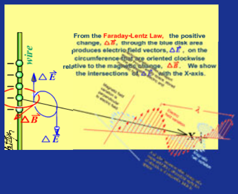

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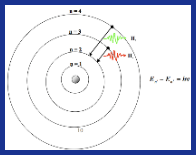

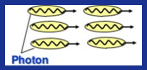

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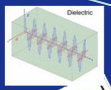
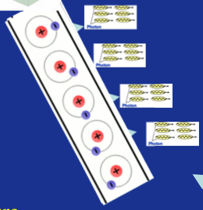
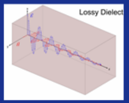
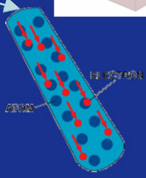


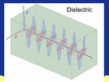


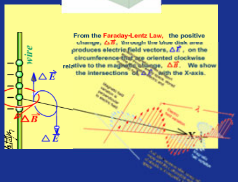
$\varepsilon = \varepsilon' - j\sigma/\omega$

+









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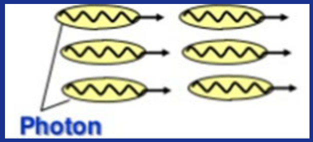
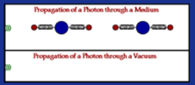
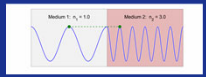


$$\frac{c_o}{c} = \frac{1/\sqrt{\mu_o \varepsilon_o}}{1/\sqrt{\mu_r \mu_o \varepsilon_r \varepsilon_o}} = \sqrt{\varepsilon_r} = n$$

The dropping of energy also involved electron which also has E and H which oscillates like the charge oscillation in the copper wire so the higher frequency radiation still behave light the picture shown here

Pump energy in to create conduction (CREATE CURRENT) and when e' dies, energy releases (PHOTON)



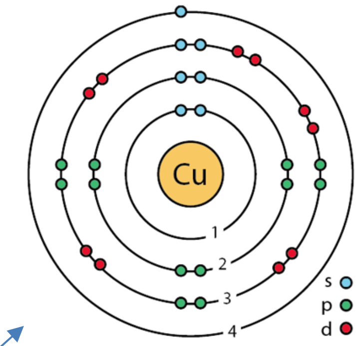


- Produce differently and detect differently but they are fundamentally the same EM wave or photons with different energy levels.

Now let's take a closer look at wave theory (first look at the current source)

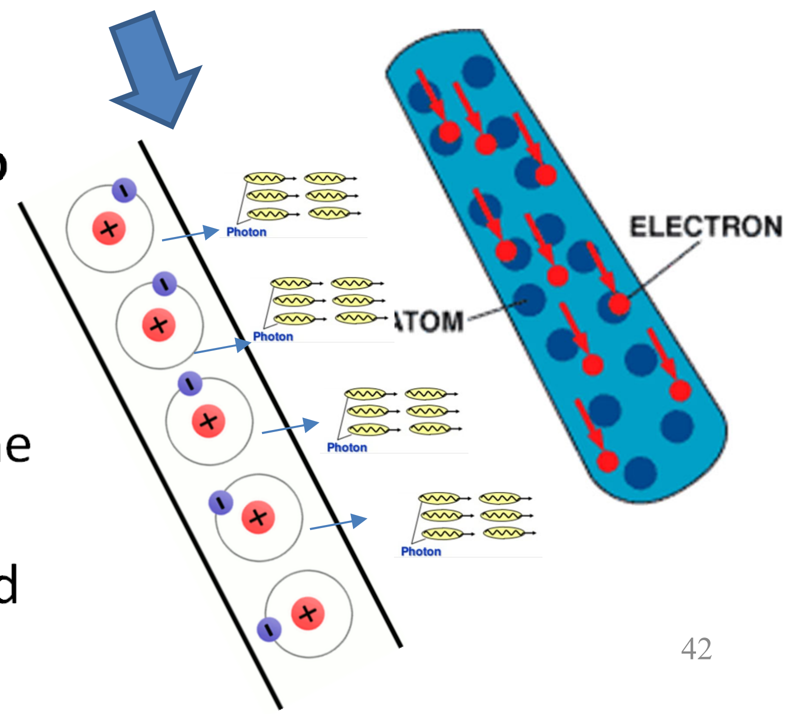
Current

- Electrons can be made to move from one atom to another. **When those electrons move between the atoms, a current of electricity is created.** The electrons move from one atom to another in a "flow." One electron is attached and another electron is lost. It is a situation that's very similar to electricity passing along a wire and a circuit. The **charge is passed from atom to atom** when **electricity is "passed."** When electrons move among the atoms of matter, a current of electricity is created. This is what happens in a piece of wire. The electrons are passed from atom to atom, creating an electrical current from one end to other, just like in the picture.



In a copper atom, the outermost 4s energy zone, or conduction band, is only half filled, so many electrons are able to carry electric current.

Pump energy in to create conduction and when e' dies, energy releases

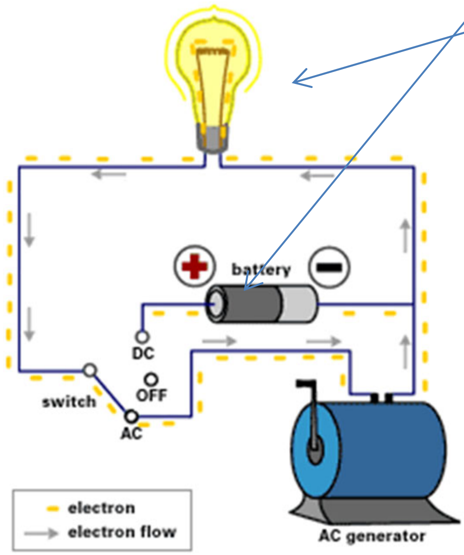


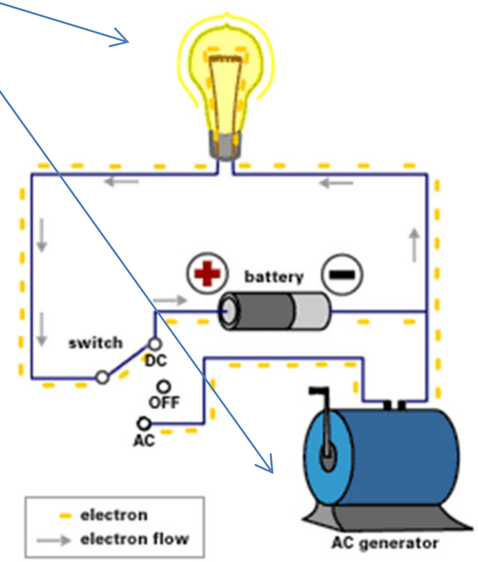
Direct and Alternating Current

$V = I \times R$ (Ohm's Law)

$P = V \times I$ (power provided by power supply)

$P_R = I^2 \times R$ (power dissipated in resistor)





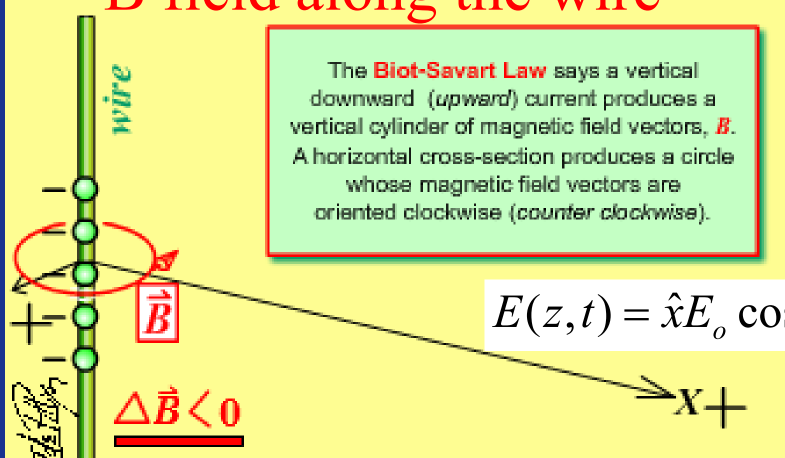
AC (e.g. household 60HZ)

DC

Electromagnetic Wave

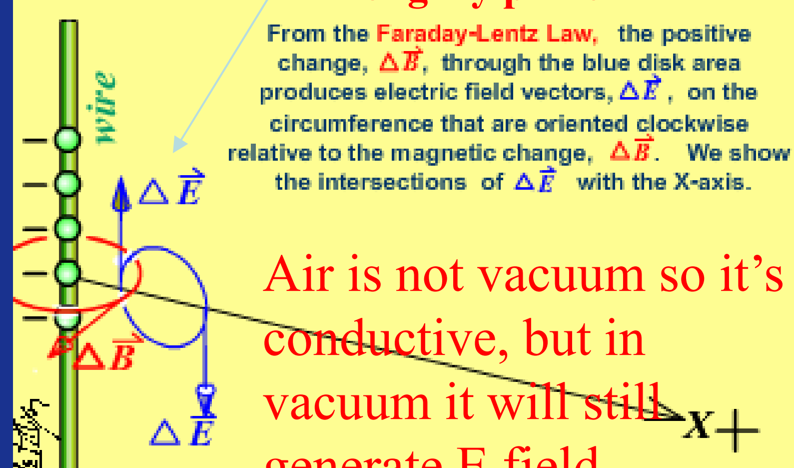
- Electromagnetic waves are created by the vibration of an electric charge. This vibration creates a wave which has both an electric and a magnetic component

Looking at a fix point of B field along the wire



$$E(z,t) = \hat{x}E_o \cos(\omega t - kz)$$

Looking only along x y plane



Air is not vacuum so it's conductive, but in vacuum it will still generate E field

Only looking at field changes along x axis

Ampere's Law

source

Integral form

Free space

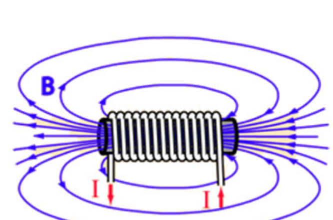
$$\boxed{\oint B \cdot ds = \mu_0 i} + \frac{1}{c^2}\frac{\partial}{\partial t}\int E \cdot dA$$

Differential form

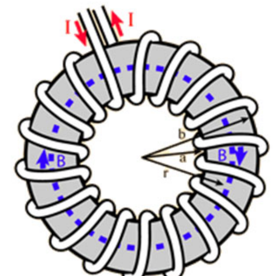
$$\nabla x B = \frac{4\pi k}{c^2} J + \frac{1}{c^2}\frac{\partial E}{\partial t}$$

$$\nabla x B = \frac{J}{\varepsilon_0 c^2} + \frac{1}{c^2}\frac{\partial E}{\partial t}$$

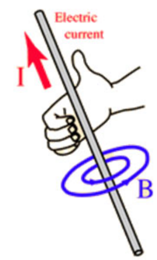
$$k = \frac{1}{4\pi\varepsilon_0} \qquad c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$



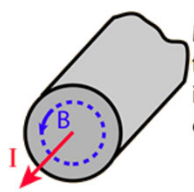
Magnetic field inside a long solenoic.



Magnetic field inside a toroidal coil.



Magnetic field from a long straight wire.



Magnetic field inside a conductor.

relating currents to magnetic effects

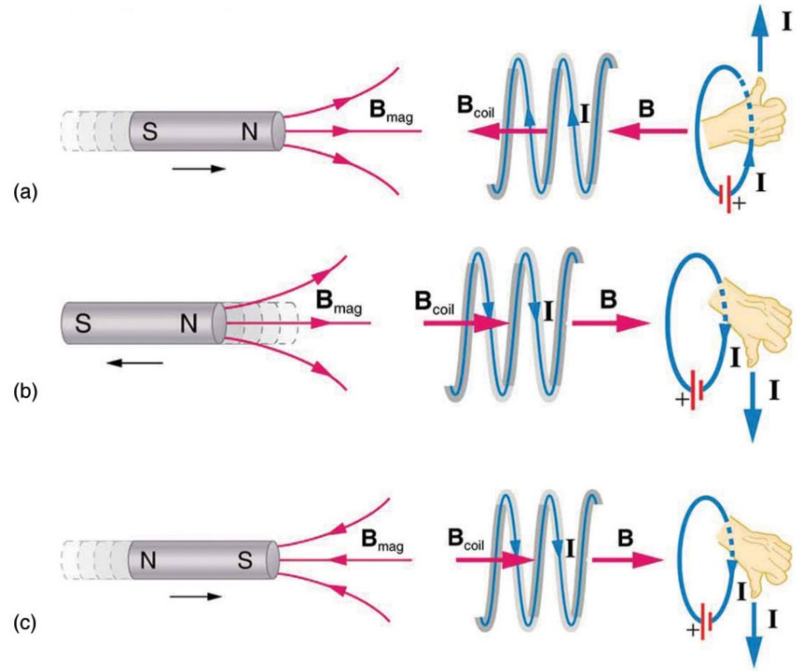
Faraday's Law of Induction

Opposing the change like Newton' third law

Integral Form

$$\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

$\vec{dl}$

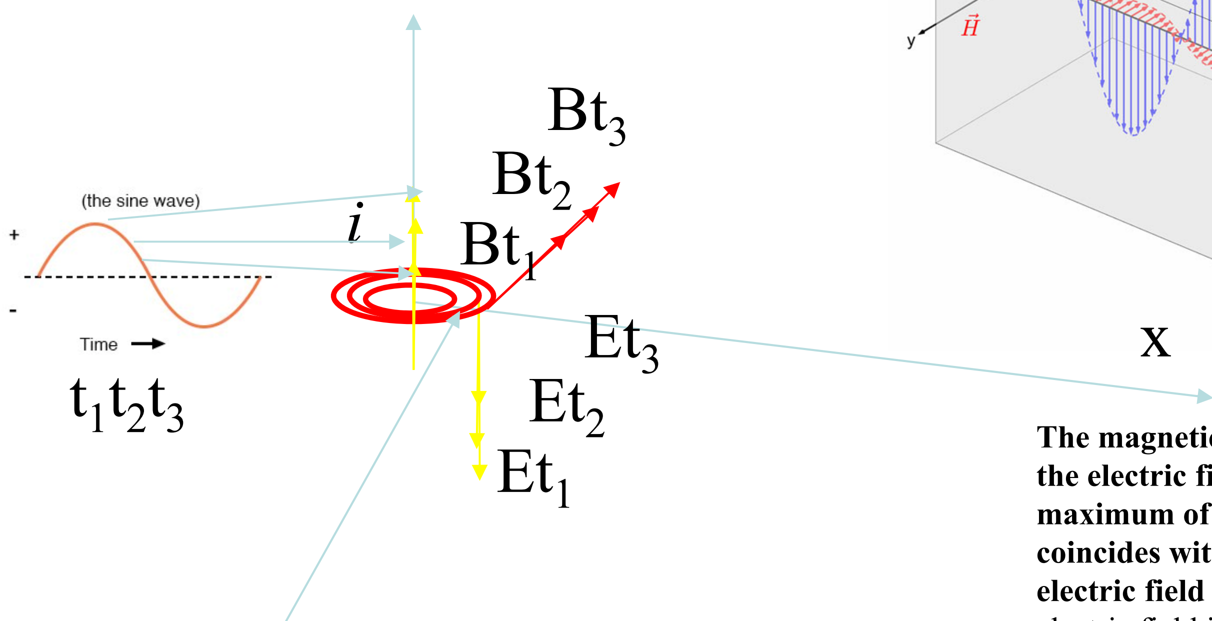


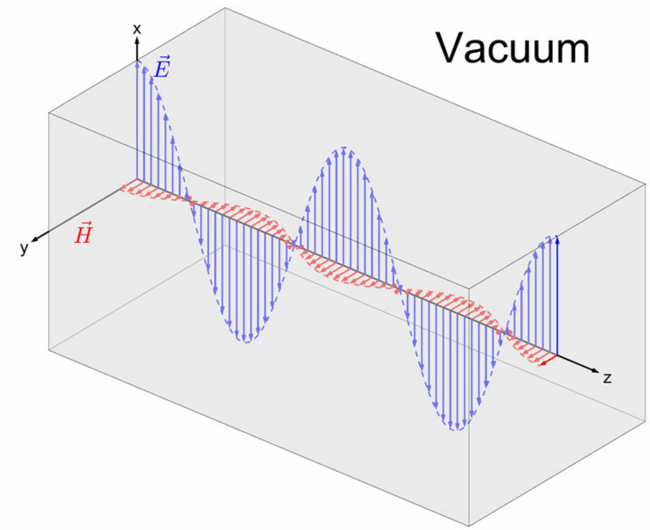
Differential Form

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

electromagnetic induction

Closer look at what happen



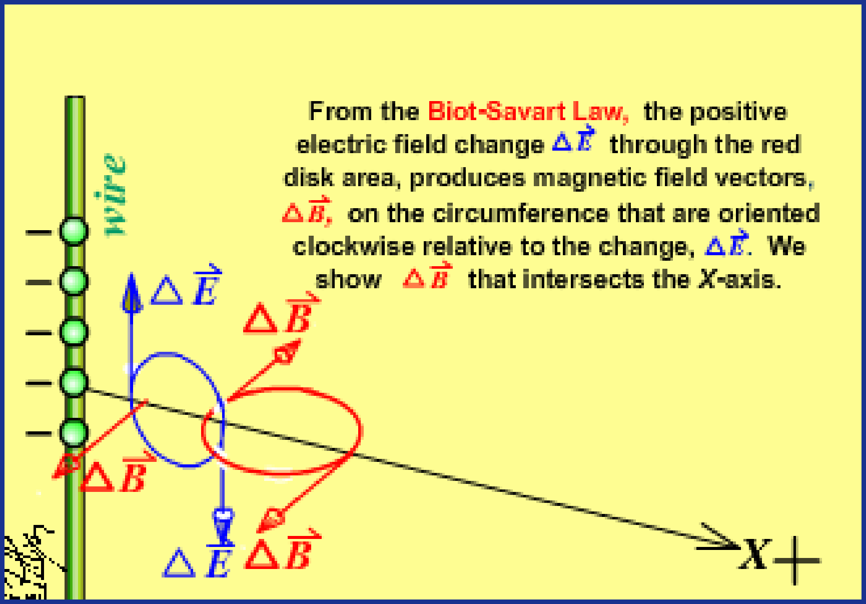


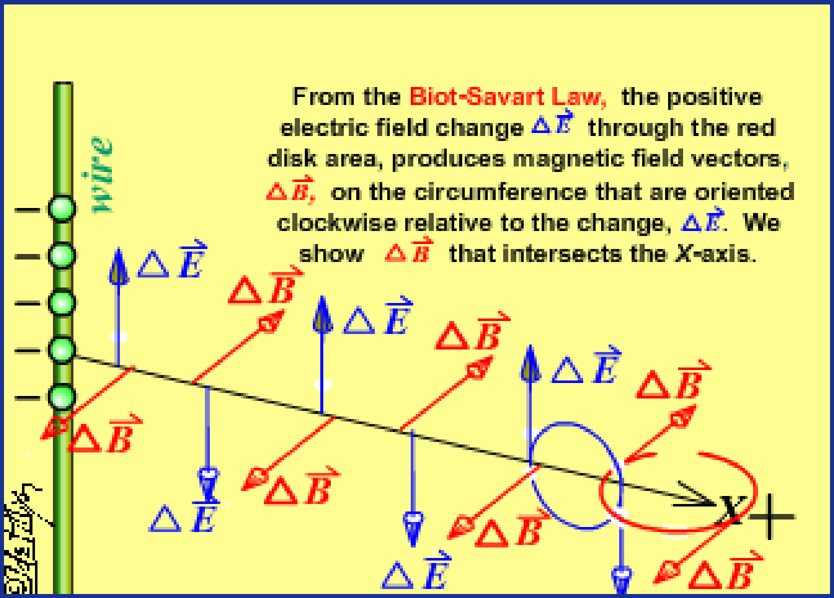
x

Looking at single point along x axis

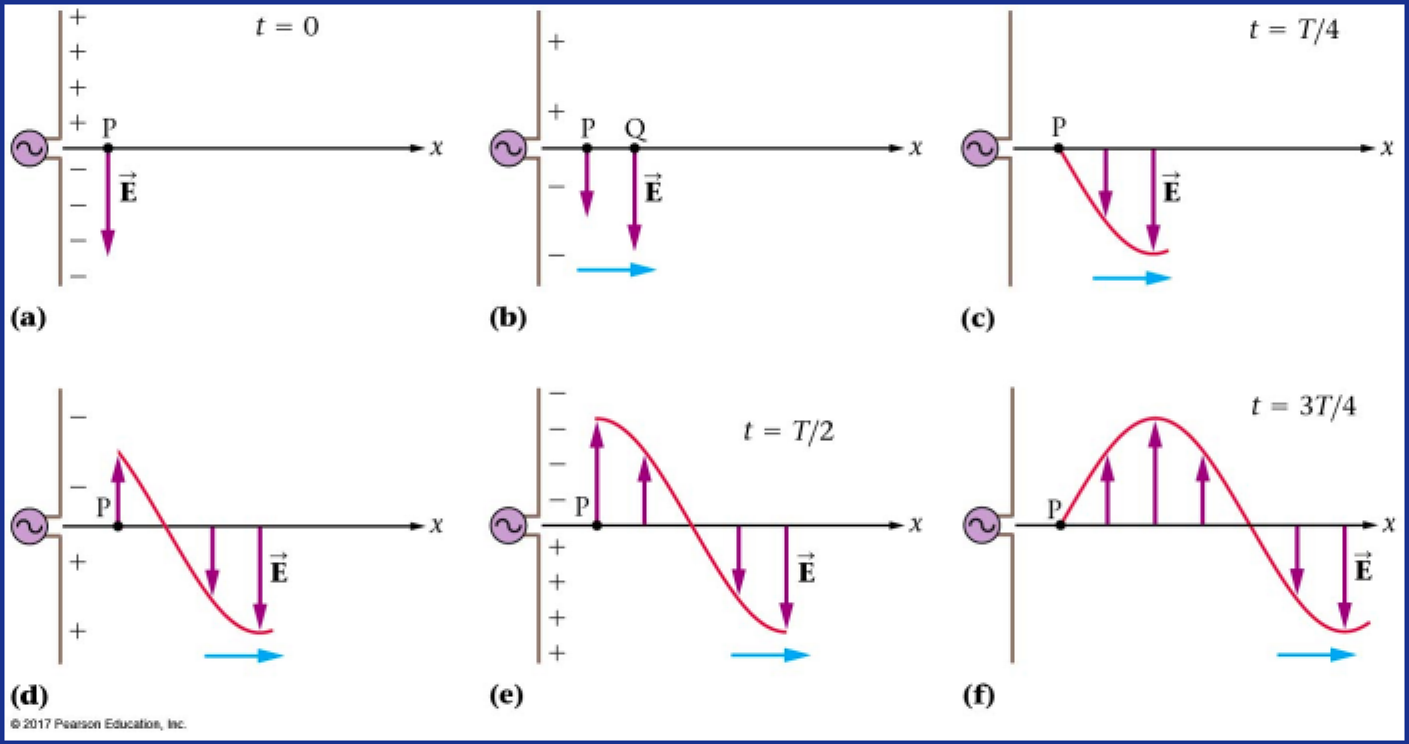
The magnetic field oscillates *in phase* with the electric field. In other words, a wave maximum of the magnetic field always coincides with a wave maximum of the electric field in both time and space. The electric field is always perpendicular to the magnetic field, and both fields are directed at right-angles to the direction of propagation of the wave. In fact, the wave propagates in the direction E×B. Electromagnetic waves are clearly a type of *transverse wav*

Electromagnetic Wave





Only looking at field changes along x axis


t = 0
P
E
x
(a)
P Q
E
x
(b)
t = T/4
P
E
x
(c)
P
E
x
(d)
t = T/2
P
E
x
(e)
t = 3T/4
P
E
x
(f)
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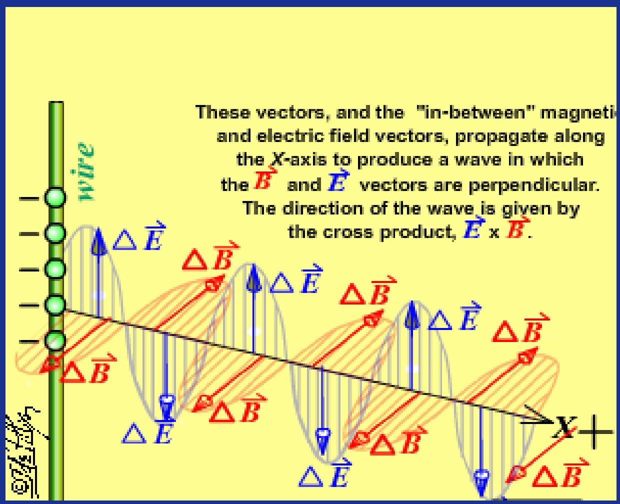
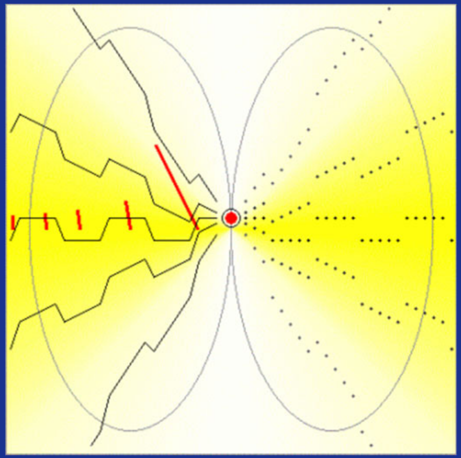
Electromagnetic Wave

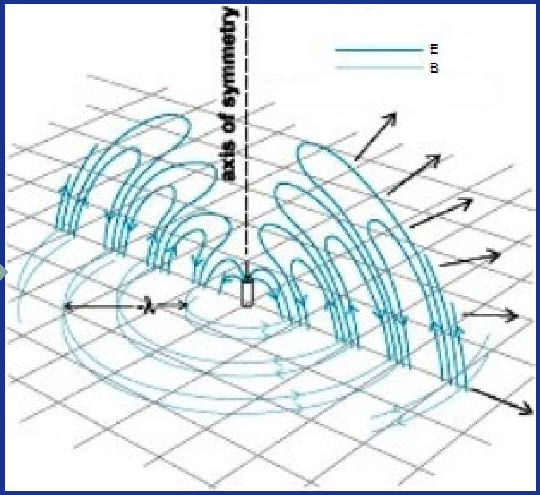


Along x axis

$$E(z,t) = \hat{x}E_o \cos(\omega t - kz)$$



Top view

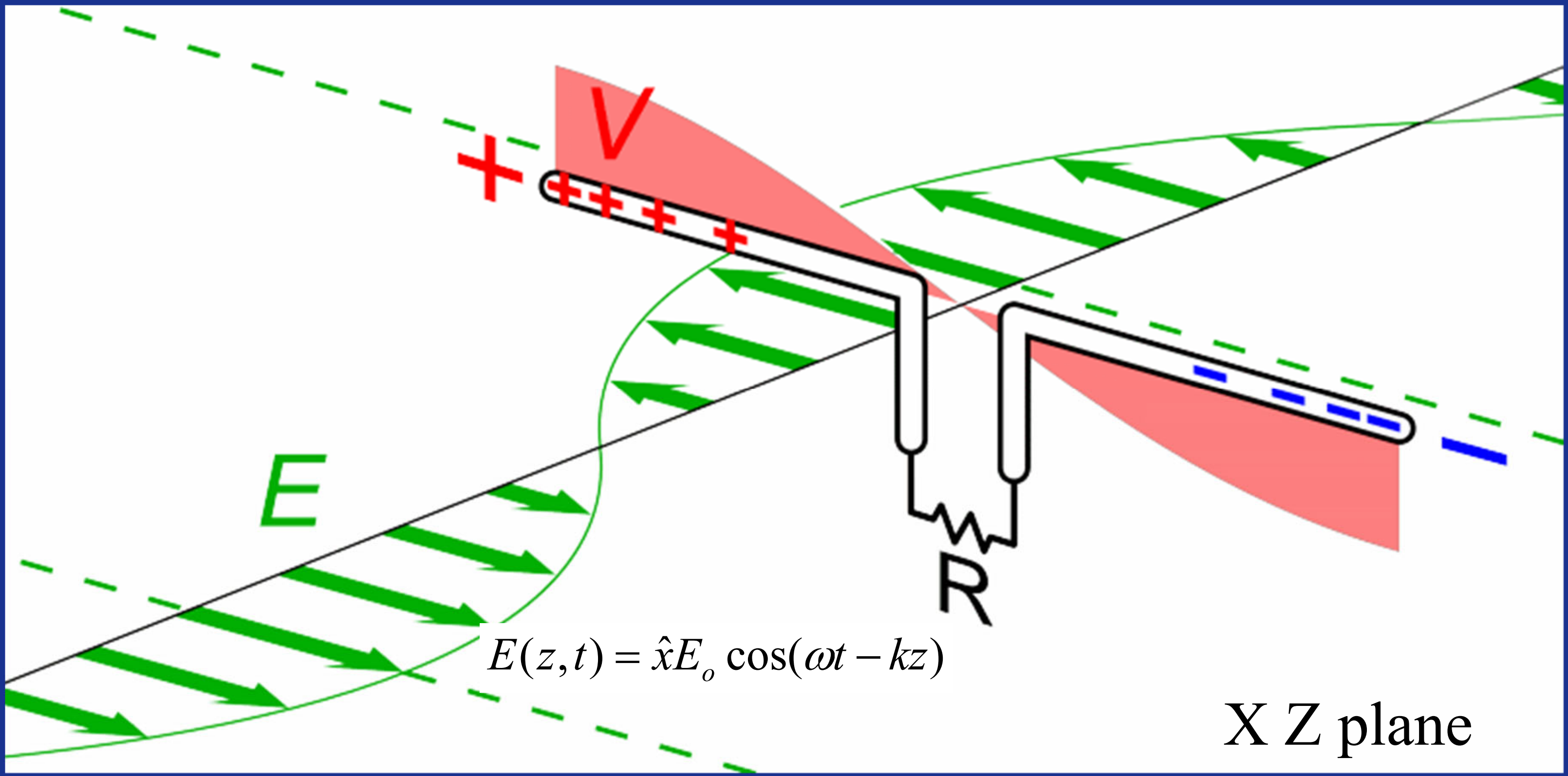


3D with cut view

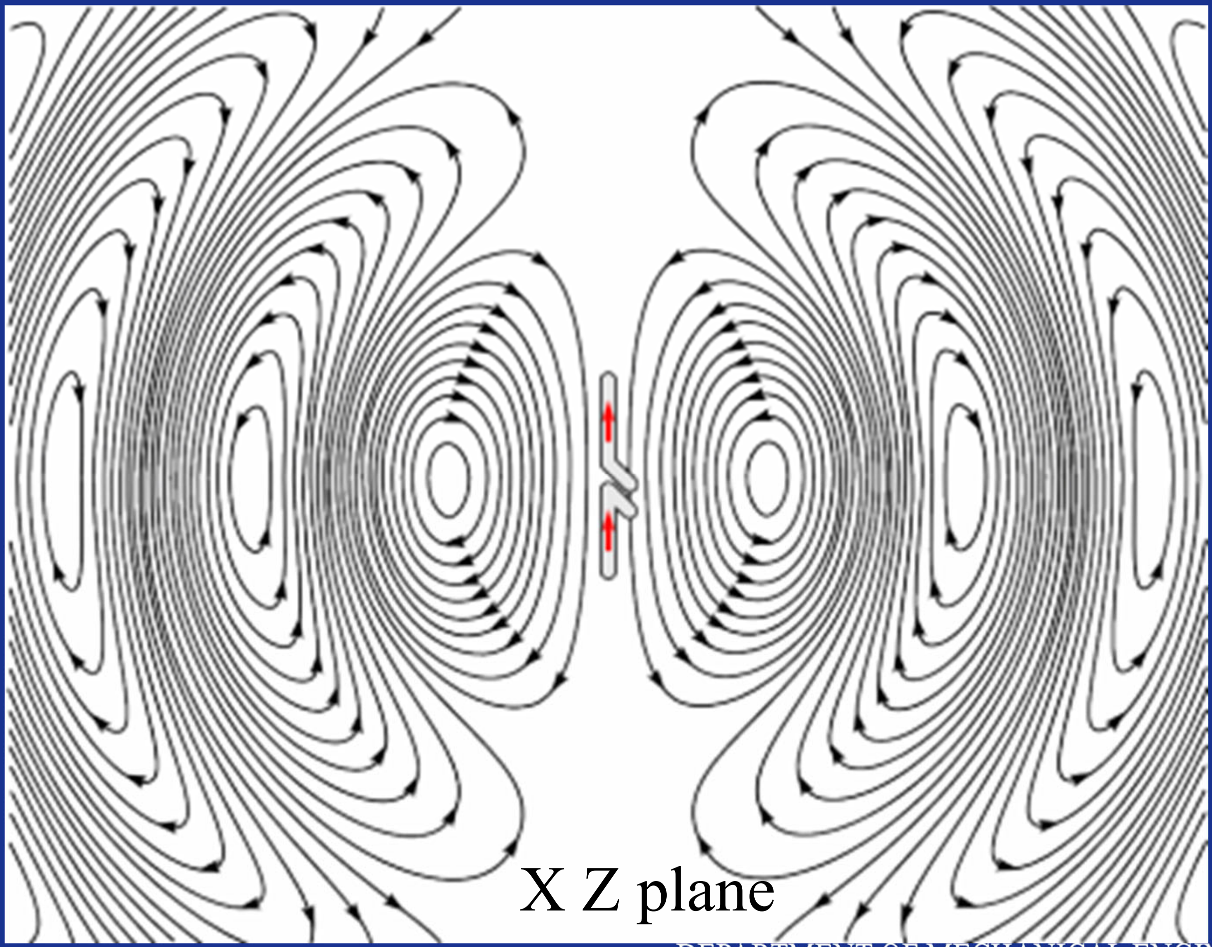
E and B field propagating from wire

Only looking at field changes along x axis

Radio wave from Dipole antenna

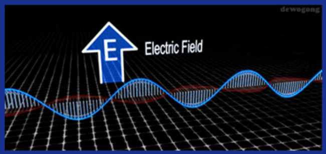


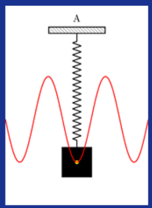
Electric Field Radiation from A Dipole antenna



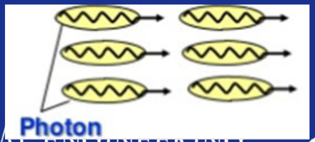
How wave propagate in vacuum

- Wave in vacuum just keep going- means energy once it's released, it just keep oscillating… not need additional interaction with any atoms and free electrons. (wave)





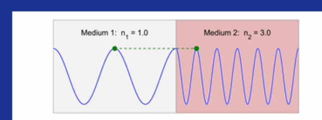
- In quantum theory, the photon is a wavelike particle that will behave just like particle while electric and magnetic energy continue to transfer back and forth while photon moving forward



MICRO TECH LAB

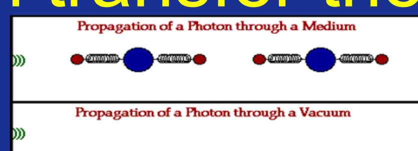
How wave or photon propagate in medium

- For wave, interaction electron and copper electron or transmitting medium atom depending on its epsilon and mu (both real and imaginary part) like describe earlier. (It could be the same as photon interaction- like below and described later in the animation)



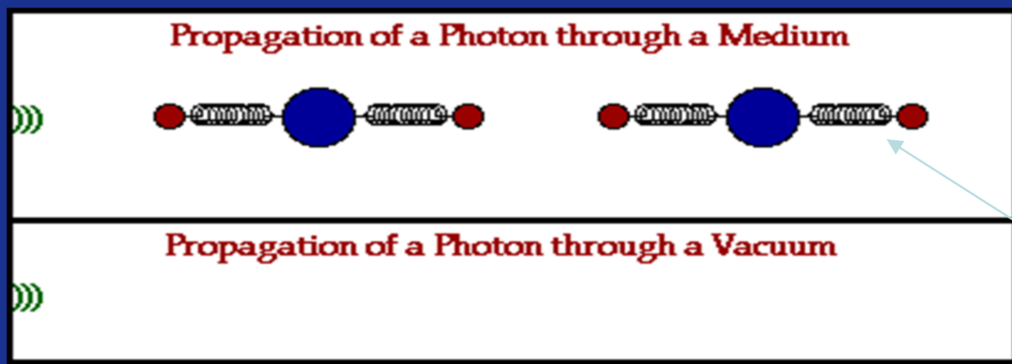
$$\frac{c_o}{c} = \frac{1/\sqrt{\mu_o \varepsilon_o}}{1/\sqrt{\mu_r \mu_o \varepsilon_r \varepsilon_o}}$$

- For photon, wave particle impinge on the atom and electron inside atom vibrates and vibration amp and speed depending on epsilon and mu before electron transfer the vibration back to photon again.



Mechanism of Energy Transport

MICRO TECH LAB



Need a damper
Vibration amplitude will decease

The mechanism of energy transport through a medium **involves the absorption and reemission of the wave energy by the atoms of the material**. When an electromagnetic wave impinges upon the atoms of a material, the energy of that wave is absorbed. The absorption of energy causes the electrons within the atoms to undergo vibrations. After a short period of vibrational motion, the vibrating electrons create a new electromagnetic wave with the same frequency as the first electromagnetic wave. While these vibrations occur for only a very short time, they delay the motion of the wave through the medium. Once the energy of the electromagnetic wave is reemitted by an atom, it travels through a small region of space between atoms. Once it reaches the next atom, the electromagnetic wave is absorbed, transformed into electron vibrations and then reemitted as an electromagnetic wave. (~C/n)

The speed of light in a medium is related to the electric and magnetic properties of the medium, and the speed of light in vacuum can be expressed as

$$c_o = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

ε_0 = electric permittivity

μ_0 = magnetic permeability

The speed of light in a material to the material "constants" permittivity ε_o of vacuum and relative permittivity ε_r , and the corresponding magnetic permeability μ_0 of vacuum and relative permeability μ_r of the material is

$$c = \frac{1}{\sqrt{\mu_r \mu_o \varepsilon_r \varepsilon_o}}$$

The **index of refraction *n*** of a non-magnetic material $\mu_r \sim= 1$ is linked to the dielectric constant $\boldsymbol{\varepsilon_r}$ via a simple relation, which is a rather direct result of the Maxwell equations.

$$\frac{c_o}{c} = \frac{1/\sqrt{\mu_o \varepsilon_o}}{1/\sqrt{\mu_r \mu_o \varepsilon_r \varepsilon_o}} = \sqrt{\varepsilon_r} = n$$

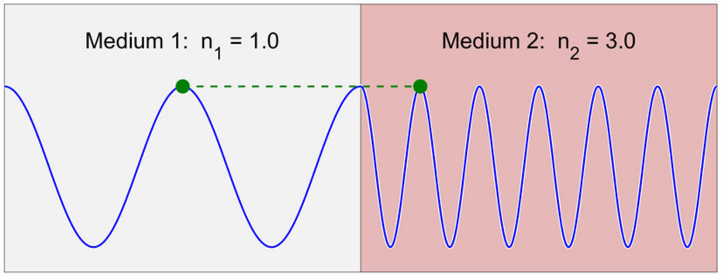
Plug back into **dispersion relation**,

$$\varepsilon = \varepsilon' - j\sigma/\omega$$

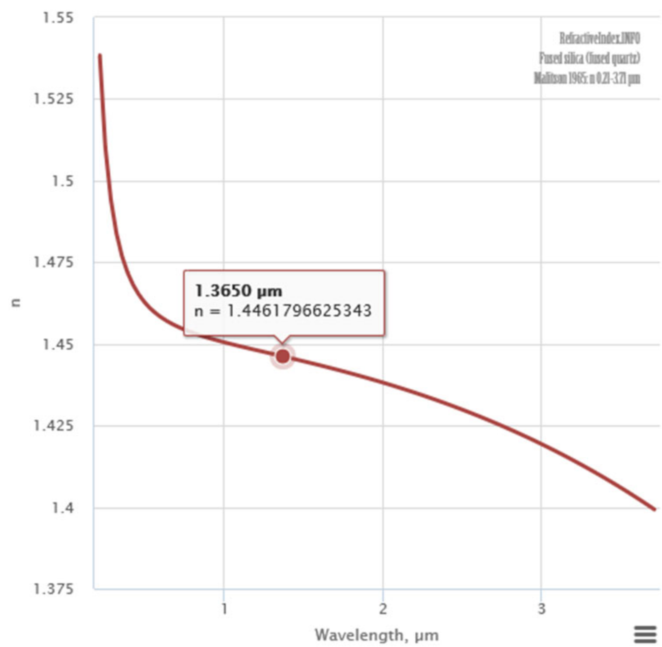
$$\frac{c_o}{c} = \frac{\lambda_i f_i}{\lambda_r f_r} = n$$

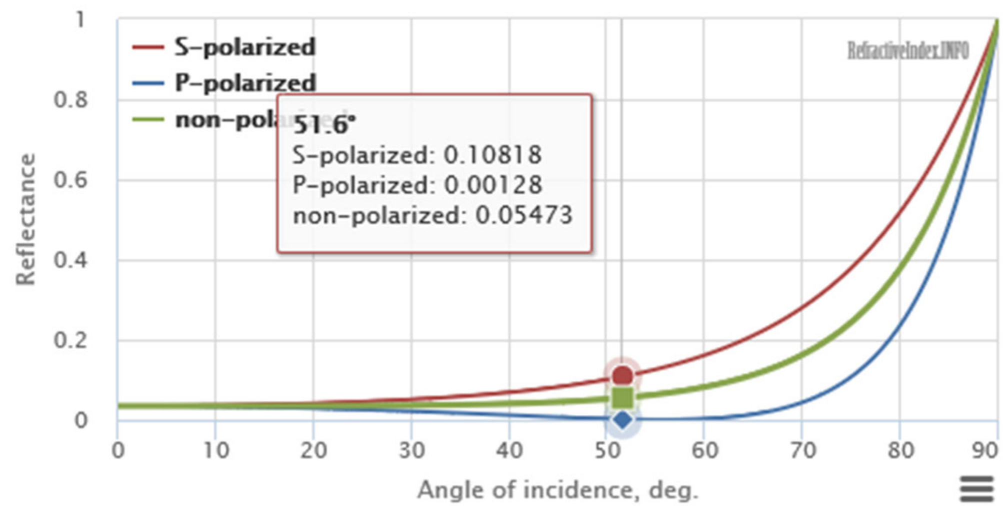
Since $f_i = f_r$,

$$n = \frac{\lambda_i}{\lambda_r}$$



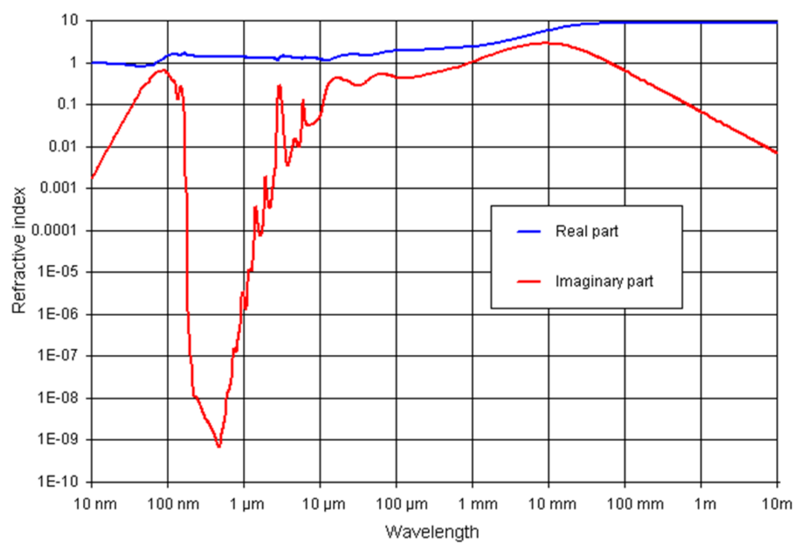
Optical constants of Fused silica (fused quartz)



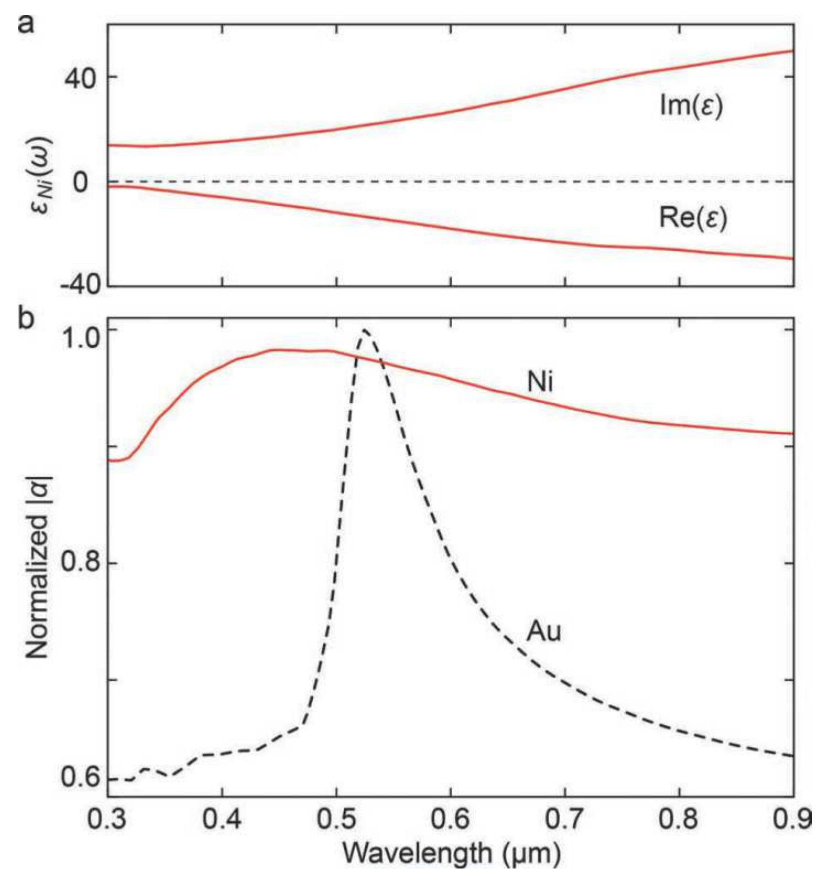


http://refractiveindex.info/?shelf=organic&book=polycarbonate&page=Sultanova

Dielectric Constant as a function of wavelengths



water



$$\varepsilon = \varepsilon' - j\sigma/\omega$$

Plane Wave in Dissipative Medium

So far we have omitted one important class of media- namely conductors. A conductor is characterized by a conductivity σ and is governed by ohm's law. For isotropic conductors, ohm's law states that $J_c = \sigma E$, as we recall J_c denotes the conduction current. For Ampere's law,

$$\nabla \times H = +J_o + j\omega D$$

Where J = J_c (conduction current) +J_o (source current). It is instructive to see that in a conducting medium, Ampere' law becomes:

$$\nabla \times H = J_o + j\omega(\underline{\varepsilon} - j\frac{\sigma}{\omega})E$$

Displacement current conductive current

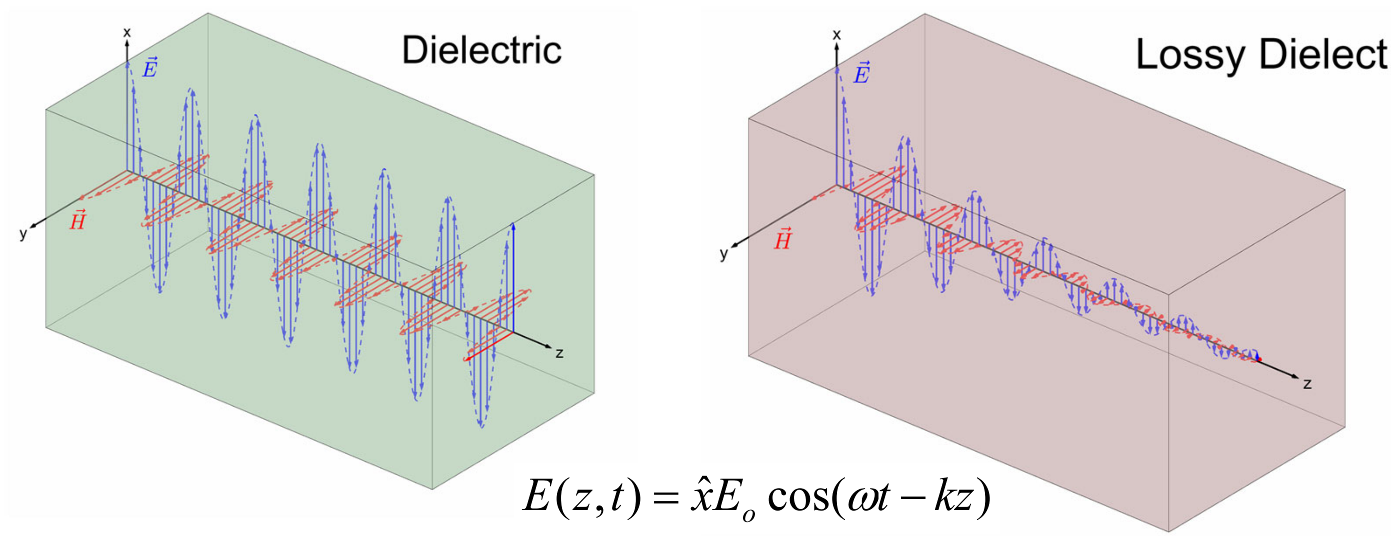
Thus, ε becomes a complex permittivity:

$$\boxed{\varepsilon = \varepsilon' - j\sigma/\omega}$$

For conducing media, the propagation constant k = 2πn/λ, where $n = \sqrt{\varepsilon_r \varepsilon_o \mu_r \mu_o}$,

$$\boxed{k = k_{real} - jk_{imaginary} = k - ja = \omega\sqrt{\mu\varepsilon}(1\text{-}j\,\frac{\sigma}{\omega\varepsilon})^{1/2}}$$

σ/ωε is called the loss tangent of the conducing media.



$$E(z,t) = \hat{x}E_o\cos(\omega t - kz)$$

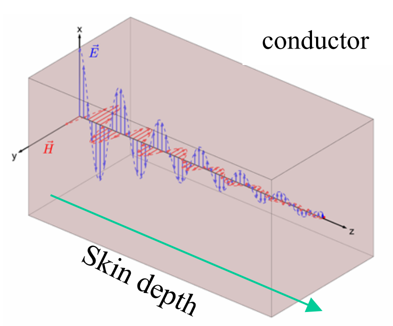
Highly Conducting Media

For highly conducting medium, $\sigma/\omega\varepsilon >> 1$, the k constant can be simplify to

$$k = k - ja = \omega\sqrt{\mu\varepsilon}\left(1-j\frac{\sigma}{\omega\varepsilon}\right)^{1/2}$$

$$k \sim \omega\sqrt{\mu\varepsilon}\left(-j\frac{\sigma}{\omega\varepsilon}\right)^{1/2} = \sqrt{\omega\mu\left(\frac{\sigma}{2}\right)}(1-j)$$

The penetration depth $\delta_p = 1/\alpha = \sqrt{\left(\frac{2}{\omega\mu\sigma}\right)} = \delta$ (skin depth) only for highly conductive media.



$$E = \hat{x}E_o e^{-i(\omega t - k_z z)}$$

For Slightly Conducting Media

For slightly conducting media, where $\sigma/\omega\varepsilon \ll 1$, the constant k can be approximated by

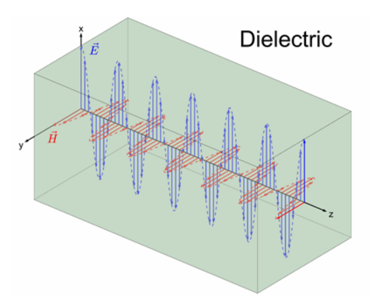
$$k = k - ja = \omega\sqrt{\mu\varepsilon}(1\text{-}j\,\frac{\sigma}{\omega\varepsilon})^{1/2} \approx \omega\sqrt{\mu\varepsilon}(1\text{-}j\,\frac{\sigma}{2\omega\varepsilon})$$

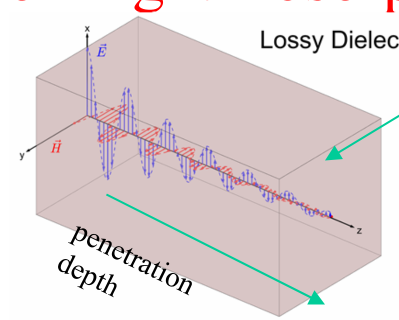
gone

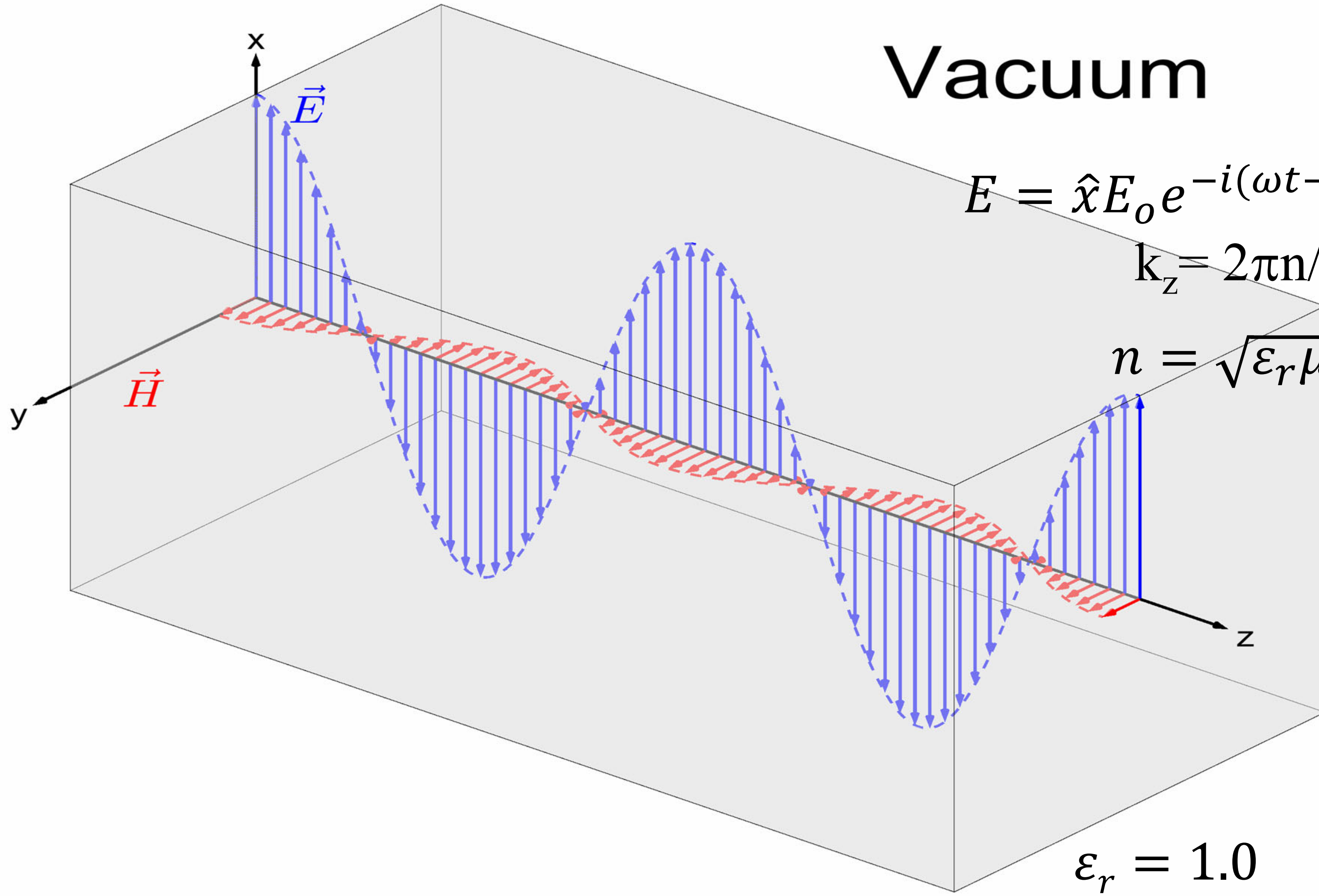
Penetration depth $\delta_p = 1/\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$ (**<u>here we don't have skin depth, skin depth only refers to metal</u>**)

Account for Light Absorption loss!!!

$$E = \hat{x}E_o e^{-i(\omega t - k_z z)}$$







Vacuum

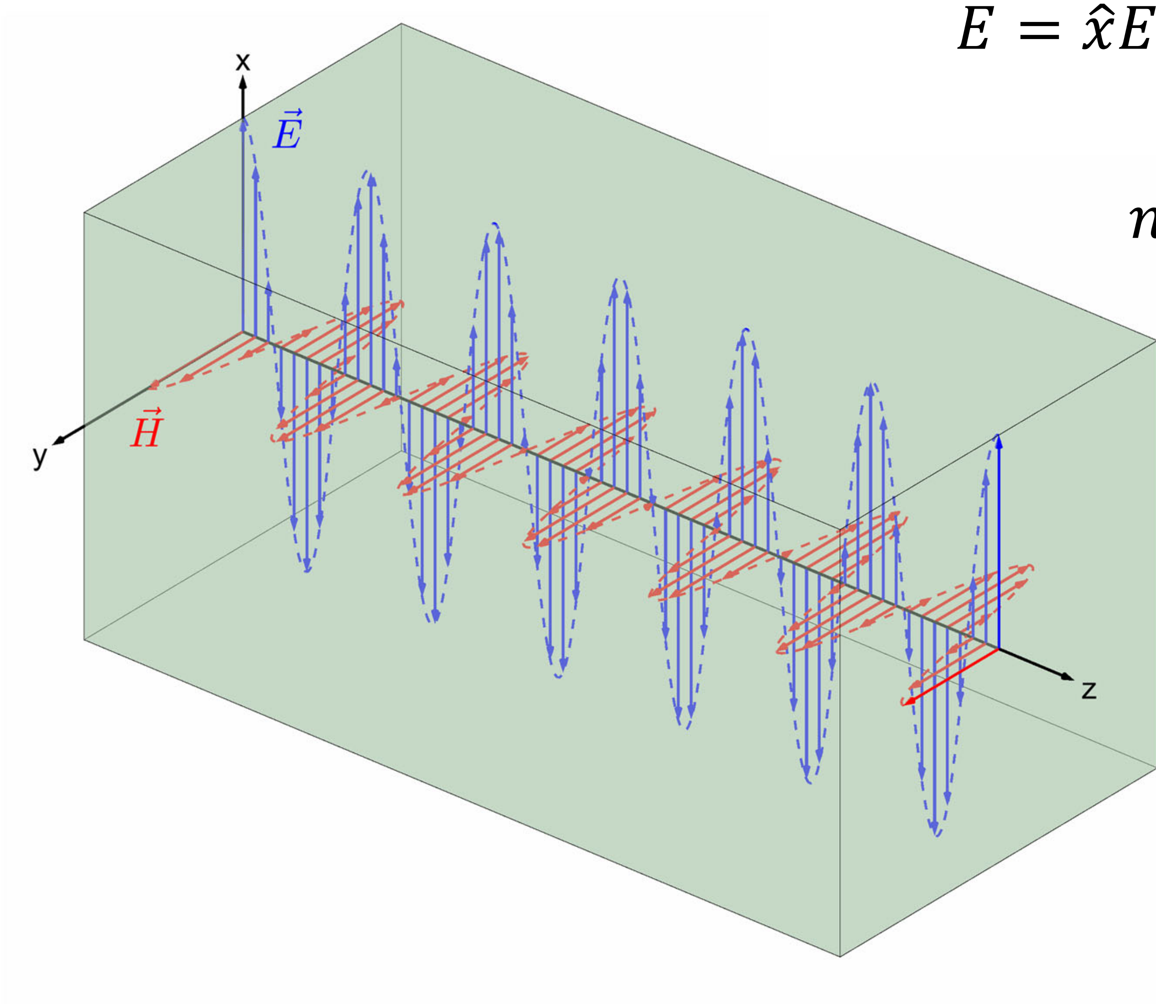
$$E = \hat{x}E_o e^{-i(\omega t - k_z z)}$$

$$k_z = 2\pi n/\lambda$$

$$n = \sqrt{\varepsilon_r \mu_r}$$

$$\varepsilon_r = 1.0$$

Wave in Lossless Dielectric



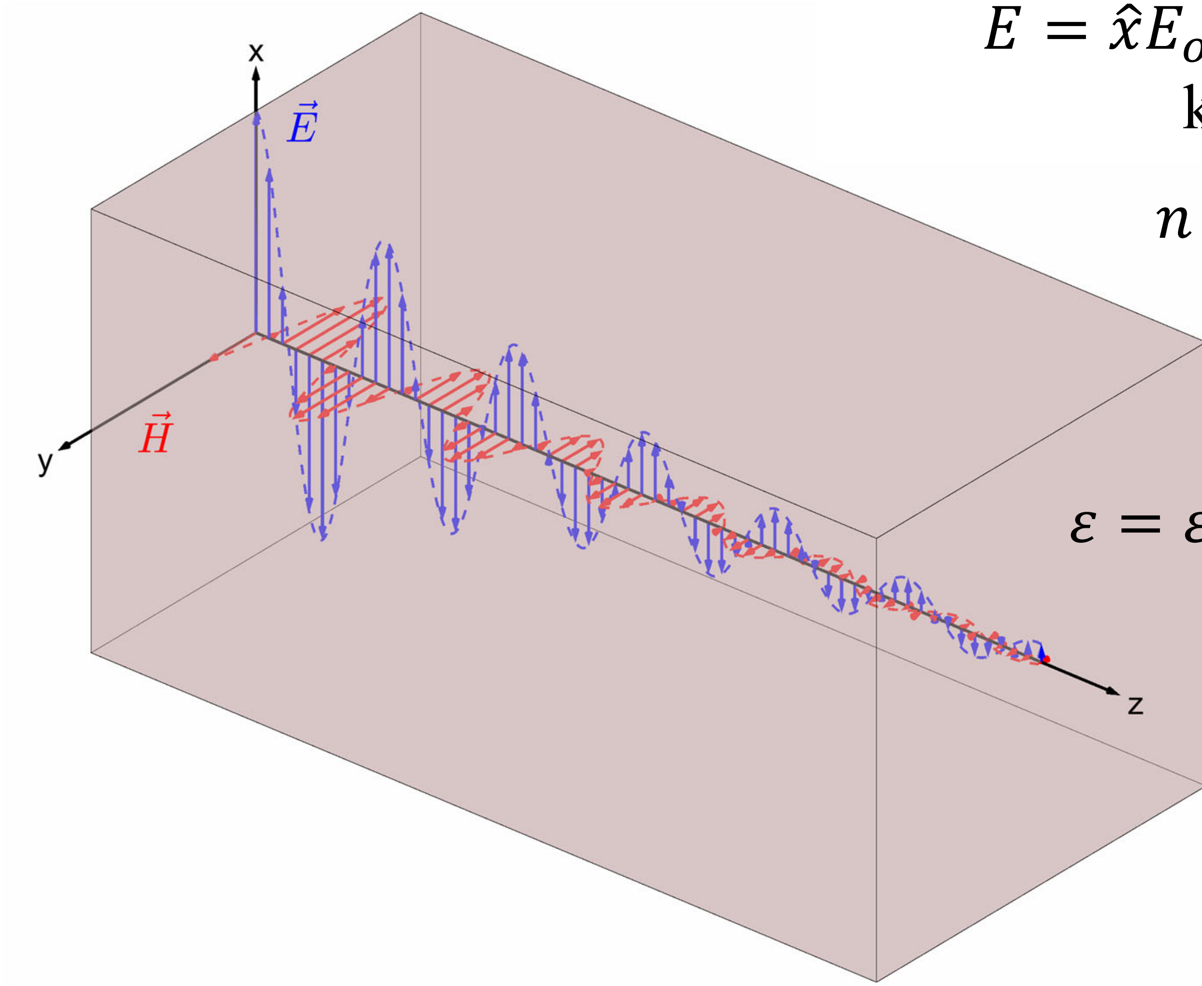
$$E = \hat{x} E_o e^{-i(\omega t - k_z z)}$$

$$k_z = 2\pi n/\lambda$$

$$n = \sqrt{\varepsilon_r \mu_r}$$

$$\varepsilon_r = \varepsilon'$$

Wave in Lossy Dielectric Material



$$E = \hat{x} E_o e^{-i(\omega t - k_z z)}$$

$k_z = 2\pi n/\lambda$

$$n = \sqrt{\varepsilon_r \mu_r}$$

$$\varepsilon = \varepsilon' - j\sigma/\omega$$

Things to know

- EM wave is $E(z,t) = \hat{x}E_o \cos(\omega t - kz)$
- A complex vector: function of space and time

- Time harmonic function: can be simplify to phasor form
- Space: can be represented by spatial vector

Complex Vectors

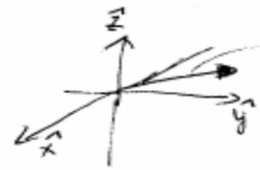
- Read Chapter 1 in Applied Electromagnetism by Kong in: http://courses.washington.edu/me557/reading/chp1-EM.pdf
- Review complex vectors (please read the hand written handout in http://courses.washington.edu/me557/reading/summary_maxwell.pdf

CR 1 Complex vectors

To explain wave, we must first talk about few definition: vector, time harmonic function, phasor and complex vector

1. Physical quantities are usually described mathematically by real variables of space and time and frequently by vector quantities.

i.e. velocity of wind



$$\vec{V} = V_x \hat{x} + V_y \hat{y} + V_z \hat{z}$$

2. physical quantities that vary periodically with time are called time-harmonic

i.e. Household electricity varies at 60 Hz

KUBE FM radio at 93.3 MHz (radio frequency)

$\Longrightarrow$ In mathematical manipulations, the time-harmonic real quantities are conviently represented by complex variables.

$$V_o e^{j\phi} e^{j\omega t} = V_o e^{j(\omega t+\phi)}$$

$$= V_o \cos(\omega t+\phi) + j V_o \sin(\omega t+\phi)$$

$$E(z,t) = \hat{x} E_o \cos(\omega t - kz)$$

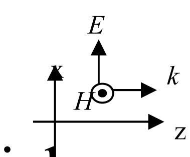
(Real part)

Mathematically, it is said when taking a function from R^2 to R so taking two real variables and give a real number out, it's said to be harmonic if it satisfies the following differential equation (2D Laplace equation example shown here) but also satisfies these second order partial derivatives here are continuous.

$$u: R^2 \rightarrow R \quad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

We will find out soon from wave equation, the wave function is a harmonic function and derived from second order differential equation.

$$\frac{\partial^2 E_x}{\partial z^2} + \omega^2 \mu_o \varepsilon_o E_x = 0 \quad \Rightarrow \quad E(z,t) = \mathrm{Re}\{E e^{j\omega t}\} = \hat{x} E_o \cos(\omega t - kz)$$



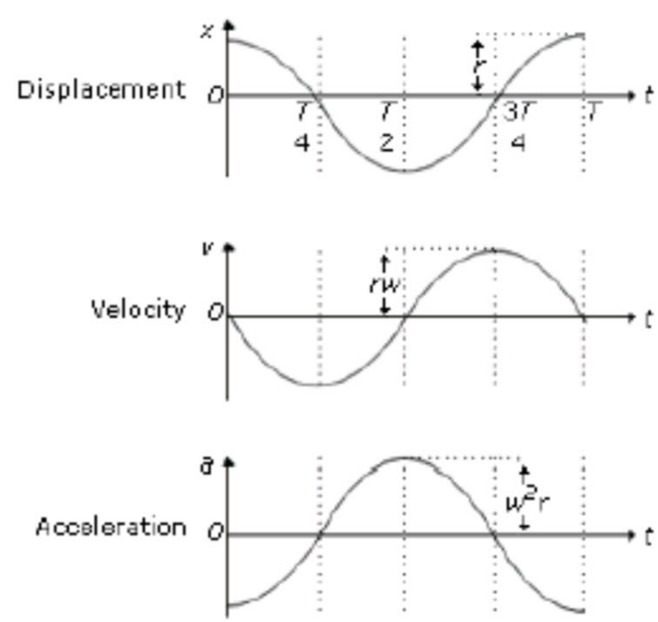
Harmonic function has a very deep link to analytic functions which are vert important in complex analysis. We got to results here:

1) If f= u+jv is analytic , u is harmonic and v is harmonic as well except we usually concentrate on the real part.

2) If u is harmonic, there is such a v such that u+jv is an analytic function, we can say this v here is a harmonic conjugate of u.

The reason is that a time-harmonic function repeat the same pattern of variation over every cycle thus the function can be represented by the pattern of a single cycle call phasor

Example:



$A\cos(\omega t+\theta)$

$\Rightarrow \ Ae^{-j\theta}$

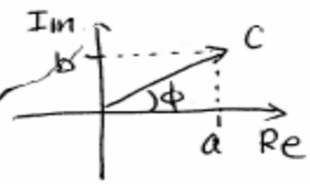
In physics and engineering, a phasor (a portmanteau of phase vector) is a complex number representing a sinusoidal function **whose amplitude (A), angular frequency (ω), and initial phase (θ) are time-invariant.**

So let's review complex algebra, it's important because time harmonic function is a complex variables

complex Algebra

1. complex number is represented by:

$$C = a + jb$$



a = real (magnitude of) part of C

b = imaginary (magnitude of) part of C

j = imaginary number $\Rightarrow \begin{cases} j^2 = -1 \\ j = \sqrt{-1} \end{cases}$

2. Polar or phasor form of a complex number:

Cartesian form of a **complex number**

$$C = a + jb = |C| e^{j\phi}$$

Polar or phasor form of a **complex number**

$$= |C| \cos\phi + j\,|C| \sin\phi$$

3. $|C| = \sqrt{a^2 + b^2}$

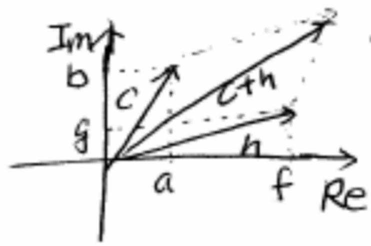
$\phi = \tan^{-1}\left(\frac{b}{a}\right)$

4. addition & subtraction :

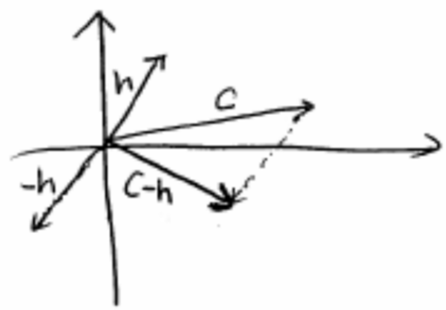
Let $c = a + jb$

$h = f + jg$

$$c + h = (a+f) + j(b+g)$$



$$c - h = (a-f) + j(b-g)$$



5. multiplication & Division

$$ch = (a+jb)(f+jg) = (af-bg) + j(bf+ag)$$

$$(\text{because } (jb)\cdot(jg) = j^2 bg = -1bg)$$
$$= -bg$$

$$\frac{c}{h} = \frac{a+jb}{f+jg} = \frac{(a+jb)(f-jg)}{(f+jg)(f-jg)} = \frac{(af+bg)}{(f^2+g^2)} + j\frac{(bf-ag)}{(f^2+g^2)}$$

(idea is to simplify the above division to the Real + j (imaginary) form)

(reason is that we can figure out exactly ~~the~~ magnitude & ~~direction~~ angle of the phasor when it's in real + j imagi...

6. ~~complex conjugate~~ complex conjugate

$$c = a + jb = |c|e^{j\phi}$$

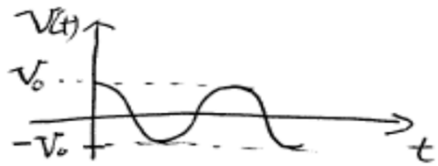
$$c^* = \text{conjugate of } c = a - jb = |c|^{-j\phi}$$

$$cc^* = (a+jb)(a-jb) = a^2 - (j^2)(b^2)$$
$$= a^2 - (-1)(b^2)$$
$$= a^2 + b^2$$

Complex representation of time-harmonic scalars: 6

wave is a scalar time harmonic function derived from wave equation (second order differential equation shown earlier)

1. $\boxed{V(t) = V_0 \cos(\omega t + \phi)}$



V_0 = amplitude

ω = angular frequency $= 2\pi f$

ϕ = phase

2. $V(t)$ can also be written as:

$$V(t) = Re\{V_0 e^{j\phi} e^{j\omega t}\}$$

Complex exponential Solution from the wave equation

$\Rightarrow$ ** $Re\{\ \}$ means taking the real part of the quantity in braces $\{\ \}$

The reason is :

$$\begin{cases} V_0 e^{j\phi} = V_0 \cos\phi + j V_0 \sin\phi \\ e^{j\omega t} = \cos\omega t + j \sin\omega t \end{cases}$$

$\longrightarrow$ since $\boxed{V(t) = V_0 \cos(\omega t + \phi)}$

$\longrightarrow$ and

$$V_0 e^{j\phi} e^{j\omega t} = V_0 e^{j(\omega t + \phi)}$$

$$= V_0 \cos(\omega t + \phi) + j V_0 \sin(\omega t + \phi)$$

(Euler's identity)

Therefore

$$V(t) = \text{Real part of } \{ V_0 e^{j\phi} e^{j\omega t} \}$$

$$= V_0 \cos(\omega t + \phi)$$

since we can only see the real part in real life and mathematically the imaginary part if just a harmonic conjugate of real part

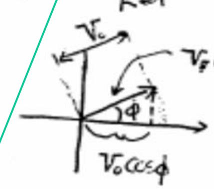
From the complex algebra, we learn we can represent $V_o e^{j\omega t+\phi} = V_o\cos(\omega t+\phi)$ in complex plane:

(a) The locus of $Ve^{j\omega t}$ is a circle of radius V_o, centered at the origin of a complex plane, and generated by a rotating vector with an angular velocity of ω rad/s. The projection of this vector on the real axis generates the sinusoidal function

Real part is $V_o\cos(\omega t+\phi)$

$$V(t) = V_o\cos(\omega t+\phi)$$

ie. Let $V(t) = V_o\cos(\omega t+\phi) = \mathrm{Re}\{Ve^{j\omega t}\}$ where $V = V_o e^{j\phi}$



$t=0$

$t = (\frac{\pi}{2}-\phi)\frac{1}{\omega}$

$V(t) = V_o\cos(\omega t+\phi) = V_o\cos\phi$

$V(t) = V_o\cos(\omega t+\phi) = V_o\cos(\pi/2) = 0$

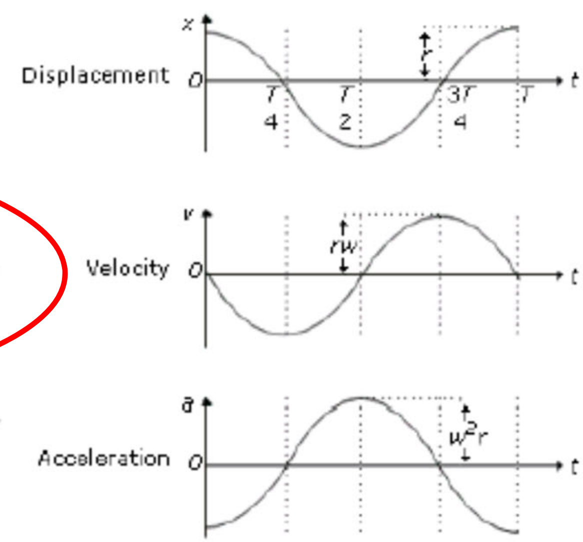
$\phi = \pi/2 - \omega t$

Phasor representation

In physics and engineering, a **phasor**, a portmanteau (twin borther) of **phase vector**, is a complex number representing a sinusoidal function whose amplitude (A), angular frequency (ω), and initial phase (θ) are time-invariant.

③ since $V(t)$ is a function/time & a time-harmonic quantity, a phasor $\boxed{V = V_0 e^{j\phi}}$ can be used to represent this sinusoidally varying function

The reason is that a time-harmonic function repeat the same pattern of variation over every cycle thus the function can be represented by the pattern of a single cycle.



The whole purpose is to look at the phase and magnitude of the sinusoidal function at a fix time

Phasor

Basically a transformation (mainly to simplify or allow one to see something more easily

phasor Identity

(1)

$$\boxed{\begin{aligned} &\vec{E}_1(t) \Longleftrightarrow \tilde{E}_1 \qquad \vec{E}_1(t) = \mathrm{Re}(\tilde{E}_1 e^{j\omega t}) \\ &\qquad\qquad\qquad\qquad \tilde{E}_1 = |E| e^{j\angle E_1} \\ &E_1(t) + E_2(t) \Longleftrightarrow \tilde{E}_1 + \tilde{E}_2 \\ &\frac{\partial}{\partial t} E(t) \Longleftrightarrow j\omega \tilde{E}_1 \end{aligned}}$$

(addition)

(derivative)

The rule of equivalent only applies to Addition, subtraction, time derivative with same frequency

(2)

Not equal

$$\boxed{E_1(t) E_2(t) \not\Longleftrightarrow \tilde{E}_1 \tilde{E}_2}$$

(but not multiplication)

Multiplication example:

$$E(t) = \cos(\omega t + \pi/2) \sin(\omega t)$$

$$= \cos(\omega t + \pi/2) \cos(\omega t - \pi/2)$$

$$= \frac{1}{2}\Big[\cos(\omega t + \pi/2 + \omega t - \pi/2)$$

$$+ \cos(\omega t + \pi/2 - \omega t + \pi/2)\Big]$$

$\cos(A)\cos(B) = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$

$$\boxed{E(t) = \frac{1}{2}[\cos(2\omega t) + \cos(\pi)]}$$

$$E_1(t) \Longleftrightarrow E_1 \cancel{e^{j\omega t}} e^{-j\pi/2}$$

$$E_2(t) \Longrightarrow E_2 \cancel{e^{j\omega t}} e^{+j\pi/2}$$

$$\Longrightarrow \boxed{E_1 e^{-j\pi/2} E_2 e^{j\pi/2} = E_1 E_2 e^{0} = E_1E_2 \quad E_1E_2\cos(0) \quad \neq E_1(t) E_2(t)}$$

(multiplication not commutable)

Addition Example:

(2)

$$\cos(\omega t + \pi/2) + 2\sin(\omega t + 3\pi/2)$$

0 0

$$= \cos\omega t \cos \pi/2 - \sin \omega t \sin \pi/2 + 2\sin\omega t \cos 3\pi/2 + 2\cos\omega t \sin 3\pi/2$$

$$= -sin\omega t + 2cos\omega t$$

use phasor rotation:

$$e^{j\pi/2} + 2e^{j3\pi/2 - \pi/2}$$

$$= j1 + 0 + 2$$

$$= j1 - 2$$

$$= -sin\omega t + 2cos\omega t$$ So it's ok

$$= \sqrt{5}\, e^{j \tan^{-1}\frac{1}{-2}} = \sqrt{5}\, e^{j\phi} \quad \phi = 153.43^\circ$$

4/2/1987

$$= \sqrt{5} \cos(\omega t + \phi)$$

calculator only

1st & 4th

quadrant ~~...~~

(4) Differentiation of a time-harmonic function with respect to time:

(a) $$\frac{\partial}{\partial t} V(t) = \mathrm{Re}\left\{ \frac{\partial (V_0 e^{j\phi} e^{j\omega})}{\partial t} \right\}$$

$$= \mathrm{Re}\left\{ j\omega \cdot V_0 e^{j\phi} e^{j\omega t} \right\}$$

$$= \mathrm{Re}\left\{ j\omega \cdot V e^{j\omega t} \right\}$$

$$= \mathrm{Re}\left\{ j\omega \cdot (V_0 \cos(\omega t + \phi) + j V_0 \sin(\omega t + \phi)) \right\}$$

or $\Downarrow$ look at page 7

(b) $$\frac{\partial}{\partial t} V(t) = \frac{\partial}{\partial t} \mathrm{Re}\left\{ V_0 \cos(\omega t + \phi) + j V_0 \sin(\omega t + \phi) \right\}$$

$$= \frac{\partial}{\partial t} \left(V_0 \cos(\omega t + \phi) \right)$$

$$= -\omega V_0 \sin(\omega t + \phi)$$

$$\boxed{\frac{\partial}{\partial t} V(t) \longleftrightarrow j\omega V}$$

$\frac{\partial}{\partial t}$ in complex representation of time harmonic quantities

complex vector space

② phasor is a vector quantity:

$$\vec{E} = \hat{x} E_x \cos(\omega t + \phi_x) + \hat{y} E_y \cos(\omega t + \phi_y) + \hat{z} E_z \cos(\omega t + \phi_z)$$

↑ real vector

$$\Leftrightarrow \tilde{E} = \hat{x} E_x e^{j\phi_x} + \hat{y} E_y e^{j\phi_y} + \hat{z} E_z e^{j\phi_z}$$

↑ complex vector (combine complex number with real vector)

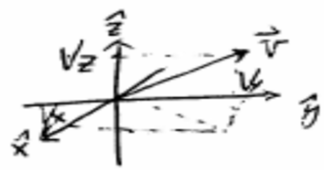
A **complex vector space** is one in which **the scalars** are complex numbers.

Real vectors

So far we have discuss a scalar time-harmonic function, which does not have spatial magnitude & direction.

Now consider an velocity in spatial coordinate is represented by

$$\vec{v} = V_x\hat{x} + V_y\hat{y} + V_z\hat{z}$$



① addition & subtraction

$$\vec{V} = V_x\hat{x} + V_y\hat{y} + V_z\hat{z}$$

$$\vec{U} = U_x\hat{x} + U_y\hat{y} + U_z\hat{z}$$

$$\vec{V} + \vec{U} = (V_x + U_x)\hat{x} + (V_y + U_y)\hat{y} + (V_z + U_z)\hat{z}$$

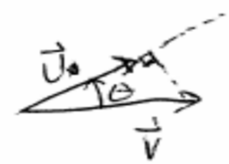
$$\vec{V} - \vec{U} = (V_x - U_x)\hat{x} + (V_y - U_y)\hat{y} + (V_z - U_z)\hat{z}$$

(2) multiplication — dot product & cross product

(a)

(dot product) $\vec{V} \cdot \vec{U} = V_x U_x + V_y U_y + V_z U_z$

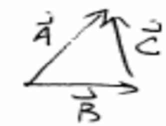
$= |\vec{V}|\,|\vec{U}| \cos\theta$



(commutative) $\vec{U} \cdot \vec{V} = \vec{V} \cdot \vec{U}$

(distributive) $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$

e.g.



$\vec{C} = \vec{A} - \vec{B}$

$C^2 = (\vec{A} - \vec{B}) \cdot (\vec{A} - \vec{B})$

$= A^2 + B^2 - 2\vec{A} \cdot \vec{B}$

$= A^2 + B^2 - 2AB\cos\theta$

(b) (cross product) $\vec{V} \times \vec{U} = (V_y U_z - V_z U_y)\hat{x} + (V_z U_x - V_x U_z)\hat{y}$

$+ (V_x U_y - V_y U_x)\hat{z}$

$$= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ V_x & V_y & V_z \\ U_x & U_y & U_z \end{vmatrix}$$

$$= \hat{x} \begin{vmatrix} V_y & V_z \\ U_y & U_z \end{vmatrix} - \hat{y} \begin{vmatrix} V_x & V_z \\ U_x & U_z \end{vmatrix} + \hat{z} \begin{vmatrix} V_x & V_y \\ U_x & U_y \end{vmatrix}$$

$= \hat{n}\, |V|\,|U| \sin\theta$

where $\hat{n}$ is a vector normal to the plane that contains both $\vec{V}$ & $\vec{U}$.

Complex vector :

Now combine the complex number with the vector

$$\vec{V}(t) = \hat{x}\, V_x \cos(\omega t + \phi_x) + \hat{y}\, V_y \cos(\omega t + \phi_y) + \hat{z}\, V_z \cos(\omega t + \phi_z)$$

$$= \mathrm{Re}\left\{ \left[V_x e^{j\phi_x} \hat{x} + V_y e^{j\phi_y} \hat{y} + V_z e^{j\phi_z} \right] e^{j\omega t} \right\}$$

A **complex vector space** is one in which the scalars are complex numbers.

i.e.

$$\vec{C} = j\,\hat{x} + (2+j)\,\hat{y} + (3+j)\,\hat{z}$$

$$\vec{D} = (2-3j)\,\hat{x} + (j-1)\,\hat{y} + (4-j)\,\hat{z}$$

$$\vec{C} + \vec{D} = \hat{x}\,(2-2j) + \hat{y}\,(1+2j) + \hat{z}\,(7)\,\hat{z}$$

$$\vec{C} - \vec{D} = \hat{x}\,(-2-4j) + \hat{y}\,(3) + \hat{z}\,(7+2j)\,\hat{z}$$

$$\vec{C} \cdot \vec{D} = (2j+3) + (j-2-1) + (12+1+j,$$

$$\vec{C} \times \vec{D} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ j & (2+j) & (3+j) \\ (2-3j) & (j-1) & (4-j) \end{vmatrix} = \hat{x}$$

(not commutative) because $\vec{V} \times \vec{U} = -\vec{U} \times \vec{V}$

(distributive) $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$

(associative) $\vec{A} \times (\vec{B} \times \vec{C}) \neq (\vec{A} \times \vec{B}) \times \vec{C}$

$$= \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$$

examples

(4) dot product applied to ~~Vector~~ complex vector as well

complex values

$$\vec{E}_1 = \hat{x} E_{x1} + \hat{y} E_{y1} + \hat{z} E_{z1}$$

$$\vec{E}_2 = \hat{x} E_{x2} + \cdots$$

$$\boxed{\vec{E}_1 \cdot \vec{E}_2 = E_{x1} E_{x2} + E_{y1} + E_{y2} + E_{z1} E_{z2}}$$

(5) Cross product

$$\vec{E}_1 \times \vec{E}_2 = \begin{vmatrix} x & y & z \\ E_{x1} & E_{y1} & E_{z1} \\ E_{x2} & E_{y2} & E_{z2} \end{vmatrix} = \hat{x} \begin{vmatrix} E_{y1} & E_{z1} \\ E_{y2} & E_{z2} \end{vmatrix} + \hat{y} \begin{vmatrix} E_{x1} & E_{z1} \\ E_{x2} & E_{z2} \end{vmatrix} + \hat{z} | \ldots$$

Exaple $\vec{E}_1 = \hat{x} 1 + \hat{y} 2$

$\vec{E}_2 = \hat{x} j + \hat{y} 1 \Rightarrow \boxed{\vec{E}_1 \cdot \vec{E}_2 = j + 2}$

Watch out!

$\cos(\omega t + \pi/2)$ $\quad \vec{E}_1(t) \cdot \vec{E}_2(t) = ?$

$\vec{E}_1(t) = \hat{x} \cos(\omega t) + \hat{y}\, 2\cos(\omega t)$

$\vec{E}_2(t) = \hat{x}$ ... $+ \hat{y} \cos(\omega t)$

② phasor is a vector quantity:

$$\vec{E} = \hat{x} E_x \cos(\omega t + \phi_x) + \hat{y} E_y \cos(\omega t + \phi_y) + \hat{z} E_z \cos(\omega t + \phi_z)$$

real vector

$$\Leftrightarrow \tilde{\vec{E}} = \hat{x} E_x e^{j\phi_x} + \hat{y} E_y e^{j\phi_y} + \hat{z} E_z e^{j\phi_z}$$

complex vector (combine complex number with real vector)

⑥ phasor only works at one frequency! transformation

So dot product doesn't work as shown from example

regular shape or sinusoidal of periodic function of time

⑦ Time Average

$= 0 \rightarrow$

$E(t) = E_0 \cos(\omega t + \phi)$

$$\begin{pmatrix}\text{time}\\ \text{Avg}\end{pmatrix} \quad \langle E(t) \rangle = \frac{1}{T}\int_0^T E(t)\,dt = \frac{1}{T}\int_0^T E_0\cos(\omega t + \phi)\,dt$$

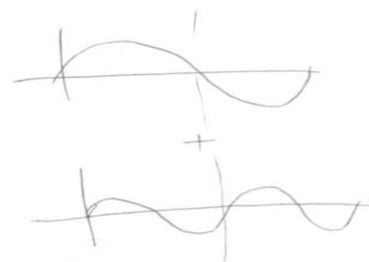
$$= \frac{1}{T}\left[\frac{E_0}{\omega}\sin(\omega t + \phi)\right]\Big|_0^T$$

$$= \frac{1}{T}\big[\underbrace{0}_{\sin(2\pi+\phi)-\sin(\phi)}\big] = 0$$

One frequency Example:

$\cos(\omega t + \pi/2) + \sin(2\omega t + 3\pi/2)$

doesn't work because freq or harmonic is diff!



We can add or substract because they are only off by some phase difference

Things to remember

- EM wave is
- A complex vector: function of <u>space</u> and <u>time</u>
- Time harmonic function: can be simplify to phasor form
- Space: can be represented by spatial vector

Week 6

- Lecture Notes (EM wave theory)
  http://courses.washington.edu/me557/sensors/week2.pdf
- Reading Materials:

Please read all materials in Week 6 in

http://courses.washington.edu/me557/reading/

And also following notes in Week 6:

- hand written lecture notes on Maxwell's Equation
- hand written lecture notes on dereivation of Wave equation
- Homework #1 due today
- No class on Thursday
- Please start working on first problem in HW #2
- **Final presentation Dec. 26 1:20 to 3:10**

Recap last week's lecture

- Light propagation as a wave
- Source made of group of electric or magnetic dipoles
- Complex Vector, time harmonic function, phasor
- Please read the hand written handout in http://courses.washington.edu/me557/reading/summary_maxwell.pdf

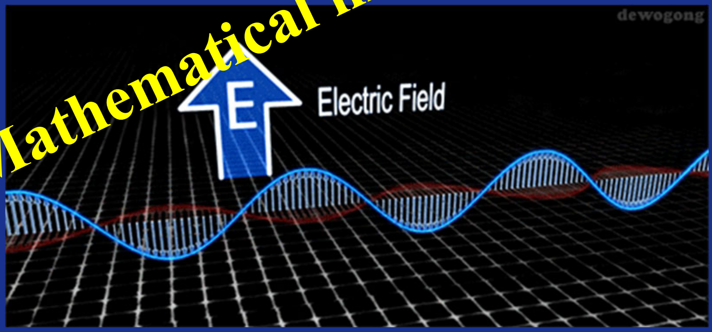
Transverse Wave

- Group velocity (net) is moving forward horizontally, phase velocity (localize particle) oscillates up and down

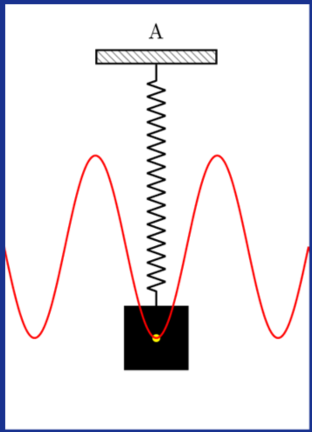
EM wave Explanation

- Energy absorb causes electrons in atom oscillates and regenerate a new EM wave
- Each electron consists of electric and magnetic components. They are perpendicular to the direction of propagation or direction where energy is transferring.
- While these vibrations occur for only a very short time, they delay the motion of the wave through the medium ~ c/n.

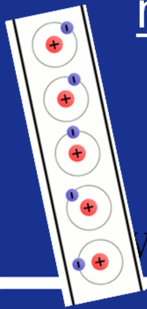
Mathematical model

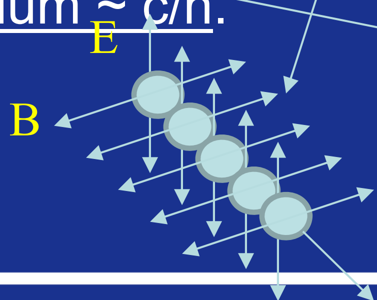


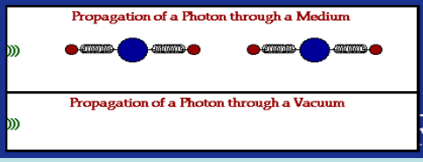
E ad B are perpendicular to propagation (energy transfer) direction



Mechanical System equivalent (Mass spring system (assume no loss)

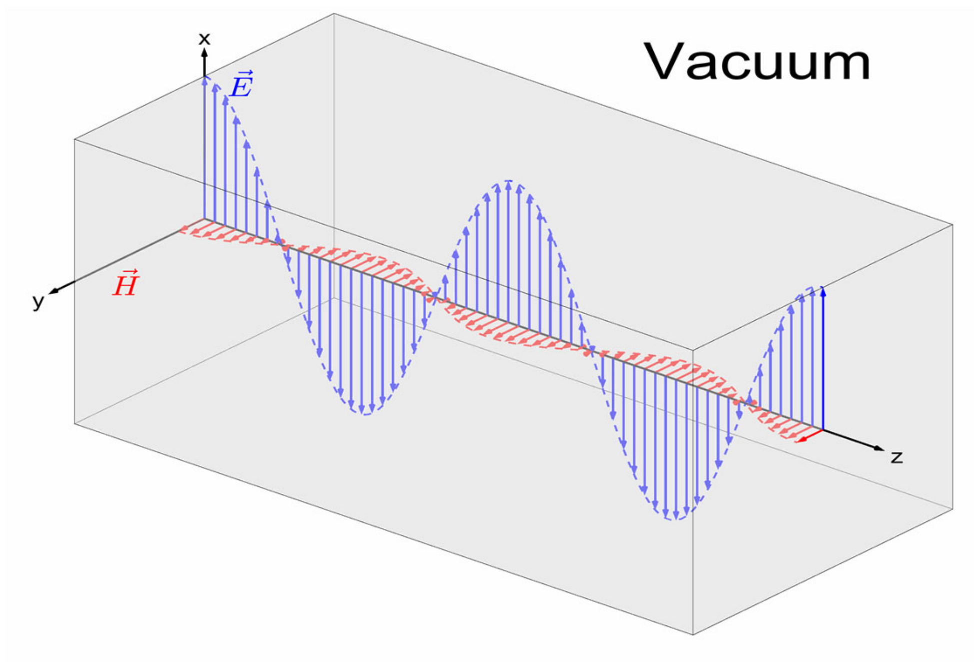






$$\frac{c_o}{c} = \frac{1/\sqrt{\mu_o \varepsilon_o}}{1/\sqrt{\mu_r \mu_o \varepsilon_r \varepsilon_o}} = \sqrt{\varepsilon_r} = n$$

Transverse Wave

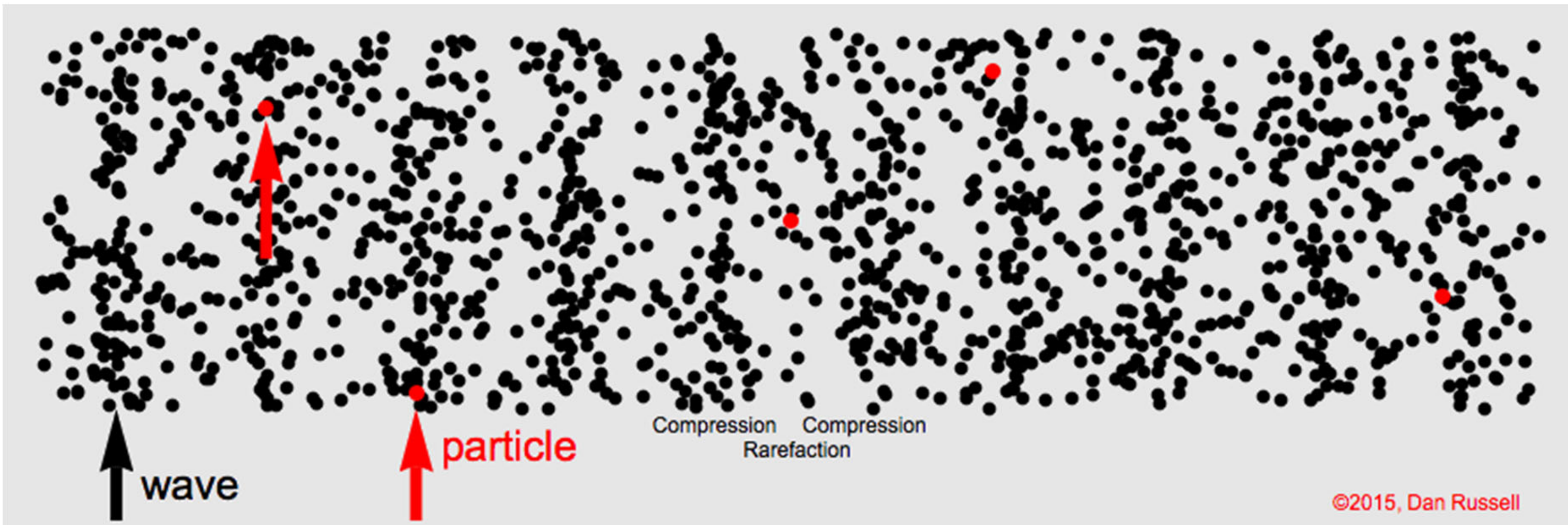


The electric field is always perpendicular to the magnetic field, and both fields are directed at right-angles to the direction of propagation of the wave. In fact, the wave propagates in the direction E×B. Electromagnetic waves are clearly a type of transverse wav

Transverse Wave

- Wave in Stadium first started at UW Husky Stadium back in 1981

Longitudinal Wave



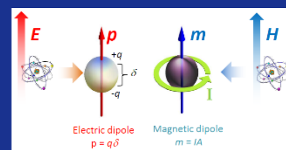
Group velocity (net) is moving forward horizontally, phase velocity (localize particle) oscillates also horizontally

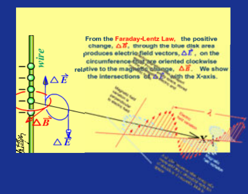
Low and high frequency EM Wave (how wave form)

MICRO TECH LAB

- EM radiation produced by <u>accelerating charge</u> (e.g. RF radiation-real ~ low frequency EM radiation)- dipole EM theory

Maxwell didn't and couldn't explain this because he didn't know quantum theory. Photon and EM wave theory are interchangeable in explaining how light or EM wave are generated and propagating





(but we don't have a charge)

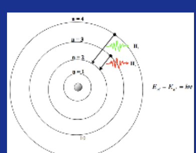
- Photon produced by electrons dropping to lower energy levels (e.g. joule heat-loss through e- with atom or energy release from dying electrons still possible with radiation from higher frequency EM Wave) – quantum theory

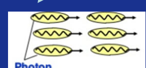
quantum explanation

Wave explanation

$\varepsilon = \varepsilon' - j\sigma/\omega$

EM wave generated basically is photon





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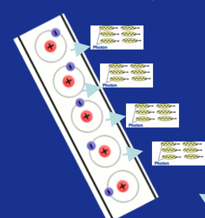

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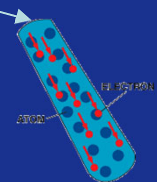

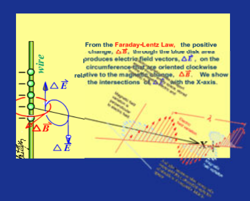

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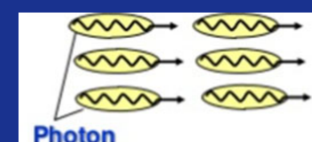


The dropping of energy also involved electron which also has E and H which oscillates like the charge oscillation in the copper wire so the higher frequency radiation still behave light the picture shown here

Pump energy in to create conduction (CREATE CURRENT) and when e' dies, energy releases (PHOTON)

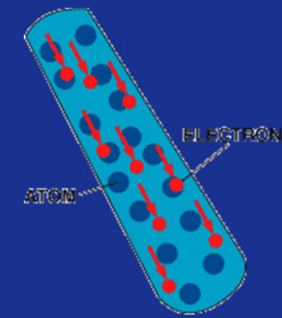
- Produce differently and detect differently but they are fundamentally the same EM wave or photons with different energy levels.

Remind what Joule heat is

Joule Heat

Joule heating is caused by interactions between charge carriers (usually electrons) and the body of the conductor (usually atomic ions).

A voltage difference between two points of a conductor creates an electric field that accelerates charge carriers in the direction of the electric field, giving them kinetic energy. When the charged particles collide with ions in the conductor, **the particles are scattered; their direction of motion becomes random rather than aligned with the electric field, which constitutes thermal motion**. Thus, energy from the electrical field is converted into thermal energy



Interaction of Electromagnetic Radiation with Matter

Electromagnetic radiation interacts with matter in different ways in different parts of the spectrum. The types of interaction can be so different that it seems justified to refer to different types of radiation. At the same time, there is a continuum containing all these *different kinds* of electromagnetic radiation. Thus, we refer to a spectrum, but divide it up based on the different interactions with matter. **Below are the regions of the spectrum and their main interactions with matter:**

- Radio: Collective oscillation of charge carriers in bulk material (plasma oscillation). An example would be the oscillation of the electrons in an antenna.
- Microwave through far infrared: Plasma oscillation, molecular rotation.
- Near infrared: Molecular vibration, plasma oscillation (in metals only).
- Visible: Molecular electron excitation (including pigment molecules found in the human retina), plasma oscillations (in metals only).
- Ultraviolet: Excitation of molecular and atomic valence electrons, including ejection of the electrons (photoelectric effect).
- X-rays: Excitation and ejection of core atomic electrons, Compton scattering (for low atomic numbers).
- Gamma rays: Energetic ejection of core electrons in heavy elements, Compton scattering (for all atomic numbers), excitation of atomic nuclei, including dissociation of nuclei.
- High-energy gamma rays: Creation of particle-antiparticle pairs. At very high energies, a single photon can create a shower of high-energy particles and antiparticles upon interaction with matter.

Perfect Absorber or Emitter (Blackbody Radiation)

What is with black body radiation then?

Blackbody Radiation represents **perfect** absorber or generator that absorb or emit all wavelength of radiation. However, this distribution of thermal radiation among varies with wavelengths and temperatures. **(ideally doesn't existing)**

Function of Frequency and temperature

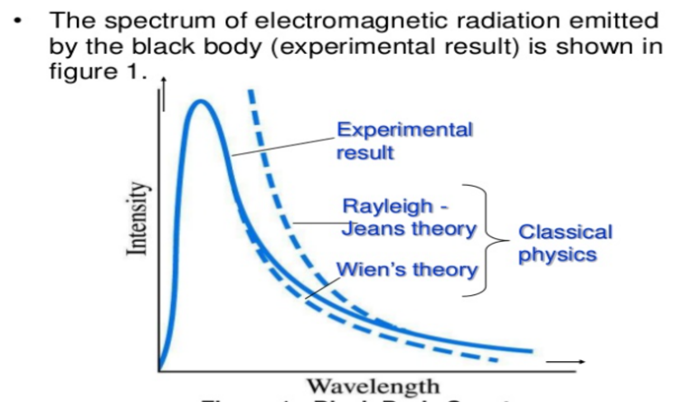


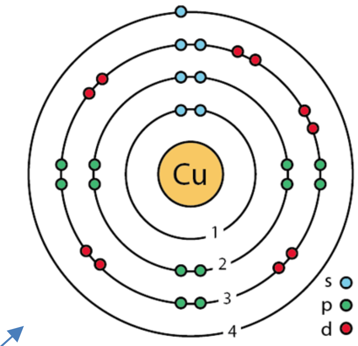
Figure 1 : Black Body Spectrum

"Sunlight heats a material such as water or a brick primarily because the long wavelength, or infrared, portion of the sun's radiation resonates well with molecules in the material, thereby setting them into motion. So the energy transfer that causes the temperature of the substance to rise takes place at the molecular rather than the electronic level."

Blackbody radiation is a theoretical concept in quantum mechanics in which a **material or substance completely absorbs all frequencies of light** or represents a conversion of **a body's internal energy into electromagnetic energy.** Because of the laws of thermodynamics, this ideal body must also re-emit as much light as it absorbs. Although there is no material that can truly be a blackbody, some have come close. **Carbon in its graphite form** is about 96% efficient in its absorption of light.

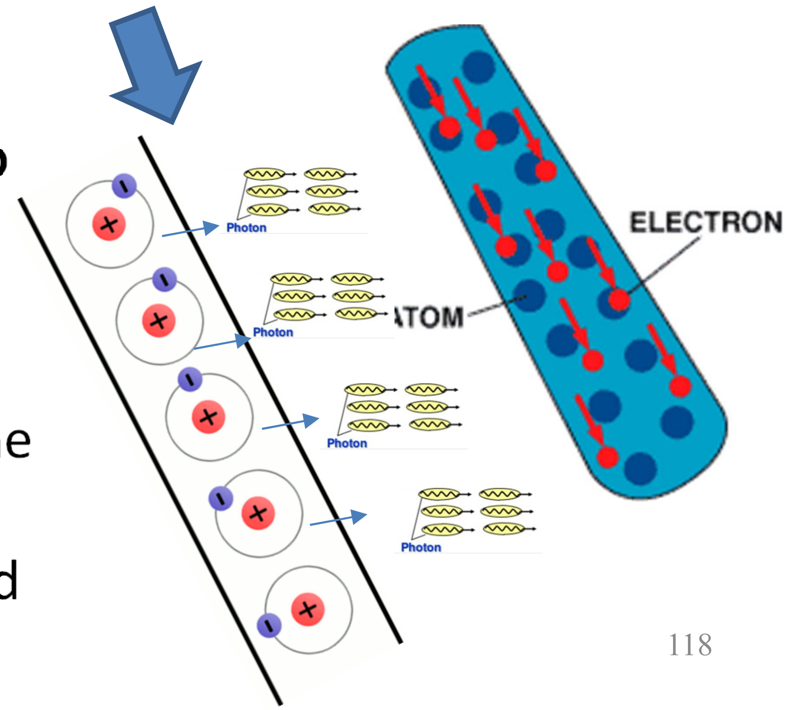
Current

- Electrons can be made to move from one atom to another. **When those electrons move between the atoms, a current of electricity is created.** The electrons move from one atom to another in a "flow." One electron is attached and another electron is lost. It is a situation that's very similar to electricity passing along a wire and a circuit. The **charge is passed from atom to atom** when **electricity is "passed."** When electrons move among the atoms of matter, a current of electricity is created. This is what happens in a piece of wire. The electrons are passed from atom to atom, creating an electrical current from one end to other, just like in the picture.

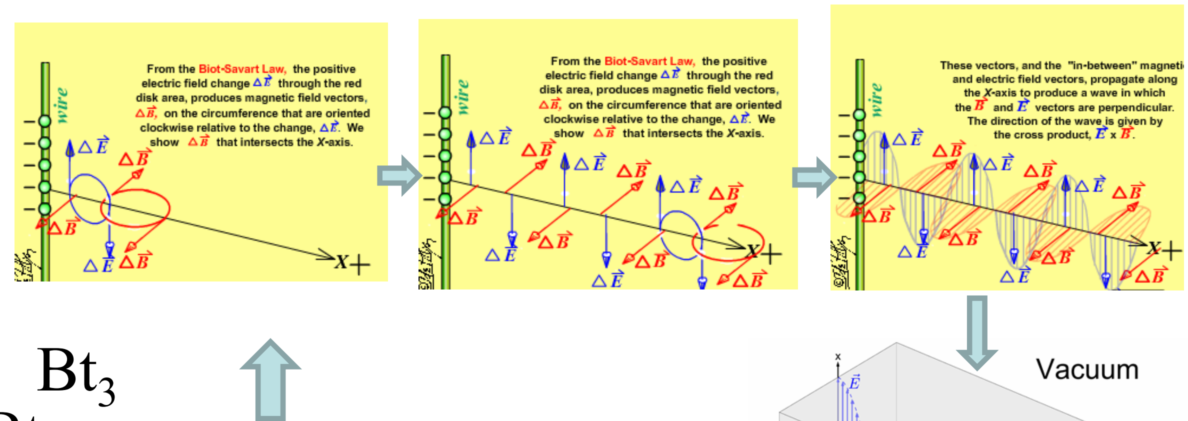


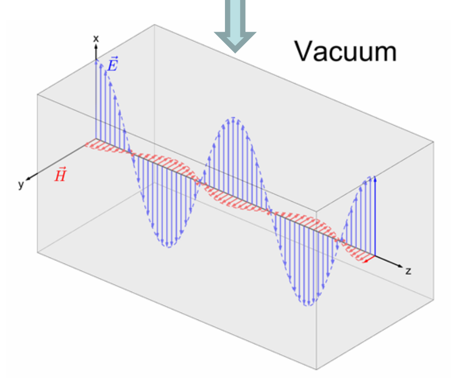
In a copper atom, the outermost 4s energy zone, or conduction band, is only half filled, so many electrons are able to carry electric current.

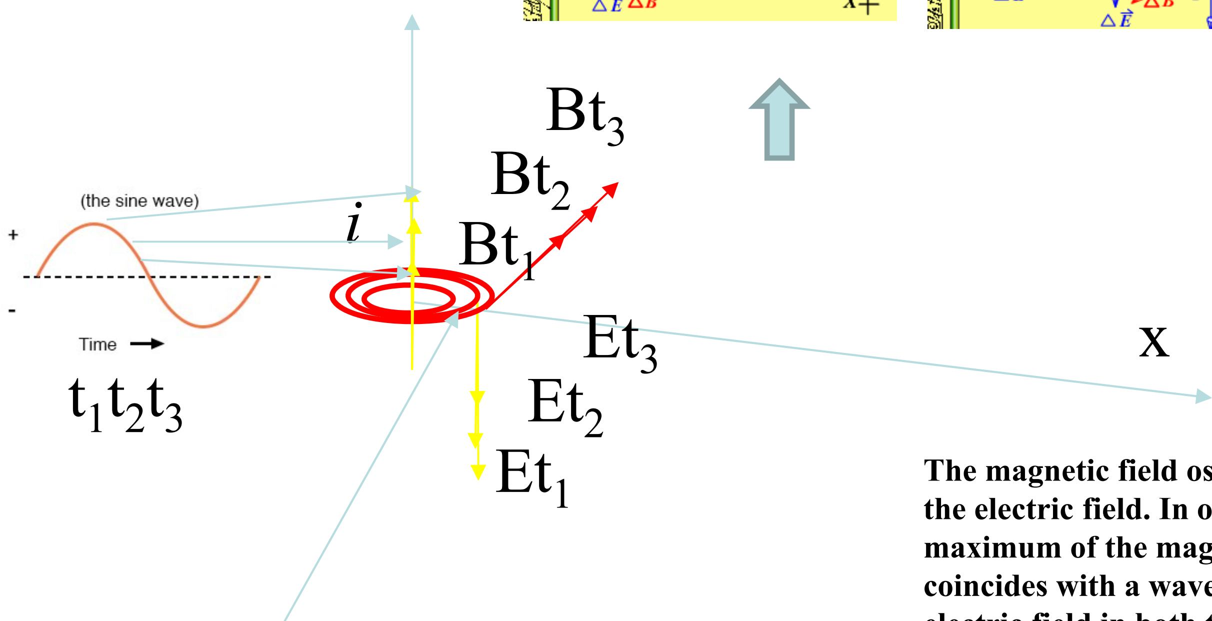
Pump energy in to create conduction (CREATE CURRENT) and when e’ dies, energy releases (PHOTON)



Closer look at what happen







Looking at single point along x axis

The magnetic field oscillates *in phase* with the electric field. In other words, a wave maximum of the magnetic field always coincides with a wave maximum of the electric field in both time and space. The electric field is always perpendicular to the magnetic field, and both fields are directed at right-angles to the direction of propagation of the wave. In fact, the wave propagates in the direction E×B. Electromagnetic waves are clearly a type of *transverse wav*

Ampere's Law

source

Integral form

Free space

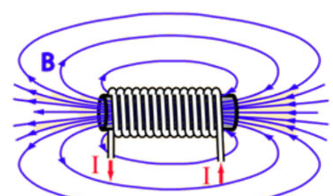
$$\boxed{\oint B \cdot ds = \mu_0 i} + \frac{1}{c^2}\frac{\partial}{\partial t}\int E \cdot dA$$

Differential form

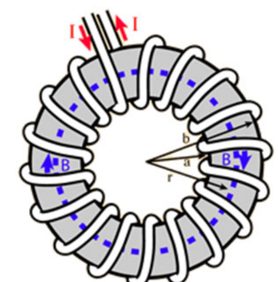
$$\nabla x B = \frac{4\pi k}{c^2} J + \frac{1}{c^2}\frac{\partial E}{\partial t}$$

$$\nabla x B = \frac{J}{\varepsilon_0 c^2} + \frac{1}{c^2}\frac{\partial E}{\partial t}$$

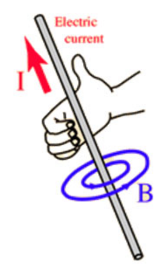
$$k = \frac{1}{4\pi\varepsilon_0} \qquad c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$



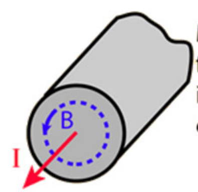
Magnetic field inside a long solenoic.



Magnetic field inside a toroidal coil.



Magnetic field from a long straight wire.



Magnetic field inside a conductor.

relating currents to magnetic effects

Faraday's Law of Induction

Opposing the change like Newton' third law

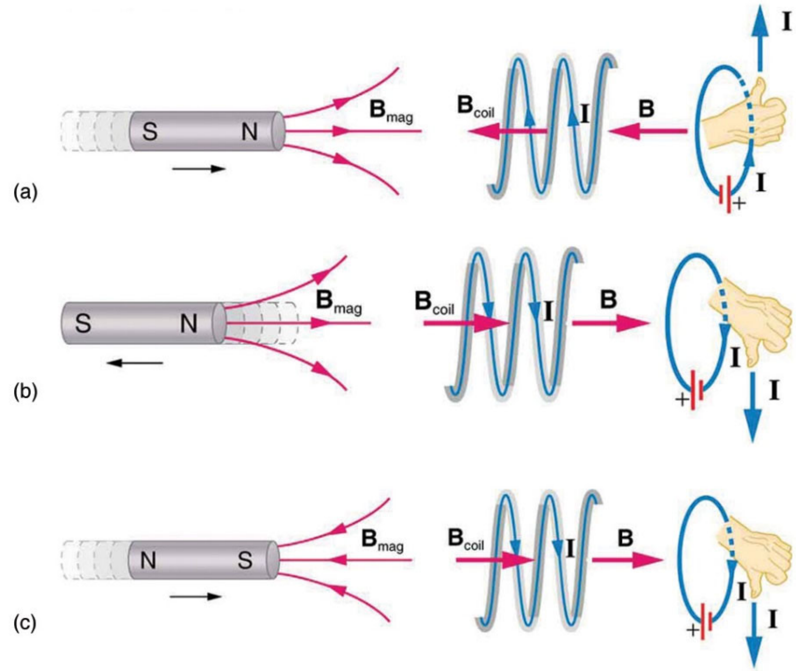
Integral Form

$$\boxed{\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}}$$

$\vec{dl}$

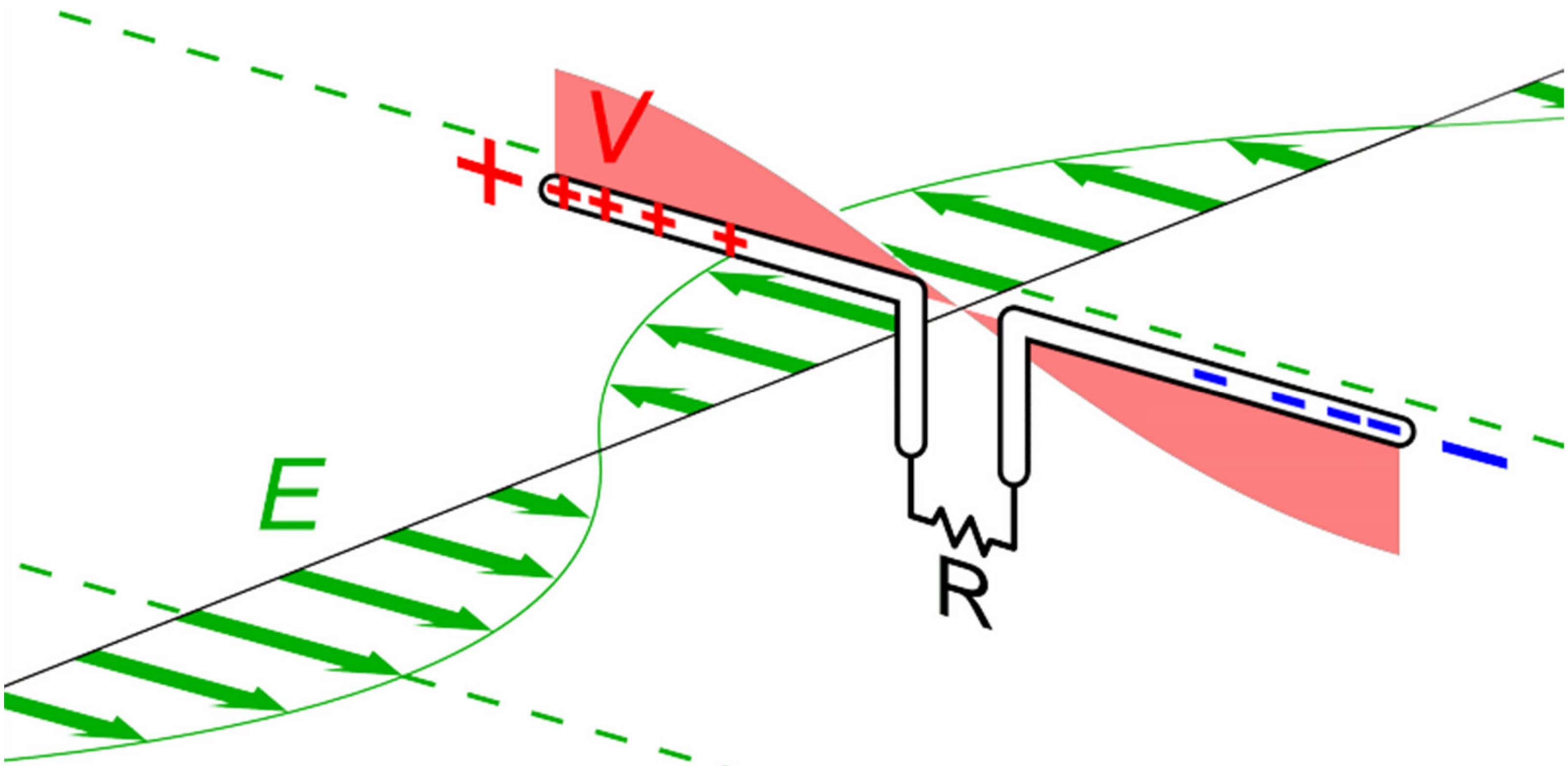
Differential Form

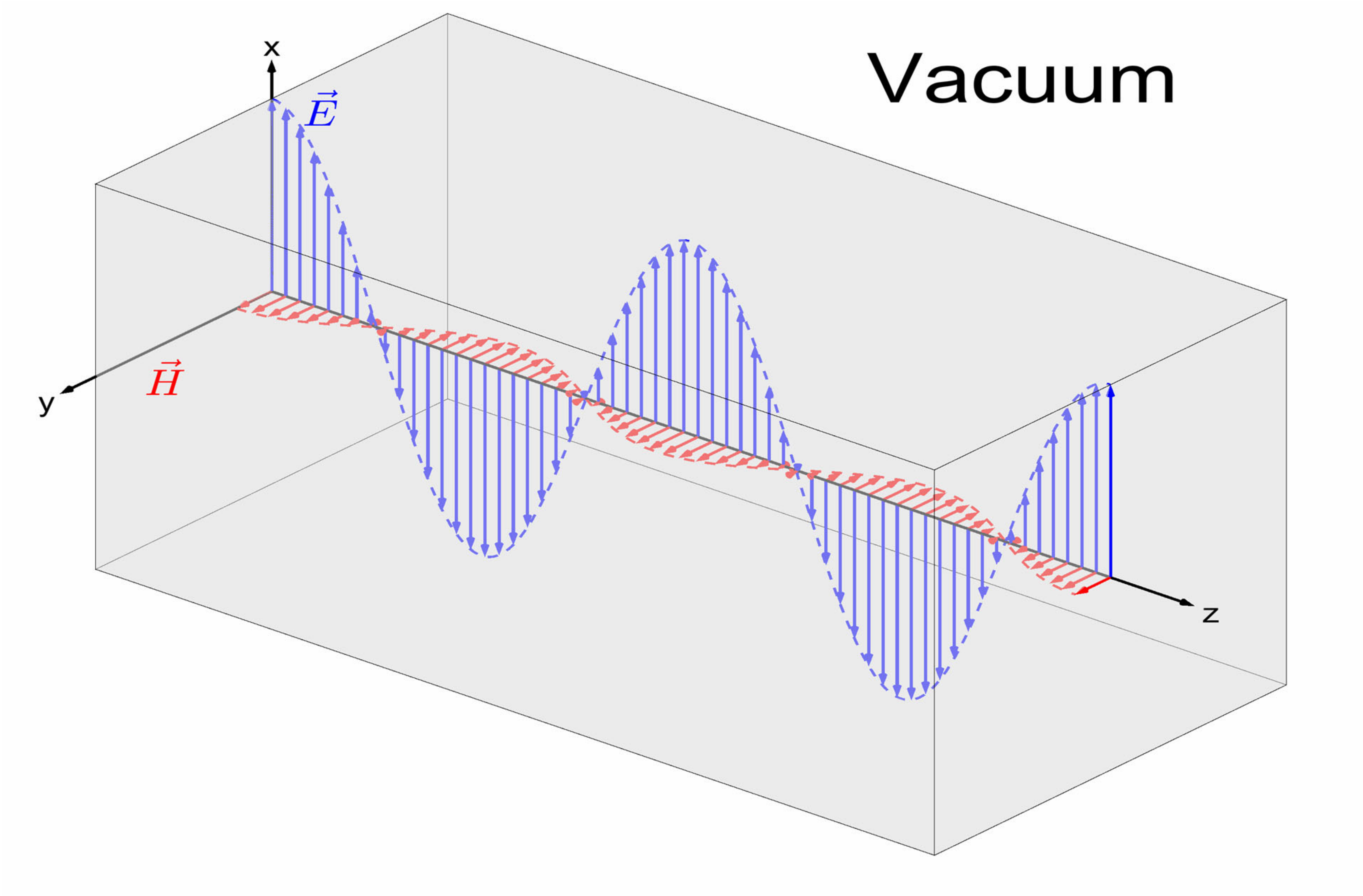
$$\nabla \times E = -\frac{\partial B}{\partial t}$$



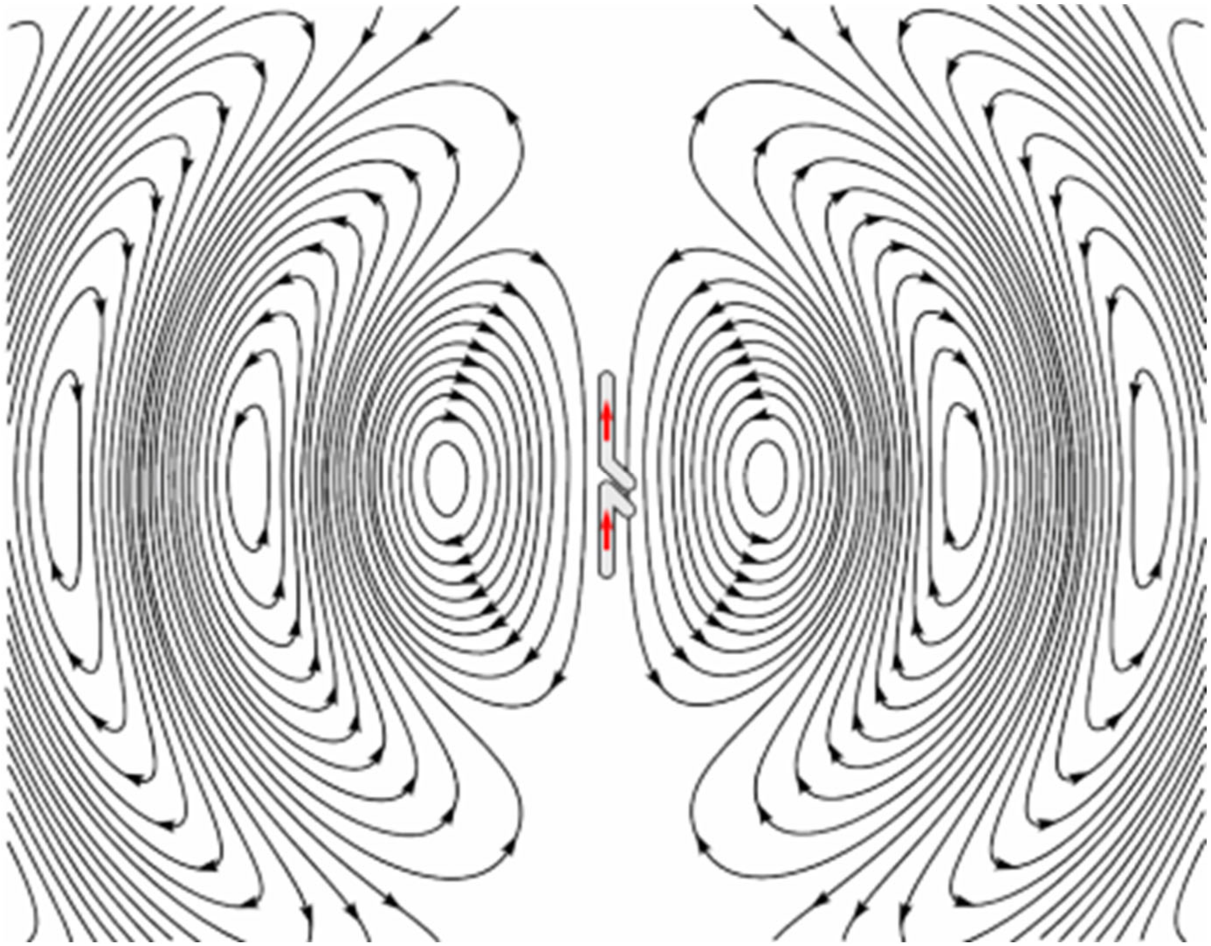
electromagnetic induction

Radio wave from Dipole antenna




Vacuum
x
$\vec{E}$
y
$\vec{H}$
z

Electric Field Radiation from A Dipole antenna

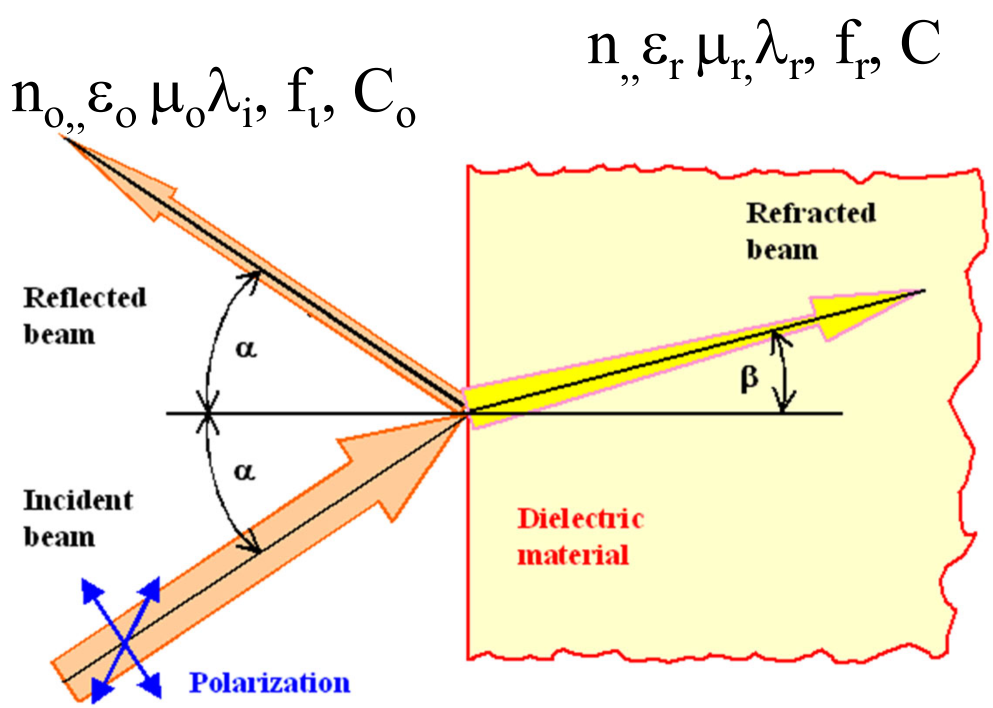


Law of Refraction and reflection (in terms of wave equation)

The *incident beam* is characterized by its wavelength λ_i, its frequency $\boldsymbol{\nu}_i$ and its velocity $\mathbf{c_0}$ **and refracted beam is** characterized by its wavelength λ_r, its frequency $\boldsymbol{\nu}_r$ and its velocity **c** , the simple **dispersion relation** for vacuum.

$$C_o = f_i \ \lambda_i$$

$$C = f_r \ \lambda_r$$



The speed of light in a medium is related to the electric and magnetic properties of the medium, and the speed of light in vacuum can be expressed as

$$c_o = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

ε_0 = electric permittivity

μ_0 = magnetic permeability

The speed of light in a material to the material "constants" permittivity ε_o of vacuum and relative permittivity ε_r , and the corresponding magnetic permeability μ_0 of vacuum and relative permeability μ_r of the material is

$$c = \frac{1}{\sqrt{\mu_r \mu_o \varepsilon_r \varepsilon_o}}$$

The **index of refraction *n*** of a non-magnetic material $\mu_r \sim= 1$ is linked to the dielectric constant $\boldsymbol{\varepsilon_r}$ via a simple relation, which is a rather direct result of the Maxwell equations.

$$\frac{c_o}{c} = \frac{1/\sqrt{\mu_o \varepsilon_o}}{1/\sqrt{\mu_r \mu_o \varepsilon_r \varepsilon_o}} = \sqrt{\varepsilon_r} = n$$

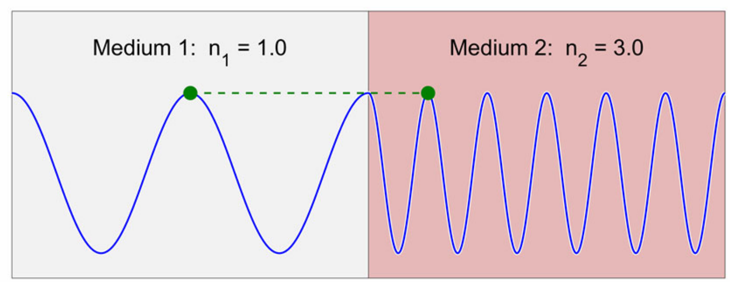
Plug back into **dispersion relation**,

$$\varepsilon = \varepsilon' - j\sigma/\omega$$

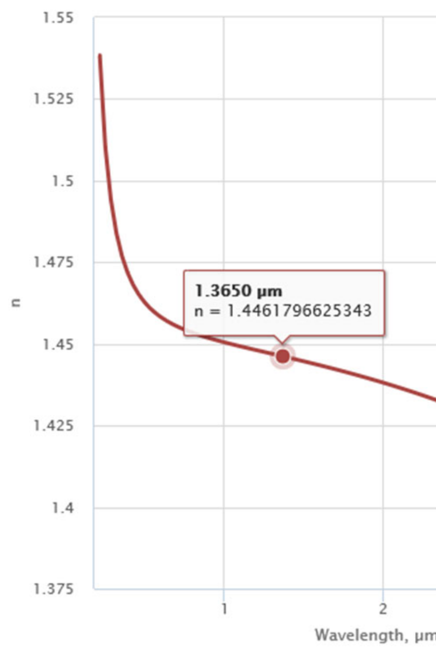
$$\frac{c_o}{c} = \frac{\lambda_i f_i}{\lambda_r f_r} = n$$

Since $f_i = f_r$,

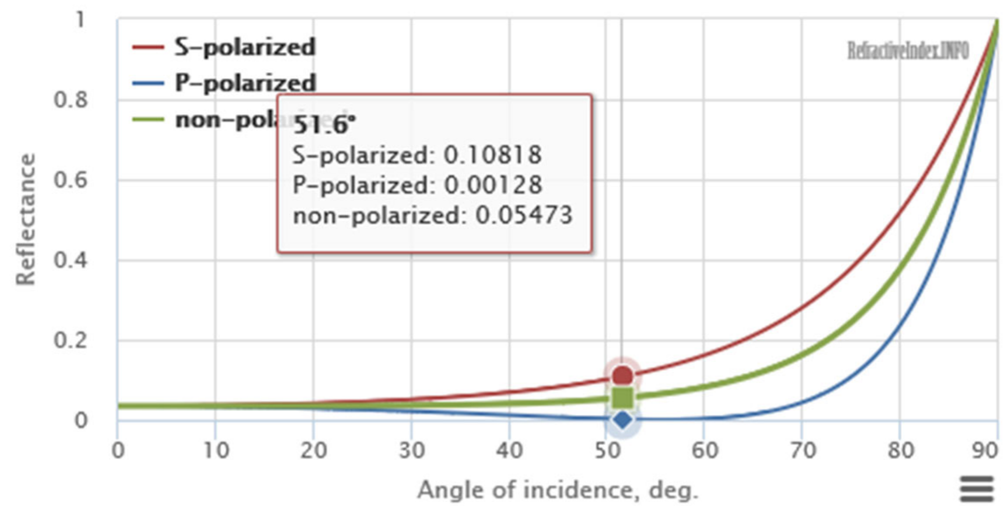
$$n = \frac{\lambda_i}{\lambda_r}$$



Optical constants of Fused silica (fused quartz)

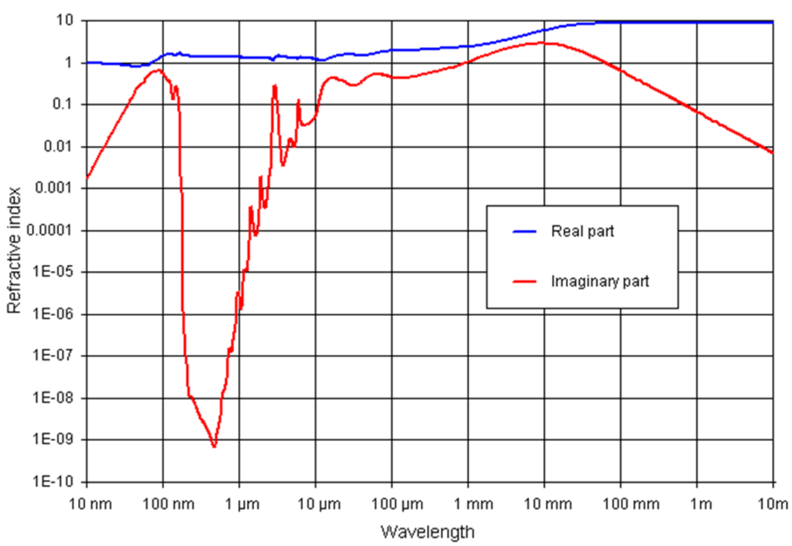


$$n = \sqrt{\varepsilon_r}$$

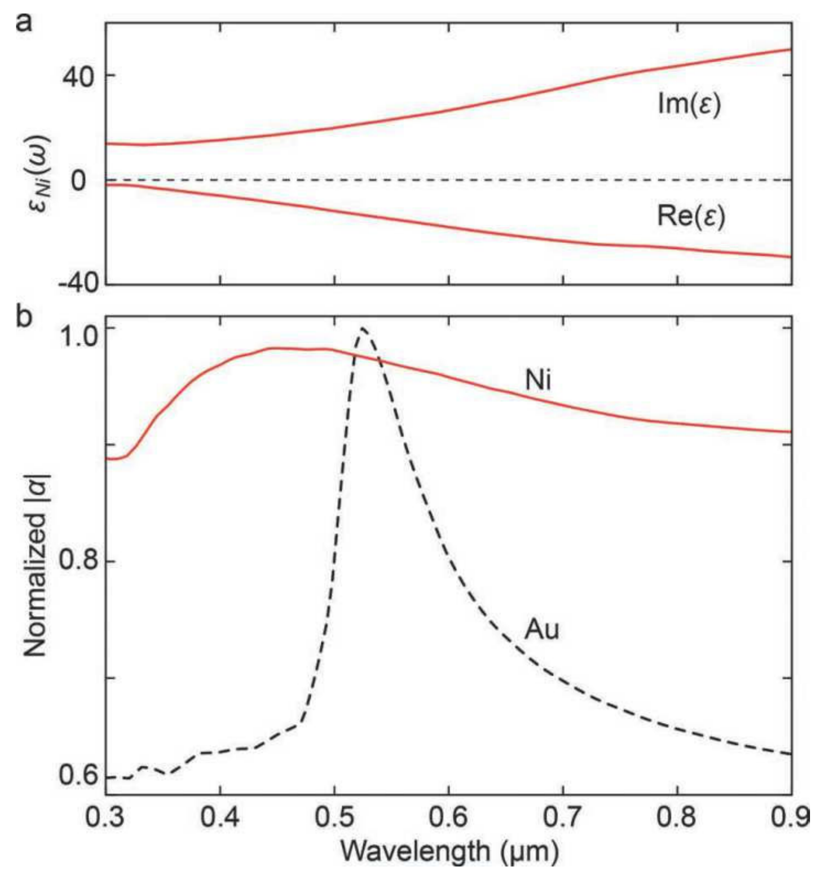


http://refractiveindex.info/?shelf=organic&book=polycarbonate&page=Sultanova

Dielectric Constant as a function of wavelengths

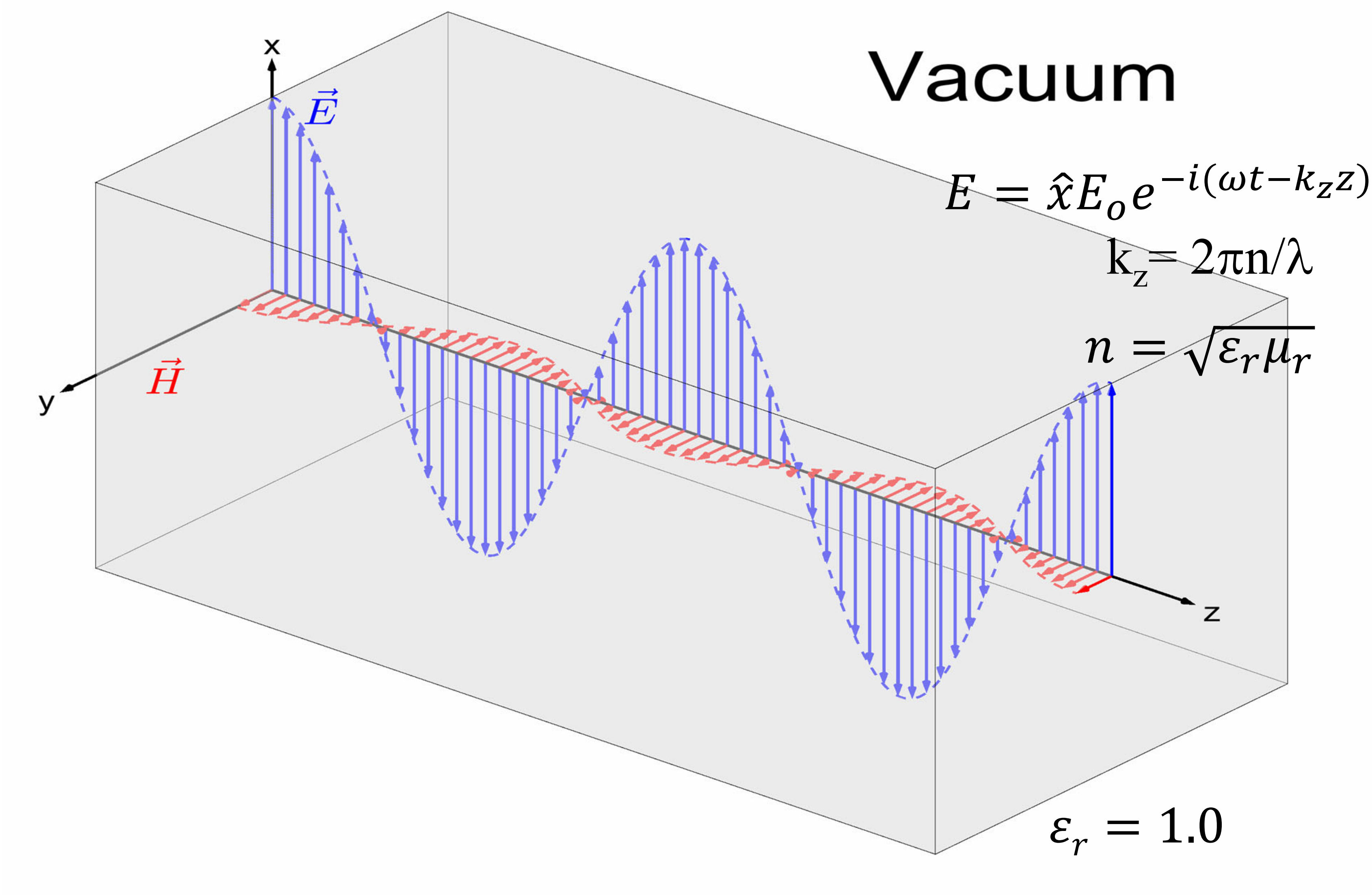


water



$$\varepsilon = \varepsilon' - j\sigma/\omega$$

$n = \sqrt{\varepsilon_r}$ If nonmagnetic

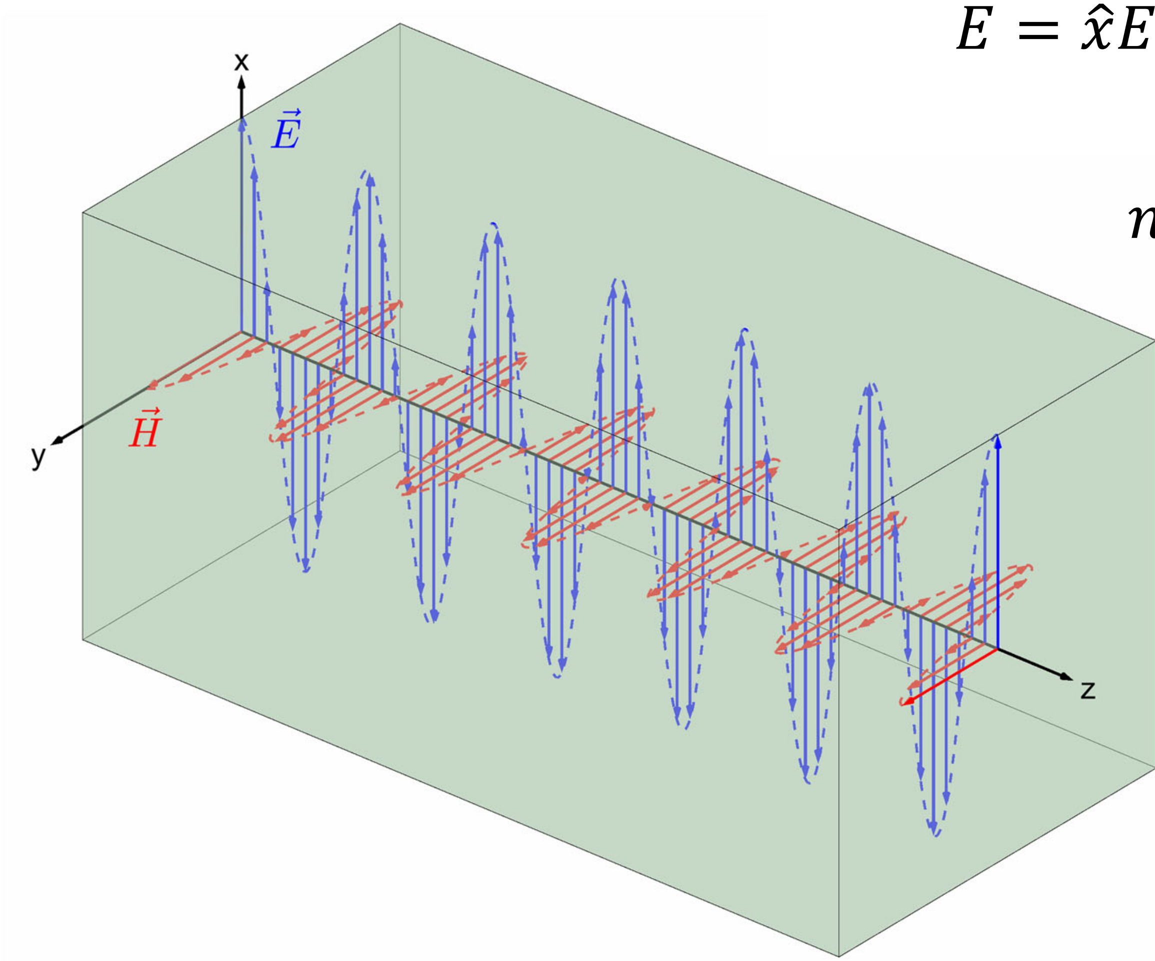

Vacuum
x
y
z
$\vec{E}$
$\vec{H}$
$E = \hat{x}E_o e^{-i(\omega t - k_z z)}$
$k_z = 2\pi n/\lambda$
$n = \sqrt{\varepsilon_r \mu_r}$
$\varepsilon_r = 1.0$

Wave in Lossless Dielectric

$$E = \hat{x} E_o e^{-i(\omega t - k_z z)}$$

$k_z = 2\pi n/\lambda$

$$n = \sqrt{\varepsilon_r \mu_r}$$



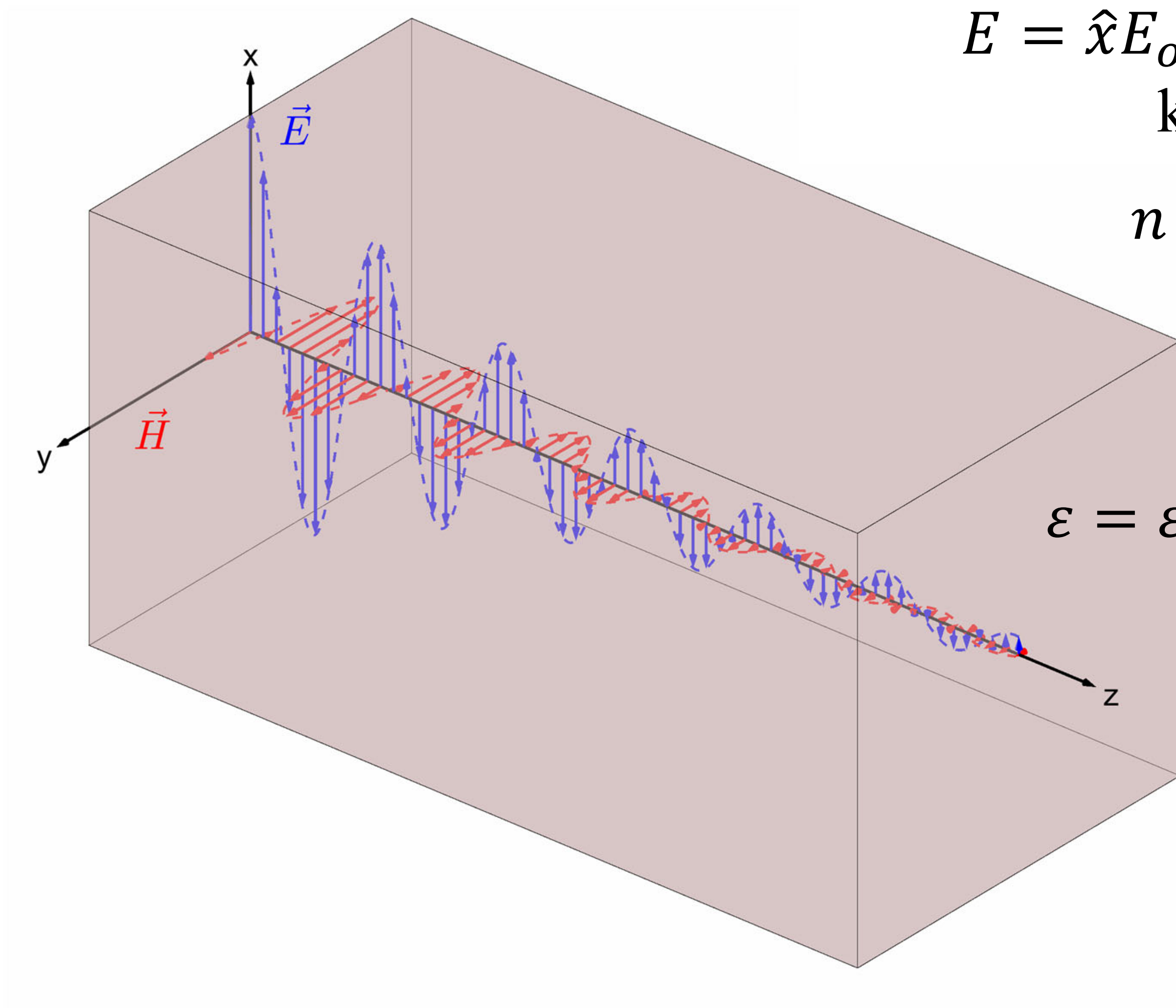
$$\varepsilon_r = \varepsilon'$$

Wave in Lossy Dielectric Material

$$E = \hat{x}E_o e^{-i(\omega t - k_z z)}$$

$$k_z = 2\pi n/\lambda$$

$$n = \sqrt{\varepsilon_r \mu_r}$$

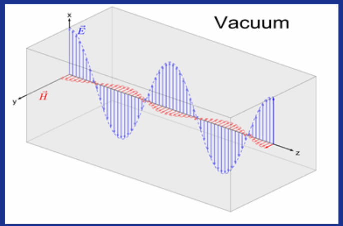


$$\varepsilon = \varepsilon' - j\sigma/\omega$$

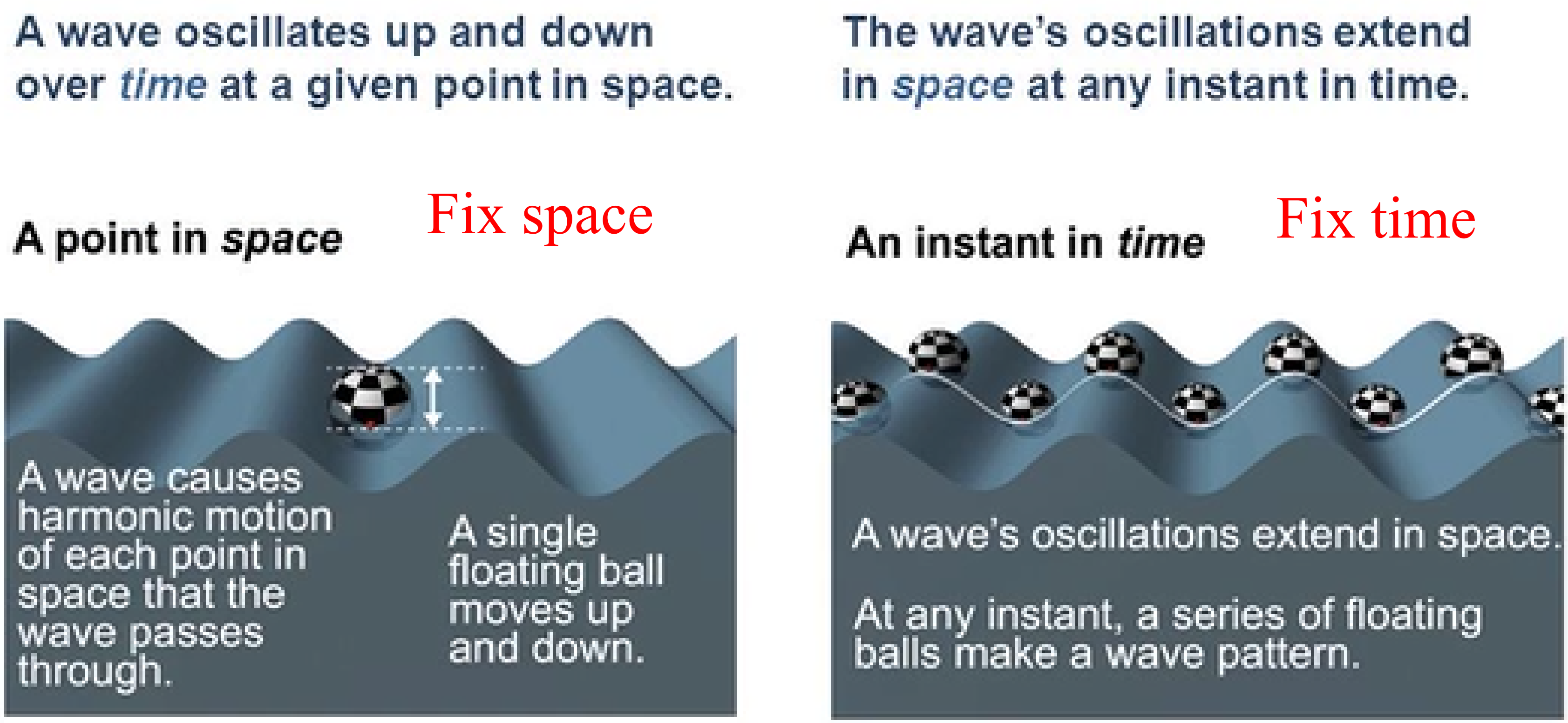
Things to know

- EM wave is $E(z,t) = \hat{x}E_o \cos(\omega t - kz)$
- A complex vector: function of space and time

- Time harmonic function: can be simplify to phasor form (fix time)
- Space: can be represented by spatial vector (fix space)



Wave in Space and Time



$$E(z,t) = \hat{x}E_o \cos(\omega t - kz)$$

This Week

- Vector Calculus (Del operator, gradient, divergence and curl)
- Maxwell's Equation (All thing EM)
- Convert integral form into differential form
- Wave Equation

Vector Calculus

Vector calculus, a branch of mathematics concerned with differentiation and integration of vector fields, primarily in 3-dimensional Euclidean space, plays an important role in differential geometry and in the study of partial differential equations. **It is used extensively in physics and engineering, especially in the description of electromagnetic fields, gravitational fields and fluid flow.**

- Del Operador"

$$\nabla = \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k$$

- Gradient of a scalar function is a **vector quantity.** $\nabla f \longrightarrow$ Vector
- Divergence of a vector is a **scalar quantity.** $\nabla \cdot A$
- Curl of a vector is a **vector quantity.** $\nabla \times A$
- The Laplacian of a scalar A $\nabla^2 A$ $(\Delta A = \nabla \cdot \nabla A = \nabla^2 \mathrm{A})$

Vector Calculus

- Cartesian coordinantes (x, y, z)

$$\nabla V = \mathbf{x}^{\sim}\frac{\partial V}{\partial x} + \mathbf{y}^{\sim}\frac{\partial V}{\partial y} + \mathbf{z}^{\sim}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

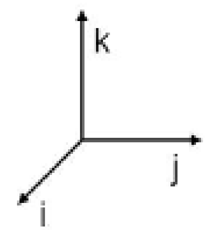
$$\nabla \times \mathbf{A} = \begin{bmatrix} \mathbf{x}^{\sim} & \mathbf{y}^{\sim} & \mathbf{z}^{\sim} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{bmatrix} = \mathbf{x}^{\sim}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \mathbf{y}^{\sim}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \mathbf{z}^{\sim}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 V = \frac{d^2 V}{dx^2} + \frac{d^2 V}{dy^2} + \frac{d^2 V}{dz^2}$$

Cross product – Curl (general form)

Can be in Cartesian and many other spatial coordinate forms

- Curl $\mathbf{A} = \nabla \mathrm{x} \mathbf{A} = \begin{vmatrix} \mathrm{i} & \mathrm{j} & \mathrm{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$



$$\nabla \mathrm{x} \mathbf{A} = \mathbf{i}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) - \mathbf{j}\left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right) + \mathbf{k}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

- Cross product of **del** with **A**

- Vector function of position

Vector Calculus

- Spherical coordinantes (r, θ, φ)

$$\nabla V = \mathbf{r}^{\sim}\frac{\partial V}{\partial x} + \theta^{\sim}\frac{1}{r}\frac{\partial}{\partial \theta}(V) + \varphi^{\sim}\frac{1}{r\sin(\theta)}\frac{\partial}{\partial \varphi}(V)$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 A_r\right) + \frac{1}{r\sin(\theta)}\frac{\partial}{\partial \theta}(A_\theta \sin(\theta)) + \frac{1}{r\sin(\theta)}\frac{\partial A_\varphi}{\partial \varphi}$$

$$\nabla \times \mathbf{A} = \frac{1}{r^2\sin(\theta)}\begin{vmatrix} \mathbf{r}^{\sim} & r\theta^{\sim} & r\sin(\theta)\varphi^{\sim} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \varphi} \\ A_r & rA_\theta & r\sin(\theta)A\varphi \end{vmatrix}$$

$$\nabla \times \mathbf{A} = \frac{\mathbf{r}^{\sim}}{r\sin(\theta)}\left(\frac{\partial}{\partial \theta}A_\varphi \sin(\theta) - \frac{\partial A_\theta}{\partial \theta}\right) + \frac{\theta^{\sim}}{r}\left(\frac{1}{\sin(\theta)}\frac{\partial A_r}{\partial \varphi} - \frac{\partial}{\partial r}(rA_\varphi)\right) + \frac{\varphi^{\sim}}{r}\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta}\right)$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin(\theta)}\frac{\partial}{\partial \theta}\left(\sin(\theta)\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2\sin^2(\theta)}\frac{\partial^2 V}{\partial \varphi^2}$$

Vector Calculus

- Useful vector relationships for the vector fields a , b, and c are

Especially the first two

$$\boxed{\begin{aligned} &\nabla \cdot (\nabla \times \mathbf{a}) = 0 \\ &\nabla \times \nabla \times \mathbf{a} = \nabla(\nabla \cdot \mathbf{a}) - \nabla^2 \mathbf{a} \end{aligned}}$$

We will use for deriving wave equation

$$\nabla \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot (\nabla \times \mathbf{a}) - \mathbf{a} \cdot (\nabla \times \mathbf{b})$$

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$$

$$\mathbf{a} \cdot \mathbf{b} \times \mathbf{c} = \mathbf{b} \cdot \mathbf{c} \times \mathbf{a} = \mathbf{c} \cdot \mathbf{a} \times \mathbf{b}$$

Maxwell's Equations

Integral form in the absence of magnetic or polarized media:

I. Faraday's law of induction $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$

II. Ampere's law $\oint \vec{B} \cdot d\vec{s} = \mu_0 i + \frac{1}{c^2} \frac{\partial}{\partial t} \int \vec{E} \cdot d\vec{A}$

III. Gauss' law for magnetism $\oint \vec{B} \cdot d\vec{A} = 0$

IV. Gauss' law for electricity $\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$

$\mu = \mu_o(1+\chi) = \mu_o$ if source like coil has iron place inside

$\varepsilon = \varepsilon_o$

E = Electric Field (V/m)	ρ *= charge density (c/m^3)*	*i = electric current (A)*
B = Magnetic flux density(Web/m^2, T)	ε_0 *= relative permittivity*	*J = current density(A/m^2)*
D = Electric flux density (c/m^2) or electric displacement field	μ_0 *= relative permeability*	*c = speed of light*
H = Magnetic Field (A/m)	Φ_B *= Magnetic flux (Web)*	*P = Polarization*

q = charge 1.6×10^{-19} *coulombs,* $\mu_o = 1.26 \times 10^{-6}$ *H/m,* $\varepsilon_o = 8.85 \times 10^{-12}$ *F/m*

Maxwell's Equations

Integral form in the absence of magnetic or polarized media:

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q = charge 1.6×10^{-19} coulombs, $\mu_o = 1.26 \times 10^{-6}$ H/m, $\varepsilon_o = 8.85 \times 10^{-12}$ F/m

Faraday’s Law of Induction

Integral Form

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

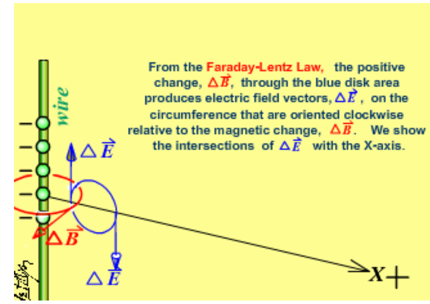
Differential Form

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

This line integral is equal to the generated voltage or emf in the loop, so Faraday's law is the basis for electric generators. It also forms the basis for inductors and transformers.

Close Contour (line integral)

It's defined direction relative to a surface.



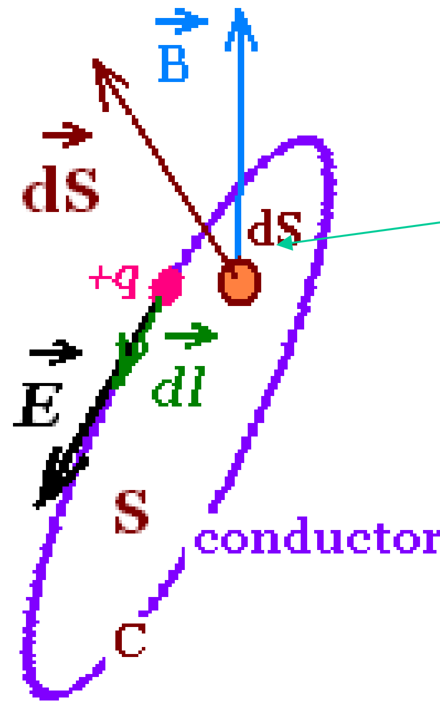
$\Phi_B = BA$

$=$*Magnetic flux (Web)*

B = Magnetic flux density(Web/m^2*, T)*

A=flux area

$$\phi_B = \int B \cdot dA$$



Close path

Magnetic flux density

$$\underbrace{\oint_C \vec{E} \cdot \vec{dl}}_{\text{V or EMF}} = -\frac{d}{dt}\int_S \vec{B} \cdot \vec{dS}$$

Negative of the Time derivative of the magnetic flux give rises the emf in a close loop

Faraday's Law of Induction

Opposing the change like Newton' third law

Integral Form

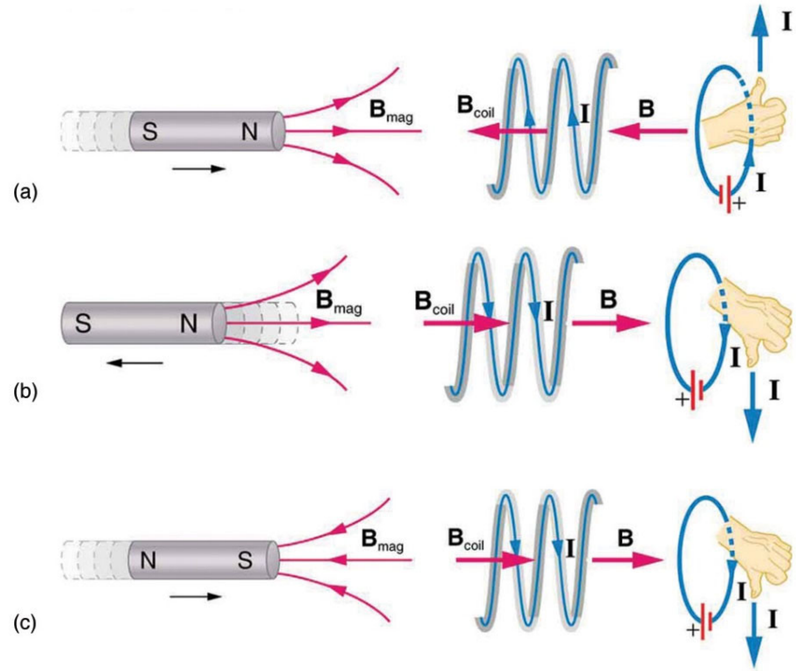
$$\oint \vec{E} \cdot dl = - \frac{d\Phi_B}{dt}$$

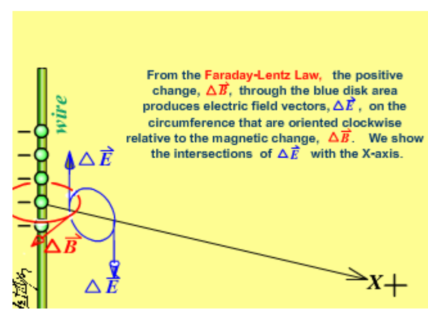
Differential Form

$$\nabla \times E = - \frac{\partial B}{\partial t}$$

curl

How?





Vector Calculus

- Del Operador”

$$\nabla = \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k$$

- Gradient of a scalar function is a **vector quantity.** $\nabla f \longrightarrow$ Vector
- Divergence of a vector is a **scalar quantity.** $\nabla \cdot A$
- Curl of a vector is a **vector quantity.** $\nabla \times A$
- The Laplacian of a scalar A $\nabla^2 A$

Vector Calculus

- Cartesian coordinantes (x, y, z)

$$\nabla V = \mathbf{x}^{\sim}\frac{\partial V}{\partial x} + \mathbf{y}^{\sim}\frac{\partial V}{\partial y} + \mathbf{z}^{\sim}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

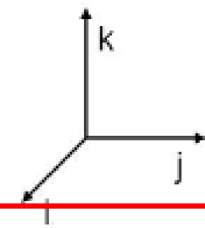
$$\nabla \times \mathbf{A} = \begin{bmatrix} \mathbf{x}^{\sim} & \mathbf{y}^{\sim} & \mathbf{z}^{\sim} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{bmatrix} = \mathbf{x}^{\sim}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \mathbf{y}^{\sim}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \mathbf{z}^{\sim}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 V = \frac{d^2 V}{dx^2} + \frac{d^2 V}{dy^2} + \frac{d^2 V}{dz^2}$$

Cross product – Curl (general form)

Can be in Cartesian and many other spatial coordinate forms

- Curl **A** = ∇x**A** = $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$



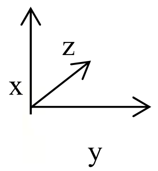
$$\boxed{\nabla \times \mathbf{A} = \mathbf{i}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) - \mathbf{j}\left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right) + \mathbf{k}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)}$$

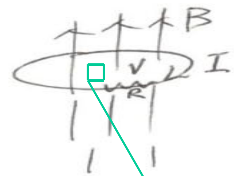
- Cross product of **del** with **A**
- Vector function of position

Faraday's law

$$V = IR = -\frac{d}{dt}\int \vec{B}\cdot d\vec{s} = \int \vec{E}\cdot d\vec{l}$$

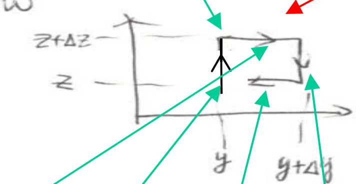
Start from integral form





We can take an small Infinitesimal area in that integrated area and defined it by delta z and y

Now



line integral

at z+Δz | at y | at z | at y+Δy

$$E_y(z+\Delta z)\cdot\Delta y - E_z(y)\cdot\Delta z - E_y(z)\cdot\Delta y - E_z(y+\Delta y)\cdot\Delta z = -\frac{d}{dt}B_x\cdot\Delta y\,\Delta z$$

Time derivative

Differential form

$$\frac{E_z(y+\Delta y)-E_z(y)}{\Delta y} - \frac{E_y(z+\Delta z)-E_y(z)}{\Delta z} = -\frac{dB}{dt}$$

$$\boxed{\nabla\times\vec{E} = -\frac{d}{dt}\vec{B}}$$

Recall

$$\nabla\times\mathbf{A} = \begin{vmatrix} \tilde{\mathbf{x}} & \tilde{\mathbf{y}} & \tilde{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \tilde{\mathbf{x}}\left(\frac{\partial A_z}{\partial y}-\frac{\partial A_y}{\partial z}\right) + \tilde{\mathbf{y}}\left(\frac{\partial A_x}{\partial z}-\frac{\partial A_z}{\partial x}\right) + \tilde{\mathbf{z}}\left(\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y}\right)$$

One of the cross product term

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\frac{dB_x}{dt}$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\frac{dB_y}{dt}$$

$$-\frac{\partial E_x}{\partial y} + \frac{\partial E_y}{\partial x} = -\frac{dB_z}{dt}$$

① $\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\frac{d}{dt} B_x$

$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\frac{d}{dt} B_y$ same as $\begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{bmatrix}$

$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\frac{d}{dt} B_z$ ~~except the sign~~

$= \nabla \times \vec{E}$

Another way to solve this using identity

② even though $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$ doesn't mean

$\vec{\nabla} \times \vec{E} \neq -\vec{E} \times \vec{\nabla}$

$= -\vec{E} \times \vec{\nabla}$

③ $\oint E \cdot dl = \frac{d}{dt} \int \vec{B}(t)\, d\vec{S}$

came from ⇓

$\boxed{\int \nabla \times \vec{E} \cdot d\vec{s} = \frac{d}{dt} \int \vec{B}(t)\, d\vec{S}}$ or $\boxed{\int (\vec{\nabla} \times \vec{E})\, ds = \int \vec{E} \cdot d}$

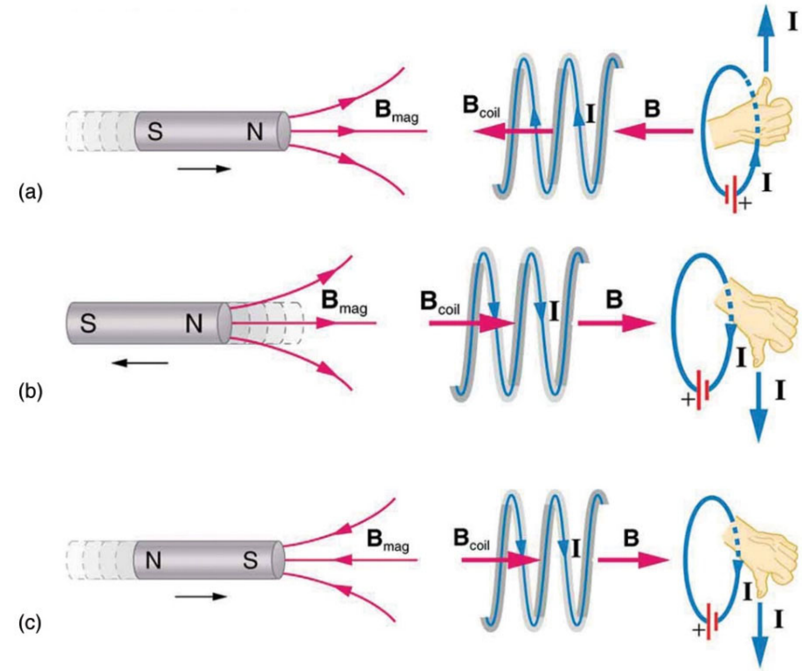
(Stoke's theorem)

Faraday's Law of Induction

Opposing the change like Newton' third law

Integral Form

$$\oint \vec{E} \cdot dl = - \frac{d\Phi_B}{dt}$$

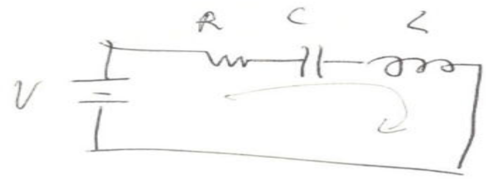


Differential Form

$$\nabla \times E = - \frac{\partial B}{\partial t}$$

curl

So now you know



$$\oint \vec{E}\cdot d\vec{l} + \frac{d}{dt}\int \vec{B}\cdot d\vec{s} = 0$$

B(t)

summation

Kirchoff voltage Law

$$\sum V = 0 = \int \vec{E}\cdot d\vec{l}$$

B(t)

Low freq means

B(t)

$\approx$

t is almost non-existance

→ thats why kirchoff is low freq limit of maxwell eqs.

This equation tells you no oscillating radiation from small signal or DC input

Maxwell Equations

Integral form in the absence of magnetic or polarized media:

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$\mu_o \Rightarrow \mu = \mu_o(1+\chi)$ if source like coil has iron place inside

E = Electric Field (V/m)	ρ *= charge density (c/m*^{3}*)*	*i = electric current (A)*
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D = Electric flux density (c/m^2*) or electric displacement field*	μ_0 *= relative permeability*	*c = speed of light*
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q = charge 1.6×10^{-19} *coulombs,* $\mu_o = 1.26\times10^{-6}$ *H/m,* $\varepsilon_o = 8.85\times10^{-12}$ *F/m*

Ampere's Law

Integral form

$$\oint B \cdot ds = \mu_0 i + \frac{1}{c^2}\frac{\partial}{\partial t}\int E \cdot dA$$

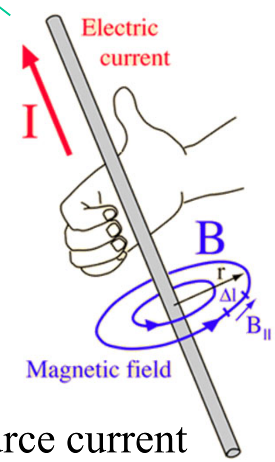
Differential form

$$\nabla x B = \frac{4\pi k}{c^2} J + \frac{1}{c^2}\frac{\partial E}{\partial t}$$

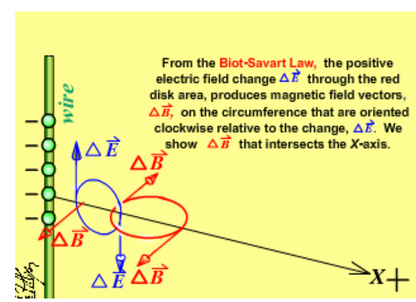
$$\nabla x B = \frac{J}{\varepsilon_0 c^2} + \frac{1}{c^2}\frac{\partial E}{\partial t}$$

$$k = \frac{1}{4\pi\varepsilon_0} \qquad c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

In the case of static electric field, the line integral of the magnetic field around a closed loop is proportional to the electric current flowing through the loop. This is useful for the calculation of magnetic field for simple geometries.



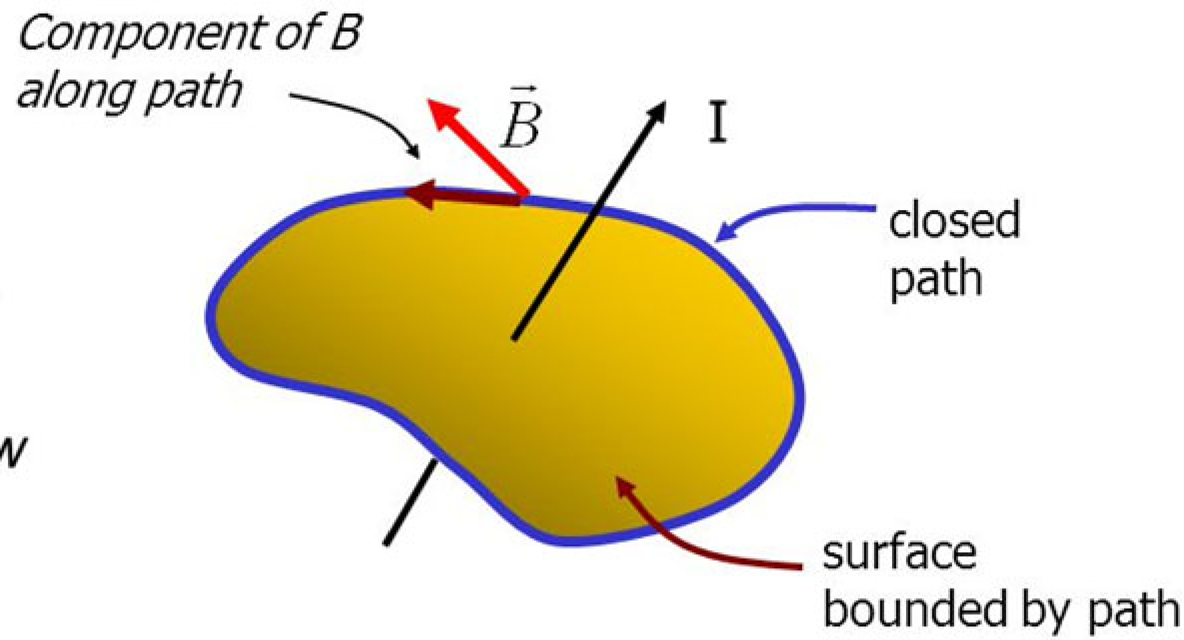
Source current



Displacement current into Free space

Close Path

Sum up component of B around path
Equals current through surface.



$$\oint \mathbf{B} \bullet d\mathbf{s} = \mu_o I$$

- *Ampere's law*

Ampere's Law

Source current

Displacement current into Free space

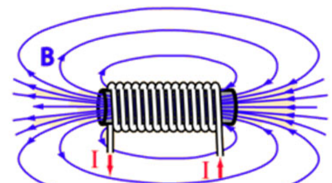
Integral form

$$\oint B \cdot ds = \mu_0 i + \frac{1}{c^2}\frac{\partial}{\partial t}\int E \cdot dA$$

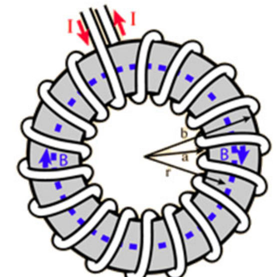
Differential form

$$\boxed{\begin{aligned} \nabla x B &= \frac{4\pi k}{c^2} J + \frac{1}{c^2}\frac{\partial E}{\partial t} \\ \nabla x B &= \frac{J}{\varepsilon_0 c^2} + \frac{1}{c^2}\frac{\partial E}{\partial t} \end{aligned}}$$

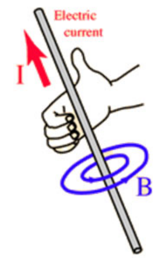
$$k = \frac{1}{4\pi\varepsilon_0} \qquad c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$



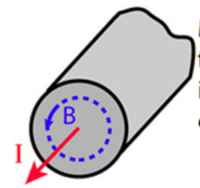
Magnetic field inside a long solenoic.



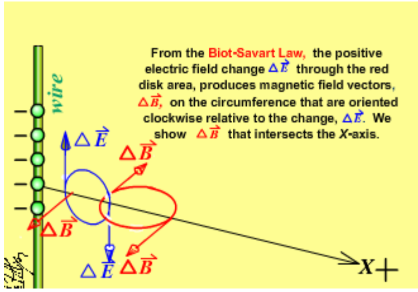
Magnetic field inside a toroidal coil.



Magnetic field from a long straight wire.

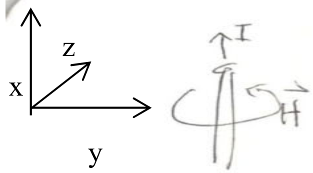


Magnetic field inside a conductor.



10/7

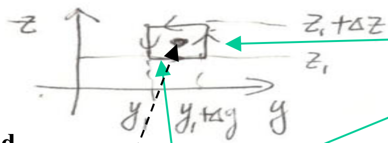
(2) Amperes Law



$\oint \vec{B}\cdot d\vec{\ell} = \mu_o I$ B = μ H = μ_o (1+χ) H = μ_o(M+H) ⇒ B = μ_o H (nonmagnetic) (copper)

$\oint \boxed{\vec{H}\cdot d\vec{\ell}} = I$

(i)



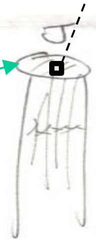
$H_y(z=z_1)\Delta y + H_z(y_1+\Delta y)\Delta z$

$- H_y(z_1+\Delta z)\Delta y - H_z(y_1)\Delta z = I = J\Delta y\Delta z$

We can take an small Infinitesimal area in that integrated area and defined it by delta x and y

(ii)

area=a



← assume non conductor

like semiconductor

$I = \int J\, d\vec{a}$ or $J = \frac{I}{\Delta y \Delta z}$

(current density, amp/m^2)

$\nabla\times\vec{H} =$

(iii)

$$\frac{H_z(y_1+\Delta y) - H_z(y_1)}{\Delta y} - \frac{H_y(y_1+\Delta y) - H_y(z_1)}{\Delta z}$$

$$= J \cancel{\Delta y \Delta z}$$

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = J_x$$

$$\nabla\times\vec{H} = J_x$$

source current — displacement current — Conductive current

(iv) Include source and free space

$$\nabla \times H = J_o + j\omega(\varepsilon' - j\frac{\sigma}{\omega})E$$

displacement current

$\vec{J}_c$ ← be caused of

Recall

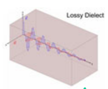
$\varepsilon = \varepsilon' - j\sigma/\omega$

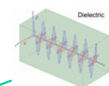
$$\nabla\times\vec{H} = \vec{J}_s + \boxed{\vec{J}_c + \vec{J}_d}$$

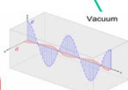
Source current — Induced current — displacement current

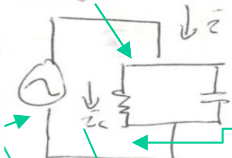
Displacement current is electric field generated by B field that was radiating in air

Induced current is from dispersive medium like dielectric material ($\varepsilon = \varepsilon' - j\sigma/\omega$)



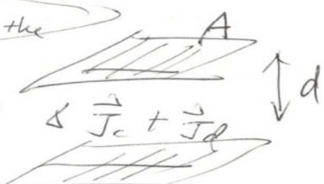






i_d (imaginary part) which is what's ~~transmitted~~ not seen possibly or

~~Lossy part~~ actually is current that induced in the medium



(v) medium J_c σ + $\boxed{\vec{J}_d\ \varepsilon'}$ = σ $\vec{J}_c + \vec{J}_d$

Lossy part

$$i = \frac{V}{R} \Rightarrow \frac{Ed}{R} = Ed\sigma A$$

$$\boxed{\vec{J}_c = \vec{E}\sigma = \frac{i_c}{A}}$$

conductivity — area

Small c is speed of light

$$\nabla x B = \frac{4\pi k}{c^2}J + \frac{1}{c^2}\frac{\partial E}{\partial t}$$

$$\nabla x B = \frac{J}{\varepsilon_0 c^2} + \frac{1}{c^2}\frac{\partial E}{\partial t}$$

(vi) use capacitor to model the displacement current because that's what happen when wave travel thru the medium

$$\nabla \times H = J_s + \frac{\partial D}{\partial t}$$

$$i_d = C\frac{dV}{dt} = \frac{\epsilon A}{d}\frac{\partial(Ed)}{\partial t}$$

use

$$k = \frac{1}{4\pi\varepsilon_0} \text{ or } c = \frac{1}{\sqrt{\mu_r\mu_o\varepsilon_r\varepsilon_o}}$$

$D = \varepsilon E$

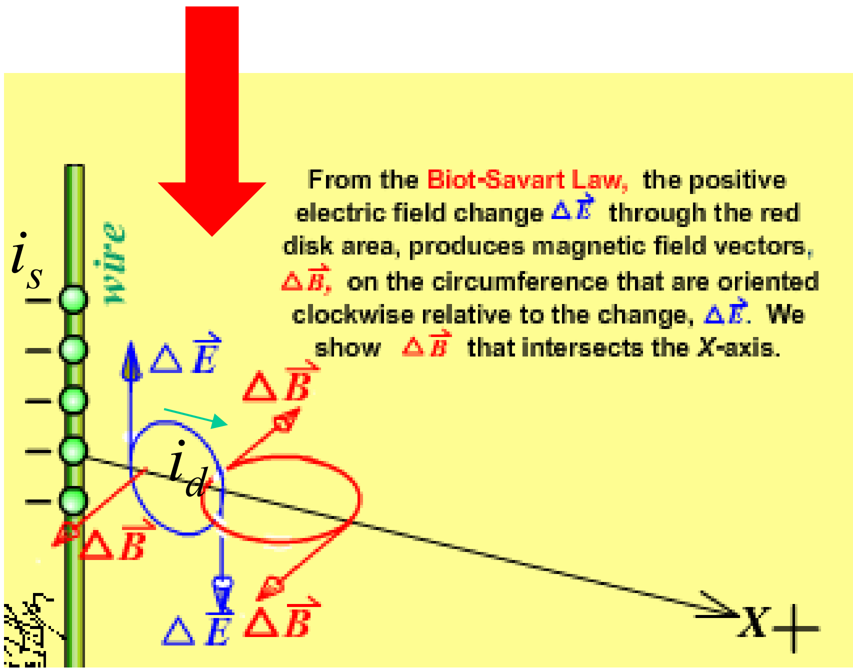
$$= \epsilon A\frac{\partial E}{\partial t}$$

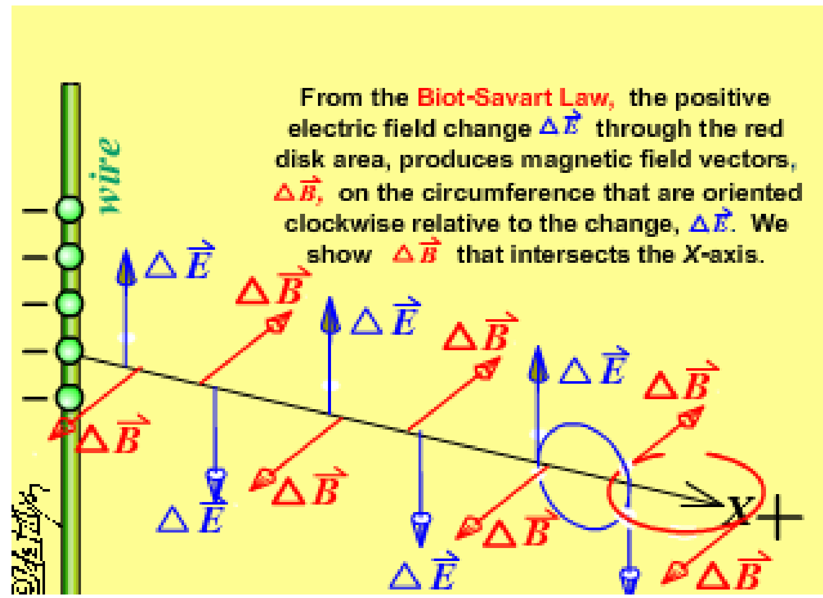
$$J_d = \epsilon\frac{\partial E}{\partial t}$$

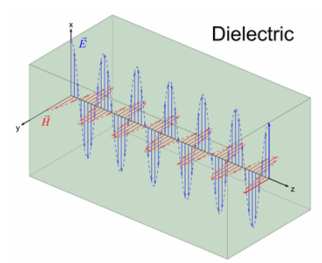
Transform into either

Electromagnetic Wave

Displacement current J_d and possibly J_δ







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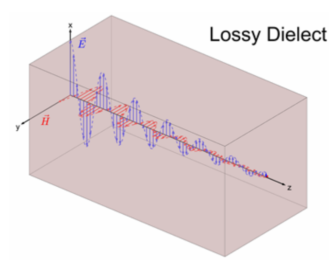
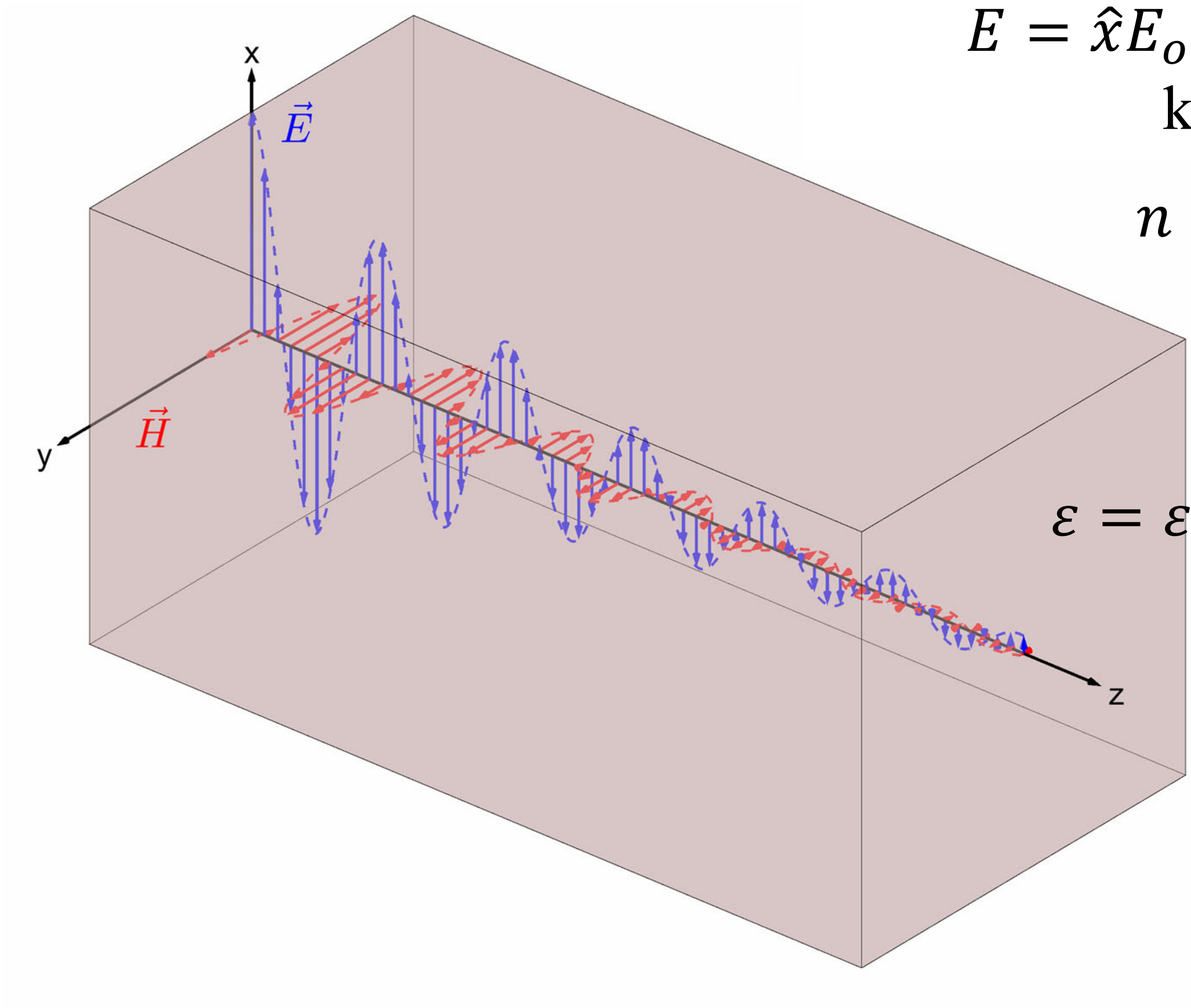


Wave in Lossy Dielectric Material

$$E = \hat{x}E_o e^{-i(\omega t - k_z z)}$$

$k_z = 2\pi n/\lambda$

$$n = \sqrt{\varepsilon_r \mu_r}$$



$$\varepsilon = \varepsilon' - j\sigma/\omega$$

Maxwell's Equations

Integral form in the absence of magnetic or polarized media:

I. Faraday's law of induction $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$

II. Ampere's law $\oint \vec{B} \cdot d\vec{s} = \mu_0 i + \frac{1}{c^2}\frac{\partial}{\partial t}\int \vec{E} \cdot d\vec{A}$

III. Gauss' law for magnetism $\oint \vec{B} \cdot d\vec{A} = 0$

$\mu_o = \mu = \mu_o(1+\chi)$ if source like coil has iron place inside

IV. Gauss' law for electricity $\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$

E = Electric Field (V/m) | *ρ = charge density (c/m^3)* | *i = electric current (A)*

B = Magnetic flux density(Web/m^2, T) | *ε_0 = relative permittivity* | *J = current density(A/m^2)*

D = Electric flux density (c/m^2) or electric displacement field | *μ_0 = relative permeability* | *c = speed of light*

H = Magnetic Field (A/m) | *Φ_B = Magnetic flux (Web)* | *P = Polarization*

q = charge 1.6×10^{-19} coulombs, *$\mu_o = 1.26 \times 10^{-6}$ H/m,* *$\varepsilon_o = 8.85 \times 10^{-12}$ F/m*

Gauss’s Law for Magnetism

Integral Form

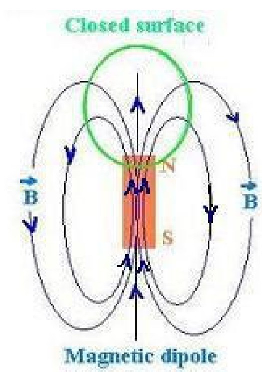
$$\oint \vec{B} \cdot d\vec{A} = 0$$

Surface normal basically is the del operator function

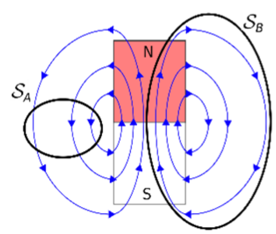
Differential Form

$$\boxed{\nabla \cdot B = 0}$$

every field line entering the volume enclosed by S or A must also exit this volume – field lines may not begin or end within the volume

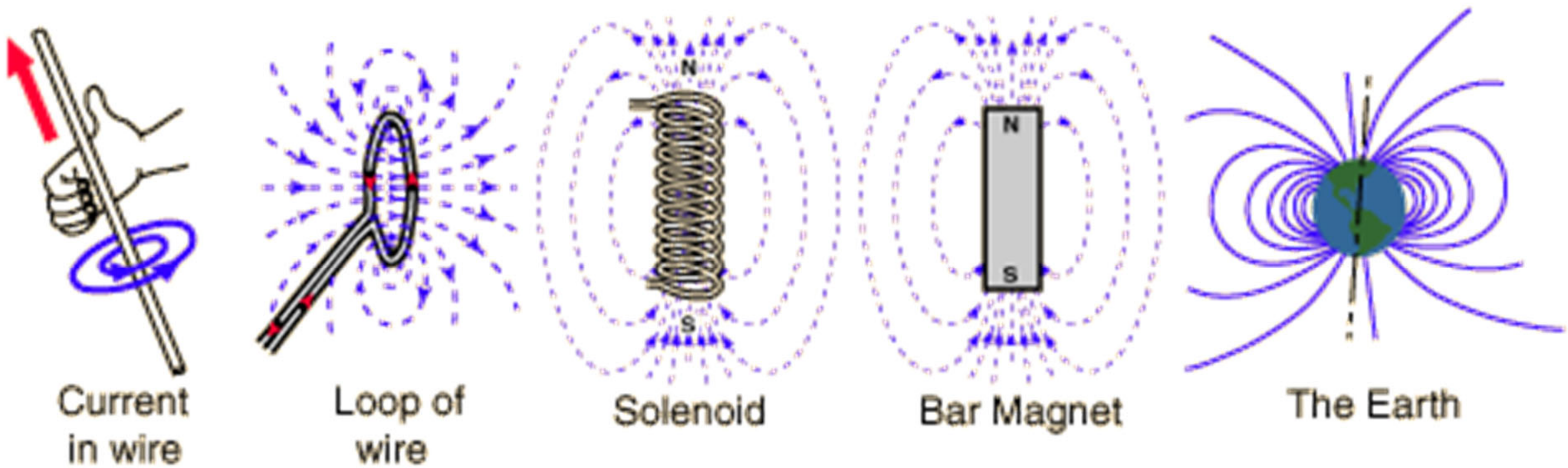


$$\oint \vec{B} \cdot d\vec{S} = 0$$



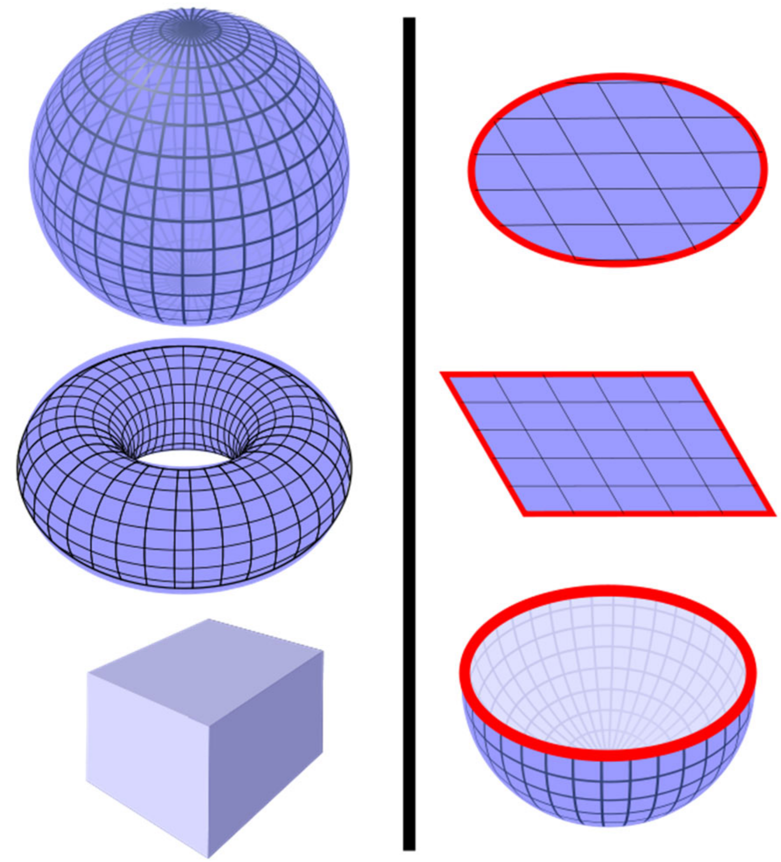
The net magnetic flux out of any closed surface is zero. This amounts to a statement about the sources of magnetic field. For a magnetic dipole, any closed surface the magnetic flux directed inward toward the south pole will equal the flux outward from the north pole. The net flux will always be zero for dipole sources. If there were a magnetic monopole source, this would give a non-zero area integral. The divergence of a vector field is proportional to the point source density, so the form of Gauss' law for magnetic fields is then a statement that there are no magnetic monopoles.

Gauss's Law for Magnetism



Magnetic Field Sources

Definition of a closed surface. Left: Some examples of closed surfaces include the surface of a sphere, surface of a torus, and surface of a cube. **The magnetic flux through any of these surfaces is zero**. Right: Some examples of non-closed surfaces include the disk surface, square surface, or hemisphere surface. They all have boundaries (red lines) and they do not fully enclose a 3D volume. The magnetic flux through these surfaces is *not necessarily zero*.



Magnetic flux

We know from Gauss's law for magnetism that in a close surface,

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Normally, the magnetic flux in an unclosed surface

$$\phi_B = \int B \cdot dA$$

Where B = magnetic flux density

But when the generated fields pass through magnetic materials which themselves contribute internal magnetic fields, ambiguities can arise about what part of the field comes from the external currents and what comes from the material itself. It has been common practice to define another magnetic field quantity, usually called the "magnetic field strength" designated by H. It can be defined by the relationship

$$B = \mu_o(H + M)$$

M = magnetization. Normally, the M = 0 for nonmagnetic material

If in air, $\mu_o = 1.26x10^{-6} H/m$

Magnetic susceptibility and permeability

In large class of materials, there exists a linear relation between M (internal magnetization) and H (external applied magnetic field)

$$M = \chi H$$

χ is positive then the material is called *paramagnetic*

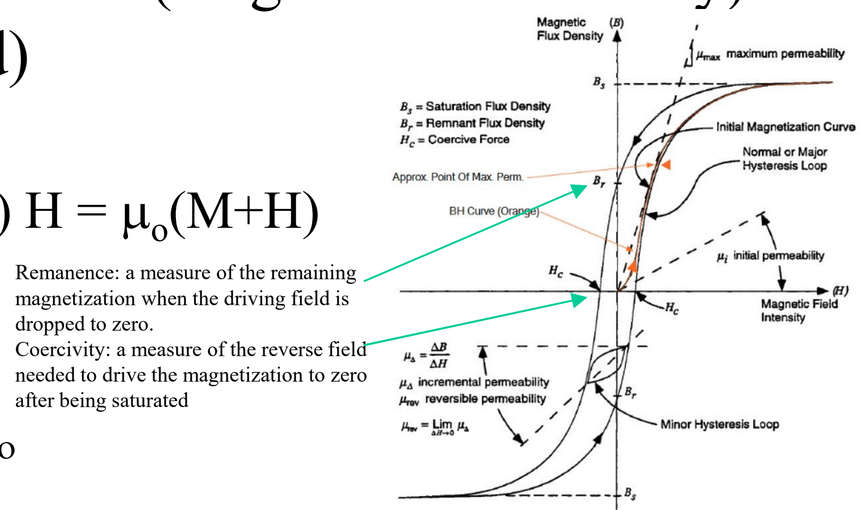
χ is negative then the material is *diamagnetic*

A linear relationship also occurs between B (magnetic flux density) and H (external applied magnetic field)

$$B = \mu H = \mu_o (1+\chi) H = \mu_o(M+H)$$

magnetic permeability is

$$\mu = (1+\chi)\, \mu_o = \mu_r\, \mu_o$$



Where μ_r = relative permeability and μ_o= free space permeability

$\mu_r \sim 1$ for paramagentic and diamagnetic, $\mu_r >> 1$ for ferromagnetic.

Permeability

Permeability is defines as

$$\mu = (1+\chi)\ \mu_o = \mu_r\ \mu_o$$

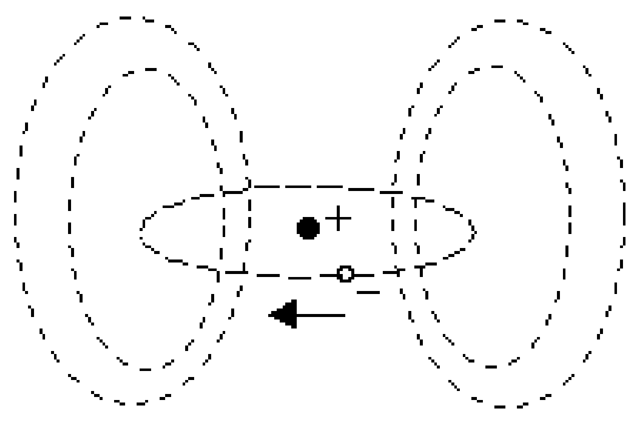
Where μ_r is relative permeability. χ is susceptibility. Typical values for ordinary liquids and solids are in the range 1.00001 to 1.003.

– $\mu_r = 1$ when the material does not respond to the magnetic field by magnetizing.

– $\mu_r > 1$ implies material magnetizes in response to the applied magnetic field.

Magnetism

- The three main types of magnetic behavior exhibited by material substances are **called diamagnetism, paramagnetism, and ferromagnetism. The first two can be explained in terms of the magnetic fields** produced by the orbital motions of the electrons in an atom. Each electron in an atom can be regarded as having some "orbital" motion about the nucleus, and this moving charge represents an electric current, which sets up a magnetic field for the atom



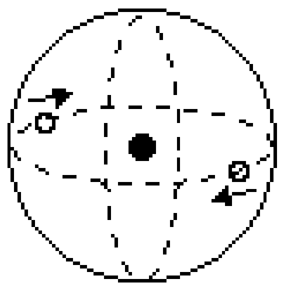
Many atoms have essentially no net magnetic dipole field, because the electrons orbit the nucleus about **different axes, so their fields cancel out**. Thus, whether or not an atom has a net dipole field depends on the structure of the electron shells surrounding the nucleus.

Three main types of magnetic behavior

- **Diamagnetism** refers to materials that are not affected by a magnetic field, but generate an opposing field when field is present.
- **Paramagnetism** refers to materials like aluminum or platinum which become magnetized in a magnetic field but their magnetism (going in same direction as the field) disappears when the field is removed.
- **Ferromagnetism** refers to materials (such as iron and nickel) that can retain their magnetic properties when the magnetic field is removed.

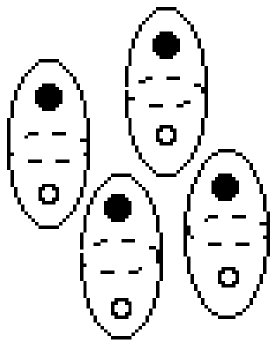
Three main kinds a magnetism

MICRO TECH LAB

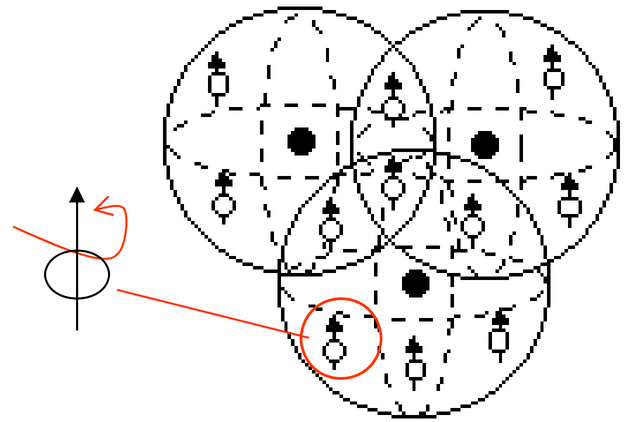


Ampere's law

Diamagnetism:
Lenz's Law applied to orbiting electrons causes atoms to be repelled from a magnetic field.



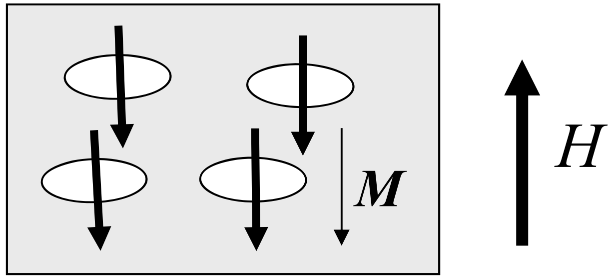
Paramagnetism:
Atomic dipoles aligned to an applied magnetic field, causing the atoms to be attracted.



Ferromagnetism:
Electron spin axes are aligned due to the "exchange" interaction. Lattice structure fixes boundaries of alignment.

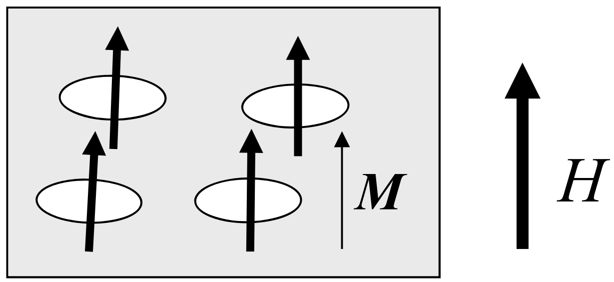
Summary of magnetic responses (pictorial explanation):

diamagnetic
(opposes *H*)



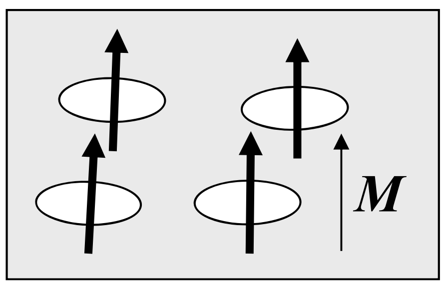
χ<<1, negative

paramagnetic
(aligns with *H*)



χ<<1, positive

ferromagnetic
(even without *H*!)



χ>1, positive

Susceptibility χ of dimagnetic and paramagnetic materials

Material	χ_m (H/m)
Aluminum	2.3×10^{-5}
Copper	-0.98×10^{-5}
Diamond	-2.2×10^{-5}
Tungsten	6.8×10^{-5}
Hydrogen (1 atm)	-0.21×10^{-8}
Oxygen (1 atm)	209.0×10^{-8}
Nitrogen (1 atm)	-0.50×10^{-8}

If χ is positive then the material is called *paramagnetic*, and the magnetic field is strengthened by the presence of the material.
if χ is negative then the material is *diamagnetic*, and the magnetic field is weakened in the presence of the material.

Material	χ_m (x 10^{-5} H/m)
Paramagnetic	
Iron aluminum alum	66
Uranium	40
Platinum	26
Aluminum	2.2
Sodium	0.72
Oxygen gas	0.19
Diamagnetic	
Bismuth	-16.6
Mercury	-2.9
Silver	-2.6
Carbon (diamond)	-2.1
Lead	-1.8
Sodium chloride	-1.4
Copper	-1.0

Magnetic susceptibility and permeability data for selected materials

Medium	Susceptibility χ_m (volumetric SI)	Permeability μ [H/m]	Relative permeability μ/μ_0	Magnetic field	Frequency (max)
Metglas 2714A (annealed)		1.26×10^{0}	1000000[7]	at 0.5 T	100 kHz
Iron (99.95% pure Fe annealed in H)		2.5×10^{-1}	200000[8]		
Nanoperm		1.0×10^{-1}	80000[9]	at 0.5 T	10 kHz
Mu-metal		2.5×10^{-2}	20000[10]	at 0.002 T	
Mu-metal		6.3×10^{-2}	50000[11]		
Cobalt-Iron (high permeability strip material)		2.3×10^{-2}	18000[12]		
Permalloy	8000	1.0×10^{-2}	8000[10]	at 0.002 T	
Iron (99.8% pure)		6.3×10^{-3}	5000[8]		
Electrical steel		5.0×10^{-3}	4000[10]	at 0.002 T	
Ferritic stainless steel (annealed)		1.26×10^{-3} - 2.26×10^{-3}	1000–1800[13]		
Martensitic stainless steel (annealed)		9.42×10^{-4} - 1.19×10^{-3}	750–950[13]		
Ferrite (manganese zinc)		>8.0×10^{-4}	640 (or more)		100 kHz ~ 1 MHz
Ferrite (nickel zinc)		2.0×10^{-5} – 8.0×10^{-4}	16–640		100 kHz ~ 1 MHz[citation needed]
Carbon Steel		1.26×10^{-4}	100[10]	at 0.002 T	
Nickel		1.26×10^{-4} - 7.54×10^{-4}	100[10] – 600	at 0.002 T	
Martensitic stainless steel (hardened)		5.0×10^{-5} - 1.2×10^{-4}	40–95[13]		
Austenitic stainless steel		1.260×10^{-6} - 8.8×10^{-6}	1.003–7 [13][14][note 1]		
Neodymium magnet		1.32×10^{-6}	1.05[15]		
Platinum		1.256970×10^{-6}	1.000265		
Aluminum	2.22×10^{-5}[16]	1.256665×10^{-6}	1.000022		
Wood		1.25663760×10^{-6}	1.00000043[16]		
Air		1.25663753×10^{-6}	1.00000037 [17]		
Concrete (dry)			1[18]		
Vacuum	0	$4\pi\times10^{-7}$ (μ_0)	1, exactly[19]		

Magnetic Characteristics

- Diamagnets are objects with a magnetic susceptibility $\chi < 0$
- This means that diamagnetic objects repel magnetic fields
- Many things are diamagnetic like water and wood and frogs

Levitation

- Is it possible for combination of gravitational and magnetic energy to have a minimum which is necessary to have stable levitation? Yes

what can we levitate?

- theoretically we can levitate anything diamagnetic with a given magnetic field
- this goes for walnuts, mice, leaves, diamonds…frogs

Analog to Lenz's law

the orbital motion of electrons creates tiny atomic current loops, which produce magnetic fields. when an external magnetic field is applied to a material, these current loops will tend to align in such a way as to oppose the applied field

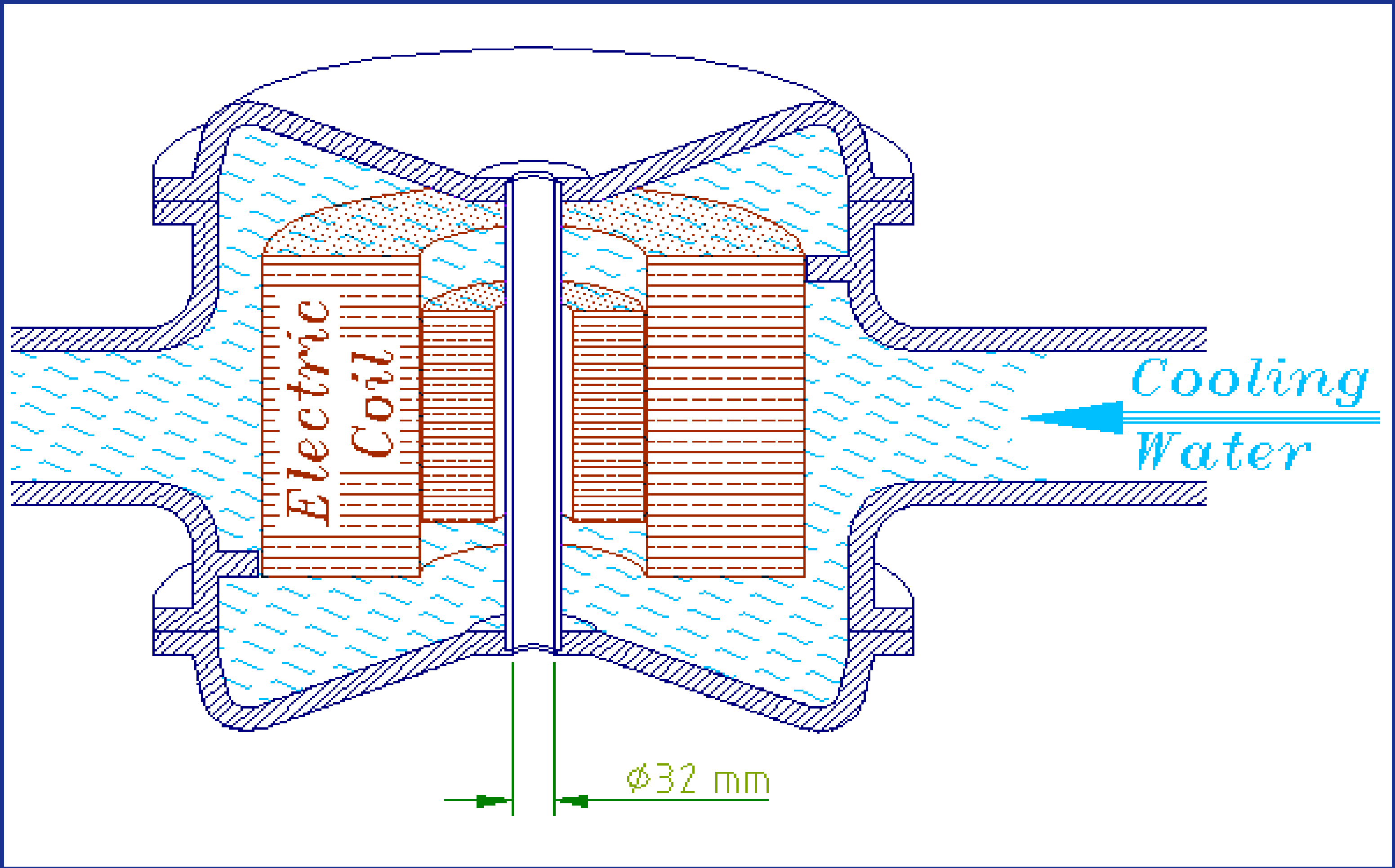
$$z = \pm\left(\frac{3\mu_o MHR^2 |M_L|}{2\rho g}\right)^{1/4}$$

Z is independent of the dimension of the levitating magnets

the experiment place

the university of nijmegen HFML

how to achieve a 16T field bitter magnet



more about the better magnet

- maximum field of 20 tesla *(T)* (this is 400000 times the earth magnetic field) consists of two concentric electrical coils.

- the coils are powered by a 6000 kW (6MW) power supply. (20 kA, 300V)
  the magnet is cooled by de-ionized water of about 10°C

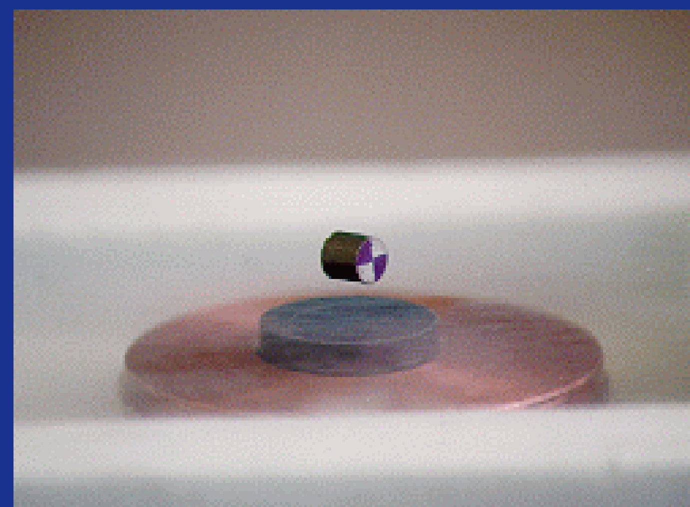
- the cooling water goes in axial direction through the small holes in the bitter plates of the coils, which line up exactly in every plate

Diamagnetic levitation

High Field
Magnet Laboratory
University of Nijmegen

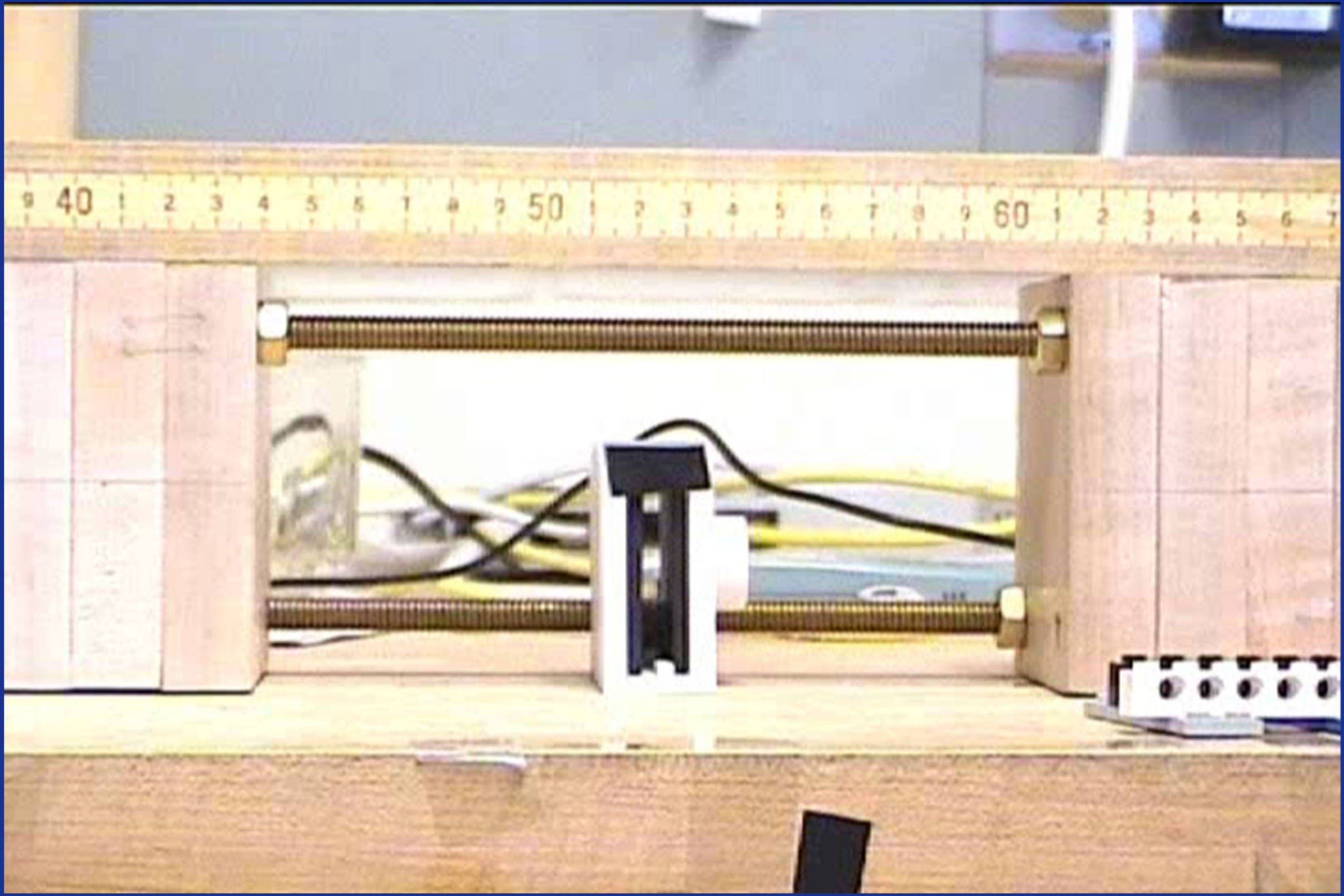
superconductivity

High Field
Magnet Laboratory
University of Nijmegen



Yitrium(1)Barium(2)Copper(3)Oxygen(6.95).
Zero resistance- perfect conductor, perfect diamagnets

Horizontal levitation



DEPARTMENT OF MECHANICAL ENGINEERING
UNIVERSITY OF WASHINGTON

Maxwell's Equations

Integral form in the absence of magnetic or polarized media:

I. Faraday's law of induction $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$

II. Ampere's law $\oint \vec{B} \cdot d\vec{s} = \mu_0 i + \frac{1}{c^2}\frac{\partial}{\partial t}\int \vec{E} \cdot d\vec{A}$

III. Gauss' law for magnetism $\oint \vec{B} \cdot d\vec{A} = 0$

$$\boxed{\text{IV. Gauss' law for electricity} \quad \oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}}$$

E = Electric Field (V/m) — *ρ = charge density (c/m^3)* — *i = electric current (A)*

B = Magnetic flux density(Web/m^2, T) — *ε_0 = relative permittivity* — *J = current density(A/m^2)*

D = Electric flux density (c/m^2) or electric displacement field — *μ_0 = relative permeability* — *c = speed of light*

H = Magnetic Field (A/m) — *Φ_B = Magnetic flux (Web)* — *P = Polarization*

q = charge 1.6×10^{-19} coulombs, *$\mu_o = 1.26 \times 10^{-6}$ H/m,* *$\varepsilon_o = 8.85 \times 10^{-12}$ F/m*

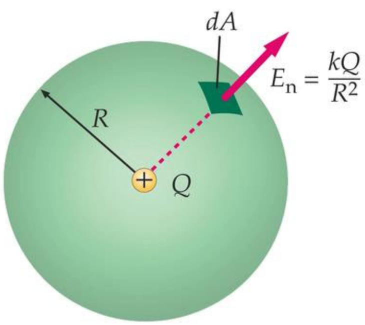
Gauss's Law for Electricity

Integral Form

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

Differential

$$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$$



<u>The electric flux out of any closed surface is proportional to the total charge enclosed within the surface.</u>

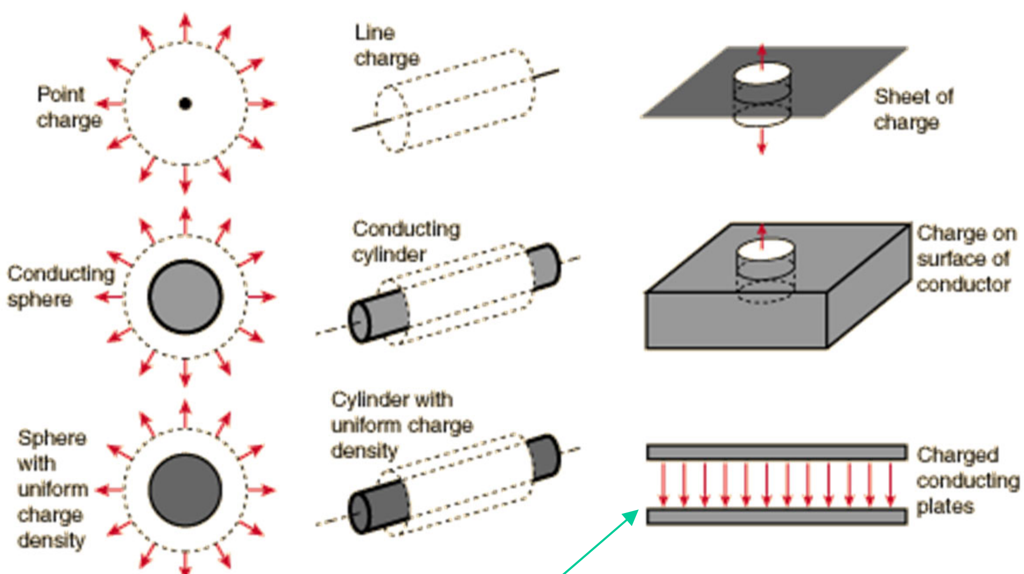
The integral form of Gauss' Law finds application in calculating electric fields around charged objects.

In applying Gauss' law to the electric field of a point charge, one can show that it is consistent with Coulomb's law.

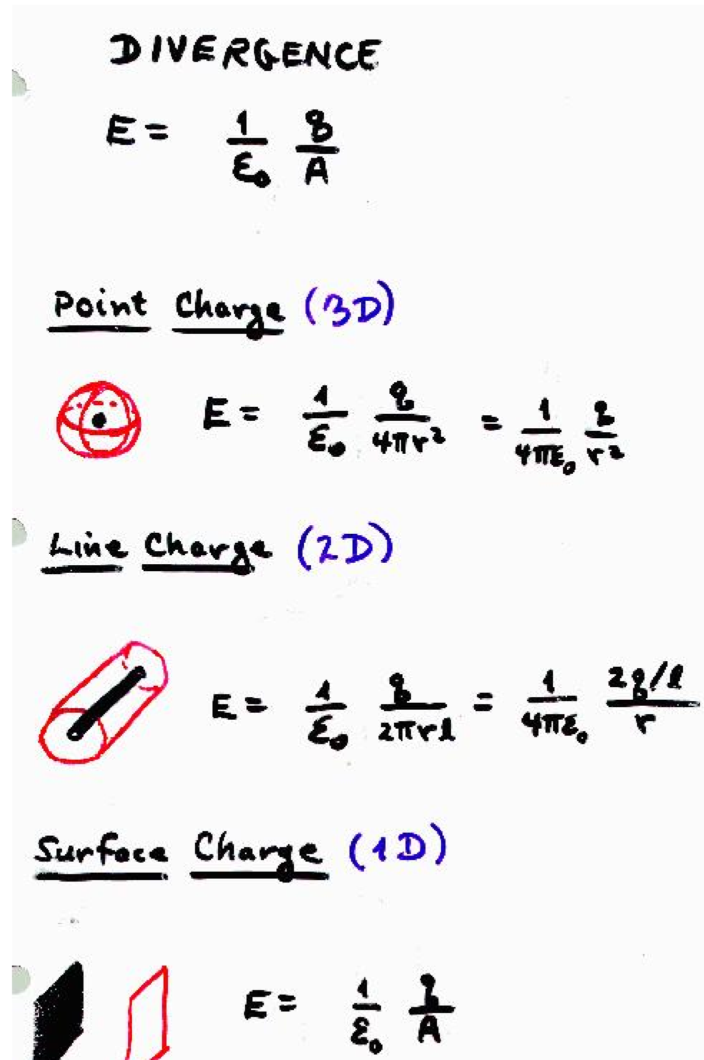
While the area integral of the electric field gives a measure of the net charge enclosed, the divergence of the electric field gives a measure of the density of sources. It also has implications for the conservation of charge.

Gauss's Law for Electricity

Flux lines



Electric
Flux lines



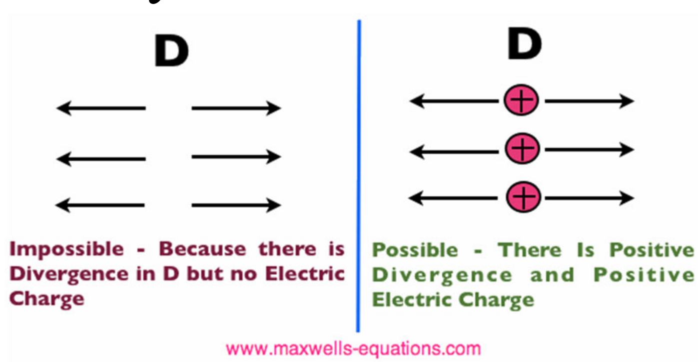
Electric flux

For instance, Gauss's law states that the flux of the electric field out of a closed surface is proportional to the electric charge enclosed in the surface (regardless of how that charge is distributed).
The constant of proportionality is the reciprocal of the permittivity of free space. Its integral form is:

$$\oint_A \epsilon_0 \mathbf{E} \cdot d\mathbf{A} = Q_A$$

The electric flux in an unclosed surface: $\phi_E = \int \varepsilon E \cdot dA$

Sometimes electric flux appears in terms of flux density D as: $\phi_E = \int D \cdot dA = \int \varepsilon E \cdot dA$



Gauss surface normal basically the del function

$$\nabla \bullet D = \rho$$

The electric elasticity equation

Displacement field (electric flux density):

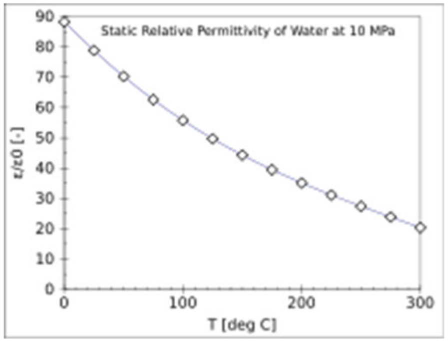
$$D = \varepsilon E$$

Where E = electric field

$\varepsilon = \varepsilon_r \varepsilon_o$ = permittivity (dielectric constant)

in air $\varepsilon_o = 8.85x10^{-12}\ F/m$

Material	ε_r
Vacuum	1 (by definition)
Air	7000100058986000000♠1.00058986 ± 6993500000000000000♠0.00000050 (at STP, for 0.9 MHz),[1]
PTFE/Teflon	2.1
Polyethylene/XLPE	2.25
Polyimide	3.4
Polypropylene	2.2–2.36
Polystyrene	2.4–2.7
Carbon disulfide	2.6
Mylar	3.1[2]
Paper	3.85
Electroactive polymers	2–12
Mica	3-6[2]
Silicon dioxide	3.9 [3]
Sapphire	8.9–11.1 (anisotropic) [4]
Concrete	4.5
Pyrex (Glass)	4.7 (3.7–10)
Neoprene	6.7[2]
Rubber	7
Diamond	5.5–10
Salt	3–15
Graphite	10–15
Silicon	11.68
Silicon nitride	7-8 (polycrystalline, 1 MHz)[5][6]
Ammonia	26, 22, 20, 17 (−80, −40, 0, 20 °C)
Methanol	30
Ethylene glycol	37
Furfural	42.0
Glycerol	41.2, 47, 42.5 (0, 20, 25 °C)
Water	88, 80.1, 55.3, 34.5 (0, 20, 100, 200 °C) for visible light: 1.77
Hydrofluoric acid	83.6 (0 °C)
Formamide	84.0 (20 °C)
Sulfuric acid	84–100 (20–25 °C)
Hydrogen peroxide	128 aq–60 (−30–25 °C)
Hydrocyanic acid	158.0–2.3 (0–21 °C)
Titanium dioxide	86–173
Strontium titanate	310
Barium strontium titanate	500
Barium titanate[7]	1200–10,000 (20–120 °C)
Lead zirconate titanate	500–6000
Conjugated polymers	1.8–6 up to 100,000[8]
Calcium copper titanate	>250,000[9][10]



W. Wang

wikipedia

Week 7

- Lecture Notes (EM wave theory)
  http://courses.washington.edu/me557/sensors/week2.pdf
- Reading Materials:

Please read all materials in Week 7 in

http://courses.washington.edu/me557/reading/

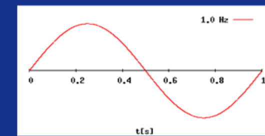
And also following notes in Week 7:

- hand written lecture notes on Maxwell's Equation
- hand written lecture notes on dereivation of Wave equation
- Homework #1 due today
- No class on Thursday
- Please start working on first problem in HW #2
- **Final presentation Dec. 26 1:20 to 3:10**

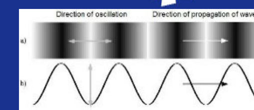
Last week

- Vector calculus (Del operator, gradient, divergence and curl)
- Maxwell’s Equations (All thing EM)
- Convert integral form to differential form

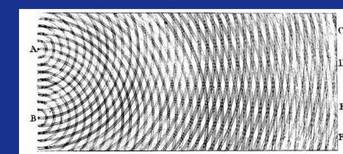
Wave Model



- A wave has a wavelength, a speed and a frequency.
- Grimaldi- observe diffraction of white light through small aperture quote, "light is a fluid that exhibits wave-like motion." (1665)

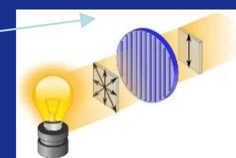


- Huygen- propose first wave model explaining reflection and refraction(1678)



Ripple tank interference

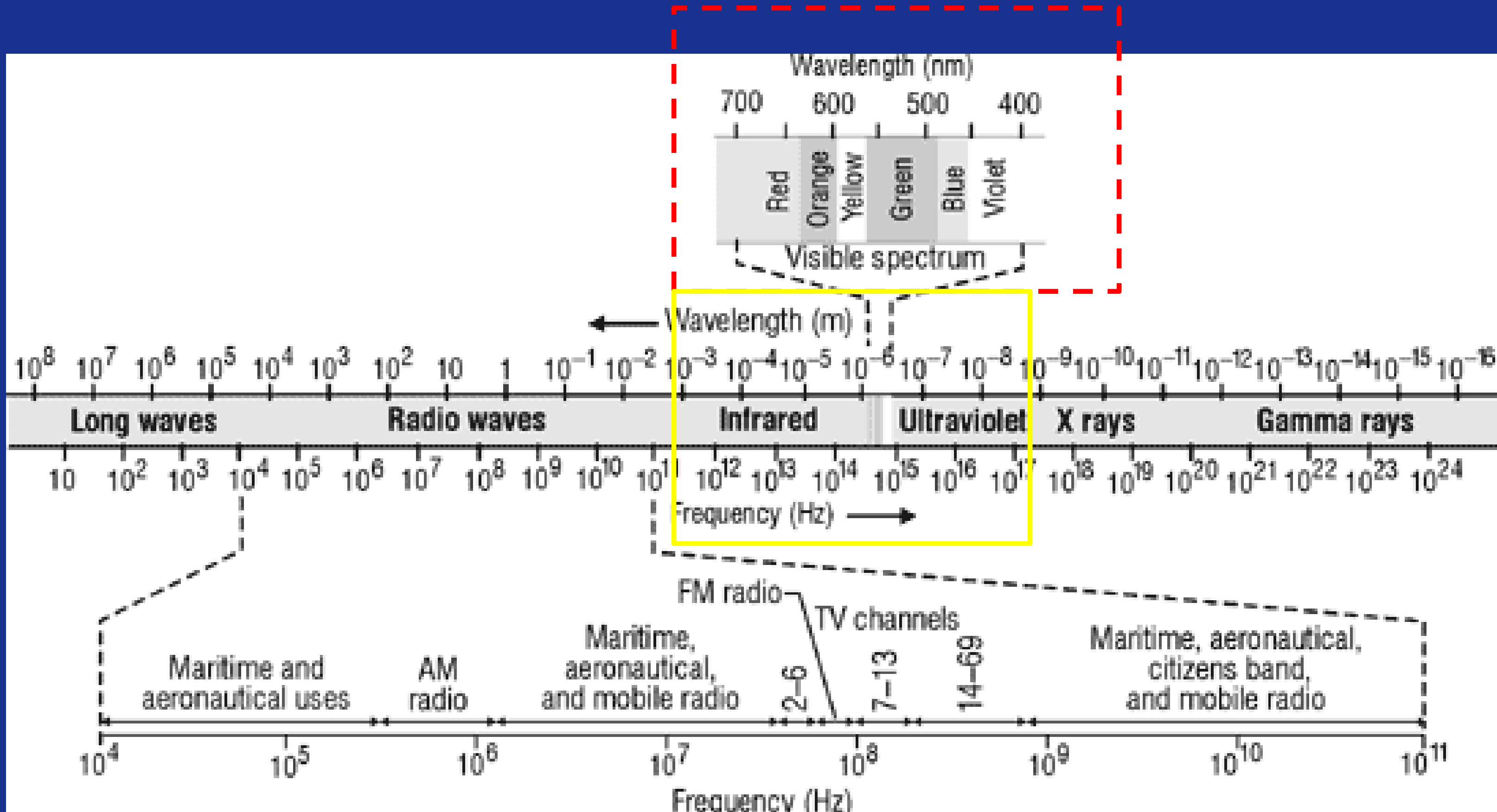
- Young- perform first interference experiment could only be explained by wave. (1801)
- Malus- observed polarization of light. (1802)

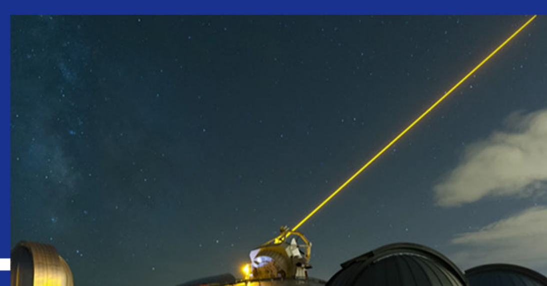


- Fresnel- gives satisfactory explanation of refraction and equation for calculating diffraction from various types of aperture (1816)
- Oersted- discover of current (1820)
- Faraday- magnetic field induces electromotive force (1830)
- Maxwell- Maxwell equation, wave equation, speed of EM wave (1830)
- Hertz- carried out experiment which produce and detect EM wave of frequencies smaller than those of light and law of reflection which can create a standing wave.

Electromagnetic Spectrum

MICRO TECH LAB





Maxwell's Equations

Integral form in the absence of magnetic or polarized media:

I. Faraday's law of induction $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$

II. Ampere's law $\oint \vec{B} \cdot d\vec{s} = \mu_0 i + \frac{1}{c^2}\frac{\partial}{\partial t}\int \vec{E} \cdot d\vec{A}$

III. Gauss' law for magnetism $\oint \vec{B} \cdot d\vec{A} = 0$

IV. Gauss' law for electricity $\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$

$\mu = \mu_o(1+\chi) = \mu_o$ if source like coil has iron place inside

$\varepsilon = \varepsilon_o$

E = Electric Field (V/m) | ρ = charge density (c/m^3) | i = electric current (A)

B = Magnetic flux density(Web/m^2, T) | ε_0 = relative permittivity | J = current density(A/m^2)

D = Electric flux density (c/m^2) or electric displacement field | μ_0 = relative permeability | c = speed of light

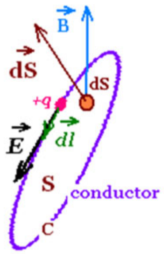
H = Magnetic Field (A/m) | Φ_B = Magnetic flux (Web) | P = Polarization

q = charge $1.6x10^{-19}$ coulombs, $\mu_o = 1.26x10^{-6}$ H/m, $\varepsilon_o = 8.85x10^{-12}$ F/m

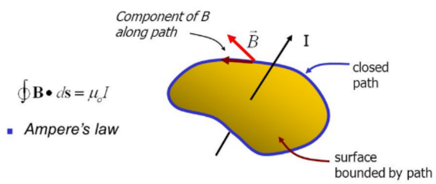
Recap

Maxwell Equations

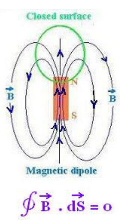
Differential form



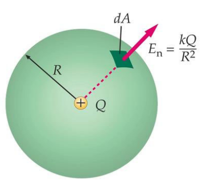
Faraday's Law $\nabla \times E = \dfrac{-\partial B}{\partial t}$



Ampere's Law $\nabla \times H = J + \dfrac{\partial D}{\partial t}$



Gauss's Law for Magnetism $\nabla \bullet B = 0$



Gauss's Law for Electricity $\nabla \bullet D = \rho$

E = Electric Field (V/m)	ρ = charge density (c/m^3)	i = electric current (A)
B = Magnetic flux density(Web/m^2, T)	ε_0 = permittivity	J = current density(A/m^2)
D = Electric flux density (c/m^2)	μ_0 = permeability	c = speed of light
H = Magnetic Field (A/m)	Φ_B = Magnetic flux (Web)	P = Polarization

Recap

Magnetic flux

We know from Gauss's law for magnetism that in a close surface,

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Normally, the magnetic flux in an unclosed surface

$$\phi_B = \int B \cdot dA$$

Where B = magnetic flux density

Recap

Magnetic susceptibility and permeability

In large class of materials, there exists a linear relation between M (internal magnetization) and H (external applied magnetic field)

$$M = \chi H$$

χ is positive then the material is called *paramagnetic*

χ is negative then the material is *diamagnetic*

A linear relationship also occurs between B (magnetic flux density) and H (external applied magnetic field)

$$B = \mu H = \mu_o (1+\chi) H = \mu_o(M+H)$$

magnetic permeability is

$$\mu = (1+\chi)\mu_o = \mu_r \mu_o$$

Where μ_r = relative permeability and μ_o= free space permeability

$\mu_r \sim 1$ for paramagentic and diamagnetic, $\mu_r >> 1$ for ferromagnetic.

Recap

Permeability

Permeability is defines as

$$\mu = (1+\chi)\,\mu_o = \mu_r\,\mu_o$$

Where μ_r is relative permeability. χ is susceptibility. Typical values for ordinary liquids and solids are in the range 1.00001 to 1.003.

– $\mu_r = 1$ when the material does not respond to the magnetic field by magnetizing.
– $\mu_r > 1$ implies material magnetizes in response to the applied magnetic field.

Susceptibility χ of dimagnetic and paramagnetic materials

Material	χ_m
Aluminum	2.3×10^{-5}
Copper	-0.98×10^{-5}
Diamond	-2.2×10^{-5}
Tungsten	6.8×10^{-5}
Hydrogen (1 atm)	-0.21×10^{-8}
Oxygen (1 atm)	209.0×10^{-8}
Nitrogen (1 atm)	-0.50×10^{-8}

If χ is positive then the material is called *paramagnetic*, and the magnetic field is strengthened by the presence of the material.
if χ is negative then the material is *diamagnetic*, and the magnetic field is weakened in the presence of the material.

Material	χ_m (x 10^{-5})
Paramagnetic	
Iron aluminum alum	66
Uranium	40
Platinum	26
Aluminum	2.2
Sodium	0.72
Oxygen gas	0.19
Diamagnetic	
Bismuth	-16.6
Mercury	-2.9
Silver	-2.6
Carbon (diamond)	-2.1
Lead	-1.8
Sodium chloride	-1.4
Copper	-1.0

Magnetic susceptibility and permeability data for selected materials

Medium	Susceptibility χ_m (volumetric SI)	Permeability μ [H/m]	Relative permeability μ/μ_0	Magnetic field	Frequency (max)
Metglas 2714A (annealed)		1.26×10^{0}	1000000[7]	at 0.5 T	100 kHz
Iron (99.95% pure Fe annealed in H)		2.5×10^{-1}	200000[8]		
Nanoperm		1.0×10^{-1}	80000[9]	at 0.5 T	10 kHz
Mu-metal		2.5×10^{-2}	20000[10]	at 0.002 T	
Mu-metal		6.3×10^{-2}	50000[11]		
Cobalt-Iron (high permeability strip material)		2.3×10^{-2}	18000[12]		
Permalloy	8000	1.0×10^{-2}	8000[10]	at 0.002 T	
Iron (99.8% pure)		6.3×10^{-3}	5000[8]		
Electrical steel		5.0×10^{-3}	4000[10]	at 0.002 T	
Ferritic stainless steel (annealed)		1.26×10^{-3} - 2.26×10^{-3}	1000–1800[13]		
Martensitic stainless steel (annealed)		9.42×10^{-4} - 1.19×10^{-3}	750–950[13]		
Ferrite (manganese zinc)		$>8.0\times10^{-4}$	640 (or more)		100 kHz ~ 1 MHz
Ferrite (nickel zinc)		2.0×10^{-5} – 8.0×10^{-4}	16–640		100 kHz ~ 1 MHz[citation needed]
Carbon Steel		1.26×10^{-4}	100[10]	at 0.002 T	
Nickel		1.26×10^{-4} - 7.54×10^{-4}	100[10] – 600	at 0.002 T	
Martensitic stainless steel (hardened)		5.0×10^{-5} - 1.2×10^{-4}	40–95[13]		
Austenitic stainless steel		1.260×10^{-6} - 8.8×10^{-6}	1.003–7 [13][14][note 1]		
Neodymium magnet		1.32×10^{-6}	1.05[15]		
Platinum		1.256970×10^{-6}	1.000265		
Aluminum	2.22×10^{-5}[16]	1.256665×10^{-6}	1.000022		
Wood		1.25663760×10^{-6}	1.00000043[16]		
Air		1.25663753×10^{-6}	1.00000037 [17]		
Concrete (dry)			1[18]		
Vacuum	0	$4\pi\times10^{-7}$ (μ_0)	1, exactly[19]		

The electric elasticity equation

Displacement field (electric flux density):

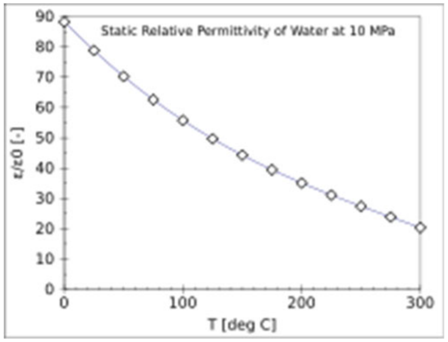
$$D = \varepsilon E$$

Where E = electric field

$\varepsilon = \varepsilon_r \varepsilon_o$ = permittivity (dielectric constant)

in air $\varepsilon_o = 8.85x10^{-12}\ F/m$

Material	ε_r
Vacuum	1 (by definition)
Air	70001000589860000001.00058986 ± 69935000000000000000.00000050 (at STP, for 0.9 MHz),[1]
PTFE/Teflon	2.1
Polyethylene/XLPE	2.25
Polyimide	3.4
Polypropylene	2.2–2.36
Polystyrene	2.4–2.7
Carbon disulfide	2.6
Mylar	3.1[2]
Paper	3.85
Electroactive polymers	2–12
Mica	3-6[2]
Silicon dioxide	3.9 [3]
Sapphire	8.9–11.1 (anisotropic) [4]
Concrete	4.5
Pyrex (Glass)	4.7 (3.7–10)
Neoprene	6.7[2]
Rubber	7
Diamond	5.5–10
Salt	3–15
Graphite	10–15
Silicon	11.68
Silicon nitride	7-8 (polycrystalline, 1 MHz)[5][6]
Ammonia	26, 22, 20, 17 (−80, −40, 0, 20 °C)
Methanol	30
Ethylene glycol	37
Furfural	42.0
Glycerol	41.2, 47, 42.5 (0, 20, 25 °C)
Water	88, 80.1, 55.3, 34.5 (0, 20, 100, 200 °C) for visible light: 1.77
Hydrofluoric acid	83.6 (0 °C)
Formamide	84.0 (20 °C)
Sulfuric acid	84–100 (20–25 °C)
Hydrogen peroxide	128 aq–60 (−30–25 °C)
Hydrocyanic acid	158.0–2.3 (0–21 °C)
Titanium dioxide	86–173
Strontium titanate	310
Barium strontium titanate	500
Barium titanate	1200–10,000 (20–120 °C)
Lead zirconate titanate	500–6000
Conjugated polymers	1.8–6 up to 100,000
Calcium copper titanate	>250,000



Electric flux

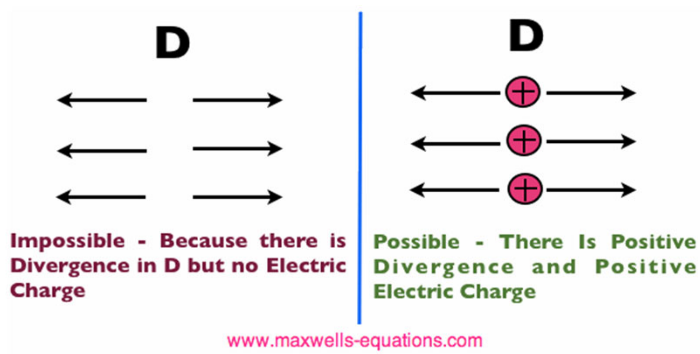
For instance, Gauss's law states that the flux of the electric field out of a closed surface is proportional to the electric charge enclosed in the surface (regardless of how that charge is distributed).
The constant of proportionality is the reciprocal of the permittivity of free space. Its integral form is:

$$\oint_A \epsilon_0 \mathbf{E} \cdot d\mathbf{A} = Q_A$$

The electric flux in an unclosed surface: $\phi_E = \int \varepsilon E \cdot dA$

Sometimes electric flux appears in terms of flux density D as:

$$\phi_E = \int D \cdot dA = \int \varepsilon E \cdot dA$$



$$\nabla \bullet D = \rho$$

Week 8

- Lecture Notes (EM wave theory)
  http://courses.washington.edu/me557/sensors/week2.pdf
- Reading Materials:

Please read materials in week 5 in:

http://courses.washington.edu/me557/reading/

- Make up classes 11/18 and 11/25 1-2PM
- Homework #1 due today.
- Final Presentation 1:20-3:10PM Dec. 27

This Week

- Derive Wave Equation from Maxwell’s Equations

  - Dispersion relation

  - Phase velocity

Vector Calculus

(differential operator) $\nabla = \hat{x}\frac{\partial}{\partial x} + \hat{y} + \frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$

(curl) $\nabla \times E = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$ where E = vector

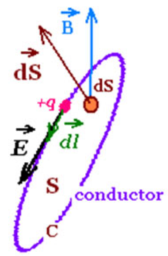
(divergent) $\nabla \cdot E = \hat{x}\frac{\partial E_x}{\partial x} + \hat{y}\frac{\partial E_y}{\partial x} + \hat{z}\frac{\partial E_z}{\partial z}$

(gradient) $\nabla \phi = \hat{x}\frac{\partial \phi}{\partial x} + \hat{y}\frac{\partial \phi}{\partial y} + \hat{z}\frac{\partial \phi}{\partial z}$

where ϕ = scalar, a function of x, y, z

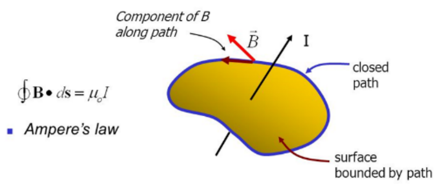
(Laplacian) $$\nabla^2 V = \frac{d^2V}{dx^2} + \frac{d^2V}{dy^2} + \frac{d^2V}{dz^2}$$

Maxwell's Equations Differential form



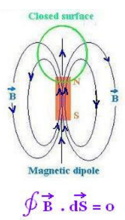
Faraday's Law

$$\nabla \times E = \frac{-\partial B}{\partial t}$$



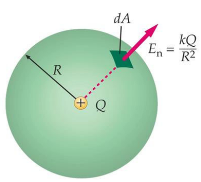
Ampere's Law

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$



Gauss's Law for Magnetism

$$\nabla \bullet B = 0$$



Gauss's Law for Electricity

$$\nabla \bullet D = \rho$$

E = *Electric Field (V/m)*	ρ = *charge density* (c/m^3)	i = *electric current (A)*
B = *Magnetic flux density*(Web/m^2, *T*)	ε_0 = *permittivity*	J = *current density*(A/m^2)
D = *Electric flux density* (c/m^2)	μ_0 = *permeability*	c = *speed of light*
H = *Magnetic Field (A/m)*	Φ_B = *Magnetic flux (Web)*	P = *Polarization*

Wave Equation

- Remember we are deriving the wave equation for wave propagation in free space

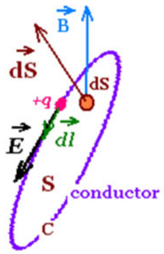
(e.g. J_s =0 and $\frac{\partial D}{\partial t} \neq 0$, M= 0, $\nabla \cdot D = \nabla \cdot \varepsilon E = 0$)

- Wave is a time harmonic function phasor can be used in representing the function

Maxwell Equations

B = μ H = μ_o (1+χ) H = μ_o(M+H)

M= 0

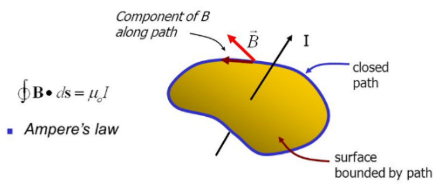


Faraday's Law

$$\nabla \times E = \frac{-\partial B}{\partial t}$$

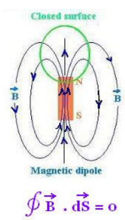
$J_s = 0$

$$\frac{\partial D}{\partial t} \neq 0$$



Ampere's Law

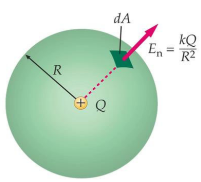
$$\nabla \times H = J + \frac{\partial D}{\partial t}$$



Gauss's Law for Magnetism

$$\nabla \bullet B = 0$$

$$\nabla \cdot D = \nabla \cdot \varepsilon E = 0$$



Gauss's Law for Electricity

$$\nabla \bullet D = \rho$$

E = Electric Field (V/m)	*ρ = charge density (c/m³)*	*i = electric current (A)*
B = Magnetic flux density(Web/m², T)	ε_0 *= permittivity*	*J = current density(A/m²)*
D = Electric flux density (c/m²)	μ_0 *= permeability*	*c = speed of light*
H = Magnetic Field (A/m)	Φ_B *= Magnetic flux (Web)*	*P = Polarization*

Wave equation
(for plane wave)

Maxwell's Equations contain the wave equation for electromagnetic waves. One approach to obtaining the wave equation:

1. Take the curl of Faraday's law:

$$\nabla \times (\nabla \times E) = -\frac{\partial(\nabla \times B)}{\partial t}$$

2. Substitute Ampere's law for a charge and current-free region:

$$\nabla \times (\nabla \times E) = -\frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$$

This is the three-dimensional wave equation in vector form. It looks more familiar when reduced a plane wave with field in the x-direction only:

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$$

Curl

The curl of a vector function is the vector product of the del operator with a vector function:

$$\nabla \times E = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)i + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)j + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)k$$

where i,j,k are unit vectors in the x, y, z directions. It can also be expressed in determinant form:

$$\begin{vmatrix} i & j & k \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

Curl in Cylindrical Polar Coordinates

The curl in cylindrical polar coordinates, expressed in determinant form is:

$$\nabla \times E = \begin{vmatrix} \frac{1_r}{r} & 1_\theta & \frac{k}{r} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ E_r & rE_\theta & E_z \end{vmatrix}$$

Curl in Spherical Polar Coordinates

The curl in spherical polar coordinates, expressed in determinant form is:

$$\nabla \times \mathbf{E} = \begin{vmatrix} \dfrac{\mathbf{1_r}}{r^2 \sin\theta} & \dfrac{\mathbf{1_\theta}}{r\sin\theta} & \dfrac{\mathbf{1_\phi}}{r} \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial \phi} \\ E_r & rE_\theta & r\sin\theta\, E_\phi \end{vmatrix}$$

Use $\nabla \times (\nabla \times E) = \nabla(\nabla \bullet E) - \nabla^2 E$ Wave equation

becomes $\nabla^2 E + \omega^2 \mu_o \varepsilon_o E = 0$

We consider the simple solution where E field is parallel to the x axis and its function of z coordinate only, the wave equation then becomes,

$$\frac{\partial^2 E_x}{\partial z^2} + \omega^2 \mu_o \varepsilon_o E_x = 0$$

A solution to the above differential equation is

$$\boxed{E = \hat{x} E_o e^{-jkz}}$$

Substitute above equation into wave equation yields,

$$(-k^2 + \omega^2 \mu\varepsilon)E = 0 \Rightarrow \boxed{k^2 = \omega^2 \mu\varepsilon}$$ (dispersion relation)

Vector Calculus

- Useful vector relationships for the vector fields a , b, and c are

$$\nabla \cdot (\nabla \times \mathbf{a}) = 0$$

recall

$$\nabla \times \nabla \times \mathbf{a} = \nabla(\nabla \cdot \mathbf{a}) - \nabla^2 \mathbf{a}$$

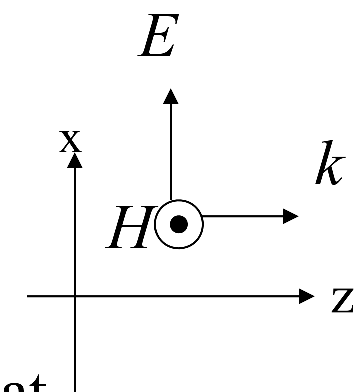
$$\nabla \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot (\nabla \times \mathbf{a}) - \mathbf{a} \cdot (\nabla \times \mathbf{b})$$

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$$

$$\mathbf{a} \cdot \mathbf{b} \times \mathbf{c} = \mathbf{b} \cdot \mathbf{c} \times \mathbf{a} = \mathbf{c} \cdot \mathbf{a} \times \mathbf{b}$$

Let's transform the solution for the wave equation into real space and time, (assume time harmonic field)

$$E(z,t) = \mathrm{Re}\{Ee^{j\omega t}\} = \hat{x}E_o \cos(\omega t - kz)$$



$k = 2\pi/\lambda$, where k = wave number

Image we riding along with the wave, we asked what Velocity shall we move in order to keep up with the wave, The answer is phase of the wave to be constant

$$\omega t - kz = \text{a constant}$$

The velocity of propagation is therefore given by,

$$\boxed{\frac{dz}{dt} = v = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_o \varepsilon_o}}}$$ (phase velocity)

Derivation of Wave Equation

- Please read the hand written handout for more complete derivation in: http://courses.washington.edu/me557/readings/summary_maxwell.pdf

$\Rightarrow ma + kx = 0$

equivalent to a mass spring mechanical system

→ Faraday's Law

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad \text{—— (1)}$$

→ take curl of Faraday's Law

$$\nabla \times (\nabla \times E) = -\frac{\partial (\nabla \times B)}{\partial t} \quad \text{—— (2)}$$

→ Substitute Ampere's Law for a charge and current free region :

Ampere's Law : $\nabla \times H = \cancel{J}^{\,0} + \frac{\partial D}{\partial t}$

since

$B = \mu_o(H + M)$

$$\left\{ \begin{array}{l} H = \frac{B}{\mu} \ \ \& \ \ \mu = \mu_o \text{ for non magnetic material}, \ M = 0 \\ D = \epsilon E \quad \text{electric elasticity equation} \\ c = \frac{1}{\sqrt{\epsilon \mu}} \quad \text{Speed of light} \end{array} \right.$$

$$\boxed{\nabla \times H = \frac{\partial D}{\partial t} \Rightarrow \nabla \times B = \mu_o \epsilon \frac{\partial E}{\partial t}} \quad \text{—— (3)}$$

$$= \frac{1}{c^2} \frac{\partial E}{\partial t}$$

$$\nabla\times(\nabla\times E) = -\frac{\partial(\nabla\times B)}{\partial t} \quad — \quad ②$$

$$\boxed{\nabla\times H = \frac{\partial D}{\partial t} \Rightarrow \nabla\times B = \mu_0\epsilon\frac{\partial E}{\partial t}} \quad — \quad ③$$

$$= \frac{1}{c^2}\frac{\partial E}{\partial t}$$

→ sub ③ back into 2 :

$$\nabla\times(\nabla\times E) = -\mu_0\epsilon\frac{\partial^2 E}{\partial t^2} = -\frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$$

$$\nabla\times\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ E_x & E_y & E_z\end{vmatrix} = -\frac{1}{c^2}\frac{\partial E}{\partial t^2} = -\frac{1}{c^2}\left[\frac{\partial E_x}{\partial t^2}\hat{x} + \frac{\partial E_y}{\partial t^2}\hat{y} + \frac{\partial E_z}{\partial t^2}\hat{z}\right.$$

The electric elasticity equation

Displacement field (electric flux density):

$$D = \varepsilon E$$

Where E = electric field

$\varepsilon = \varepsilon_r \varepsilon_o$ = permittivity (dielectric constant)

in air $\varepsilon_o = 8.85x10^{-12}\ F/m$

But when the generated fields pass through magnetic materials which themselves contribute internal magnetic fields, ambiguities can arise about what part of the field comes from the external currents and what comes from the material itself. It has been common practice to define another magnetic field quantity, usually called the "magnetic field strength" designated by H. It can be defined by the relationship

$$B = \mu_o (H + M)$$

M = magnetization. Normally, the $M = 0$ for nonmagnetic material

If in air, $\mu_o = 1.26 \times 10^{-6} H/m$

Curl

The curl of a vector function is the vector product of the del operator with a vector function:

$$\nabla \times E = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) i + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) j + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) k$$

where i,j,k are unit vectors in the x, y, z directions. It can also be expressed in determinant form:

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

Curl in Cylindrical Polar Coordinates

The curl in cylindrical polar coordinates, expressed in determinant form is:

$$\nabla \times E = \begin{vmatrix} \dfrac{1_r}{r} & 1_\theta & \dfrac{k}{r} \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial z} \\ E_r & rE_\theta & E_z \end{vmatrix}$$

Curl in Spherical Polar Coordinates

The curl in spherical polar coordinates, expressed in determinant form is:

$$\nabla \times E = \begin{vmatrix} \dfrac{1_{\mathbf{r}}}{r^2 \sin\theta} & \dfrac{1_{\boldsymbol{\theta}}}{r \sin\theta} & \dfrac{1_{\boldsymbol{\phi}}}{\mathbf{r}} \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial \phi} \\ E_{\mathbf{r}} & rE_{\boldsymbol{\theta}} & r \sin\boldsymbol{\theta}\, E_{\boldsymbol{\phi}} \end{vmatrix}$$

$$\nabla \times (\nabla \times E) = -\mu_0 \epsilon \frac{\partial^2 E}{\partial t^2} = -\frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$$

Use an identity

$$\boxed{\nabla \times (\nabla \times E) = \nabla(\nabla \cdot E) - \nabla^2 E} \quad — (4)$$

above equation becomes

$$\boxed{\nabla(\nabla \cdot E) - \nabla^2 E = -\frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}} \quad — (5)$$

Vector Calculus

- Useful vector relationships for the vector fields a , b, and c are

$$\nabla \cdot (\nabla \times \mathbf{a}) = 0$$

recall

$$\nabla \times \nabla \times \mathbf{a} = \nabla(\nabla \cdot \mathbf{a}) - \nabla^2 \mathbf{a}$$

$$\nabla \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot (\nabla \times \mathbf{a}) - \mathbf{a} \cdot (\nabla \times \mathbf{b})$$

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$$

$$\mathbf{a} \cdot \mathbf{b} \times \mathbf{c} = \mathbf{b} \cdot \mathbf{c} \times \mathbf{a} = \mathbf{c} \cdot \mathbf{a} \times \mathbf{b}$$

$$\boxed{\nabla(\nabla\cdot E) - \nabla^2 E = -\frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}} \quad — (5)$$

since $\nabla\cdot E = 0$ for charge free region (Gauss's Law of electricity)

(travel in free space)

$\frac{\partial E}{\partial t} = j\omega E$ assume time harmonic function

[redacted] equation (5) now becomes

$$0 - \nabla^2 E = \frac{\omega^2}{c^2} E$$

(wave equation) $$\boxed{\nabla^2 E + \frac{\omega^2}{c^2} E = 0} \quad — (6)$$

recall $\nabla^2 V = \frac{d^2 V}{dx^2} + \frac{d^2 V}{dy^2} + \frac{d^2 V}{dz^2}$

We consider a simple solution where E field is parallel to the x axis & is function of z coordinate only, the wave equation then becomes

recall

$$\nabla^2 V = \frac{d^2V}{dx^2} + \frac{d^2V}{dy^2} + \frac{d^2V}{dz^2}$$

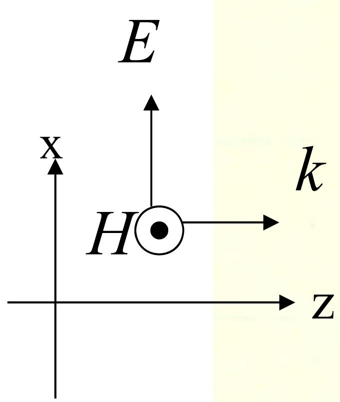
Also you can define you own xyz in free space

$$\boxed{\frac{\partial^2 E_x}{\partial z^2} + \frac{\omega^2}{c^2} E_x = 0} \quad \text{— (7)}$$

A solution to the above differential equation is

$$\boxed{E = \hat{x} E_0 e^{-jkz}} \quad \text{— (8)}$$

(in phasor form because of time harmonic function)

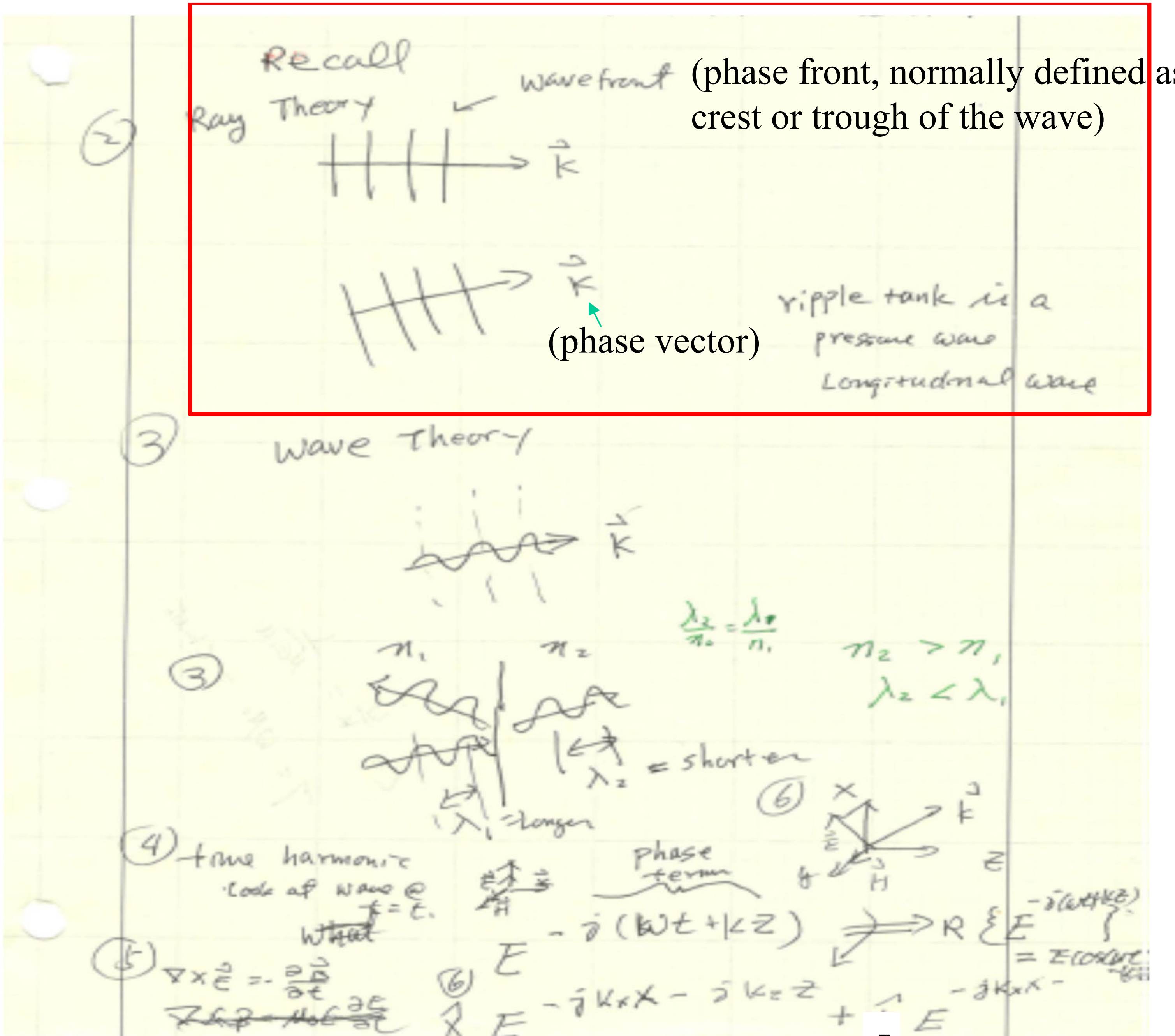


Substitute above equation into wave equation (eq 7)

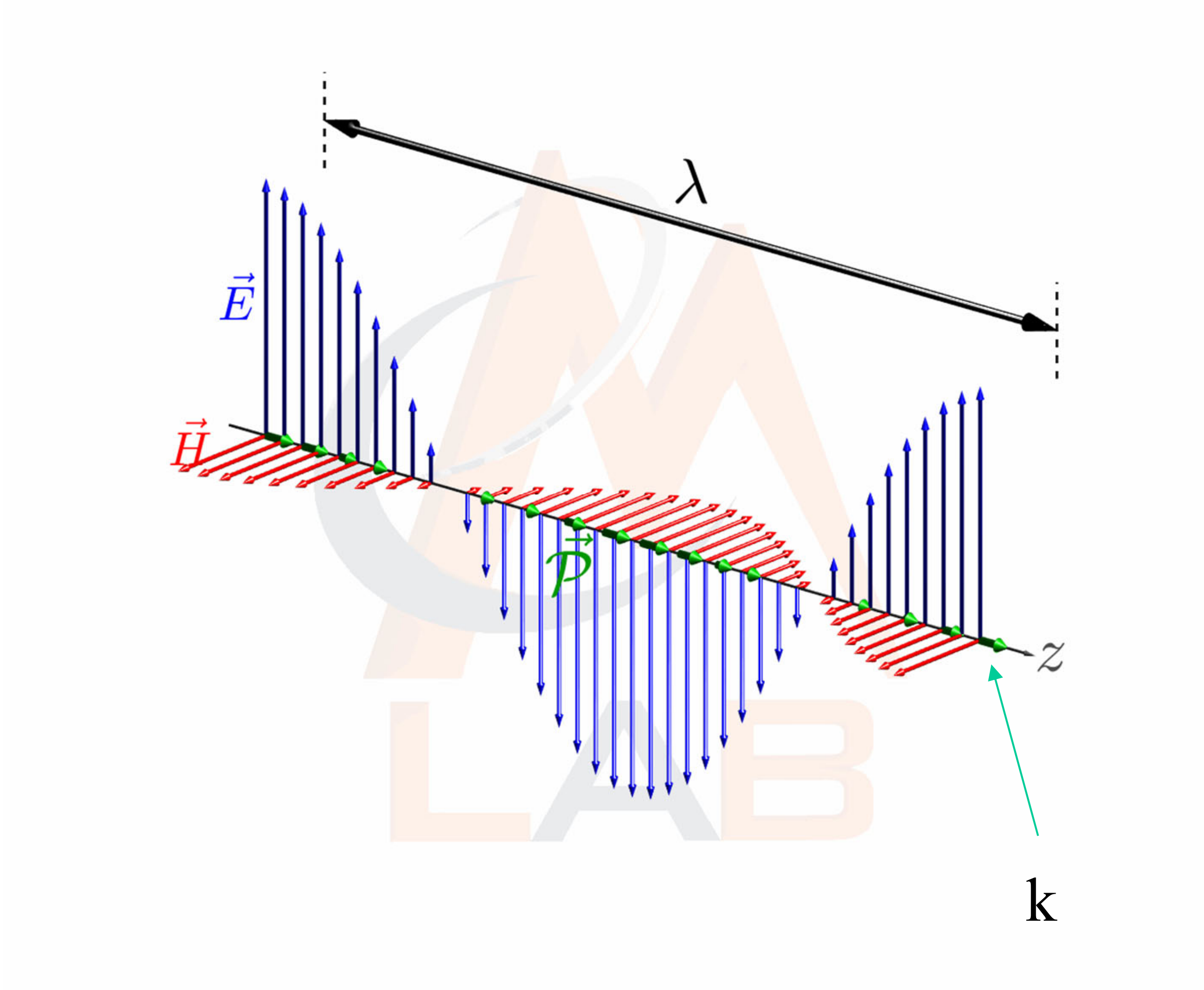
$$\left(-k^2 + \frac{\omega^2}{c^2}\right) E = 0 \Rightarrow \boxed{k^2 = \frac{\omega^2}{c^2}} \quad \text{(dispersion relation)}$$

$$k = \frac{\omega}{c} = \frac{2\pi f}{c} \quad (k = \text{wave number or propagation constant})$$

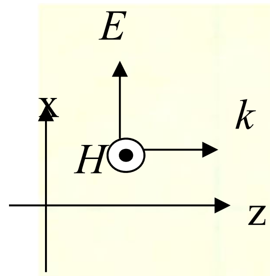
Let's transform the solution for the wave equation into real space & time (assume time harmonic field)


Recall
(2) Ray Theory
wavefront
(phase front, normally defined as crest or trough of the wave)
$\vec{k}$
$\vec{k}$
(phase vector)
ripple tank is a pressure wave
Longitudinal wave
(3) Wave Theory
$\vec{k}$
(3)
n_1
n_2
$\frac{\lambda_2}{n_o} = \frac{\lambda_o}{n_1}$
$n_2 > n_1$
$\lambda_2 < \lambda_1$
λ_2 = shorter
λ_1 = longer
(4) time harmonic
look at wave @ t = t.
phase term
$E^{-j(\omega t + kz)}$
$\Rightarrow R\{E^{-j(\omega t + kz)}\}$
$= E\cos(\omega t \ldots)$
(6)
$\vec{k}$
$\vec{E}$
$\vec{H}$
(5) $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
(6) $\hat{x}\, E^{-jk_x x - jk_z z}$
z

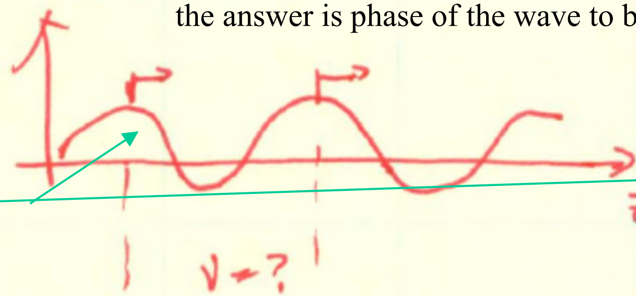
Poynting Vector (Propagation vector)



$$E(z,t) = Re\{E e^{j\omega t}\} = \hat{x} E_0 \cos(\underbrace{\omega t - kz}_{\text{phase}})$$



Imagine we riding along with the wave, we asked what velocity shall we move in order to keep up with the wave, the answer is phase of the wave to be constant



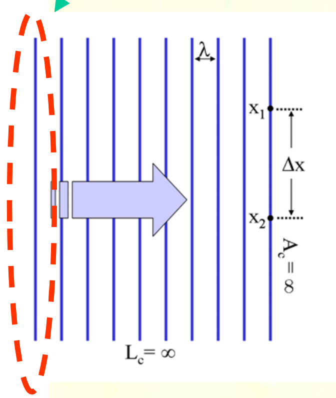
$$\omega t - kz = \text{phase} = \text{constant}$$

Phase of this wave varying with time but spacing of phase front is constant

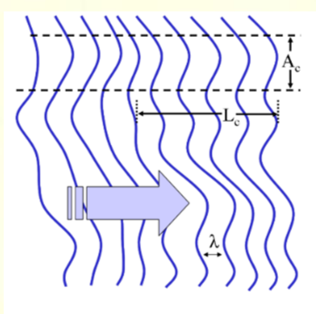
take time derivative

$$\frac{d(\omega t)}{dt} - k\frac{dz}{dt} = 0$$

Plane wave



phase velocity $\boxed{\frac{dz}{dt} = v = \frac{\omega}{k}}$



Phase velocity not constant
Longitudinally and laterally!!
k(z) – longitudinally
Not plane wave- laterally

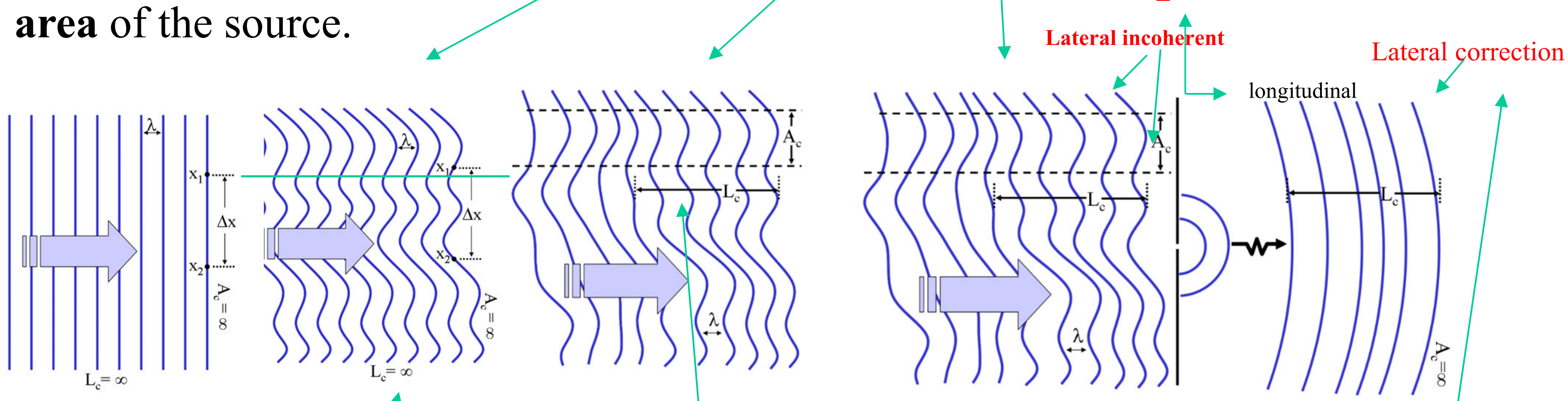
$$\omega = k\frac{dz}{dt}$$

$$\frac{\omega}{k} = \frac{dz}{dt}$$

If w is not constant or kz kx ky are not constant then the phase term will not be the same over time!!!!!! (this is called spatial or time incoherence)

Spatial Coherence

Spatial coherence is a measure of the correlation between the phases of a light wave at different points transverse to the direction of propagation. Spatial coherence tells us how uniform the phase of the wave front is. A distance L from a thermal monochromatic (line) source whose linear dimensions are on the order of δ, two slits separated by a distance greater than $d_c = 0.16\lambda L/\delta$ will no longer produce a recognizable interference pattern. We call $\pi d_c^2/4$ the **coherence area** of the source.



A plane wave with an infinite coherence length.

A wave with a varying profile (wavefront) and infinite coherence length (spatial conherent in longitudinal direction)

A wave with a varying profile (wavefront) and finite coherence length. (spatial inconherent in longitudinal direction)

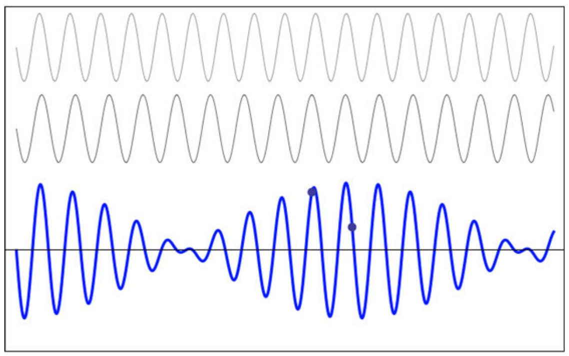
A wave with finite coherence area (both longitudinal and transverse directions are incoherent) is incident on a pinhole (small aperture). The wave will diffract out of the pinhole. Far from the pinhole the emerging spherical **wavefronts are approximately flat. The coherence area is now infinite while the longitudinal coherence length is unchanged.**

w.wang

Temporal coherent ~ varying coherent length (longitudinal) Spatial coherent ~ varying coherent length (transverse)

Group Velocity

Group velocity is trickier. The word 'group' suggests that the concept involves more than one wave. Because two is the first whole number larger than one, the simplest illustration uses to two waves:



$$\sin A+\sin B = 2\sin(A+B)/2 * \underline{\cos(A-B)/2}$$

Let $A = k_1x+\omega_1 t \quad k_1 = 2\pi n_1/\lambda_1$

$B = k_2x+\omega_2 t \quad k_2 = 2\pi n_1/\lambda_2= k_1+\Delta k$

$\omega_2=\omega_1+\Delta\omega$

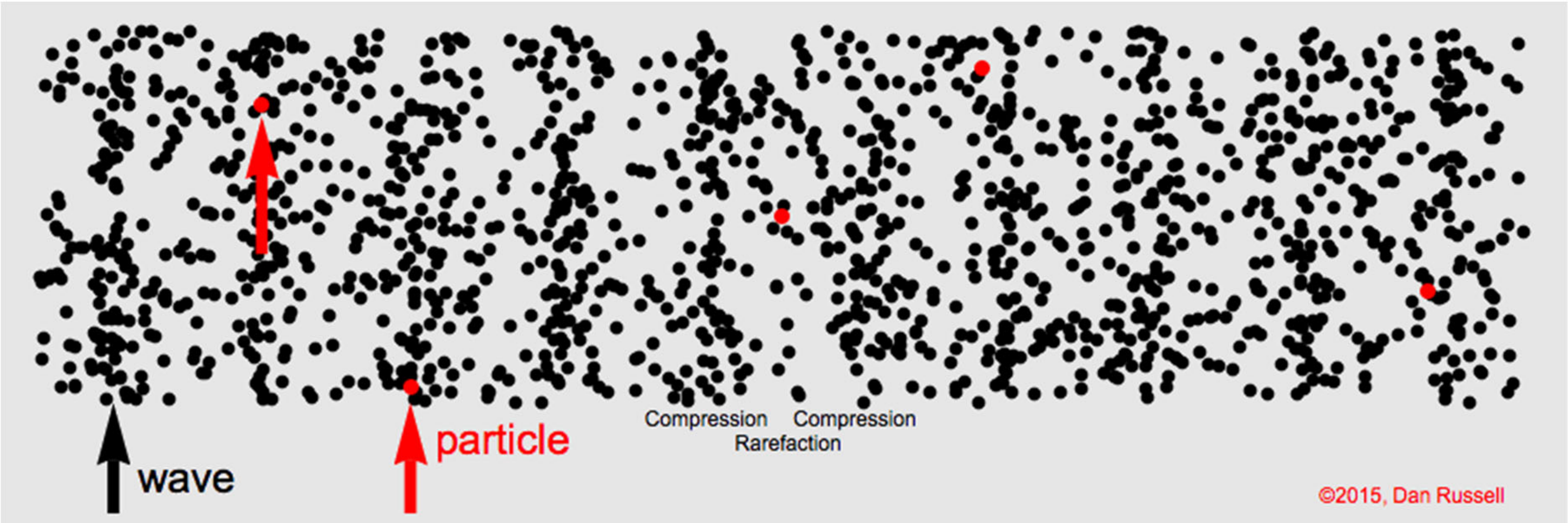
So it is a wave with wavenumber $\Delta k/2$ and frequency $\Delta\omega/2$. The **envelope's phase velocity** is the group velocity of f_1 and f_2:

$$v_g= \omega/k = (\Delta\omega/2)/(\Delta k/2)= \Delta\omega/\Delta k$$

In the limit where $\Delta\omega$ -> 0 and Δk-> 0, the group velocity becomes :

$$v_g= d\omega/dk$$

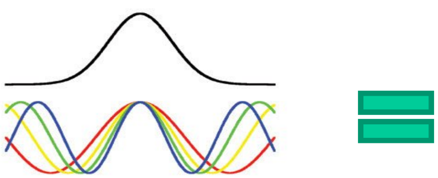
Longitudinal Wave

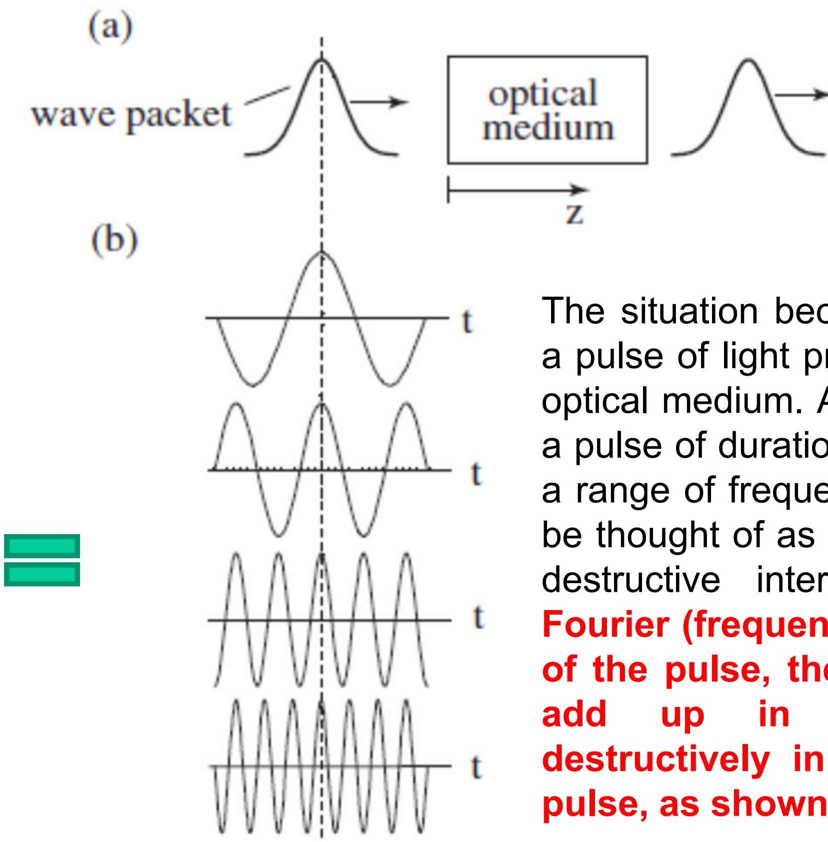


Group velocity (net) is moving forward horizontally, phase velocity (localize particle) oscillates also horizontally

Traverse and Longitudinal Wave

Multi-wavelengths





The situation becomes more complicated when a pulse of light propagates through a dispersive optical medium. According to Fourier's theorem, a pulse of duration τ is necessarily composed of a range of frequencies. In a sense, a pulse can be thought of as resulting from constructive and destructive interference among the various **Fourier (frequency) components. At the peak of the pulse, these components will tend to add up in phase, while interfering destructively in the temporal wings of the pulse, as shown in Figure 2.**

Figure 2: Schematic representation of an optical pulse in terms of its various spectral components. Note that these contributions add in phase at the peak of the pulse.

RELATIONSHIP BETWEEN GROUP VELOCITY V_g AND PHASE VELOCITY V_p

Based on the definition $\mathbf{Vp = \omega/k = C_o/n}$, we can replace $\boldsymbol{\omega}$ with **kxVp**, then we get

$$\boxed{V_g = \frac{\partial \omega}{\partial k} = \frac{\partial (k V_p)}{\partial k} = V_p + k \frac{dV_p}{dk}}$$

(In terms of k and V_p)

Since

$$k = \frac{2\pi}{\lambda}$$

and

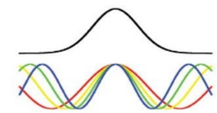
$$dk = -\frac{2\pi}{\lambda^2} d\lambda$$

the we get

$$\boxed{V_g = V_p - \lambda \frac{dV_p}{d\lambda}}$$

(In terms of λ and V_p)

Group velocity



Let us next consider the propagation of a pulse through a material system. **A pulse is necessarily composed of a spread of optical frequencies**, as illustrated symbolically in Figure next page. At the peak of the pulse, the **various Fourier components will tend to add up in phase**. If this pulse is to propagate without distortion, these components must add in phase for all values of the propagation distance z. To express this thought mathematically, we first write the phase of the wav

$$\phi = \frac{n\omega z}{c} - \omega t$$

and require that there be no change in φ to first order in ω. That is, $d\varphi/d\omega = 0$ or

$$\frac{dn}{d\omega}\frac{\omega z}{c} + \frac{nz}{c} - t = 0\ ,$$

which can be written as $z = v_g t$ where the group velocity is given by

$$\boxed{v_g = \frac{c}{n + \omega\, dn/d\omega} = \frac{d\omega}{dk}\ .}$$

(In terms of n and ω)

The last equality in this equation results from the use of the relation $k = n\omega/c$. Alternatively, we can express this result in terms of a group refraction index n_g defined by

$$\boxed{v_g = \frac{c}{n_g}}$$

With

$$\boxed{n_g = n + \omega\,\frac{dn}{d\omega}\ .}$$

(In terms of n and ω)

We see that the group index differs from the phase index by a term that depends on the dispersion dn/dω of the refractive index.

Calculating Group Velocity vs. Wavelength

We more often think of the refractive index in terms of wavelength,so let's write the group velocity in terms of the vacuum wavelength λ_0.

Use the chain rule: $\dfrac{dn}{d\omega} = \dfrac{dn}{d\lambda_0}\dfrac{d\lambda_0}{d\omega}$

Now, $\lambda_0 = \dfrac{2\pi c_0}{\omega}$, so: $\dfrac{d\lambda_0}{d\omega} = \dfrac{-2\pi c_0}{\omega^2} = \dfrac{-2\pi c_0}{(2\pi c_0/\lambda_0)^2} = \dfrac{-\lambda_0^2}{2\pi c_0}$

Recalling that: $\boxed{v_g = \left(\dfrac{c_0}{n}\right) \Big/ \left[1 + \dfrac{\omega}{n}\dfrac{dn}{d\omega}\right]}$ In terms of n and ω

we have: $v_g = \left(\dfrac{c_0}{n}\right) \Big/ \left[1 + \dfrac{\cancel{2\pi c_0}}{n\cancel{\lambda_0}}\left\{\dfrac{dn}{d\lambda_0}\left(\dfrac{-\lambda_0^{\cancel{2}}}{\cancel{2\pi c_0}}\right)\right\}\right]$

or: $\boxed{v_g = \left(\dfrac{c_0}{n}\right) \Big/ \left(1 - \dfrac{\lambda_0}{n}\dfrac{dn}{d\lambda_0}\right)} = \dfrac{c_0}{n - \lambda_0 \dfrac{dn}{d\lambda_0}}$ In terms of n and λ

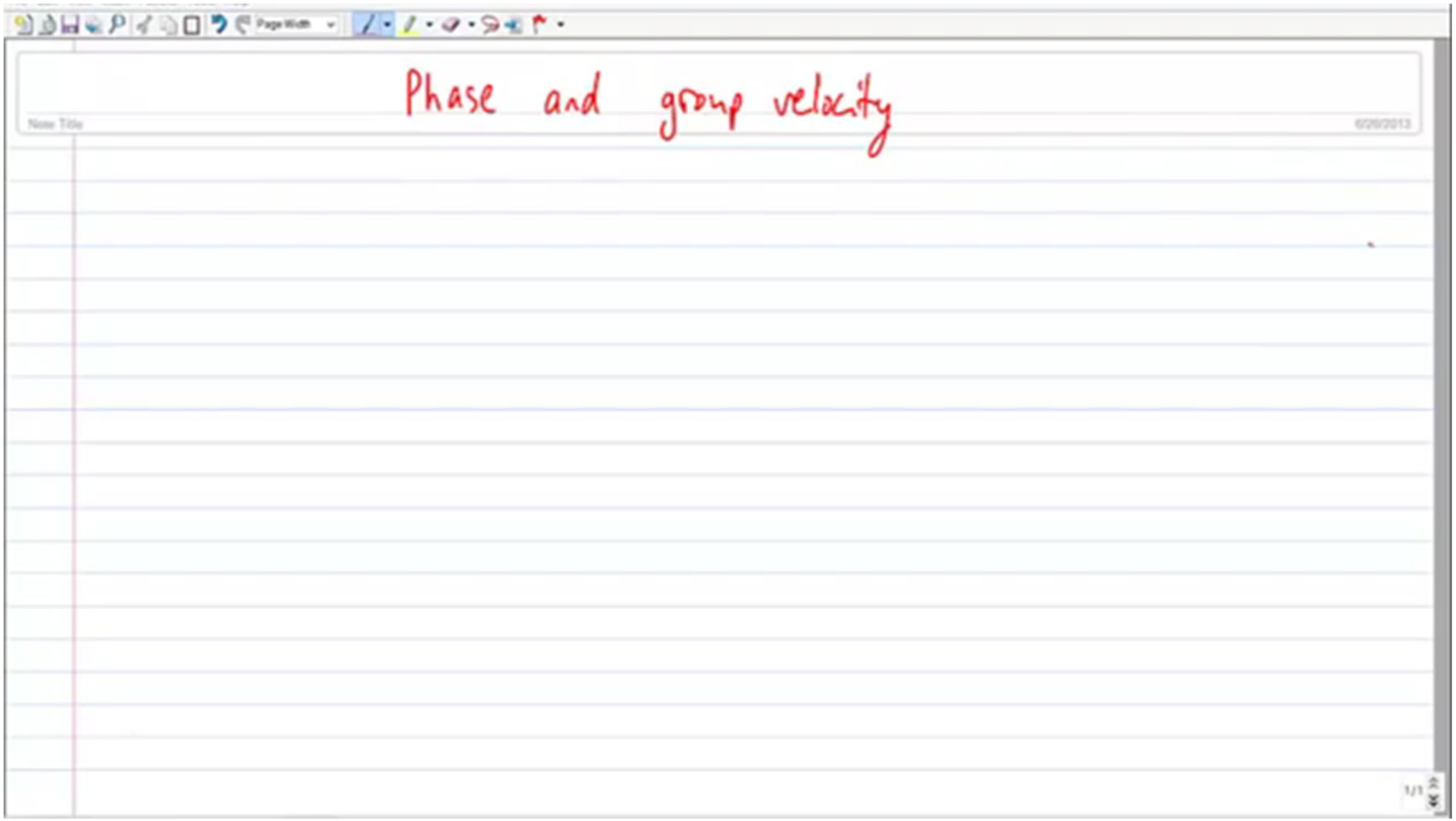
Phase and Group Velocity

Phase velocity ($v_p = \omega/k$) : motion of point on underlining sinusoidal wave (v_p could $\geq$ c)

Group velocity ($v_g = d\omega/dk$): velocity of overall pulses (Physically relevant velocity only one that is limited by speed of light in vacuum C_o because it's a propagation of energy and <u>information</u>, v_g always $\underline{\leq \boldsymbol{c_0/n < c_0}}$)

True only for information… but not energy… so above statement only true for energy because in anomalous material, Vg can be greater than Co

Phase and Group Velocity



Phase and Group Velocity

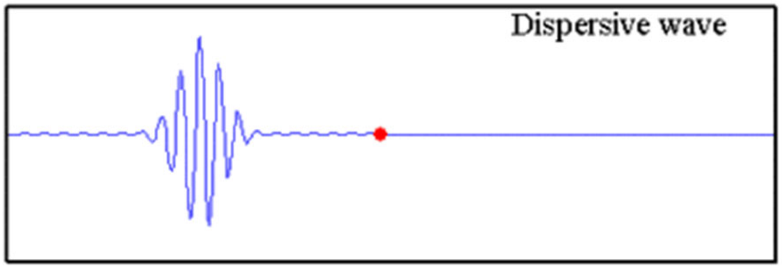
Dispersive and Nondispersive (function of λ and not)

math

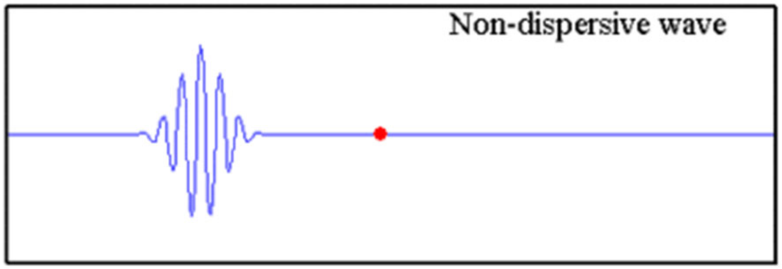
$$V_g = V_p - \lambda \frac{dV_p}{d\lambda}$$

Type of wave	Condition	Formula
Dispersive wave	dV_p/dk or $dV_p/d\lambda \neq 0$	$V_p \neq V_g$
Non-dispersive wave	dV_p/dk or $dV_p/d\lambda = 0$	$V_p = V_g$

V_p is different for different λ
V_g is more spread out



V_p is same for different λ
V_g is more compact because
V_p -=V_g



Dispersive and Nondispersive

math

Use two waves to explain

Usually, group velocity **is not equal to phase velocity, except in empty space.**

For our example, $$v_g \equiv \frac{\Delta\omega}{\Delta k} = \frac{c_0 k_1 - c_0 k_2}{n_1 k_1 - n_2 k_2}$$

Or use

$$v_g = \frac{c}{n + \omega\, dn/d\omega} = \frac{d\omega}{dk} .$$

$$n_g = n + \omega \frac{dn}{d\omega} .$$

$$v_g = \frac{c_o}{n_g}$$

$v_p = \omega/k = C_o/n$

where the subscripts 1 and 2 refer to the values at ω_1 and at ω_2. k_1 and k_2 are the k-vectors in vacuum.

If $n_1 = n_2 = n$, $\quad v_g = \frac{c_0}{n}\frac{k_1 - k_2}{k_1 - k_2} = \frac{c_0}{n} =$ phase velocity

If $n_1 \neq n_2$, $\quad v_g \neq$ phase velocity fact

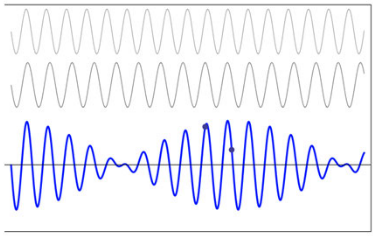
Phase and Group velocity

When these two monochrome waves are propagating in vacuum, they have the same phase velocity c (the speed of light in vacuum), and the superposed wave's phase velocity equals its group velocity(both are c).

$$V_p = V_g = C$$

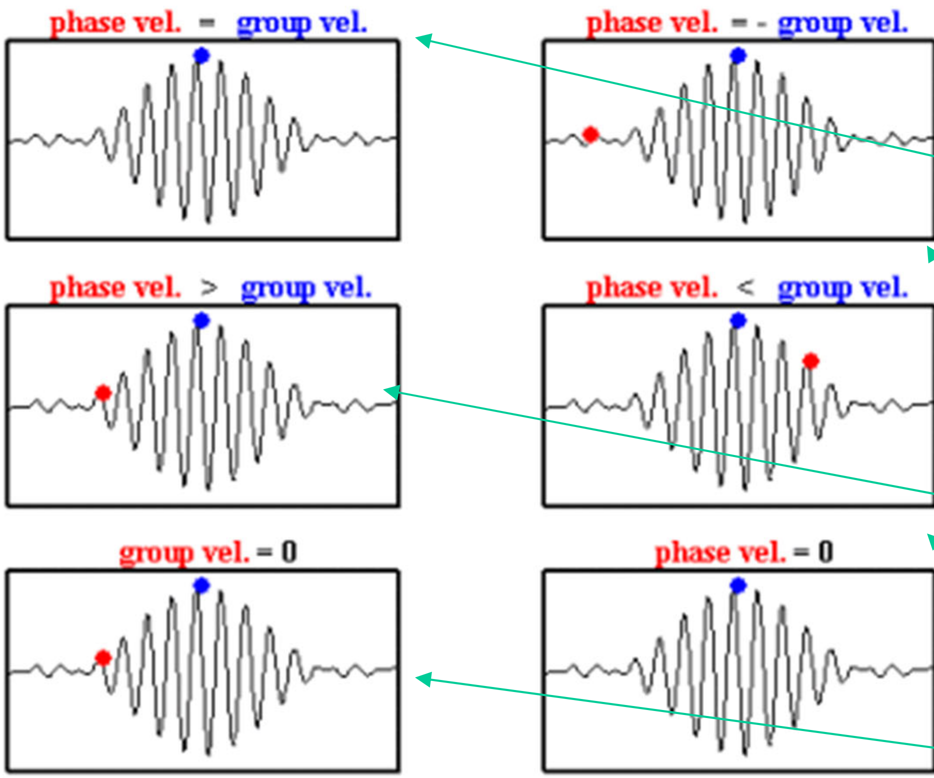
However, when these two monochrome waves are propagating in a dispersive medium, they will have different velocities and thus the superposed wave will have a phase velocity V_p that is different from its group velocity V_g.

So that means the bigger $dV_p/d\lambda$, the bigger the difference of velocities for different wavelengths, and the bigger the difference between the superposed wave's V_p and V_g.



$$V_g = V_p - \lambda \boxed{\frac{dV_p}{d\lambda}}$$

Dispersive and Nondispersive Medium



https://web.bryanston.co.uk/physics/Applets/Wave%20animations/Sound%20waves/Dispersive%20waves.htm

The group velocity is the speed of the wavepacket and the phase velocity is the speed of the individual waves. The following movies show wave packets with various combinations of phase and group velocities.

Phase velocity= Group Velocity

The entire waveform—the component waves and their envelope—moves as one. This is an example of a non-dispersive wave.

Phasevelocity = -Group Velocity

The envelope moves in the opposite direction of the component waves.

Phasevelocity > Group Velocity

The component waves move more quickly than the envelope.

Phasevelocity < Group Velocity

The component waves move more slowly than the envelope.

GroupVelocity = 0

The envelope is stationary while the component waves move through it.

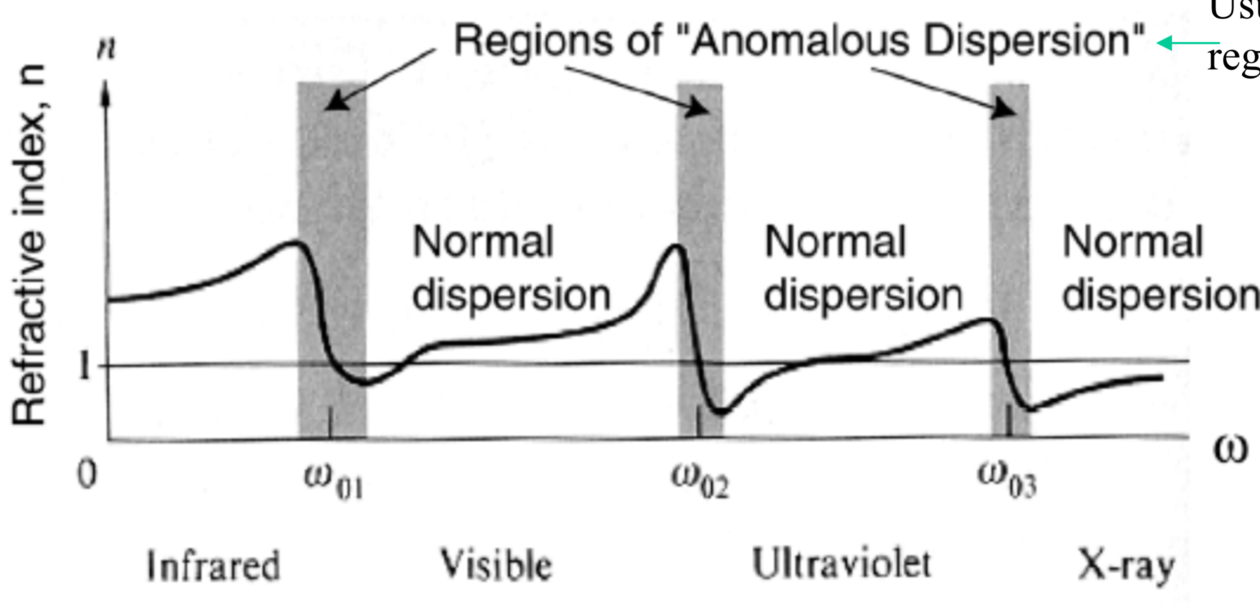
Phasevelocity = 0

Now only the envelope moves over stationary component waves.

NORMAL DISPERSION

In regions of normal dispersion, $dn/d\omega$ is positive. So $\underline{v_g < c_0/n < c_0}$ for these frequencies.

$$\boxed{v_g = \frac{c_0}{n + \omega \dfrac{dn}{d\omega}}}$$



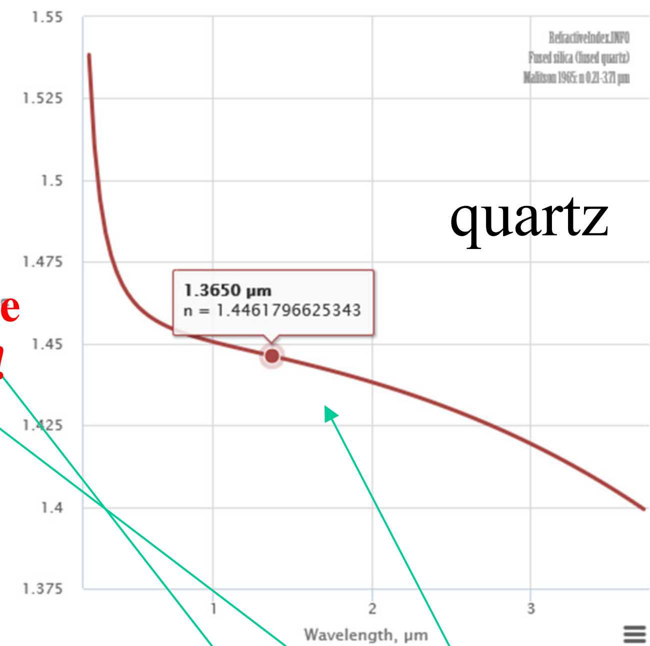
Usually resonating region and lossy

$V_p > V_g$

NORMAL DISPERSION

If in the dispersive medium, **$dV_p/d\lambda > 0$**, then **longer wavelength lights propagate faster than shorter wavelength lights, this is called normal dispersion.**

In this case, the superposed wave's group velocity V_g is smaller than its phase velocity V_p, and in some cases, the group velocity can even be negative (travels backward)!

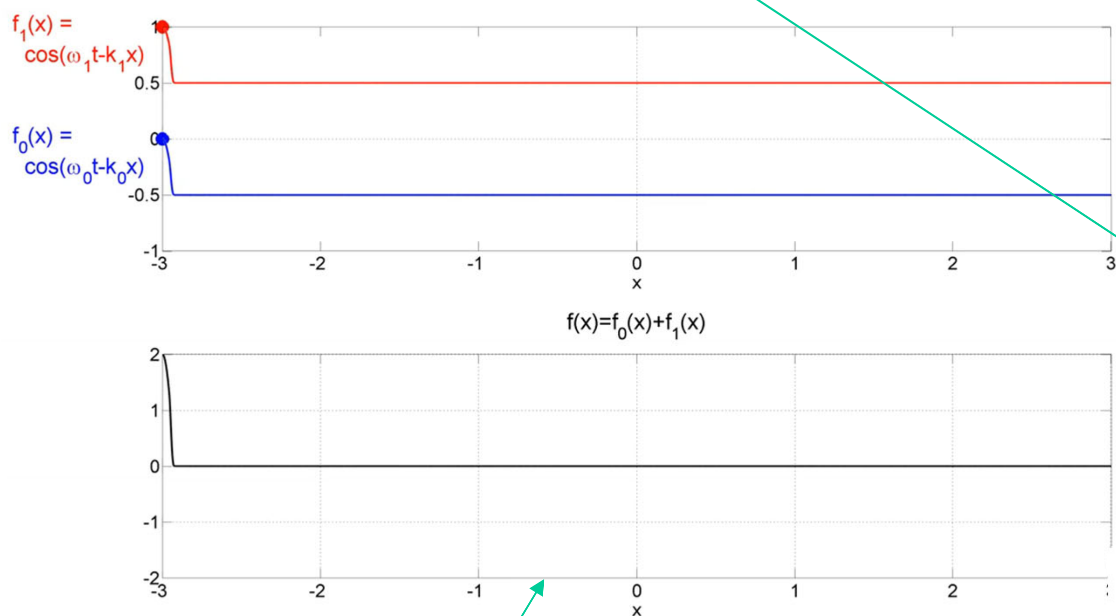


$$V_p = C_o/n$$

$$V_g = V_p - \lambda \boxed{\frac{dV_p}{d\lambda}}$$

$$v_g = \left(\frac{c_0}{n}\right) \Big/ \left(1 - \frac{\lambda_0}{n}\frac{dn}{d\lambda_0}\right) = \frac{c_0}{n - \lambda_0 \boxed{\frac{dn}{d\lambda_0}}}$$

$$v_g = \frac{d\omega}{dk} = \frac{1}{dk/d\omega} = \frac{c}{n + \omega \boxed{\frac{dn}{d\omega}}}.$$

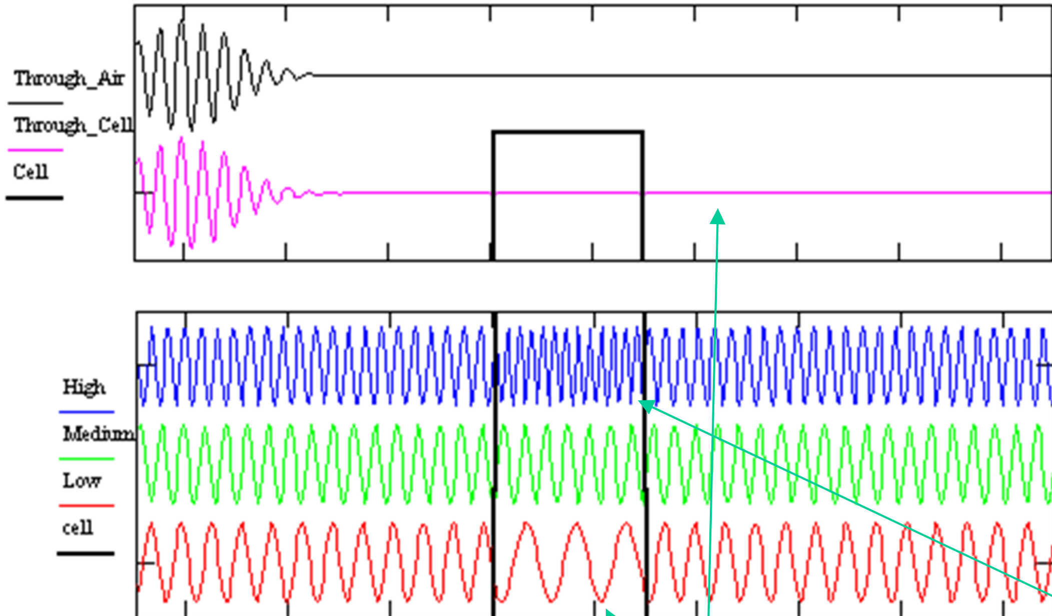


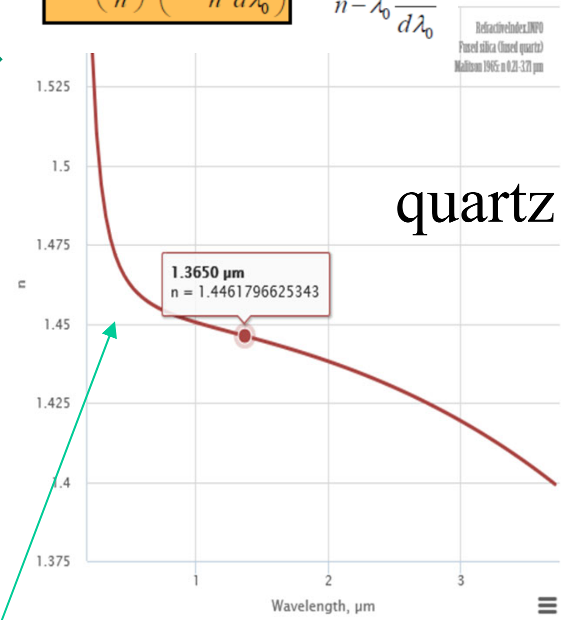
W. Wang

$V_p > 0$ and $V_g < 0$ $\quad V_p > V_g$

NORMAL DISPERSION

$$v_g = \left(\frac{c_0}{n}\right) \Big/ \left(1 - \frac{\lambda_0}{n}\frac{dn}{d\lambda_0}\right) = \frac{c_0}{n - \lambda_0 \frac{dn}{d\lambda_0}}$$

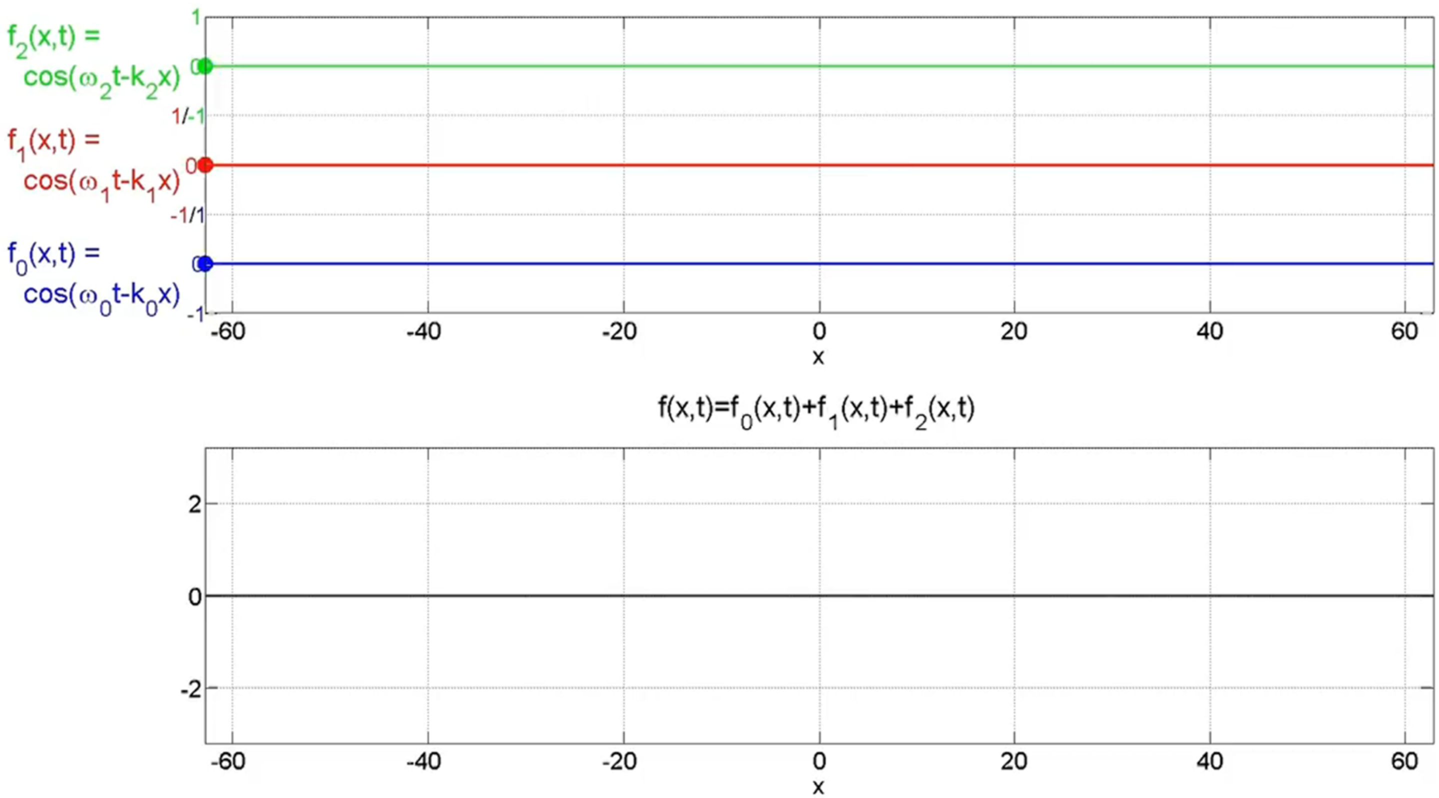




Positive (normal) dispersion. **This is what happens in most light-matter interaction**. The **medium has higher index of refraction for higher frequencies**; the frequency components of a pulse are dispersed such that **the long wavelengths become even longer**, the **short ones even shorter**. When they recombine to create a new pulse after going through the cell, **the pulse appears delayed (the red pulse, travelling through medium, is delayed from the black pulse, travelling through air)**. Hence, **the group velocity (velocity of the pulse) is smaller than c.**

$$V_g < C_o$$

Negative Group velocity and Positive Phase Velocity



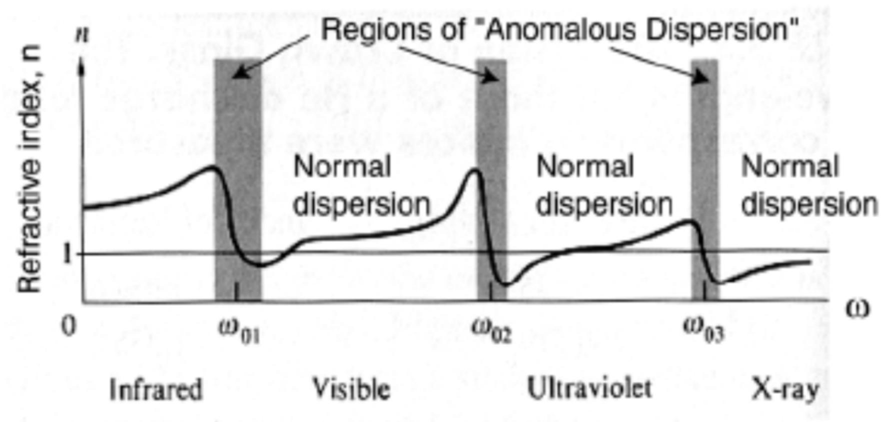
$V_p > 0$ and $V_g < 0$ $V_p > V_g$

ANOMALOUS DISPERSION (resonant structure)

$V_p = C_o/n$

$$v_g = \frac{c_0}{n + \omega \frac{dn}{d\omega}}$$

dn/dω **is negative** in regions of **anomalous dispersion**, that is, **near a resonance**. So **v_g exceeds v_p, and can even exceed c_0 in these regions**!



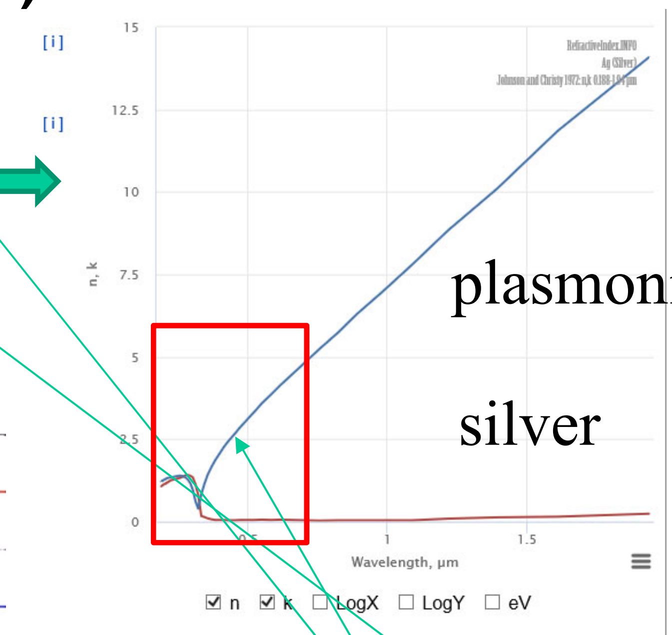
Meta or plasmonic materials

We note that **absorption is strong in these regions.** *dn/dω* is only steep when the resonance is narrow, so only a narrow range of frequencies has $v_g > c_0$. Frequencies outside this range have $v_g < c_0$.

ANOMALOUS DISPERSION (material)

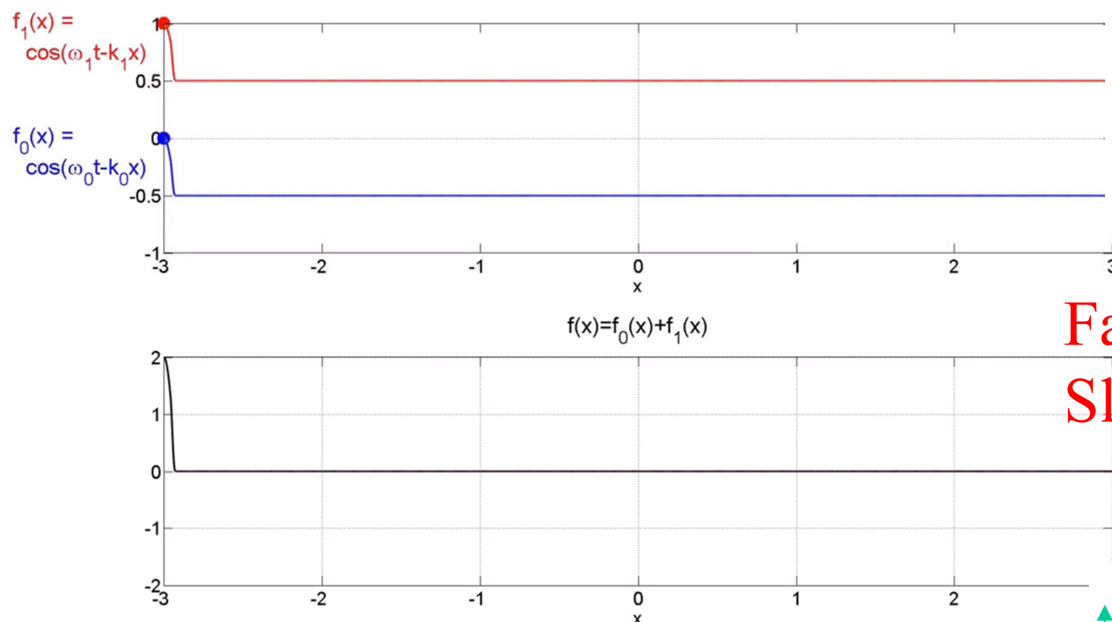
And on the other hand, if **dVp/dλ** < 0, then **<u>longer wavelength lights propagate slower than shorter wavelength lights, this is called anomalous dispersion.</u>**

In this case, the superposed wave's **<u>group veloicty Vg is larger than its phase velocity Vp.</u>**



plasmonic

silver



Fast light

Slow wave

$V_p = C_o/n$

$$V_g = V_p - \lambda \boxed{\frac{dV_p}{d\lambda}}$$

$$v_g = \left(\frac{c_0}{n}\right) / \left(1 - \frac{\lambda_0}{n}\frac{dn}{d\lambda_0}\right) = \frac{c_0}{n - \lambda_0 \boxed{\frac{dn}{d\lambda_0}}}$$

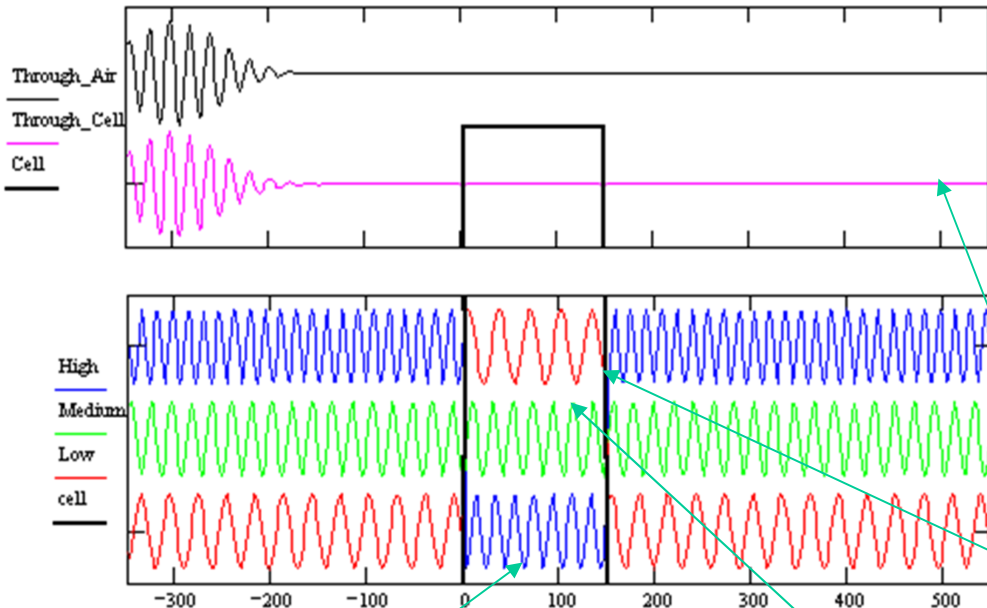
$$v_g = \frac{d\omega}{dk} = \frac{1}{dk/d\omega} = \frac{c}{n + \omega \boxed{\frac{dn}{d\omega}}}.$$

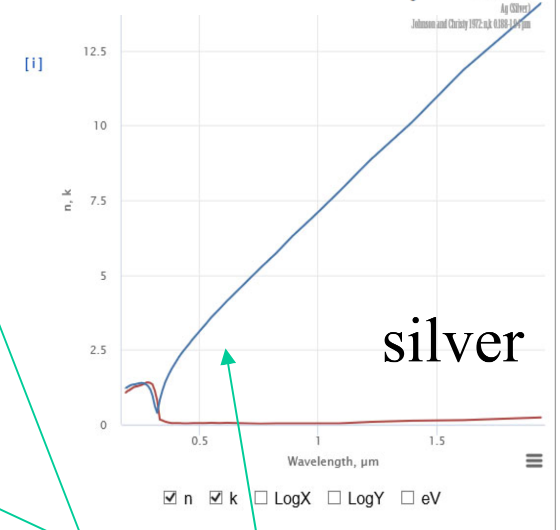
Decrease or negative

$V_p > 0$ and $V_g > 0$ and $V_g > V_p$

ANOMALOUS DISPERSION

$$v_g = \left(\frac{c_0}{n}\right) \Big/ \left(1 - \frac{\lambda_0}{n}\frac{dn}{d\lambda_0}\right) = \frac{c_0}{n - \lambda_0 \dfrac{dn}{d\lambda_0}}$$

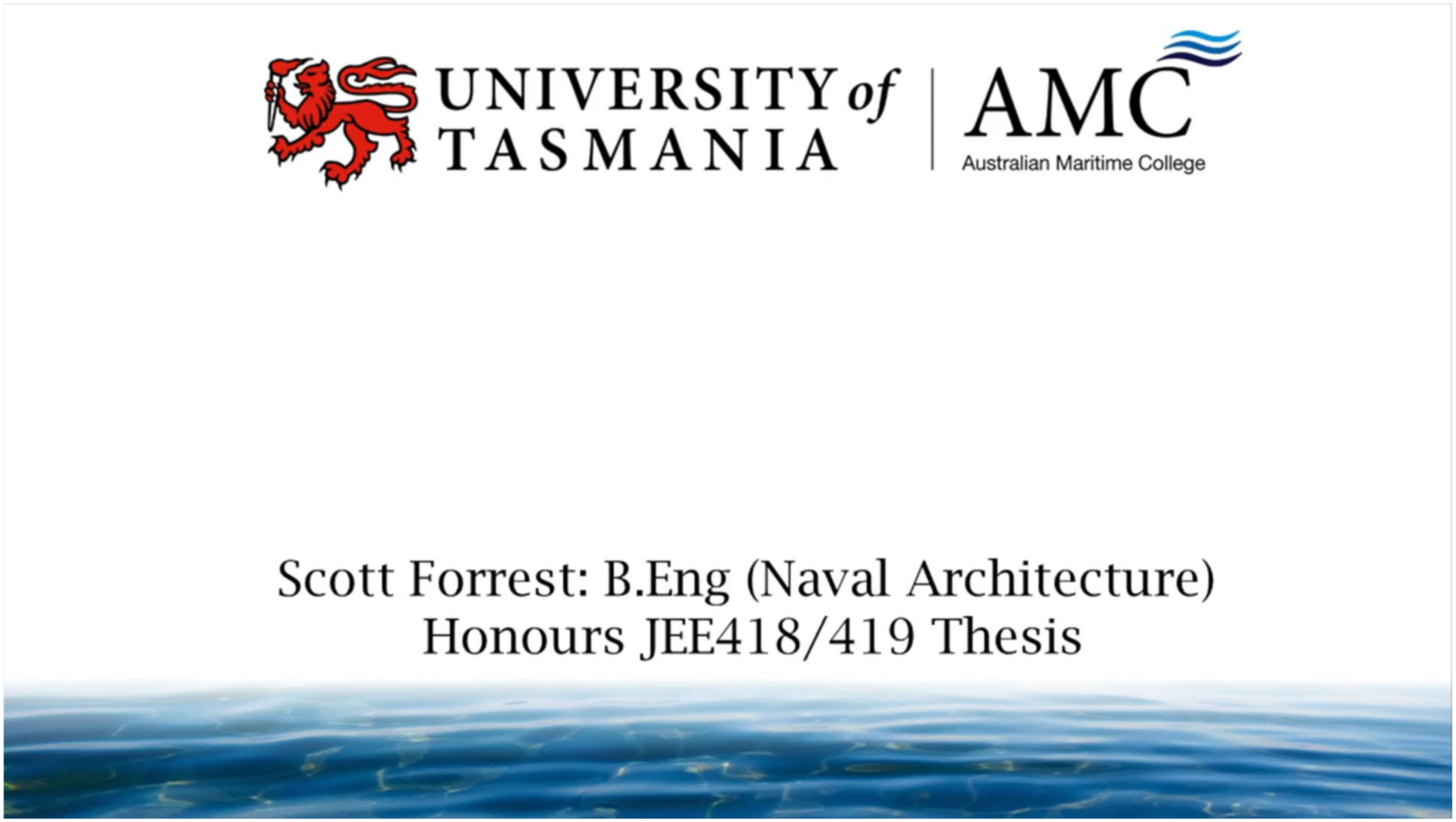




Negative (anomalous) dispersion. If the medium has lower index of refraction for higher frequencies; the frequency components of a pulse are dispersed such that the long wavelengths become shorter, the short ones become longer. When they recombine to create a new pulse after going through the cell, the pulse appears advanced, in fact there is another pulse already in the cell. The output pulse appears even before the input pulse entered the cell fully. Hence, the group velocity (velocity of the pulse) is higher than c.

Example of
group Vg> Vp

Soliton Waves



Negative Dispersion

Propagation of light in a negative dispersion regime (antiparallel phase and group velocities) may be attributed to either **fast light or a backward wave**.

$n_p < 0$

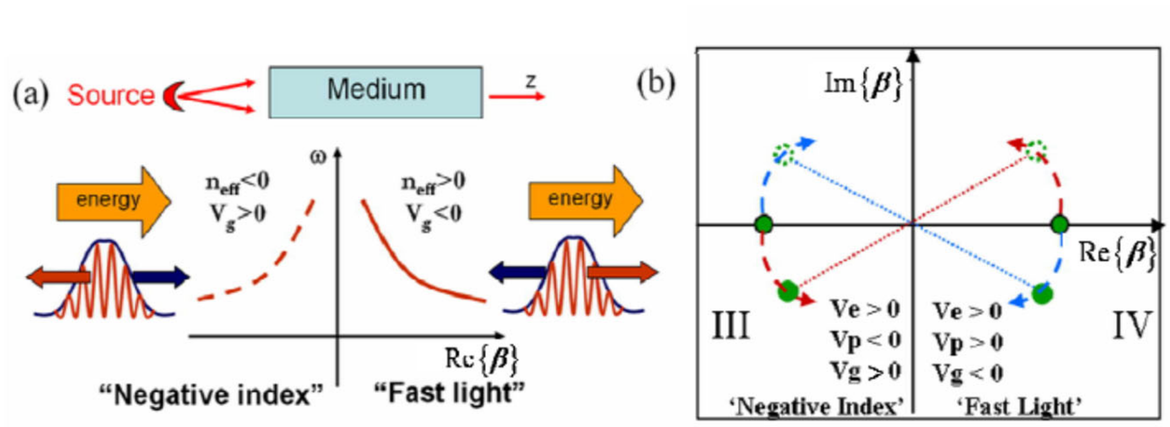
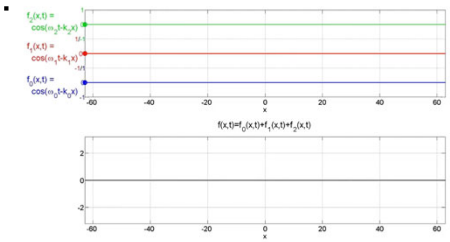


Fig. 1. (a) Schematic presentation of a negative dispersion curve. Block blue arrows, group direction; block red arrows, phase direction. (b) Schematic roots in the complex plane of modal propagation constant for negative dispersion. Green points on the real axis indicate the solution pair for the lossless case, and the red/blue arrows indicate the revolution of the roots into the causal fast-light/backward-wave quadrants.

Negative dispersion, defined here as a spectral range where the **phase and group velocities are opposite in direction**, is a subject of seemingly two disjointed research areas: one is the intriguing field of **fast light [parallel phase and energy and antiparallel pulse (group) velocity, and the other is the highly active field of negative-index metamaterials (parallel pulse group and energy and antiparallel phase velocity) .**

Fast Light and Slow Light

In a material with a frequency-dependent refractive index, each frequency propagates with a different phase velocity, thereby modifying the nature of the interference. **If n(ω) varies linearly with frequency ω, the effect of the modified interference is to shift the peak of the pulse in time, but with the pulse shape staying the same**. The fact that the pulse is temporally shifted implies that it is traveling with a velocity different from the phase velocity. This new velocity is known as the group velocity and is defined as:



$$v_g = \left. \frac{c}{n(\omega) + \omega \dfrac{dn(\omega)}{d\omega}} \right|_{\omega = \omega_c} = \frac{c}{n_g}$$

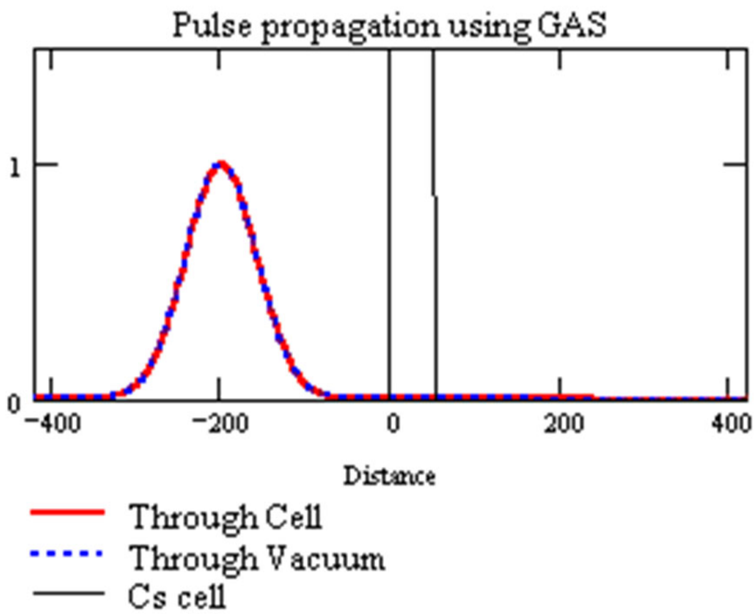
where ωc is the central frequency and **<u>ng is the group index of the material.</u>** We see that **<u>ng differs from the phase index by a term that depends on the dispersion dn/dω</u>** of the refractive index.

- **For slow light, which occurs for n_g >1,** the point of **<u>constructive interference occurs at a later time</u>**.

- **For fast light,** which **<u>occurs for n_g <1, interference or beat occurs at an earlier time.</u>**

A crucial observation is that the physics behind fast light is identical to the physics behind slow light. Although most of us readily accept the notion of a pulse of light moving through a dispersive material at a group velocity less than c, many of us are uncomfortable with the fast light case. We shouldn't be. Both arise from the same effect: the shifting of the point of constructive interference to another point in space-time.

Fast Light



Propagation of light in a **negative dispersion regime** (antiparallel phase and group velocities) may be attributed to either **fast light (anamolous) or a backward wave (nomal)**.

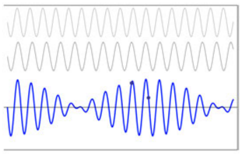
A pulse propagates with a group velocity which is not bound by the speed of light c, hence **it can be higher than c**, **or even negative**. Superluminescent light (Slow wave structure)

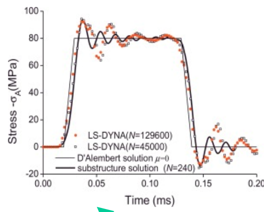
Ways to prove group velocity can be negative

Velocities of Wave

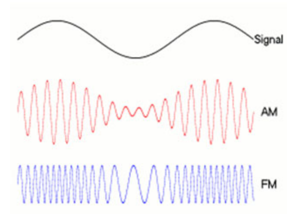
The **standard group velocity fails because in the derivation an assumption was made that in general is not true in a region of anomalous dispersion.** That is it was assumed that

$$k(\omega)-k(\bar{\omega})\approx(\omega-\bar{\omega})\left[\delta k(\omega)/\delta\omega\right]\big|_{\bar{\omega}}.$$





*Group Velocity



By its nature, the group velocity is a **mathematical entity** which may **not have any real physical significance associate with it**. There is no physical particle, mass, energy or signal which necessarily travels at the group velocity. **This is clearly the case in a region of anomalous dispersion as well as for a region of amplification**. In a **region of absorption** Vg may **become negative, zero or infinity.** In fact, $v_g(\mathbf{r}) = |\nabla(\delta\phi_\omega(\mathbf{r})/\delta\omega|_{\bar{\omega}})|^{-1}$, may **not longer yield a unique value for Vg**. To see this one need only consider the case where one wave packet enters, and after passing a distance in the medium, **there are several packets separated in space at a given instance**. In this case one cannot define a group velocity by the use of above equation.

$$\psi(\mathbf{r}, t) = \int_0^\infty A_\omega(\mathbf{r}) \cos[\omega t - \phi_\omega(\mathbf{r})] d\omega.$$

A wave form which has more than one maximum at a given time may have one, two, or more maxima at some later time. There is no law of conservation of the number of such maxima. Furthermore, we cannon associate with this maximum any unique physical entity which we can use as a tag and thereby follow its progress.

The reported demonstrations of the equality of the group velocity and the velocity of energy transport have been **limited to special cases**, the most important restriction **being that the medium is loss free**. We note that this restriction is equivalent to the abandonment of the principle of causality, We wish to show that the standard definition of the **group velocity fails to describe the motion of the peak of an arbitrary pulse in a region** of **anomalous dispersion.** **According to the standard definition the group velocity in a region of anomalous dispersion can exceed c, go to positive infinity, negative) infinity, and assume a large range of negative values.** Needless to say, the behavior of the group velocity in this region is not consistent with what one would consider reasonable. In the derivation of the expression for the group velocity found in modern texts, the position of the maximum of the pulse is given by

$$t = \delta\phi_\omega(r)/\delta\omega \,|_{\bar{\omega}}. \tag{8}$$

If we have

$$\phi_\omega(r) = n\omega x/c, \tag{9}$$

it follows that

$$t = c^{-1}(\delta n\omega/\delta\omega)_{\bar{\omega}}\, x. \tag{10}$$

The standard definition of the group velocity fails whenever Eq. (10) yields a value for the time position of the maximum such that

$$\boxed{t - x/c < 0} \tag{11}$$

or

$$(\delta n\omega/\delta\omega)|_{\bar{\omega}} - 1 < 0 \tag{12}$$

since we know the field is zero for all t's that satisfies Eq. (11).[10]

Even though the standard definition of the group velocity implies that the peak of the wave group has arrived, it has not. **The standard group velocity fails because in the derivation an assumption was made that in general is not true in a region of anomalous dispersion**. That is it was assumed that

$$k(\omega) - k(\bar{\omega}) \approx (\omega - \bar{\omega})[\delta k(\omega)/\delta\omega]|_{\bar{\omega}}.$$

This approximation **is not in general valid in a region where there is a resonance.** This is why the standard expression for the group velocity does not describe the motion of the maximum of the pulse **in a region where one has gain or absorption.**

A general expression for the group velocity, i.e., the velocity of the maximum of the intensity of the pulse, in a region of anomalous dispersion is not readily apparent. The conventional one is clearly unacceptable. Furthermore, the group velocity of a pulse is a function of the gain or absorption, the depth in the medium, and the pulse shape. Thus, the group velocity is a much more complex quantity than it is normally assume to be.

New group velocity model

Because of the **distortion due to dispersion**, a new definition has recently been proposed for the group velocity. It was proposed that the group velocity be the **velocity of motion of** the **temporal center of gravity of the amplitude of the wave packet.** Under this definition the group velocity could be written as

$$v_{gt} = \left| \nabla \left(\int_{-\infty}^{\infty} t \, |A(r,t)| \, dt \Big/ \int_{-\infty}^{\infty} |A(r,t)| \, dt \right) \right|^{-1} \tag{14}$$

For a **quasimonochromatic pulse**, this definition reduces to that of $v_g(\mathbf{r}) = |\nabla(\delta g_\omega(\mathbf{r})/\delta\omega|_{\bar{\omega}})|^{-1}$, This definition has the advantage that for any case there exists a unique temporal center of gravity as long as the integrals converge. This is true even in the case where the original pulse splits into several parts. **Furthermore, the pulse need not be quasimonochromatic as the previous definition required**. For the experimentalist, an additional amount of work may be required to determine the temporal center of gravity of the amplitude, but there is seemingly no serious difficulty.

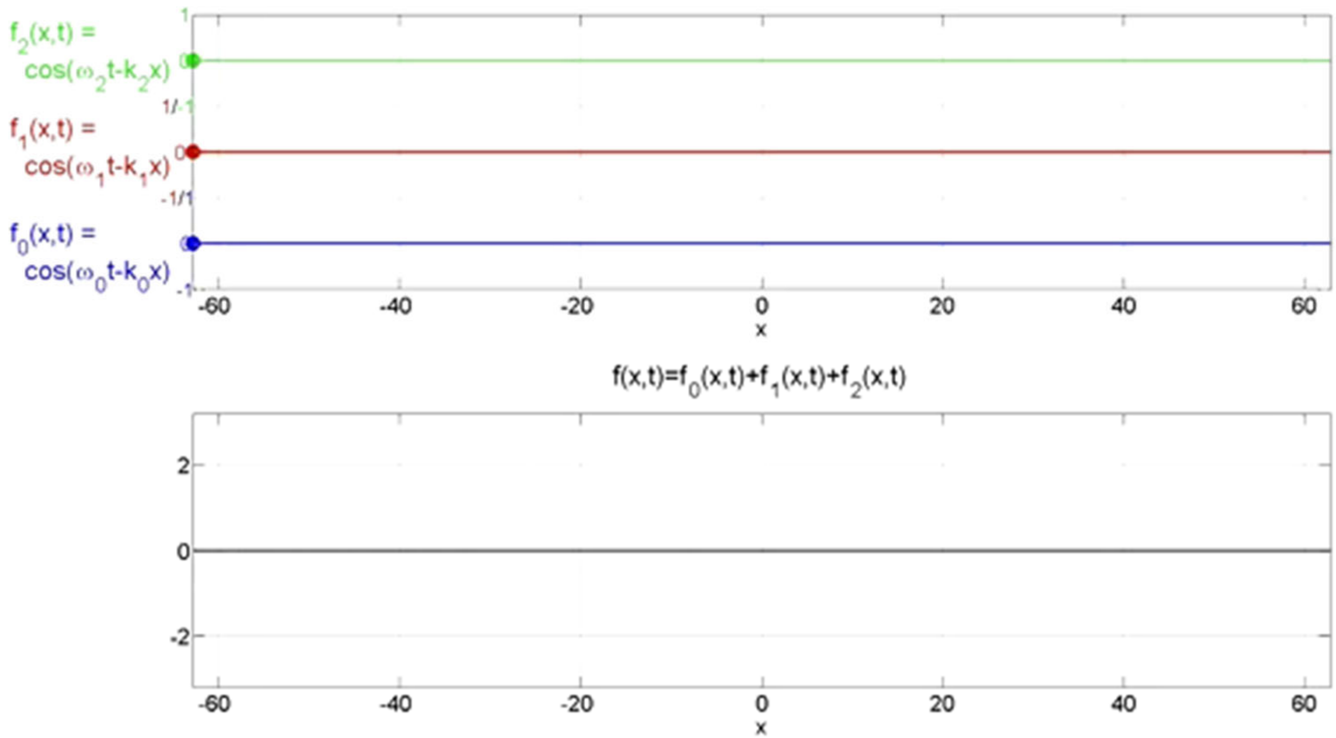
The Velocities of Light
Richard L. Smith

The group velocity can exceed c_o when dispersion is anomalous

There is a more fundamental reason why $v_g > c_0$ doesn't necessarily bother us. **The interpretation of the group velocity as the speed of energy propagation is only valid in the case of normal dispersion!** In fact, mathematically we can superpose waves to make any group velocity we desire - even zero!

You could ideally get all these negative phase, group velocity, but they are not real signal velocity. Read https://www.mathpages.com/home/kmath210/kmath210.htm

Zero group velocity
(standing wave)



Sum of all Phase velocity of different wavelengths equal to zero

Vp > 0 and Vg = 0

Zero group velocity (standing wave)

Incidentally, **since we can contrive to make the "groups" propagate in either direction, it's not surprising that we can also make them stationary. Two identical waves propagating in opposite directions at the same speed are given by**

$$A_0 \cos(kx \pm \omega t) = A_0\left[\cos(kx)\cos(\omega t) \mp \sin(kx)\sin(\omega t)\right]$$

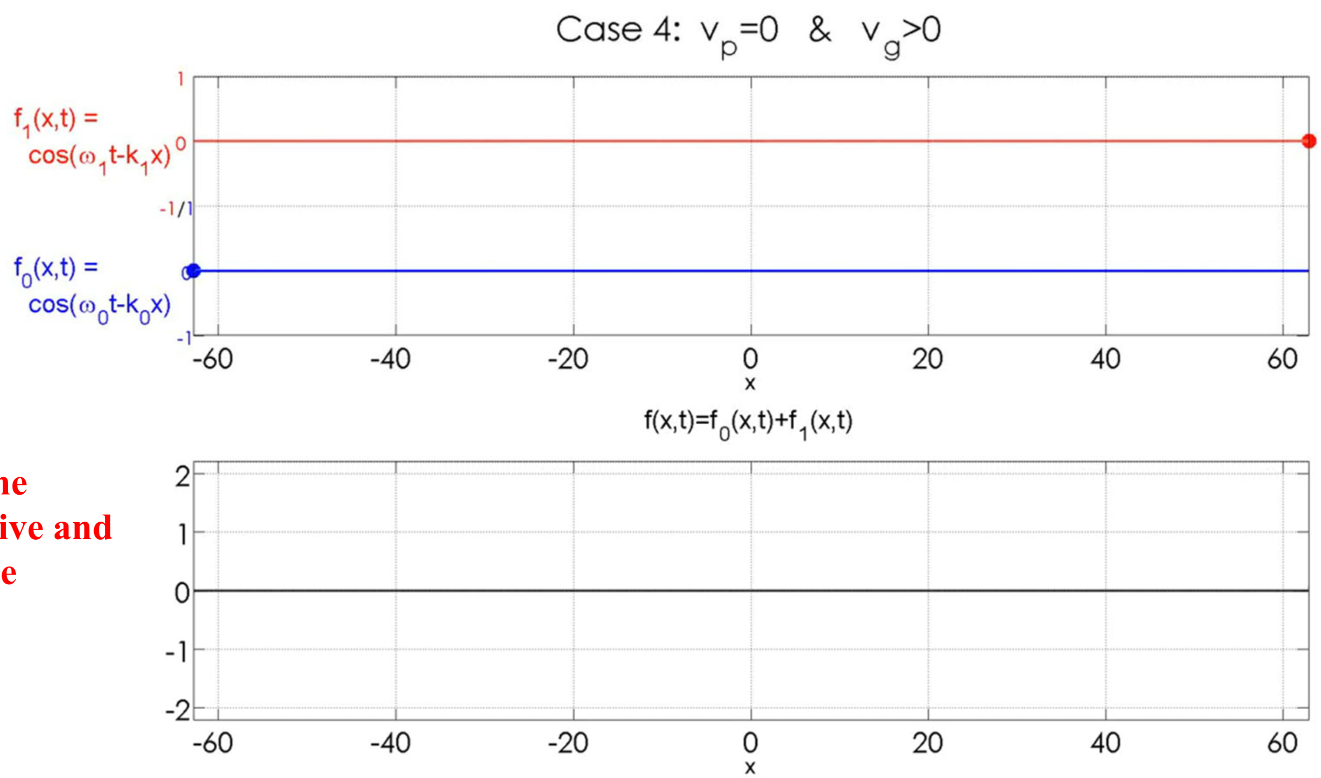
Superimposing these two waves propagating (with synchronized nodes) in opposite directions yields a **standing pure wave**

$$A_0 \cos(kx \pm \omega t) + A_0 \cos(kx \pm \omega t) = 2A_0 \cos(kx)\cos(\omega t)$$

Fabry-Perot

ZERO PHASE VELOCITY

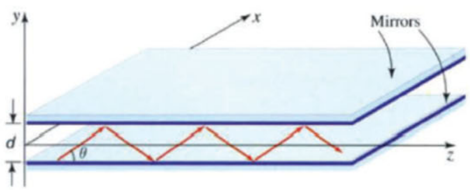
When the **monochrome waves are propagating in opposite directions**, they can make the **phase velocity Vp of the superposed wave to be 0.**

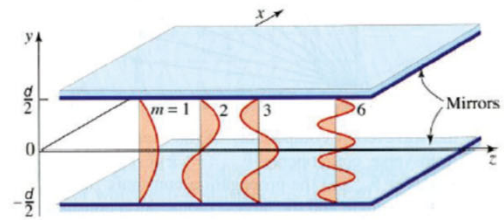


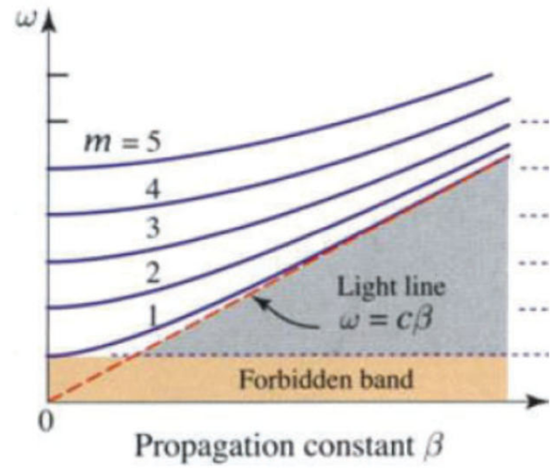
Phase velocity of one wavelength is positive and the other is negative

$Vp = 0$ and $Vg > 0$

Planar Waveguide







TE modes

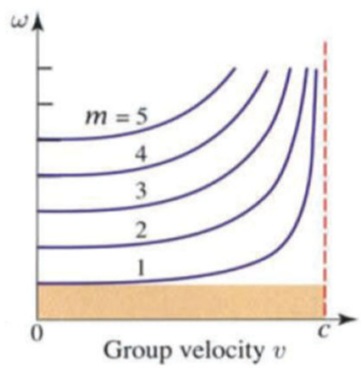
$$E_x(y,z) = a_m u_m(y)\exp(-j\beta_m z)$$

$a_m = \sqrt{2d}\,A_m$ or $j\sqrt{2d}\,A_m$

$$u_{m(y)} = \begin{cases} \sqrt{\dfrac{2}{d}}\cos(\dfrac{m\pi y}{d}), & m = 1,3,5\ldots \\ \sqrt{\dfrac{2}{d}}\sin(\dfrac{m\pi y}{d}), & m = 2,4,6\ldots \end{cases}$$

Dispersion relation for waveguide

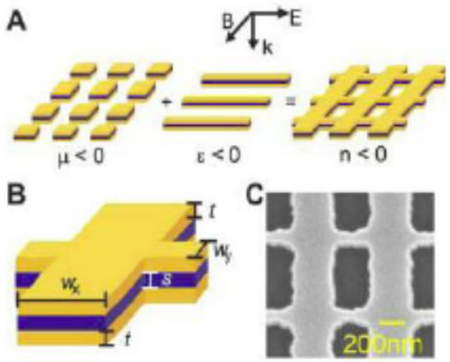
$$\beta_m^2 = (\omega / c)^2 - m^2\pi^2 / d^2$$



Waveguide provides anomalous dispersion without an atomic resonance

In artificially designed materials, almost any behavior is possible

Here's one recent example:



A metal/dielectric composite structure

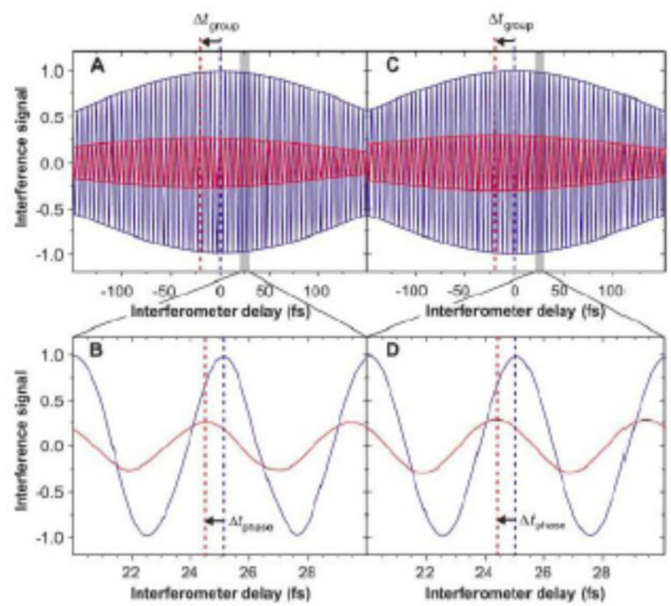
In this material, a light pulse appears to exit the medium before entering it.

Of course, relativity and causality are *never* violated.

Simultaneous Negative Phase and Group Velocity of Light in a Metamaterial

Gunnar Dolling,[1*] Christian Enkrich,[1] Martin Wegener,[1,2] Costas M. Soukoulis,[3,4] Stefan Linden[2]

Science, vol. 312, p. 892 (2006)



experiment theory

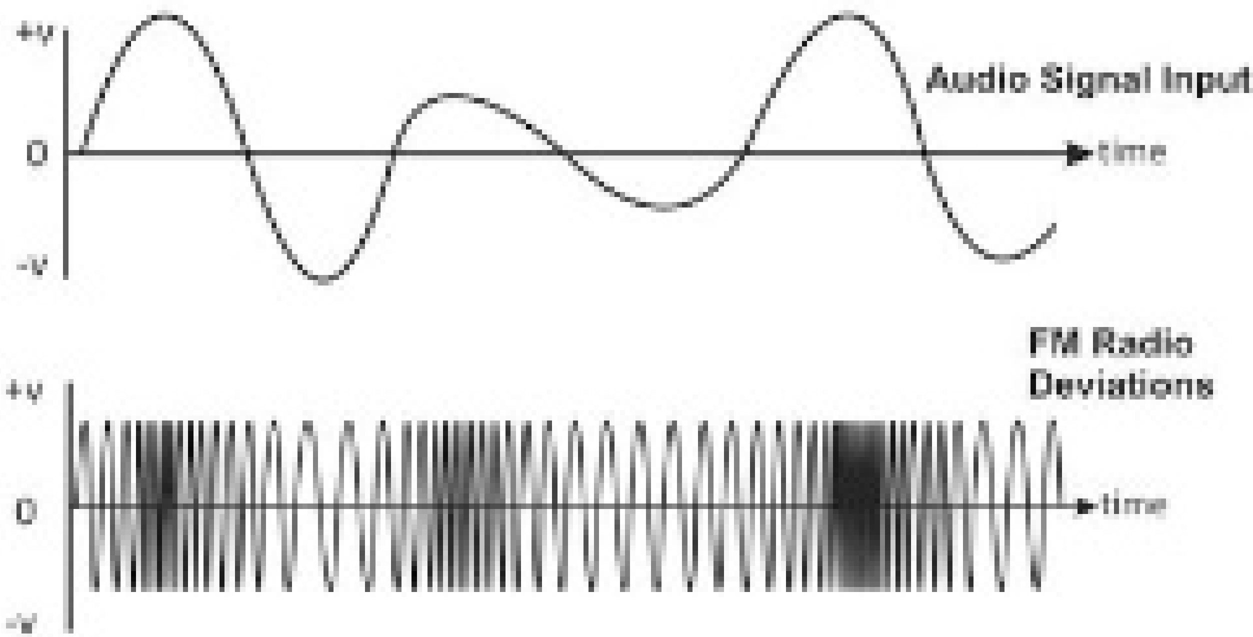
Summary

- V_g and V_p can be positive or negative and even $V_g > C_o$, we are not travelling faster than speed of light because signal velocity is not there.
- Normal and anomalous dispersion can give both positive and negative n or dn/dw
- Standing wave (V_g=0) = Fabry-Perot interferometer
- Fast light (dn/dω large and negative) n_g<1 and slow light n_g>1 at strong absorption resonance (high loss so energy is conserved)
- Metamaterial can have negative np an n_g because the resonance is not just monochromatic, it's a band

$$v_g = \left. \frac{c}{n(\omega) + \omega \frac{dn(\omega)}{d\omega}} \right|_{\omega = \omega_c} = \frac{c}{n_g}$$

- **The interpretation of the group velocity as the speed of energy propagation is only valid in the case of normal dispersion!**

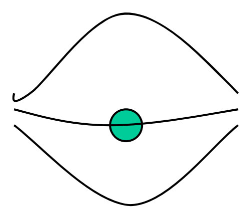
FM

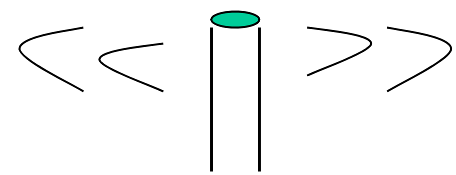


Velocities of Light

More discussion of what wave velocity is

Plane, Spherical and Cylindrical Wave





$$\psi = \psi_0 \cos(\omega t - \mathbf{k} \cdot \mathbf{r} + \phi),$$

$$2Bl = \mu_0 K l,$$

$$\psi(r,t) = \frac{\psi_0}{r} \cos(\omega t - kr + \phi).$$

Poynting's Theorem

For a time –harmonic electromagnetic wave, the power density per unit area associate with the wave is defined in complex representation by vector S,

$$\nabla \times E = \frac{-\partial B}{\partial t} = -j\omega B$$

$H = B/\mu$

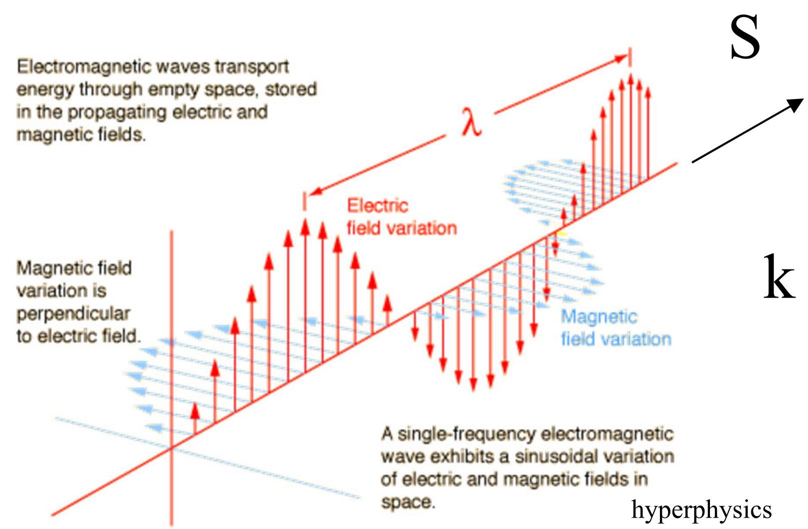
$B = kE/\omega$

$C = \omega/\kappa$

$B = E/C$

$$S = E \times H^{*} \ (\mathrm{W/m^2})$$

Again Ponyting's vector is derived using Ampere's and Faraday's and couple vector calculus identities.



hyperphysics

(or)

Use Faraday's law and phase velocity equation

$$S = \frac{E x B^{*}}{2\mu_o} = \frac{|E|^2}{2C\mu_o}\hat{z} = \frac{|E|^2}{2Z_o}\hat{z} \Rightarrow \boxed{I(P) = \frac{|E|^2}{2Z_o}}$$

(Intensity)

where z_o = air impedance $= \sqrt{\frac{\mu_o}{\varepsilon_o}}$

Time average pontying vector <s> is defined as average of the Time domain Poynting vector S over a period T=2π/ω.

$$< S >= \frac{1}{2\pi}\int_0^{2\pi} d(\omega t) E(x,y,z,t) \times H(x,y,z,t)$$

(or) $$< S >= \frac{1}{2}\mathrm{Re}\{E \times H\}$$

More complete derivation of Ponyting vector

Ponyting Theory derivation

From Faraday's and Ampere's ⇒

$$\boxed{\begin{aligned}\nabla \times E &= \frac{-\partial B}{\partial t} \\ \nabla \times H &= J + \frac{\partial D}{\partial t}\end{aligned}}$$

We obtain

$$H \cdot (\nabla \times E) = -H \cdot \frac{\partial B}{\partial t}$$

$$\mathrm{E} \cdot (\nabla \times H) = J \cdot E + E \cdot \frac{\partial D}{\partial t}$$

Subtract above two equations and use this vector identity

$$\mathrm{H} \cdot (\nabla \times E) - \mathrm{E} \cdot (\nabla \times H) = \nabla \cdot (E \times H)$$

We get

$$\nabla \cdot (E \times H) = -J \cdot E - H \cdot \frac{\partial B}{\partial t} - E \cdot \frac{\partial D}{\partial t}$$

Vector Calculus

- Useful vector relationships for the vector fields a , b, and c are

$$\nabla \cdot (\nabla \times \mathbf{a}) = 0$$

$$\nabla \times \nabla \times \mathbf{a} = \nabla(\nabla \cdot \mathbf{a}) - \nabla^2 \mathbf{a}$$

recall

$$\nabla \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot (\nabla \times \mathbf{a}) - \mathbf{a} \cdot (\nabla \times \mathbf{b})$$

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$$

$$\mathbf{a} \cdot \mathbf{b} \times \mathbf{c} = \mathbf{b} \cdot \mathbf{c} \times \mathbf{a} = \mathbf{c} \cdot \mathbf{a} \times \mathbf{b}$$

$$\nabla \cdot (\underline{E} \times \underline{H}) = -\underline{J} \cdot \underline{E} - \underline{H} \cdot \frac{\partial \underline{B}}{\partial t} - \underline{E} \cdot \frac{\partial \underline{D}}{\partial t}$$

Next, assume that Ohm's law applies for the electric current:

$$\underline{J} = \sigma \underline{E}$$

$$\Rightarrow \quad \nabla \cdot (\underline{E} \times \underline{H}) = -\sigma (\underline{E} \cdot \underline{E}) - \underline{H} \cdot \frac{\partial \underline{B}}{\partial t} - \underline{E} \cdot \frac{\partial \underline{D}}{\partial t}$$

or

$$\nabla \cdot (\underline{E} \times \underline{H}) = -\sigma |\underline{E}|^2 - \underline{H} \cdot \frac{\partial \underline{B}}{\partial t} - \underline{E} \cdot \frac{\partial \underline{D}}{\partial t}$$

From calculus (chain rule), we have that

$$\underline{E} \cdot \frac{\partial \underline{D}}{\partial t} = \varepsilon \left(\underline{E} \cdot \frac{\partial \underline{E}}{\partial t} \right) = \varepsilon \frac{1}{2} \frac{\partial}{\partial t} (\underline{E} \cdot \underline{E})$$

$$\underline{H} \cdot \frac{\partial \underline{B}}{\partial t} = \mu \left(\underline{H} \cdot \frac{\partial \underline{H}}{\partial t} \right) = \mu \frac{1}{2} \frac{\partial}{\partial t} (\underline{H} \cdot \underline{H})$$

Hence we have

$$\nabla \cdot (\underline{E} \times \underline{H}) = -\sigma |\underline{E}|^2 - \mu \frac{1}{2} \frac{\partial}{\partial t} (\underline{H} \cdot \underline{H}) - \varepsilon \frac{1}{2} \frac{\partial}{\partial t} (\underline{E} \cdot \underline{E})$$

$$\nabla \cdot (\underline{E} \times \underline{H}) = -\sigma |\underline{E}|^2 - \mu \frac{1}{2} \frac{\partial}{\partial t} (\underline{H} \cdot \underline{H}) - \varepsilon \frac{1}{2} \frac{\partial}{\partial t} (\underline{E} \cdot \underline{E})$$

This may be written as

$$\nabla \cdot (\underline{E} \times \underline{H}) = -\sigma |\underline{E}|^2 - \mu \frac{1}{2} \frac{\partial}{\partial t} |\underline{H}|^2 - \varepsilon \frac{1}{2} \frac{\partial}{\partial t} |\underline{E}|^2$$

or

Final differential (point) form of the Poynting theorem:

$$\boxed{\nabla \cdot (\underline{E} \times \underline{H}) = -\sigma |\underline{E}|^2 - \frac{\partial}{\partial t} \left(\frac{1}{2} \mu |\underline{H}|^2 \right) - \frac{\partial}{\partial t} \left(\frac{1}{2} \varepsilon |\underline{E}|^2 \right)}$$

Volume (integral) form

Integrate both sides over a volume and then apply the divergence theorem:

$$\nabla \cdot (\underline{E} \times \underline{H}) = -\sigma |\underline{E}|^2 - \frac{\partial}{\partial t}\left(\frac{1}{2}\mu |\underline{H}|^2\right) - \frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon |\underline{E}|^2\right)$$

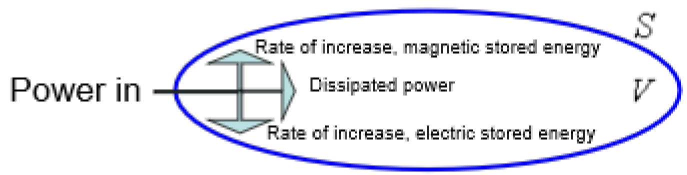
$$\boxed{\int_V \nabla \cdot (\underline{E} \times \underline{H})\, dV} = -\int_V \sigma |\underline{E}|^2\, dV - \int_V \frac{\partial}{\partial t}\left(\frac{1}{2}\mu |\underline{H}|^2\right) dV - \int_V \frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon |\underline{E}|^2\right) dV$$

Final volume form of Poynting theorem:

$$\oint_S (\underline{E} \times \underline{H}) \cdot \hat{n}\, dS = -\int_V \sigma |\underline{E}|^2\, dV - \int_V \frac{\partial}{\partial t}\left(\frac{1}{2}\mu |\underline{H}|^2\right) dV - \int_V \frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon |\underline{E}|^2\right) dV$$

For a stationary surface:

$$\oint_S (\underline{E} \times \underline{H}) \cdot \hat{n}\, dS = -\int_V \sigma |\underline{E}|^2\, dV - \frac{\partial}{\partial t}\int_V \left(\frac{1}{2}\mu |\underline{H}|^2\right) dV - \frac{\partial}{\partial t}\int_V \left(\frac{1}{2}\varepsilon |\underline{E}|^2\right) dV$$



Physical interpretation:

(Assume that S is stationary.)

$$-\oint_S (\underline{E}\times\underline{H})\cdot\hat{n}\,dS = \int_V \sigma|\underline{E}|^2\,dV + \frac{\partial}{\partial t}\int_V\left(\frac{1}{2}\mu|\underline{H}|^2\right)dV + \frac{\partial}{\partial t}\int_V\left(\frac{1}{2}\varepsilon|\underline{E}|^2\right)dV$$

$$I(P) = \frac{|E|^2}{2Z_o}$$

Power dissipation as heat (Joule's law)

Rate of change of stored magnetic energy

Rate of change of stored electric energy

Resistance

Inductance

Capacitance

$\Longrightarrow$ Right-hand side = power flowing into the volume of space.

Hence

$$-\oint_S (\underline{E} \times \underline{H}) \cdot \hat{n}\, dS = \text{power flowing into the region}$$

Or, we can say that

$$\oint_S (\underline{E} \times \underline{H}) \cdot \hat{n}\, dS = \text{power flowing } \textit{out} \text{ of the region}$$

Define the Poynting vector: $\underline{S} = \underline{E} \times \underline{H}$

Analogy:

$$\oint_S \underline{S} \cdot \hat{n}\, dS = \text{power flowing out of the region}$$

$$\oint_S \underline{J} \cdot \hat{n}\, dS = \text{current flowing out of the region}$$

$\underline{J}$ = current density vector

$\underline{S}$ = power flow vector

Poynting's Theorem

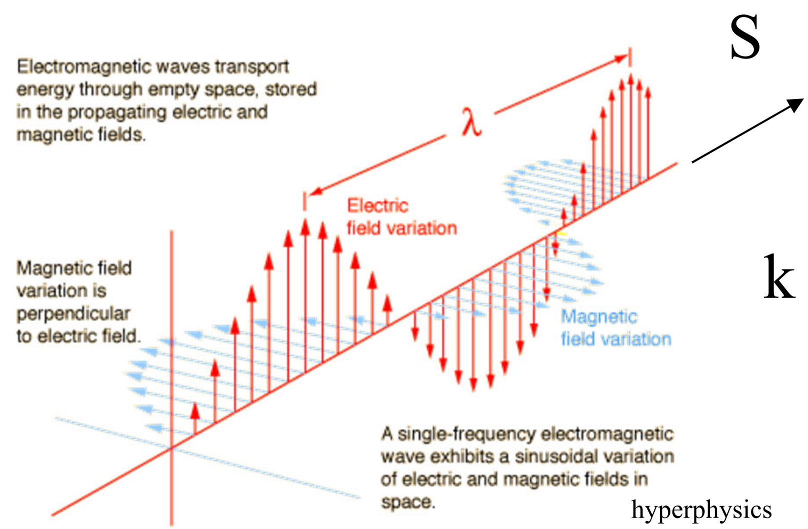
For a time –harmonic electromagnetic wave, the power density per unit area associate with the wave is defined in complex representation by vector S,

$$\left\{\begin{array}{l} \nabla \times E = \dfrac{-\partial B}{\partial t} \\ H = B/\mu \\ B = kE/\omega \\ C = \omega/\kappa \\ B = E/C \end{array}\right.$$

$$S = E \times H^* \ (\mathrm{W/m^2})$$

Again Ponyting's vector is derived using Ampere's and Faraday's and couple vector calculus identities.

(or)



hyperphysics

Use Faraday's law and phase velocity equation

$$S = \frac{ExB^*}{2\mu_o} = \frac{|E|^2}{2C\mu_o}\hat{z} = \frac{|E|^2}{2Z_o}\hat{z} \Rightarrow \boxed{I(P) = \frac{|E|^2}{2Z_o}}$$

where z_o = air impedance $= \sqrt{\dfrac{\mu_o}{\varepsilon_o}}$

(Intensity)

Week 8

- Lecture Notes (EM wave theory)
  http://courses.washington.edu/me557/sensors/week2.pdf
- Reading Materials:

Please read materials in week 5 in:

http://courses.washington.edu/me557/reading/

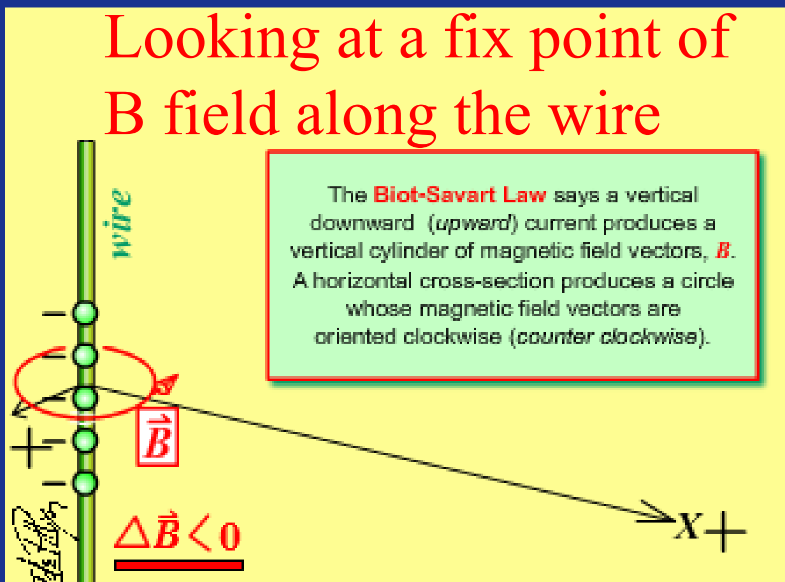
- Homework #1 due today.
- Final Presentation 1:20-3:10PM Dec. 28

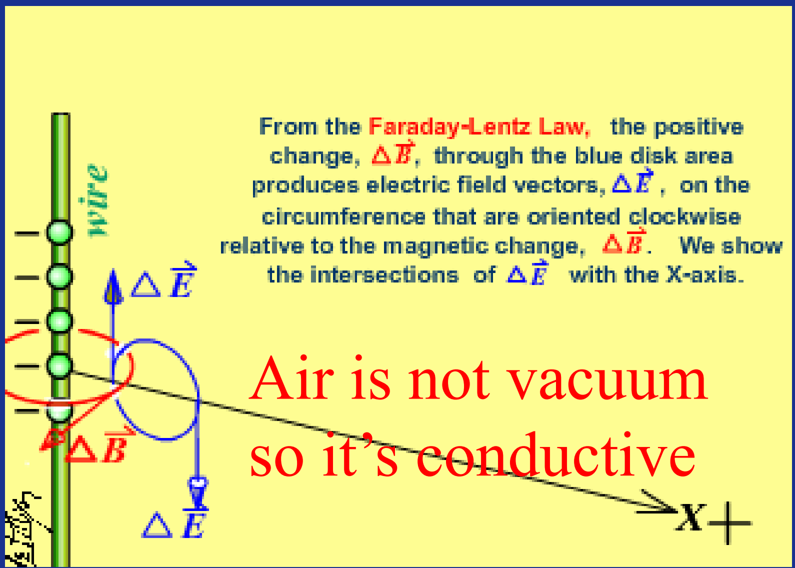
Recap

- How wave is generated
- Maxwell's equations
- Derive Wave propagation in free space from Maxwell's

Electromagnetic Wave

- Electromagnetic waves are created by the vibration of an electric charge. This vibration creates a wave which has both an electric and a magnetic component





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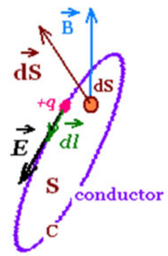
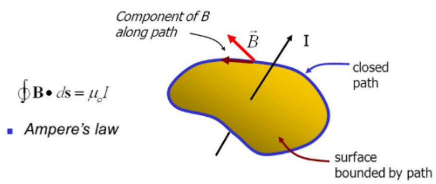
UNIVERSITY OF WASHINGTON

Maxwell Equations

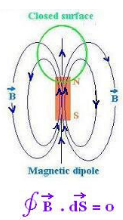
Differential form



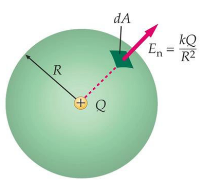
Faraday's Law $$\nabla \times E = \frac{-\partial B}{\partial t}$$



Ampere's Law $$\nabla \times H = J + \frac{\partial D}{\partial t}$$



Gauss's Law for Magnetism $$\nabla \bullet B = 0$$



Gauss's Law for Electricity $$\nabla \bullet D = \rho$$

E = *Electric Field (V/m)*	ρ = *charge density* (c/m^3)	i = *electric current (A)*
B = *Magnetic flux density*(Web/m^2, *T*)	ε_0 = *permittivity*	J = *current density*(A/m^2)
D = *Electric flux density* (c/m^2)	μ_0 = *permeability*	c = *speed of light*
H = *Magnetic Field (A/m)*	Φ_B = *Magnetic flux (Web)*	P = *Polarization*

From Ampere's and Faraday's law and a vector calculus identity,

$$\nabla \times (\nabla \times E) = \nabla(\nabla \bullet E) - \nabla^2 E$$, Wave equation

becomes $$\boxed{\nabla^2 E + \omega^2 \mu_o \varepsilon_o E = 0}$$

We consider the simple solution where E field is parallel to the x axis and its function of z coordinate only, the wave equation then becomes,

$$\frac{\partial^2 E_x}{\partial z^2} + \omega^2 \mu_o \varepsilon_o E_x = 0$$

A solution to the above differential equation is

Wave propagation in free space based on above condition

$$\boxed{E = \hat{x} E_o e^{-jkz}}$$

Substitute above equation into wave equation yields,

$$(-k^2 + \omega^2 \mu\varepsilon)E = 0 \Rightarrow \boxed{k^2 = \omega^2 \mu\varepsilon}$$ (dispersion relation)

Trigonometric Functions in Terms of Exponential Functions

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i} \qquad \csc x = \frac{2i}{e^{ix} - e^{-ix}}$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2} \qquad \sec x = \frac{2}{e^{ix} + e^{-ix}}$$

$$\tan x = \frac{e^{ix} - e^{-ix}}{i\left(e^{ix} + e^{-ix}\right)} \qquad \cot x = \frac{i\left(e^{ix} + e^{-ix}\right)}{e^{ix} - e^{-ix}}$$

Exponential Function vs. Trigonometric and Hyperbolic Functions

$$\boxed{e^{ix} = \cos x + i \sin x} \qquad e^{x} = \cosh x + \sinh x$$

Remember exponential term can be put in terms o trigonometric function

Hyperbolic Functions in Terms of Exponential Functions

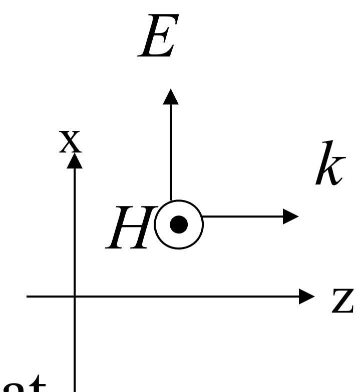
$$\sinh x = \frac{e^{x} - e^{-x}}{2} \qquad \operatorname{csch} x = \frac{2}{e^{x} - e^{-x}}$$

$$\cosh x = \frac{e^{x} + e^{-x}}{2} \qquad \operatorname{sech} x = \frac{2}{e^{x} + e^{-x}}$$

$$\tanh x = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}} \qquad \coth x = \frac{e^{x} + e^{-x}}{e^{x} - e^{-x}}$$

Let's transform the solution for the wave equation into real space and time, (assume time harmonic field)

$$E(z,t) = \mathrm{Re}\{Ee^{j\omega t}\} = \boxed{\hat{x}E_o \cos(\omega t - kz)}$$



$k = 2\pi/\lambda$, where k = wave number
Image we riding along with the wave, we asked what Velocity shall we move in order to keep up with the wave, The answer is phase of the wave to be constant

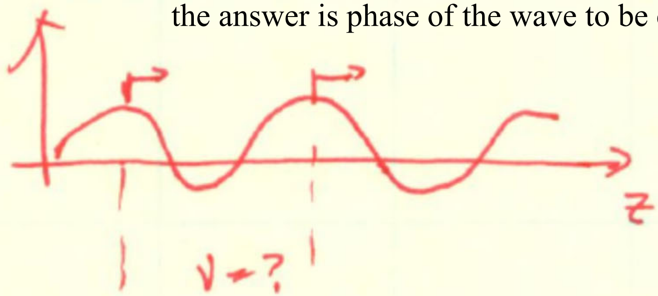
$$\omega t - kz = \text{a constant}$$

The velocity of propagation is therefore given by,

$$\boxed{\frac{dz}{dt} = v = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_o \varepsilon_o}}} \quad \text{(phase velocity)}$$

$$E(z,t) = Re\{E e^{j\omega t}\} = \hat{x}E_0 \cos(\underbrace{\omega t - kz}_{\text{phase}})$$

Imagine we riding along with the wave, we asked what velocity shall we move in order to keep up with the wave, the answer is phase of the wave to be constant



$$\omega t - kz = \text{phase} = \text{constant}$$

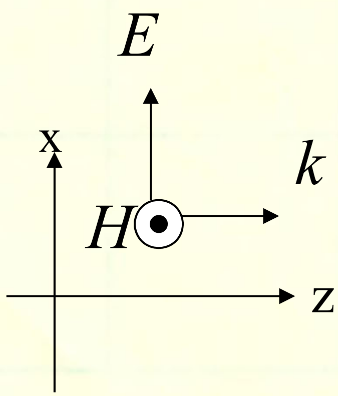
take time derivative

$$\frac{d(\omega t)}{dt} - k\frac{dz}{dt} = 0$$

$$\omega = k\frac{dz}{dt}$$

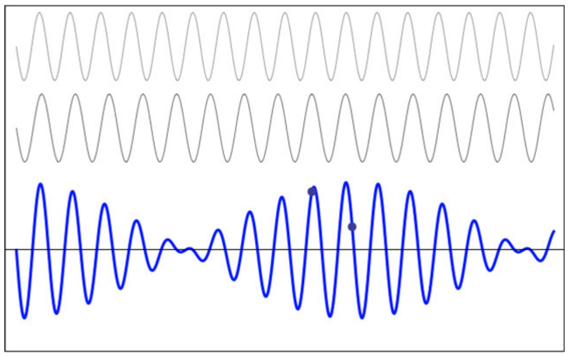
$$\frac{\omega}{k} = \frac{dz}{dt}$$

Phase velocity $\boxed{\frac{dz}{dt} = v = \frac{\omega}{k}}$



Group Velocity

Group velocity is trickier. The word 'group' suggests that the concept involves more than one wave. Because two is the first whole number larger than one, the simplest illustration uses to two waves:



$$\sin A+\sin B = 2\sin(A+B)/2 \ {*}\underline{\cos(A-B)/2}$$

$$\text{Let } A = k_1x+\omega_1 t \qquad k_1 = 2\pi n_1/\lambda_1$$

$$B = k_2x+\omega_2 t \qquad k_2 = 2\pi n_1/\lambda_2= k_1+\Delta k$$

$$\omega_2=\omega_1+\Delta\omega$$

So it is a wave with wavenumber $\Delta k/2$ and frequency $\Delta\omega/2$. The **envelope's phase velocity** is the group velocity of f_1 and f_2:

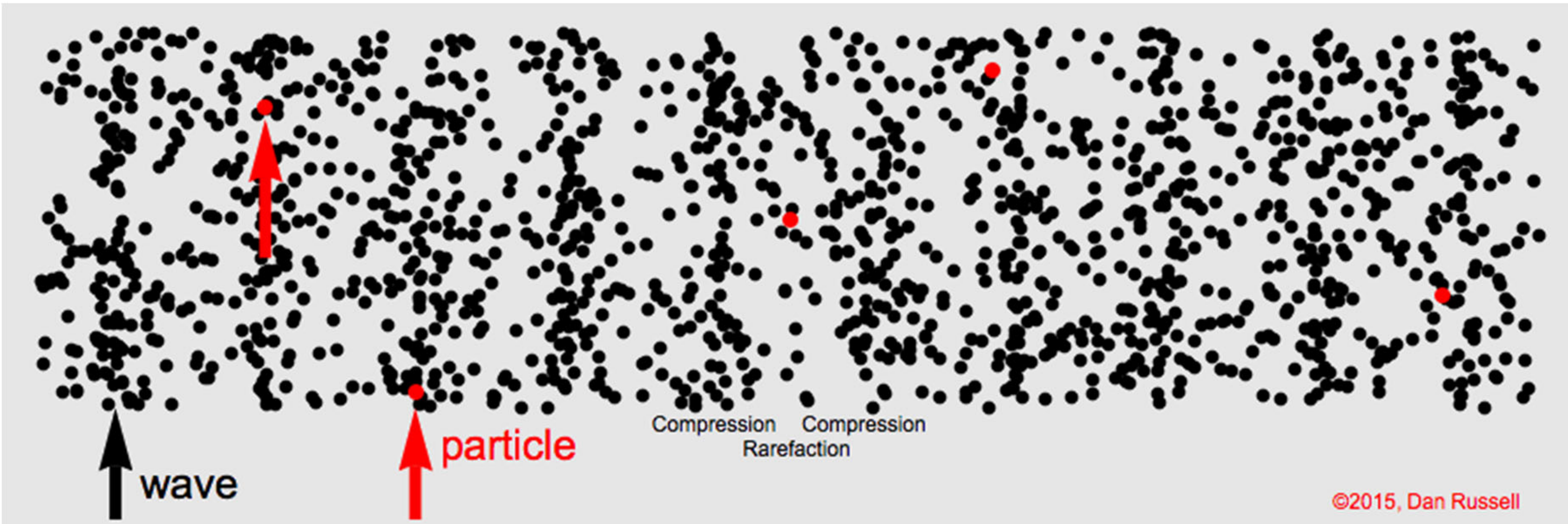
$$v_g= \omega/k = (\Delta\omega/2)/(\Delta k/2)= \Delta\omega/\Delta k$$

In the limit where $\Delta\omega$ -> 0 and Δk-> 0, the group velocity becomes :

$$v_g= d\omega/dk$$

Traverse and Longitudinal Wave

Longitudinal Wave



Group velocity (net) is moving forward horizontally, phase velocity (localize particle) oscillates also horizontally

Poynting's Theorem

For a time –harmonic electromagnetic wave, the power density per unit area associate with the wave is defined in complex representation by vector S,

$$\nabla \times E = \frac{-\partial B}{\partial t}$$

$H = B/\mu$

$B = kE/\omega$

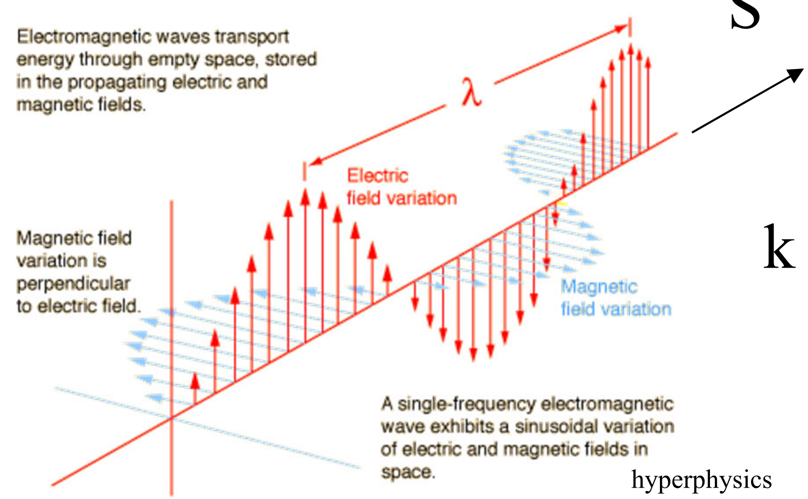
$C = \omega/\kappa$

$B = E/C$

$$S = E \times H^* \ (\mathrm{W/m^2})$$

Again Ponyting's vector is derived using Ampere's and Faraday's and couple vector calculus identities.

(or)



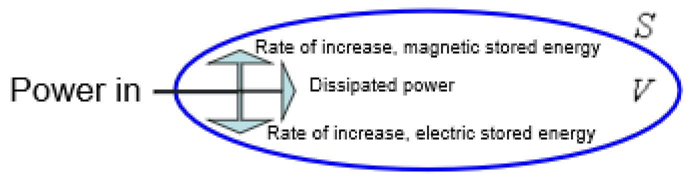
hyperphysics

Use Faraday's law and phase velocity equation

$$S = \frac{E x B^*}{2\mu_o} = \frac{|E|^2}{2C\mu_o}\hat{z} = \frac{|E|^2}{2Z_o}\hat{z} \Rightarrow \boxed{I(P) = \frac{|E|^2}{2Z_o}}$$

(Intensity)

where z_o = air impedance $= \sqrt{\frac{\mu_o}{\varepsilon_o}}$



Physical interpretation: (Assume that S is stationary.)

$$-\oiint_S (\underline{E} \times \underline{H}) \cdot \hat{n}\, dS = \int_V \sigma |\underline{E}|^2 \, dV + \frac{\partial}{\partial t} \int_V \left(\frac{1}{2} \mu |\underline{H}|^2 \right) dV + \frac{\partial}{\partial t} \int_V \left(\frac{1}{2} \varepsilon |\underline{E}|^2 \right) dV$$

Power dissipation as heat (Joule's law)

Rate of change of stored magnetic energy

Rate of change of stored electric energy

$\Longrightarrow$ Right-hand side = power flowing into the volume of space.

Time average pontying vector <s> is defined as average of the Time domain Poynting vector S over a period T=2π/ω.

$$< S >= \frac{1}{2\pi}\int_0^{2\pi} d(\omega t) E(x,y,z,t) \times H(x,y,z,t)$$

(or) $$< S >= \frac{1}{2}\mathrm{Re}\{E \times H\}$$

Week 9

- Course Website: http://courses.washington.edu/me557/sensors
- Reading Materials:

  \- Week 9 reading materials can be found:

  http://courses.washington.edu/me557/reading/
- Homework #2 is due Week 13
- Sign up for Lab #1
- Makeup classes (11/18 and 11/25) 1-2PM
- Proposal meeting Week 12 (Dec. 2) afternoon
- Proposal due Week 13
- Final Presentation 12/27 1:20 to 3:10PM

This week lecture

- Phase and group velocity
- Boundary Condition derivation from Maxwell's
- Linear circular and elliptical polarization
- Law of refraction and reflection from Wave equation and B.C.
- Critical and Brewster's angles
- Wave in dissipative medium

Boundary Condition

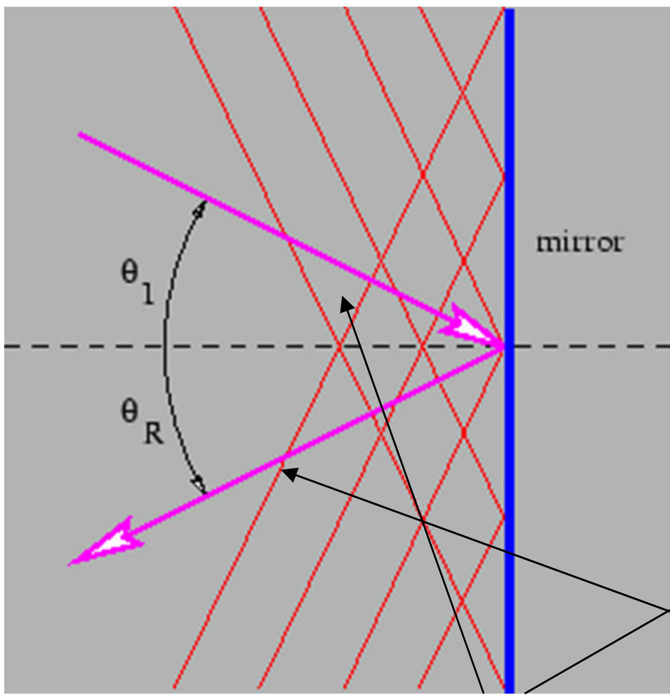
- To solve wave propagating from one medium to another, we need to find out how wave is transferring at the interface

Previously we assume total transmission or total reflection but

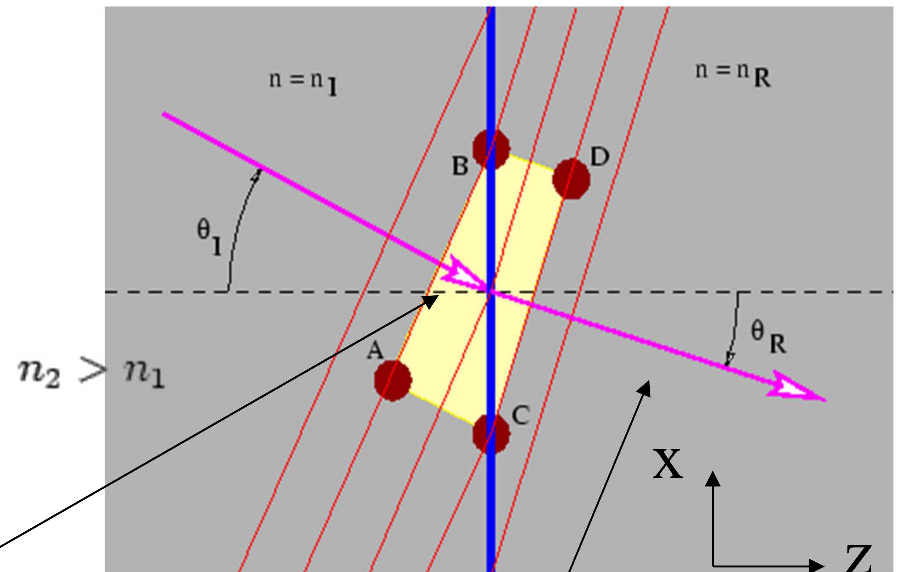
Most of what we need to know about geometrical optics can be summarized in two rules:

1) the laws of reflection
2) The law of refraction.

$$\theta_i = \theta_r$$



$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$



$$k_x = k\sin\theta_i = k_{rx} = k_r \sin\theta_r = k_{tx} = k_t \sin\theta_t$$

Reflection and Transmission (TE, S wave, I, perpendicular)

Fresnel Equation

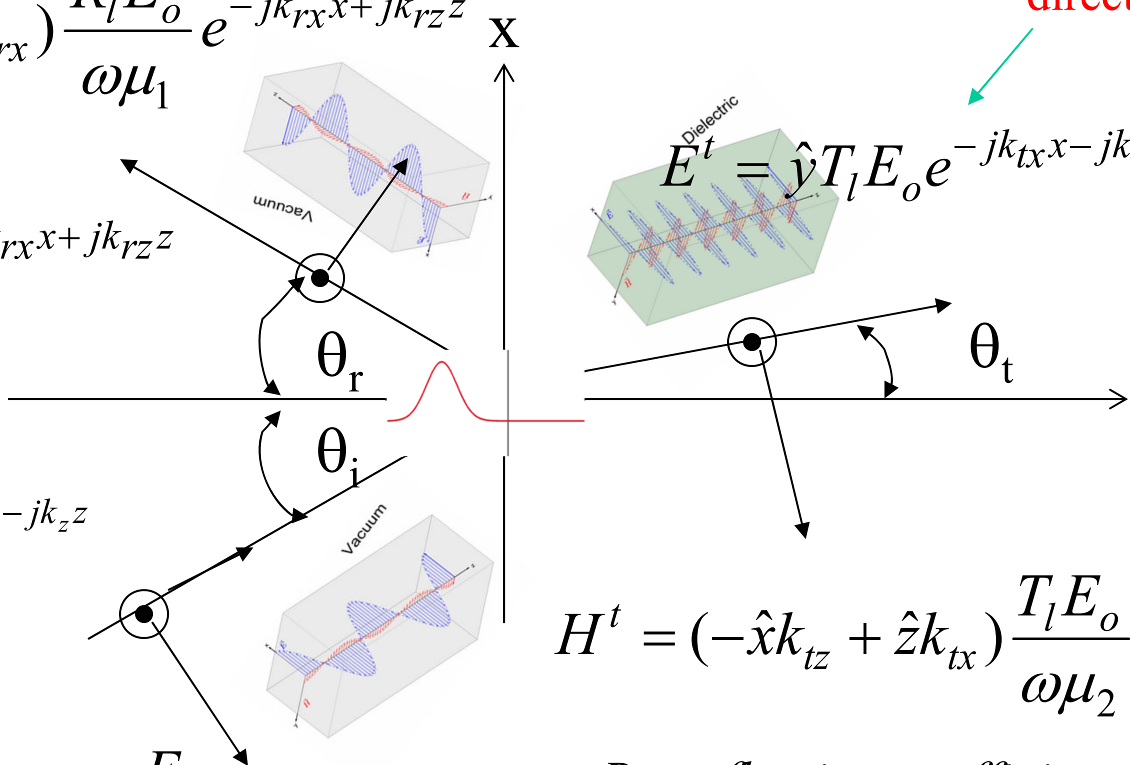
μ1, ε1, n1 $\quad\quad$ μ2, ε2, n2

Negative sign means positive propagating direction

$$H^r = (\hat{x}k_{rz} + \hat{z}k_{rx})\frac{R_l E_o}{\omega\mu_1}e^{-jk_{rx}x + jk_{rz}z}$$

$$E^t = \hat{y}T_l E_o e^{-jk_{tx}x - jk_{tz}z}$$

$$E^r = \hat{y}R_l E_o e^{-jk_{rx}x + jk_{rz}z}$$



Phasor form

$$E^i = \hat{y}E_o e^{-jk_x x - jk_z z}$$

$$H^t = (-\hat{x}k_{tz} + \hat{z}k_{tx})\frac{T_l E_o}{\omega\mu_2}e^{-jk_{tx}x - jk_{tz}z}$$

$$H^i = (-\hat{x}k_z + \hat{z}k_x)\frac{E_o}{\omega\mu_1}e^{-jk_x x - jk_z z}$$

R_l=reflection coefficient

T_l=transmission coefficient

TE = transverse electric, perpendicularly polarized (E perpendicular to plan of incident)

Reflection and Transmission (TM, P wave, II, Parallel)

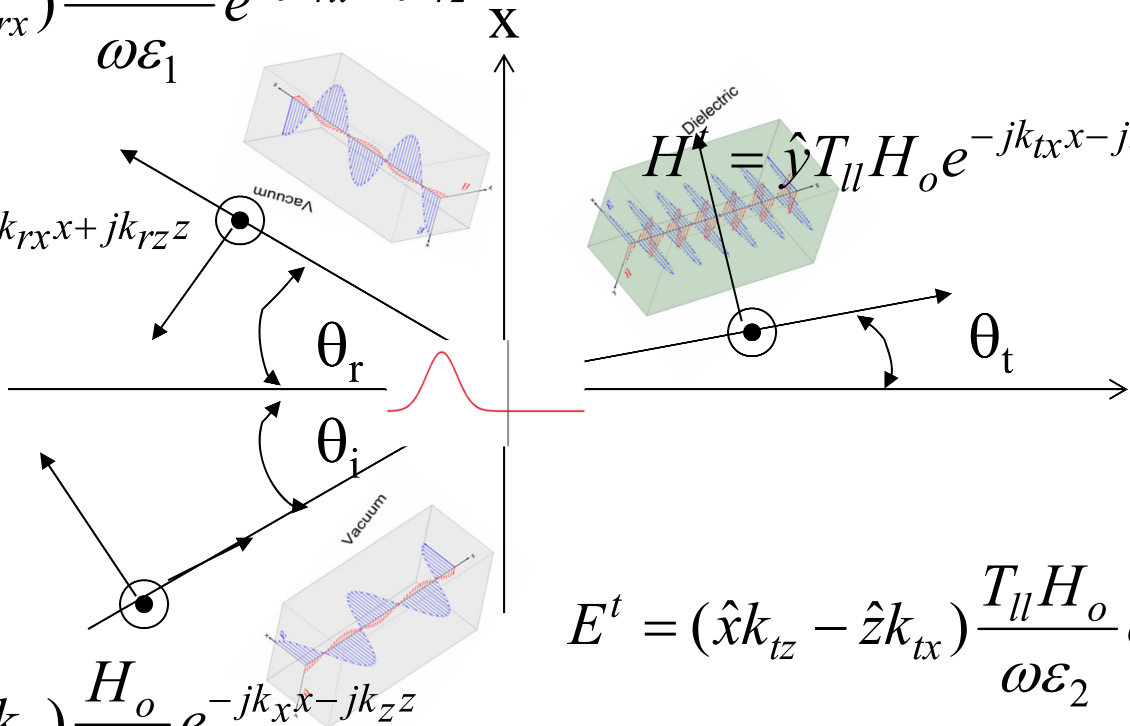
$\mu_1, \varepsilon_1, n_1$

$\mu_2, \varepsilon_2, n_2$

$$E^r = (-\hat{x}k_{rz} - \hat{z}k_{rx})\frac{R_{ll}H_o}{\omega\varepsilon_1}e^{-jk_{rx}x + jk_{rz}z}$$

$$H^r = \hat{y}R_{ll}H_o e^{-jk_{rx}x + jk_{rz}z}$$

$$H^t = \hat{y}T_{ll}H_o e^{-jk_{tx}x - jk_{tz}z}$$



$$E^t = (\hat{x}k_{tz} - \hat{z}k_{tx})\frac{T_{ll}H_o}{\omega\varepsilon_2}e^{-jk_{tx}x - jk_{tz}z}$$

$$E^i = (\hat{x}k_z - \hat{z}k_x)\frac{H_o}{\omega\varepsilon_1}e^{-jk_x x - jk_z z}$$

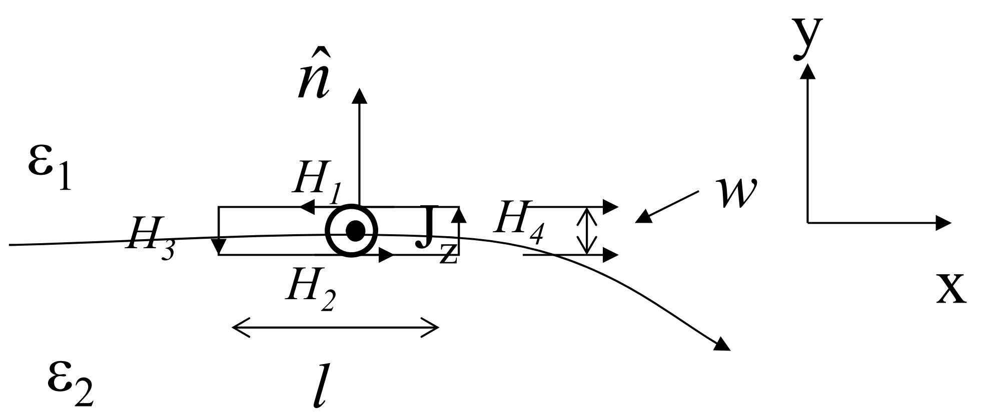
$$H^i = \hat{y}H_o e^{-jk_x x - jk_z z}$$

R_{ll}= reflection coefficient

T_{ll}= transmission coefficient

TM = transverse magnetic, parallel polarized (E parallel to plan of incident)

Boundary Conditions



At an interface between two media, the file quantities must satisfy Certain conditions. Consider an interface between two dielectric media With dielectric constants ε_1 and ε_2, in the z component Ampere's Law, we have,

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = J_z + jwD_z$$

or

$$\frac{H_4 - H_3}{l} - \frac{H_1 - H_2}{w} = J_z + jwD_z$$

Now let area shrink to a point where *w* goes to zero before *l* does. So $J_z = J_s \sim J_v w$, then

$$H_2 - H_1 = J_z$$

Or in general,

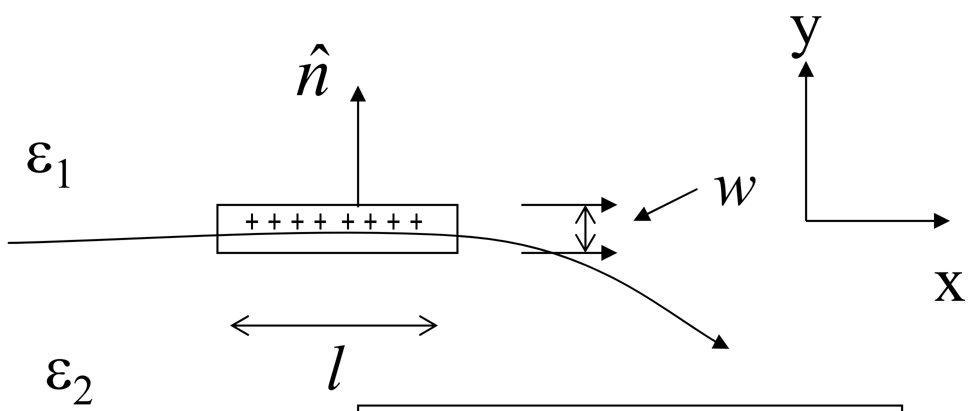
$$\boxed{\hat{n} \times (H_2 - H_1) = J_s}$$

Applying the same above argument to Faraday's Law and we get,

$$\boxed{\hat{n} \times (E_1 - E_2) = 0}$$

<u>The tangential electric field *E* is continuous across the boundary surface.The discontinuity in the tangential component of *H* is equal to the surface current density J_s.</u>

Apply the divergence theory $\nabla \bullet B = 0$ and $\nabla \bullet D = \rho$ for The pillbox volume shown



As w -> 0, we get

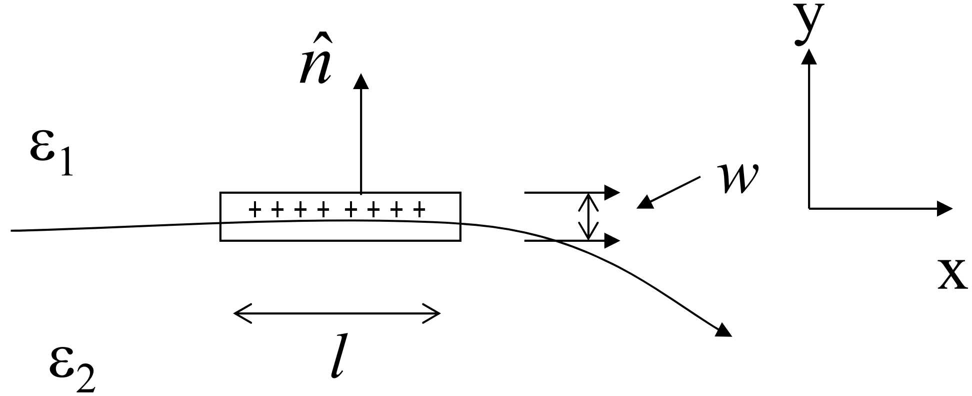
$$\boxed{(B_1 - B_2) \bullet \hat{n} = 0}$$

$$\boxed{(D_1 - D_2) \bullet \hat{n} = \rho}$$

The normal component of B is continuous across the boundary surface. The discontinuity in the normal component of D is equal to the surface charge density ρ

Boundary Condition for Perfect Conductor

On the surface of a perfect conductor, $E_2 = 0$ *and* $H_2 = 0$



Additional Handwritten note on BC

- Additional handwritten notes on how boundary conditions ae derived can be found:

http://courses.washington.edu/me557/readings/BC-EM.pdf

Finding Corresponding E or B field components in TE and TM mode

Reflection and Transmission (TE, S wave, I, perpendicular)

Fresnel Equation

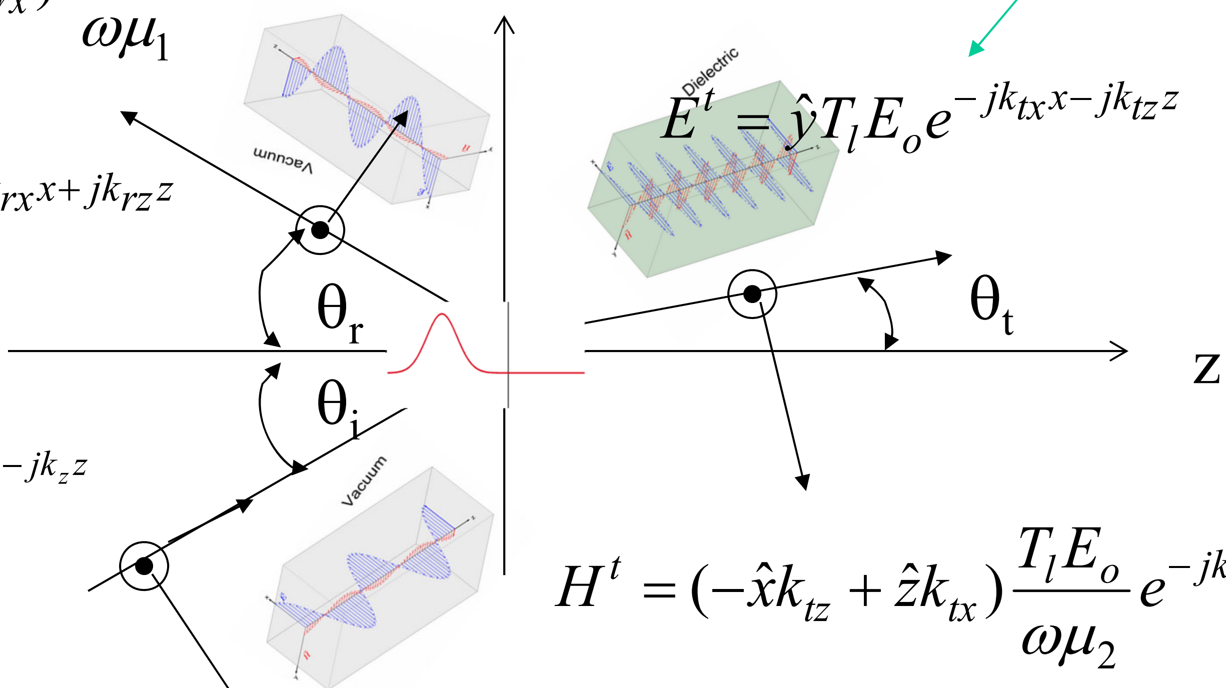
$\mu_1, \varepsilon_1, n_1$ $\quad\quad$ $\mu_2, \varepsilon_2, n_2$

Negative sign means positive propagating direction

$$H^r = (\hat{x}k_{rz} + \hat{z}k_{rx})\frac{R_l E_o}{\omega\mu_1}e^{-jk_{rx}x + jk_{rz}z}$$

$$E^r = \hat{y}R_l E_o e^{-jk_{rx}x + jk_{rz}z}$$

$$E^t = \hat{y}T_l E_o e^{-jk_{tx}x - jk_{tz}z}$$



$$E^i = \hat{y}E_o e^{-jk_x x - jk_z z}$$

$$H^t = (-\hat{x}k_{tz} + \hat{z}k_{tx})\frac{T_l E_o}{\omega\mu_2}e^{-jk_{tx}x - jk_{tz}z}$$

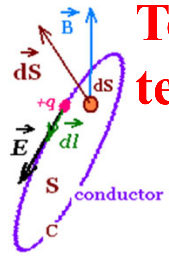
$$H^i = (-\hat{x}k_z + \hat{z}k_x)\frac{E_o}{\omega\mu_1}e^{-jk_x x - jk_z z}$$

R_l=reflection coefficient
T_l=transmission coefficient

TE = transverse electric, perpendicularly polarized (E perpendicular to plan of incident)

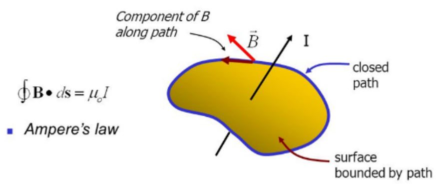
Maxwell's Equations

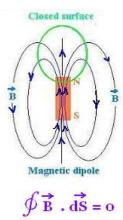
Differential form

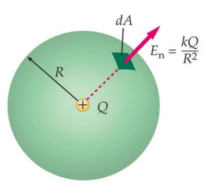


To fine H component in terms of E

Faraday's Law	$\boxed{\nabla \times E = \dfrac{-\partial B}{\partial t}}$
Ampere's Law	$\nabla \times H = J + \dfrac{\partial D}{\partial t}$
Gauss's Law for Magnetism	$\nabla \bullet B = 0$
Gauss's Law for Electricity	$\nabla \bullet D = \rho$







E = Electric Field (V/m)	ρ = charge density (c/m^3)	i = electric current (A)
B = Magnetic flux density(Web/m^2, T)	ε_0 = permittivity	J = current density(A/m^2)
D = Electric flux density (c/m^2)	μ_0 = permeability	c = speed of light
H = Magnetic Field (A/m)	Φ_B = Magnetic flux (Web)	P = Polarization

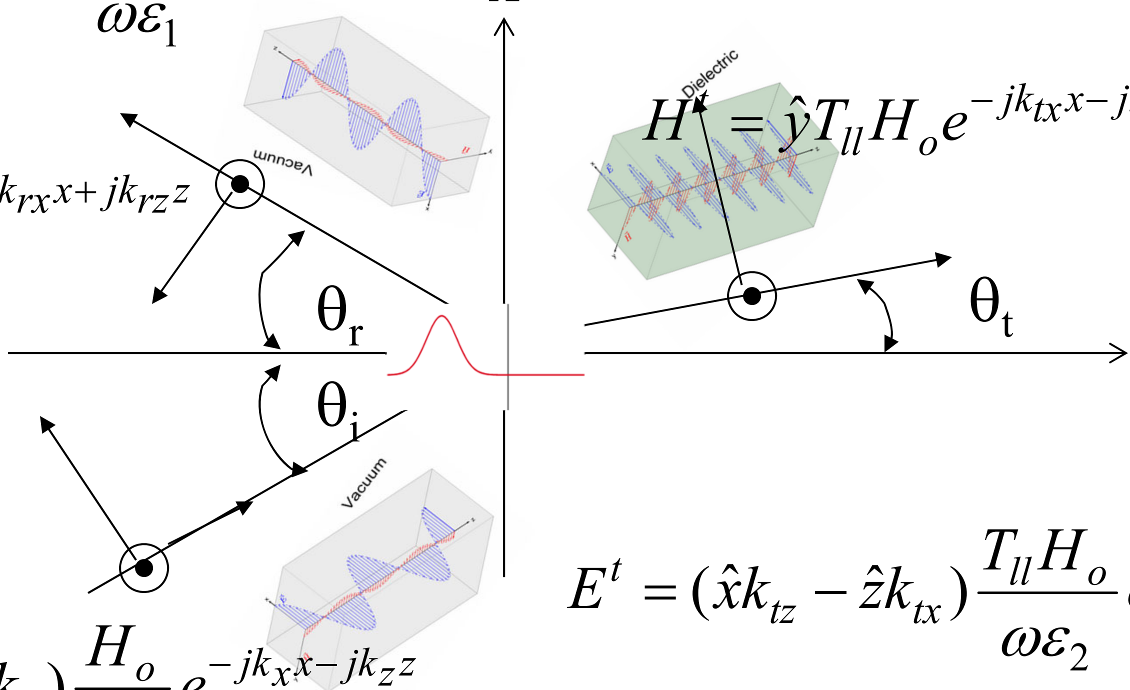
Reflection and Transmission (TM, P wave, II, Parallel)

$\mu_1, \varepsilon_1, n_1$ $\mu_2, \varepsilon_2, n_2$

$$E^r = (-\hat{x}k_{rz} - \hat{z}k_{rx})\frac{R_{ll}H_o}{\omega\varepsilon_1}e^{-jk_{rx}x + jk_{rz}z}$$

$$H^r = \hat{y}R_{ll}H_o e^{-jk_{rx}x + jk_{rz}z}$$

$$H^t = \hat{y}T_{ll}H_o e^{-jk_{tx}x - jk_{tz}z}$$



$$E^t = (\hat{x}k_{tz} - \hat{z}k_{tx})\frac{T_{ll}H_o}{\omega\varepsilon_2}e^{-jk_{tx}x - jk_{tz}z}$$

$$E^i = (\hat{x}k_z - \hat{z}k_x)\frac{H_o}{\omega\varepsilon_1}e^{-jk_x x - jk_z z}$$

$$H^i = \hat{y}H_o e^{-jk_x x - jk_z z}$$

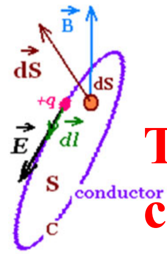
R_{ll} = *reflection coefficient*

T_{ll} = *transmission coefficient*

TM = transverse magnetic, parallel polarized (E parallel to plan of incident)

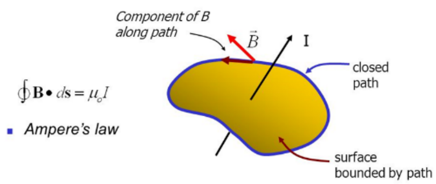
Maxwell's Equations

Differential form

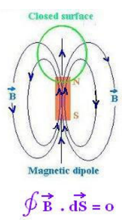


To fine E component in H

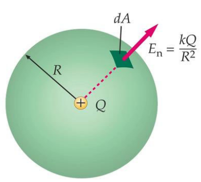
Faraday's Law $\nabla \times E = \frac{-\partial B}{\partial t}$



Ampere's Law $\nabla \times H = J + \frac{\partial D}{\partial t}$



Gauss's Law for Magnetism $\nabla \bullet B = 0$



Gauss's Law for Electricity $\nabla \bullet D = \rho$

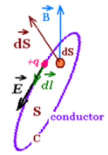
E = *Electric Field (V/m)*	ρ = *charge density (c/m^3)*	i = *electric current (A)*
B = *Magnetic flux density(Web/m^2, T)*	ε_0 = *permittivity*	J = *current density(A/m^2)*
D = *Electric flux density (c/m^2)*	μ_0 = *permeability*	c = *speed of light*
H = *Magnetic Field (A/m)*	Φ_B = *Magnetic flux (Web)*	P = *Polarization*

Boundary Condition

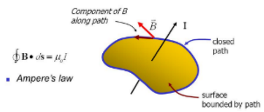
- To solve wave propagating from one medium to different mediums, we need to find out how wave is transferring at the interface

Maxwell's Equations

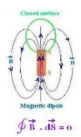
Integral form in the absence of magnetic or polarized media:



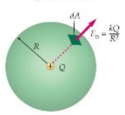
I. Faraday's law of induction $\oint \vec{E} \cdot \vec{dl} = -\frac{d\Phi_B}{dt}$



II. Ampere's law $\oint \vec{B} \cdot d\vec{s} = \mu_0 i + \frac{1}{c^2}\frac{\partial}{\partial t}\int \vec{E} \cdot d\vec{A}$



III. Gauss' law for magnetism $\oint \vec{B} \cdot d\vec{A} = 0$



IV. Gauss' law for electricity $\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$

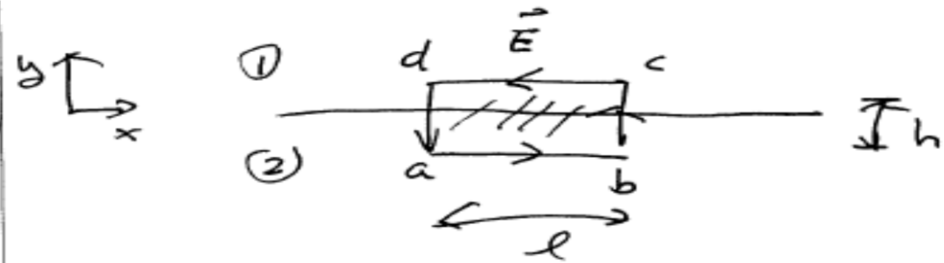
E = Electric Field (V/m) | *ρ = charge density (c/m^3)* | *i = electric current (A)*

B = Magnetic flux density(Web/m^2, T) | *ε_0 = relative permittivity* | *J = current density(A/m^2)*

D = Electric flux density (c/m^2) or electric displacement field | *μ_0 = relative permeability* | *c = speed of light*

H = Magnetic Field (A/m) | *Φ_B = Magnetic flux (Web)* | *P = Polarization*

q = charge $1.6x10^{-19}$ coulombs, *$\mu_o = 1.26x10^{-6} H/m$,* *$\varepsilon_o = 8.85x10^{-12} F/m$*



(i) Use Faraday's Law $\oint \vec{E}\cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t}\, ds$

Remember faraday's law tells us that Electric field along the close path equal to the negative of the magnetic flux density over the area the path enclosed

$$E_{ab}\cdot l + E_{bc}\, h - E_{cd}\, l - E_{da}\cdot h = -\frac{\partial B_z}{\partial t}\, l\, h$$

(ii) Let $h \Rightarrow 0$ close to surface

$$E_{ab}\, l - E_{cd}\, l = 0$$

← because area $\simeq 0$

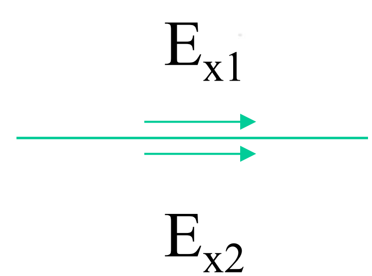
tangential E field is continuous but opposite direction

$$\boxed{E_{x2} - E_{x1} = 0}$$

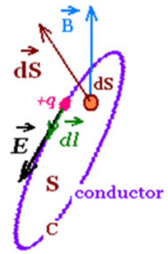
$E_{x2} = E_{ab}$, $E_{x1} = E_{cd}$

OR in general

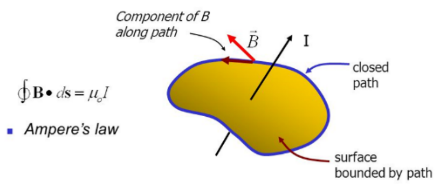
$$\hat{n}\times(E_2 - E_1) = 0$$



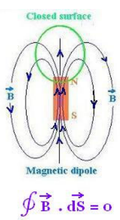
Maxwell's Equations



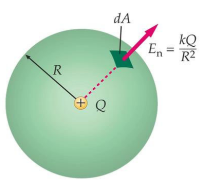
Faraday's Law $$\nabla \times E = \frac{-\partial B}{\partial t}$$



Ampere's Law $$\nabla \times H = J + \frac{\partial D}{\partial t}$$



Gauss's Law for Magnetism $$\nabla \bullet B = 0$$



Gauss's Law for Electricity $$\nabla \bullet D = \rho$$

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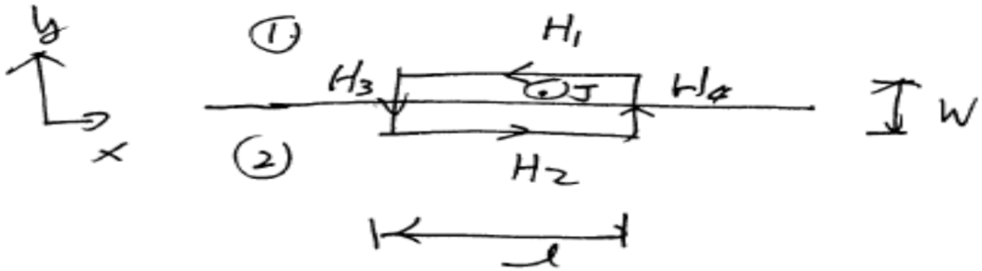


D= εE

(i) using Ampere's Law

J_z = volume current density

$$\vec{\nabla}\times\vec{H} = \vec{J}_z + j\omega\underbrace{\epsilon\vec{E}}_{D_z}$$

$$\boxed{\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}} = \boxed{J_z + j\omega D_z}$$

$$\boxed{\frac{H_4 - H_3}{l} - \frac{H_2 - H_1}{w}} = \boxed{J_z + j\omega D_z}$$

$$-H_2 + H_1 = J_z w + \left(j\omega D_z - \frac{H_4 - H_3}{l}\right) w$$

Let area shrink to a point

$$\lim_{w\to 0} w \Rightarrow -H_2 + H_1 = J_z w = J_s$$

J_s = surface current density

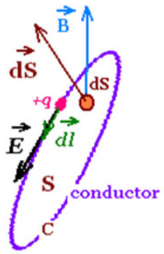
or in differential form:

$$\boxed{\hat{n}\times(H_1 - H_2) = J_s}$$

e.g. If neither two are perfect conductors, J_s=0,

Tangential H is continuous if no current density

Maxwell's Equations



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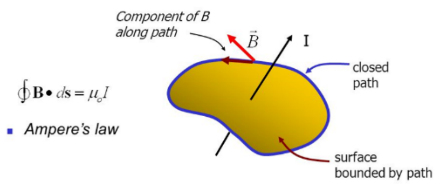

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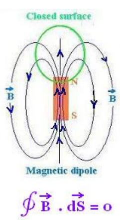

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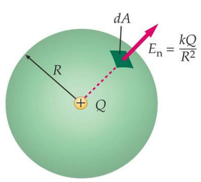

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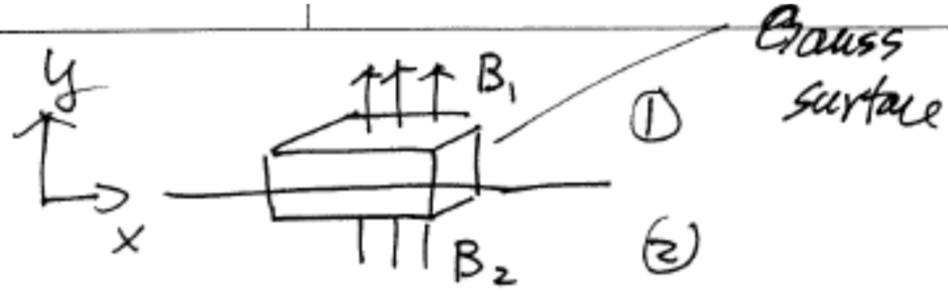


(i) Using ~~divergence~~ ~~the~~ magnetic Gauss's Law

$$\nabla \cdot \vec{B} = 0$$

$$B_{1y} - B_{2y} = 0$$ no magnetic monopole inside

$$(B_1 - B_2) \cdot \hat{n} = 0$$

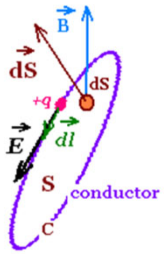
Remember

B= μH

$$\left(\frac{H_{1y}}{\mu_1} - \frac{H_{2y}}{\mu_2}\right) \cdot \widehat{n} = 0$$

Flux is continuous but field is not Unless $\mu_1=\mu_2$

Maxwell's Equations



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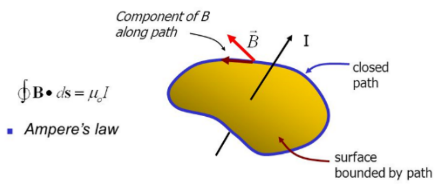

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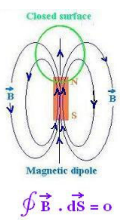
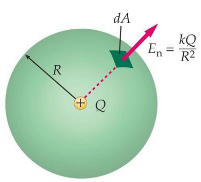


Gauss's Law for Magnetism $\nabla \bullet B = 0$



Gauss's Law for Electricity $\boxed{\nabla \bullet D = \rho}$

E = Electric Field (V/m)

B = Magnetic flux density(Web/m², T)

D = Electric flux density (c/m²)

H = Magnetic Field (A/m)

ρ = *charge density (c/m³)*

ε_0 = *permittivity*

μ_0 = *permeability*

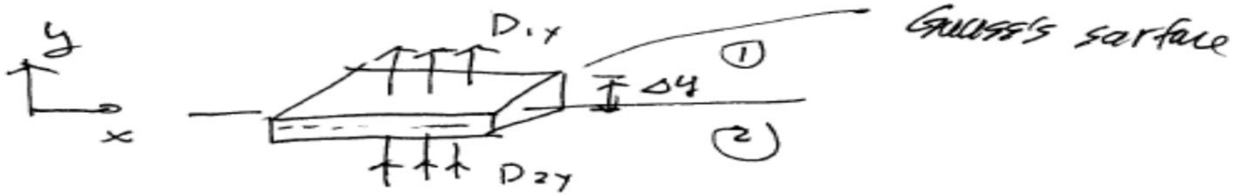
Φ_B = *Magnetic flux (Web)*

i = electric current (A)

J = current density(A/m²)

c = speed of light

P = Polarization



(i) using electric Gauss's Law

$$\vec{\nabla} \cdot \vec{D} = \rho_v$$

if no charge inside gauss's surface

$$\rho_v = 0$$

$$\nabla \cdot \vec{D} = 0$$

$$D_{1y} - D_{2y} = 0$$ ← * normal component of E flux density is continuous across Boundary

or in general $(D_1 - D_2) \cdot \hat{n} = 0$

$$\epsilon_1 E_{1y} - \epsilon_2 E_2 = 0$$

$$E_{1y} = \frac{\epsilon_2}{\epsilon_1} E_{2y}$$ ← normal component of electric field is discontinuous if $\epsilon_2 \neq \epsilon_1$

(ii) If $\rho_v \neq 0$

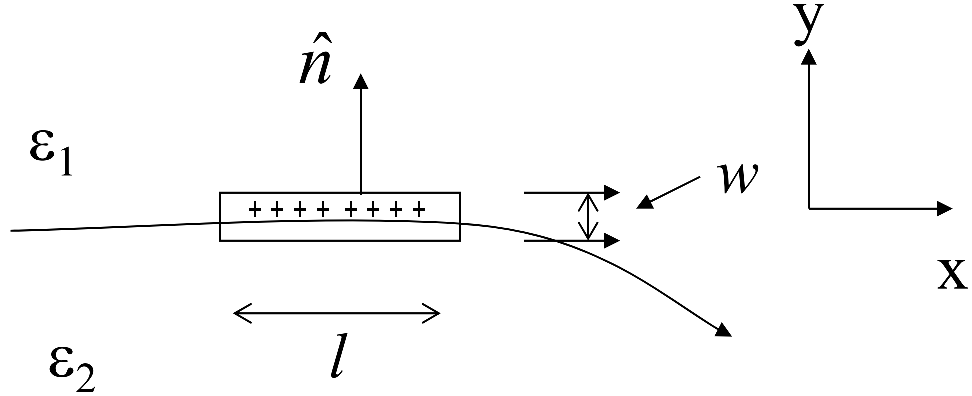
$$\nabla \cdot \vec{D} = \rho_v$$

$$D_{1y} - D_{2y} = \rho_v$$

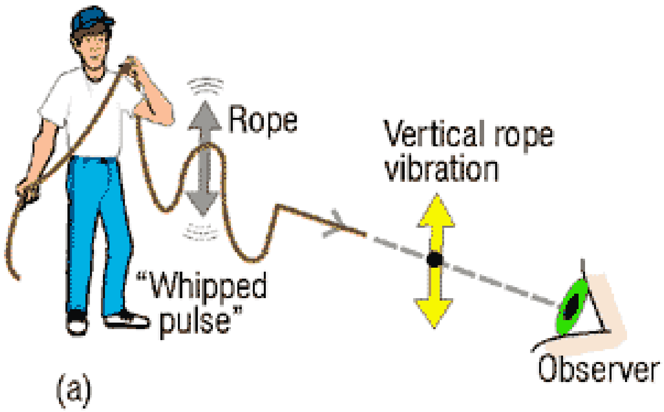
or in general $(D_1 - D_2) \cdot \hat{n} = \rho_v$

Boundary Condition for Perfect Conductor

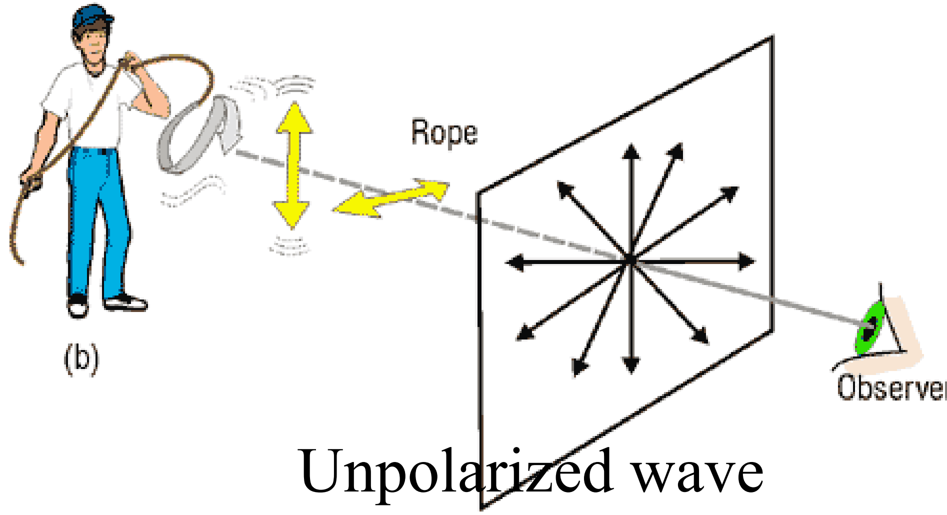
On the surface of a perfect conductor, $E_2 = 0$ *and* $H_2 = 0$



Polarization

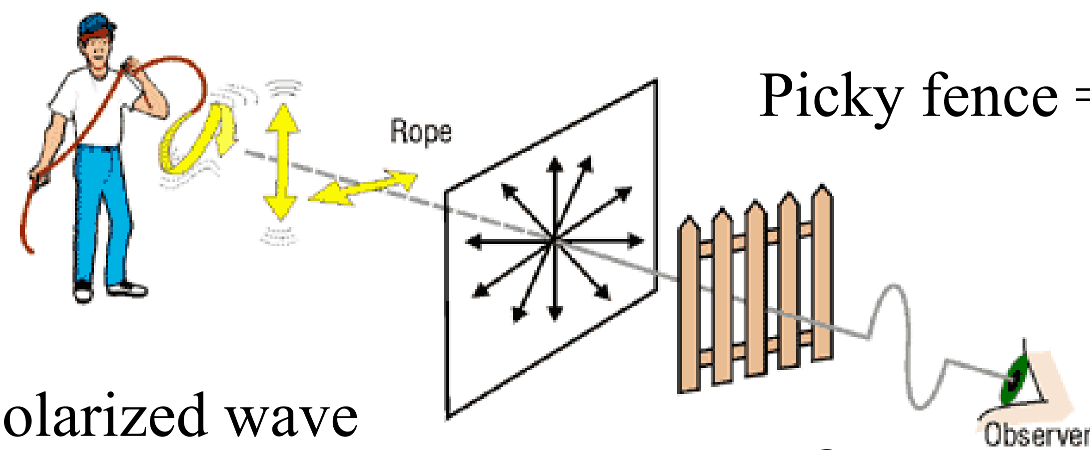


Polarized wave



Unpolarized wave

Imagine a "magic" rope that you can whip up and down at one end, thereby sending a *transverse* "whipped pulse" (vibration) out along the rope.



Picky fence = polarizer

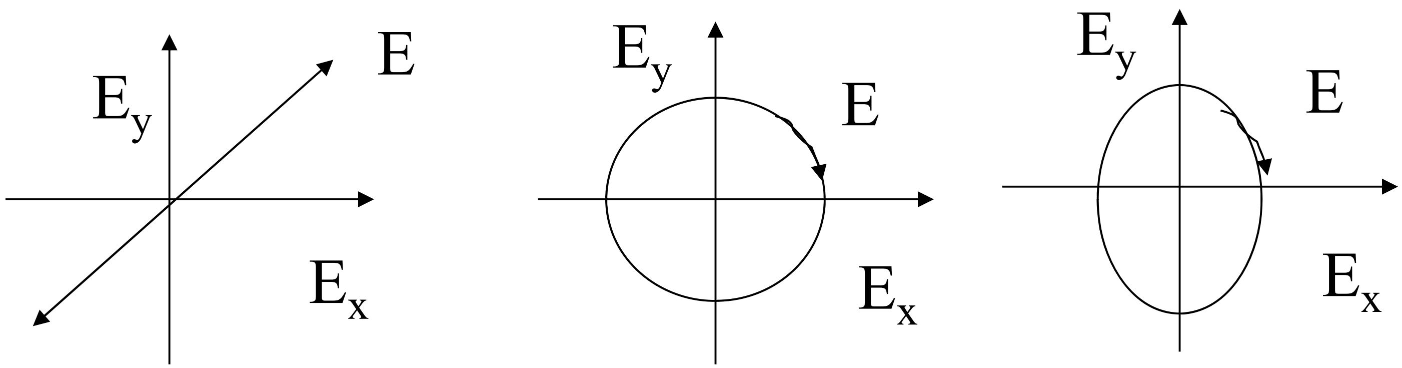
Input put = Unpolarized wave

Output wave = polarized

Polarization

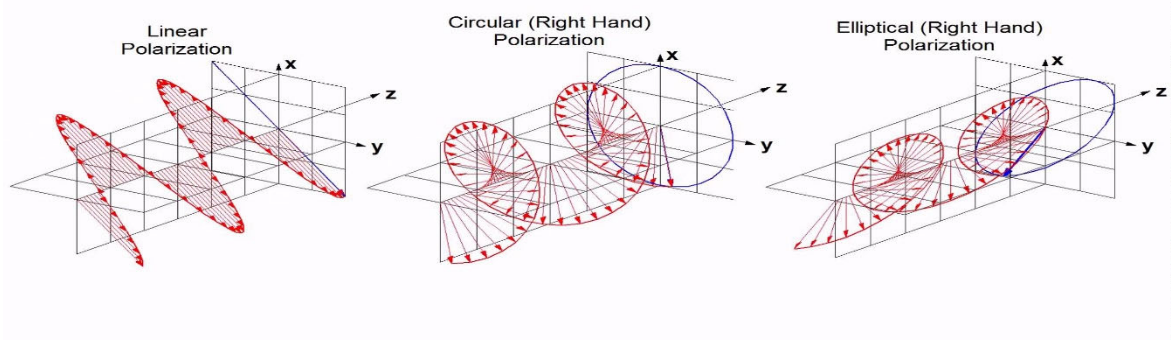
A fixed point in space, E vector of a time-harmonic electromagnetic wave varies sinusoidally with time. The polarization of the wave is described by the locus of the tip of the E vector as time progress. when the locus is a straight line, the wave is said to be linearly polarized. If the locus is a circle then the wave is said to be circularly polarized and if locus is elliptical then the wave is elliptically polarized. You can also fix time and see how it varies in space

Fix space



$$E(z,t) = a\cos(\omega t - kz + \phi_a)\hat{x} + b\cos(\omega t - kz + \phi_b)\hat{y}$$

Fix time



Let's assume the real time-space E vector has x and y components:

$$E(z,t) = a\cos(\omega t - kz + \phi_a)\hat{x} + b\cos(\omega t - kz + \phi_b)\hat{y}$$

$E_y/E_x = Ae^{j\phi}$

linearly polarized: $\phi_b - \phi_a = 0..or\,\pi$ $\quad E_y = \pm(\frac{b}{a})E_x$

circularly polarized: $\phi_b - \phi_a = \pm\frac{\pi}{2}$ $\quad \frac{E_y}{E_x} = \frac{b}{a} = 1$

Elliptically polarized: $\phi_b - \phi_a = anything..except..0, \pi, \pm\frac{\pi}{2}$

$$\frac{E_y}{E_x} = \frac{b}{a} = anything$$

polarization examples :

$$\vec{E} = \hat{x}E_x + \hat{y}E_y$$

$$E_x = a\cos(\omega t - kz + \phi_a)$$

$$E_y = b\cos(\omega t - kz + \phi_b)$$

Take a snap shot at different z at fix time

$t = 0$ $\qquad$ $\phi = \phi_b - \phi_a$

$= 0$ or π

ϕ_a & ϕ_b

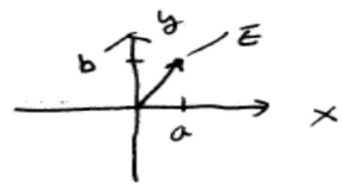
like some delay

not represented by the plot?

$E = |E| e^{j\phi}$

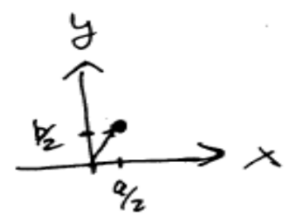
$|E| = \sqrt{E_x^2 + E_y^2} = \sqrt{a^2+b^2}$

$\phi = \tan^{-1}\frac{E_y}{E_x}$

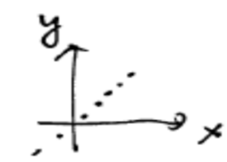


ie $\begin{cases} \phi = 0 & z = 0 \\ E_x = a & t = 0 \\ E_y = b & \end{cases}$

$\begin{cases} \phi = 0 & z = \frac{\lambda}{4} \\ E_x = 0 & kz = \frac{\pi}{2} \\ E_y = 0 & \end{cases}$

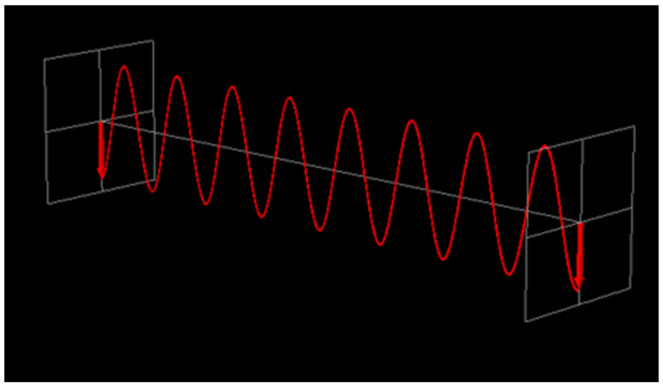


$\begin{cases} \phi = 0 & z = \frac{\lambda}{6} \\ E_x = \frac{a}{2} & kz = \frac{\pi}{3} \\ E_y = \frac{b}{2} & \end{cases}$

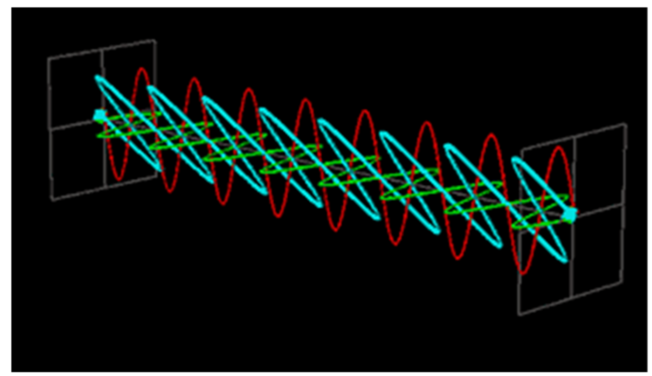


Linearly Polarized

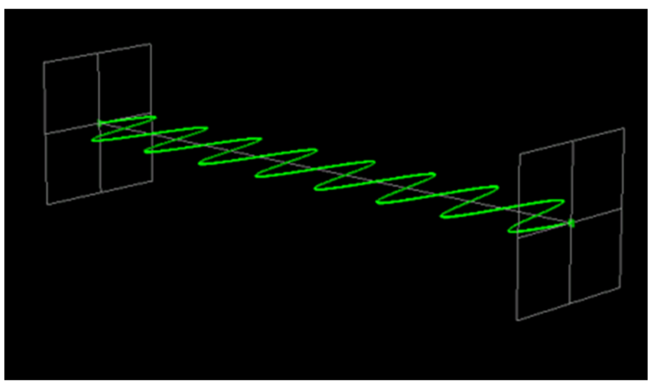
$$\phi_b - \phi_a = 0..or\ \pi \qquad E_y = \pm(\frac{b}{a})E_x$$



vertical



45°



horizontal

Circularlly Polarized

Take a snap shot at different z at fix time

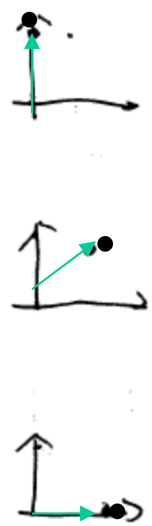
$E_x = a e^{j\omega t - kz + \phi_a}$ $E_x = a\cos(\omega t - k\,z + \phi_a)$

$E_y = b e^{j\omega t - kz + \phi_b}$ $E_y = a\cos(\omega t - k\,z + \phi_b)$

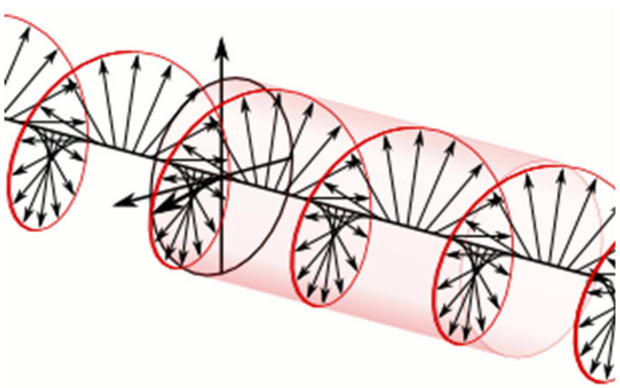
cos vs sin

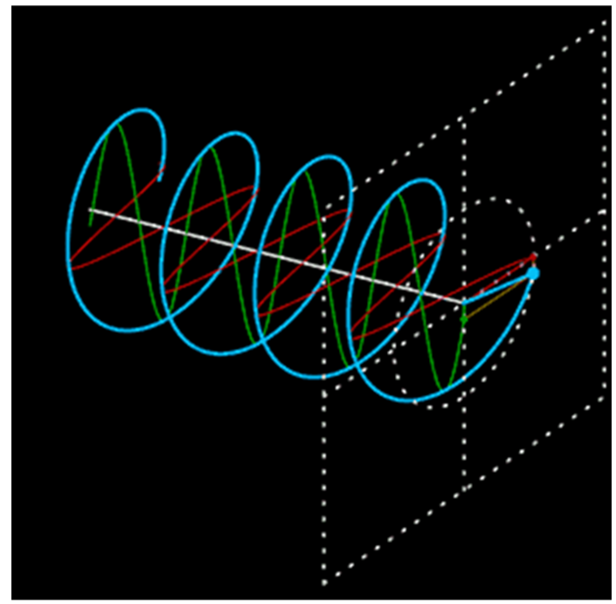
circl

$\phi_b - \phi_a = \phi = \pm\pi/2$



Circular polarized light

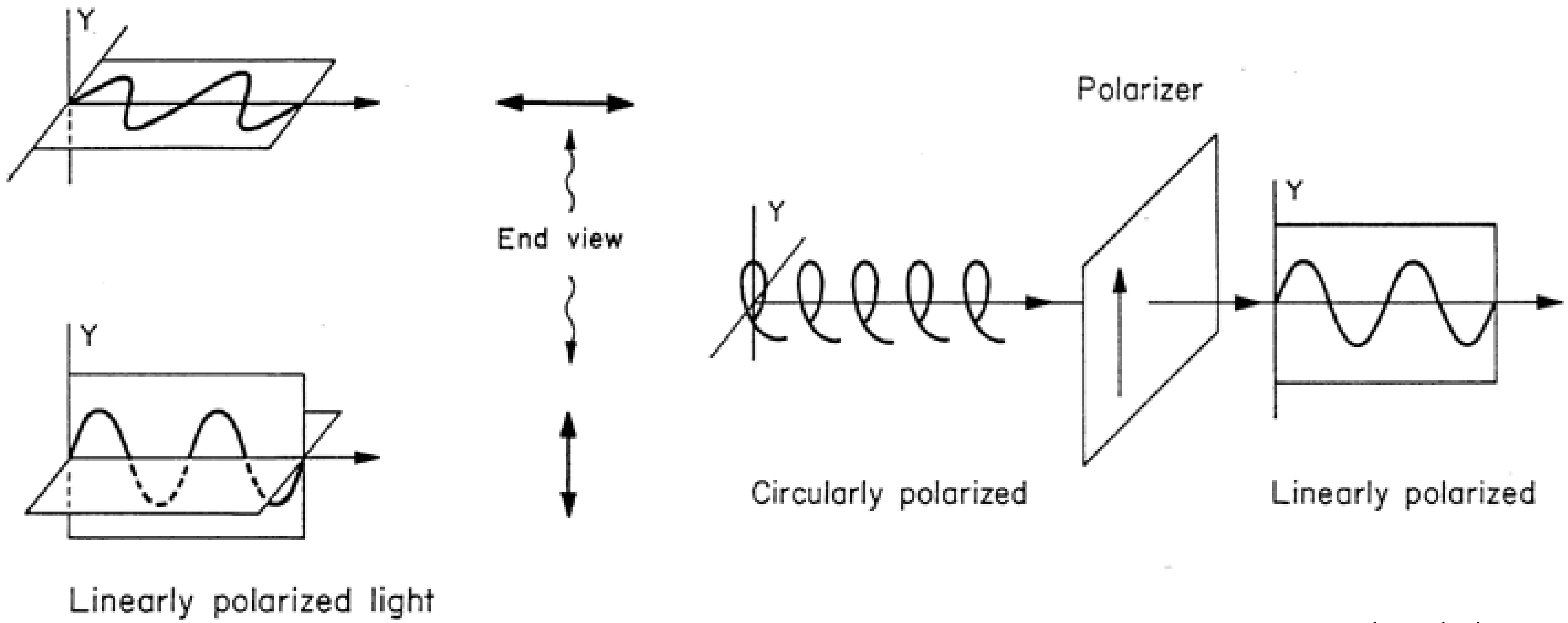




$$\phi_b - \phi_a = \pm\frac{\pi}{2}$$

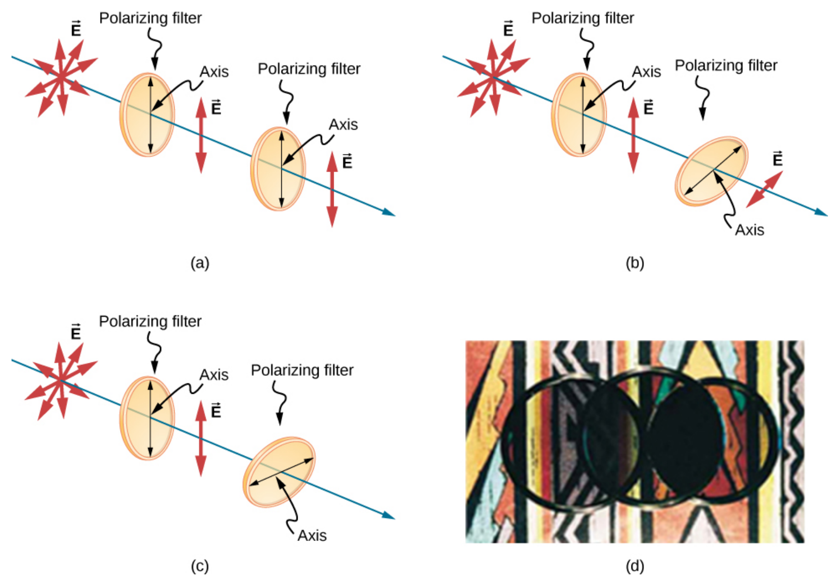
$$\frac{E_y}{E_x} = \frac{b}{a} = 1$$

Linear and circular polarization



hyperphysics

Polarizing Light



Reduce Intensity

Reduce glare!!!

Linear polarizer

Recap

Polarization

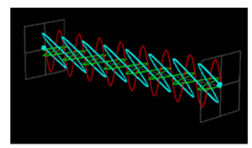
Let's assume the real time-space *E* vector has x and y components:

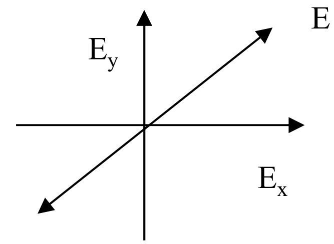
$$E(z,t) = a\cos(\omega t - kz + \phi_a)\hat{x} + b\cos(\omega t - kz + \phi_b)\hat{y}$$

$E_y/E_x = Ae^{j\phi}$

linearly polarized: $\phi_b - \phi_a = 0..or\,\pi$

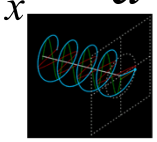
$$E_y = \pm(\frac{b}{a})E_x$$

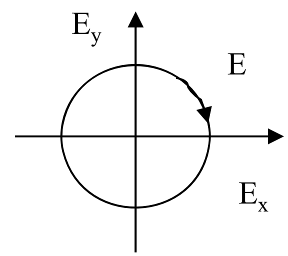




circularly polarized: $\phi_b - \phi_a = \pm\frac{\pi}{2}$

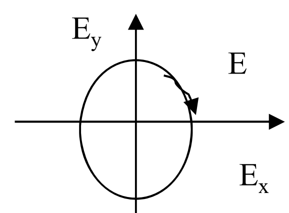
$$\frac{E_y}{E_x} = \frac{b}{a} = 1$$





Elliptically polarized: $\phi_b - \phi_a = anything..except..0, \pi, \pm\frac{\pi}{2}$

$$\frac{E_y}{E_x} = \frac{b}{a} = anything$$



Summary of EM Wave

Maxwell's Equations

$$\nabla \bullet \vec{D} = 0, \quad \nabla \bullet \vec{B} = 0 \qquad \vec{D}, \vec{E} \perp \vec{k} \qquad \vec{B}, \vec{H} \perp \vec{k}$$

$$\left.\begin{aligned} \nabla \times \vec{E} &= -j\omega\mu\vec{H} \\ \nabla \times \vec{H} &= j\omega\varepsilon\vec{E} \end{aligned}\right\} \text{Predicts waves}$$

Wave Equation

Helmholtz Wave Equation

$$\nabla^2 u + \left(\frac{\omega}{v}\right)^2 u = 0$$

$u \equiv$ disturbance, $\omega \equiv$ frequency, $v \equiv$ velocity

Inhomogeneous Media

$$\nabla \times \left(\frac{1}{\mu} \nabla \times \vec{E}\right) = \omega^2 \varepsilon \vec{E}$$

Used mostly in numerical analysis.

Homogeneous Media

$$\nabla^2 \vec{E} + \underbrace{\omega^2 \mu\varepsilon}_{} \vec{E} = 0$$

Used mostly in closed-form analysis.

$$k^2 = \left(\frac{\omega}{v}\right)^2 = \omega^2 \mu\varepsilon$$

$k \equiv$ wave number

EM Wave Velocity

$$v = 1/\sqrt{\mu\varepsilon} \qquad c_0 = 1/\sqrt{\mu_r \varepsilon_r}$$

$$v = c_0 / n \qquad c_0 = 299{,}792{,}458 \text{ m/s}$$

$$n = \sqrt{\mu_r \varepsilon_r} \qquad n \equiv \text{refractive index}$$

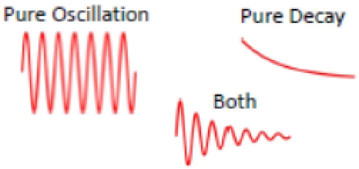
Solution to Wave Equation

$$\nabla^2 \vec{E} + k^2 \vec{E} = 0 \rightarrow \begin{aligned} \nabla^2 E_x + k^2 E_x &= 0 \\ \nabla^2 E_y + k^2 E_y &= 0 \\ \nabla^2 E_z + k^2 E_z &= 0 \end{aligned} \rightarrow E_i(z) = Ae^{-jkz} + Be^{+jkz}$$

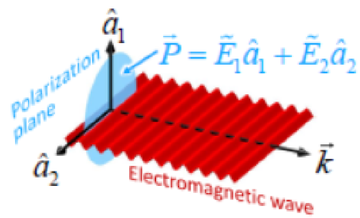
Forward Wave — Backward Wave

Solution is complex exponentials.

Waves can only do 2.5 things:



Plane Waves



Time-Domain

$$\vec{E}(t) = \vec{P}\cos\left(\omega t - \vec{k} \bullet \vec{r}\right)$$

Frequency-Domain

$$\vec{E}(\omega) = \vec{P}\exp\left(-j\vec{k} \bullet \vec{r}\right)$$

Wave Vector

$\left|\vec{k}\right| = 2\pi/\lambda$ — Conveys wavelength λ inside medium

$\left|\vec{k}\right| = k_0 = 2\pi/\lambda_0$ — In vacuum.

$\left|\vec{k}\right| = k_0 n$ — Also conveys refractive index n when frequency is known.

Relation Between E&H

Directionality: $\vec{E} \perp \vec{k} \perp \vec{H}$

Magnetic Field

$$\vec{H}(\omega) = \frac{\vec{k} \times \vec{P}}{\omega\mu} \exp\left(-j\vec{k} \bullet \vec{r}\right)$$

Impedance

$$\eta = \frac{E_0}{H_0} = \sqrt{\frac{\mu/\varepsilon}{1 + \sigma/j\omega\varepsilon}}$$

$$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\left[1 + (\sigma/\omega\varepsilon)^2\right]^{1/4}}$$

$$\measuredangle\eta = 0.5\tan(\sigma/\omega\varepsilon)$$

Polarization

$$\vec{P} = \left(E_1\hat{a}_1 + E_2 e^{j\delta}\hat{a}_2\right)e^{j\theta}$$

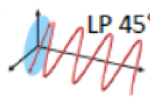
Expanded Polarization Vector

Linear Polarization (LP)

Test: $\delta = 0°$

LP 90°

LP 0°



LP 45°

RCP

Circular Polarization (CP)

Test: $\delta = +90°$ and $E_1 = E_2$

Right-Hand CP (RCP): $\delta = +90°$

Left-Hand CP (LCP): $\delta = -90°$

LCP

Elliptical Polarization (EP)

Test: not LP or CP

LP and CP are just special cases of EP.

EP

Properties

Loss Tangent

$$\tan\delta = \varepsilon''/\varepsilon' \qquad P(z) = P_0 e^{-k\delta z}$$

Propagation Constant

$$\gamma = \alpha + \beta \qquad E(z) = E_0 e^{-\gamma z}$$

Attenuation Coefficient

$$\alpha = \sqrt{(k/2)\left[\sqrt{1 + (\sigma/\omega\varepsilon)^2} - 1\right]}$$

Phase Constant

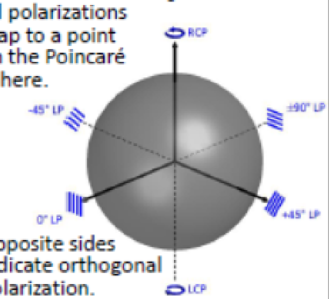
$$\beta = \sqrt{(k/2)\left[\sqrt{1 + (\sigma/\omega\varepsilon)^2} + 1\right]}$$

Absorption Coefficient

$$\alpha_{abs} = k\delta = 2\alpha \qquad P(z) = P_0 e^{-\alpha_{abs} z}$$

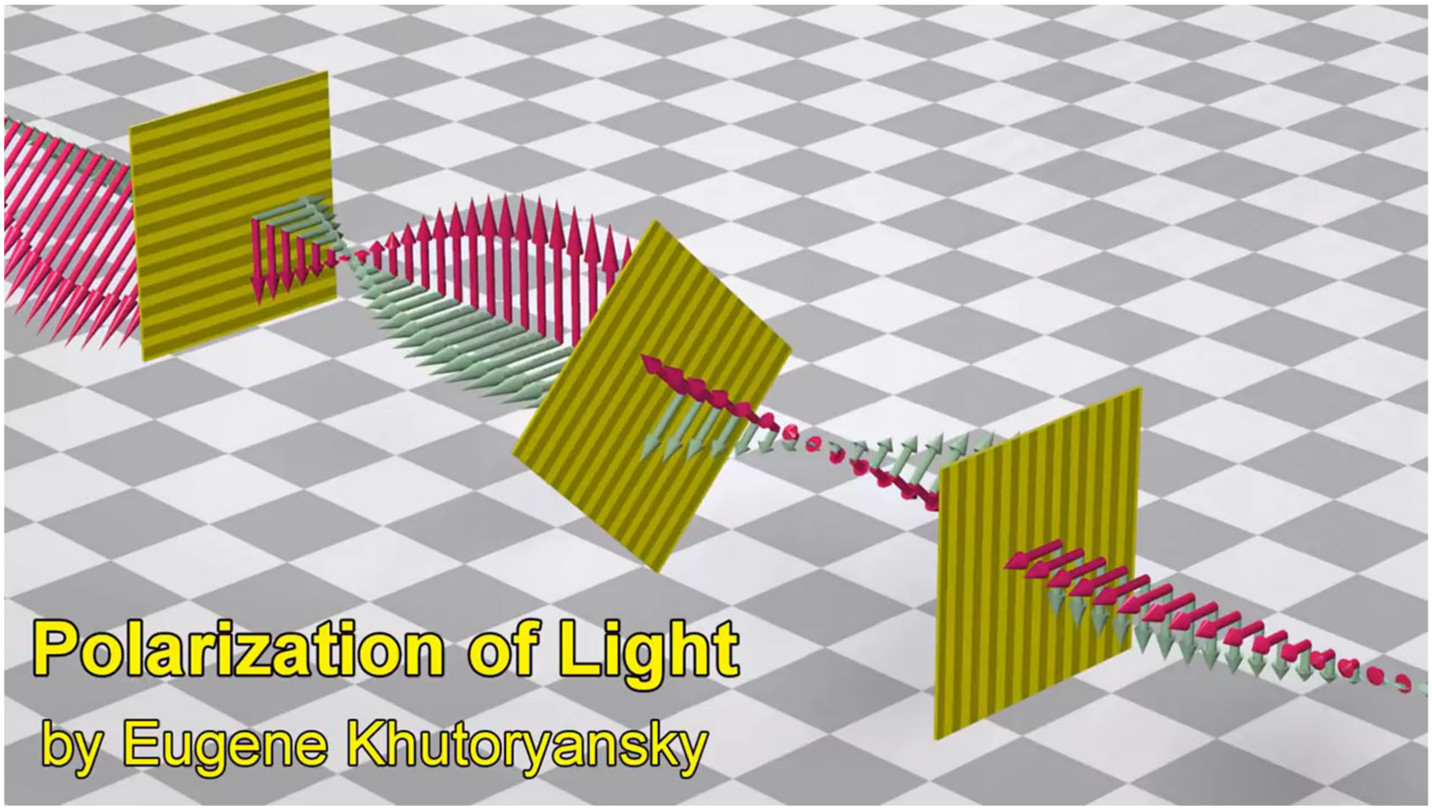
Poincaré Sphere

All polarizations map to a point on the Poincaré sphere.



Opposite sides indicate orthogonal polarization.

Polarization Video



Recap

Boundary Conditions

Tangential Components:

- Faraday's Law, $\boxed{\hat{n}\times(E_1 - E_2) = 0}$

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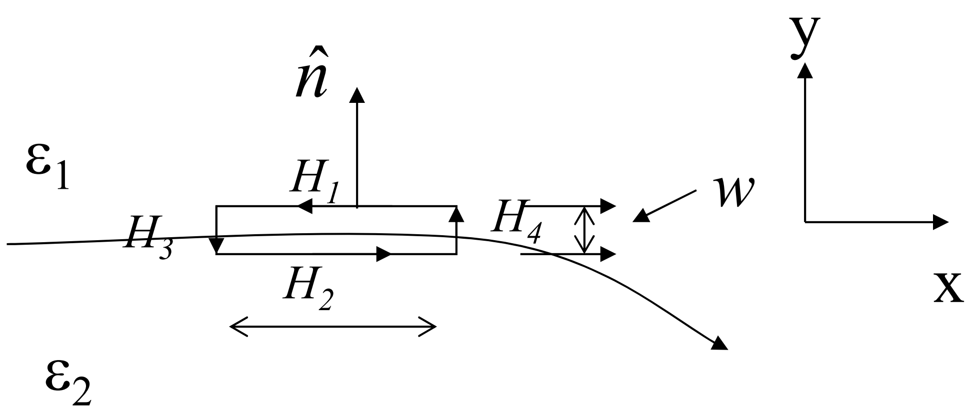

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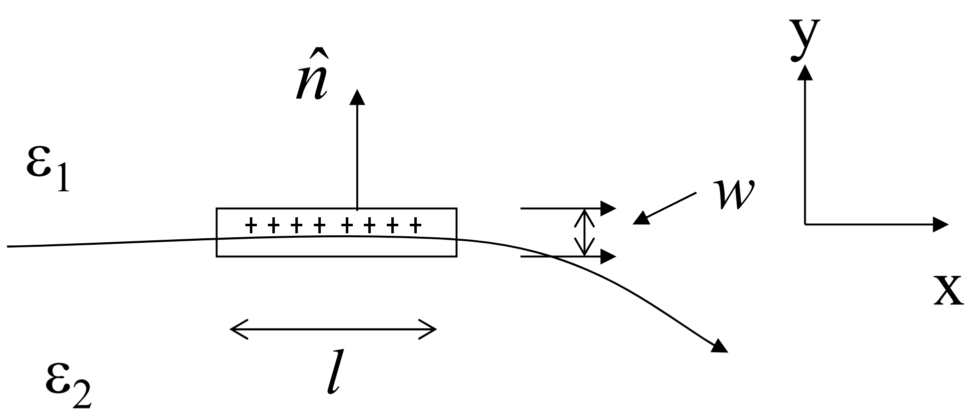

On the surface of a perfect conductor, $E_{2//} = 0$ *and* $B_{2\perp} = 0$

Recap

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Let's assume the real time-space *E* vector has x and y components:

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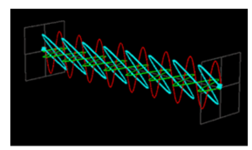

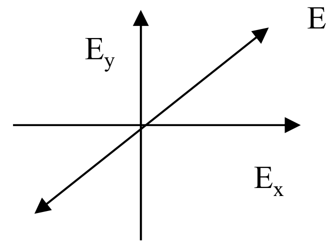

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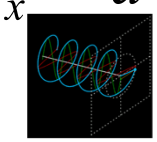

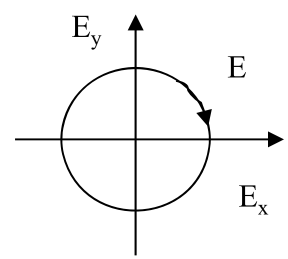

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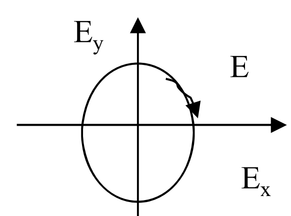

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Used mostly in numerical analysis.

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Used mostly in closed-form analysis.

$k \equiv$ wave number $\quad k^2 = \left(\frac{\omega}{v}\right)^2 = \omega^2 \mu\varepsilon$

EM Wave Velocity

$v = 1/\sqrt{\mu\varepsilon} \qquad c_0 = 1/\sqrt{\mu_r \varepsilon_r}$

$v = c_0/n \qquad c_0 = 299{,}792{,}458\ \text{m/s}$

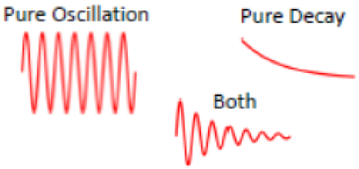
$n = \sqrt{\mu_r \varepsilon_r} \qquad n \equiv$ refractive index

Solution to Wave Equation

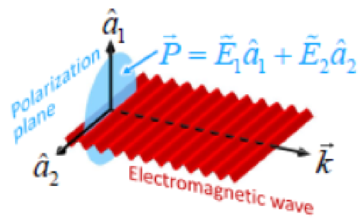
$$\nabla^2 \vec{E} + k^2 \vec{E} = 0 \rightarrow \begin{array}{l} \nabla^2 E_x + k^2 E_x = 0 \\ \nabla^2 E_y + k^2 E_y = 0 \\ \nabla^2 E_z + k^2 E_z = 0 \end{array} \rightarrow E_i(z) = \underbrace{Ae^{-jkz}}_{\text{Forward Wave}} + \underbrace{Be^{+jkz}}_{\text{Backward Wave}}$$

Solution is complex exponentials.

Waves can only do 2.5 things:



Plane Waves



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Relation Between E&H

Directionality: $\vec{E} \perp \vec{k} \perp \vec{H}$

Magnetic Field

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$$\measuredangle \eta = 0.5\tan(\sigma/\omega\varepsilon)$$

Polarization

$$\vec{P} = \left(E_1 \hat{a}_1 + E_2 e^{j\delta}\hat{a}_2\right)e^{j\theta}$$

Expanded Polarization Vector

Linear Polarization (LP)

Test: $\delta = 0°$

Circular Polarization (CP)

Test: $\delta = +90°$ and $E_1 = E_2$

Right-Hand CP (RCP): $\delta = +90°$
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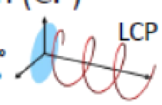


Elliptical Polarization (EP)

Test: not LP or CP
LP and CP are just special cases of EP.

Properties

Loss Tangent

$\tan\delta = \varepsilon''/\varepsilon' \quad P(z) = P_0 e^{-k\delta z}$

Propagation Constant

$\gamma = \alpha + \beta \quad E(z) = E_0 e^{-\gamma z}$

Attenuation Coefficient

$$\alpha = \sqrt{(k/2)\left[\sqrt{1 + (\sigma/\omega\varepsilon)^2} - 1\right]}$$

Phase Constant

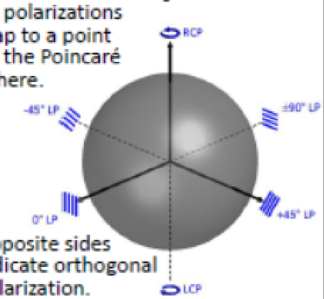
$$\beta = \sqrt{(k/2)\left[\sqrt{1 + (\sigma/\omega\varepsilon)^2} + 1\right]}$$

Absorption Coefficient

$\alpha_{\text{abs}} = k\delta = 2\alpha \quad P(z) = P_0 e^{-\alpha_{\text{abs}} z}$

Poincaré Sphere

All polarizations map to a point on the Poincaré sphere.



Opposite sides indicate orthogonal polarization.

Reflection and Refraction

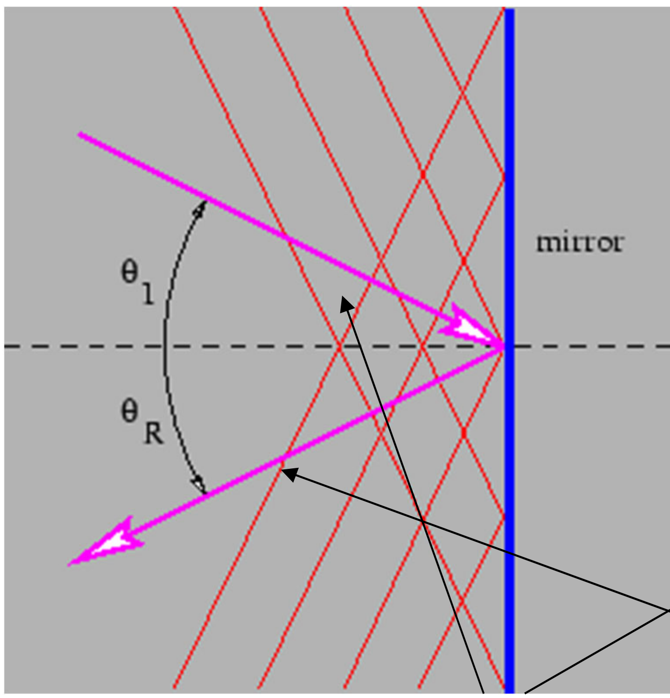
- We have derive the reflection and refraction using **ray optics particle theory.**
- Next we will derive law of reflection, refraction and transmission and reflection coefficient based on wave equation, boundary condition and polarization we just derived

Most of what we need to know about geometrical optics can be summarized in two rules:

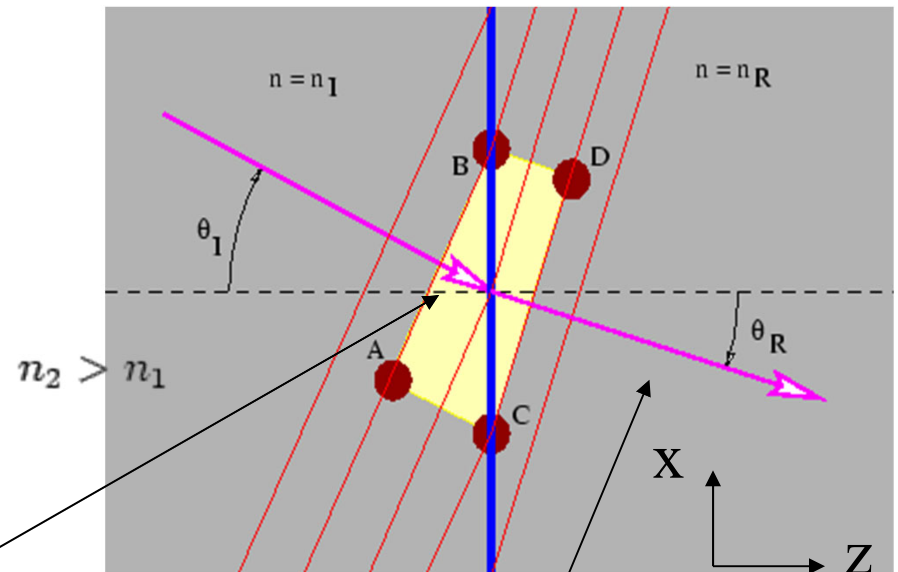
1) the laws of reflection
2) The law of refraction.

Assume total reflection or refraction

$$\theta_i = \theta_r$$



$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$



$$k_x = k \sin\theta_i = k_{rx} = k_r \sin\theta_r = k_{tx} = k_t \sin\theta_t$$

Reflection and Transmission (TE, S wave, I, perpendicular)

Fresnel Equation

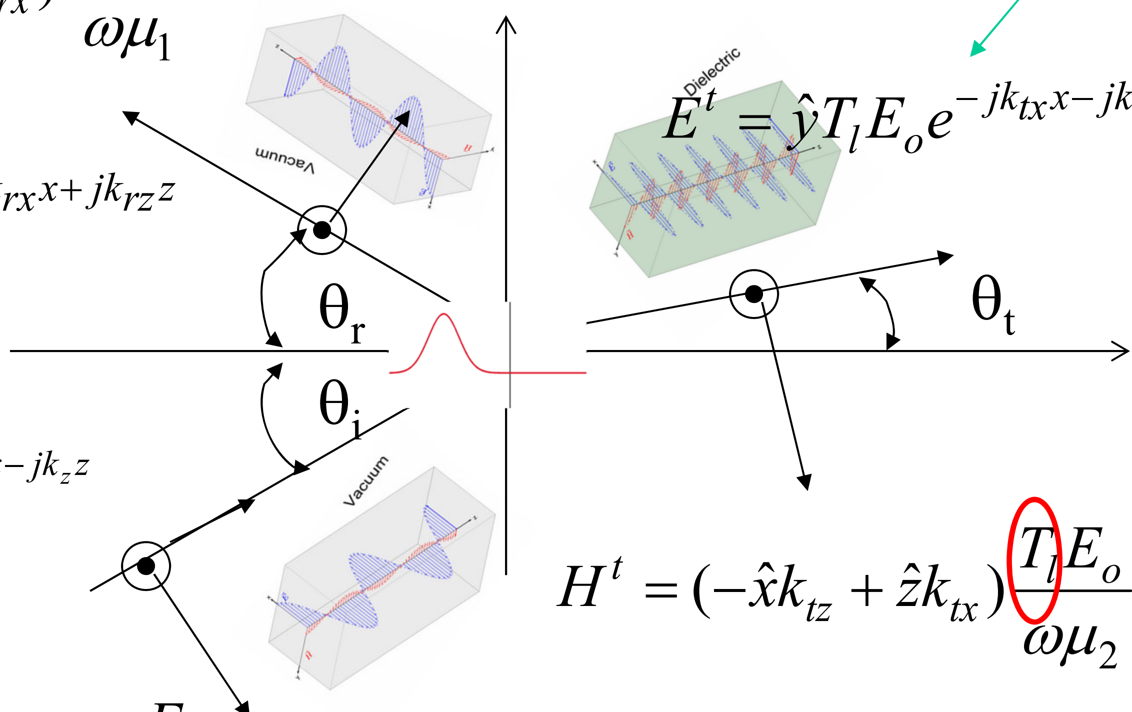
$\mu_1, \varepsilon_1, n_1$ $\mu_2, \varepsilon_2, n_2$

Negative sign means positive propagating direction

$$H^r = (\hat{x}k_{rz} + \hat{z}k_{rx})\frac{R_l E_o}{\omega\mu_1}e^{-jk_{rx}x + jk_{rz}z}$$

$$E^r = \hat{y}R_l E_o e^{-jk_{rx}x + jk_{rz}z}$$

$$E^t = \hat{y}T_l E_o e^{-jk_{tx}x - jk_{tz}z}$$



$$E^i = \hat{y}E_o e^{-jk_x x - jk_z z}$$

$$H^t = (-\hat{x}k_{tz} + \hat{z}k_{tx})\frac{T_l E_o}{\omega\mu_2}e^{-jk_{tx}x - jk_{tz}z}$$

$$H^i = (-\hat{x}k_z + \hat{z}k_x)\frac{E_o}{\omega\mu_1}e^{-jk_x x - jk_z z}$$

R_l=*reflection coefficient*

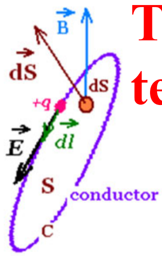
T_l=*transmission coefficient*

TE = transverse electric, perpendicularly polarized (E perpendicular to plan of incident)

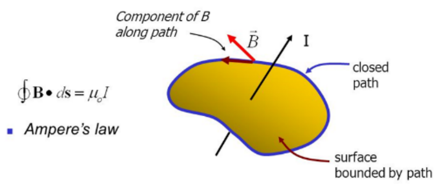
Maxwell's Equations

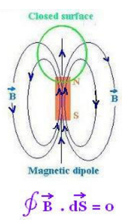
Differential form

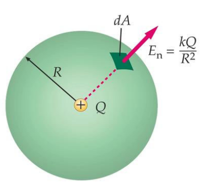
To fine H component in terms of E



Faraday's Law	$\boxed{\nabla \times E = \dfrac{-\partial B}{\partial t}}$
Ampere's Law	$\nabla \times H = J + \dfrac{\partial D}{\partial t}$
Gauss's Law for Magnetism	$\nabla \bullet B = 0$
Gauss's Law for Electricity	$\nabla \bullet D = \rho$







E = Electric Field (V/m)	ρ = charge density (c/m^3)	i = electric current (A)
B = Magnetic flux density(Web/m^2, T)	ε_0 = permittivity	J = current density(A/m^2)
D = Electric flux density (c/m^2)	μ_0 = permeability	c = speed of light
H = Magnetic Field (A/m)	Φ_B = Magnetic flux (Web)	P = Polarization

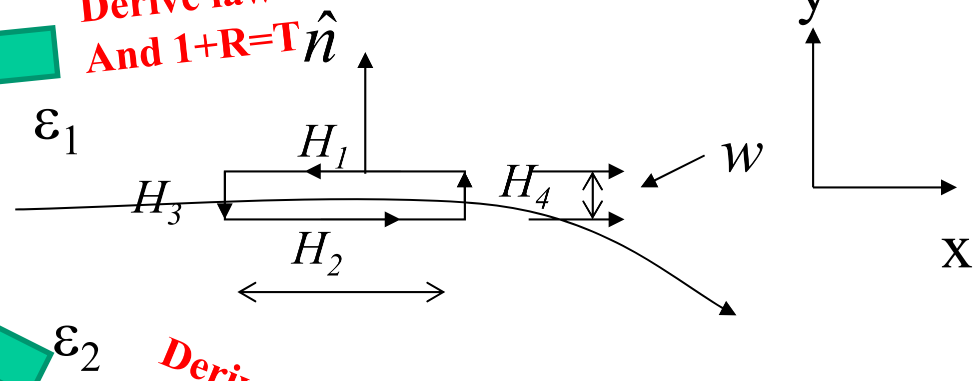
Remember

Boundary Conditions

Tangential Components:

- Faraday's Law, $\boxed{\hat{n} \times (E_1 - E_2) = 0}$
- Ampere's Law $\boxed{\hat{n} \times (H_2 - H_1) = J_s}$

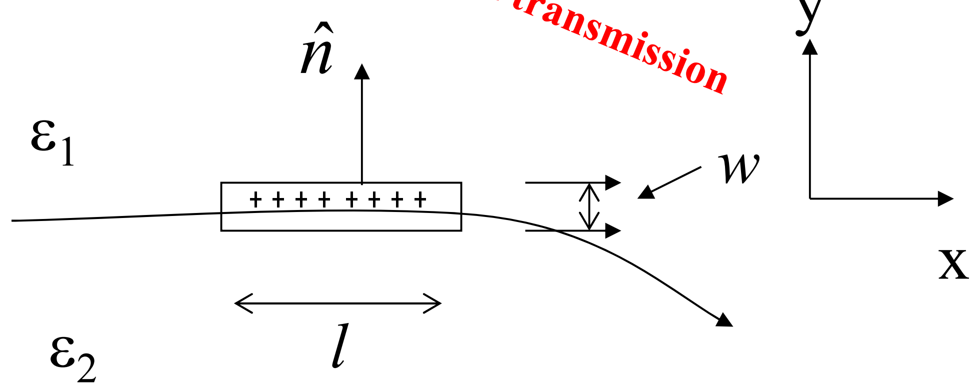
Derive law of reflection and refraction And 1+R=T



Derive reflection and transmission coefficient

Normal components:

- Gauss Law of Magnetism $\boxed{(B_1 - B_2) \bullet \hat{n} = 0}$
- Gauss Law of Electricity $\boxed{(D_1 - D_2) \bullet \hat{n} = \rho}$



On the surface of a perfect conductor, $E_{2//} = 0$ and $B_{2\perp} = 0$

If neither two are perfect conductors, J_s=0, then boundary conditions requires <u>both the tangential electric-filed and magnetic-field components be continuous</u> at **<u>z =0</u>** thus,

$$\boxed{e^{-jk_x x} + R_l e^{-jk_{rx} x} = T_l e^{-jk_{tx} x} \qquad \text{(E component)}}$$

$$\boxed{\frac{-k_z}{\omega\mu_1} e^{-jk_x x} + \frac{k_{rz}}{\omega\mu_1} R_l e^{-jk_{rx} x} = \frac{-k_{tz}}{\omega\mu_2} T_l e^{-jk_{tx} x} \qquad \text{(B component)}}$$

For the above equations to hold at all x, all components must be the same, thus we get the phase matching condition:

$$k_x = k\sin\theta_i = k_{rx} = k_r \sin\theta_r = k_{tx} = k_t \sin\theta_t$$

From this we obtain law of reflection:

$$\boxed{\theta_i = \theta_r}$$

Since $k = k_r$ *because* $k^2 = k_r^2 = \omega^2\mu_1\varepsilon_1 = k_1$

And Snell's Law:

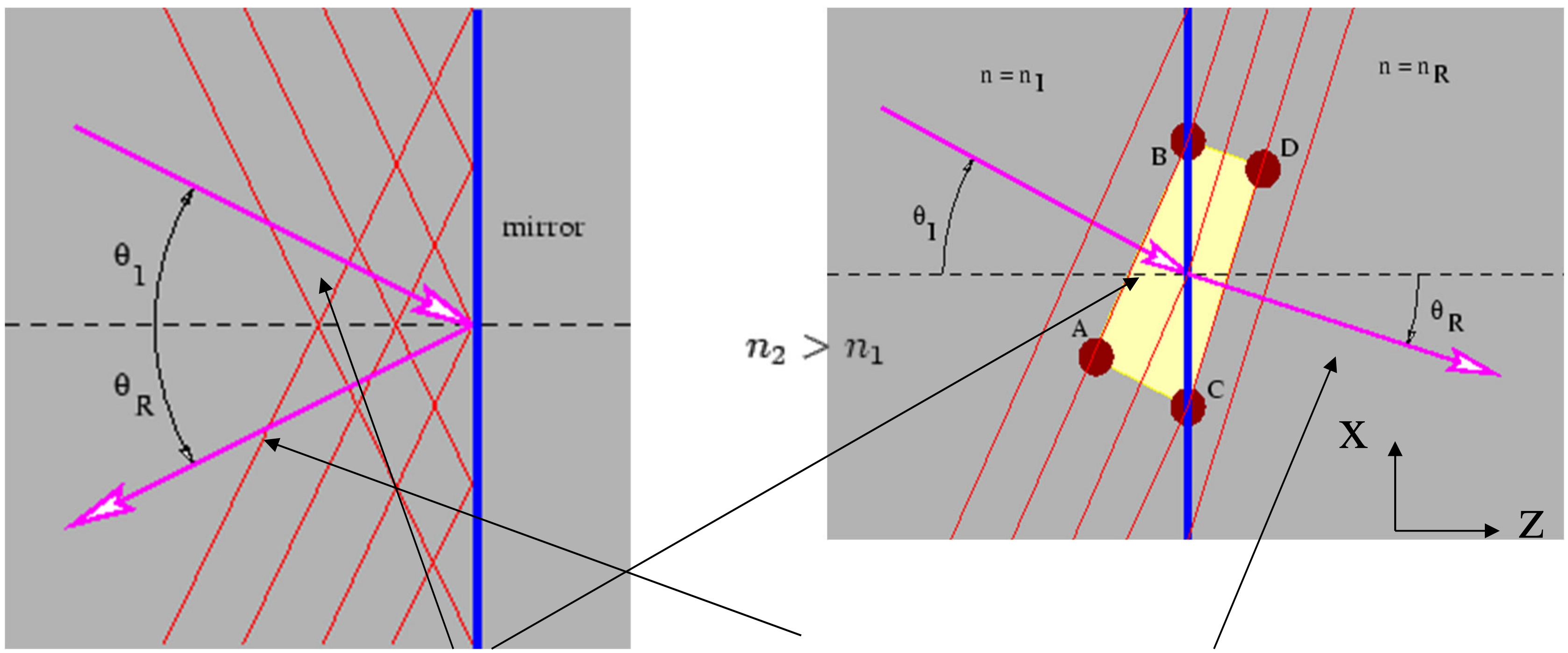
$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

$$\begin{cases} n_1 = c\sqrt{\mu_1\varepsilon_1} = \dfrac{c}{\omega}k_1 \\ n_2 = c\sqrt{\mu_2\varepsilon_2} = \dfrac{c}{\omega}k_2 \end{cases}$$

Most of what we need to know about geometrical optics can be summarized in two rules:

1) the laws of reflection
2) The law of refraction.

$$\theta_i = \theta_r \qquad\qquad n_1 \sin\theta_1 = n_2 \sin\theta_2$$



$$k_x = k\sin\theta_i = k_{rx} = k_r \sin\theta_r = k_{tx} = k_t \sin\theta_t$$

To Find Reflection and Transmission Coefficient, substitute solution for E^i, E^r, E^t, into wave equation

$$\nabla^2 E^i + \omega^2 \mu_1 \varepsilon_1 E^i = 0$$

$$\nabla^2 E_r + \omega^2 \mu_1 \varepsilon_1 E_r = 0$$

$$\nabla^2 E^t + \omega^2 \mu_2 \varepsilon_2 E^t = 0$$

We find,

$$k_x^2 + k_z^2 = k_1^2 = k_{rx}^2 + k_{rz}^2$$

$$k_{tx}^2 + k_{tz}^2 = k_2^2$$

Using phase matching condition, we get,

$$1 + R_l = T_l$$

$$1 - R_l = \frac{\mu_1 k_{tz}}{\mu_2 k_z} T_l$$

$$\Longrightarrow$$

Combine with H component, we find

$$R_l = \frac{\mu_2 k_z - \mu_1 k_{tz}}{\mu_2 k_z + \mu_1 k_{tz}}$$

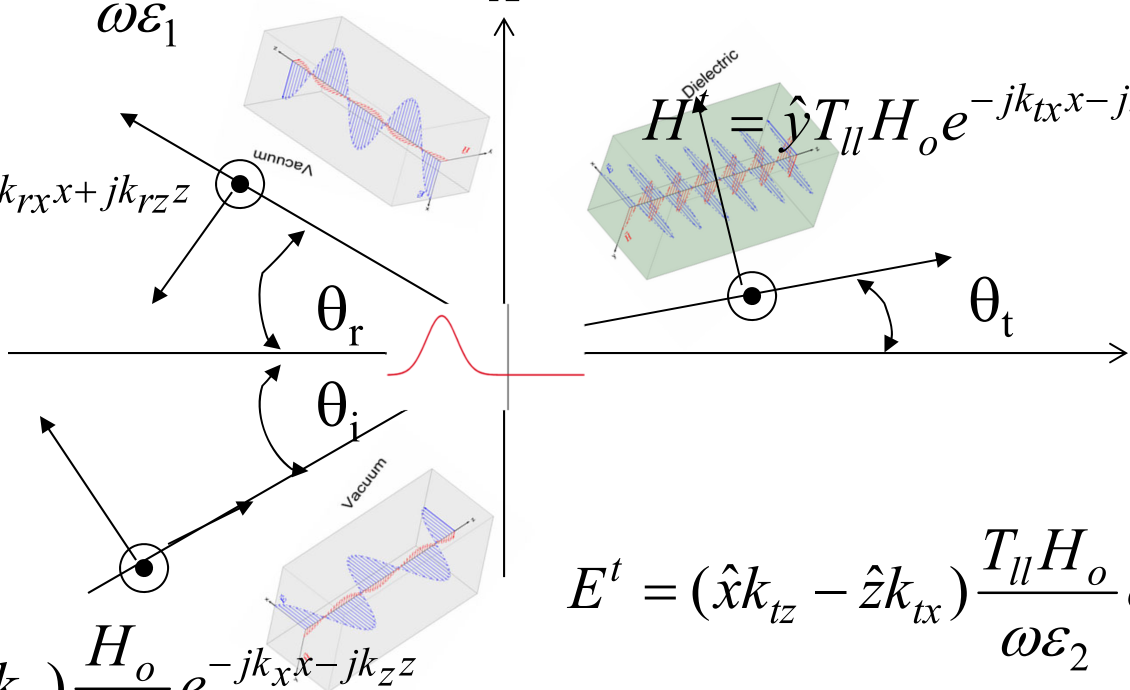
$$T_l = \frac{2\mu_2 k_z}{\mu_2 k_z + \mu_1 k_{tz}}$$

Reflection and Transmission (TM, P wave, II, parallel)

$\mu_1, \varepsilon_1, n_1$ $\qquad$ $\mu_2, \varepsilon_2, n_2$

$$E^r = (-\hat{x}k_{rz} - \hat{z}k_{rx})\frac{R_{ll}H_o}{\omega\varepsilon_1}e^{-jk_{rx}x + jk_{rz}z}$$

$$H^r = \hat{y}R_{ll}H_o e^{-jk_{rx}x + jk_{rz}z}$$



$$H^t = \hat{y}T_{ll}H_o e^{-jk_{tx}x - jk_{tz}z}$$

$$E^t = (\hat{x}k_{tz} - \hat{z}k_{tx})\frac{T_{ll}H_o}{\omega\varepsilon_2}e^{-jk_{tx}x - jk_{tz}z}$$

$$E^i = (\hat{x}k_z - \hat{z}k_x)\frac{H_o}{\omega\varepsilon_1}e^{-jk_x x - jk_z z}$$

$$H^i = \hat{y}H_o e^{-jk_x x - jk_z z}$$

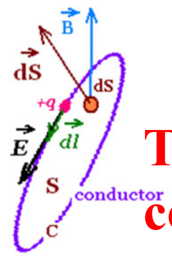
R_{ll}= reflection coefficient

T_{ll}= transmission coefficient

TM = transverse magnetic, parallel polarized (E parallel to plan of incident)

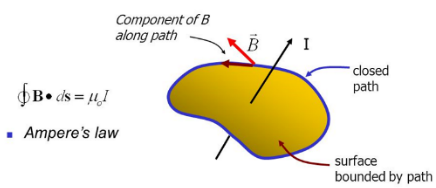
Maxwell's Equations

Differential form

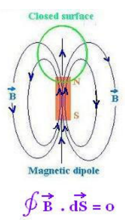


To fine E component in H

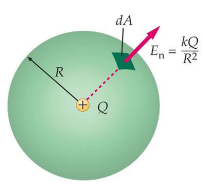
Faraday's Law $$\nabla \times E = \frac{-\partial B}{\partial t}$$



Ampere's Law $$\boxed{\nabla \times H = J + \frac{\partial D}{\partial t}}$$



Gauss's Law for Magnetism $$\nabla \bullet B = 0$$



Gauss's Law for Electricity $$\nabla \bullet D = \rho$$

E = Electric Field (V/m) | *ρ = charge density (c/m^3)* | *i = electric current (A)*

B = Magnetic flux density(Web/m^2, T) | *ε_0 = permittivity* | *J = current density(A/m^2)*

D = Electric flux density (c/m^2) | *μ_0 = permeability* | *c = speed of light*

H = Magnetic Field (A/m) | *Φ_B = Magnetic flux (Web)* | *P = Polarization*

Remember

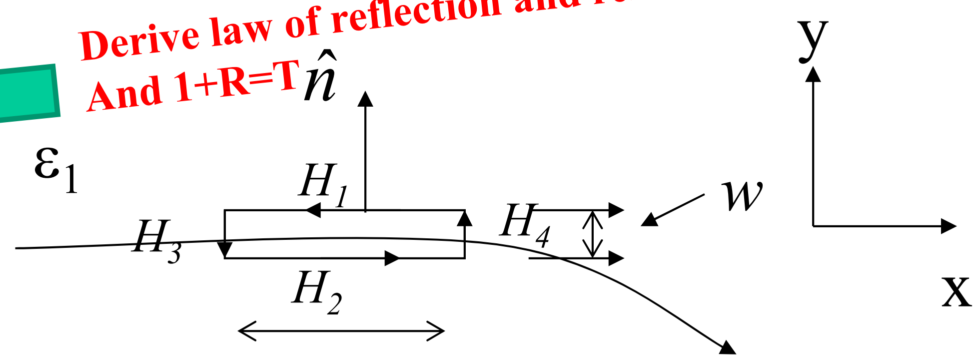
Boundary Conditions

Tangential Components:

- Faraday's Law, $\boxed{\hat{n}\times(E_1 - E_2) = 0}$

- Ampere's Law $\boxed{\hat{n}\times(H_2 - H_1) = J_s}$

Derive law of reflection and refraction
And 1+R=T

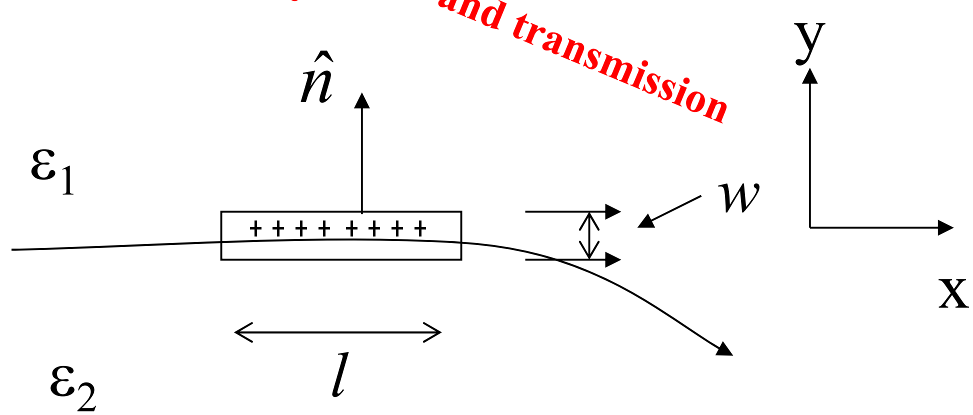


Derive reflection and transmission coefficient

Normal components:

- Gauss Law of Magnetism $\boxed{(B_1 - B_2)\bullet\hat{n} = 0}$

- Gauss Law of Electricity $\boxed{(D_1 - D_2)\bullet\hat{n} = \rho}$



On the surface of a perfect conductor, $E_{2//} = 0$ and $B_{2\perp} = 0$

Substitute solution for E^i, E^r, E^t, into wave equation along with B.C. and Faraday's law, we get:

$$1 + R_{ll} = T_{ll}$$

$$1 - R_{ll} = \frac{\varepsilon_1 k_{tz}}{\varepsilon_2 k_z} T_{ll}$$

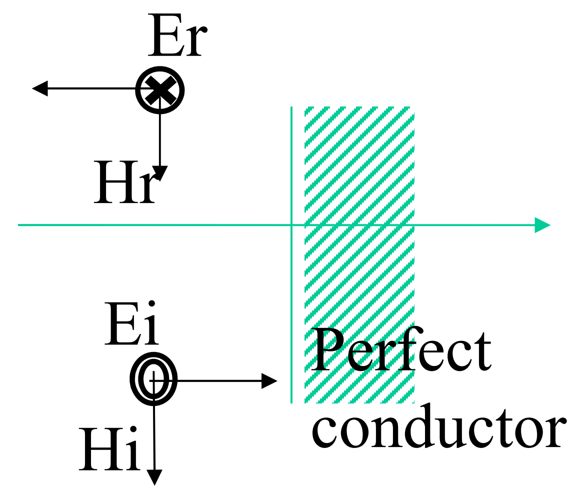
$$\Longrightarrow$$

$$R_{ll} = \frac{\varepsilon_2 k_z - \varepsilon_1 k_{tz}}{\varepsilon_2 k_z + \varepsilon_1 k_{tz}}$$

$$T_{ll} = \frac{2\varepsilon_2 k_z}{\varepsilon_2 k_z + \varepsilon_1 k_{tz}}$$

Reflection from Perfect Conductor

On the surface of a perfect conductor, $E_{2//} = 0$ and $B_2 = 0$

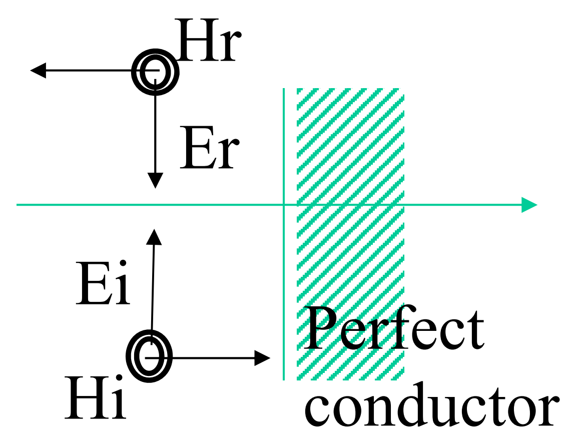


For TE polarization: $$R_l = \frac{\mu_2 k_z - \mu_1 k_{tz}}{\mu_2 k_z + \mu_1 k_{tz}}$$

$$\varepsilon_{2\,p.c.} = \varepsilon_2 - j\frac{\sigma}{\omega} \approx \infty$$

$$k_2 \propto \sqrt{\varepsilon_{2\,p.c.}} \approx \infty \Rightarrow k_2 \approx \infty$$

$$so \qquad R_I = -1$$



For TM polarization: $$R_{ll} = \frac{\varepsilon_2 k_z - \varepsilon_1 k_{tz}}{\varepsilon_2 k_z + \varepsilon_1 k_{tz}}$$

$$\varepsilon_{2\,p.c.} \propto \frac{\sigma}{\omega}$$

$$k_2 \propto \sqrt{\varepsilon_{2\,p.c.}}$$

$$so \qquad R_{II} = 1$$

Difference between reflection, backward travelling wave and phase conjugated wave

$$E_i = E_o e^{-jk_{ix}x}$$
$$E_r = RE_o e^{jk_{rx}x}$$
$$E_t = TE_o e^{-jk_{tx}x}$$

- perfect conductor TE reflection: R = -1 $E_r = e^{j\pi} E_o e^{jk_{rx}x}$ and $E_t = 0$
- perfect conductor TM reflection: R = 1 $E_r = e^{j0} E_o e^{jk_{rx}x}$ and $E_t = 0$
- plain reflection, $k_{rx} = k_{ix}$ and n = positive, $E_r = RE_o e^{jk_{rx}x}$ and $E_t = TE_o e^{-jk_{tx}x}$
- backward travelling wave, **n_t = negative**, $E_o e^{-jk_{ix}x}$ $E_r = RE_o e^{jk_{tx}x}$ *and* $\boxed{E_t = TE_o e^{jk_{tx}x}}$
- phase conjugated wave, $\boxed{E_r = E_o e^{jk_{ix}x}}$ (reflected wave is same as input!!!)

Derivation of refraction and reflection

- Please read the hand written handout for more complete derivation in: http://courses.washington.edu/me557/reading/TE+TM.pdf

Reflection and Transmission (TE, S wave, I, perpendicular)

Fresnel Equation

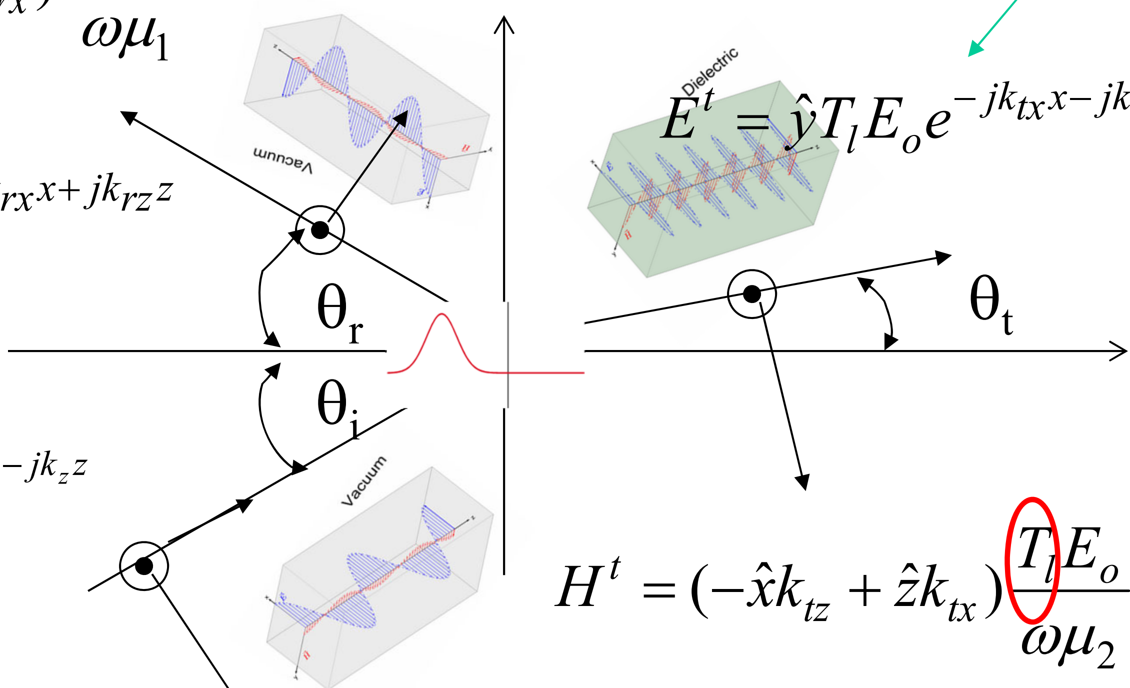
$\mu_1, \varepsilon_1, n_1$ $\quad\quad$ $\mu_2, \varepsilon_2, n_2$

Negative sign means positive propagating direction

$$H^r = (\hat{x}k_{rz} + \hat{z}k_{rx})\frac{R_l E_o}{\omega\mu_1} e^{-jk_{rx}x + jk_{rz}z}$$

$$E^t = \hat{y}T_l E_o e^{-jk_{tx}x - jk_{tz}z}$$

$$E^r = \hat{y}R_l E_o e^{-jk_{rx}x + jk_{rz}z}$$



$$E^i = \hat{y}E_o e^{-jk_x x - jk_z z}$$

$$H^t = (-\hat{x}k_{tz} + \hat{z}k_{tx})\frac{T_l E_o}{\omega\mu_2} e^{-jk_{tx}x - jk_{tz}z}$$

$$H^i = (-\hat{x}k_z + \hat{z}k_x)\frac{E_o}{\omega\mu_1} e^{-jk_x x - jk_z z}$$

R_l=*reflection coefficient*
T_l=*transmission coefficient*

TE = transverse electric, perpendicularly polarized (E perpendicular to plan of incident)

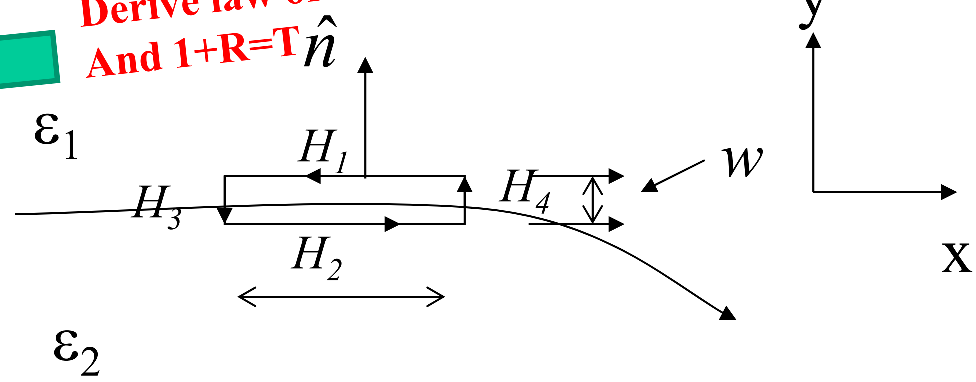
Recap

Boundary Conditions

Tangential Components:

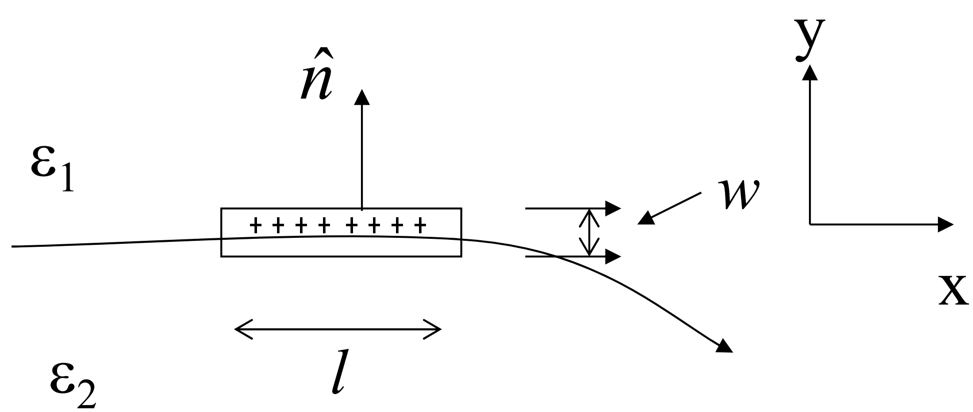
Derive law of reflection and refraction
And 1+R=T

- Faraday's Law, $\boxed{\hat{n}\times(E_1 - E_2) = 0}$
- Ampere's Law $\hat{n}\times(H_2 - H_1) = J_s$



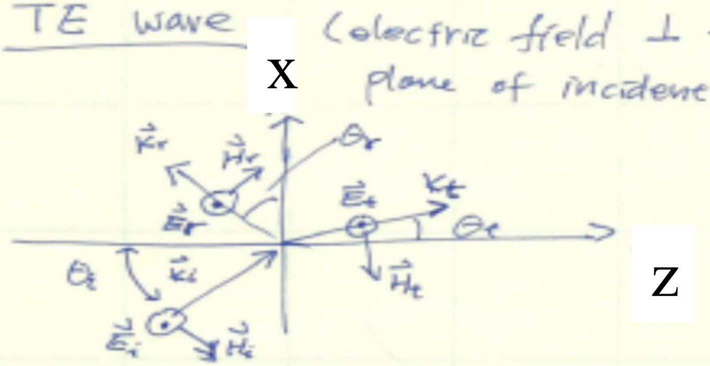
Normal components:

- Gauss Law of Magnetism $\boxed{(B_1 - B_2)\bullet\hat{n} = 0}$
- Gauss Law of Electricity $\boxed{(D_1 - D_2)\bullet\hat{n} = \rho}$



On the surface of a perfect conductor, $E_{2//} = 0$ *and* $B_{2\perp} = 0$

TE wave (electric field $\perp$ to the plane of incidence)



$$\vec{E}_i = \hat{y}\, E_0\, e^{-j k_i \sin\theta_i x - j k_i \cos\theta_i z}$$

$$k_i = \omega\sqrt{\epsilon_1\mu_1} \quad \vec{k}_i = \hat{x}\, \overbrace{k_i \sin\theta_i}^{k_{ix}} + \hat{z}\, \overbrace{k_i \cos\theta_i}^{k_{iz}}$$

$$\vec{E}_t = \hat{y}\, T_e E_0\, e^{-j k_t \sin\theta_t x - j k_t \cos\theta_t z}$$

$$k_t = \omega\sqrt{\epsilon_2\mu_2} \quad \vec{k}_t = \hat{x}\, k_t \sin\theta_t + \hat{z}\, k_t \cos\theta_t$$

$$\vec{E}_r = \hat{y}\, R_e E_0\, e^{-j k_r \sin\theta_r x + j k_r \cos\theta_r z}$$

$$k_r = \omega\sqrt{\epsilon_1\mu_1} \quad \vec{k}_r = \hat{x}\, k_r \sin\theta_r + \hat{z}\, k_r \cos\theta_r$$

① use B.C. at surface ($z=0$) $\quad \hat{n}\times(E_1 - E_2) = 0$

ⓒ tangential E-field continuous

$$\vec{E}_i + \vec{E}_r = \vec{E}_t \quad \boxed{\text{at } z=0}$$

$$E_{iy} + E_{ry} = E_{ty}$$

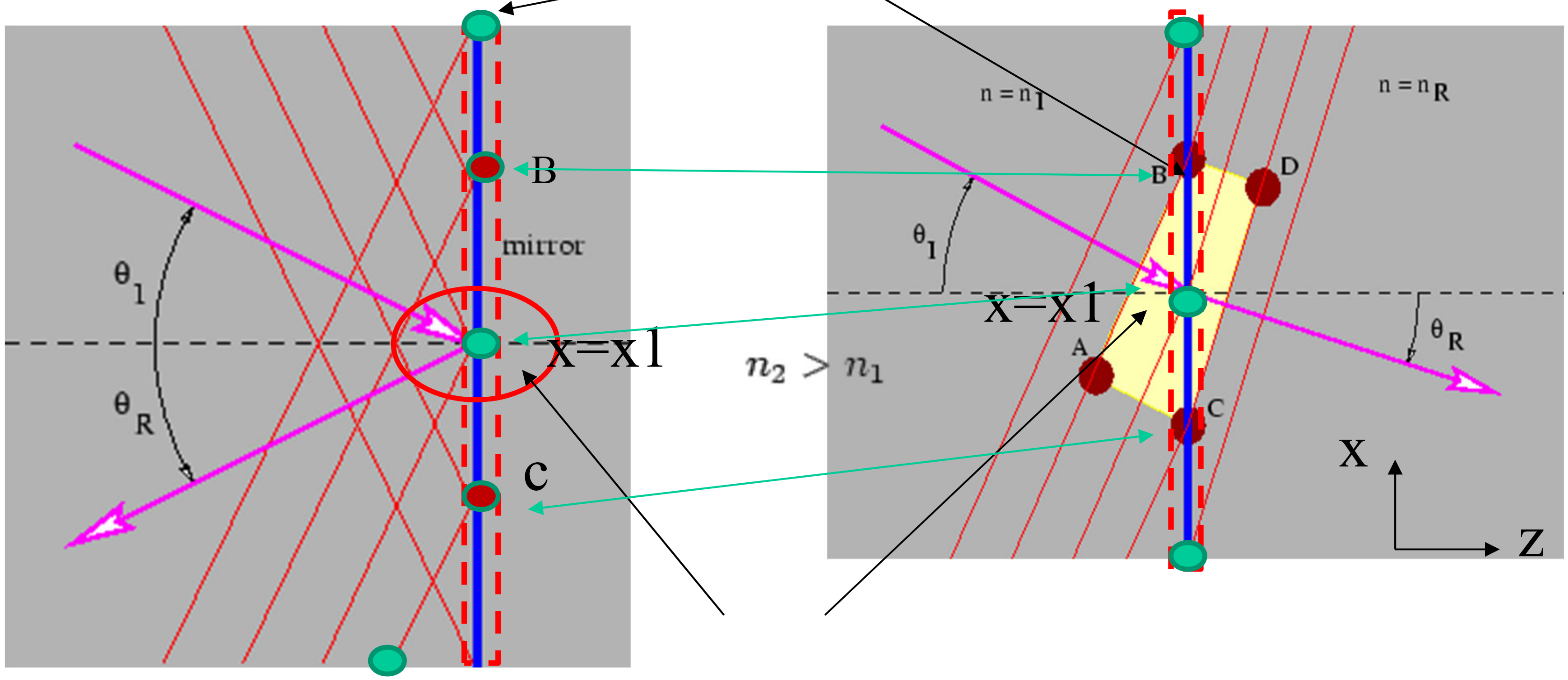
(normalized) $\quad e^{-j k_{ix} x} + R_e\, e^{-j k_{rx} x} = T_e\, e^{-j k_{tx} x}$

This is only true if all x value satisfied

$\Rightarrow$ $\boxed{\text{phase matching cond. } k_{ix} = k_{rx} = k_{tx}}$

graphic explanation

Recall the phase map in the ray optics at $z = 0$, all phase in x direction must be the same:



Because $\hat{n}\times(E_1 - E_2) = 0 \Rightarrow \quad E_o e^{-jk_x x} + R_l E_o e^{-jk_{rx} x} = T_l E_o e^{-jk_{tx} x}$

We know $\phi_{ix} = \phi_{rx} = \phi_{tx}$ therefore $k_x x = k_{rx} x = k_{tx} x$ for any x at z=0.

Then $k_x = k \sin\theta_i = k_{rx} = k_r \sin\theta_r = k_{tx} = k_t \sin\theta_t$ and $1 + R_l = T_l$

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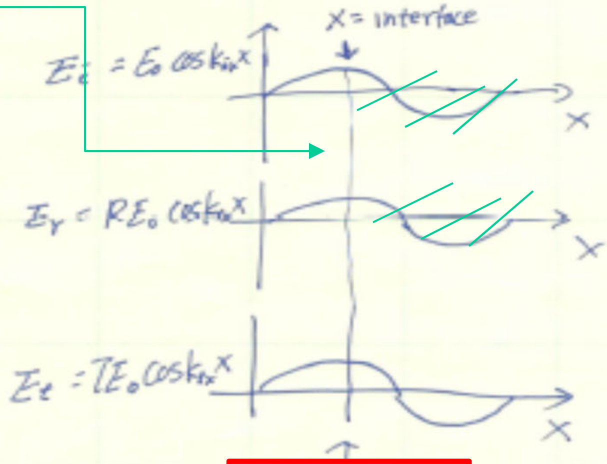
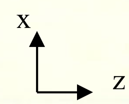
Mathematic explanation

Has to be the same

The reason is because we need to have same phase front! because plane wave incident





from $\boxed{\text{phase is same at all } x}$ and $\hat{n} \times (E_1 - E_2) = 0$

$$\Rightarrow \boxed{1 + R_e = T_e} \quad \text{— ①}$$

(ii) From phase matching condition

$k_{ix} = k_{rx} = k_{tx}$

$k_i = \omega\sqrt{\mu_1 \epsilon_1}$

$k_r = \omega\sqrt{\mu_1 \epsilon_1}$

$k_t = \omega\sqrt{\mu_2 \epsilon_2}$

$\boxed{k_i \sin\theta_i} = k_i \sin\theta_r = \boxed{k_t \sin\theta_t}$

$\Rightarrow \boxed{\theta_i = \theta_r}$ — ②

(same as Law of reflection)

$\omega\sqrt{\mu_1 \epsilon_1} \sin\theta_i = \omega\sqrt{\mu_2 \epsilon_2} \sin\theta_t$

$\sqrt{\mu_1 \epsilon_1} \sin\theta_i = \sqrt{\mu_2 \epsilon_2} \sin\theta_t \Rightarrow \boxed{n_1 \sin\theta_i = n_2 \sin\theta_t}$ ③

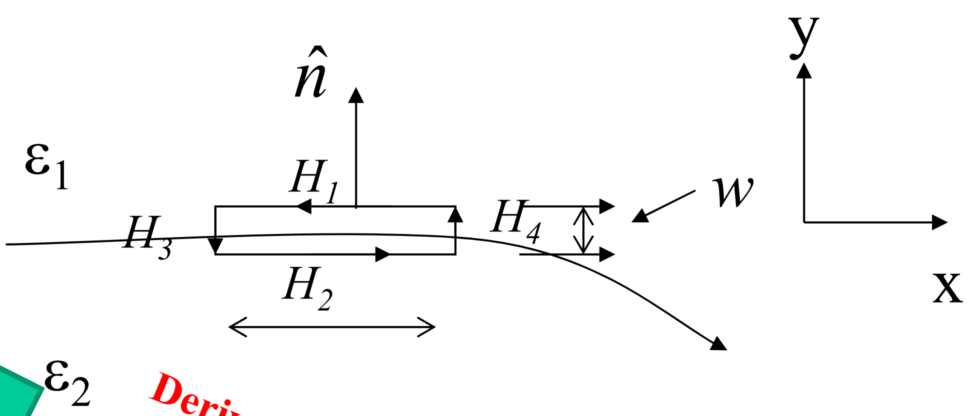
[same as Law of refraction]

Recap

Boundary Conditions

Tangential Components:

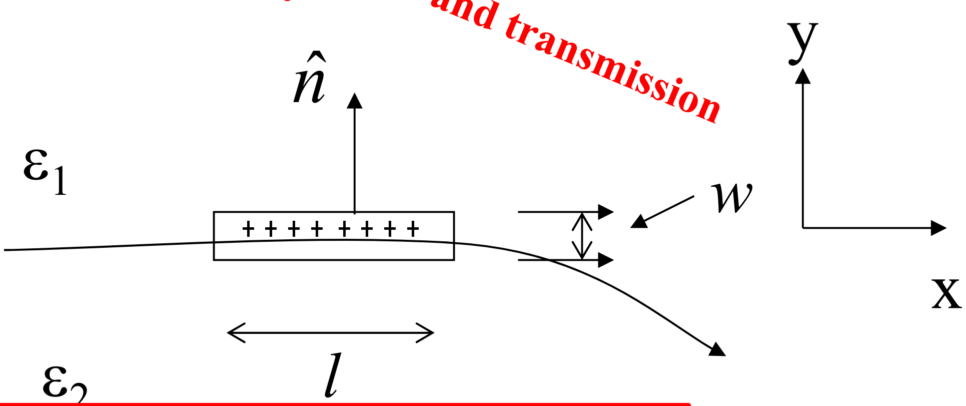
- Faraday's Law, $\hat{n}\times(E_1-E_2)=0$
- Ampere's Law $\boxed{\hat{n}\times(H_2-H_1)=J_s}$



Derive reflection and transmission coefficient

Normal components:

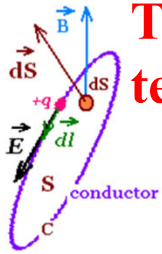
- Gauss Law of Magnetism $\boxed{(B_1-B_2)\bullet\hat{n}=0}$
- Gauss Law of Electricity $\boxed{(D_1-D_2)\bullet\hat{n}=\rho}$



$\boxed{\text{On the surface of a perfect conductor, } E_{2//} = 0 \text{ and } B_{2\perp} = 0}$

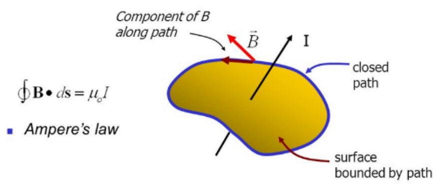
Maxwell's Equations

Differential form

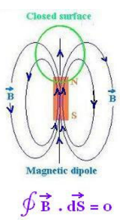


To fine H component in terms of E

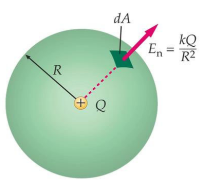
$$\boxed{\text{Faraday's Law} \qquad \nabla \times E = \frac{-\partial B}{\partial t}}$$



Ampere's Law
$$\nabla \times H = J + \frac{\partial D}{\partial t}$$



Gauss's Law for Magnetism
$$\nabla \bullet B = 0$$



Gauss's Law for Electricity
$$\nabla \bullet D = \rho$$

E = Electric Field (V/m)	*ρ = charge density (c/m^3)*	*i = electric current (A)*
B = Magnetic flux density(Web/m^2, T)	*ε_0 = permittivity*	*J = current density(A/m^2)*
D = Electric flux density (c/m^2)	*μ_0 = permeability*	*c = speed of light*
H = Magnetic Field (A/m)	*Φ_B = Magnetic flux (Web)*	*P = Polarization*

(iii) using B.C. for magnetic field

If neither two are perfect conductors, J_s=0,

$\hat{n} \times (H_1 - H_2) = 0$ tangential B field is continuous

η = impedance

$$= \sqrt{\frac{\mu}{\varepsilon}}$$

$\eta = \sqrt{\frac{\mu_0}{\varepsilon_0}}$

$\vec{H}_0 = \frac{\hat{k} \times \vec{E}}{\eta}$ ← (based on Faraday's law)

solved using Faraday's or Ampere's Law

z=0@ interface

$\vec{H}_i = \frac{-k_{iz}}{\omega\mu_1} E_0 e^{-jk_{ix}x} e^{-jk_{iz}z}$

$\vec{H}_r = \frac{k_{rz}}{\omega\mu_1} E_0 R_e e^{-jk_{rx}x} e^{jk_{rz}z}$

$\vec{H}_t = \frac{-k_{tz}}{\omega\mu_2} E_0 T_e e^{-jk_{tx}x} e^{-jk_{tz}z}$

at z = 0 & using phase matching condition

$$\boxed{\frac{-k_{iz}}{\mu_1} + \frac{k_{rz}}{\mu_1} R_e = \frac{-k_{tz}}{\mu_2} T_e} \quad (4)$$

$k_{ix} = k_{rx} = k_{tx}$

(iv) To get R_ℓ & T_ℓ:

multiply $\mu_1 k_{tz}$ to $1 + R_\ell = T_\ell$ (eq 1)

$$\boxed{\mu_1 k_{tz} + \mu_1 k_{tz} R_\ell = \mu_1 k_{tz} T_\ell}$$

& multiply μ_2/μ_1 to equation (4)

$$\frac{-k_{iz}}{\mu_1} + \frac{k_{rz}}{\mu_1} R_\ell = \frac{-k_{tz}}{\mu_2} T_\ell \quad (4)$$

$$\boxed{-\mu_2 k_{iz} + \mu_2 k_{rz} R_\ell = -\mu_1 k_{tz} T_\ell}$$

Combine

we get

$$\boxed{(\mu_2 k_{rz} + \mu_1 k_{tz}) R_\ell = (-\mu_1 k_{tz} + \mu_2 k_{iz})}$$

$$\Rightarrow \boxed{R_\ell = \frac{-\mu_1 k_{tz} + \mu_2 k_{iz}}{\mu_1 k_{tz} + \mu_2 k_{rz}}}$$

$$\boxed{T_\ell = 1 + R_\ell = \frac{2\mu_2 k_{iz}}{\mu_1 k_{tz} + \mu_2 k_{iz}}}$$

$$\Rightarrow R_\ell = \frac{-\mu_1 \omega \sqrt{\mu_2 \epsilon_2}\cos\theta_t + \mu_2 \omega \sqrt{\mu_1 \epsilon_1}\cos\theta_i}{\mu_1 \omega \sqrt{\mu_2 \epsilon_2}\cos\theta_t + \mu_2 \omega \sqrt{\mu_1 \epsilon_1}\cos\theta_r} = \frac{-\sqrt{\epsilon_2}\cos\theta_t + \sqrt{\epsilon_1}\cos\theta_i}{\sqrt{\epsilon_2}\cos\theta_t + \sqrt{\epsilon_1}\cos\theta_r}$$

multiply top & bottom by c_0

$$= \frac{-n_2 \cos\theta_t + n_1 \cos\theta_i}{n_2 \cos\theta_t + n_1 \cos\theta_r}$$

Recall

$$\begin{cases} n_1 = \sqrt{\mu_1 \varepsilon_1} = \dfrac{c}{\omega} k_1 \\ n_2 = \sqrt{\mu_2 \varepsilon_2} = \dfrac{c}{\omega} k_2 \end{cases}$$

Do the same derivation like TE for TM to get the TM polarization R an T

(V) For TE wave (perpendicular polarization)

$$|R_e|^2 = \left| \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} \right|^2$$

For TM (parallel polarization)

$$|R_{ee}|^2 = \left| \frac{n_1 \cos\theta_t - n_2 \cos\theta_i}{n_1 \cos\theta_t + n_2 \cos\theta_i} \right|^2$$

For TE wave

$$1 + |R_e|^2 = |T_e|^2$$

For TM wave

$$1 + |R_{ee}|^2 = |T_{ee}|^2$$

$$I(P) = \frac{|E|^2}{2Z_o}$$

Recall Intensity is square of E field

Plane Wave in Dissipative Medium

So far we have omitted one important class of media- namely conductors. A conductor is characterized by a conductivity σ and is governed by ohm's law. For isotropic conductors, ohm's law states that $J_c = \sigma E$, as we recall J_c denotes the conduction current. For Ampere's law,

$$\nabla \times H = +J_o + j\omega D$$

Where J = J_c (conduction current) +J_o (source current). It is instructive to see that in a conducting medium, Ampere' law becomes:

$$\nabla \times H = J_o + j\omega(\varepsilon - j\frac{\sigma}{\omega})E$$

Displacement current

conductive current

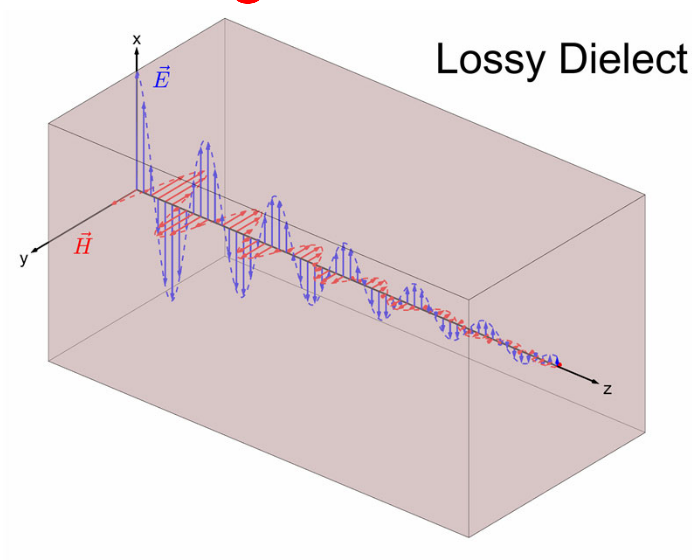
Thus, ε becomes a complex permittivity:

$$\boxed{\varepsilon = \varepsilon' - j\sigma/\omega}$$

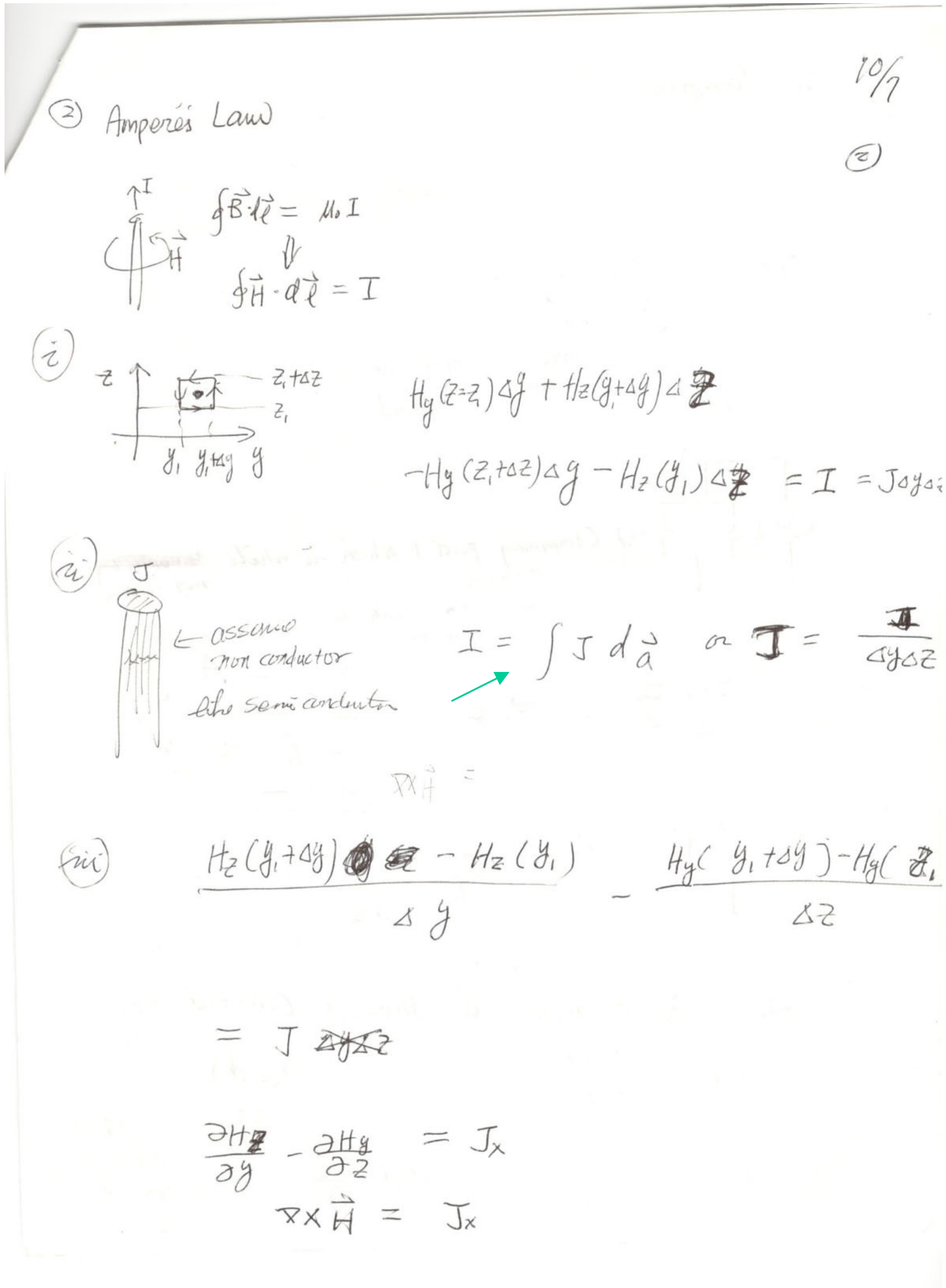
For conducing media, the propagation constant k = 2πn/λ, where $n = \sqrt{\varepsilon_r \varepsilon_o \mu_r \mu_o}$,

$$\boxed{k = k_{real} - jk_{imaginary} = k - ja = \omega\sqrt{\mu\varepsilon}(1\text{-}j\,\frac{\sigma}{\omega\varepsilon})^{1/2}}$$

σ/ωε is called the loss tangent of the conducing media.



Recall derivation of Ampere's law:



10/7

② Ampere's Law ②

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$

$\Downarrow$

$\oint \vec{H}\cdot d\vec{l} = I$

(i)

$H_y(z=z_1)\Delta y + H_z(y_1+\Delta y)\Delta z$

$- H_y(z_1+\Delta z)\Delta y - H_z(y_1)\Delta z = I = J\Delta y\Delta z$

(ii) J

← assume non conductor like semiconductor

$I = \int J\, d\vec{a}$ or $J = \dfrac{I}{\Delta y\Delta z}$

$\nabla\times\vec{H} =$

(iii)

$$\frac{H_z(y_1+\Delta y) - H_z(y_1)}{\Delta y} - \frac{H_y(y_1+\Delta y) - H_y(z_1}{\Delta z}$$

$= J\ \cancel{\Delta y\Delta z}$

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = J_x$$

$$\nabla\times\vec{H} = J_x$$

(iv) In freespace

displacement current

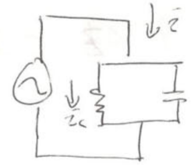
$\vec{J}_c$ ← be called of

$$\nabla \times \vec{H} = \vec{J}_s + \vec{J}_c + \vec{J}_d$$

source current — induced current — displacement current

Displacement current is electric field generated by B field that was radiating in air

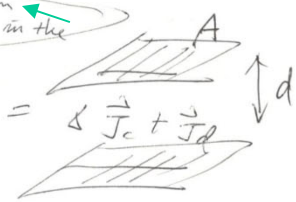
Now you understand where the conductive part came from… is from the eddy current induce on the surface which causes that additional loss and the real part of dielectric constant is from air and source is only there if there is a source on the conductor.



i

i_d (imaginary part) which is what ~~[illegible]~~ not seem

possibly or lossy part actually is current that induced in the medium



(v)

$$\frac{\text{medium}}{J_c\,\sigma} + J_d\,\varepsilon = \sigma \vec{J}_c + \vec{J}_d$$

$$\boxed{\vec{J}_c = \vec{E}\sigma = \frac{i_c}{A}}$$

A ← area

conductivity

$$\nabla x B = \frac{4\pi k}{c^2} J + \frac{1}{c^2}\frac{\partial E}{\partial t}$$

$$\nabla x B = \frac{J}{\varepsilon_0 c^2} + \frac{1}{c^2}\frac{\partial E}{\partial t}$$

use

$$k = \frac{1}{4\pi\varepsilon_0} \qquad c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

(vi) use capacitor to model the displacement current because that's what happen when wave travel thru the medium

$$i_d = C\frac{dV}{dt} = \frac{\epsilon A}{d}\frac{\partial(Ed)}{\partial t} = \epsilon A \frac{\partial E}{\partial t}$$

$$\boxed{J_d = \epsilon \frac{\partial E}{\partial t}}$$

Like liquid, dye, and solid state laser, radiating energy instead of decay by collisions

Highly Conducting Media

For highly conducting medium, σ/ωε >>1, the k constant can be simplify to

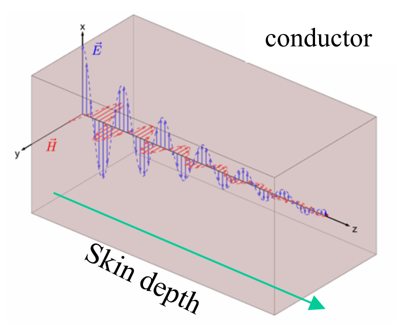
$$k = k - ja = \omega\sqrt{\mu\varepsilon}(1\text{-}j\,\frac{\sigma}{\omega\varepsilon})^{1/2}$$

$$k \sim \omega\sqrt{\mu\varepsilon}(\text{-}j\,\frac{\sigma}{\omega\varepsilon})^{1/2} = \sqrt{\omega\mu(\frac{\sigma}{2})(1-j)}$$

The penetration depth $\delta_p = 1/\alpha = \sqrt{(\frac{2}{\omega\mu\sigma})} = \delta$ (skin depth) only for highly conductive media.

High conductivity lower penetration but **high attenuation**

$$E = \hat{x}E_o e^{-i(\omega t - k_z z)}$$



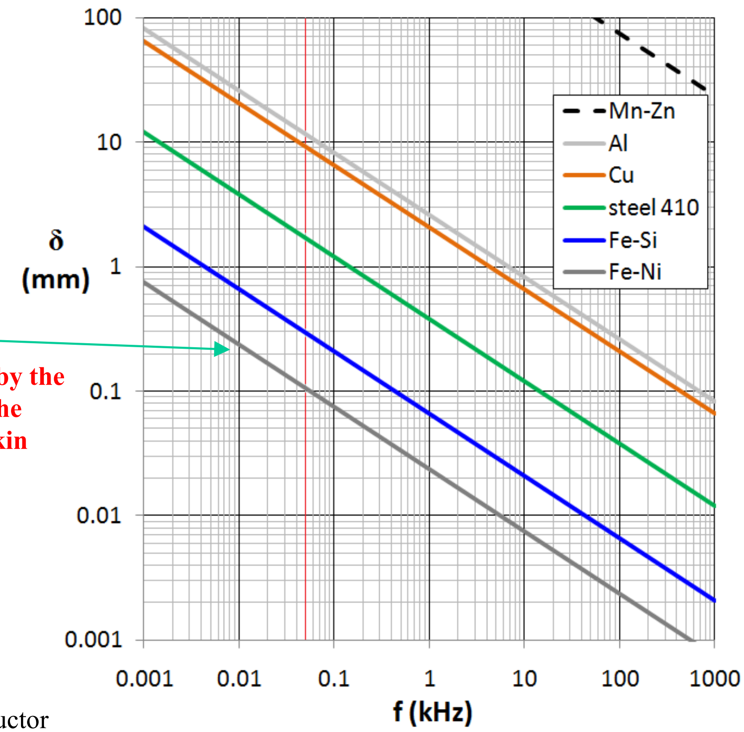
Skin effect in conductor

Absorption is part of attenuation but not the other way around.

We can derive a practical formula for skin depth :

$$\delta_p = 1/\alpha = \sqrt{\left(\frac{2}{\omega\mu\sigma}\right)} = \delta$$

$$\Rightarrow \quad \delta = \sqrt{\frac{2\rho}{(2\pi f)(\mu_o \mu_r)}}$$



Higher opposing eddy currents induced by the changing magnetic field resulting from the alternating current creating shallower skin depth. higher the u lower the skin depth

Where

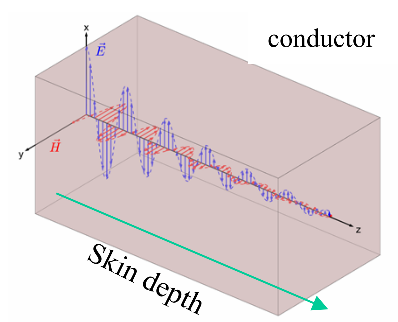
δ = the skin depth in meters

μ_r = the relative permeability of the medium

ρ = the resistivity of the medium in Ω·m, also equal to the reciprocal of its conductivity: $\rho = 1/\sigma$

(for copper, $\rho = 1.68\times10^{-8}$ Ω·m)

f = the frequency of the current in Hz



Metal Conductivity (σ)

The common metals that have the highest resistivity (lowest conductivity) are:

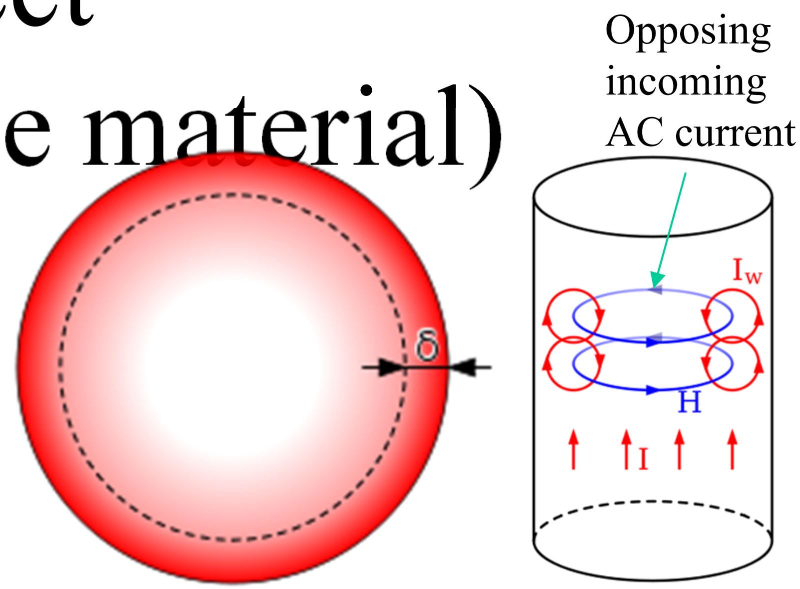
1. Mercury
2. Stainless steel varieties
3. Titanium
4. Lead
5. Carbon
6. Carbon steel
7. Tungsten

The common metals that have the lowest resistivity (highest conductivity) are:

1. Silver
2. Copper
3. Gold
4. Aluminum
5. Zinc
6. Brass
7. Nickel

Keep in mind, too, that purity of the metals effects conductivity and it inverse property, resistivity.

Skin effect (highly conductive material)



Skin effect is the tendency of an alternating electric current (AC) to become distributed within a conductor such that the current density is largest near the surface of the conductor, and decreases with greater depths in the conductor. The electric current flows mainly at the "skin" of the conductor, between the outer surface and a level called the skin depth. The skin effect causes the effective resistance of the conductor to increase at higher frequencies where the skin depth is smaller, thus reducing the effective cross-section of the conductor. The skin effect is **due to opposing eddy currents induced by the changing magnetic field resulting from the alternating current.** At 60 Hz in copper, the skin depth is about 8.5 mm. At high frequencies the skin depth becomes much smaller. Increased AC resistance due to the skin effect can be mitigated by using specially woven litz wire. Because the interior of a large conductor carries so little of the current, tubular conductors such as pipe can be used to save weight and cost.

Distribution of current flow in a cylindrical conductor, shown in cross section. For alternating current, most (63%) of the electric current flows between the surface and the skin depth, δ, which depends on the frequency of the current and the electrical and magnetic properties of the conductor

$$\delta = \sqrt{\frac{2\rho}{(2\pi f)(\mu_o \mu_r)}}$$

Induction current in the lecture note

For Slightly Conducting Media

For slightly conducting media, where σ/ωε <<1, the constant k can be approximated by

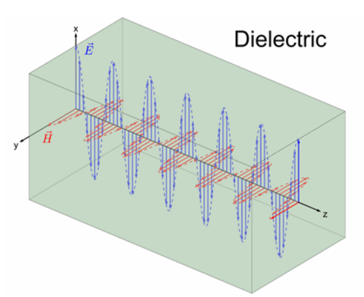
$$k = k - ja = \omega\sqrt{\mu\varepsilon}(1\text{-}j\,\frac{\sigma}{\omega\varepsilon})^{1/2} \approx \omega\sqrt{\mu\varepsilon}(1\text{-}j\,\frac{\sigma}{2\omega\varepsilon})$$

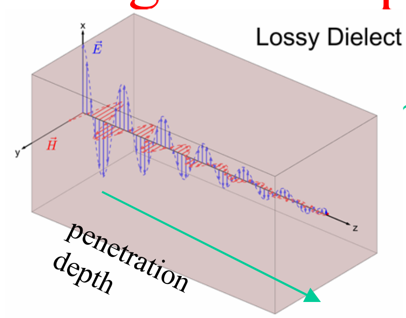
gone

Penetration depth $\delta_p = 1/\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$ (**here we don't have skin depth, skin depth only refers to metal**)

Account for Light Absorption loss!!!

$$E = \hat{x}E_o e^{-i(\omega t - k_z z)}$$





penetration depth

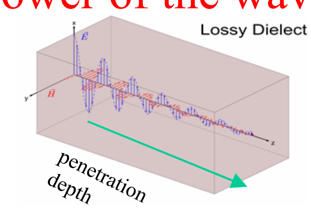
Penetration Depth (dielectric and slight conductive)

According to Beer-Lambert law, the intensity of an **electromagnetic wave inside a material falls off exponentially from the surface** as

Beer Lambert's law

$$I(z) = I_o e^{-\alpha z}$$

If δ_p denotes the penetration we have $\delta_p = 1/\alpha$ "Penetration depth" is one term that describes the decay of electromagnetic waves inside of a material. The above definition refers to the depth δ_p at which the intensity or power of the field decays to 1/e of its surface value. In many contexts one is concentrating on the field quantities themselves: the electric and magnetic fields in the case of electromagnetic waves. Since the power of a wave in a particular medium is proportional to the square of a field quantity, one may speak of a penetration depth at which the magnitude of the electric (or magnetic) field has decayed to 1/e of its surface value, and at which point the power of the wave has thereby decreased to 1/e or about 13% of its surface value:



$$\delta_e = \frac{1}{\alpha/2} = \frac{2}{\alpha} = 2\delta_p$$ (slightly conductive)

$$\delta_p = 1/\alpha = \sqrt{\left(\frac{2}{\omega\mu\sigma}\right)} = \delta$$ (highly conductive)

Note that δ is identical to the skin depth, the latter term usually applying to metals in reference to the decay of electrical currents or we only use penetration depth to describe the media

Transmission: Beer-Lambert or Bouger's Law

Absorption by a filter glass varies with wavelength and filter thickness. Bouger's law states the logarithmic relationship between internal transmission at a given wavelength and thickness.

$$\log_{10}(\tau_1) / d_1 = \log_{10}(\tau_2) / d_2$$

Internal transmittance, τ_i, is defined as the transmission through a filter glass after the initial reflection losses are accounted for by dividing external transmission, T, by the reflection factor P_d.

$$\tau_i = T / P_d$$

The law that the change in intensity of light transmitted through an absorbing substance is related exponentially to the thickness of the absorbing medium and a constant which depends on the sample and the wavelength of the light. Also known as Lambert's law.

Example

The external transmittance for a nominal 1.0 mm thick filter glass is given as $T_{1.0}$ = 59.8 % at 330 nm. The reflection factor is given as P_d = 0.911. Find the external transmittance $T_{2.2}$ for a filter that is 2.2 mm thick.

Solution:

$\tau_{1.0} = T_{1.0} / P_d = 0.598 / 0.911 = 0.656$

$\tau_{2.2} = [\tau_{1.0}]^{2.2/1.0} = [0.656]^{2.2} = 0.396$

$T_{2.2} = \tau_{2.2} * P_d = (0.396)(0.911) = 0.361$

So, for a 2.2 mm thick filter, the external transmittance at 330 nm would be 36.1%

Beer–Lambert law

The absorbance of a beam of collimated monochromatic radiation in a homogeneous isotropic medium is proportional to the absorption path length, l, and to the concentration, c, or — in the gas phase — to the pressure of the absorbing species. The law can be expressed as:

$$A = \log_{10}\left(\frac{P_\lambda^0}{P_\lambda}\right) = \varepsilon\, c\, l$$

or

$$P_\lambda = P_\lambda^0\, 10^{-\varepsilon\, c\, l}$$

where the proportionality constant, ε, is called the molar (decadic) absorption coefficient. For l in cm and c in mol dm $^{-3}$ or M, ε will result in dm^{-3} mol $^{-1}$ cm $^{-1}$ or M cm $^{-1}$, which is a commonly used unit. The SI unit of ε is m ^{2} mol $^{-1}$. Note that spectral radiant power must be used because the Beer–Lambert law holds only if the spectral bandwidth of the light is narrow compared to spectral linewidths in the spectrum.
See: absorbance, extinction coefficient, Lambert law

Transmittance and Absorbance

A spectrophotometer is an apparatus that measures the intensity, energy carried by the radiation per unit area per unit time, of the light entering a sample solution and the light going out of a sample solution. The two intensities can be expressed as transmittance: the ratio of the intensity of the exiting light to the entering light or percent transmittance (%T). Different substances absorb different wavelengths of light. Therefore, the wavelength of maximum absorption by a substance is one of the characteristic properties of that material. A completely transparent substance will have $I_t = I_0$ and its percent transmittance will be 100. Similarly, a substance which allows no radiation of a particular wavelength to pass through it will have $I_t = 0$, and a corresponding percent transmittance of 0.

Transmittance

$T = I_t / I_0$

% Transmittance: $\%T = 100\ T$

Absorbance

$A = \log 10\ (I_0/I_t)$

$A = \log 10\ (1/T) = -\log 10\ (T)$

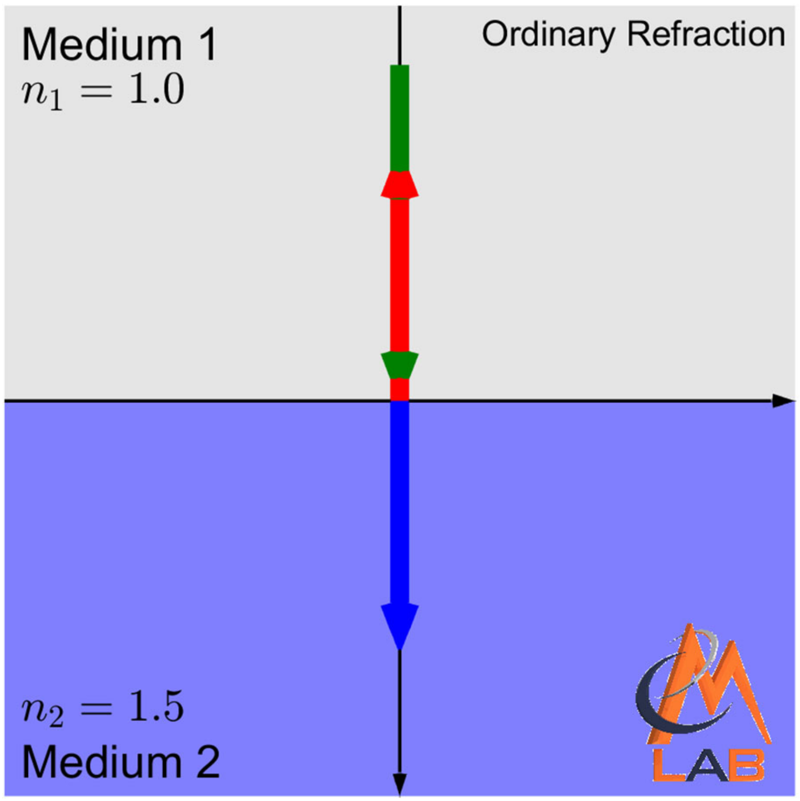
$A = \log 10\ (100/\%T)$

$A = 2 - \log 10\ (\%T)$

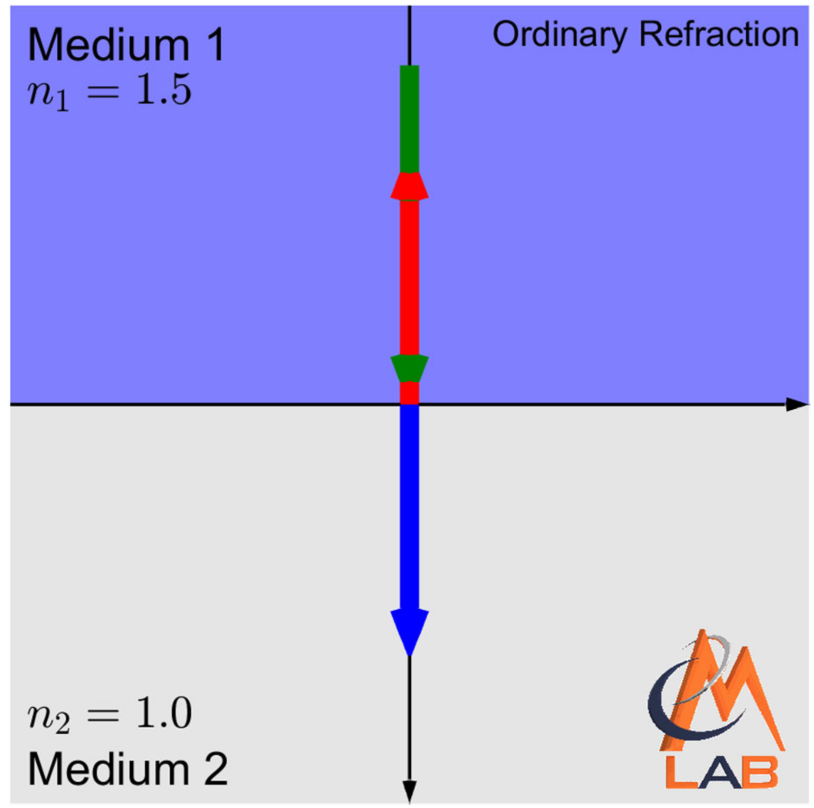
Transmittance for liquids is usually written as: $T = I/I_0 = 10^{-\alpha l} = 10^{\Sigma l c'}$
Transmittance for gases is written as $T = I/I_0 = 10^{-\alpha} l = e^{-\sigma l N}$
I_0 and I are the intensity (or power) of the incident light and the transmitted light, respectively.
Absorbance for liquids is written as $A = -\log 10 = (I/I_0)$
Absorbance for gases it is written as $A' = -\ln(I/I_0)$

Refraction/Reflection at different interfaces

$n_1 < n_2$



$n_1 > n_2$



Critical Angle

In case of $n_1 > n_2$ and TE wave, when incident angle is greater than critical angle θ_c, k_x is larger than the magnitude of k_2

z

$$k_{tz}^2 = k_2^2 - k_x^2 < 0 \quad \Rightarrow \quad \boxed{k_{tz}^2 = -j\alpha}$$

x

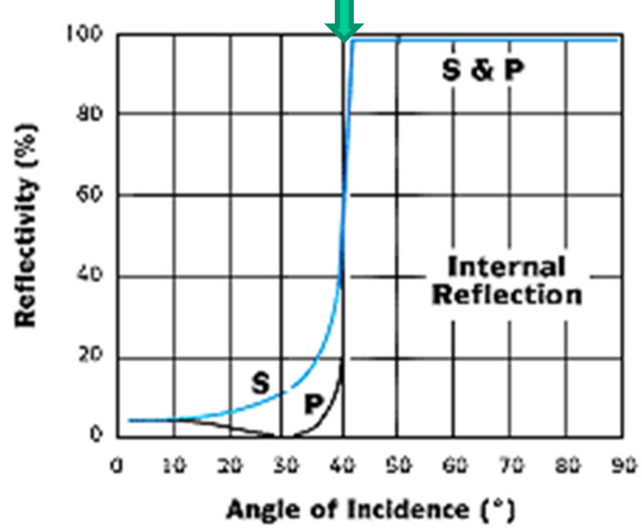
$$E^t = \hat{y} T_l E_o e^{-jk_{tx}x - j\alpha z} \quad \Rightarrow \quad E^t = \hat{y} T_l \boxed{E_o e^{-j\alpha z} \cos(\omega t - k_x x)}$$

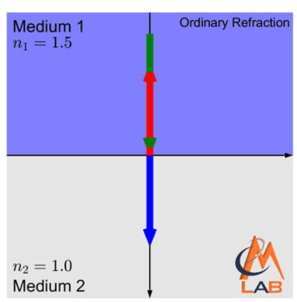
Because it decays away from interface and because the wave propagating along the interface, the wave is also called surface wave.

Critical angle is defined as

$$\theta_c = \theta_1 = \sin^{-1}\frac{k_2}{k_1}$$

$$\theta_c = \theta_1 = \sin^{-1}\frac{n_2}{n_1}$$





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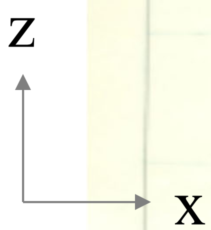
Total reflection



$\theta_2 = 90°$ (refraction angle $= 90°$)

$k_1 \sin\theta_1 = k_2 \sin\theta_2$ (use Snell's law)

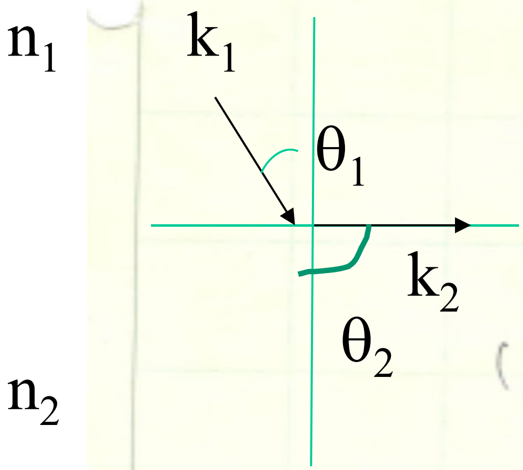
or phase matching condition

$$\boxed{\theta_{critical} = \sin^{-1} \frac{k_2}{k_1}}$$

if $\theta_1 > \theta_{critical}$

How?

Then $\sin\theta_2 > 1$



since $k_2^2 = k_{2z}^2 + k_{2x}^2$

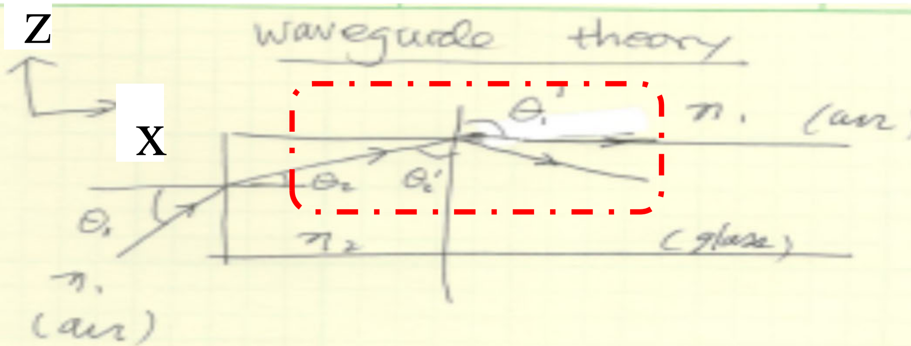
(z direction propagation constant) $k_{2z}^2 = k_2^2 - k_{2x}^2$

$k_{2z}^2 = k_2^2 - (k_2 \sin\theta_2)^2$, $\sin\theta_2 > 1$

$e^{jk_{2z} z}$ $k_{2z}^2 < 0$

~~Plane~~ $k_{2z} =$ imaginary

waveguide theory



Snell's law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$n_2 \sin\theta_2' = n_1 \sin\theta_1'$$

$\theta_1' = 90°$ ∴ total reflection occurs

then $\theta_2' = \theta_{critical}$

no wave travel outside the glass substrate, therefore glass substrate becomes a wave guide.

However if $\theta_2' > \theta_{critical}$

Then

$$n_2 \sin\theta_2' = n_1 \underbrace{\sin\theta_1'}_{>1}$$

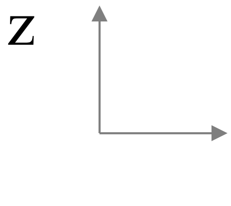
$$1 = \underbrace{\sin^2\theta_1'}_{>1} + \boxed{\underbrace{\cos^2\theta_1'}_{<0}}$$

since $k_1^2 = k_{1z}^2 + k_{1x}^2$

$$k_{1z}^2 = k_1^2 - k_{1x}^2 = k_1^2 - (k_1 \sin\theta_1')^2 \Rightarrow k_{1z}^2 < 0$$

$$k_1^2 = k_1^2\sin^2\theta_1' + \boxed{k_1^2\cos^2\theta_1'}$$

TE mode



Using physics notation easier to see the attenuation

k_{1z} = imaginary

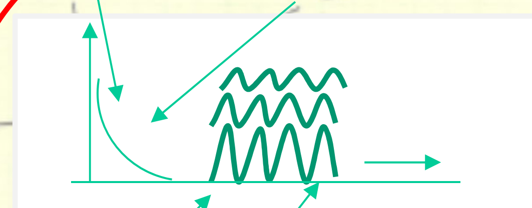
$\hat{y}\, E e^{j k_{1x} x + j k_{1z} z}$

$\hat{y}\, \mathrm{Re}\left\{ \left(E e^{j k_{1x} x}\right) e^{j \overset{\text{imaginary}}{k_{1z}} z} \right\}$

$= \hat{y}\, E_0 \cos k_{1x} x \;\; e^{-\alpha_z z}$

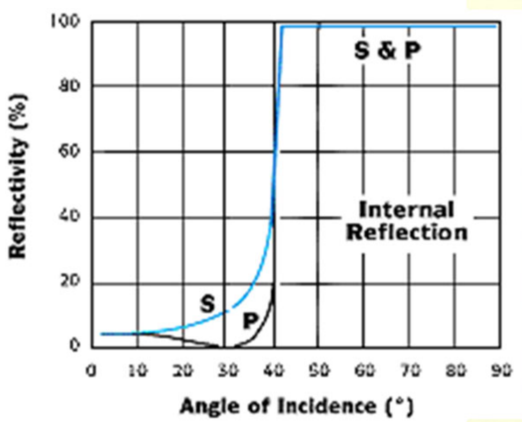
where α_z = ~~imaginary~~ k_{1z} = imaginary

$e^{-\alpha_z z}$



$\cos k_{1x} x$

Evanescent wave
(surface wave)
(leaky wave)
use for sensing and
wave coupling



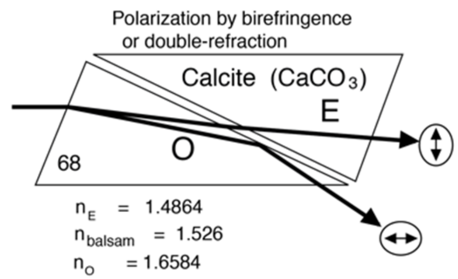
cosk_{1x}x at different z

Attenuation

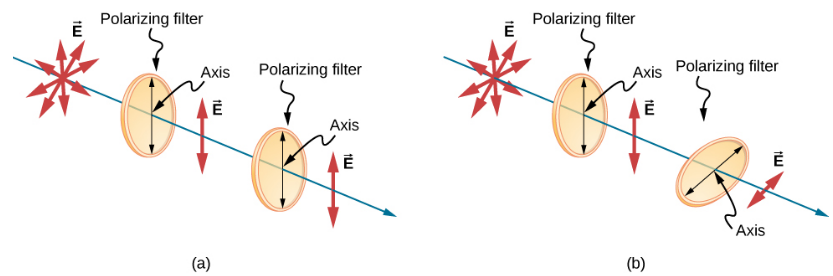
- Due to material properties (dissipative material- resistivity)
- Boundary condition

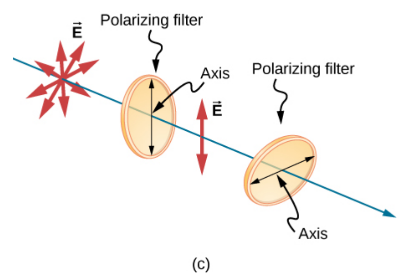
Recall

Polarizing Light



Polarizing light





(d)

Reduce Intensity

Reduce glare!!!

W. Wang

Linear polarizer

Recall

Polarization

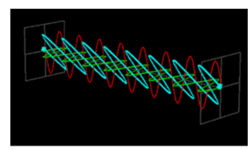
Let's assume the real time-space *E* vector has x and y components:

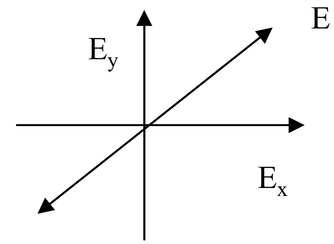
$$E(z,t) = a\cos(\omega t - kz + \phi_a)\hat{x} + b\cos(\omega t - kz + \phi_b)\hat{y}$$

$E_y/E_x = Ae^{j\phi}$

linearly polarized: $\phi_b - \phi_a = 0..or\,\pi$

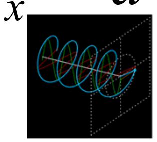
$$E_y = \pm(\frac{b}{a})E_x$$

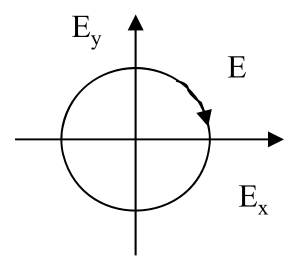




circularly polarized: $\phi_b - \phi_a = \pm\frac{\pi}{2}$

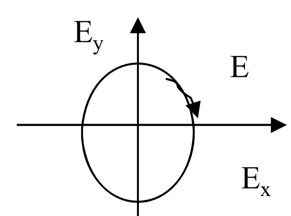
$$\frac{E_y}{E_x} = \frac{b}{a} = 1$$





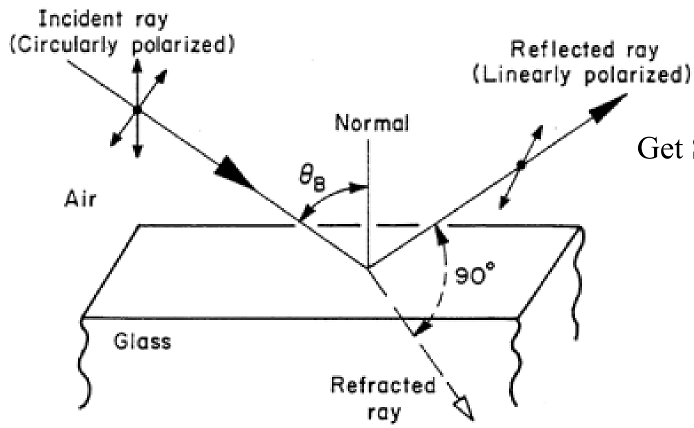
Elliptically polarized: $\phi_b - \phi_a = anything..except..0,\pi,\pm\frac{\pi}{2}$

$$\frac{E_y}{E_x} = \frac{b}{a} = anything$$

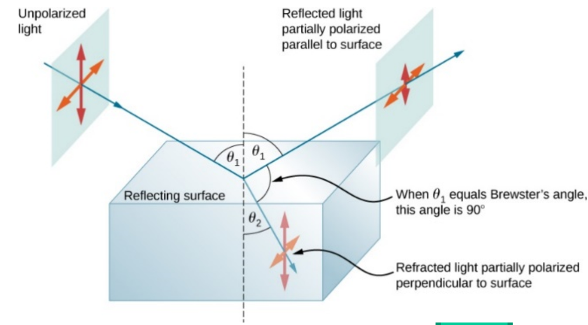


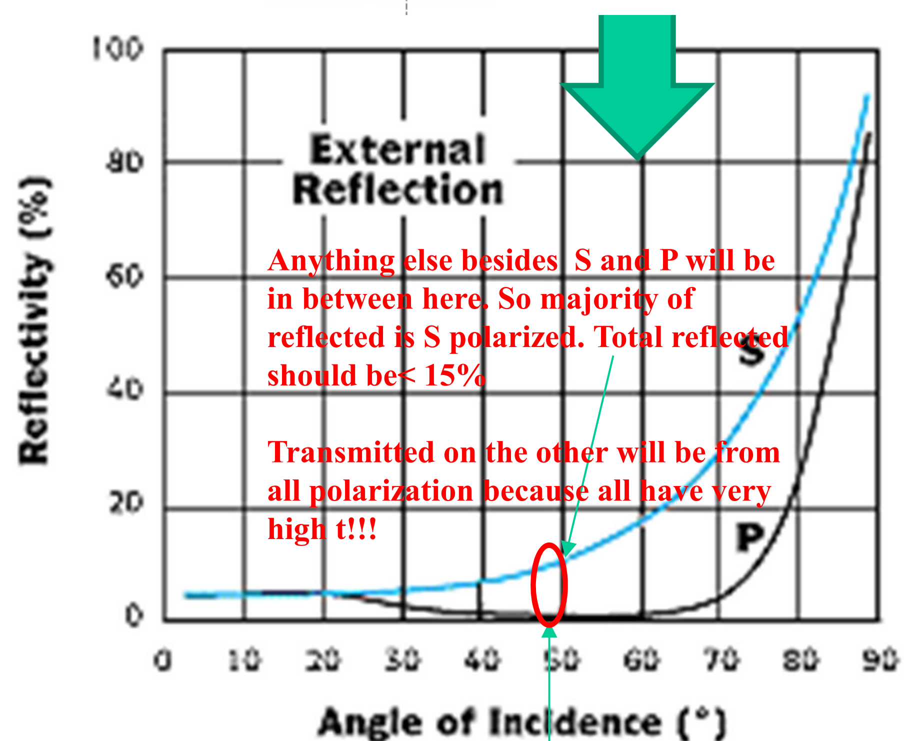
Reflection of Unpolarized Light from Dielectrics

Either circular or unpolarized input



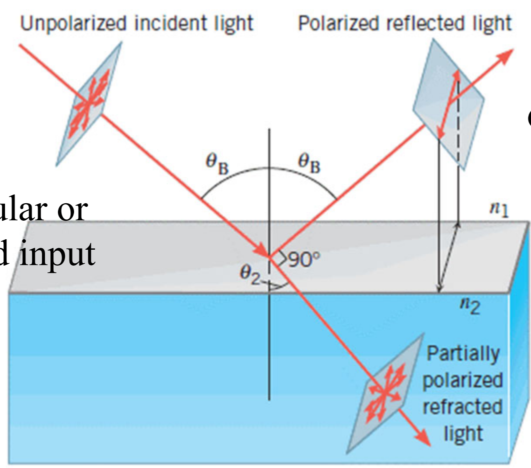
Get S wave





Anything else besides S and P will be in between here. So majority of reflected is S polarized. Total reflected should be< 15%

Transmitted on the other will be from all polarization because all have very high t!!!



Get S wave
TE wave

Either circular or unpolarized input

Brewster angle

Polarizing unpolarized wave

Method for achieving polarizing light

Recall TE and TM mode reflection and transmission coefficient:

If we put $n_2 = n_1 \sin\theta_i / \sin\theta_t$ (Snell's law and multiply the numerator and denominator by $(1/n_1)\sin\theta_t$

$$r_s = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t},$$

$$t_s = \frac{2n_1 \cos\theta_i}{n_1 \cos\theta_i + n_2 \cos\theta_t},$$

$$r_p = \frac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t},$$

$$t_p = \frac{2n_1 \cos\theta_i}{n_2 \cos\theta_i + n_1 \cos\theta_t}.$$

$$r_s = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}.$$

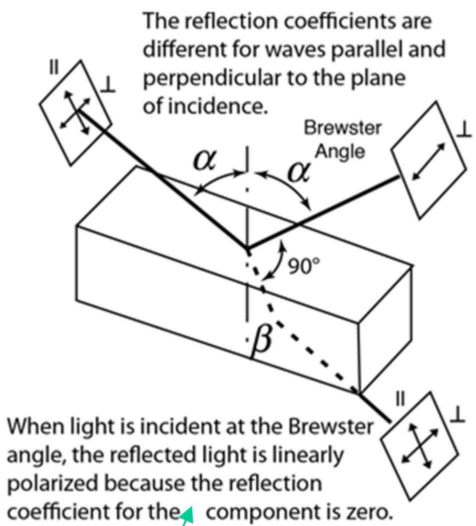
$$t_\perp = \frac{2\sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)}$$

If we do likewise with the formula for r_p, the result is easily shown to be equivalent to

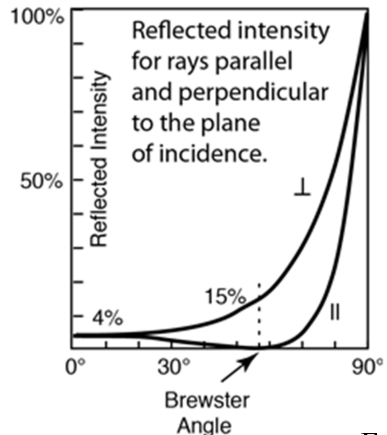
$$r_p = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}.$$

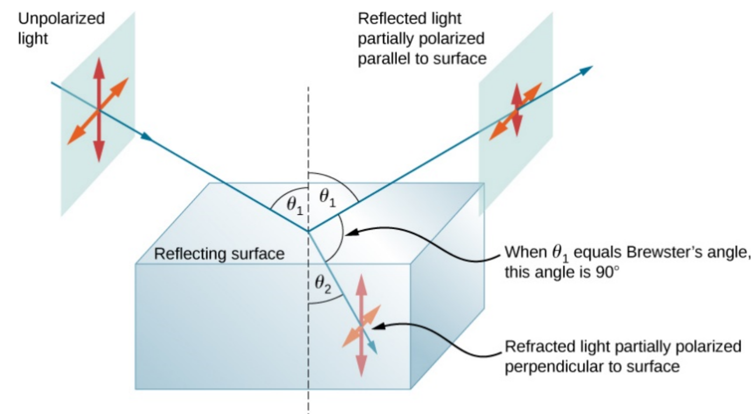
$$t_\parallel = \frac{2\sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)}$$

Method for achieving polarizing light

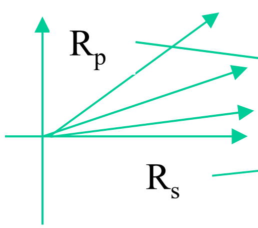


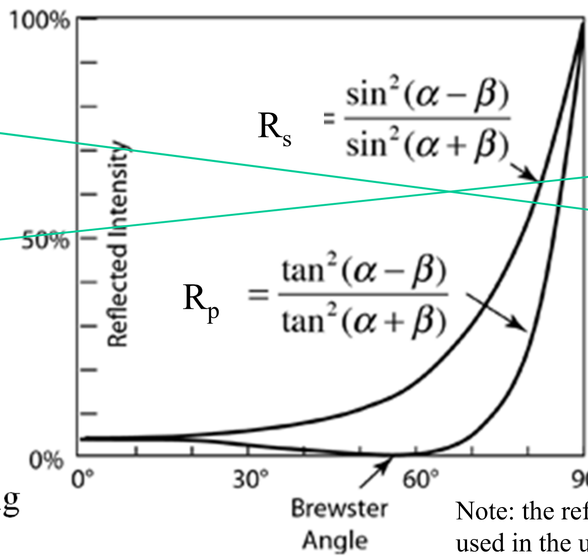
parallel





For n=1.5, The Brewster angle is 56.309932474020215 °.





$$R_s = \frac{\sin^2(\alpha-\beta)}{\sin^2(\alpha+\beta)}$$

$$R_p = \frac{\tan^2(\alpha-\beta)}{\tan^2(\alpha+\beta)}$$

R can be broken down to Rs and Rp components less than brester angel

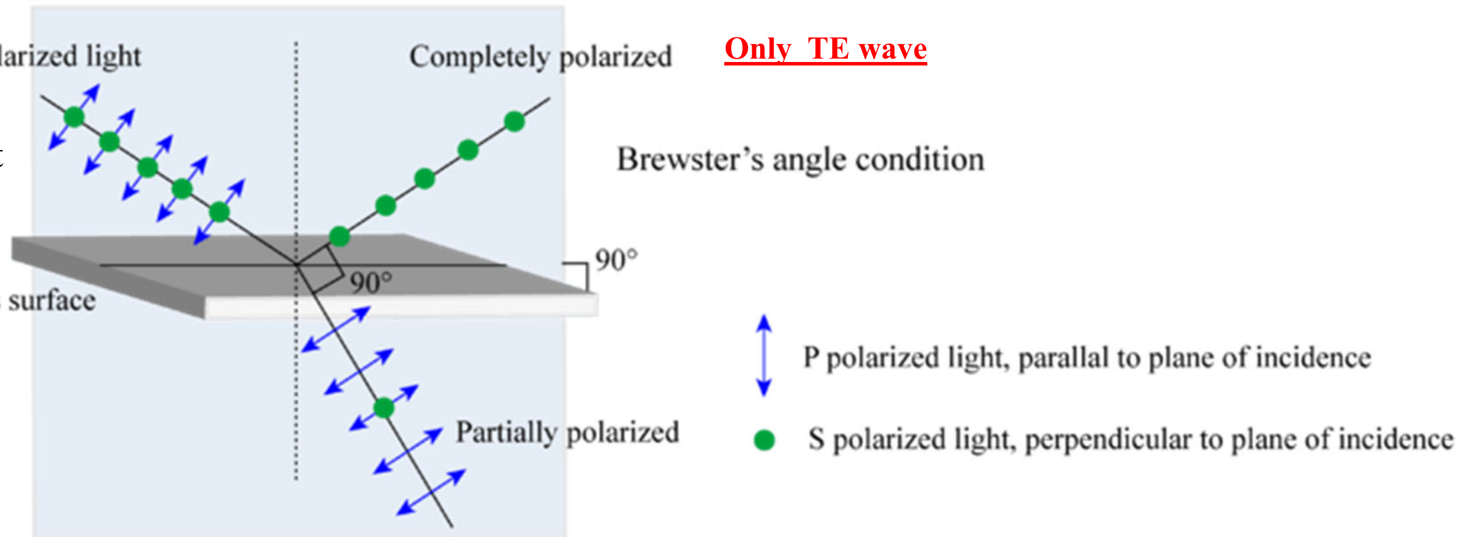
W. Wang

If we have an unpolrized light with an incident angle of α = 56°, the transmitted angle is β = 33.55187256201638°.

The reflection coefficients are (intensity):
R_s= 0.14581593211226546
R_p= 0.000010442342353900743

The overall reflected intensity is
Int_{reflec}=7.291318722730968 % of the incident
and R_s ()x $Int_{reflect}$= 99.99283919497213% of that is in the perpendicular plane.

Note: the reflection coefficients used here are the intensities and not the amplitudes as used in the usual presentation of the Fresnel equations. That is, these reflection coefficients are the square of those in the Fresnel expressions.



For a case where $\boxed{\mu_1 = \mu_2}$ and parallel polarization (TM), there is always a angle θ_b such that wave is totally transmitted and the **reflection coefficient is zero *$R_{//}$=0***, $\omega\sqrt{\mu_1\varepsilon_2}\cos\theta_1 = \omega\sqrt{\mu_1\varepsilon_1}\cos\theta_2$

Phase matching conduction gives $\omega\sqrt{\mu_1\varepsilon_1}\sin\theta_1 = \omega\sqrt{\mu_1\varepsilon_2}\sin\theta_2$

$$\boxed{\begin{aligned}\theta_b &= \tan^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\\ \theta_b &= \tan^{-1}\sqrt{\frac{n_2}{n_1}}\end{aligned}}$$

(Brewster Angle)

Recall TM mode reflection and transmission coefficient:

We get,

$$1 + R_{ll} = T_{ll}$$

$$1 - R_{ll} = \frac{\varepsilon_1 k_{tz}}{\varepsilon_2 k_z} T_{ll}$$

$$\Rightarrow$$

$$R_{ll} = \frac{\varepsilon_2 k_z - \varepsilon_1 k_{tz}}{\varepsilon_2 k_z + \varepsilon_1 k_{tz}}$$

$$T_{ll} = \frac{2\varepsilon_2 k_z}{\varepsilon_2 k_z + \varepsilon_1 k_{tz}}$$

page 109 Brewster angle

Two equations derived from Maxwell's or B.C.

Derivation in terms of ε and μ

① $R_{\parallel} = 0 = \dfrac{\varepsilon_2 k_z - \varepsilon_1 k_{tz}}{\varepsilon_2 k_z + \varepsilon_1 k_{tz}}$

$0 = \dfrac{\varepsilon_2 k_1 \cos\theta_1 - \varepsilon_1 k_2 \cos\theta_2}{\varepsilon_2 k_1 \cos\theta_1 + \varepsilon_1 k_2 \cos\theta_2}$

$\varepsilon_1 k_0 n_2 \cos\theta_2 = \varepsilon_2 k_0 n_1 \cos\theta_1$

Divide $\sqrt{\varepsilon_1 \varepsilon_2}$

$\varepsilon_1 \omega\sqrt{\mu_2\varepsilon_2}\cos\theta_2 = \varepsilon_2 \omega\sqrt{\mu_1\varepsilon_1}\cos\theta_1$

$\omega\sqrt{\mu_2\varepsilon_1}\cos\theta_2 = \omega\sqrt{\mu_1\varepsilon_2}\cos\theta_1$

$\boxed{\omega\sqrt{\mu_1\varepsilon_1}\cos\theta_2 = \omega\sqrt{\mu_1\varepsilon_2}\cos\theta_1}$ ⟵ $\boxed{\mu_1 = \mu_2}$

Works only if

② $k_{1x} = k_{2x}$ ⟵ (phase matching cond)

$\boxed{\omega\sqrt{\mu\varepsilon_1}\sin\theta_1 = \omega\sqrt{\mu_1\varepsilon_2}\sin\theta_2}$

From Last page

(1) $R_{\parallel}=0 \Rightarrow$ $\boxed{\omega\sqrt{\mu_1\epsilon_2}\cos\theta_1 = \omega\sqrt{\mu_1\epsilon_1}\cos\theta_2}$

(2) phase matching

$$\boxed{\omega\sqrt{\mu_1\epsilon_1}\sin\theta_1 = \omega\sqrt{\mu_1\epsilon_2}\sin\theta_2}$$

$$\begin{cases} \cos\theta_2 = \sqrt{\dfrac{\epsilon_2}{\epsilon_1}}\cos\theta_1 \\ \sin\theta_2 = \sqrt{\dfrac{\epsilon_1}{\epsilon_2}}\sin\theta_1 \end{cases}$$

$$\cos^2\theta_2 + \sin^2\theta_2 = 1 = \cos^2\theta_1 + \sin^2\theta_1 =$$

$$\left(\sqrt{\frac{\epsilon_2}{\epsilon_1}}\right)^2\cos^2\theta_1 + \frac{\epsilon_1}{\epsilon_2}\sin^2\theta_1 = 1$$

$$= \cos^2\theta_1 + \sin^2\theta_1$$

We sub the above two equations for $\sin^2\theta_2$ and $\cos^2\theta_2$ and set it equal to $\sin^2\theta_1+\cos^2\theta_1$ we get :

$$\boxed{\frac{\epsilon_2-\epsilon_1}{\epsilon_1}\cos^2\theta_1 = \frac{\epsilon_2-\epsilon_1}{\epsilon_2}\sin^2\theta_1}$$

$$\frac{\epsilon_2}{\epsilon_1} = \tan^2\theta_1$$

$$\boxed{\theta_1 = \tan^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}}}$$

$$\boxed{\theta = \tan^{-1}(n_2/n_1)}$$

Brewster angle

Derivation in terms of n

(1) $n_1 \sin\theta_1 = n_2 \sin\theta_2$ (plane match)

$$(\sin\theta_2)^2 = \left(\frac{n_1 \sin\theta_1}{n_2}\right)^2$$

(2) $n_2 \cos\theta_1 = n_1 \cos\theta_2$ ($R_\parallel = 0$)

$$\cos^2\theta_2 = \left(\frac{n_2 \cos\theta_1}{n_1}\right)^2$$

We sub the above two equations for $\sin^2\theta$ and $\cos^2\theta$

$$\sin^2\theta_1 + \cos^2\theta_1 = \sin^2\theta_2 + \cos^2\theta_2 = 1 = \frac{n_2^2}{n_1^2}\cos^2\theta_1 + \frac{n_1^2}{n_2^2}\sin^2\theta_1$$

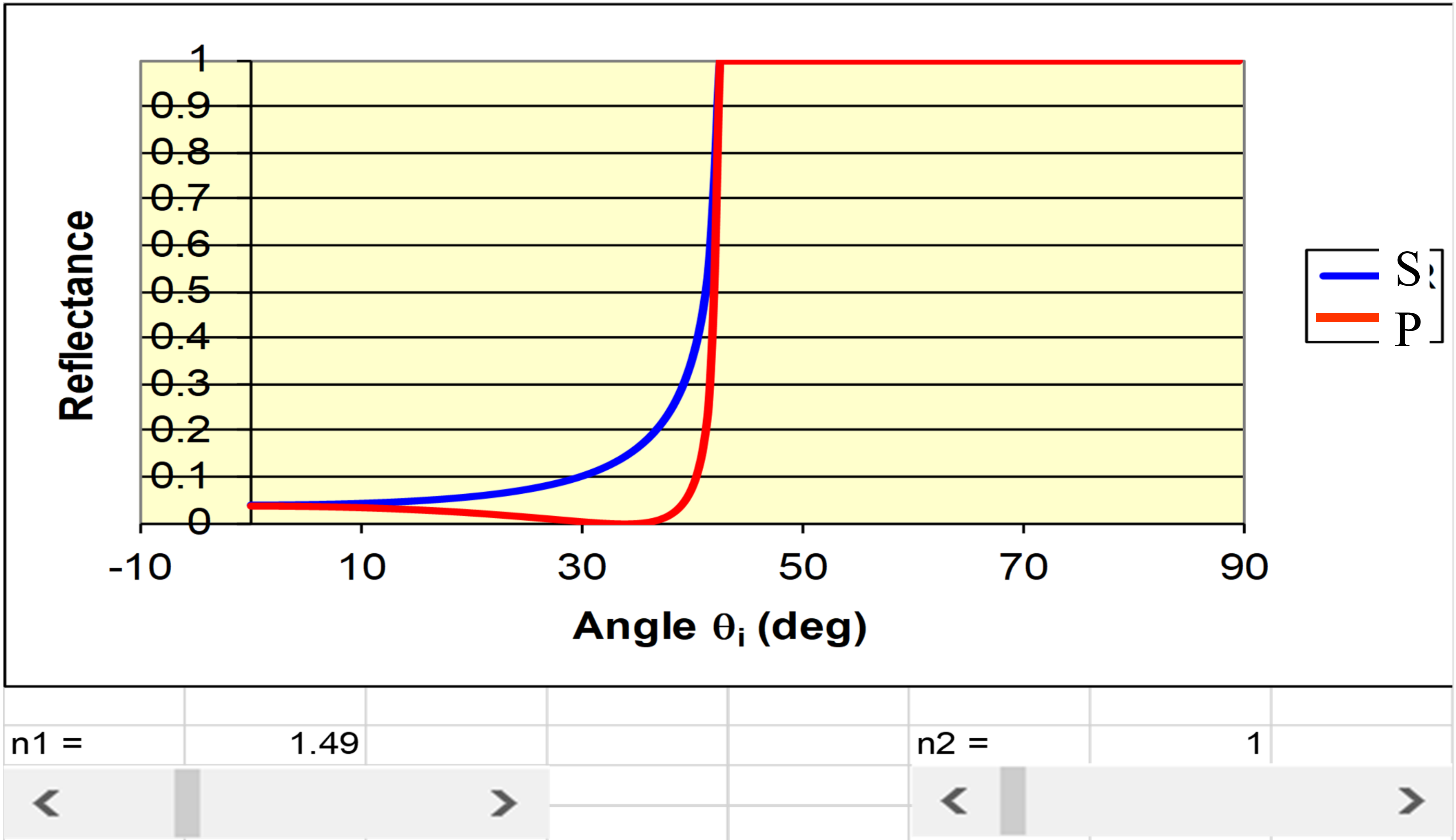
equate the sub with $\sin^2\theta + \cos^2\theta$ we get

$$\boxed{\sin^2\theta_1\left(\frac{n_2^2 - n_1^2}{n_2^2}\right) = \cos^2\theta_1\left(\frac{n_2^2 - n_1^2}{n_1^2}\right)}$$

$$\tan^2\theta_1 = \frac{n_2^2}{n_1^2}$$

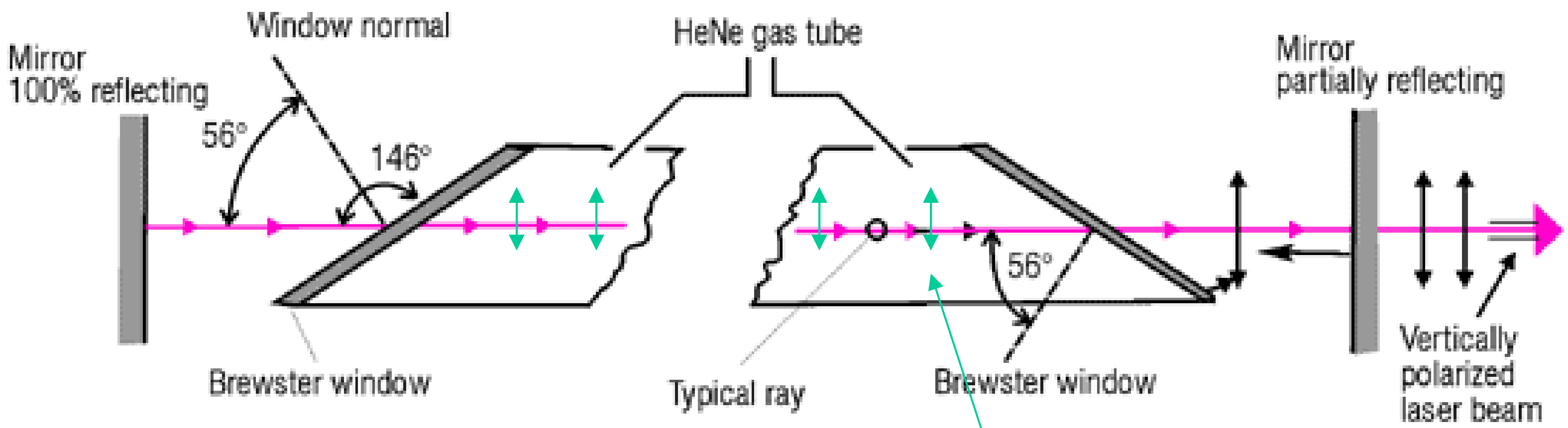
$$\tan\theta_1 = \frac{n_2}{n_1}$$

$$\theta = \tan^{-1}(n_2/n_1)$$



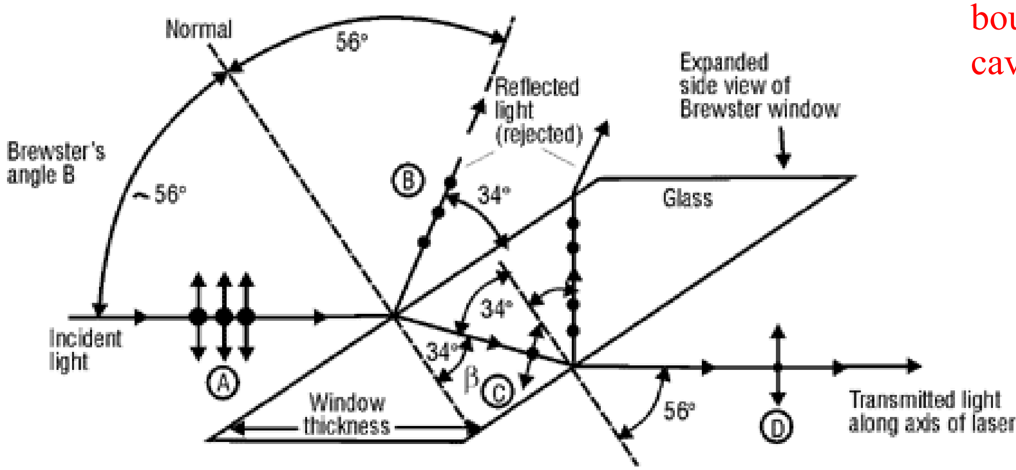
graph of the reflectance R for s- and p-polarized light as a function of n_1, n_2, and θ_1 $n_{acrylate} = 1.488 \Rightarrow \theta_{critical} = 42.4^o$

Brewster windows in a laser cavity



Brewster windows are used in laser cavities to ensure that the laser light after bouncing back and forth between the cavity mirrors emerges as linearly polarized light.

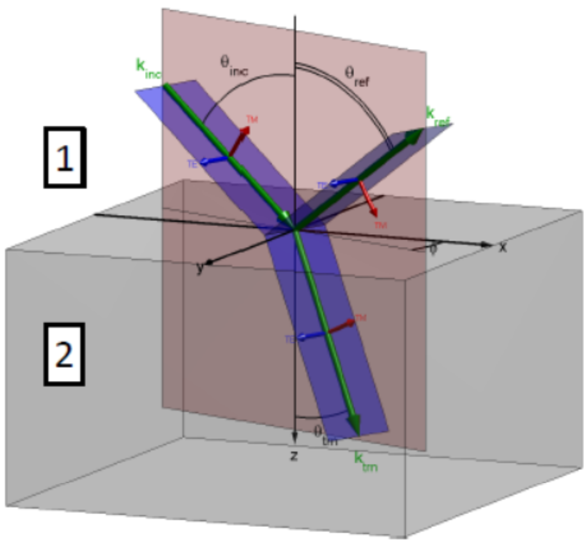
Remember initially transmission is all I but after bouncing around for awhile, all the light inside the cavity is all vertically polarized



Unpolarized light passing through both faces at a Brewster angle

Summary of Scattering at Interface

Geometry and Polarization



$$\hat{a}_{\text{TE}} = \frac{\hat{a}_z \times \vec{k}_{inc}}{\left|\hat{a}_z \times \vec{k}_{inc}\right|} \qquad \hat{a}_{\text{TM}} = \frac{\hat{a}_{\text{TE}} \times \vec{k}_{inc}}{\left|\hat{a}_{\text{TE}} \times \vec{k}_{inc}\right|}$$

Law of Reflection and Refraction

$\theta_{\text{ref}} = \theta_{\text{inc}}$ $\qquad n_1 \sin\theta_1 = n_2 \sin\theta_2$ (Snell's Law)

Normal Incidence (θ_1 = 0°)

$$r = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad t = \frac{2\eta_2}{\eta_2 + \eta_1} \qquad 1 + r = t$$

$$R = |r|^2 \qquad T = |t|^2 \frac{\eta_1}{\eta_2} \qquad R + T = 1$$

There is no distinction between TE and TM for LHI media.

Fresnel Equations (Amplitude)

$$r_{\text{TE}} = \frac{\eta_2\cos\theta_1 - \eta_1\cos\theta_2}{\eta_2\cos\theta_1 + \eta_1\cos\theta_2} \qquad t_{\text{TE}} = \frac{2\eta_2\cos\theta_1}{\eta_2\cos\theta_1 + \eta_1\cos\theta_2} \qquad 1 + r_{\text{TE}} = t_{\text{TE}}$$

$$r_{\text{TM}} = \frac{\eta_2\cos\theta_2 - \eta_1\cos\theta_1}{\eta_2\cos\theta_2 + \eta_1\cos\theta_1} \qquad t_{\text{TM}} = \frac{2\eta_2\cos\theta_1}{\eta_2\cos\theta_2 + \eta_1\cos\theta_1} \qquad 1 + r_{\text{TM}} = t_{\text{TM}}\frac{\cos\theta_2}{\cos\theta_1}$$

Reflectance and Transmittance (Power)

$$R_{\text{TE}} = |r_{\text{TE}}|^2 \qquad T_{\text{TE}} = |t_{\text{TE}}|^2 \frac{\eta_1\cos\theta_2}{\eta_2\cos\theta_1} \qquad R_{\text{TE}} + T_{\text{TE}} = 1$$

$$R_{\text{TM}} = |r_{\text{TM}}|^2 \qquad T_{\text{TM}} = |t_{\text{TM}}|^2 \frac{\eta_1\cos\theta_2}{\eta_2\cos\theta_1} \qquad R_{\text{TM}} + T_{\text{TM}} = 1$$

$$\frac{T_{\text{TE}}}{|t_{\text{TE}}|^2} = \frac{T_{\text{TM}}}{|t_{\text{TM}}|^2}$$

Brewster's Angle

$$\sin^2\theta_B\big|_{\text{TE}} = \frac{1 - \mu_{r1}\varepsilon_{r2}/\mu_{r2}\varepsilon_{r1}}{1 - \left(\mu_{r1}/\mu_{r2}\right)^2}$$

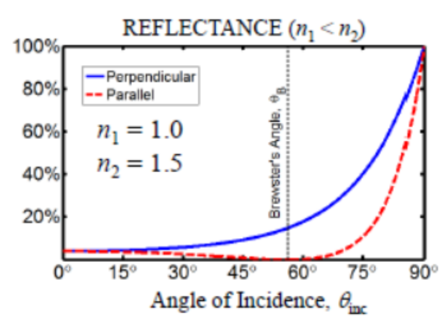
No Brewster's angle for TE polarization if there is no magnetic response.

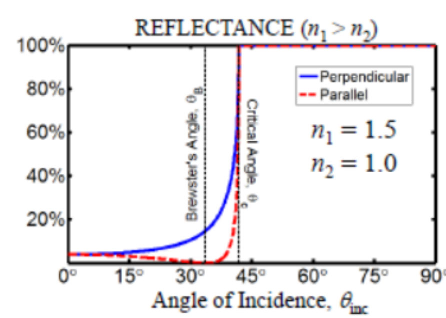
$$\sin^2\theta_B\big|_{\text{TM}} = \frac{1 - \mu_{r2}\varepsilon_{r1}/\mu_{r1}\varepsilon_{r2}}{1 - \left(\varepsilon_{r1}/\varepsilon_{r2}\right)^2} \qquad \tan\theta_B\big|_{\text{TM}} = \sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}} = \frac{n_2}{n_1}$$

for no magnetic response

Critical Angle

$$\theta_c = \sin^{-1}\left(n_2/n_1\right)$$

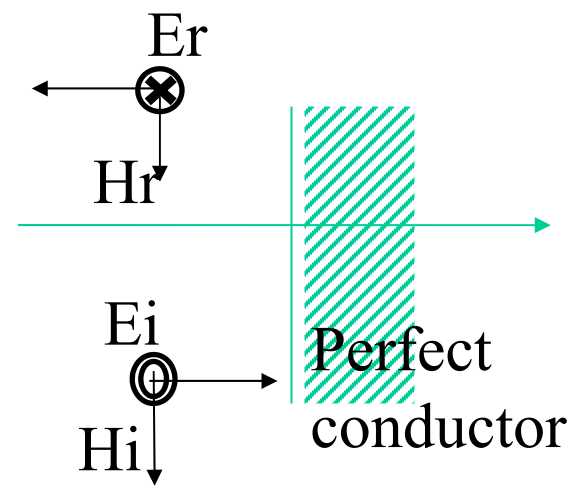




Reflection from Perfect Conductor

On the surface of a perfect conductor, $E_{2//} = 0$ and $B_2 = 0$

For TE polarization: $R_l = \dfrac{\mu_2 k_z - \mu_1 k_{tz}}{\mu_2 k_z + \mu_1 k_{tz}}$

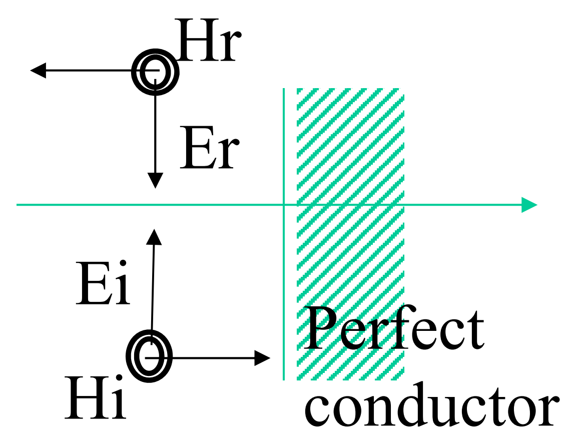


$$\varepsilon_{2\,p.c.} = \varepsilon_2 - j\frac{\sigma}{\omega} \approx \infty$$

$$k_2 \propto \sqrt{\varepsilon_{2\,p.c.}} \approx \infty \Rightarrow k_2 \approx \infty$$

$$so \qquad R_I = -1$$

For TM polarization: $R_{ll} = \dfrac{\varepsilon_2 k_z - \varepsilon_1 k_{tz}}{\varepsilon_2 k_z + \varepsilon_1 k_{tz}}$



$$\varepsilon_{2\,p.c.} \propto \frac{\sigma}{\omega}$$

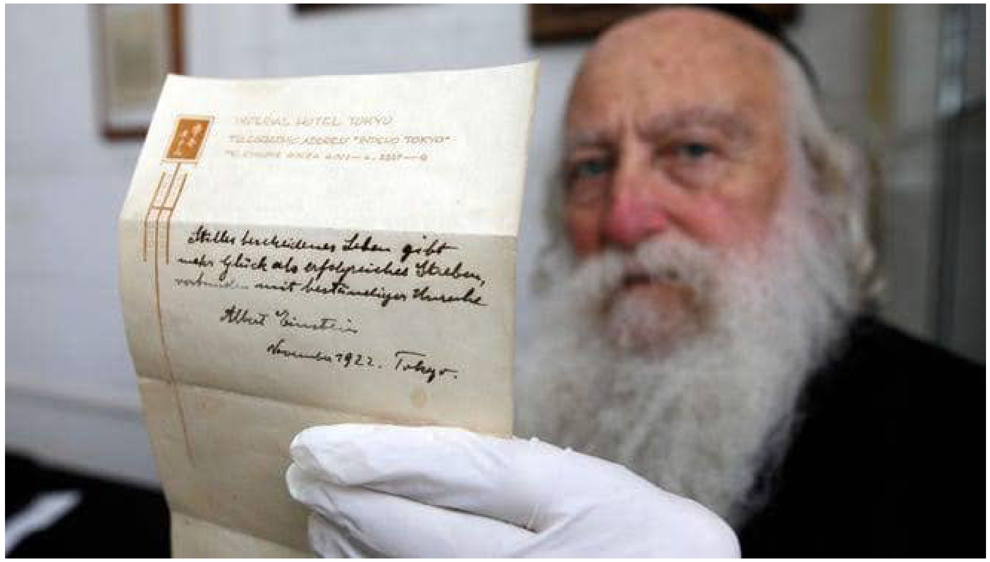
$$k_2 \propto \sqrt{\varepsilon_{2\,p.c.}}$$

$$so \qquad R_{II} = 1$$

Difference between reflection, backward travelling wave and phase conjugated wave

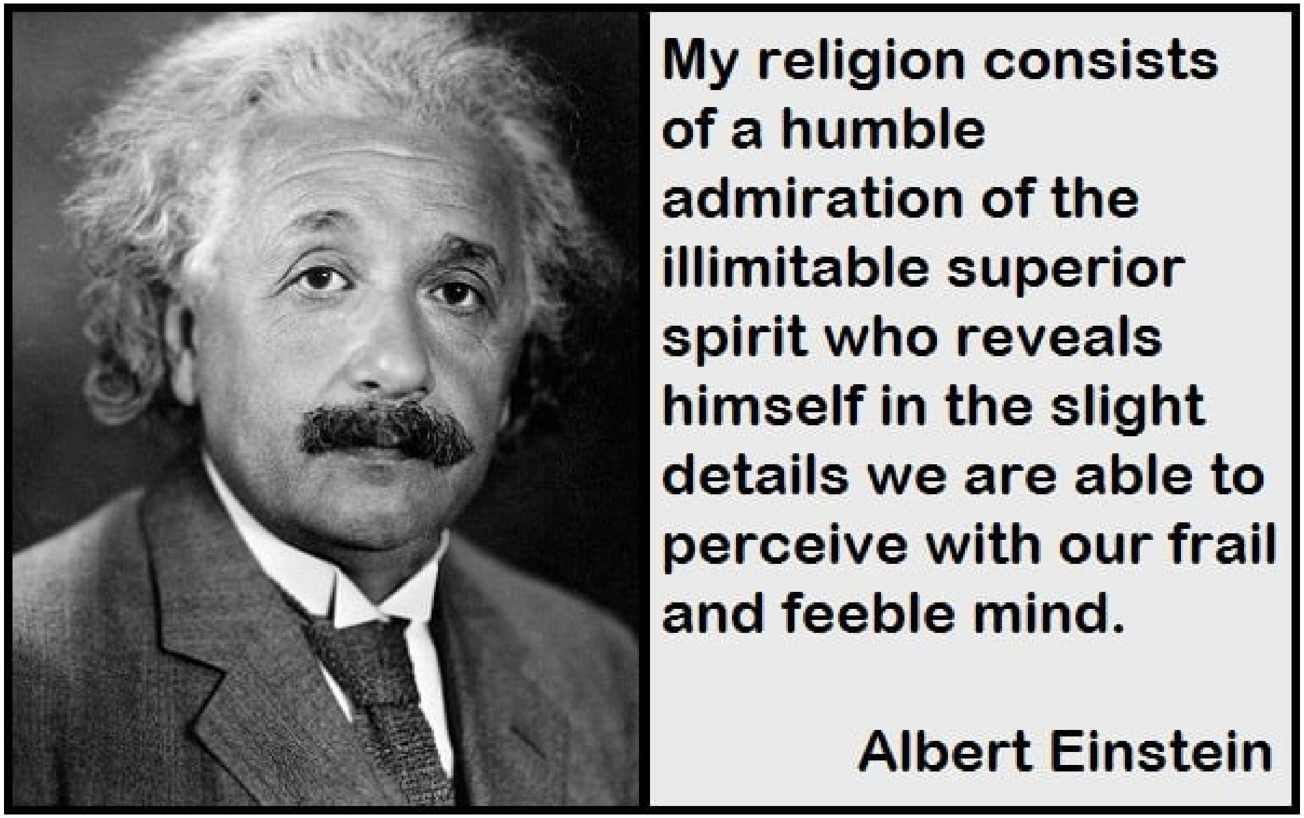
$$E_i = E_o e^{-jk_{ix}x}$$
$$E_r = RE_o e^{jk_{rx}x}$$
$$E_t = TE_o e^{-jk_{tx}x}$$

- perfect conductor TE reflection: R = -1 $E_r = e^{j\pi} E_o e^{jk_{rx}x}$ and $E_t = 0$
- perfect conductor TM reflection: R = 1 $E_r = e^{j0} E_o e^{jk_{rx}x}$ and $E_t = 0$
- regular reflection, $k_{rx} = k_{ix}$ and $n_{i,t,r}$ = positive, $E_r = RE_o e^{jk_{rx}x}$ and $E_t = TE_o e^{-jk_{tx}x}$
- backward travelling wave, **n_t = negative**, $E_o e^{-jk_{ix}x}$ $E_r = RE_o e^{jk_{tx}x}$ *and* $\boxed{E_t = TE_o e^{jk_{tx}x}}$
- phase conjugated wave, $\boxed{E_r = E_o e^{jk_{ix}x}}$ (reflected wave is same as input!!!)



"A calm and humble life will bring more happiness than the pursuit of success and the constant restlessness that comes with it."

This note was written by Albert Einstein in 1922.
Ironically, it sold for $1.6m at an auction in 2017.


My religion consists of a humble admiration of the illimitable superior spirit who reveals himself in the slight details we are able to perceive with our frail and feeble mind.
Albert Einstein

"A human being is a part of the whole called by us universe, a part limited in time and space. He experiences himself, his thoughts and feeling as something separated from the rest, a kind of optical delusion of his consciousness. This delusion is a kind of prison for us, restricting us to our personal desires and to affection for a few persons nearest to us. Our task must be to free ourselves from this prison by widening our circle of compassion to embrace all living creatures and the whole of nature in its beauty.

Albert Einstein

Week 10

- Course Website: http://courses.washington.edu/me557/sensors
- Reading Materials:
  - Week 10 reading materials can be found:
    http://courses.washington.edu/me557/reading/
- Makeup classes on 11/18 and 11/22 1-2PM
- Homework #2 is due Week 13
- Sign up for Lab #1
- Discuss Final Project Proposal (set up a time to go over your proposal, Week 12, Proposal due week 13)
- Final Presentation 12/27 1:20 to 3:10PM

Last week lecture

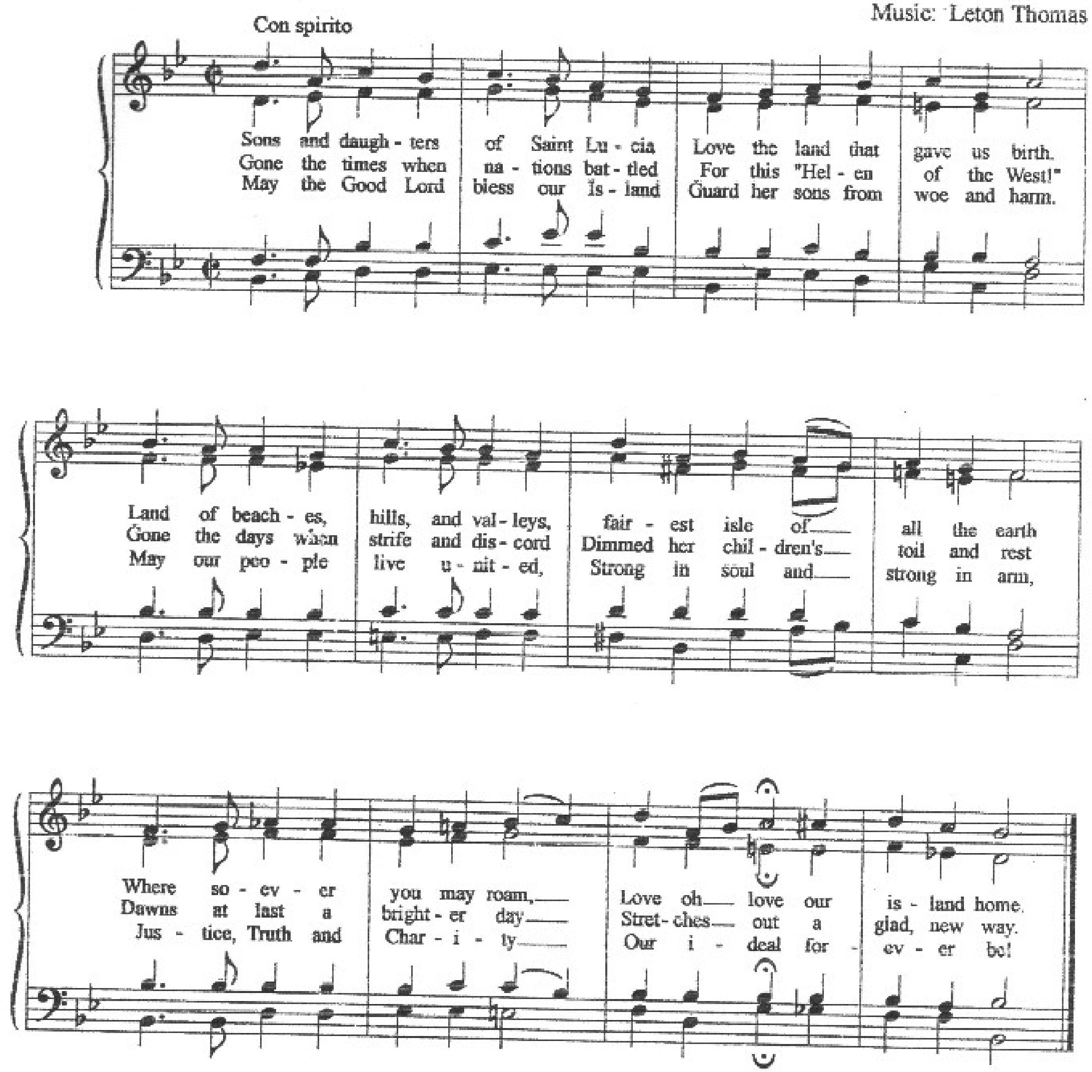
- Reflection and refraction using wave theory
- Wave in dispersive and dissipative medium
- Critical and Brewster's angles
- Diffraction and interference
- Slit and Grating

This week lecture

- Reflection and refraction using wave theory
- Wave in dispersive and dissipative medium
- Critical and Brewster's angles
- Diffraction and interference
- Slit and Grating

Saint Lucia's National Anthem

Words: Rev. Father Charles Jesse
Music: Leton Thomas



Diffraction and Interference

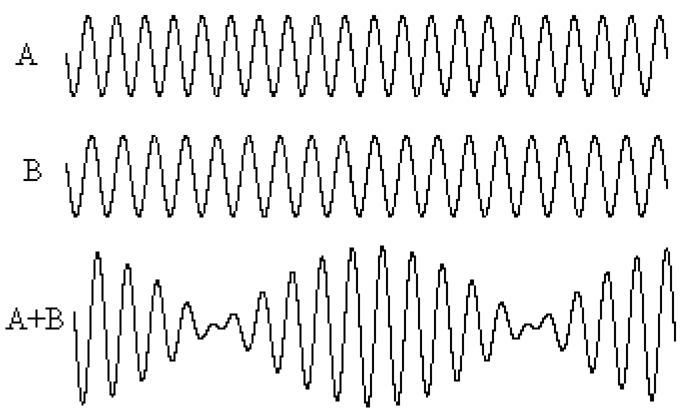
Two Approaches

- Full wave method (EM theory)
- Phase only (Physics class)

Most important summary

Power of sin(a)+sin(b)

Everything can be expanded or explained in a series of sine function either a summation, multiplication or convolution. Many of our optical theory and sensor concept exploit this concept to allow us to study small physical changes using resolution of optical wavelength but observed at relatively lower frequency or longer wavelength or simplify the way we calculate them. e.g. interference or beats



$\sin A+\sin B = 2\sin(A+B)/2 * \cos(A-B)/2$

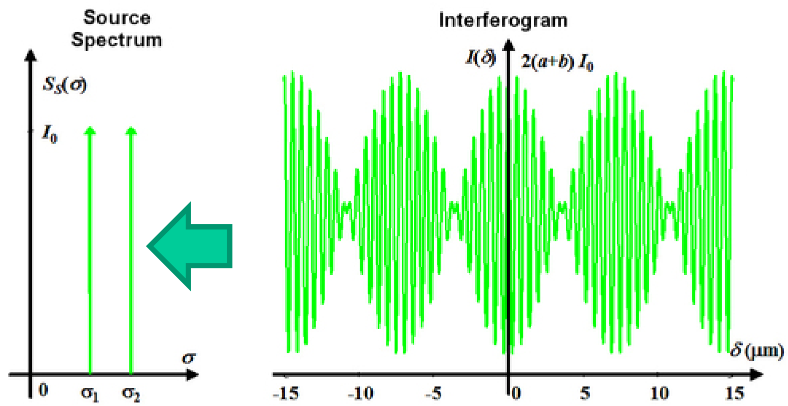
Let $A = k_1x+\omega_1t+ \phi_1 \quad k_1 = 2\pi n_1/\lambda$

$B = k_2x+\omega_1t+ \phi_2 \quad k_2 = 2\pi n_2/\lambda$

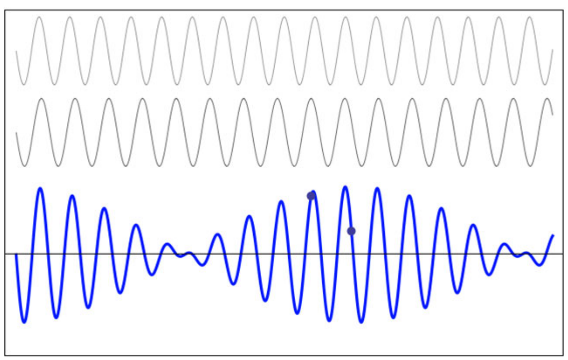
Light phenomena is just a superposition of waves with different wave lengths, phases, or indices etc. (ambient light)

Beats (example)

The interferogram shows beats. They are represented in figure by two <u>**slightly different wavelengths**</u>.



Coherent length ~ beat length



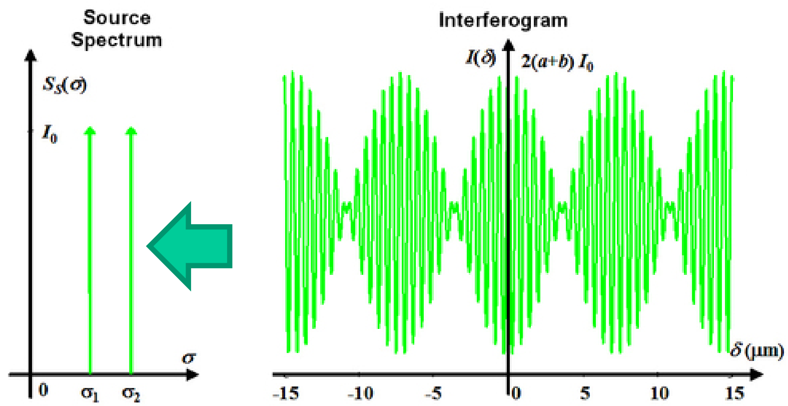
$$\sin A + \sin B = 2\sin(A+B)/2 \; * \underline{\cos(A-B)/2}$$

Let $A = k_1 x + \omega_1 t \qquad k_1 = 2\pi n_1/\lambda_1$

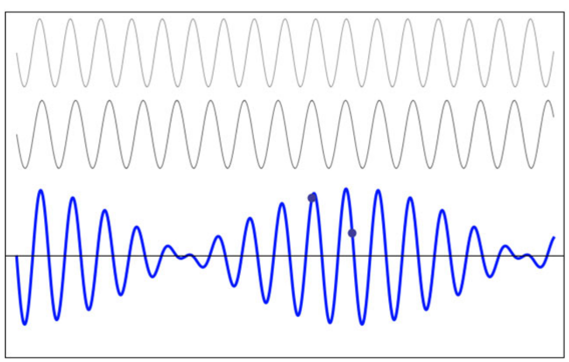
$B = k_2 x + \omega_2 t \qquad k_2 = 2\pi n_1/\lambda_2$

Beats (example)

The interferogram shows beats. They are represented in figure by two **slightly different indices**



Coherent length ~ beat length



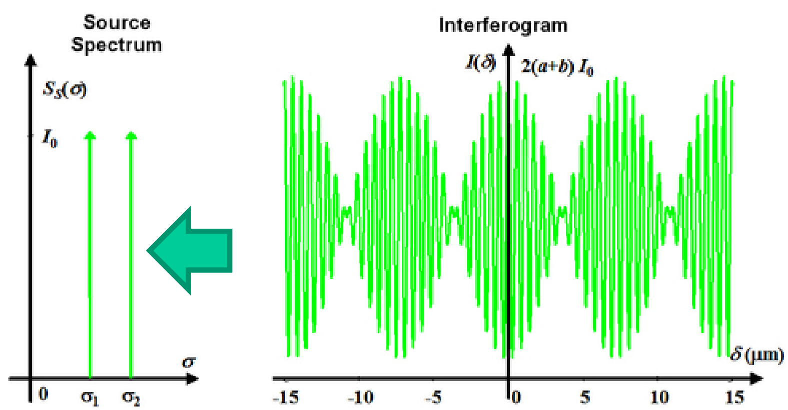
$$\sin A+\sin B = 2\sin(A+B)/2 * \cos(A-B)/2$$

Let $A = k_1x+\omega_1 t \quad k_1 = 2\pi n_1/\lambda_1$

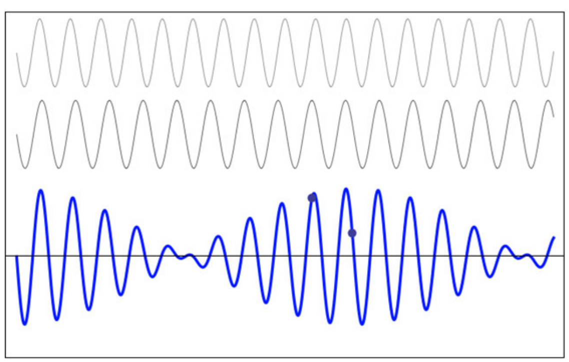
$B = k_2x+\omega_1 t \quad k_2 = 2\pi n_2/\lambda_1$

Beats (example)

The interferogram shows beats. They are represented in figure by two slightly different in **travel distance x.**



Coherent length ~ beat length



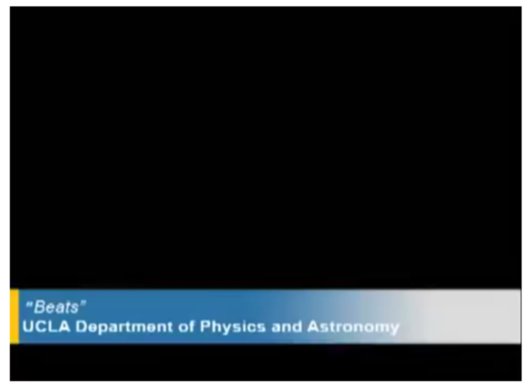
$$\sin A+\sin B = 2\sin(A+B)/2 * \underline{\cos(A-B)/2}$$

Let $A = k_1 x_1 + \omega_1 t \quad k_1 = 2\pi n_1/\lambda_1$

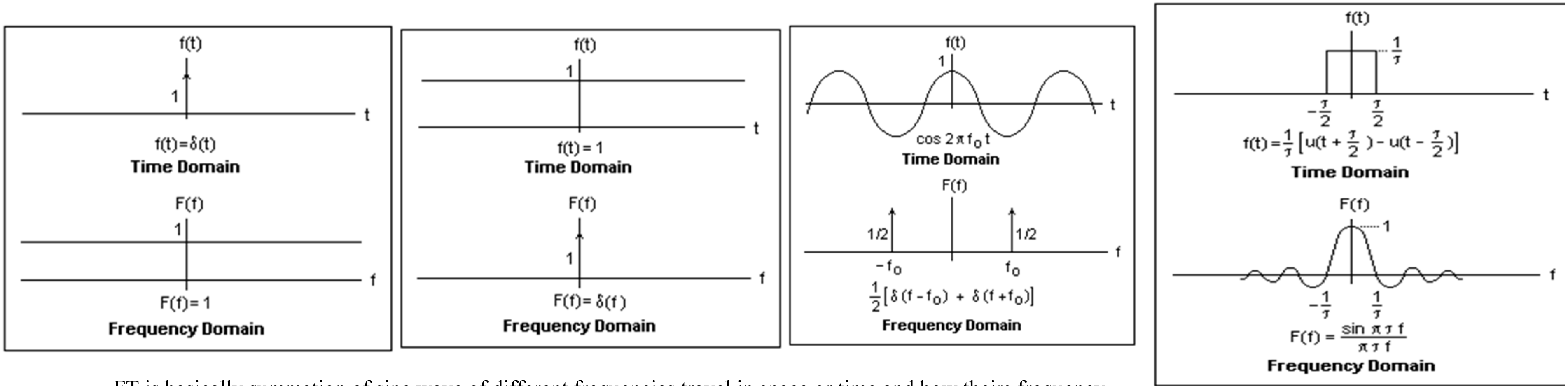
$B = k_1 x_2 + \omega_2 t \quad k_2 = 2\pi n_1/\lambda_1$

Beat Example Videos

When Choir member singing off key



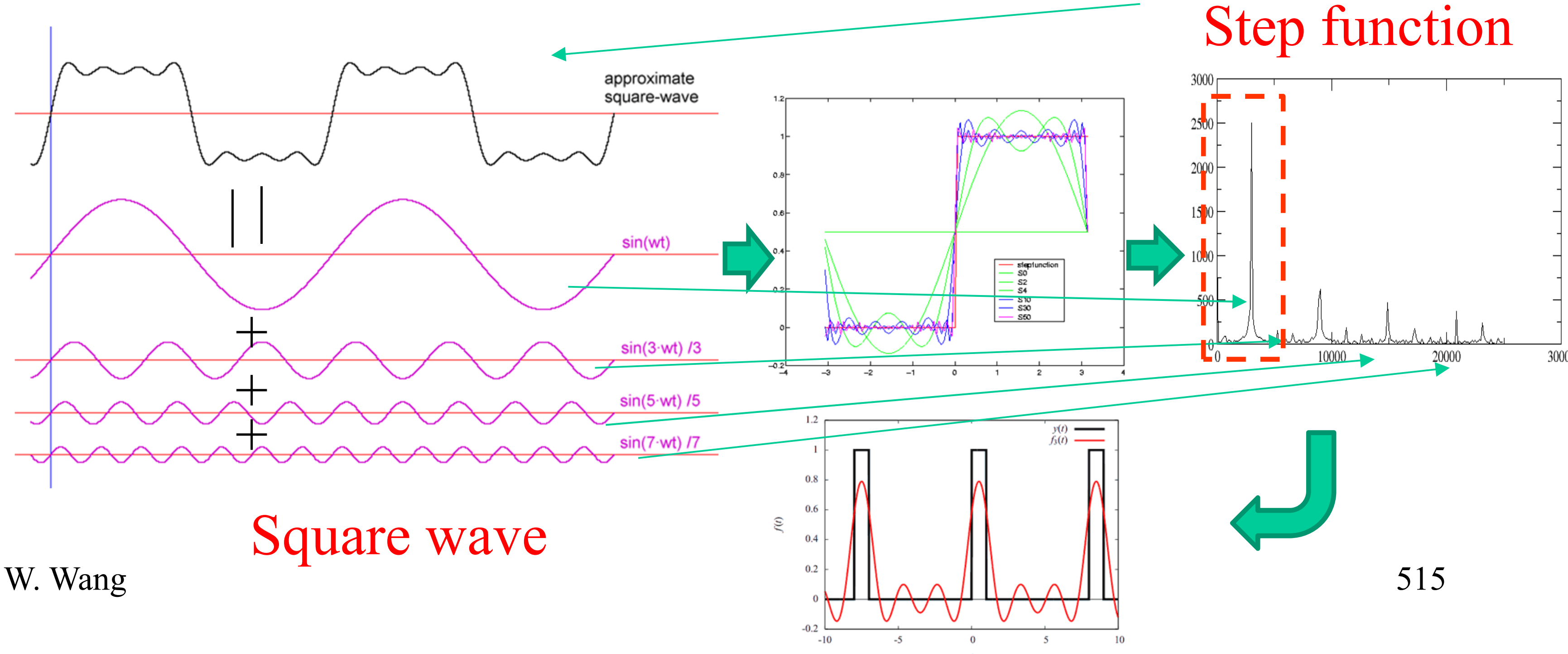
Fourier Transformation and Series



FT is basically summation of sine wave of different frequencies travel in space or time and how theirs frequency, phase and magnitudes difference creates signals in frequency or spatial domain.

Step function

Recall



Square wave

Fourier Transformation

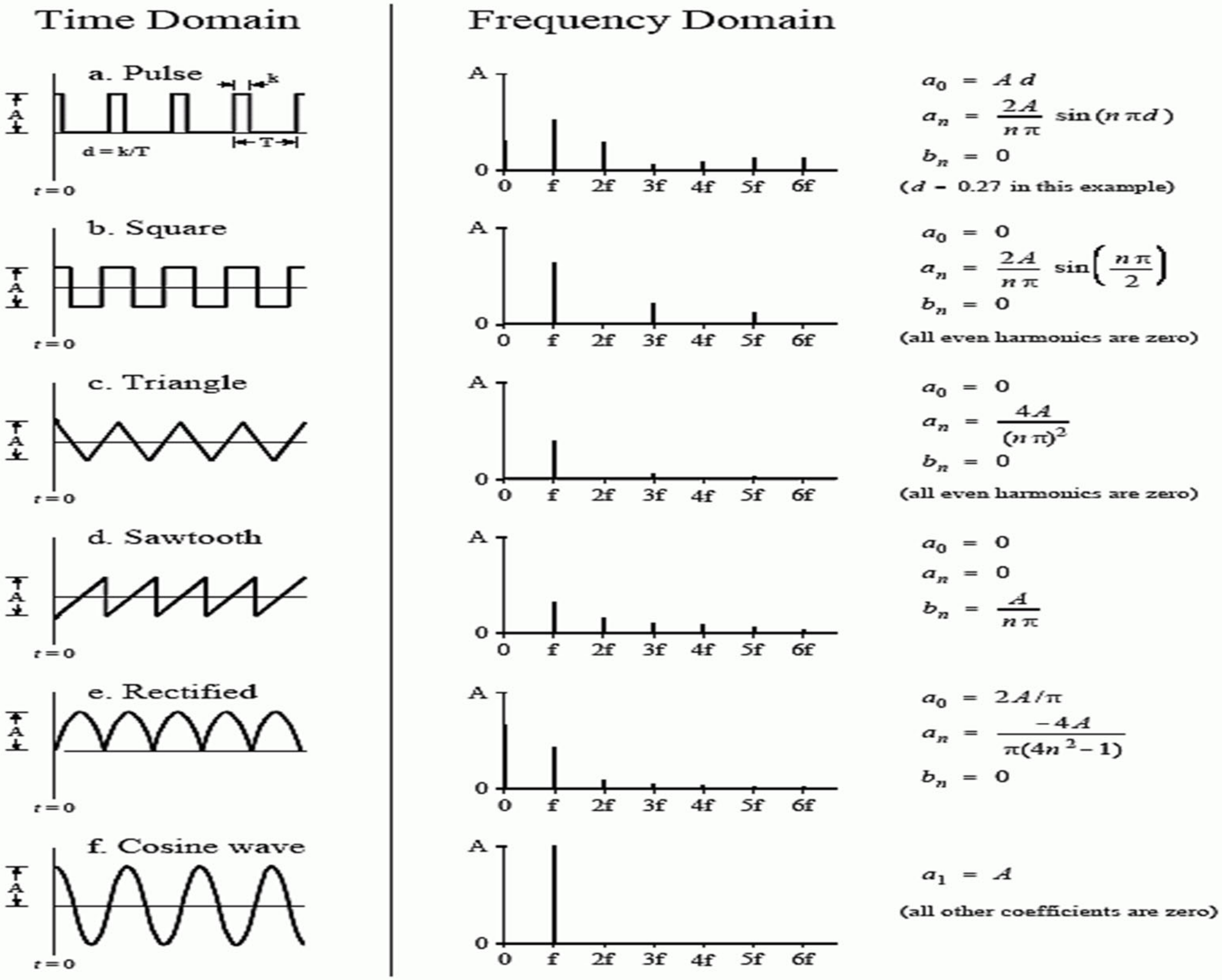
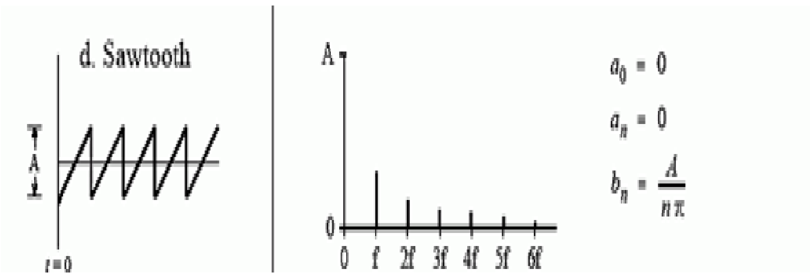
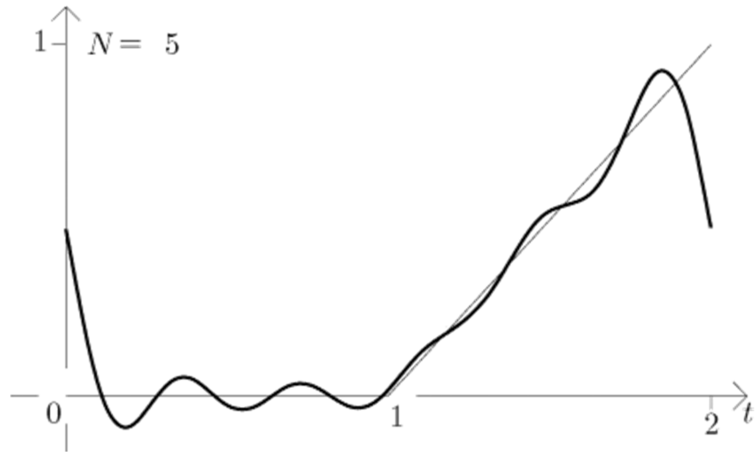


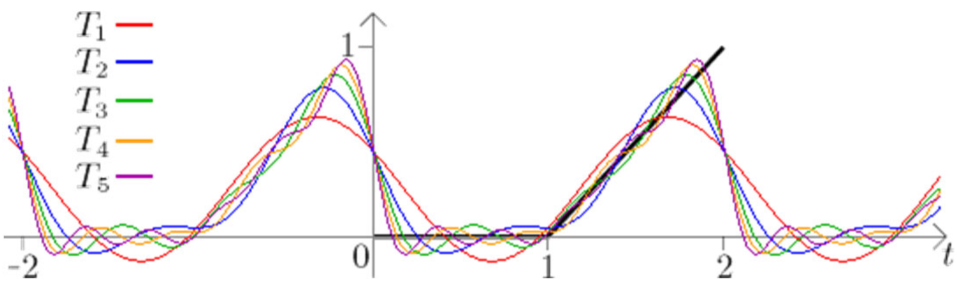
FIGURE 13-10
Examples of the Fourier series. Six common time domain waveforms are shown, along with the equations to calculate their "a" and "b" coefficients.

Fourier Transformation and Series



Numbers of expansion

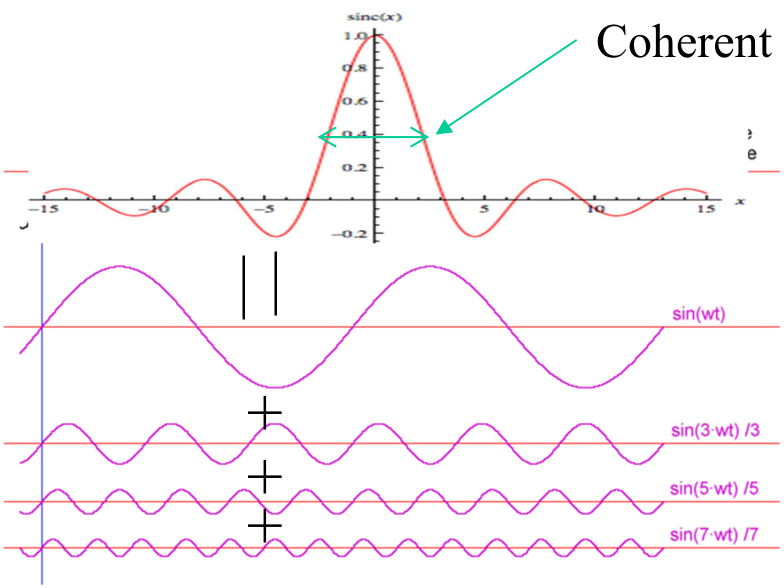


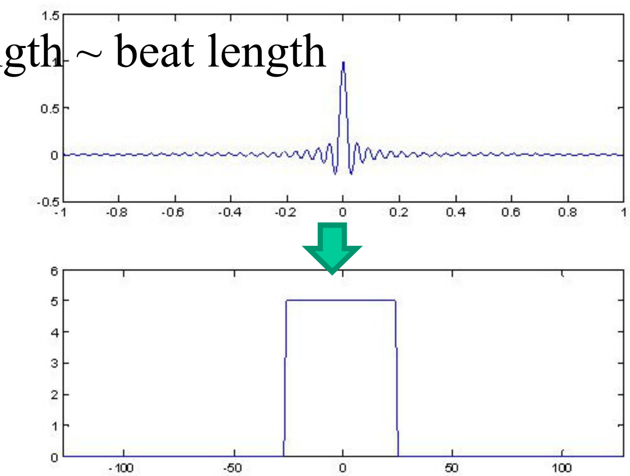


Broadband light

- Ambient light we see is a broadband light interference
- We see interference everywhere

Coherent length ~ beat length

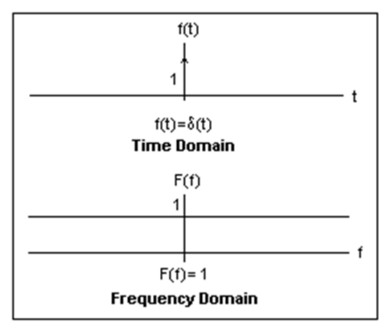




Sinc function

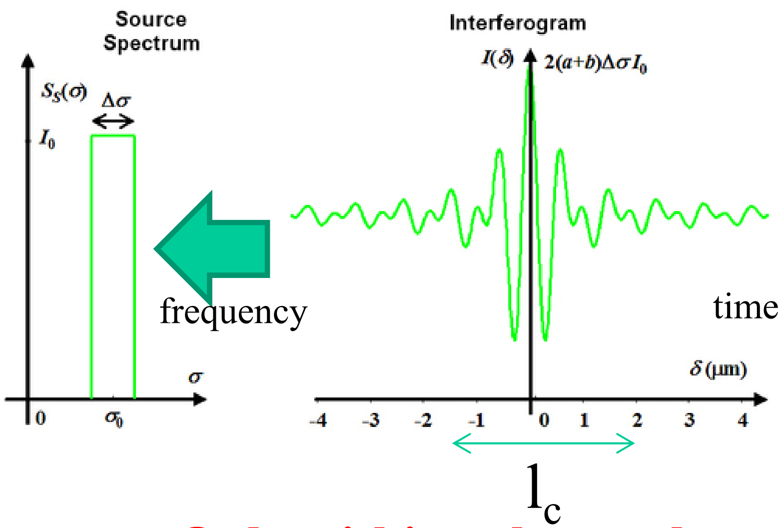
Square wave

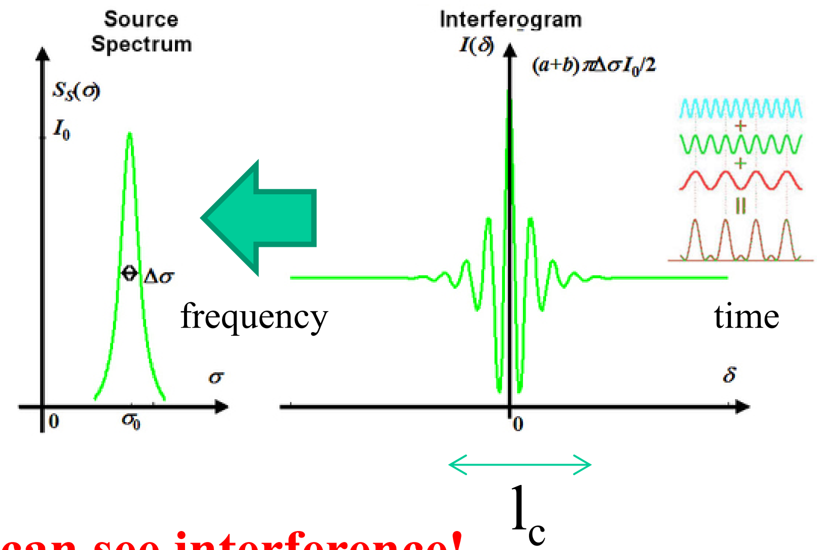
Ambient light is a broadband interference (impulse response)



Coherence and Beats (example)

If there are **more than two wavelength in case of a broadband light source**, then the sum Of all wavelengths in time domain is shown as

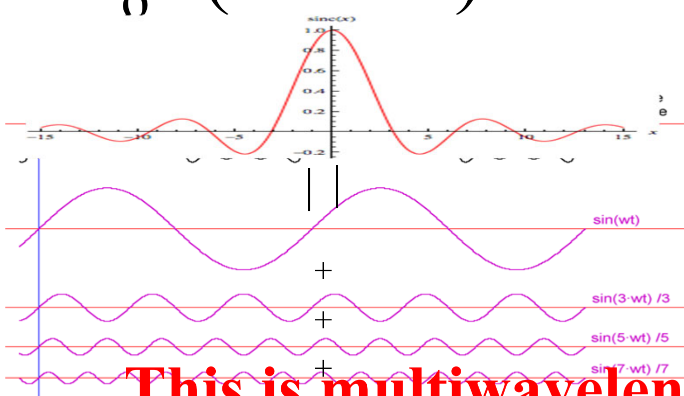


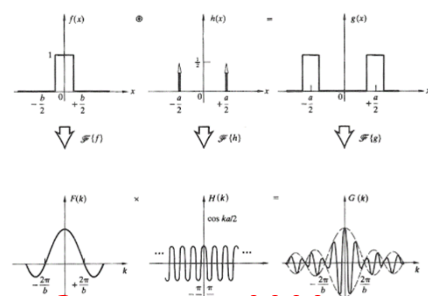


Only within coherent length you can see interference!

$$l_c = \lambda_o^2/(2\pi n\Delta\lambda) \qquad l_c = (2\ln 2/\pi)(\lambda_o^2/(2\pi n\Delta\lambda))$$

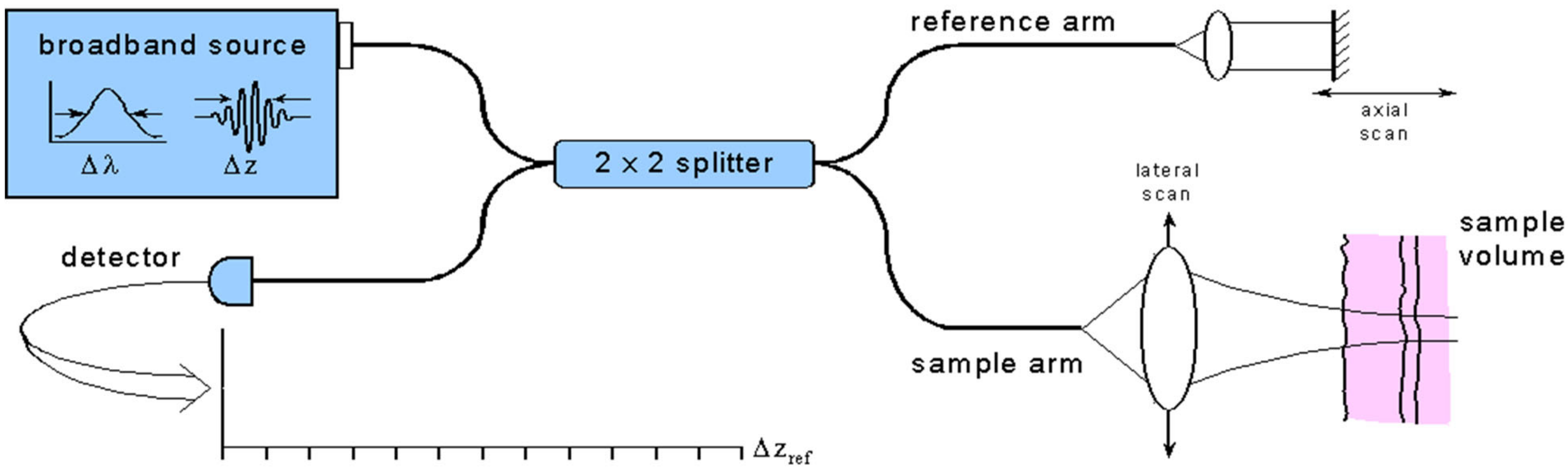
Recall



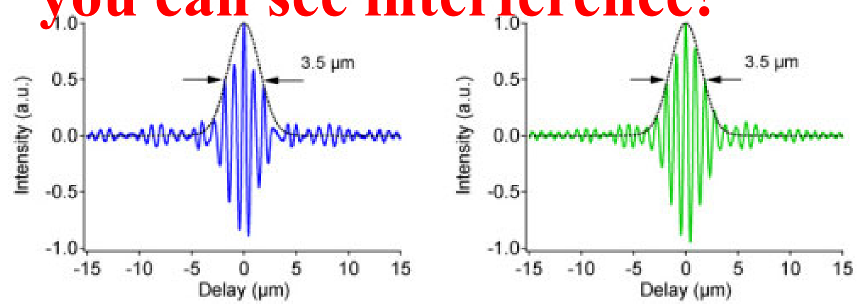


This is multiwavelength interference!!!!

Optical Coherence Tomography (example of low coherent light source)



Only within coherent length you can see interference!



Different coherent lengths with different wavelengths

Resolution proportional to coherent length

Trigonometric Functions in Terms of Exponential Functions

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i} \qquad \csc x = \frac{2i}{e^{ix} - e^{-ix}}$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2} \qquad \sec x = \frac{2}{e^{ix} + e^{-ix}}$$

$$\tan x = \frac{e^{ix} - e^{-ix}}{i\left(e^{ix} + e^{-ix}\right)} \qquad \cot x = \frac{i\left(e^{ix} + e^{-ix}\right)}{e^{ix} - e^{-ix}}$$

Exponential Function vs. Trigonometric and Hyperbolic Functions

$$\boxed{e^{ix} = \cos x + i \sin x} \qquad e^{x} = \cosh x + \sinh x$$

Hyperbolic Functions in Terms of Exponential Functions

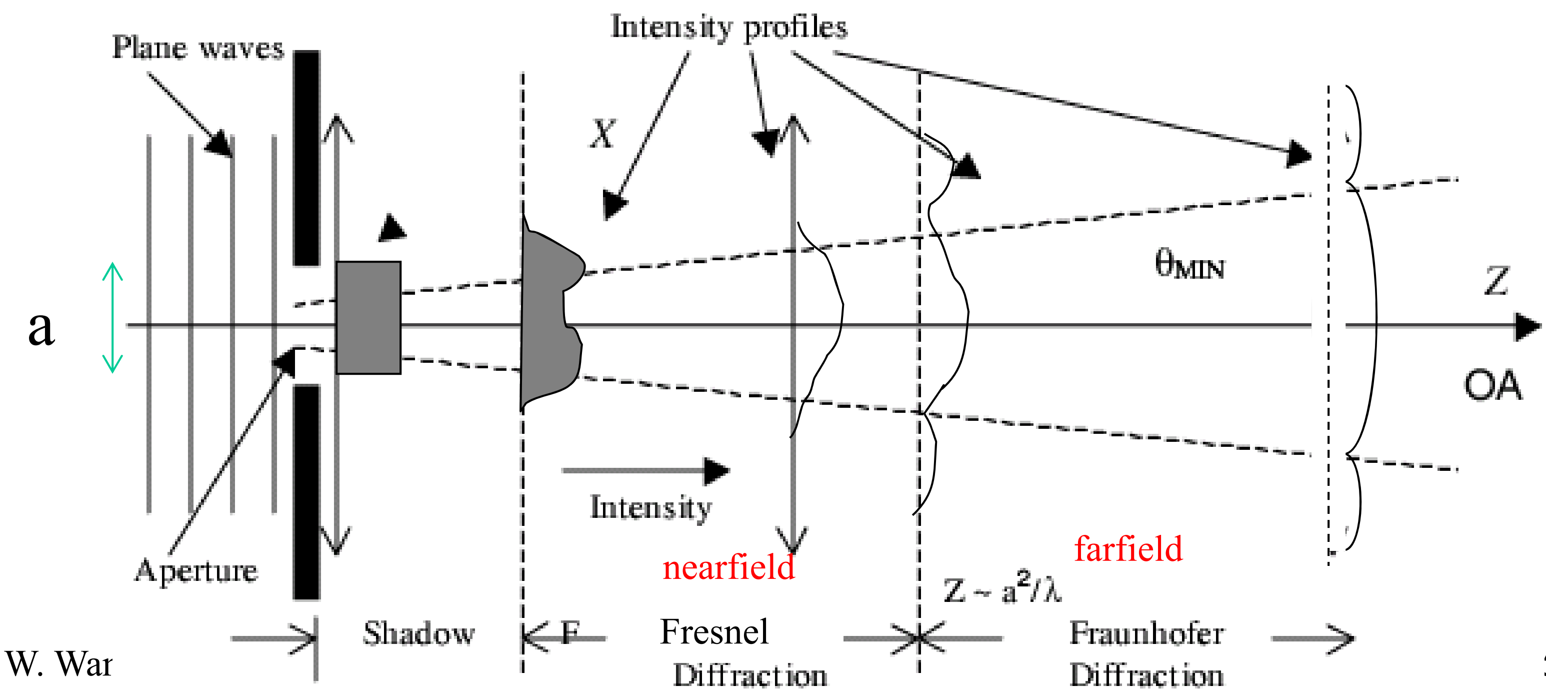
$$\sinh x = \frac{e^{x} - e^{-x}}{2} \qquad \operatorname{csch} x = \frac{2}{e^{x} - e^{-x}}$$

$$\cosh x = \frac{e^{x} + e^{-x}}{2} \qquad \operatorname{sech} x = \frac{2}{e^{x} + e^{-x}}$$

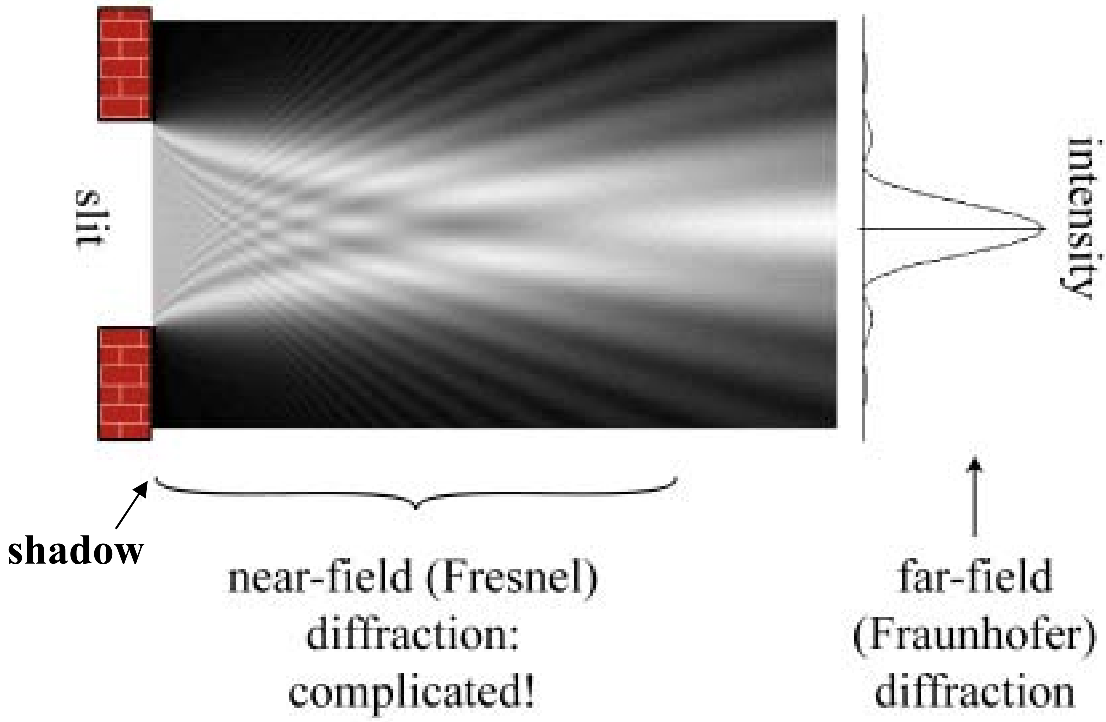
$$\tanh x = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}} \qquad \coth x = \frac{e^{x} + e^{-x}}{e^{x} - e^{-x}}$$

Diffraction

Diffraction refers to the spread of waves and appearance of fringes that occur when a wave front is constricted by an aperture in a a screen that is otherwise opaque. The light pattern changes as you move away from the aperture, being characterized by three regions



The intensity pattern behind a narrow single slit under uniform monochromatic illumination looks something like this:



Simulation if you have questions, you can ask Chileung or Karthick. Mainly the meshing size will be affected by wavelength you are simulating in.. It will take a long time or the solution won't converge so this is more of a numerical analysis problem.. There is a handbook you should get called "numerical recipe". You should start trying to read and see if you can learn stuff on your own. Also CST website has a lot of examples and also video instruction to deal with stuff like this.We will cover the concept of shadow region, near field and far field in the class, but the region between far field and near field is defined by the aperture of the struct relative to the wavelength it's operating... the threshold is when distance is greater than aperture diameter ^2/wavelengthAperture size is basically the size of the source.. Anyway, this is derived using some diffraction limit but not covered in the class since we don't cover it in the class. If you look at different paper, this limit also is different again depending on how the person use this limit for what application. it's function of the structure and wavelength. Our metamaterial, this approximation is based on where we stop seeing any interaction of waves from the structure interacting with the incident or transmitted wave.. this is more empirically derived since meta seems to do some funny stuff within its effective region... but quite interesting...Anyway, in class you will see that diffraction from a slit actually looks like interference in nearfield region instead of sinc wave.. this has to do with time and distance you integrate the function as well... anyway... will show you in class so don't worry. You will see everything can be explained by math and integration time and space.

Aperture size is basically = the size of the source.. Anyway, this is derived using some diffraction limit but not covered in this class. If you look at different papers, this limit also is different because it depends on the application. it's function of the structure and wavelength regardless. Anyway, in class you will see that diffraction from a slit actually looks like interference in the nearfield region instead of sinc wave in fairfield.. Mathematically, you can actually see this by the limit you set on the integral and the solution will be different.

You can see this by the fourier series expansion as well.

How far away before far field?

- a^2/λ = (3microns)^2/0.632.8microns = 14.8microns (~20x wavelengths)

- Depending on the aperture, larger it is, the further away it needs to be
- For 1mm aperture 1580mm or 1.58m

most use $\sim 2a^2/\lambda$

Derive an quantities expression for the diffraction can be done using

* Kirchhoff Fresnel– Derivation of diffraction from wave equation

* Fourier Optics (slit = square wave TF, Lens = sin TF ,etc.)

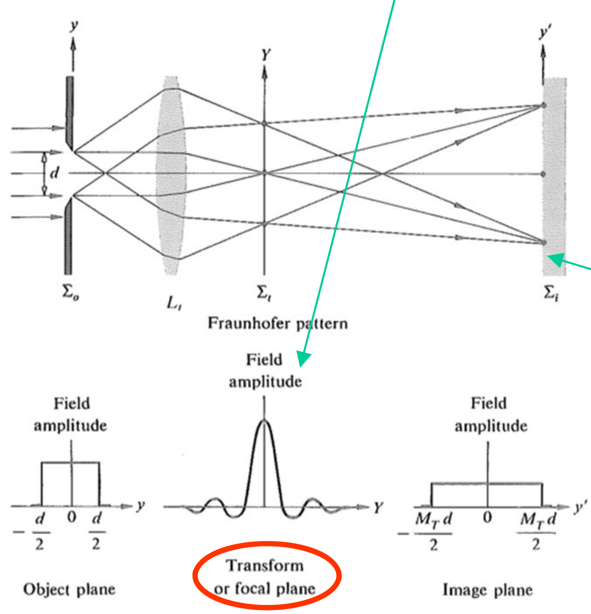
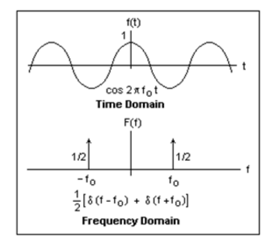
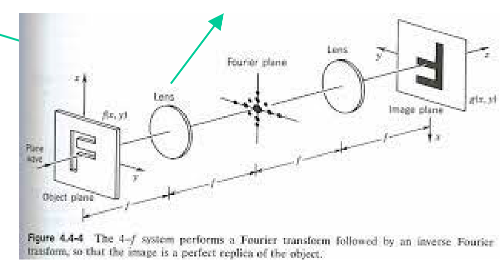
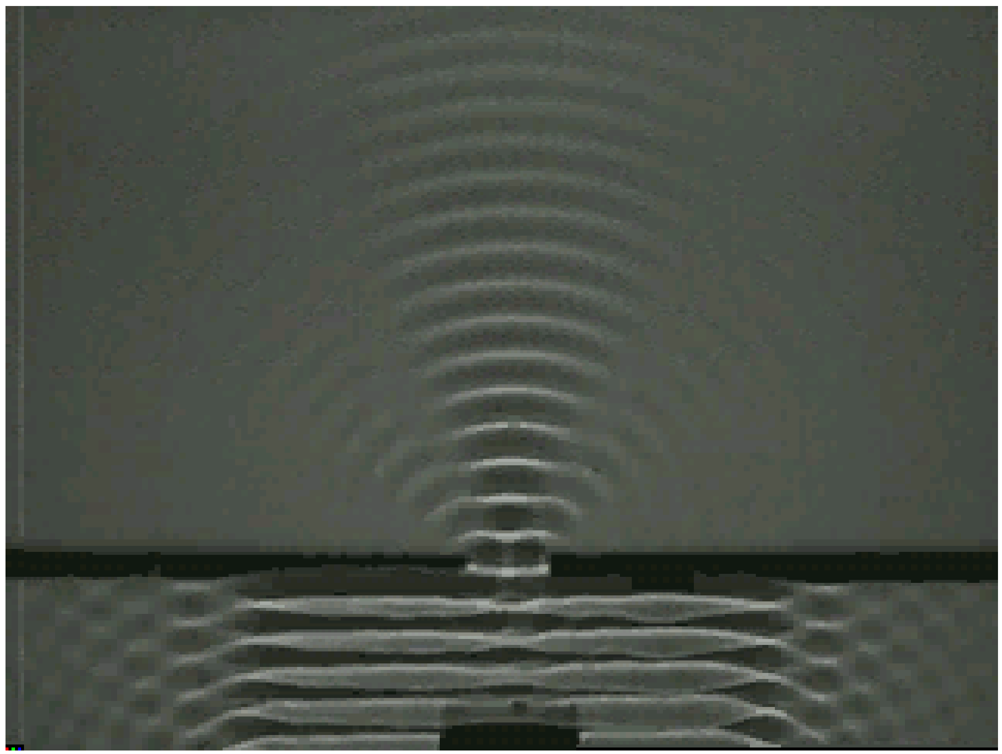


Figure 14.7 The image of a slit.





Single Slit Ripple Tank Experiment (Diffraction)



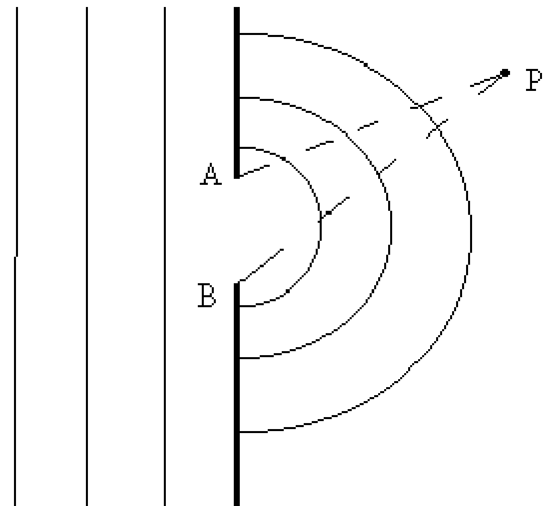
http://www.phy.davidson.edu/introlabs/labs220-230/html/lab10diffract.htm

Diffraction

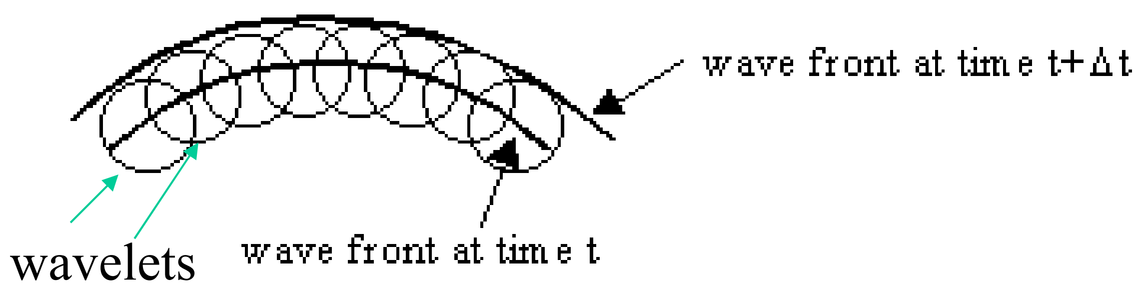
- Diffraction occurs when light passing through an opening or edge that has a *different index from the adjacent obstruction*. Diffraction effects increase as the physical dimension of the aperture approaches the wavelength of the radiation. Diffraction of radiation results in interference that produces dark and bright rings, lines, or spots, depending on the geometry of the object causing the diffraction.

HuygensFresnel principle

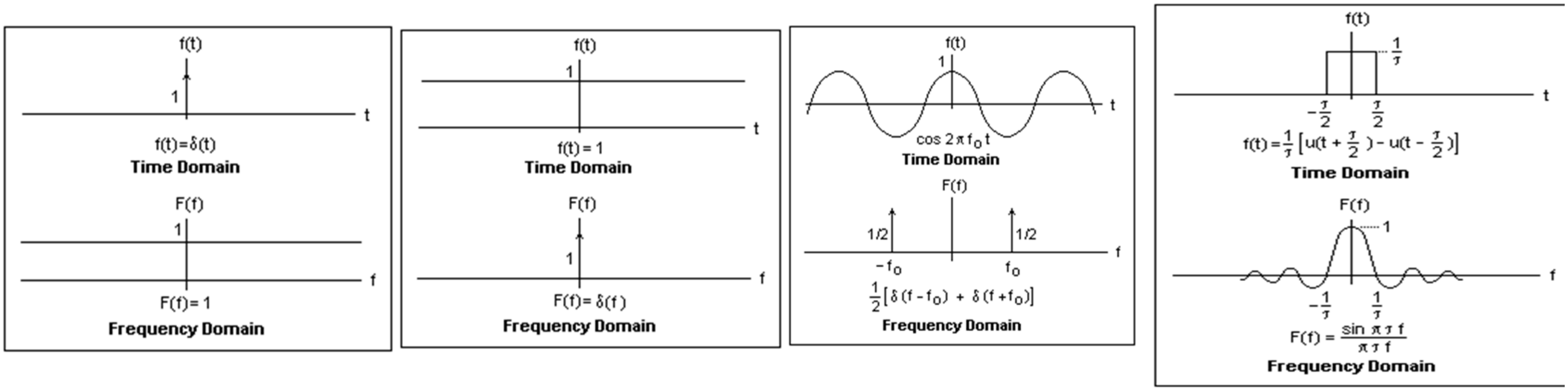
Huygens Fresnel principle, states that **<u>every unobstructed point of a wavefront, at a given instant in time, serves as a source of spherical secondary wavelets, with the same frequency as that of the primary wave. The amplitude of the optical field at any point beyond is the superposition of all these wavelets, taking into consideration their amplitudes and relative phases.</u>**



Treat each as spherical source and see how they interfere with each other at different point in space and time

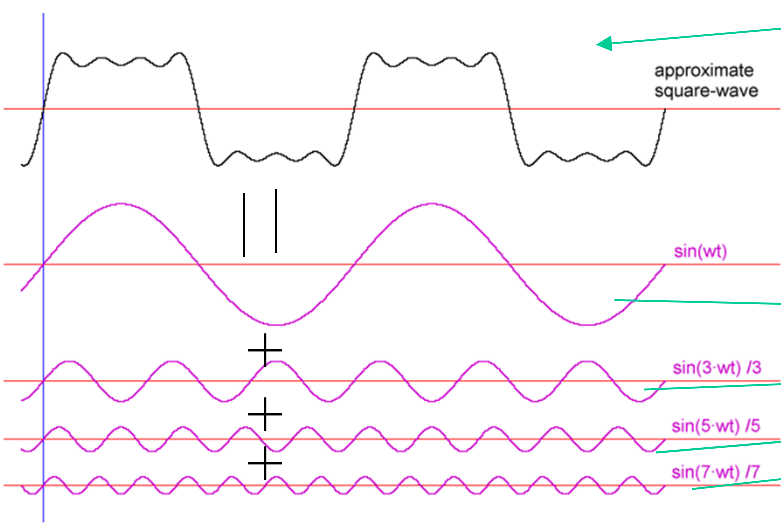


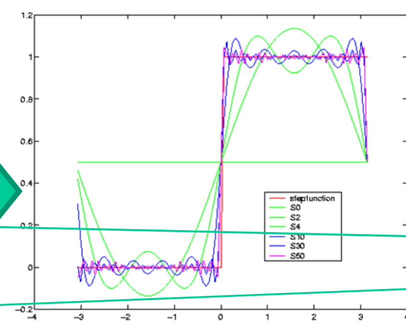
Fourier Transformation and Series

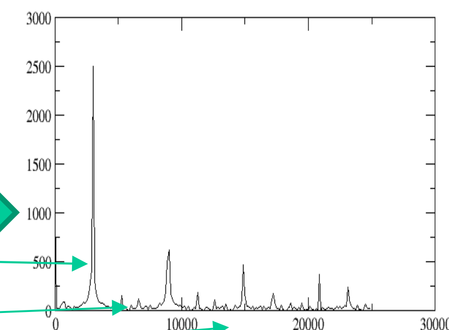


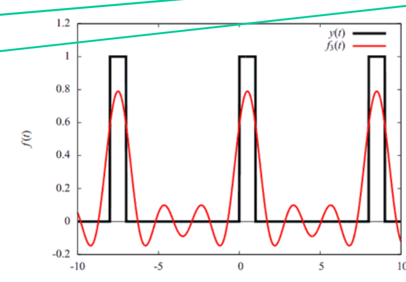
Step function

Recall









Square wave

HuygensFresnel principle

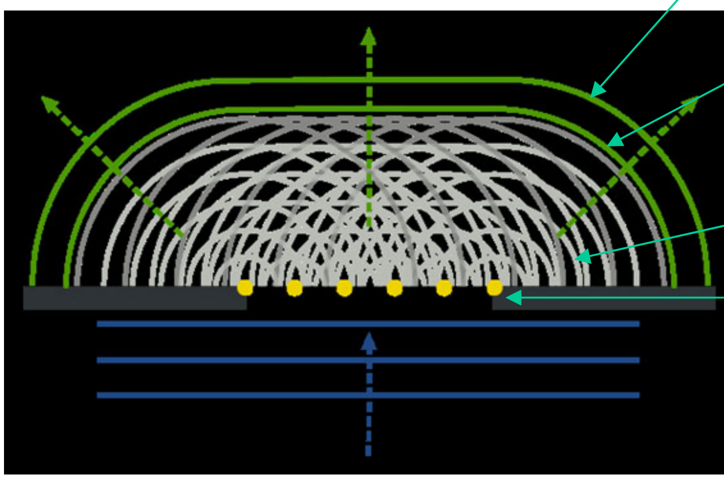
Plane wave at aperture can be re-represented by <u>a series of spherical wave</u> operating at the **same amplitude and frequency** as original plane wave

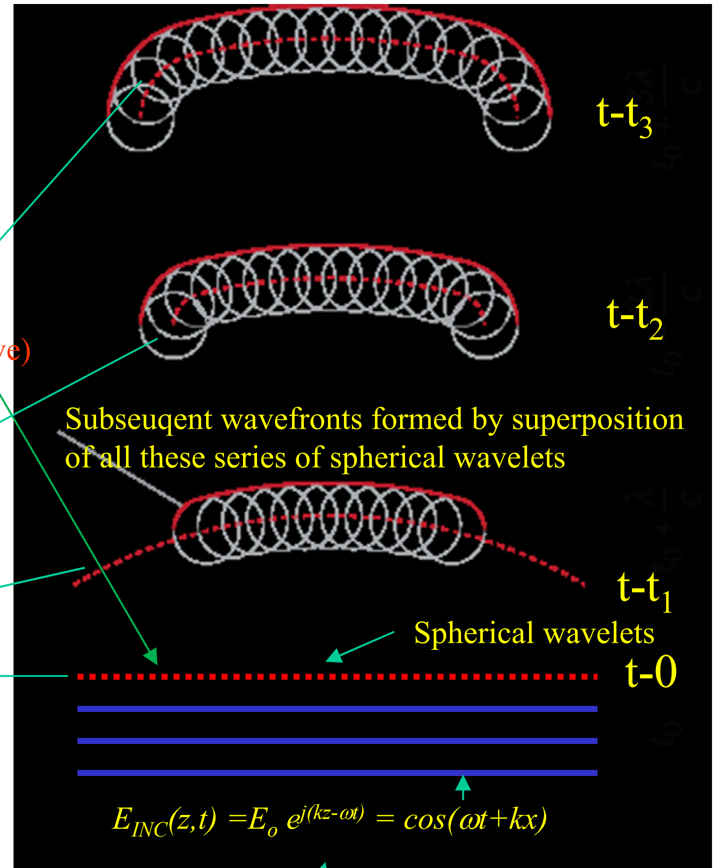
(Source strength)

$$E(P) = \iint\limits_{aperture\ \ area} \left[\frac{E_o e^{-j\omega t}}{\lambda} dA' \right] \times \left[\frac{e^{jkr}}{r} \right]$$

(Source strength) x (Huygens spherical wave)

(point source, Green's function,spherical wave)





Plane wave equation

Trigonometric Functions in Terms of Exponential Functions

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i} \qquad \csc x = \frac{2i}{e^{ix} - e^{-ix}}$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2} \qquad \sec x = \frac{2}{e^{ix} + e^{-ix}}$$

$$\tan x = \frac{e^{ix} - e^{-ix}}{i\left(e^{ix} + e^{-ix}\right)} \qquad \cot x = \frac{i\left(e^{ix} + e^{-ix}\right)}{e^{ix} - e^{-ix}}$$

Exponential Function vs. Trigonometric and Hyperbolic Functions

$$\boxed{e^{ix} = \cos x + i \sin x} \qquad e^{x} = \cosh x + \sinh x$$

Remember exponential term can be put in terms o trigonometric function

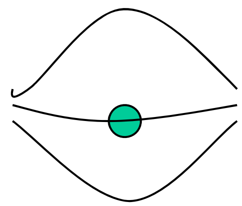
Hyperbolic Functions in Terms of Exponential Functions

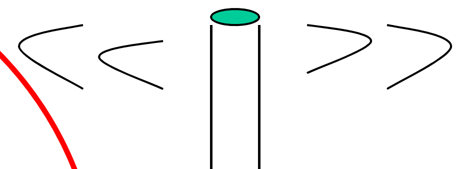
$$\sinh x = \frac{e^{x} - e^{-x}}{2} \qquad \operatorname{csch} x = \frac{2}{e^{x} - e^{-x}}$$

$$\cosh x = \frac{e^{x} + e^{-x}}{2} \qquad \operatorname{sech} x = \frac{2}{e^{x} + e^{-x}}$$

$$\tanh x = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}} \qquad \coth x = \frac{e^{x} + e^{-x}}{e^{x} - e^{-x}}$$

Plane, Spherical and Cylindrical Wave



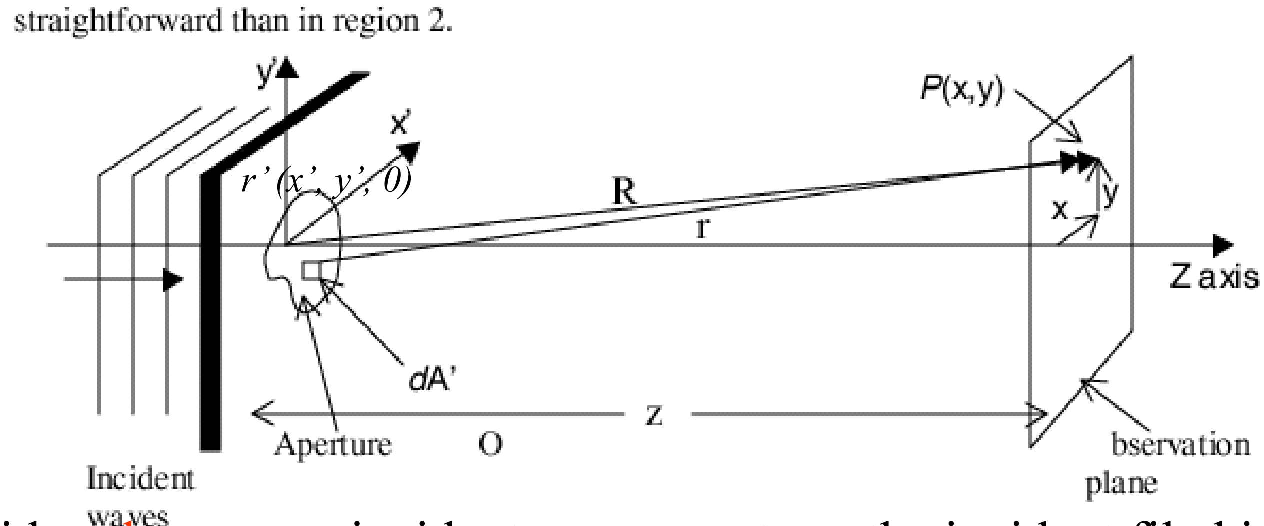


$$\psi = \psi_0 \cos(\omega t - \mathbf{k} \cdot \mathbf{r} + \phi),$$

$$2Bl = \mu_0 K l,$$

$$\psi(r,t) = \frac{\psi_0}{r} \cos(\omega t - kr + \phi).$$

Using Kirchoff-Fresnel Diffraction Integral, we can derive an quantitative expression for the irradiating field of a finite aperture.



Consider plane wave incident on an aperture, the incident filed is described as

$$E_{INC}(z,t) = E_o\, e^{j(kz-\omega t)} = \cos(\omega t + kx)$$

At $z = 0$ => $E_{INC}(z,t) = E_o\, e^{j(-\omega t)}$. A typical element of the wave front of the area dA' and at position $r\ (x, y, 0)$ then act as a source of Huygens wavelets. Assume we are interested in detecting light at point P, the distance from element dA' to P is given by

$$r = |R - r'|$$

The field at P due to the element dA' is then equal to

$$dE(P) = \left[\frac{E_o e^{-j\omega t}}{\lambda} dA' \right] \times \left[\frac{e^{jkr}}{r} \right] \qquad e^{ix} \cong \cos x$$

= (Source strength) x (Huygens spherical wave)
(point source, Green's function)

The field at P due to the entire aperture is then a superposition of the wavelets from all elements areas,

$$E(P) = \iint_{aperture\ \ area} \left[\frac{E_o e^{-j\omega t}}{\lambda} dA' \right] \times \left[\frac{e^{jkr}}{r} \right]$$

Since the detector measures the light intensity at P, E field is convert to intensity using the time averaged Polynting vector

$$S = \frac{ExB}{2\mu_o} = \frac{|E|^2}{2Z_o} \hat{z} \quad \Rightarrow \quad I(P) = \frac{|E|^2}{2Z_o}$$

where z_o = air impedance

$$= \sqrt{\frac{\mu_o}{\varepsilon_o}}$$

Usually Fraunhofer condtion applied when **z>> a²/ λ**. The parallel rays is adequately assume at a distance of **z ~ 10 a²/ λ**

W. Wang

Single Slit Diffraction Intensity

Under the Fraunhofer conditions, the wave arrives at the single slit as a plane wave. Divided into segments, each of which can be regarded as a point source, the amplitudes of the segments will have a constant phase displacement from each other, and will form segments of a circular arc when added as vectors. The resulting relative intensity will depend upon the total phase displacement according to the relationship:

$$I = I_o \frac{\sin^2(\frac{\delta}{2})}{(\frac{\delta}{2})^2}$$

Where total phase angle

Relate to derivation of θ

$$\delta = \frac{2\pi a \sin\theta}{\lambda}$$

Intensity as a function of θ

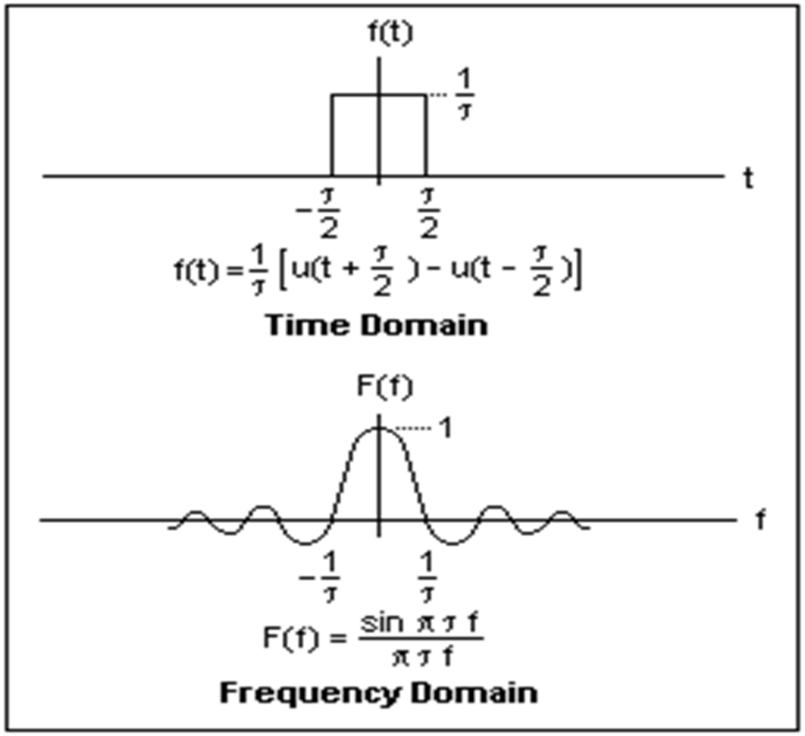
$$I = I_o \frac{\sin^2(\frac{\pi a \sin\theta}{\lambda})}{(\frac{\pi a \sin\theta}{\lambda})^2}$$

Intensity as a function of y

$$I = I_o \frac{\sin^2(\frac{\pi a y}{\lambda D})}{(\frac{\pi a y}{\lambda D})^2}$$

Sinc function

Spatial Transformation



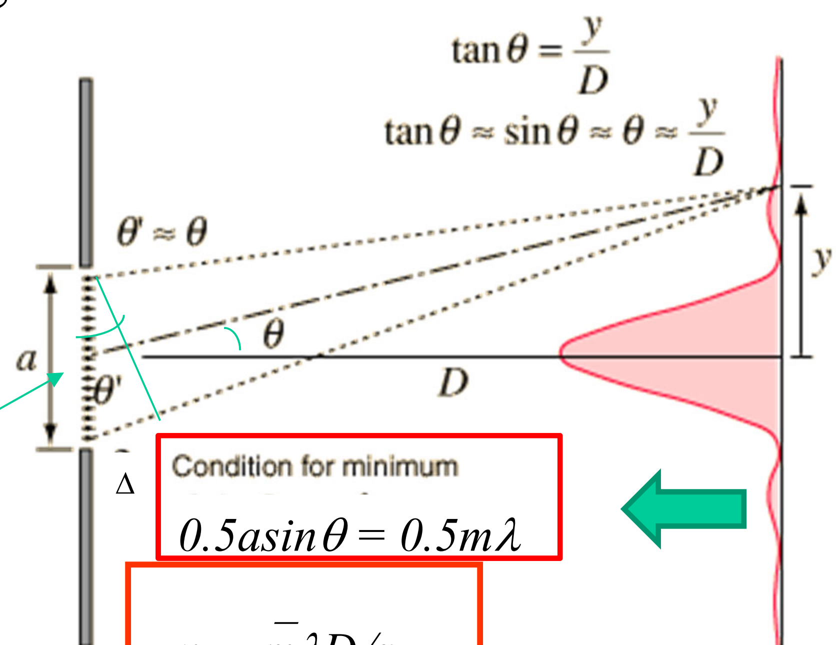
Slit ~ Step function

Far field interference
~ Sinc function

Quicker way to find the peaks (look at the phase only)

Fraunhofer diffraction

Single slit

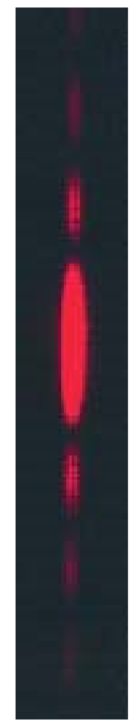


Condition for minimum

$$\boxed{0.5a\sin\theta = 0.5m\lambda}$$

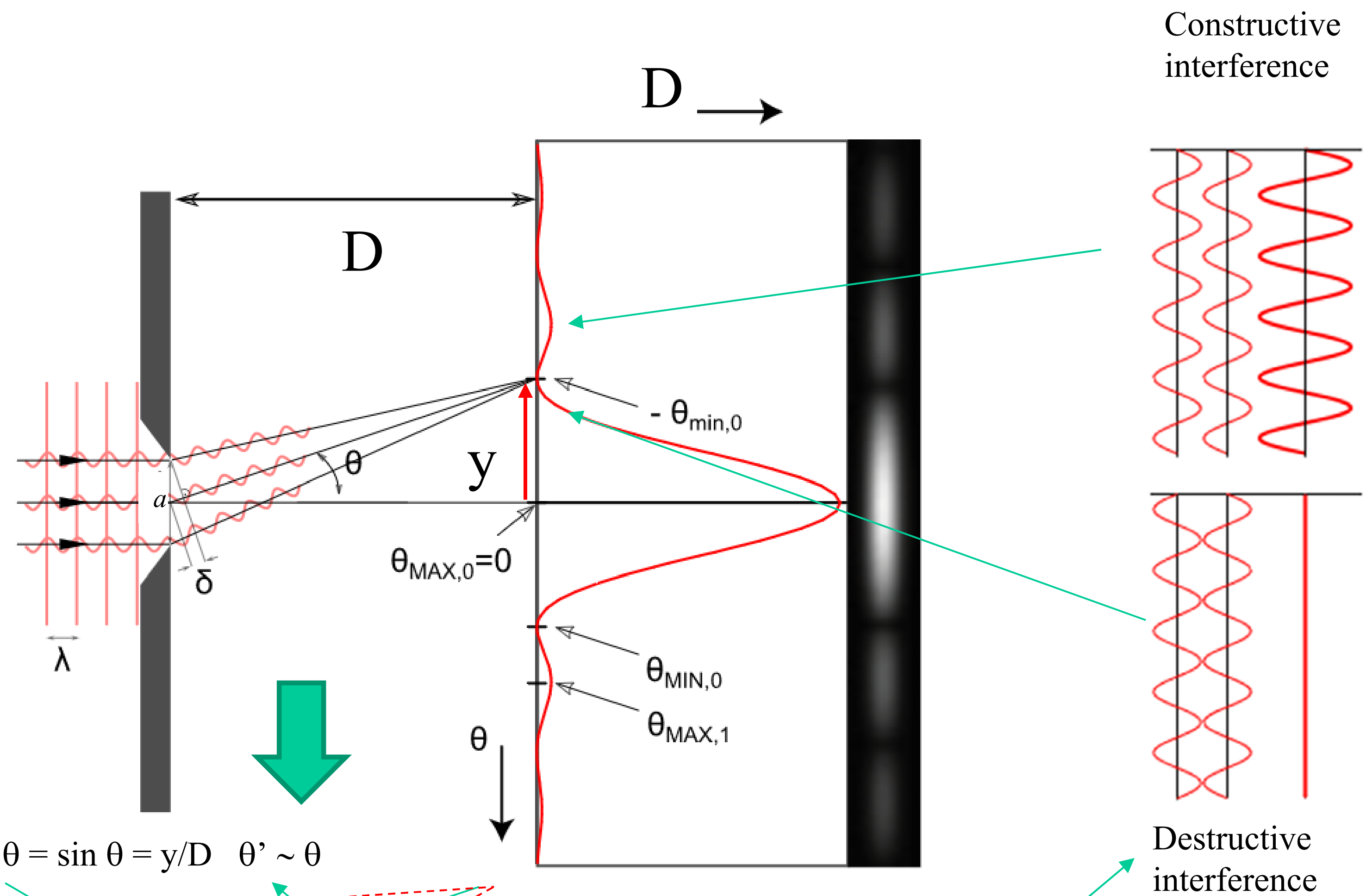
$$\boxed{y \sim \bar{m}\lambda D/a}$$

All phases
From each
Wavelet is
Complimentary to the other side of the slit

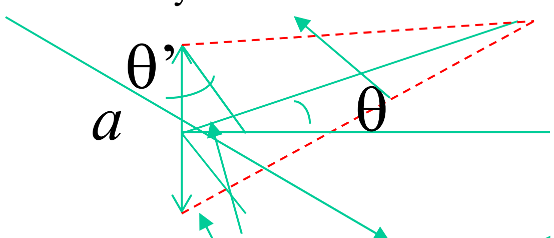


hyperphysics

The diffraction pattern at the right is taken with a helium-neon laser and a narrow single slit. To obtain the expression for the displacement y above, the small angle approximation was used.



$\tan\theta = \sin\theta = y/D \quad \theta' \sim \theta$



Every ray on the upper half of the slit (a) will be cancel with a ray on the bottom half of the slit at min.

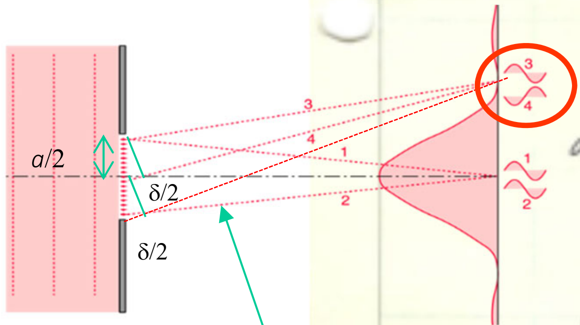
$$\delta = a/2 \sin\theta$$

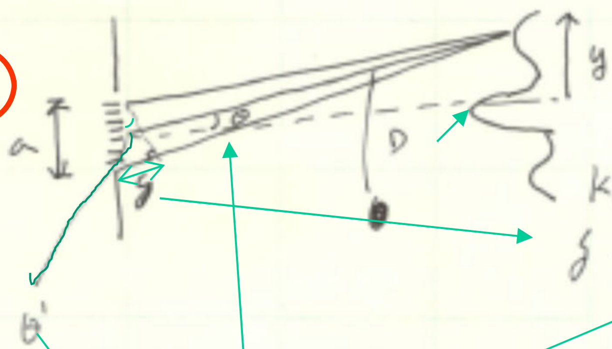
$$\kappa\delta = \kappa a/2 \sin\theta = m\pi \Rightarrow \delta = m\lambda$$

$$y = m\lambda D/a$$

W. Wang

Handout derivation
Using phase function





$kz = \phi$

$2\pi n/\lambda$, n=1 (air)

mth minimum
(diffraction order) $\boxed{k\delta/2 = m\pi}$

Condition for 2π phase shift

$k\delta = 2\pi \cdot m \Rightarrow \frac{2\pi}{\lambda}\delta = 2\pi m$

$\delta = a\sin\theta' = m\lambda$

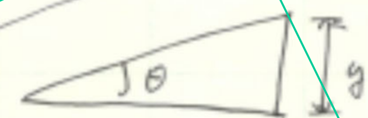
$a\theta = m\lambda$

$a\frac{y}{D} = m\lambda$

$\boxed{y = \frac{m\lambda D}{a}}$

Condition: $D >> a$

$\boxed{\theta' \simeq \theta}$

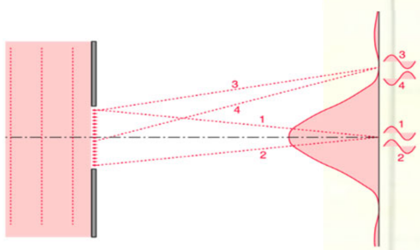


$\tan\theta = \frac{y}{D}$

D

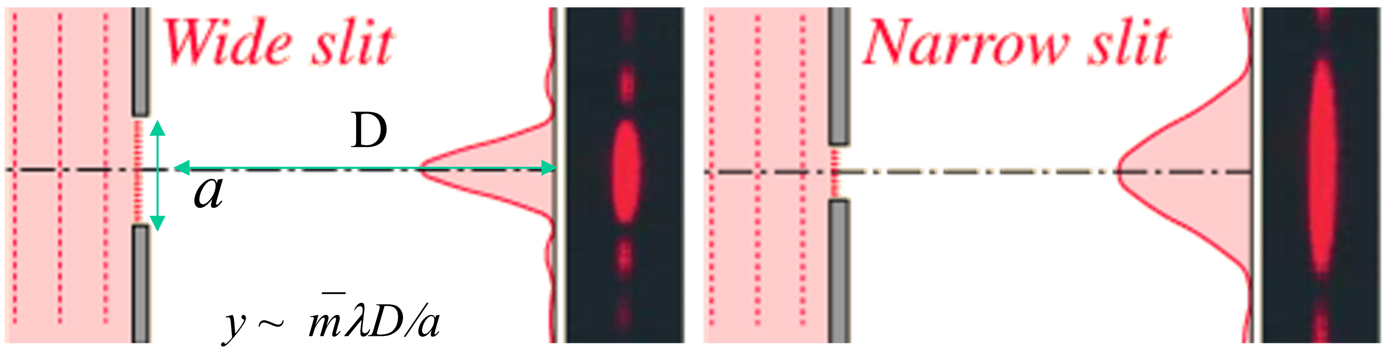
$\tan\theta \simeq \sin\theta \simeq \theta \simeq y/D$

Assumption of infinite source distance gives plane wave at slit so that all amplitude element are in phase



…ery Ray from the upper half of slit will canceled by a ray from lower half, originating at a pt. a/2 below first ray. The minimum intensity

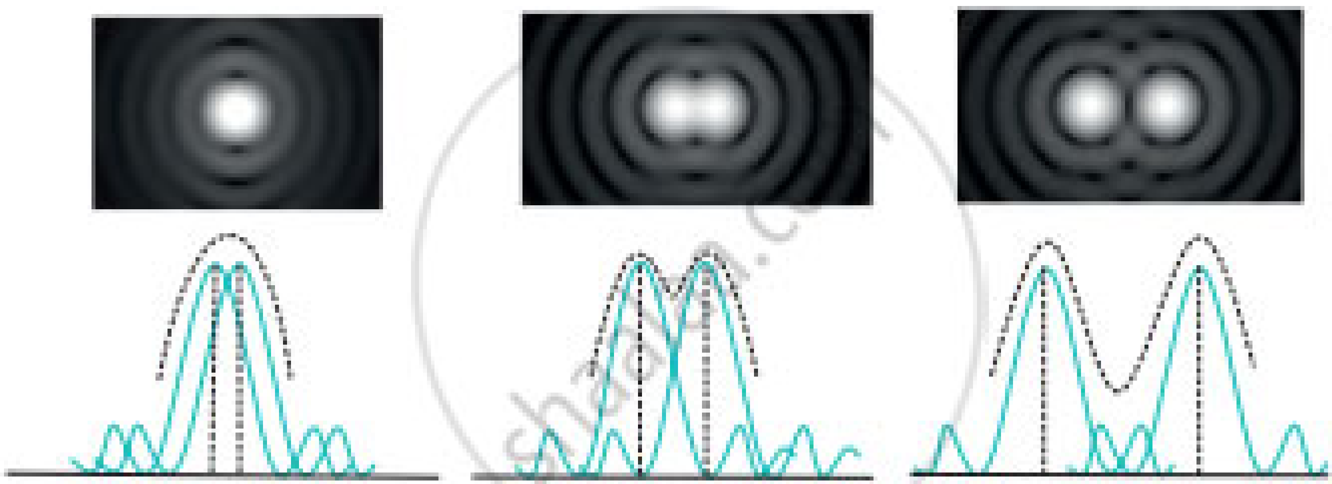
Example of Fraunhofer Diffraction



hyperphysics

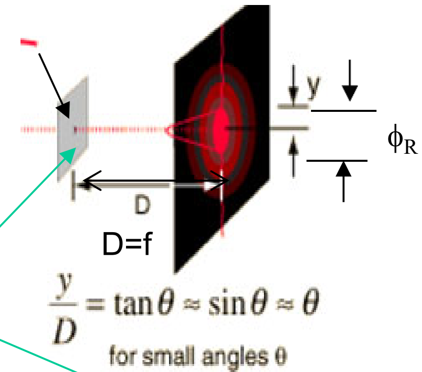
The diffraction patterns were taken with a helium-neon laser and a narrow single slit. The slit widths used were on the order of 100 micrometers, so their widths were 100 times the laser wavelength or more. A slit width equal to the wavelength of the laser light would spread the first minimum out to 90° so that no minima would be observed. The relationships between slit width and the minima and maxima of diffraction can be explored in the single slit calculation.

How to resolve two spot pattern



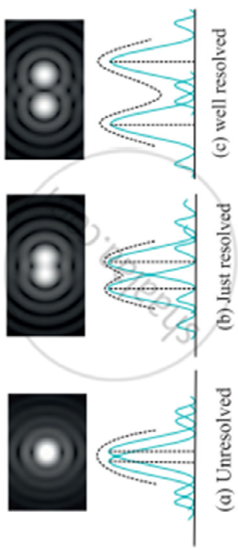
(a) Unresolved (b) Just resolved (c) well resolved

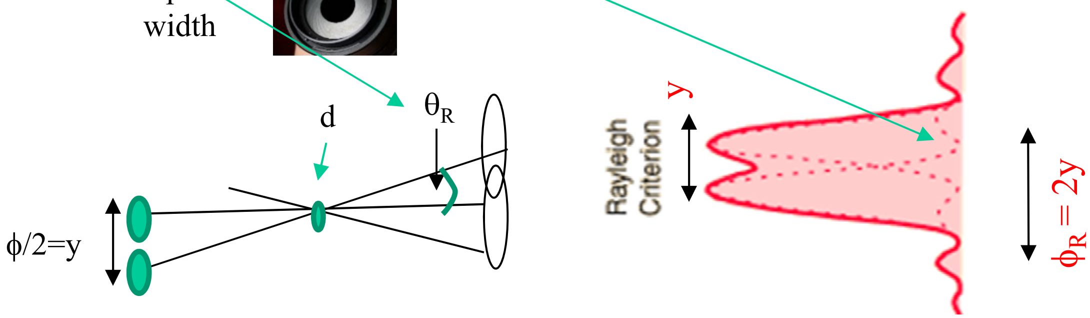
Aperture will diffract the objects, theta from diffraction is the same as the theta from the lens because diffraction observed at focal plan and also the first minimum happens to occurs at the same sin (theta). We can just use diffraction to calculate the spatial resolution!!

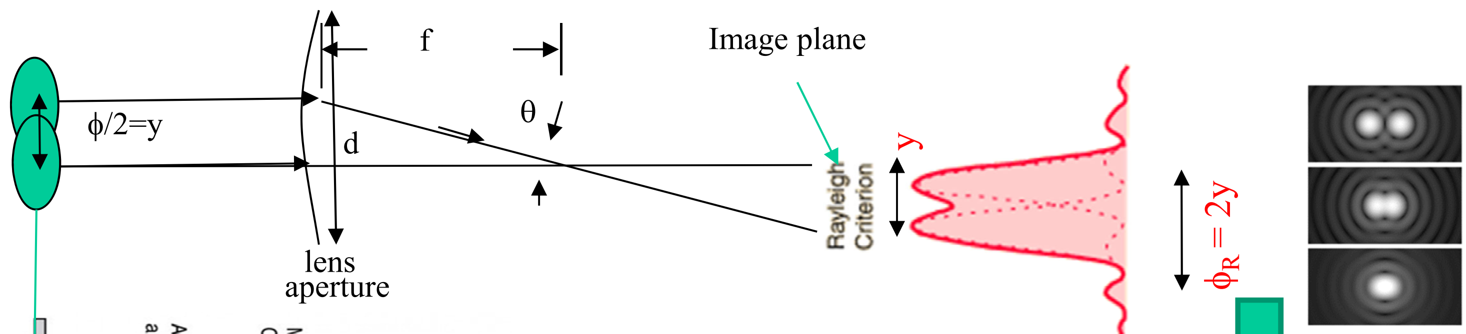


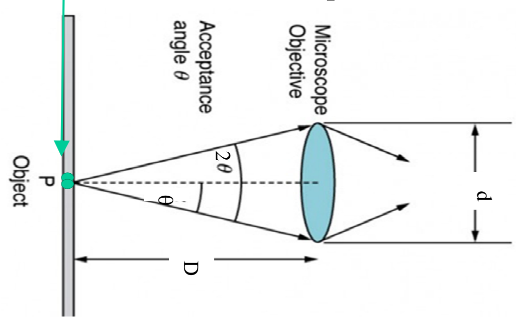
$y \sim \bar{m}\lambda D/a$

Aperture width









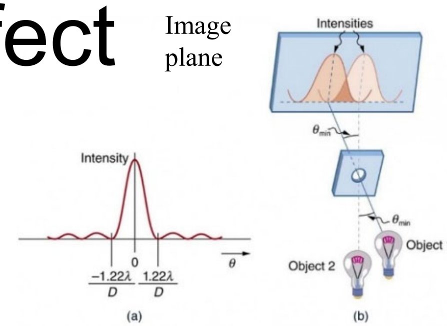
$y = 1.22\lambda\ f/d$

Bigger the (NA), smaller the y (higher the spatial resolution)

W. Wang

Spot size and spatial resolution

Example of Diffraction Effect



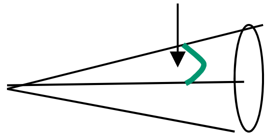
Rayleigh criterion- spatial resolution is limited by diffraction.

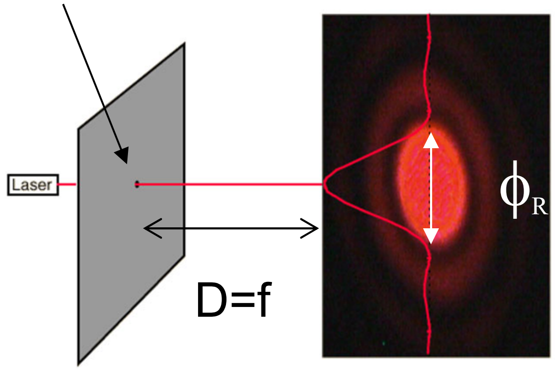
Assume two separate point sources can be resolved when the center of the airy disc from one overlaps the first dark ring in the diffraction pattern of second one

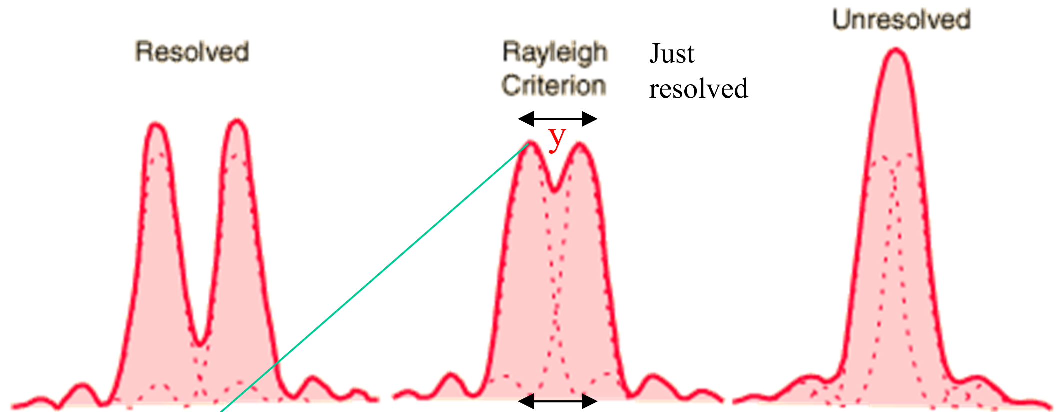
θ_R

Aperture diameter or slit width

d







Since this is the radius of the Airy disk, the resolution is better estimated by the diameter

θ_R (angular separation between two images)

NA= $\sin\theta_R = \frac{\lambda}{d}$

NA= $\sin\theta_R = 1.22\frac{\lambda}{d}$

Spatial resolution $\boxed{y = \lambda\ f/\ d}$

$\phi_R = 2\lambda\ f/\ d$

$\boxed{y = 1.22\lambda\ f/d}$

$\phi_R = 2.44\lambda\ f/d$

Single slit

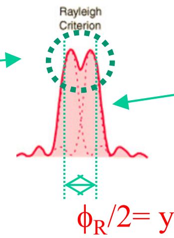
Circular aperture

y= 1.27λ f/d (Gaussian beam)

ϕ_R represents the smallest **spot size** that can be achieved by an optical system with a circular or slit aperture of a given f-number (diffraction-limit spot size)

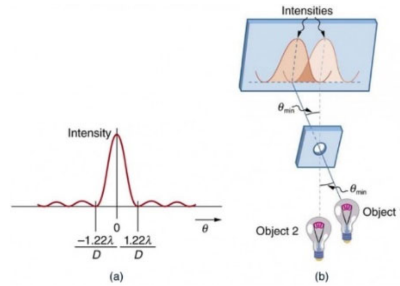
How to resolve two line or spot pattern

Our eye(s) or microscope lens can resolve

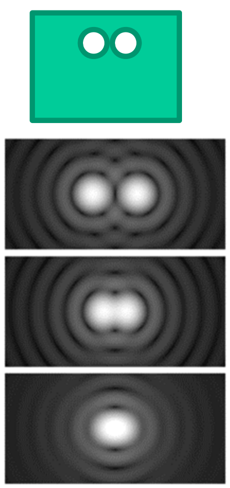


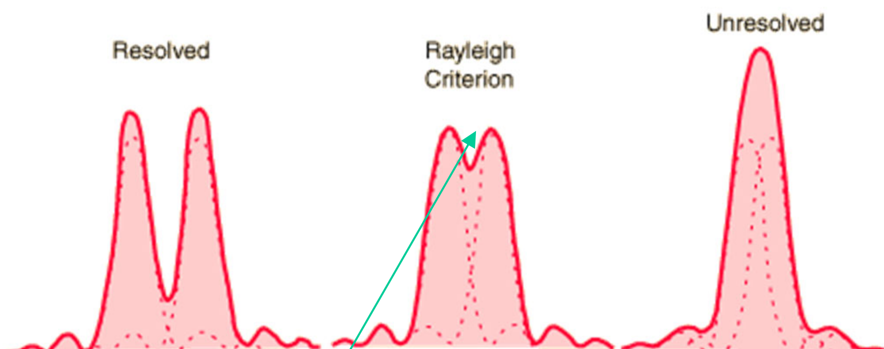
Far field diffraction pattern
Within diffraction limit formed by the two lines
(**that's what we see or at microscope)**

Resolving power is the ability of an imaging device to separate (i.e., to see as distinct) points of an object that are located at a small angular distance or it is the power of an optical instrument to separate far away objects to separate, that are close together, into individual images



- Remember lines once beyond far field appears as diffraction pattern





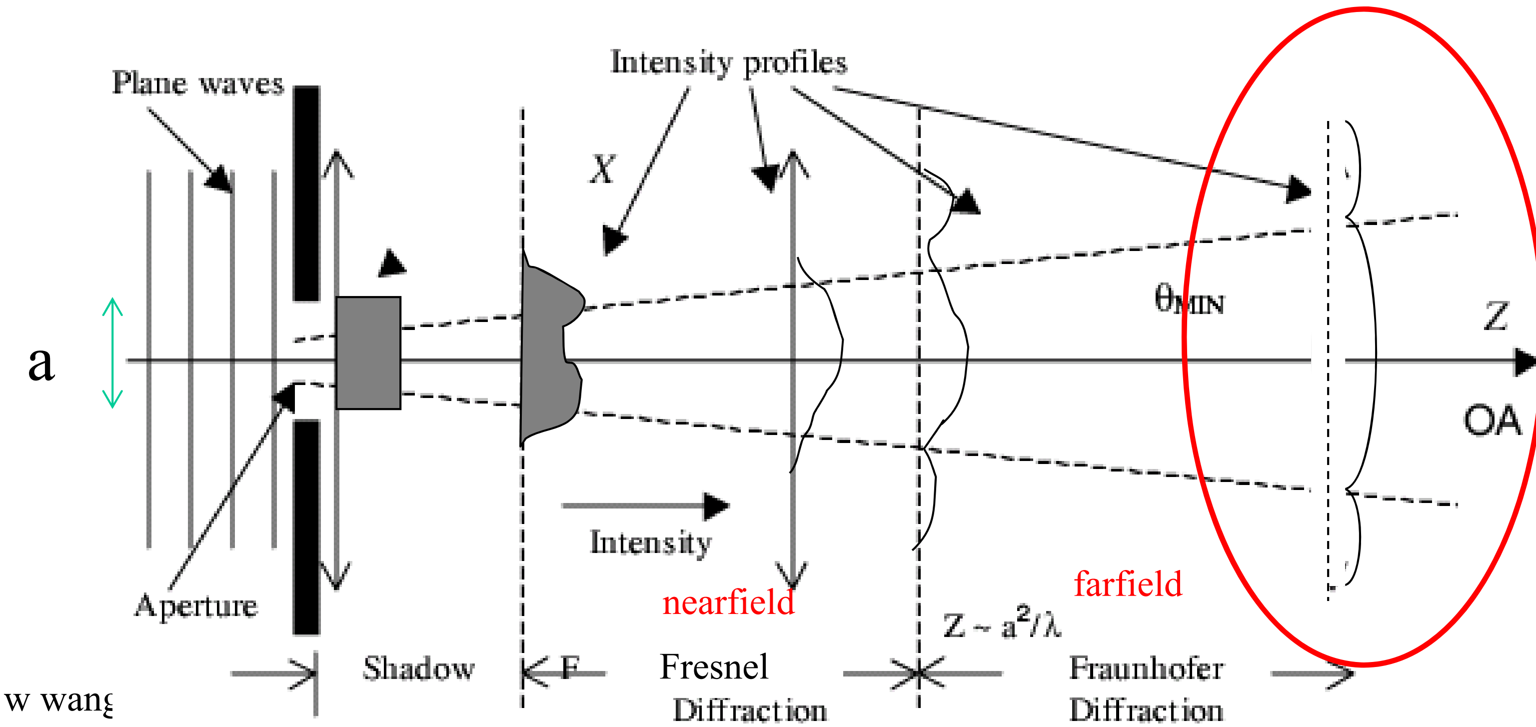
The imaging system's resolution can be limited either by aberration or by diffraction causing blurring of the image.

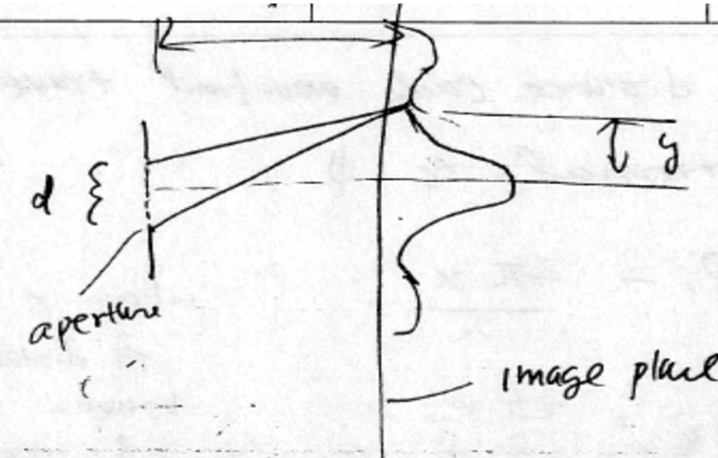
Central max of one line diffraction overlapping the other first minimum of the other line

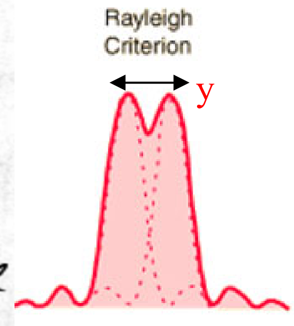
W. Wang

Diffraction

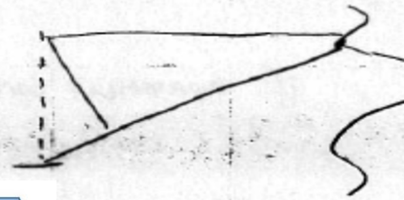
Diffraction refers to the spread of waves and appearance of fringes that occur when a wave front is constricted by an aperture in a a screen that is otherwise opaque. The light pattern changes as you move away from the aperture, being characterized by three regions



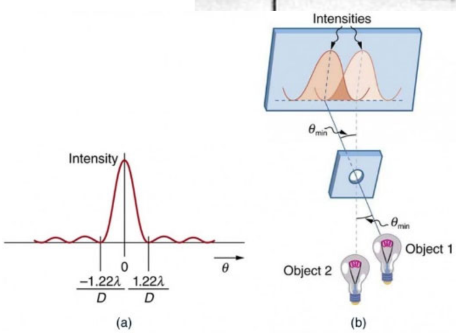


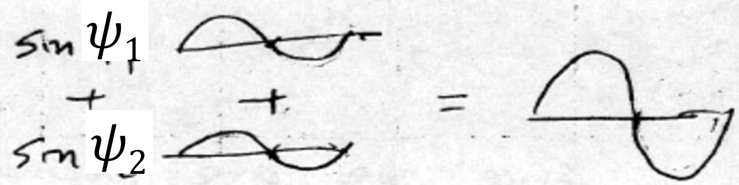


We know light is a wave, assume we divide the wave fronts into infinite tiny wavefronts each one is represented by a wave function.



for the lower corner wavefront to add up constructively with top corner wavefront

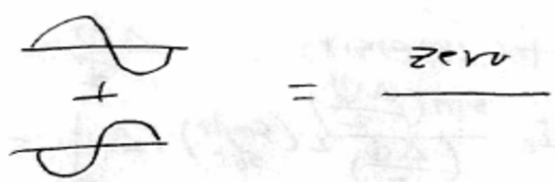




they must be in phase

$$\psi_2 = \psi_1 + 2m\pi$$

For $\psi_2 = \psi_1 + m\pi$



$$I = I_o \frac{\sin^2(\frac{\delta}{2})}{(\frac{\delta}{2})^2}$$

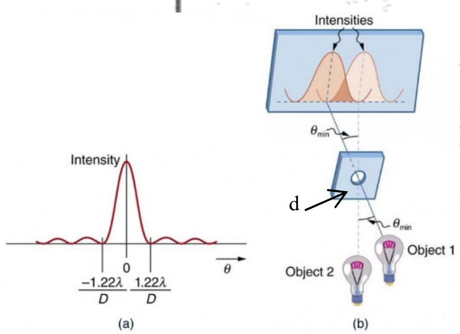
$$\delta = \frac{2\pi a \sin\theta}{\lambda}$$

Since the distance each wavefront travel is proportional to ψ

& phase $\psi_1 = \frac{2\pi x_1}{\lambda}$ where x is distance between aperture & image plane

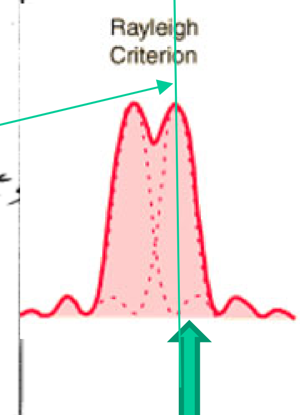
$\psi_2 = \frac{2\pi x_2}{\lambda}$

$$\Delta\psi = \psi_2 - \psi_1 = \frac{2\pi(x_2 - x_1)}{\lambda} = \frac{2\pi \Delta x}{\lambda}$$



if aperture width is d & angle is θ

$\Delta x = d \sin\theta$



Since we are looking at the pt of diffraction pattern where the 2 wavefronts distructively interfere

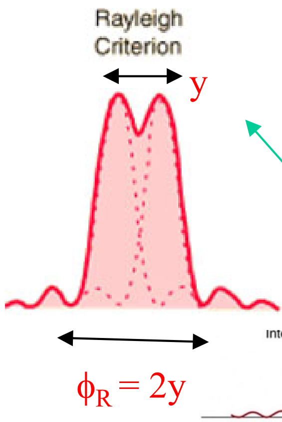
for intensity $\frac{\Delta\psi}{2} = \frac{\pi \Delta x}{\lambda}$

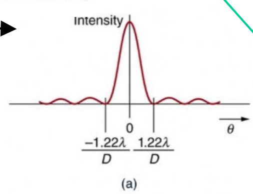
$I = I_0 \frac{\sin(\frac{\Delta\psi}{2})}{(\frac{\Delta\psi}{2})^2}$ (single slit) $\boxed{\frac{\Delta\psi}{2} = \frac{\pi d \sin\theta}{\lambda}}$ — eq 1

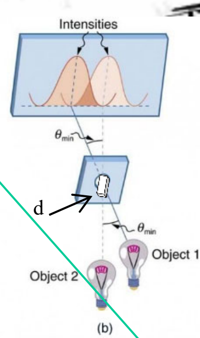
W. Wang

$I = I_d\, J_1(\frac{\Delta\psi}{2})/\Delta\psi$ (Circular aperture)

Recall $I = I_o\,[\mathrm{sinc}(\psi/2)]^2$ Where $\psi = 2\pi d \sin\theta / m\lambda$







For single slit

For any minimum to occure

$$\frac{\Delta\psi}{2} = m\pi = \frac{\pi d \sin\theta}{\lambda}$$

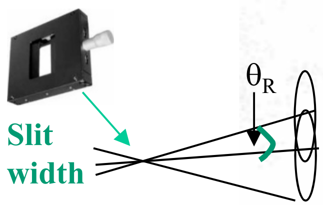
$$\boxed{\sin\theta = \frac{\frac{\Delta\psi}{2}\lambda}{m\pi d}}$$

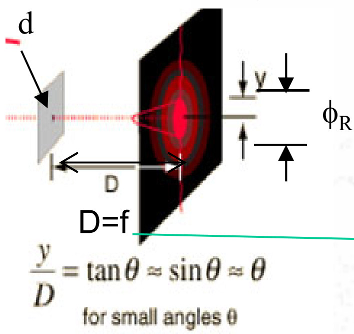
$\frac{\Delta\psi}{2} = \pi$ for 1st min.

$$\sin\theta = \frac{\lambda}{d}$$

(remember one max is overlapping the other min)

Because f ~ D





(separation distance) $\boxed{y = f\sin\theta = D\frac{\lambda}{d},}$ **(spatial resolution of the image)**

Recall $f = \frac{2y}{f\text{-number}}$

$$\frac{2y}{D} = f\text{-number}$$

$$y = \frac{f\text{-number}\cdot D}{2}$$

$$\sin\theta = \frac{\phi}{2f} = \frac{1}{2\,f\text{-number}} \Rightarrow 2f\sin\theta = \phi$$

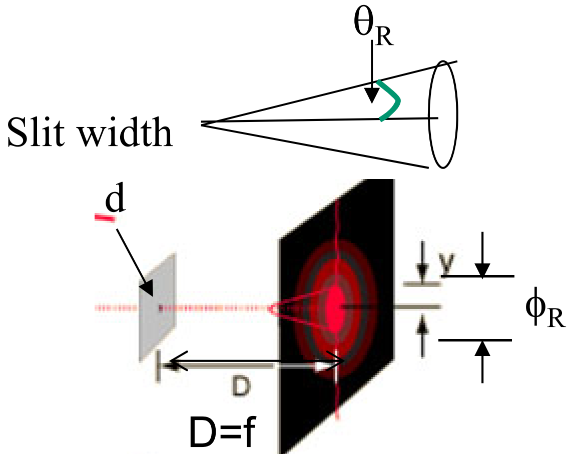
Since this is the radius of the Airy disk, the resolution is better estimated by the diameter

(total separation distance) $\boxed{\phi = 2f\sin\theta = 2\frac{\lambda f}{d}}$ (spot size)

Diffraction–limited spot size

As we will see later when we derive irradiance distribution in the diffraction pattern of a slit is defined as

$$I = I_o \frac{\sin^2(\frac{\delta}{2})}{(\frac{\delta}{2})^2} = I_o\,[\mathrm{sinc}(\delta/2)]^2$$



$$\frac{y}{D} = \tan\theta \approx \sin\theta \approx \theta$$

for small angles θ

Where $\delta = 2\pi d\sin\theta/m\lambda$

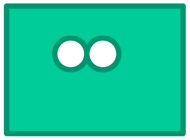
Since m=1 for 1st min. and NA=Sinθ =ϕ/2f= 1/2f-number

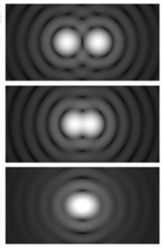
$$\boxed{NA = \sin\theta_R = \lambda/d}$$

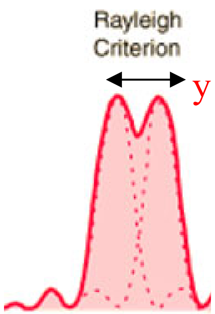
Please read the hand written derivation for more detail:
http://courses.washington.edu/me557/readings/reflection+refraction.pdf

aperture

Recall $I = I_o\,[2J_1(\psi/2)/\delta/2]^2$ Where $\psi = 2\pi d\sin\theta/m\lambda$





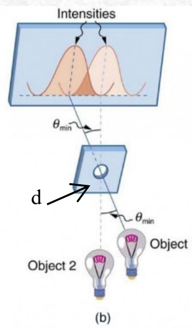


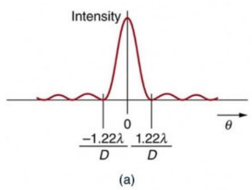
Since it's bessel function distructive interference occur slightly off, we found 1st min occur at $\boxed{\frac{\Delta\psi/2}{\pi} = 1.22}$

$m = 1$

Now rearrange eg 1 $\frac{\Delta\psi}{2} = \frac{\pi d\sin\theta}{\lambda}$

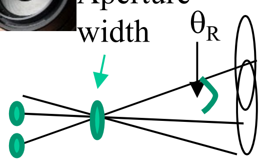
$$\boxed{\sin\theta = \frac{\Delta\psi/2\,\lambda}{\pi d}} \quad \text{— eg 2}$$

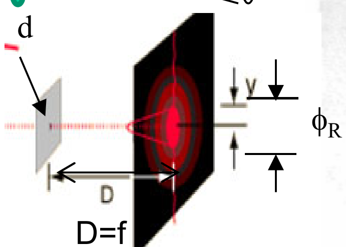




$\phi_R = 2y$

Aperture width





D=f

$\frac{y}{D} = \tan\theta \approx \sin\theta \approx \theta$

for small angles θ

Because f ~ D

$y = D\tan\theta$

$\tan\theta \approx \sin\theta = \frac{y}{D}$

$y = D\sin\theta = D\frac{\Delta\psi/2\,\lambda}{\pi d}$

Recall

$f = \frac{2y}{f\text{-number}}$

$\frac{2y}{D} = f\text{-number}$

$y = \frac{f\text{-number}\,D}{2}$

$\sin\theta = \frac{\phi}{2f} = \frac{1}{2f\text{-number}}$

separation distance

Since $\frac{\Delta\psi/2}{\pi} = m = 1.22$ for 1st min

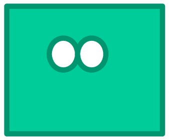
$\sin\theta = 1.22\frac{\lambda}{d}$

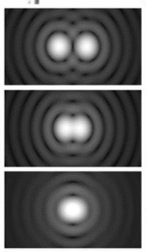
$y = D\left(\frac{1.22\lambda}{d}\right) = f\sin\theta = f\frac{1.22\lambda}{d}$ (spatial resolution of the image)

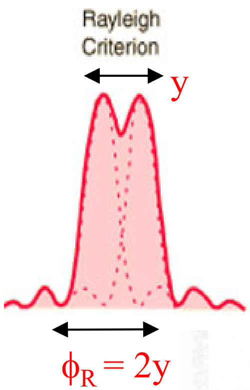
total separation dist $\phi = 2f\sin\theta = \boxed{\phi_R = 2.44\lambda\, f/d}$ (Airy disk diameter)

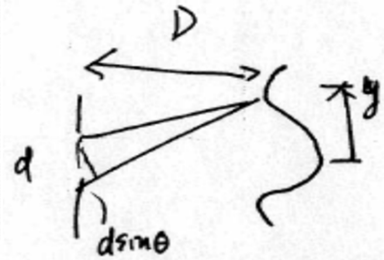
Recall $I = I_o\,[2J_1(\delta/2)/\delta/2]^2$ Where $\delta = 2\pi d\sin\theta/m\lambda$ page 44

$x = \delta/2$



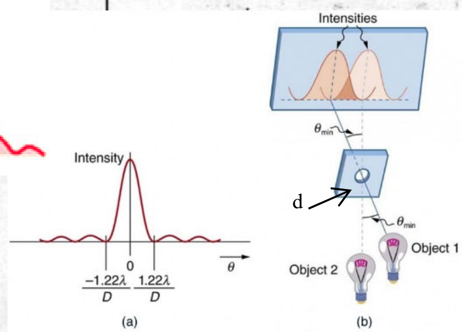






$x = \frac{\pi d\sin\theta}{\lambda}$

$$\boxed{\sin\theta = \frac{x\lambda}{\pi d}}$$



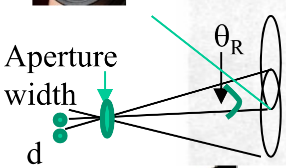
$y = D\tan\theta$

$\tan\theta \approx \sin\theta = \frac{y}{D}$

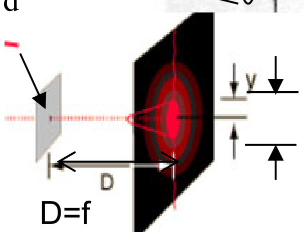
$y = D\sin\theta = D\frac{x\lambda}{\pi d} \Rightarrow$ [redacted]

Since $m = 1.22$ for 1st min. $\frac{x}{\pi} = 1.22$

$$\boxed{\sin\theta = 1.22\frac{\lambda}{d}}$$



(spatial resolution of the image)



$\frac{y}{D} = \tan\theta \approx \sin\theta \approx \theta$

for small angles θ

$f = \frac{2y}{f\text{-number}}$

$\frac{2y}{D} = f\text{-number}$

$y = \frac{f\text{-number}\,D}{2}$

$\sin\theta = \frac{\phi}{2f} = \frac{1}{2f\#}$

Separation distance $$\boxed{y = \frac{D\lambda\,(1.22)}{d} \approx f\,\frac{1.22\lambda}{d}}$$

Because f ~ D

total separation distance $$\boxed{\begin{aligned}\phi &= 2f\sin\theta \\ &= 2f\sin\theta = 2.44\,\frac{\lambda}{d}f\end{aligned}}$$

Since this is the radius of the Airy disk, the resolution is better estimated by the diameter

W. Wang

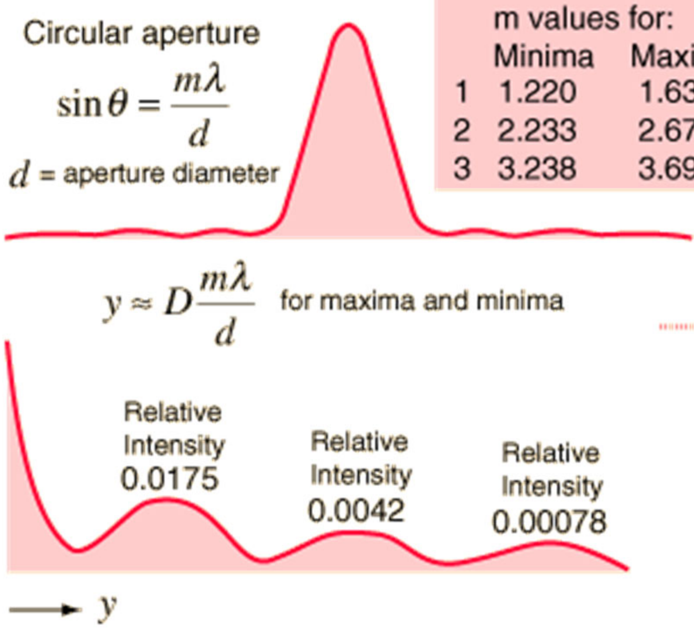
Diffraction–limited spot size

As we will see later when we derive irradiance distribution in the diffraction for a circular aperture

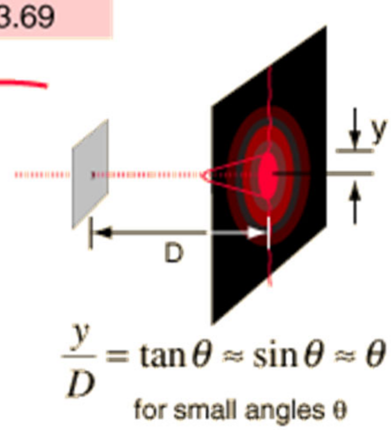
Circular aperture

$$I = I_o\,[2J_1(\delta)/\delta]^2$$

Where $\delta = \pi d\sin\theta / m\lambda$



m values for:	Minima	Maxima
1	1.220	1.635
2	2.233	2.679
3	3.238	3.69



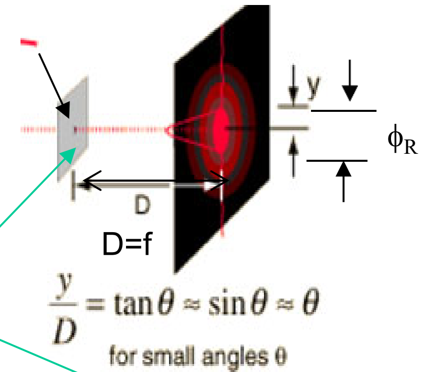
Since m=1.22 for 1st min. and

NA=Sinθ =φ/2f= 1/2f-number

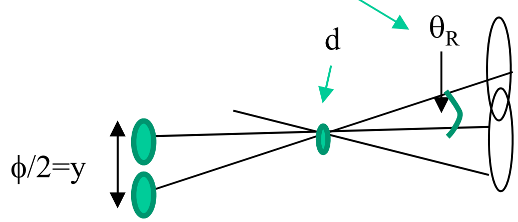
$$\boxed{NA = \sin\theta_R = 1.22\lambda/d}$$

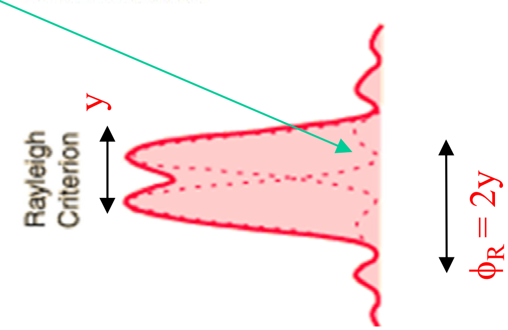
Please read the hand written derivation for more detail: http://courses.washington.edu/me557/readings/reflection+refraction.pdf

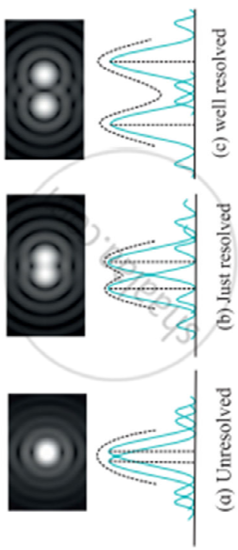
Aperture will diffract the objects, theta from diffraction is the same as the theta from the lens because diffraction observed at focal plan and also the first minimum happens to occurs at the same sin (theta). We can just use diffraction to calculate the spatial resolution!!

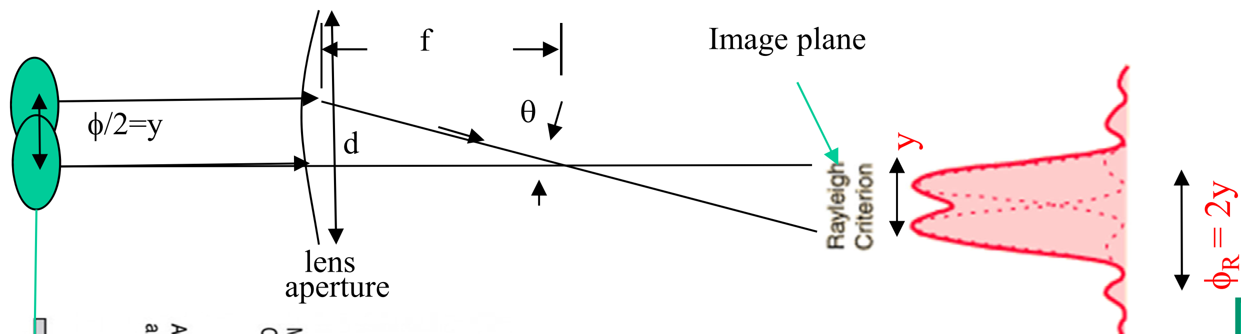


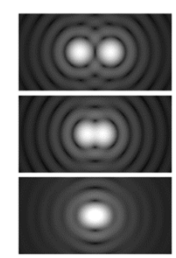
Aperture width

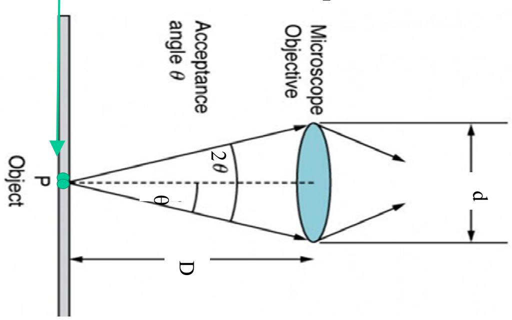












y= 1.22λ f/d

Bigger the (NA), smaller the y (higher the spatial resolution)

W. Wang

Lens and spatial resolution

In most biology laboratories, resolution is presented when the use of the microscope is introduced. **The ability of a lens to produce sharp images of two closely spaced point objects is called resolution.** The smaller the distance *y* by which two objects can be separated and still be seen as distinct, the greater the resolution. The resolving power of a lens is defined as that distance *y*. An expression for resolving power is obtained from the Rayleigh criterion. In Figure, we have two point objects separated by a distance *y*. According to the **Rayleigh criterion, resolution is possible when the minimum angular separation is**

From diffraction limit

$$\sin\theta = 1.22\lambda/d = y/D$$

where D is the distance between the specimen and the objective lens, and we have used the small angle approximation (i.e., we have assumed that y is much smaller than D), so that $\tan\theta \approx \sin\theta \approx y/D$. Therefore, the resolving power is

$$y = 1.22\lambda D/d$$

From diffraction limit

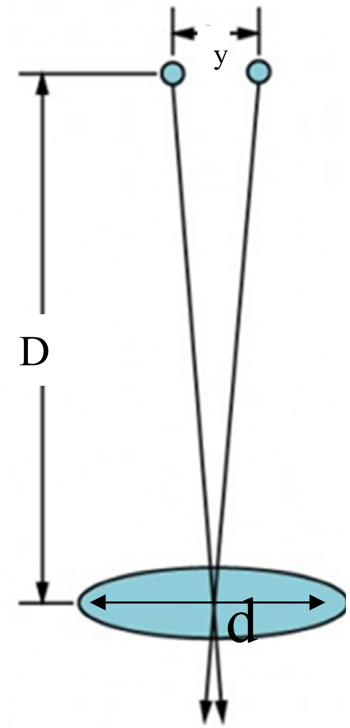


Figure. Two points separated by at distance y and a positioned a distance D away from the objective. (credit: Infopro, Wikimedia Commons)

Lens and spatial resolution

Another way to look at this is by re-examining the concept of Numerical Aperture (*NA*) discussed in Microscopes. There, *NA* is a measure of the maximum acceptance angle at which the fiber will take light and still contain it within the fiber. Figure shows a lens and an object at point P. **The *NA* here is a measure of the ability of the lens to gather light and resolve fine detail**. The angle subtended by the lens at its focus is defined to be 2θ. From the Figure and again using the small angle approximation, we can write

$$\sin\theta = (d/2)/D = d/(2D) = NA$$

From lens

The *NA* for a lens is ***NA* = *n* sin *θ*, where *n* is the index of refraction of the medium between the objective lens and the object at point P.** From this definition for *NA*, we can see that from last page:

$$y = 1.22\lambda D/d = 1.22\lambda/2\sin\theta = 0.61\lambda n/NA$$

combined

In a microscope, NA is important because it relates to the resolving power of a lens. **A lens with a large NA will be able to resolve finer details. Lenses with larger NA will also be able to collect more light and so give a brighter image.** Another way to describe this situation is that the larger the NA, the larger the cone of light that can be brought into the lens, and so more of the diffraction modes will be collected. Thus the microscope has more information to form a clear image, and so its resolving power will be higher.

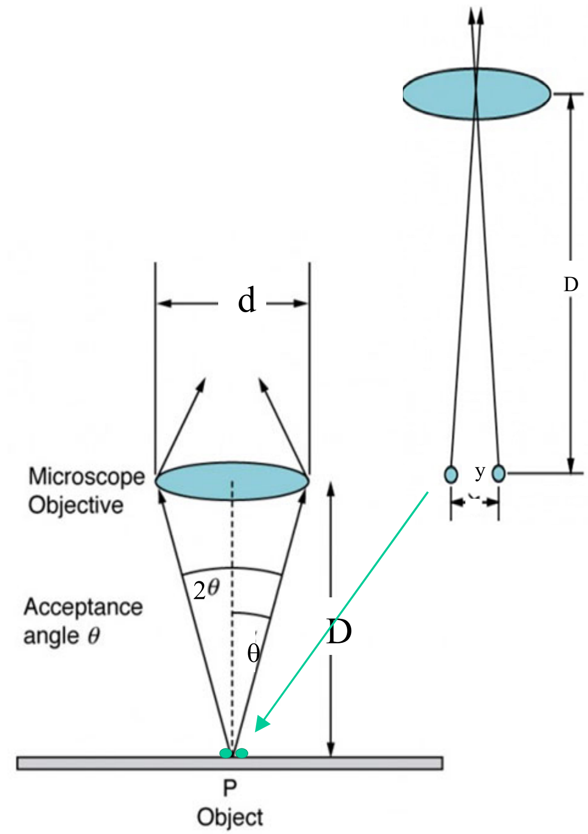


Figure. Terms and symbols used in discussion of resolving power for a lens and an object at point P. (credit: Infopro, Wikimedia Commons)

F-number and Numerical Aperture of Lens

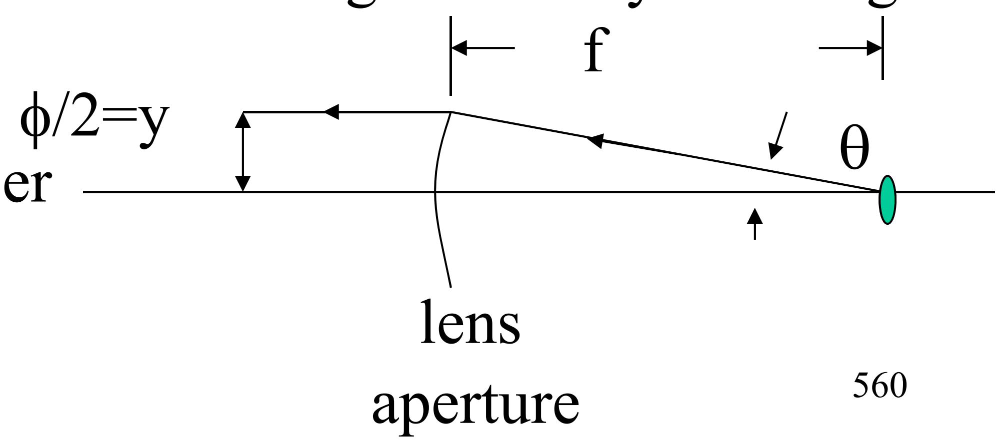
The f-number (focal ratio) is the ratio of the focal length f of the lens to its clear aperture ϕ (effective diameter). The f-number defines the angle of the cone of light leaving the lens which ultimately forms the image. This is important concept when the throughput or light-gathering power of an optical system is critical, such as when **focusing light into a monochromator or projecting a high power image.)**:

$$\text{f-number} = f/\phi$$

Numeric aperture is defined as sine of the angle made by the marginal ray with the optical axis:

$$\boxed{NA = \sin\theta = \phi/(2f)} = 1/2\text{f-number}$$

Acceptance angle



Geometric and Wave lens

One of the consequences of diffraction is that the focal point of a beam has a finite width and intensity distribution. Consider focusing when only considering geometric optics, shown in Figure a. The focal point is infinitely small with a huge intensity and the capacity to incinerate most samples irrespective of the *NA* of the objective lens. For wave optics, due to diffraction, the focal point spreads to become a focal spot (see Figure b) with **the size of the spot decreasing with increasing *NA*.** Consequently, **the intensity in the focal spot increases with increasing *NA*. The higher the *NA*, the greater the chances of photodegrading the specimen**. However, the spot never becomes a true point.

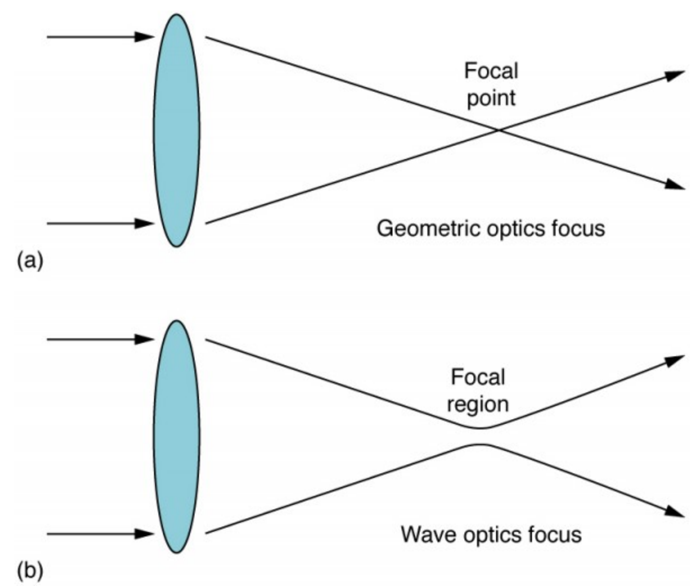
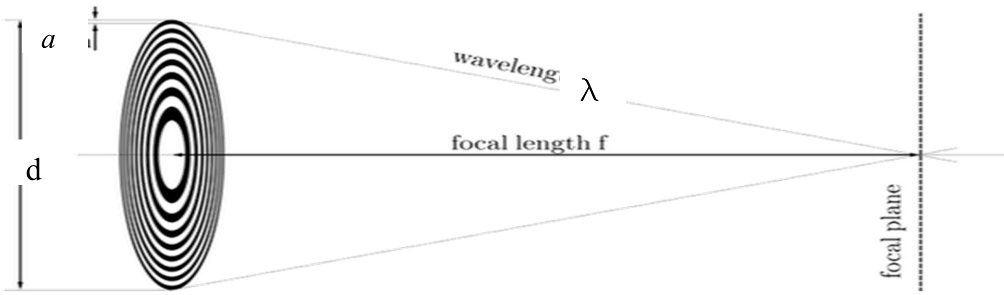
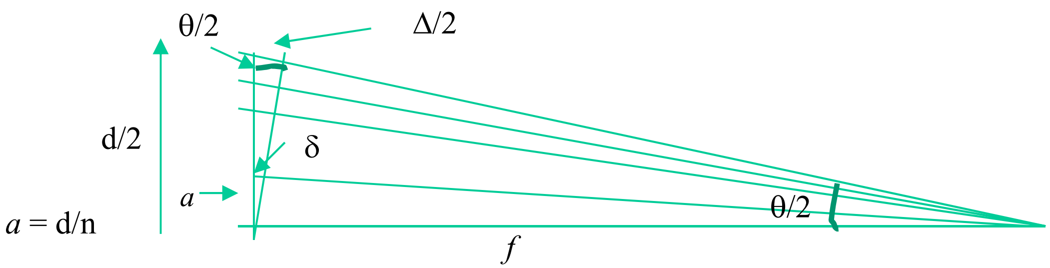
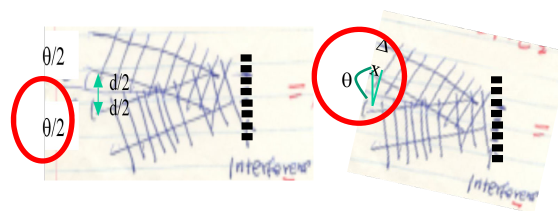


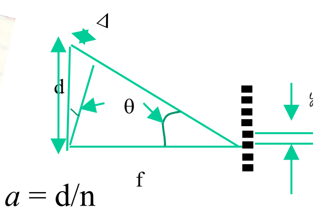
Figure . (a) In geometric optics, the focus is a point, but it is not physically possible to produce such a point because it implies infinite intensity. (b) In wave optics, the focus is an extended region.

Fresnel diffraction Lens









Design to create
constructive inference

$k\Delta = kd\sin\theta = 2\pi d\sin\theta/\lambda = 2\pi$

$\Rightarrow \quad \Delta = \lambda \;\; => d\sin\theta = \lambda$

$\Rightarrow \quad (an)\sin\theta = \lambda$

$\Rightarrow \quad (an)d/f = \lambda \quad => \quad f = d^2/\lambda$

$d/f = \tan\theta \sim \sin\theta$

focal spot size = 1.22a

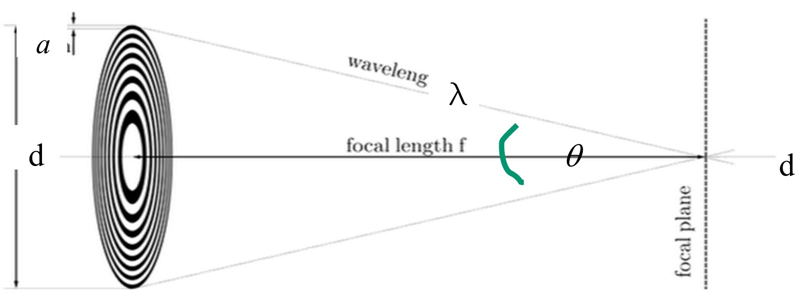
Line of point sources (pinholes), all in phase with same amplitude (gap space 1/l)

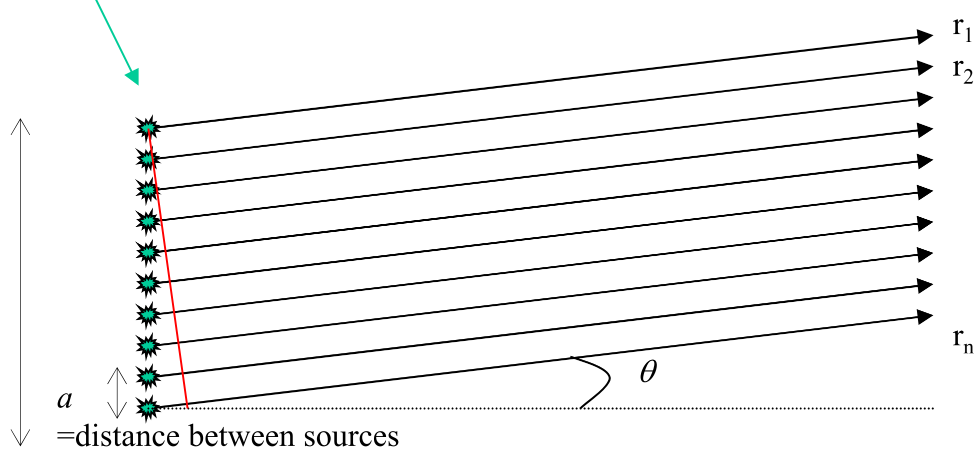
If the spatial extent of the oscillator array is small compared to the wavelength of the radiation, then the amplitudes of the separate waves arriving at some observation point P will be essentially equal, $E_0(r_1) = E_0(r_2) = E_0(r_3) = \cdots = E_0(r_N) = E_0(r)$

$$E = E_o(r)e^{i(kr_1-\omega t)}[1 + e^{ik(r_2-r_1)} \dots\dots + e^{ik(r_N-r_1)}]$$

$r_2 - r_1 = a\sin\theta$ $\quad r_3 - r_1 = 2a\sin\theta$

$r_4 - r_1 = 3a\sin\theta$ $\quad r_N - r_1 = (N-1)a\sin\theta$





Assume all in phase (meaning the ring actually **is evenly spaced by 1/2λ** and wave arrive at central max all constructively interfere, then each wave must be in phase with the other , it means $k\delta = 2\pi$.

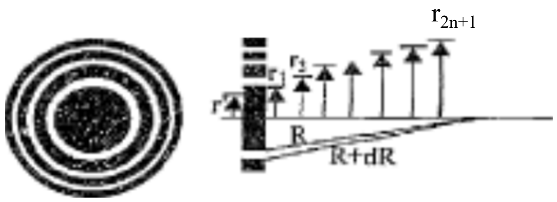
$k(r_1 - r_2) = k\delta = = 2\pi a\sin\theta/\lambda = 2\pi$ $\qquad d/f = \tan\theta \sim \sin\theta$

$\Rightarrow \quad \delta = \lambda \; \Rightarrow a\sin\theta = \lambda$

$\Rightarrow \quad (d/n)\sin\theta = \lambda$

$\Rightarrow \quad d^2/nf = \lambda \; \Rightarrow \; \boxed{f = ad/\lambda}$ $\qquad$ *focal spot size = 1.22a*

Half wave band source

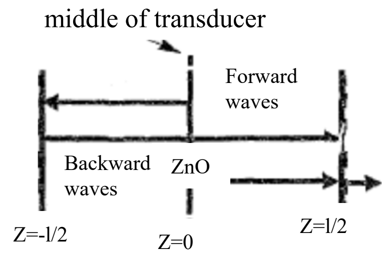


The acoustic waves generated by the successive annular sources are designed to arrive at the focal point (f) with finite delays (equal to multiples of the wavelength) by ensuring that the radii r_n satisfy the following relation

$$\sqrt{r_n^2 + f^2} - f = n\lambda/2$$

$$r_n = \sqrt{n\lambda \ \left(f + \frac{n\lambda}{4}\right)} \Rightarrow \boxed{f = (r_n^2/n\lambda) - n\lambda/4}$$

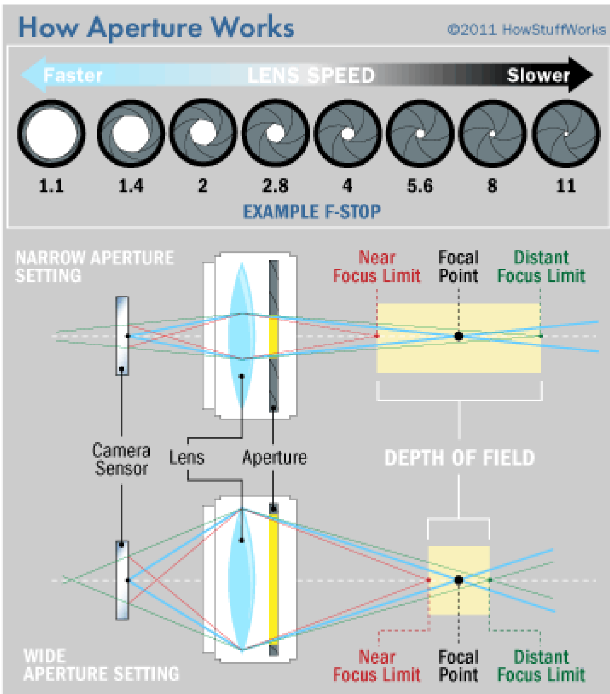
where n=l,3,5, ... 2n+1 .. ; λ is the wave length of acoustic wave in a liquid. The waves, generated by each successive sources, arrive at the focal point in phase resulting in constn1ctive interference. Such sources are referred to as half-wave-band sources.



Thus, to obtain maximum acoustic wave generation, the thickness of the transducer should be odd-multiple of a half wavelength (i.e., l=(2n+ 1)λ/2).

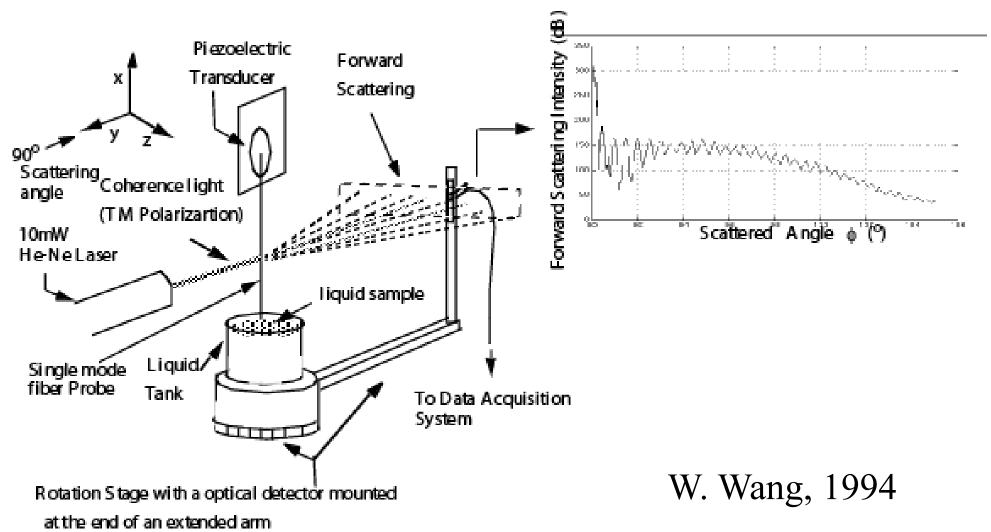
The above results are valid for an air-liquid terminated transducer. For an air-air terminated transducer (i.e., Z1<<Zc and Z2<<Zc), the transducer should be a half wavelength long for a strongest excitation. If Z1<<Zc and Z2>>Zc, the transducer should be near a quarter wavelength long. Z is impemdance of liquid . An air-backed transducer (i.e., either the back or the front acoustic port is terminated with air) can be :represented with medium 1 (i.e., air) having an acoustic impedance Z1=0 and medium 2 with acoustic impedance Z2 Assuming that layers (such as the SixNy mem. brane, top and bottom electrodes) other than the piezoelectric film in the transducer are thin compared to the acoustic wavelength, and their effects are negligible.

More Diffraction Examples



Camera aperture

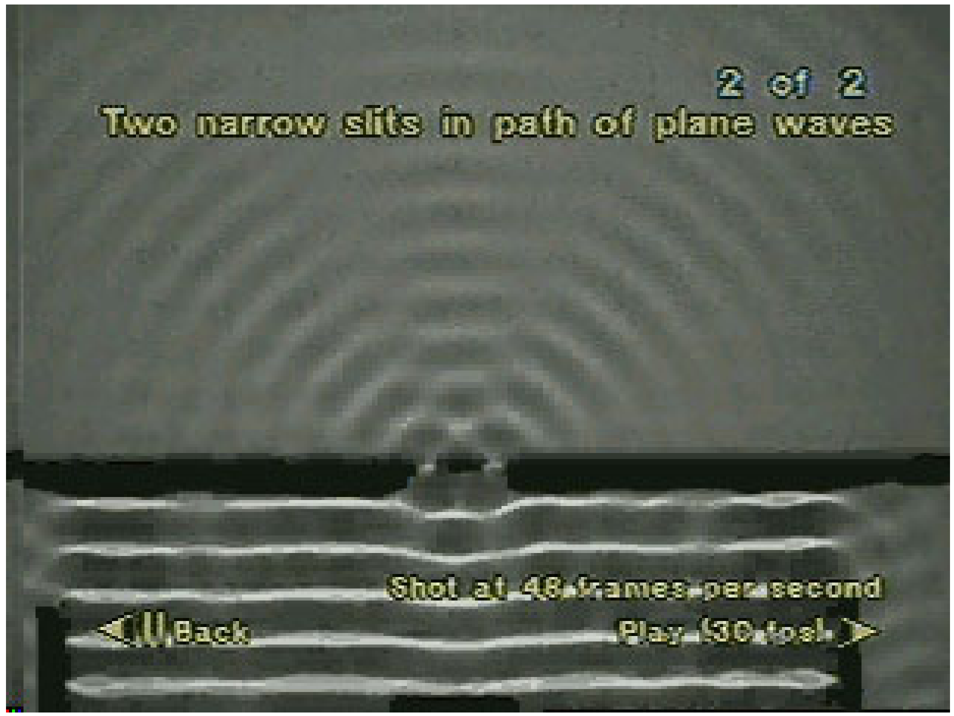
Punk glasses



W. Wang, 1994

Diffraction + interference from optical fiber

Double slits Ripple Tank Experiment (Interference)



http://www.phy.davidson.edu/introlabs/labs220-230/html/lab10diffract.htm

d >> a, otherwise we get diffraction modulation
d = separation between two slits, a=slit width

Two Approaches

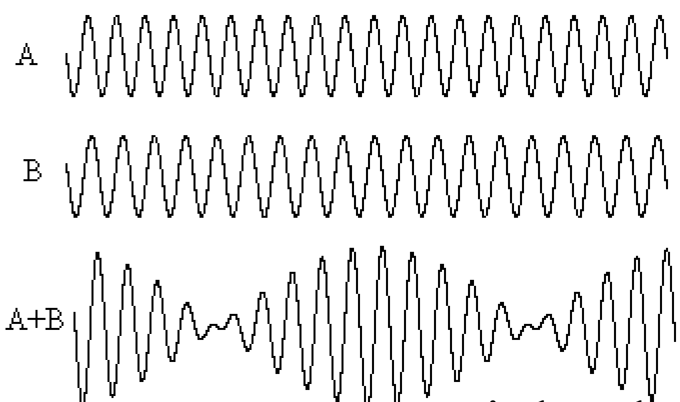
- Full wave method (EM theory)
- Phase only (Physics class)

Two Approaches

- Full wave method (EM theory)
- Phase only (Physics class)

Power of sin(a)+sin(b)

Everything can be expanded or explained in a series of sin function either a summation, multiplication or convolution. Many of our optical theory and sensor concept exploit this concept to allow us to study small physical changes using resolution of optical wavelength but observed at relatively lower frequency or longer wavelength or simplify the way we calculate them. e.g. interference or beats



$\sin A + \sin B = 2\sin(A+B)/2 * \cos(A-B)/2$

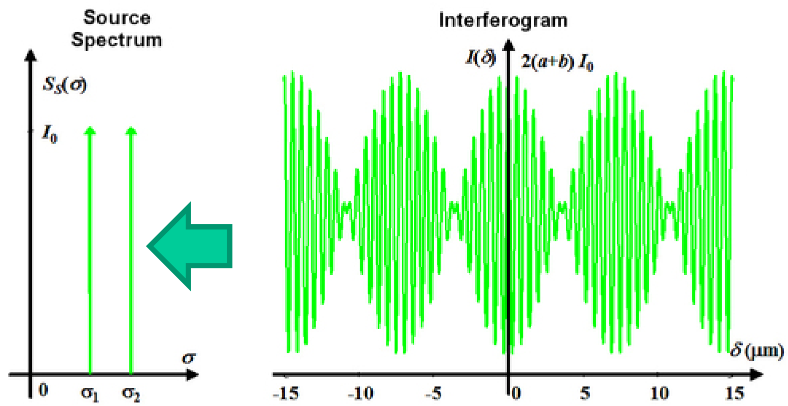
Let $A = k_1 x + \omega_1 t + \phi_1 \quad k_1 = 2\pi n_1/\lambda$

$B = k_2 x + \omega_1 t + \phi_2 \quad k_2 = 2\pi n_2/\lambda$

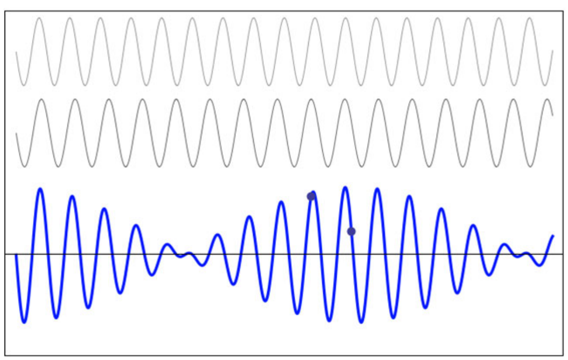
Light phenomena is just a superposition of waves with different wav lengths, phase, etc. (ambient light)

Beats (example)

The interferogram shows beats. They are represented in figure by two **slightly different wavelengths**.



Coherent length ~ beat length



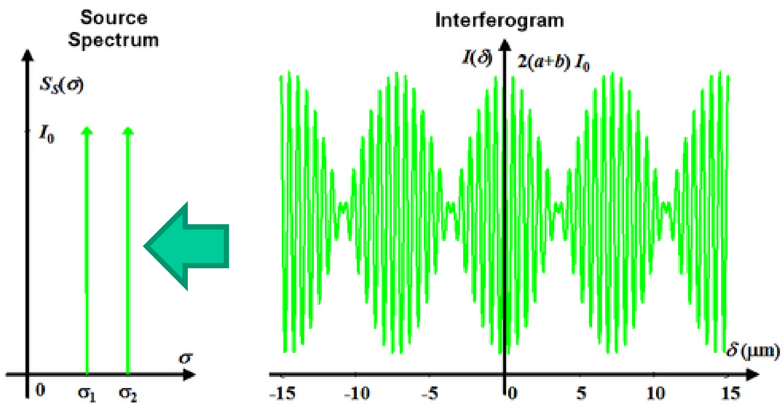
sinA+sinB = 2sin(A+B)/2 *cos(A-B)/2

Let $A = k_1x+\omega_1t$ $\quad k_1 = 2\pi n_1/\lambda_1$

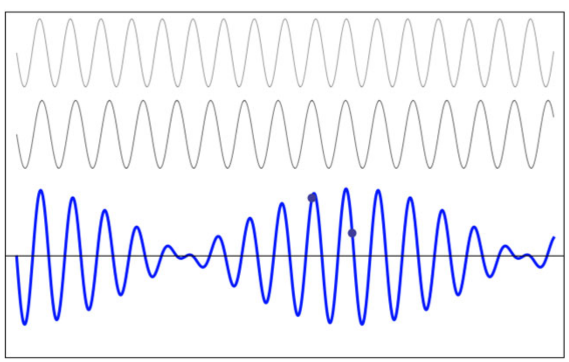
$B = k_2x+\omega_2t$ $\quad k_2 = 2\pi n_1/\lambda_2$

Beats (example)

The interferogram shows beats. They are represented in figure by two **slightly different indices**



Coherent length ~ beat length



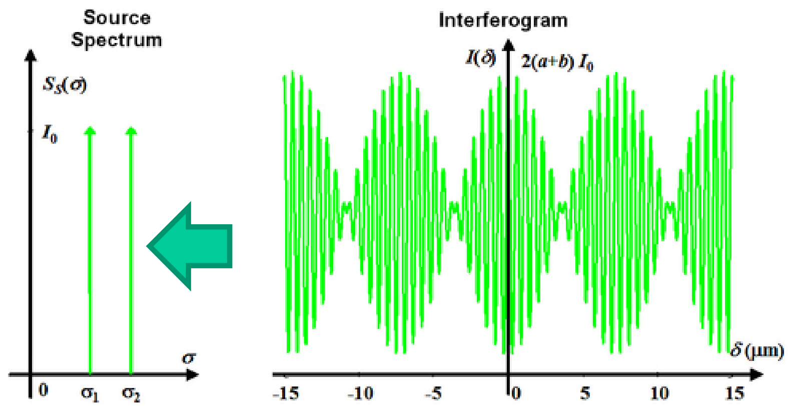
sinA+sinB = 2sin(A+B)/2 *cos(A-B)/2

Let $A = k_1 x + \omega_1 t \quad k_1 = 2\pi n_1/\lambda_1$

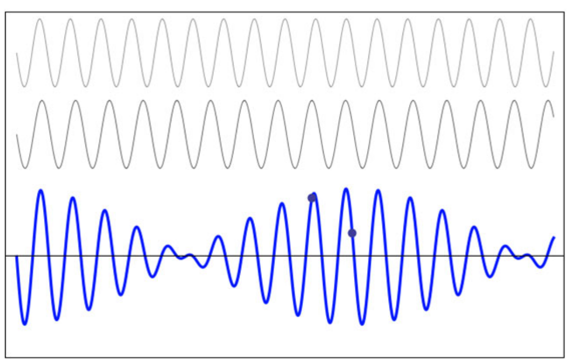
$B = k_2 x + \omega_1 t \quad k_2 = 2\pi n_2/\lambda_1$

Beats (example)

The interferogram shows beats. They are represented in figure by two slightly different in **travel distance x.**



Coherent length ~ beat length



$$\sin A+\sin B = 2\sin(A+B)/2 * \cos(A-B)/2$$

Let $A = k_1 x_1 + \omega_1 t \quad k_1 = 2\pi n_1/\lambda_1$

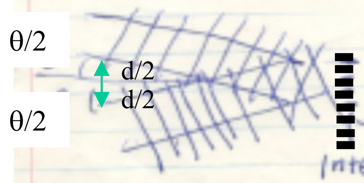
$B = k_1 x_2 + \omega_2 t \quad k_2 = 2\pi n_1/\lambda_1$

Spatial Interference due to different incident angles

$$E_1 = E_0 \sin(\omega t + kx_1)$$
$$E_2 = E_0 \sin(\omega t + kx_2)$$

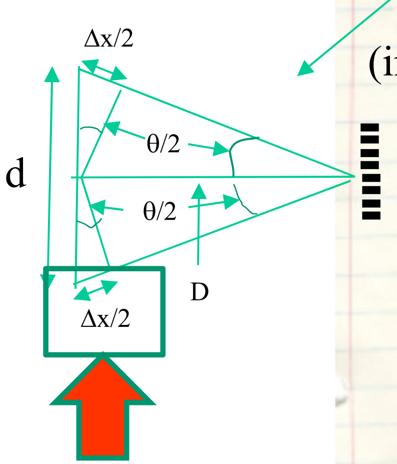
$$E_1 = \sqrt{I_0}\, e^{-jkx_1}$$
$$E_2 = \sqrt{I_0}\, e^{-jkx_2}$$

$$E_1 + E_2 = \sqrt{I_0}\, e^{-jkx_1} + \sqrt{I_0}\, e^{-jkx_2}$$



$$= \sqrt{I_0}\, e^{-jkx_1}\left(1 + e^{-jk(x_2 - x_1)}\right)$$

Interference
$$= \sqrt{I_1}\left(1 + e^{-jk(x_2 - x_1)}\right)$$



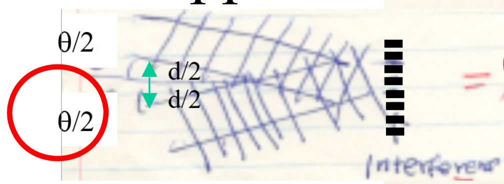
(intensity) $$|E_1 + E_2|^2 = I_1^2\, \left|1 + e^{-jk\Delta x}\right|^2$$

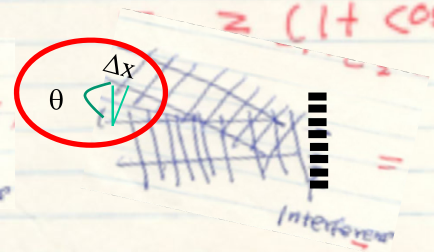
$$\left|1 + e^{-jk\Delta x}\right|^2 = \left(1 + e^{-jk\Delta x}\right)^* \left(1 + e^{-jk\Delta x}\right)$$
$$= \left(1 + e^{-jk\Delta x}\right)\left(1 + e^{jk\Delta x}\right)$$
$$= 1 + e^{-jk\Delta x} + e^{jk\Delta x} + e^{-jk\Delta x} e^{jk\Delta x}$$
$$= 1 + (\cos k\Delta x - j \sin k\Delta x) + (\cos k\Delta x + j \sin k\Delta x) + 1$$

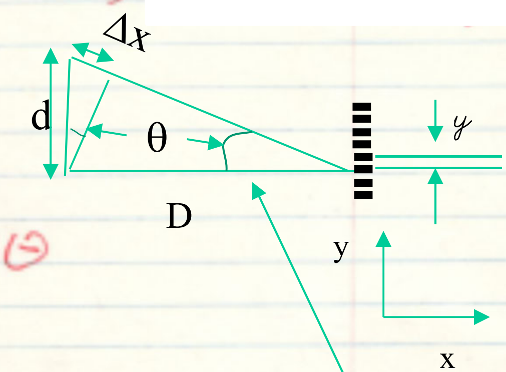
Phase approach

$$= 2 + 2\cos k\Delta x$$

$$= 2(1 + \cos k\Delta x)$$







$$k\Delta x = kd\sin\Theta$$

since ~~minimum~~ maximum occurs @ $2m\pi$

$$2m\pi = kd\sin\Theta$$

$$2m\pi = \frac{2\pi d\sin\theta}{\lambda}$$

$$\boxed{\sin\theta \approx \tan\theta = \frac{y}{D}}$$

$$\boxed{y \cong \frac{m\lambda D}{d}}$$

fringe spacing

From above figure d/D is sinθ

From next page where y/D is sinθ

d/D= sinθ ~ y/D
when m=1
then d~y
(interesting) only
at specific D

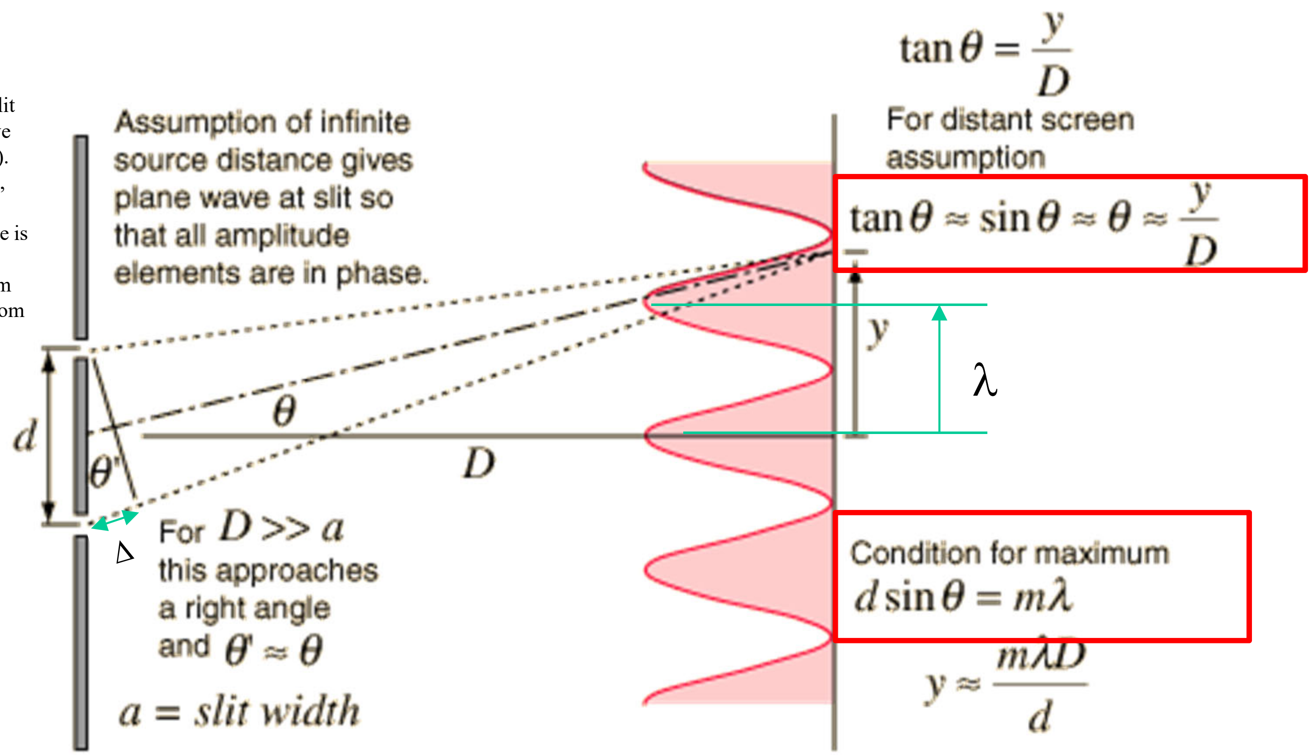
G = λ/(2sinϕ) (grating spacing)

Two Approaches

- Phase only (Physics class)

Double Slits Interference

Adding all the phase from one slit to the other actually give additive phase (constructive interference). Why? We just happen to do that, we can also find y where it's minimum. But what's difference is we don't have to divide the distance between two slits to sum the phase from top half and bottom half.



hyperphysics

$$\Delta = d\sin\theta = m\lambda$$

$$\boxed{y \sim \bar{m}\lambda D/d}$$

$$\boxed{I = I_0 \left[\frac{\sin\left(\frac{Nkd}{2}\sin\theta\right)}{\sin\left(\frac{kd}{2}\sin\theta\right)} \right]^2 \quad \text{Where } N = 2}$$

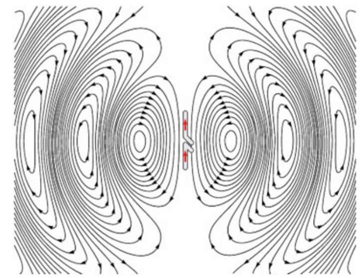
λ/d or d/2D= sin θ ~ y/D

when m=1 then d~y

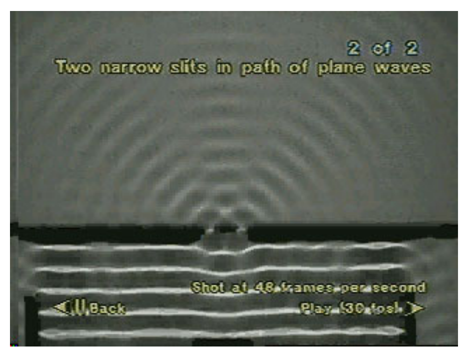
G = λ/(2sinϕ) (grating spacing)

$$\boxed{d >> a}$$

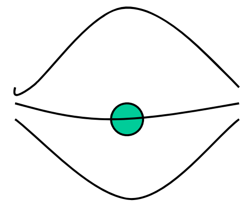
How to create spherical interference pattern



- Diverging beam
- Projected surface is spherical, however, once it's far away enough, it becomes planar
- Two coplanar interference or two coaxial interference

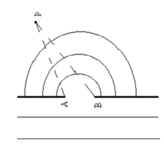


Plane, Spherical and Cylindrical Wave

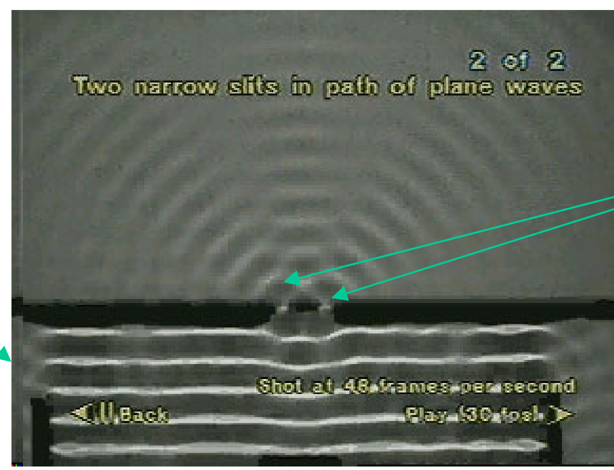


$$\psi = \psi_0 \cos(\omega t - \mathbf{k}\cdot\mathbf{r} + \phi),$$

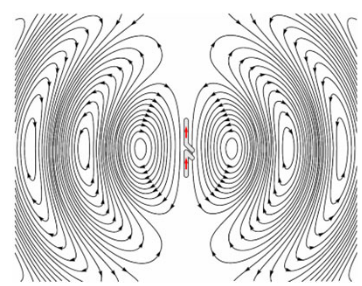
$$\psi(r,t) = \frac{\psi_0}{r}\cos(\omega t - kr + \phi).$$



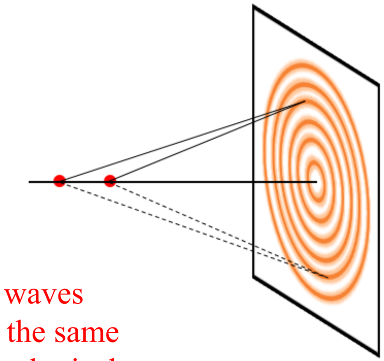
Plane wave

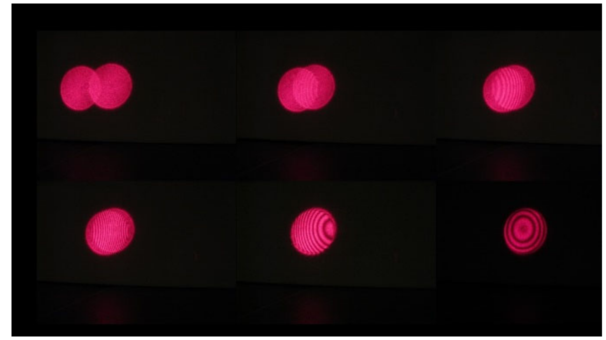


Spherical wave
point source



Interference Due to arbitrary phase plate





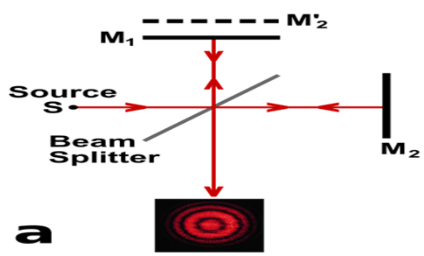
true spherical waves diverrge from the same axis creates a spherical shape interfere (point sources)

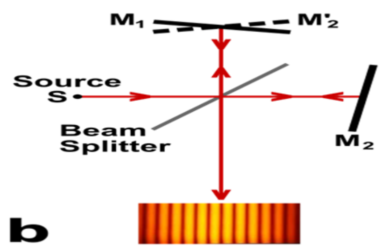
S'2 •
S'1 •

S'2 S'1

Temporal

Spatial





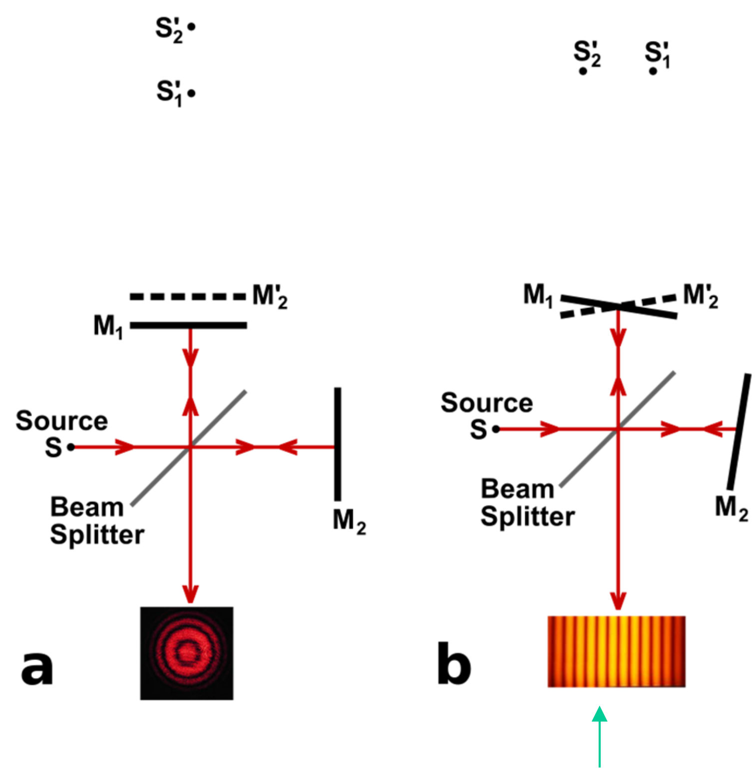
Pont source is assumed
But because it's diverged into plane wave so interference becomes like line interference

Basically anytime two wave fronts along the same axis is offset, you will see a phase shift between them and thus creates spatial interference in the plane so you see circular if they are spaced out and like shown with tilted mirror, the you get hyperbolic fringes (straight lines) instead. If like third case, you get an arbitrary shaped interference and spacing between fringes.

Michaelson Interferometer (Coplanar interference)

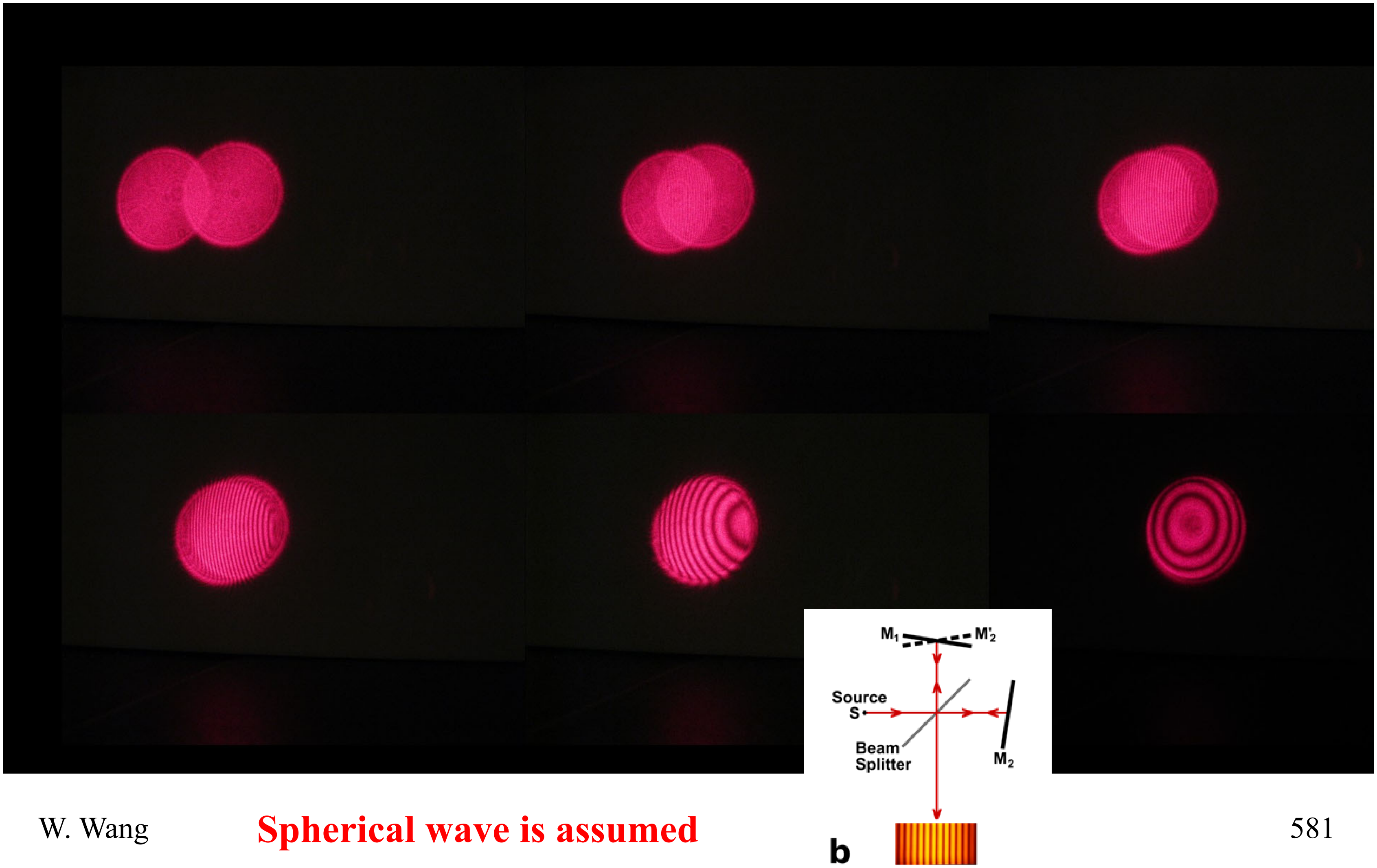
spherical wave interference (point source)

the observer has a direct view of mirror M1 seen through the beam splitter, and sees a reflected image M'2 of mirror M2. The fringes can be interpreted as the result of interference between light coming from the two virtual images S'1 and S'2 of the original source S. The characteristics of the interference pattern depend on the nature of the light source and the precise orientation of the mirrors and beam splitter. In Fig. a, the optical elements are oriented so that S'1 and S'2 are in line with the observer, and the resulting interference pattern consists of circles centered on the normal to M1 and M'2 (fringes of equal inclination). If, as in Fig. b, M1 and M'2 are tilted with respect to each other, the interference fringes will generally take the shape of conic sections (hyperbolas), but if M1 and M'2 overlap, the fringes near the axis will be straight, parallel, and equally spaced (fringes of equal thickness). If S is an extended source rather than a point source as illustrated, the fringes of Fig. a must be observed with a telescope set at infinity, while the fringes of Fig. b will be localized on the mirrors.



hyperbolic

larger the angle the straighter the interference fringes

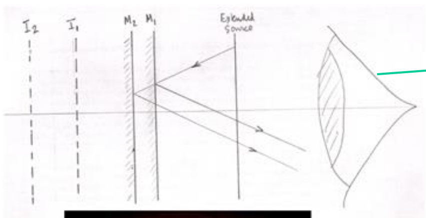


Spherical wave is assumed

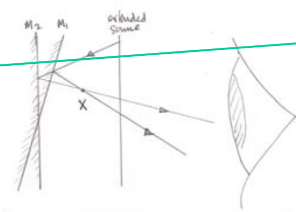
Tilt and Straight

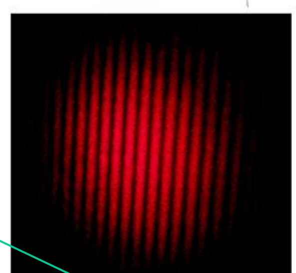
spherical wave interference (point source)

Fringes of equal inclination

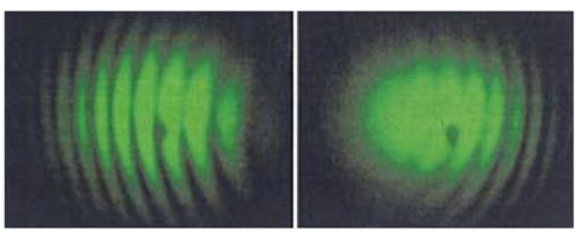


Fringes of equal thickness





A bit of each



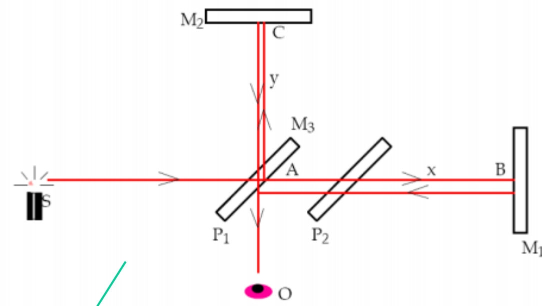
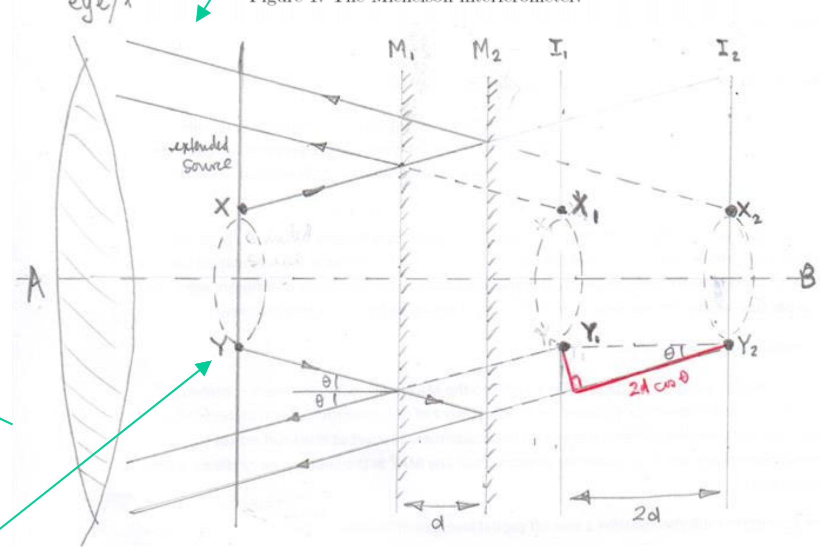


Figure 1: The Michelson interferometer.

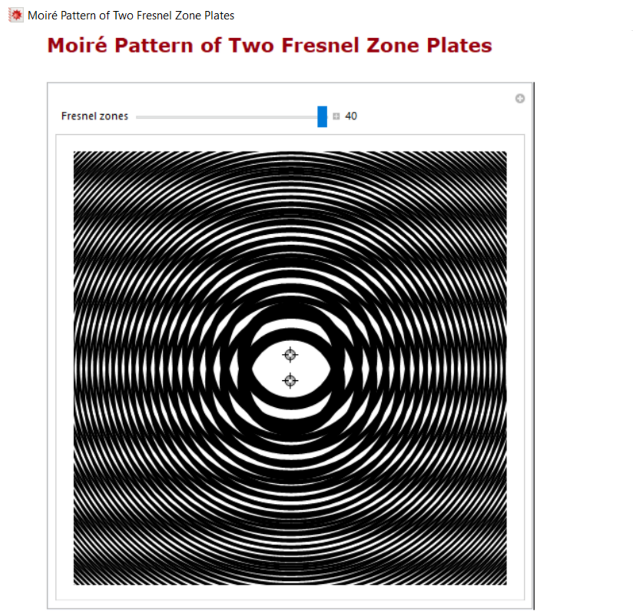


Here is a schematic diagram to show how the circular fringes are formed using the Michelson interferometer.

Spherical wave is assumed due to lens focusing or defocusing

Interference using Circular Moire Pattern
(representing interference from 2 point sources)

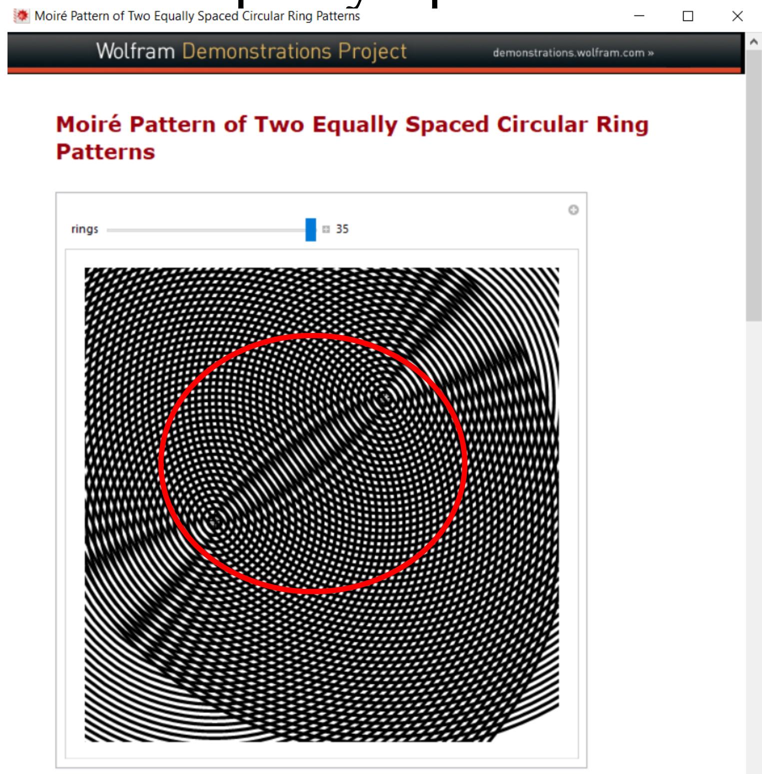
Moiré Pattern of Two Fresnel Zone Plates



Inference due to space separation

The Moiré pattern formed using two displaced Fresnel zone plates illustrates the straight line interference fringes produced by interfering two spherical waves having the same radius of curvature. The spacing of the straight line pattern depends on the separation between the centers of the two Fresnel zone plates, just like the spacing of interference fringes depends upon the separation between the two point sources.

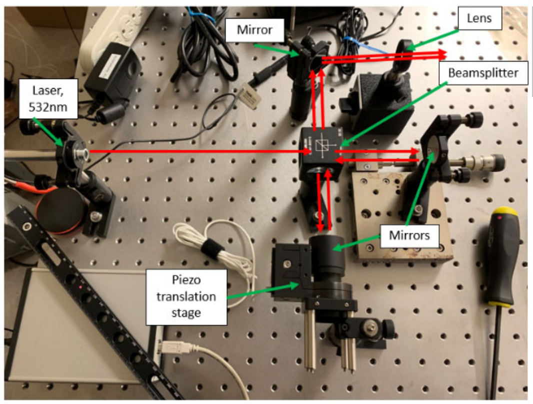
Moiré Pattern of Two Equally Spaced Circular Ring Patterns

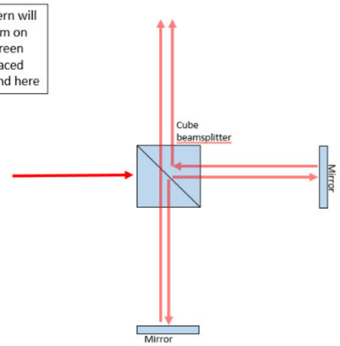


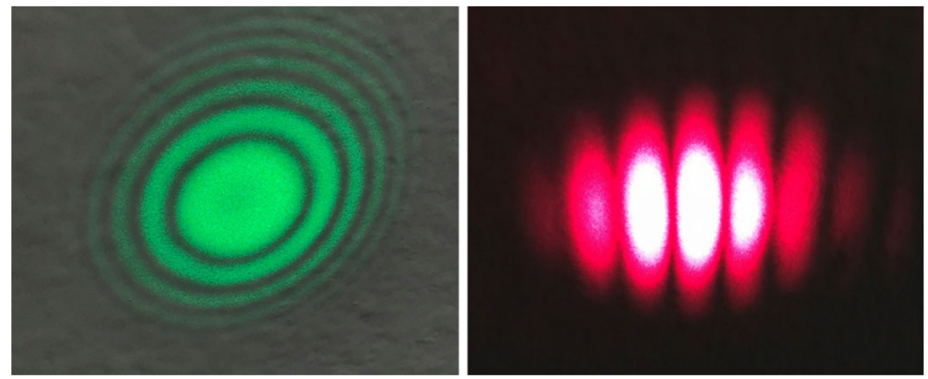
Inference due to different frequency

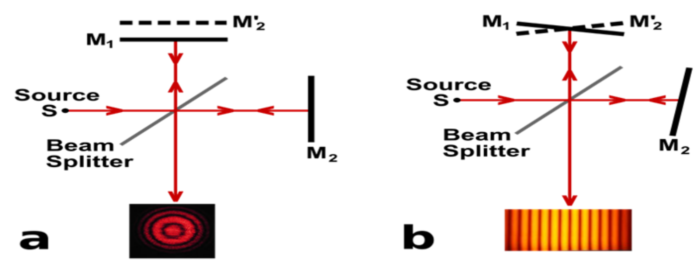
The Moiré pattern formed using two displaced equally spaced circular ring patterns illustrates the interference fringes produced by interfering two spherical waves. The spacing of the Moiré pattern depends on the separation between the centers of the two circular ring patterns, just like the spacing of interference fringes depends upon the separation between the two point sources. This Demonstration shows the shape of the interference fringes in a plane containing the two point sources.

Collimated and Diverging beam interference







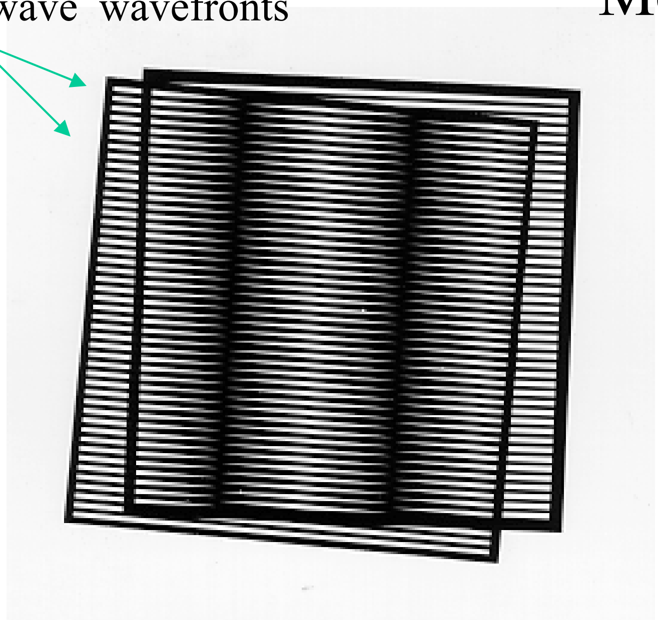


Light from a 532-nm diode laser is directed to the cube beamsplitter. Optionally, a lens (50 mm in the case of the picture above) is placed directly before the beamsplitter -- this will result in spherical waves interfering and, therefore, interference patterns looking like bulls-eyes. The beamsplitter directs the light along two different paths. Each path has a flat mirror. The mirror on the bottom arm can be mounted on a translation stage (either a manual stage or a piezo driven stage) so that the bottom path length can be varied. After reflecting off the mirrors in the two arms, the light reenters the beamsplitter and the light that is directed to the left in the figure above will interfere on the wall.

The interference fringes you will observe on the wall will look like those above. **You'll get straight fringes when interfering diverging beams and circular fringes when collimating beam interferes.**

Plane wave Interference

Plane wave wavefronts



Moire patterns

Plane waves going in slightly different direction=> still straight line pattern

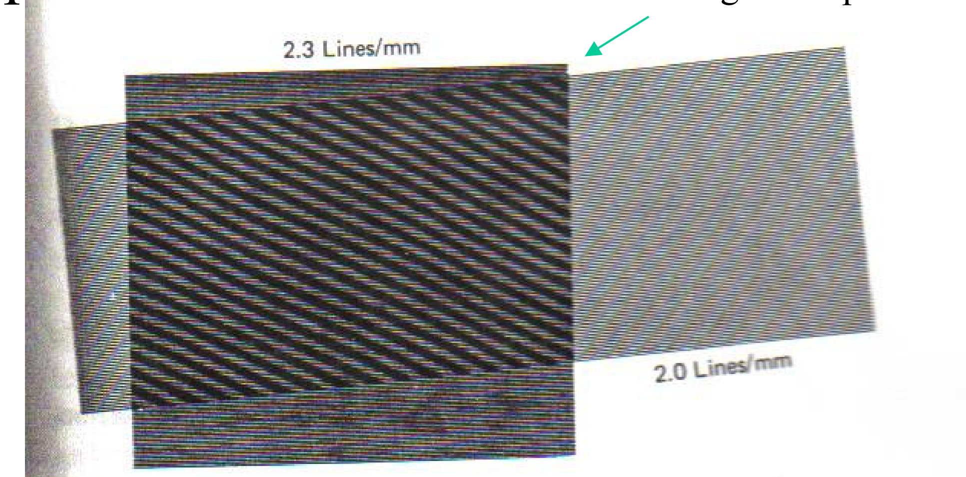


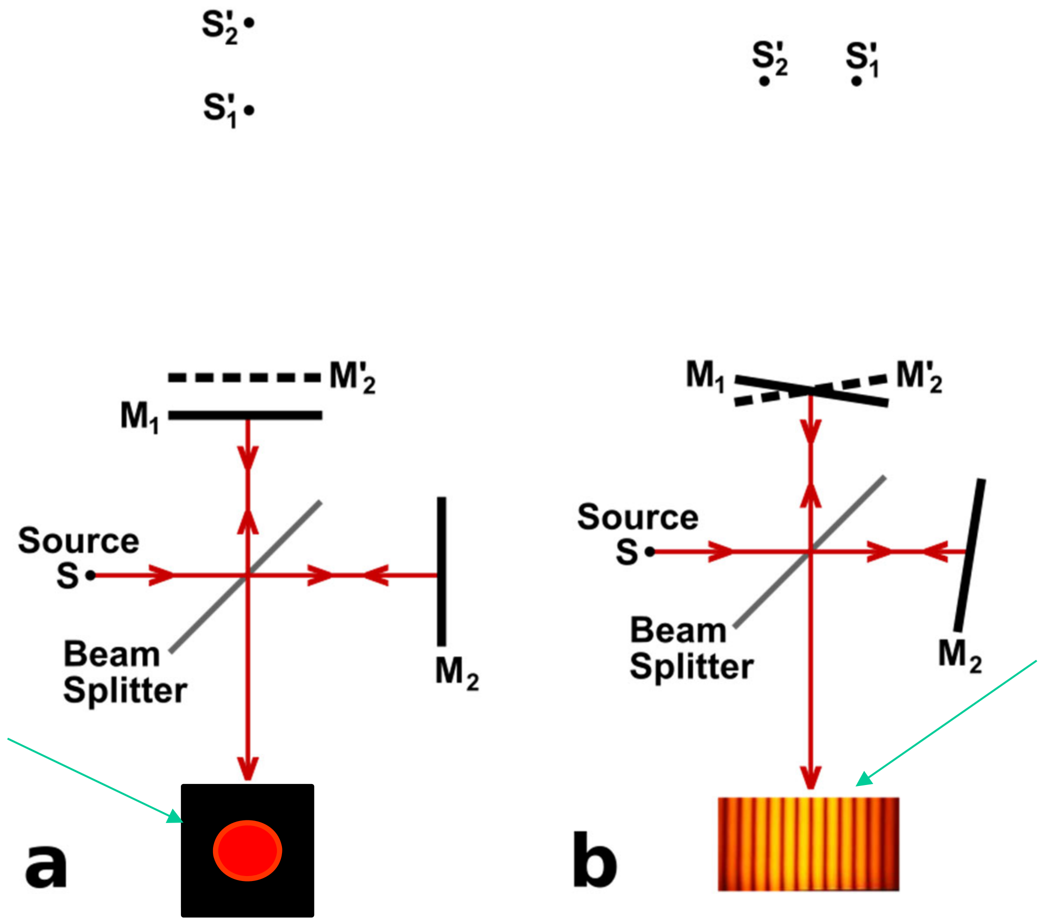
FIGURE 11.4
Moiré fringes formed by a combination of rotation and difference in pitch.

If both incident waves are plane waves then resulting interference pattern is "straight" line interference. Examples are moiré pattern (spatial interference) created by two plane wave line pattern

Coplanar interference

If true plane wave interfere

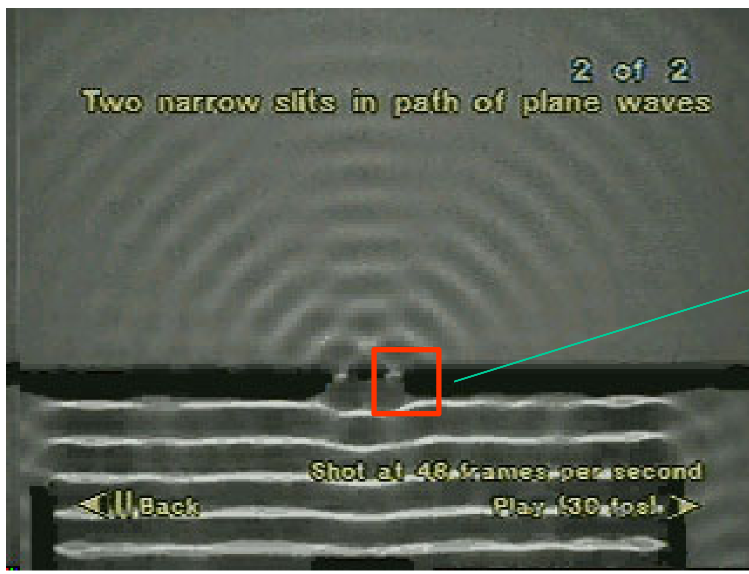
Intensity will vary moving mirror according to the interference equation

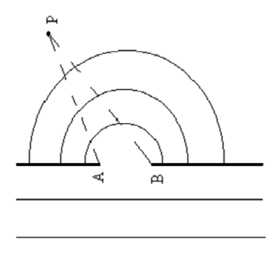


Spatial interference pattern will vary with moving mirror

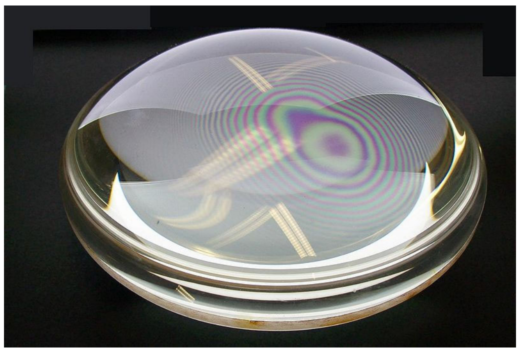
Diverging beam interference

Two diverging beams (spherical waves) will eventually meet up and interfere





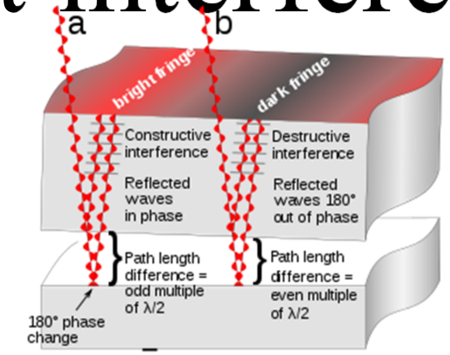
Newtonian Rings (Fabry-Perot interference)

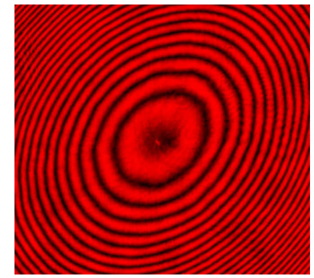


"Newton's rings" interference fringes between 2 plano convex lenses, placed together with their plane surfaces in contact. The rings are created by interference between the light reflected off the two surfaces, caused by a slight gap between them, showing that these surfaces are not precisely plane but are slightly convex.

For glass surfaces that are not spherical, the fringes will not be rings but will have other shapes.

W. Wang





For illumination from above, with a dark center, the radius of the Nth bright ring is given by

$$r_N = \left[\lambda R \left(N - \frac{1}{2}\right)\right]^{1/2},$$

where N is the bright-ring number, R is the radius of curvature of the glass lens the light is passing through, and λ is the wavelength of the light.

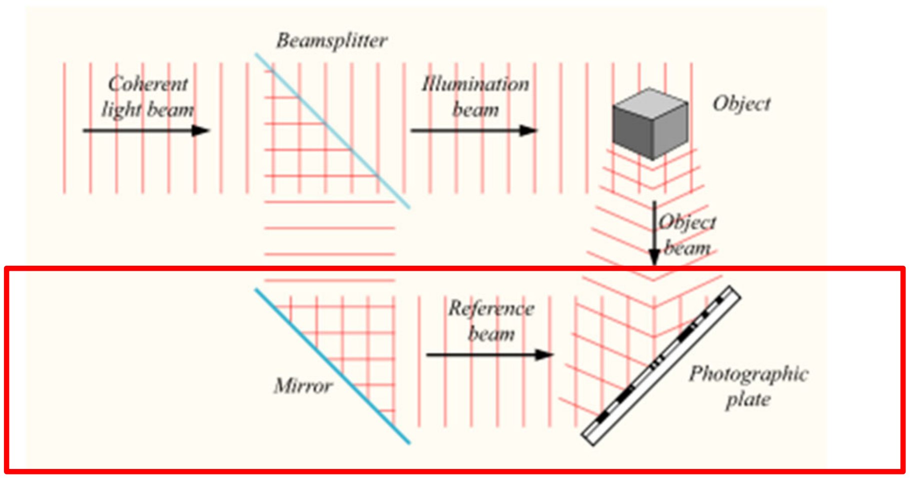
Given the radial distance of a bright ring, *r*, and a radius of curvature of the lens, *R*, the air gap between the glass surfaces, *t*, is given to a good approximation by

$$t = \frac{r^2}{2R}$$

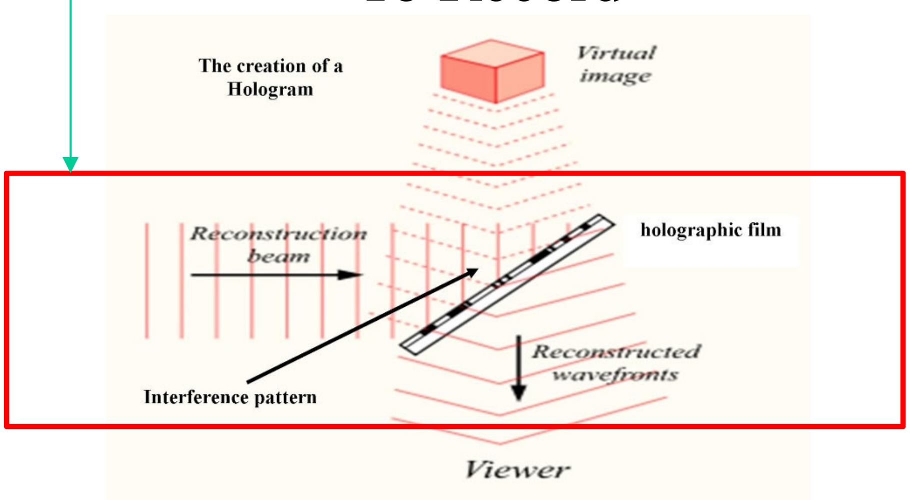
where the effect of viewing the pattern at an angle oblique to the incident rays is ignored.

Spatial Interference Examples

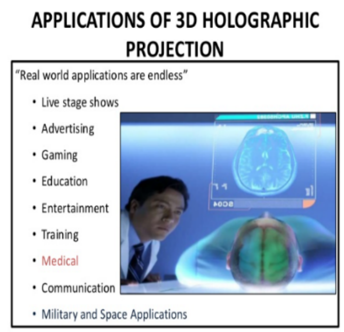
Holography

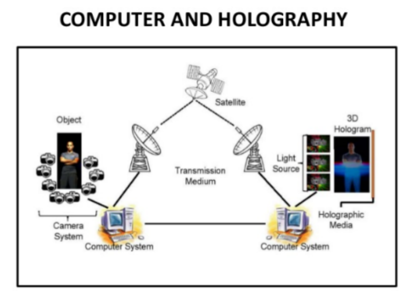


To Record



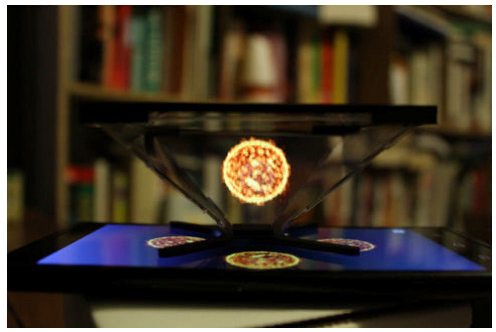
To view

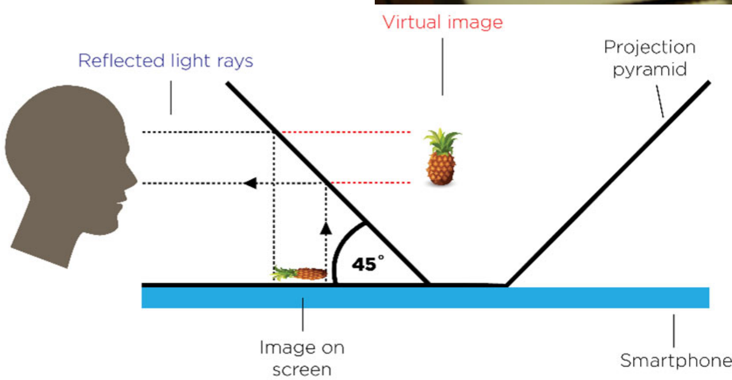


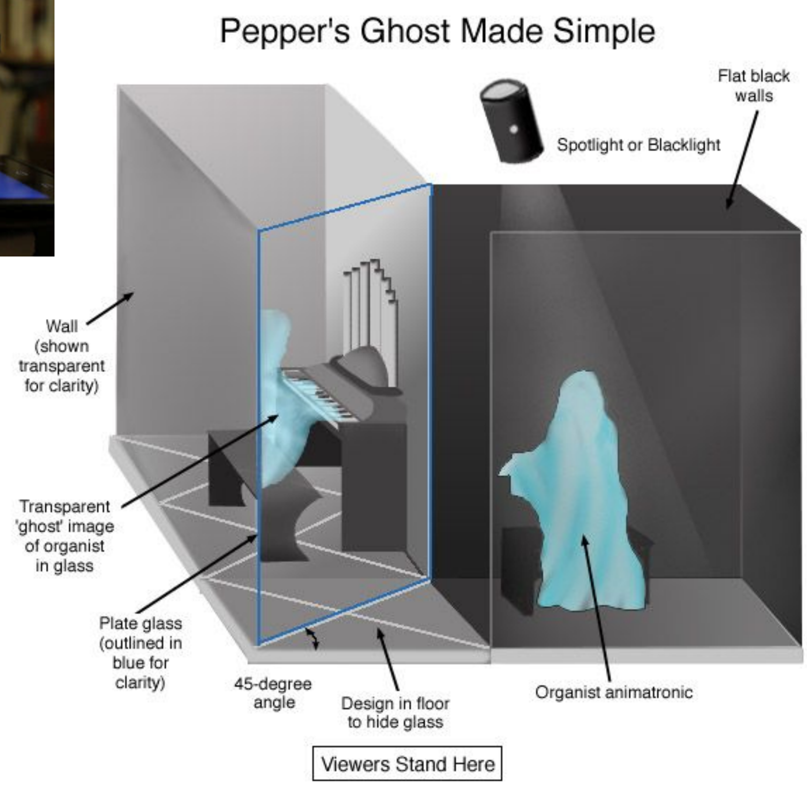


Pepper's Ghost Illusion (not interference hologram)

Due to reflection







Pepper's Ghost Illusion, when a real or recorded image is reflected in a transparent screen at a 45° angle, viewers see a reflected virtual image that seems to have depth and appear out of nowhere

Pepper's Ghost Illusion continues to be used in amusement parks and theatres

Hologram

basic **transmission hologram** setup for now.

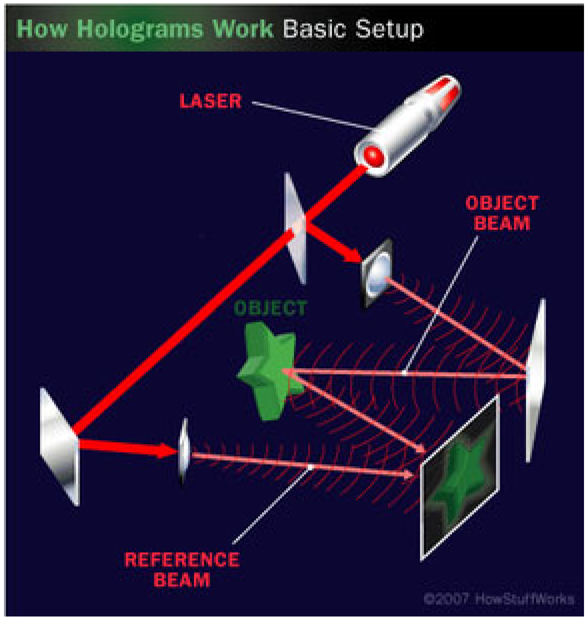
1.The laser points at the beam splitter, which divides the beam of light into two parts.

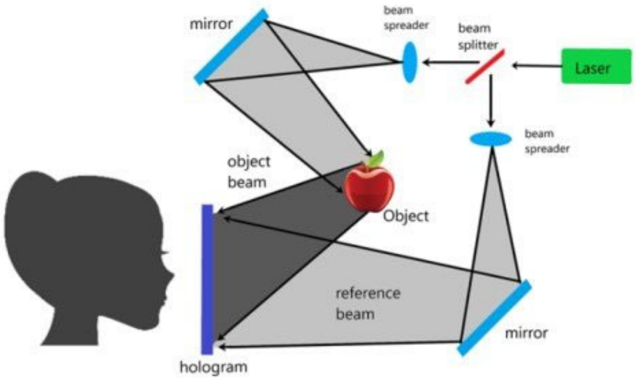
2.Mirrors direct the paths of these two beams so that they hit their intended targets.

3.Each of the two beams passes through a diverging lens and becomes a wide swath of light rather than a narrow beam.

4.One beam, the **object** beam, reflects off of the object and onto the photographic emulsion.

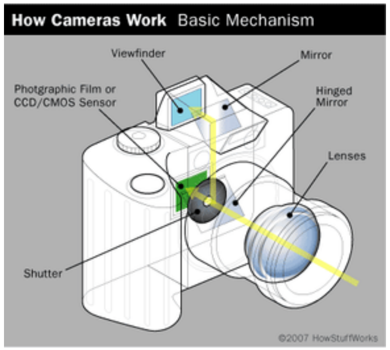
5.The other beam, the **reference** beam, hits the emulsion without reflecting off of anything other than a mirror.





There are holograms on most driver's licenses, ID cards and credit cards. If you're not old enough to drive or use credit, you can still find holograms around your home. They're part of CD, DVD and software packaging, as well as just about everything sold as "official merchandise."

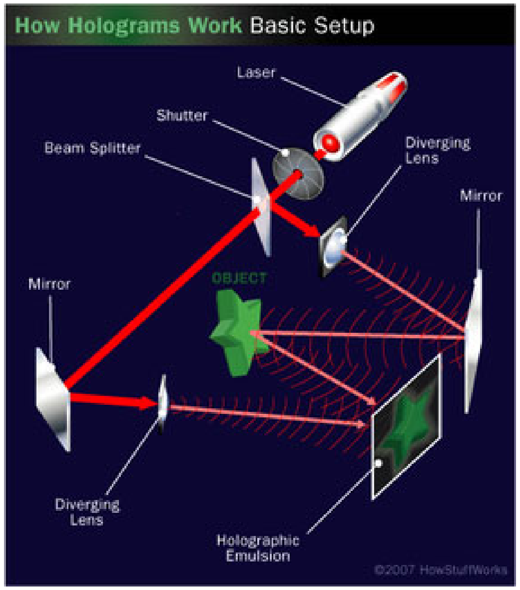
Hologram



When you take a picture with a film camera, four basic steps happen in an instant:

1. A shutter opens.
2. Light passes through a lens and hits the photographic emulsion on a piece of film.
3. A light-sensitive compound called **silver halide** reacts with the light, recording its **amplitude,** or intensity, as it reflects off of the scene in front of you.
4. The shutter closes.

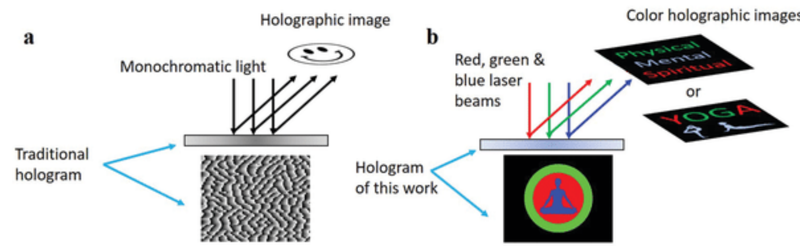
Recording holograms requires steps that are similar to what it takes to make a photograph:
A shutter opens or moves out of the path of a laser. (In some setups, a **pulsed** laser fires a single pulse of light, eliminating the need for a shutter.)
The light from the object beam reflects off of an object. The light from the reference beam bypasses the object entirely.
The light from both beams comes into contact with the photographic emulsion, where light-sensitive compounds react to it.
The shutter closes, blocking the light.



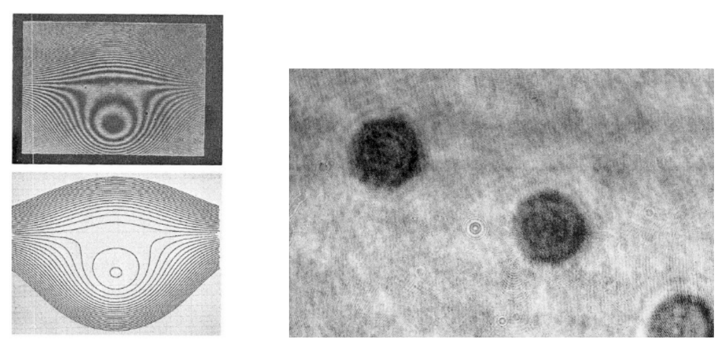
In holography, light passes through a shutter and lenses before striking a light-sensitive piece of holographic film.

Hologram

Turning this frame of film into an image requires the right **illumination**. In a **transmission** hologram, monochromatic light shines through the hologram to make an image. In a **reflection** hologram, monochromatic or white light reflects off of the surface of the hologram to make an image. Your eyes and brain interpret the light shining through or reflecting off of the hologram as a representation of a three-dimensional object. The holograms you see on credit cards and stickers are reflection holograms. But when you look at a developed piece of film used to make a hologram, you don't see anything that looks like the original scene. Instead, you might see a dark frame of film or a random pattern of lines and swirls.



You need the right light source to see a hologram because it records the light's **phase and amplitude like a code.** Rather than recording a simple pattern of reflected light from a scene**, it records the interference between the reference beam and the object beam.** It does this as a pattern of tiny interference fringes. **Each fringe can be smaller than one wavelength of the light used to create them. Decoding these interference fringes requires a key -- that key is the right kind of light.**



Holographic interference microscopy

Decoding the Fringes

The diffraction grating and reflective surfaces inside the hologram **recreate the original object beam.** This beam is absolutely identical to the original object beam before it was combined with the reference wave. **This is what happens when you listen to the radio. Your radio receiver removes the sine wave that carried the amplitude- or frequency-modulated information. The wave of information returns to its original state, before it was combined with the sine wave for transmission.**

Since the object was on the other side of the holographic plate, the beam travels toward you. Your eyes focus this light, and your brain interprets it as a three-dimensional image located behind the transparent hologram. This may sound far-fetched, but you encounter this phenomenon every day. Every time you look in a mirror, you see yourself and the surroundings behind you as though they were on the other side of the mirror's surface. But the light rays that make this image aren't on the other side of the mirror -- they're the ones that bounce off of the mirror's surface and reach your eyes. Most holograms also act like color filters, so you see the object as the same color as the laser used in its creation rather than its natural color.

Decoding the Fringes

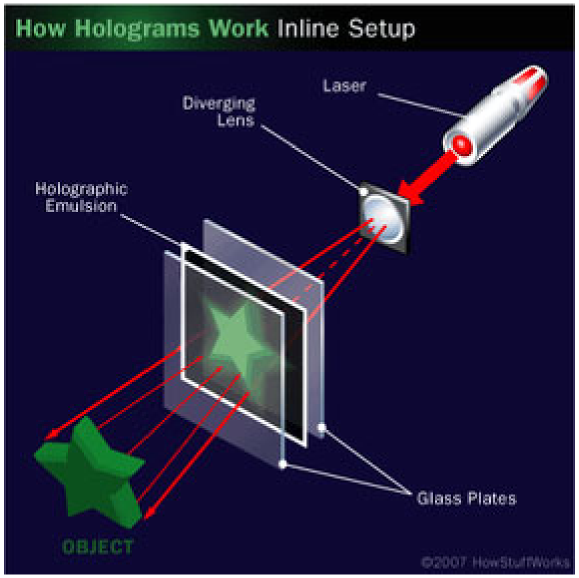
This virtual image comes from the **light that hits the interference fringes and spreads out on the way to your eyes.** However, **light that hits the reverse side of each fringe does the opposite**. Instead of **moving upward and diverging, it moves downward and converges.** It turns into a focused reproduction of the object -- a real image that you can see if you put a screen in its path. The real image **is pseudoscopic, or flipped back to front** -- it's the **opposite of the virtual image that you can see without the aid of a screen**. With the right illumination, holograms can display both images at the same time. However, in some cases, whether you see the real or the virtual image depends on what side of the hologram is facing you.

Your brain plays a big role in your perception of both of these images. When your eyes detect the light from the virtual image, your brain interprets it as a beam of light reflected from a real object. Your brain uses multiple cues, including, shadows, the relative positions of different objects, distances and parallax, or differences in angles, to interpret this scene correctly. It uses these same cues to interpret the pseudoscopic real image.

Basically when wave coming through are constructively interfere, image will appears

Decoding the Fringes

Reflection holograms are **often thicker** than transmission holograms. There is more physical space for recording interference fringes. This also means that there are more layers of reflective surfaces for the light to hit. You can think of holograms that are made this way as having multiple layers that are only about half a wavelength deep. When light enters the first layer, some of it reflects back toward the light source, and some continues to the next layer, where the process repeats. The light from each layer interferes with the light in the layers above it. This is known as the **Bragg effect**, and it's a necessary part of the reconstruction of the object beam in reflection holograms. In addition, holograms with a strong Bragg effect are known as thick holograms, while those with little Bragg effect are thin.



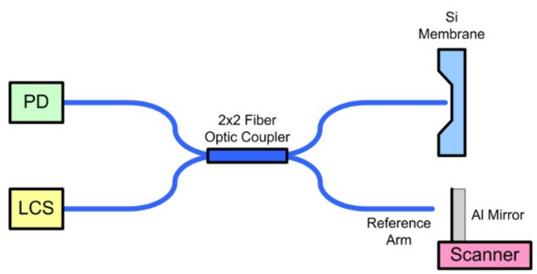
Reflection holograms

The holograms found on credit cards and other everyday objects are mass-produced by stamping the pattern of the hologram onto the foil.
IMAGE COURTESY DREAMSTIME

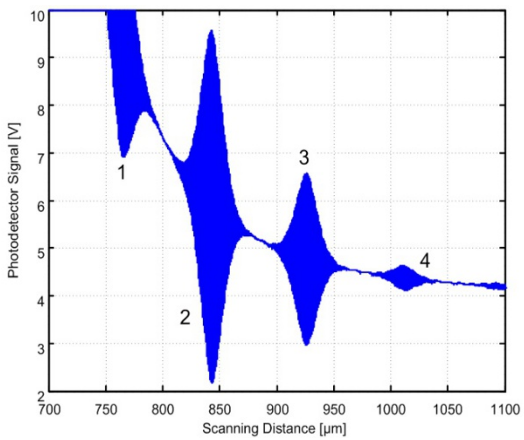
The Bragg effect can also change the way the hologram reflects light, especially in holograms that you can view in white light. At different viewing angles, the Bragg effect can be different for different wavelengths of light. This means that you might see the hologram as one color from one angle and another color from another angle.

Fibroptics Interferometer Examples

Temporal



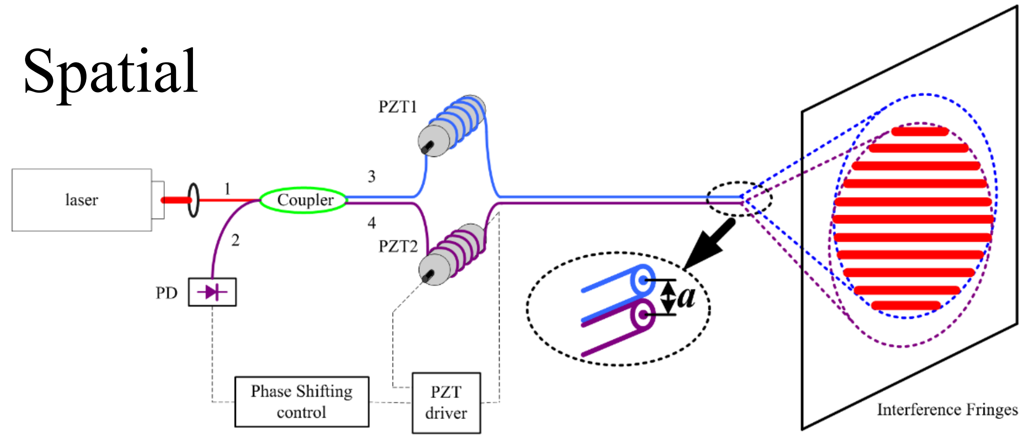
Zoran Djinovic



Fiber-Optic Low-Coherence interferometer

W. Wang

Spatial



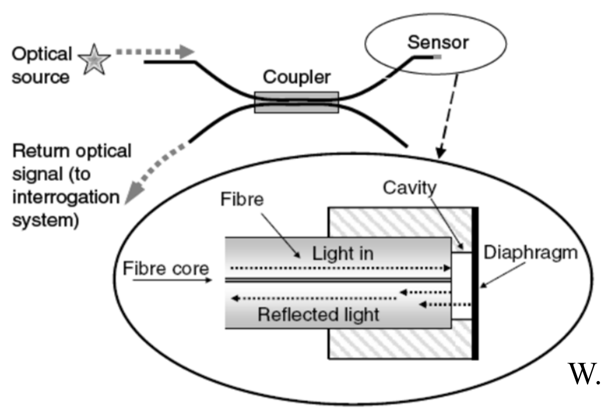
F. Zhang

Mach–Zehnder interferometer

Application: FIBER DOPPLER DIFFERENTIAL VELOCIMETER

W. Wang

Temporal



Fabry Perot Microphone

W. Wang

Multiple slits

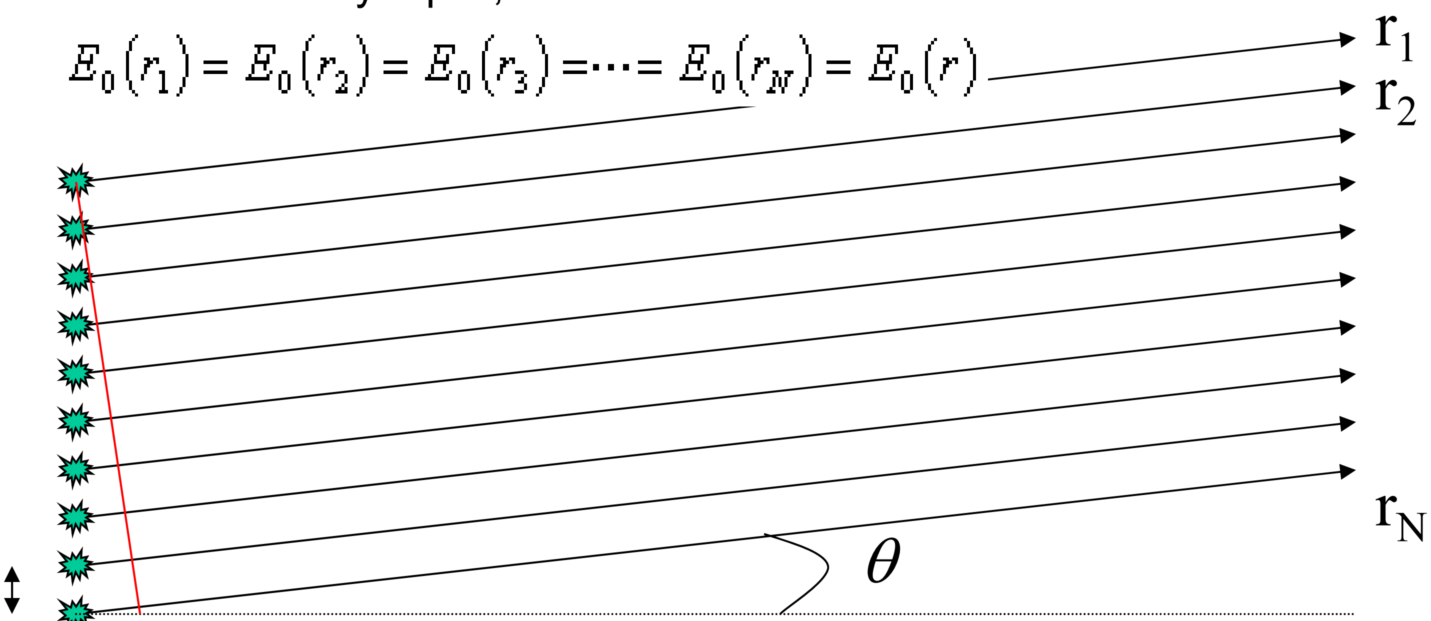
Two Approaches

- Full wave method (EM theory)
- Phase only (Physics class)

Line of point sources (pinholes), all in phase with same amplitude

If the spatial extent of the oscillator array is small compared to the wavelength of the radiation, then the amplitudes of the separate waves arriving at some observation point *P* will be essentially equal,

$$E_0(r_1) = E_0(r_2) = E_0(r_3) = \cdots = E_0(r_N) = E_0(r)$$



Series of spherical sources with a gap space (d)

d=distance between sources

Note that:

$$r_2 - r_1 = d\sin\theta \qquad r_3 - r_1 = 2d\sin\theta$$

$$r_4 - r_1 = 3d\sin\theta \qquad r_N - r_1 = (N-1)d\sin\theta$$

The sum of the interfering spherical wavelets yields a composite electric field at *P* that is the real part of

$$E = E_o(r)e^{i(kr_1-\omega t)} + E_o(r)e^{i(kr_2-\omega t)} + \cdots. + E_o(r)e^{i(kr_N-\omega t)}$$

Rearrange to get

$$E = E_o(r)e^{i(kr_1-\omega t)}[1 + e^{ik(r_2-r_1)} + e^{ik(r_3-r_1)} \dots \dots. + e^{ik(r_N-r_1)}$$

The phase difference between adjacent sources is obtained from the expression $\delta = k_0\Lambda = 2\pi\Lambda/\lambda$ where the maximum optical-path length difference is $\Lambda = nd \sin\theta$ in a medium with an index of refraction *n*.

But, since d is the distance between two adjacent oscillators, it can be easily seen that $\delta = d\,sin\theta = r_2 - r_1$. Thus, the field at P becomes

$$E = E_o(r)e^{i(kr_1-\omega t)}\left[1 + \left(e^{i\delta}\right) + \left(e^{i\delta}\right)^2 + \cdots \ldots + \left(e^{i\delta}\right)^{N-1}\right]$$

$$= E_o(r)e^{i(kr_1-\omega t)}\,\frac{e^{iN\delta}-1}{e^{i\delta}-1}$$

$$= E_o(r)e^{i(kr_1-\omega t)}\,\frac{e^{\frac{iN\delta}{2}}\boxed{\left(e^{\frac{iN\delta}{2}}-e^{\frac{-iN\delta}{2}}\right)}}{e^{i\delta/2}\boxed{\left(e^{\frac{i\delta}{2}}-e^{\frac{-i\delta}{2}}\right)}}$$

$$= E_o(r)e^{i(kr_1-\omega t)}e^{\frac{i(N-1)\delta}{2}}\,\boxed{\frac{(\sin(N\delta/2)}{\sin(\delta/2)}}$$

Look at next page, ½ is from the identity

$$= E_o(r)e^{i(kR-\omega t)}\,\frac{(\sin(N\delta/2)}{\sin(\delta/2)}$$

$$I(P) = \frac{|E|^2}{2Z_o} \Rightarrow I = I_0\left[\frac{\sin\left(\frac{Nkd}{2}\sin\theta\right)}{\sin\left(\frac{kd}{2}\sin\theta\right)}\right]^2$$

where $\boxed{R = r_1 + \frac{1}{2}(N-1)d\sin\theta}$ is the distance from the center of the line of oscillators to the point P.

Trigonometric Functions in Terms of Exponential Functions

$$\boxed{\sin x = \frac{e^{ix} - e^{-ix}}{2i}} \qquad \csc x = \frac{2i}{e^{ix} - e^{-ix}}$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2} \qquad \sec x = \frac{2}{e^{ix} + e^{-ix}}$$

$$\tan x = \frac{e^{ix} - e^{-ix}}{i\left(e^{ix} + e^{-ix}\right)} \qquad \cot x = \frac{i\left(e^{ix} + e^{-ix}\right)}{e^{ix} - e^{-ix}}$$

Exponential Function vs. Trigonometric and Hyperbolic Functions

$$e^{ix} = \cos x + i \sin x \qquad e^{x} = \cosh x + \sinh x$$

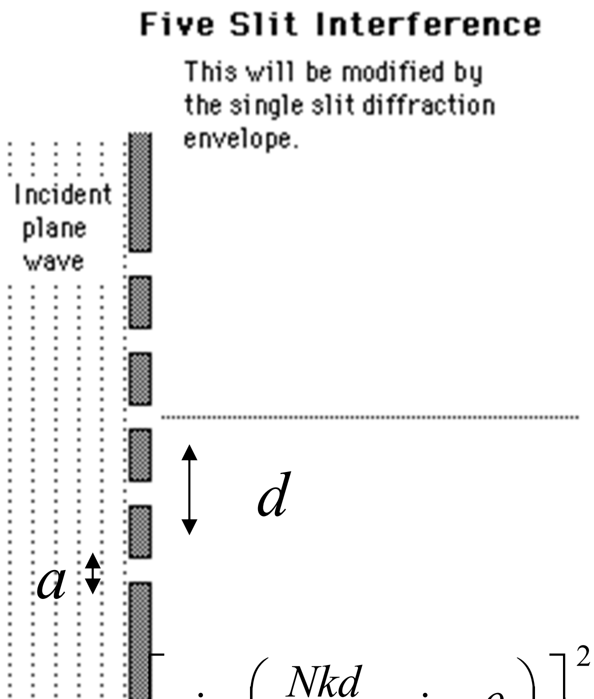
Hyperbolic Functions in Terms of Exponential Functions

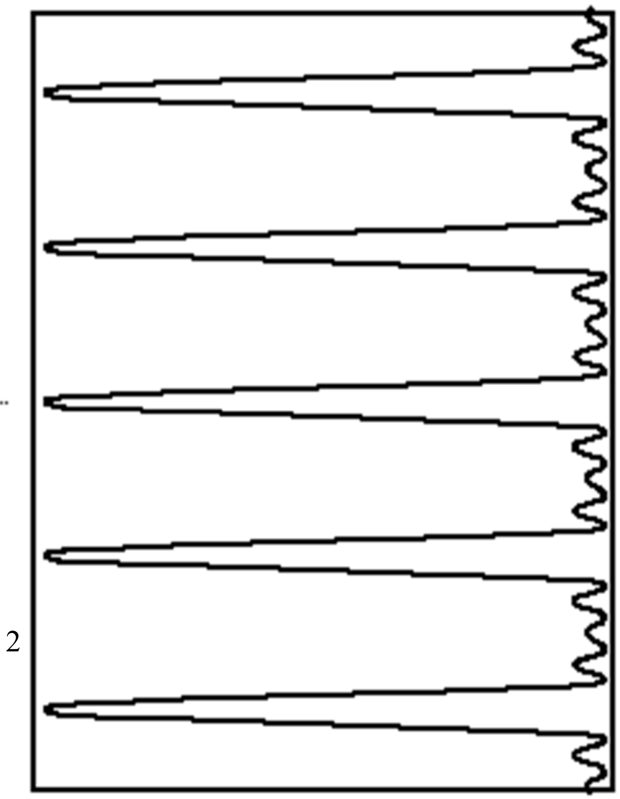
$$\sinh x = \frac{e^{x} - e^{-x}}{2} \qquad \operatorname{csch} x = \frac{2}{e^{x} - e^{-x}}$$

$$\cosh x = \frac{e^{x} + e^{-x}}{2} \qquad \operatorname{sech} x = \frac{2}{e^{x} + e^{-x}}$$

$$\tanh x = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}} \qquad \coth x = \frac{e^{x} + e^{-x}}{e^{x} - e^{-x}}$$

Multiple Slits



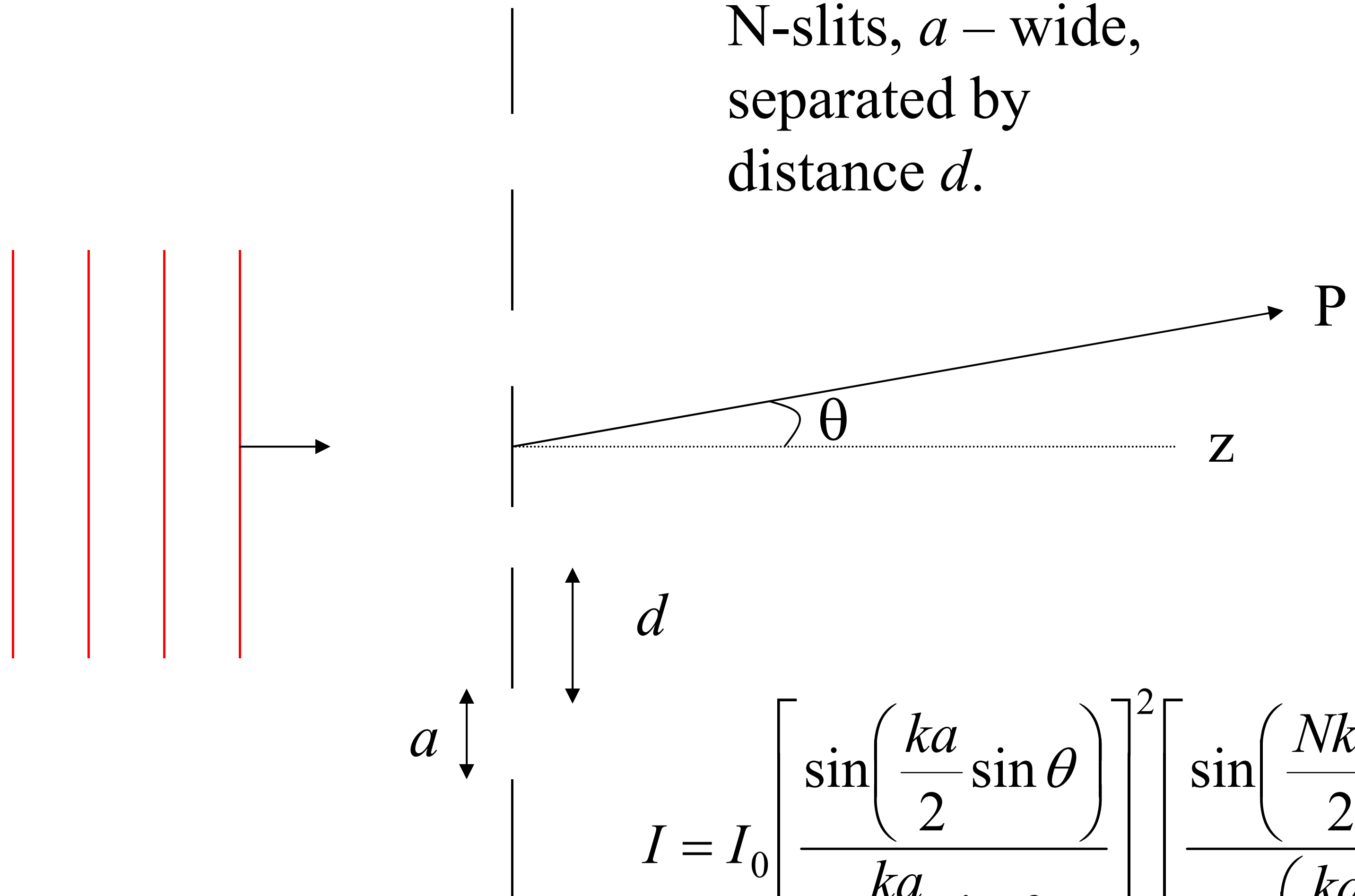


$$I = I_0 \left[\frac{\sin\left(\frac{Nkd}{2}\sin\theta\right)}{\sin\left(\frac{kd}{2}\sin\theta\right)} \right]^2$$

Under the Fraunhofer conditions, the light curve of a multiple slit arrangement will be the interference pattern multiplied by the single slit diffraction envelope. This assumes that all the slits are identical. In this case $a <<< d$.

Use Fraunhofer to model a transmission grating of N-slits

N-slits, a – wide, separated by distance d.



$$I = I_0 \left[\frac{\sin\left(\frac{ka}{2}\sin\theta\right)}{\frac{ka}{2}\sin\theta} \right]^2 \left[\frac{\sin\left(\frac{Nkd}{2}\sin\theta\right)}{\sin\left(\frac{kd}{2}\sin\theta\right)} \right]^2$$

diffraction　　　interference

Slit separation d <~ slit width a

Grating Intensity

The intensity is given by the interference intensity expression

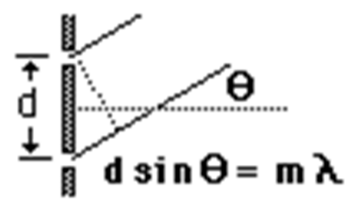
$$I = I_0 \left[\frac{\sin\left(\frac{Nkd}{2}\sin\theta\right)}{\sin\left(\frac{kd}{2}\sin\theta\right)} \right]^2$$

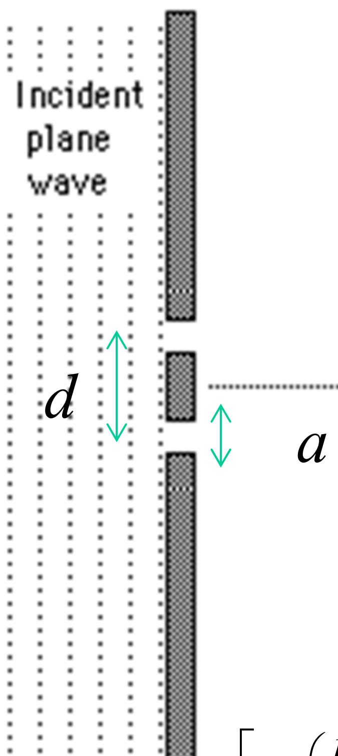
Modulated by the single slit diffraction envelope for the slits which make up the grating:

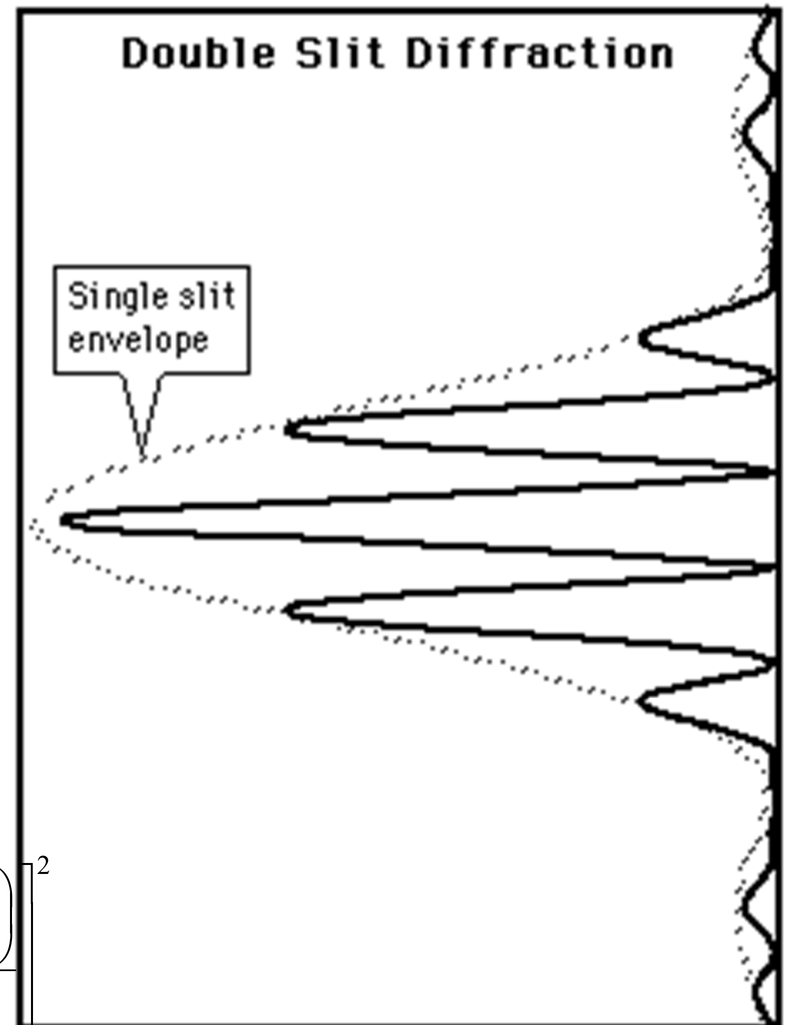
$$I = I_0 \left[\frac{\sin\left(\frac{ka}{2}\sin\theta\right)}{\frac{ka}{2}\sin\theta} \right]^2$$

The given total intensity expression,

$$I = I_0 \left[\frac{\sin\left(\frac{ka}{2}\sin\theta\right)}{\frac{ka}{2}\sin\theta} \right]^2 \left[\frac{\sin\left(\frac{Nkd}{2}\sin\theta\right)}{\sin\left(\frac{kd}{2}\sin\theta\right)} \right]^2$$

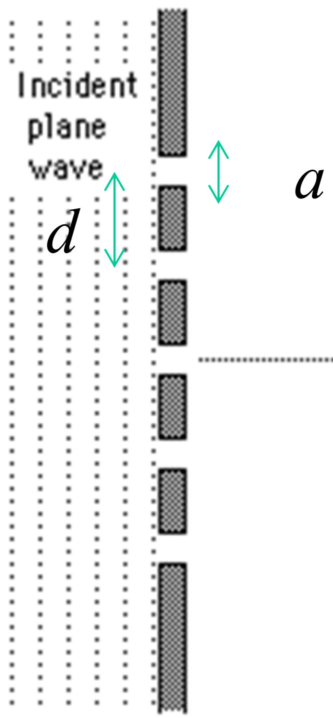


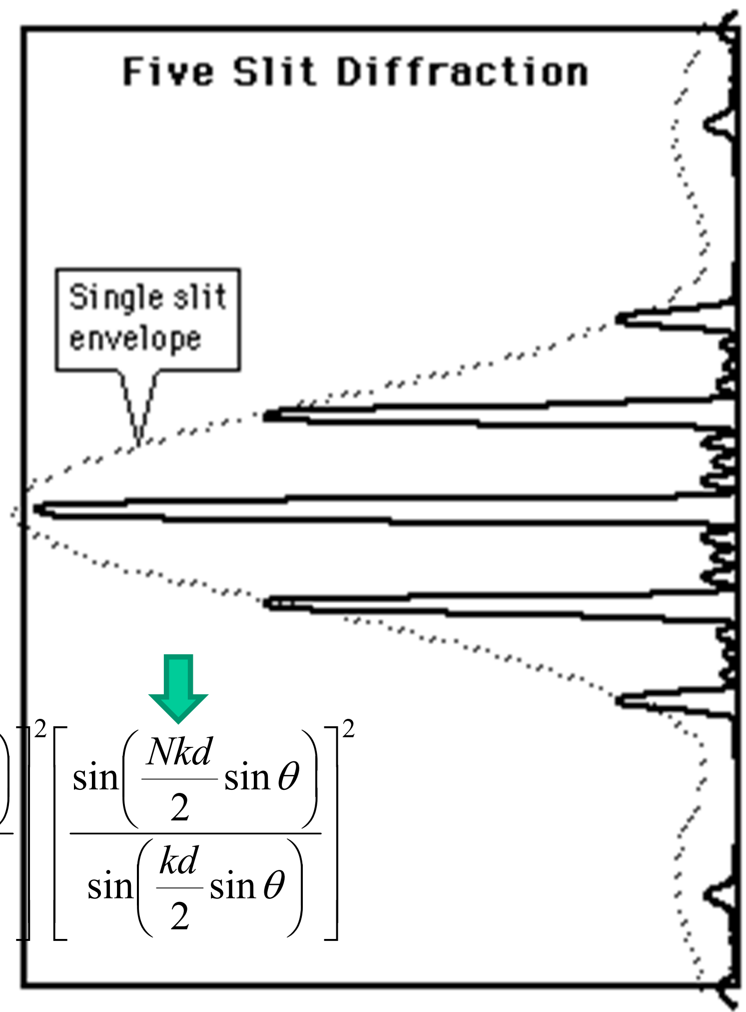




$$I = I_0 \left[\frac{\sin\left(\frac{ka}{2}\sin\theta\right)}{\frac{ka}{2}\sin\theta} \right]^2 \left[\frac{\sin\left(\frac{Nkd}{2}\sin\theta\right)}{\sin\left(\frac{kd}{2}\sin\theta\right)} \right]^2$$

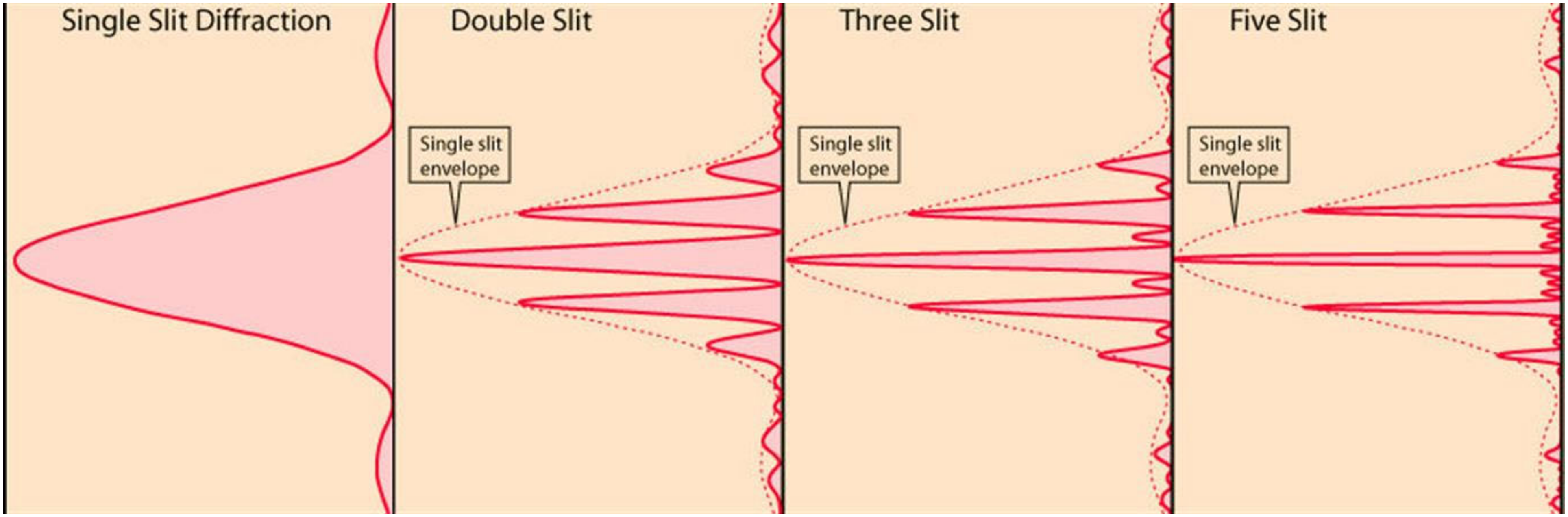
Slit separation d <~ slit width *a*





$$I = I_0 \left[\frac{\sin\left(\frac{ka}{2}\sin\theta\right)}{\frac{ka}{2}\sin\theta} \right]^2 \left[\frac{\sin\left(\frac{Nkd}{2}\sin\theta\right)}{\sin\left(\frac{kd}{2}\sin\theta\right)} \right]^2$$

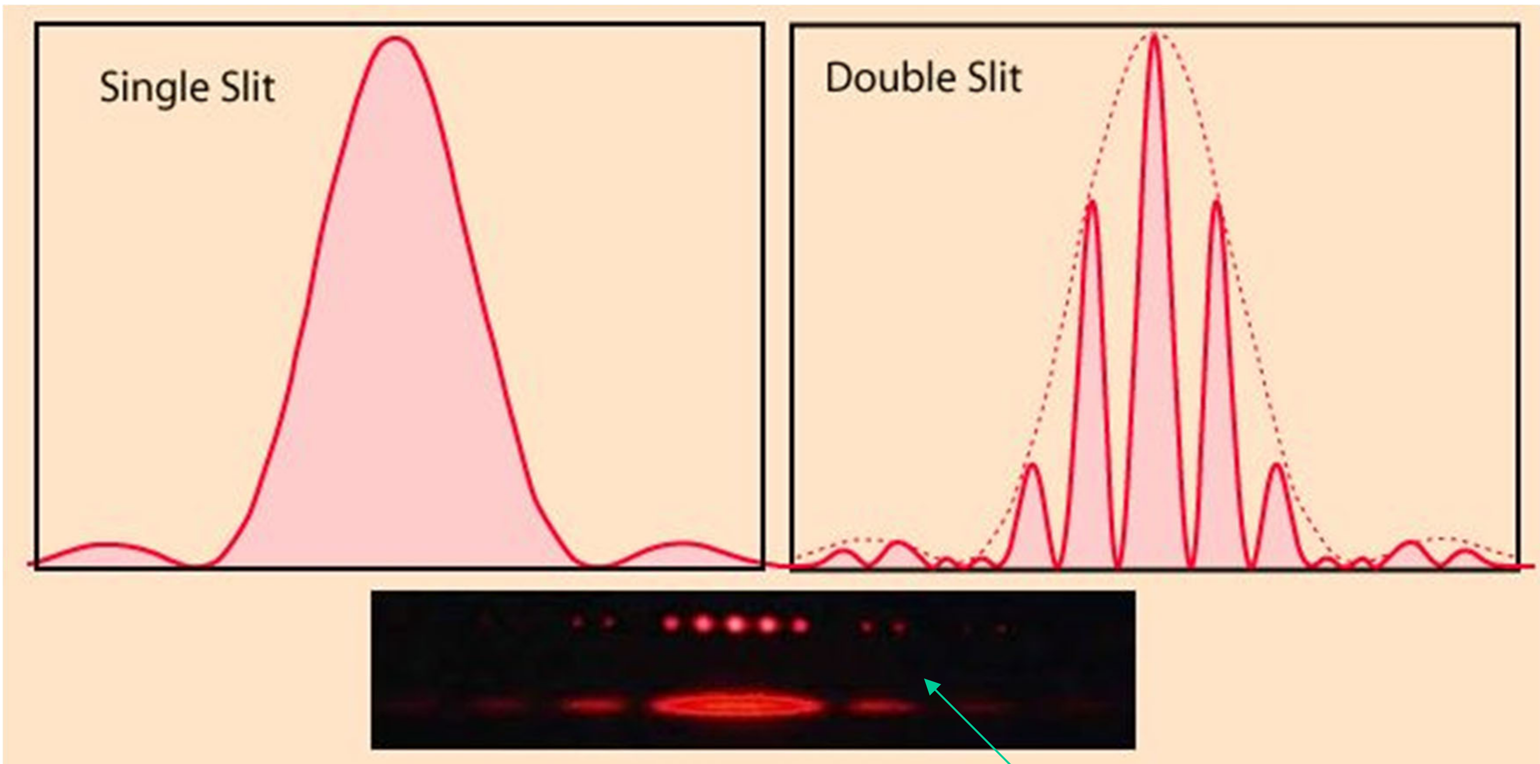
Slit separation d <~slit width a



$a \sim> d.$

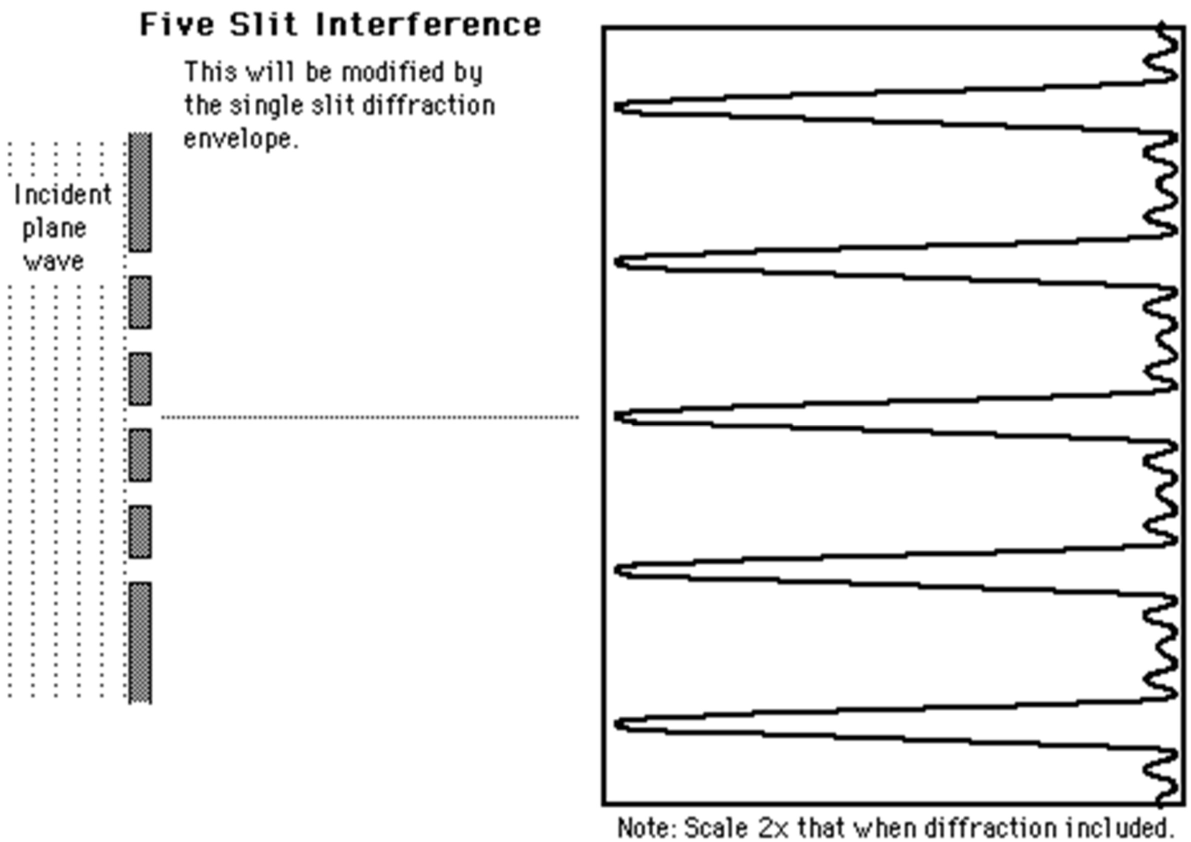
Modulated interference pattern for one slit, two slits, three slits and five slits with <u>all slits the same width</u> and with the same slit separation.

Diffraction modulated interference



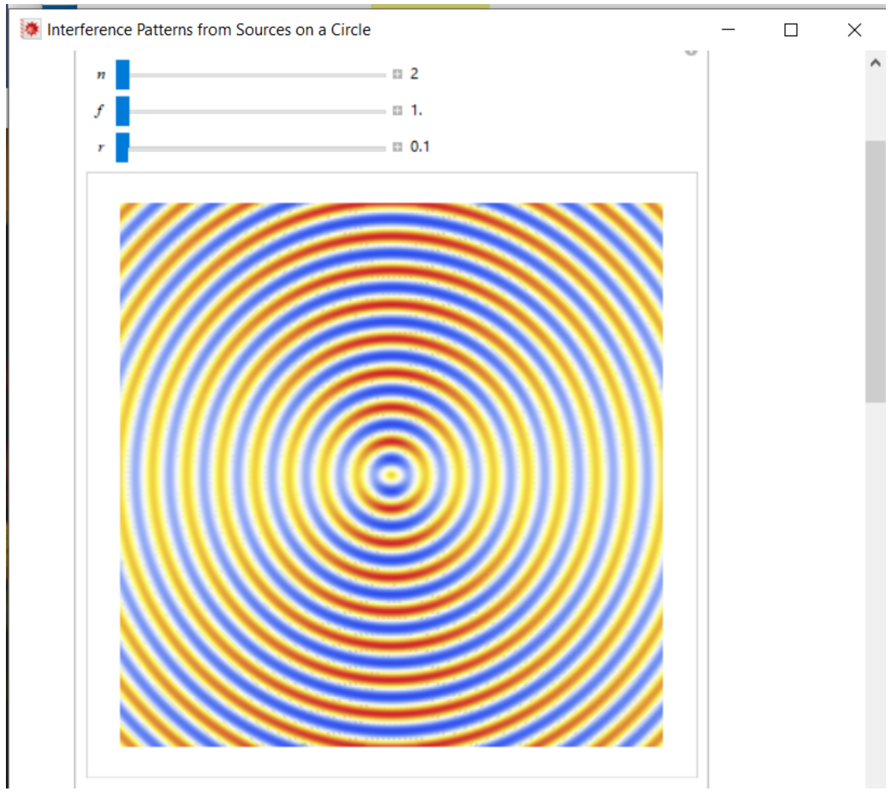
$a \sim > d$.

Multiple Slits



Under the Fraunhofer conditions, the light curve of a multiple slit arrangement will be the interference pattern multiplied by the single slit diffraction envelope. This assumes that all the slits are identical. In this case $a <<< d$.

Interference Patterns from Sources on a Circle



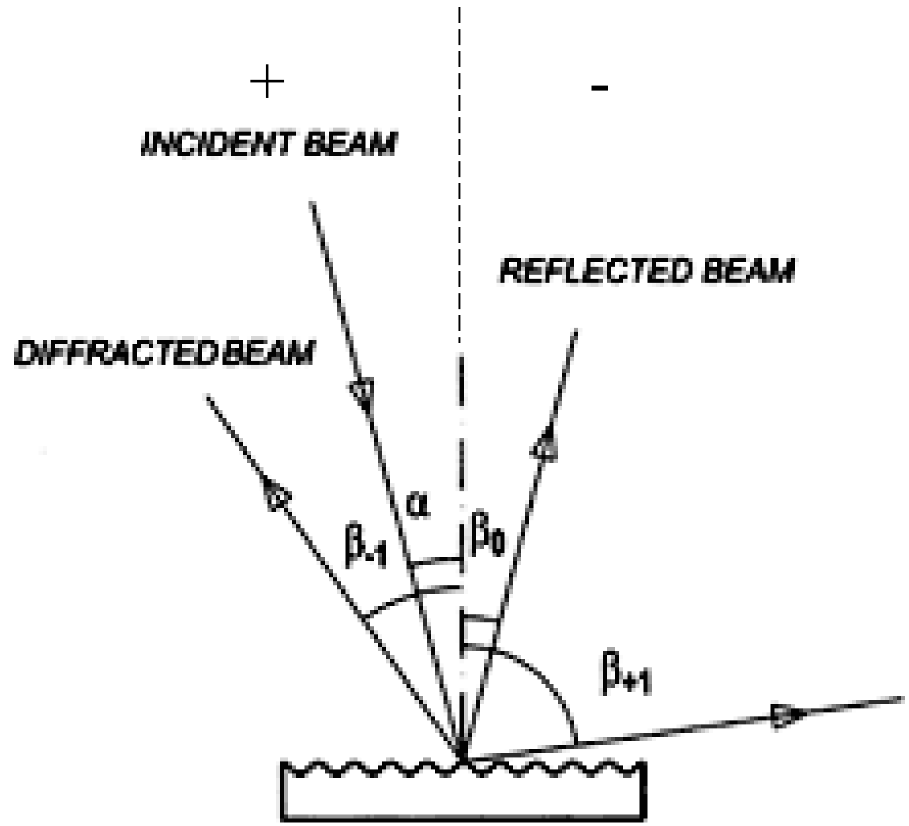
See the interference patterns produced by waves from n sources with frequency f arranged on a circle with radius r.

Two Approaches

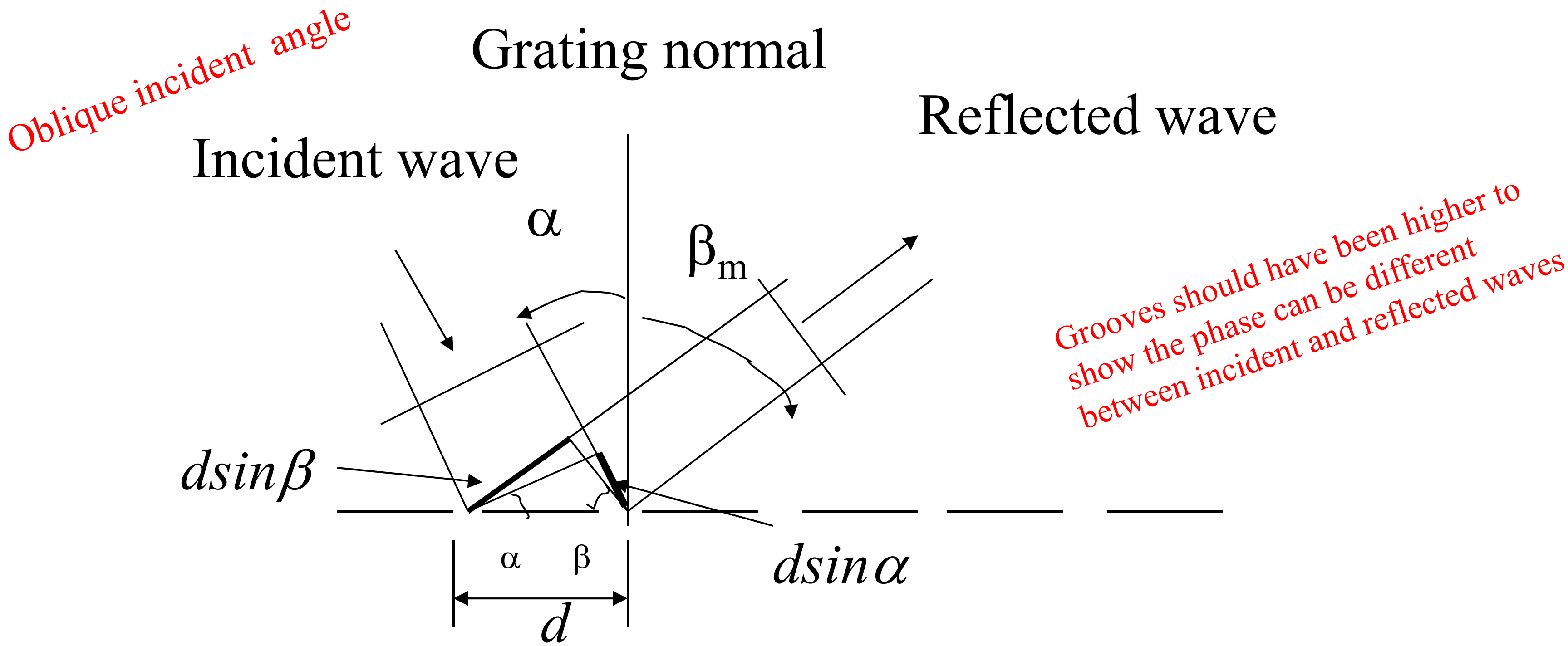
- Full wave method (EM theory)
- Phase only (Physics class)

Reflective Grating

Oblique incident angle



Grating can be made into reflective type and diffractive Grating theory still hold.



The geometrical path difference between light from adjacent grooves is seen to be $d\sin\alpha + d\sin\beta$. The principle of interference dictates that only when this difference equals the wavelength λ of the light, or some integral multiple thereof, will the light from adjacent grooves be in phase (lead to constructive interference)

Path length difference creates **constructive** interference:

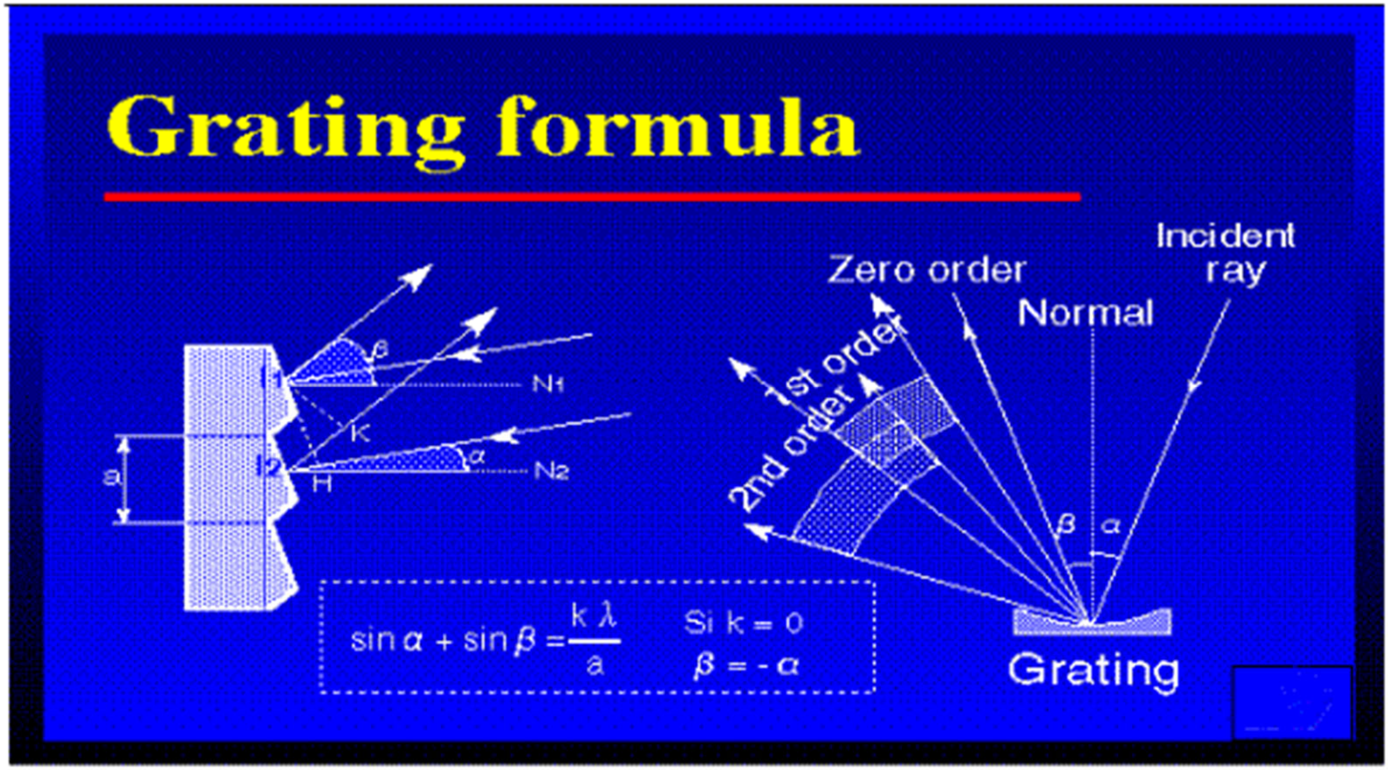
$$d sin\alpha + d sin\beta_m = m\lambda$$

Where m = diffraction order

For a ray arriving with an angle of incidence α, the angle β under which it will be diffracted by a grating of N lines per millimeter depends on the wavelength λ by the grating equation:

$$sin\alpha + sin\beta_m = Nm\lambda$$

Frequency of the grating structure is defined N (lines per millimeter)

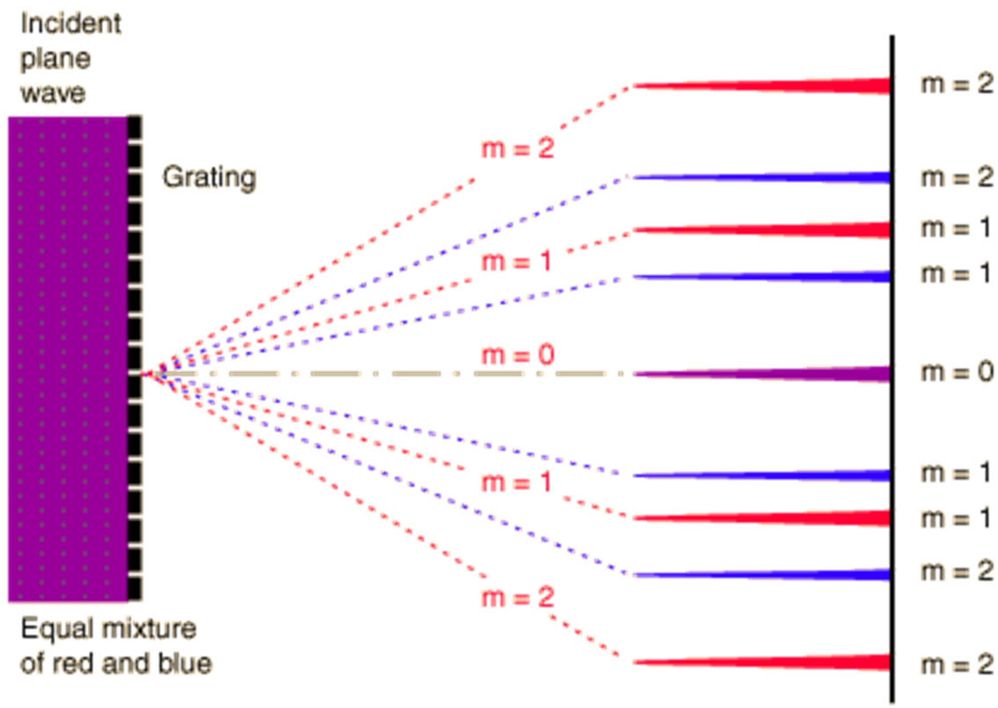


Surface Analytical

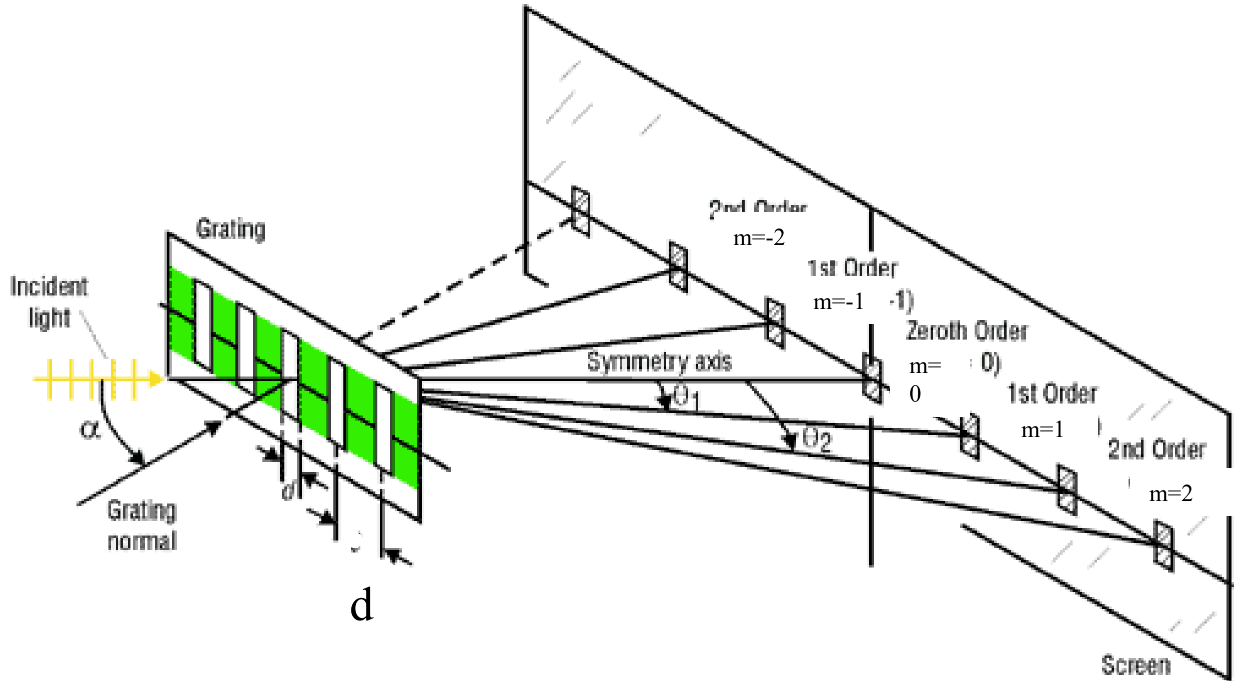
Order zero represents about 40% of the total energy.The rest of the energy is distributed amongst the various orders. Generally, the higher the order, the lower the brightness of its spectrum. The highest orders carry almost no energy. In practice, only the first and second orders are usable.

Diffraction Grating

A diffraction grating is an optical component that serves to periodically modulate the phase or the amplitude of the incident wave. It can be made of a transparent plate with periodically varying thickness or periodically graded refractive index



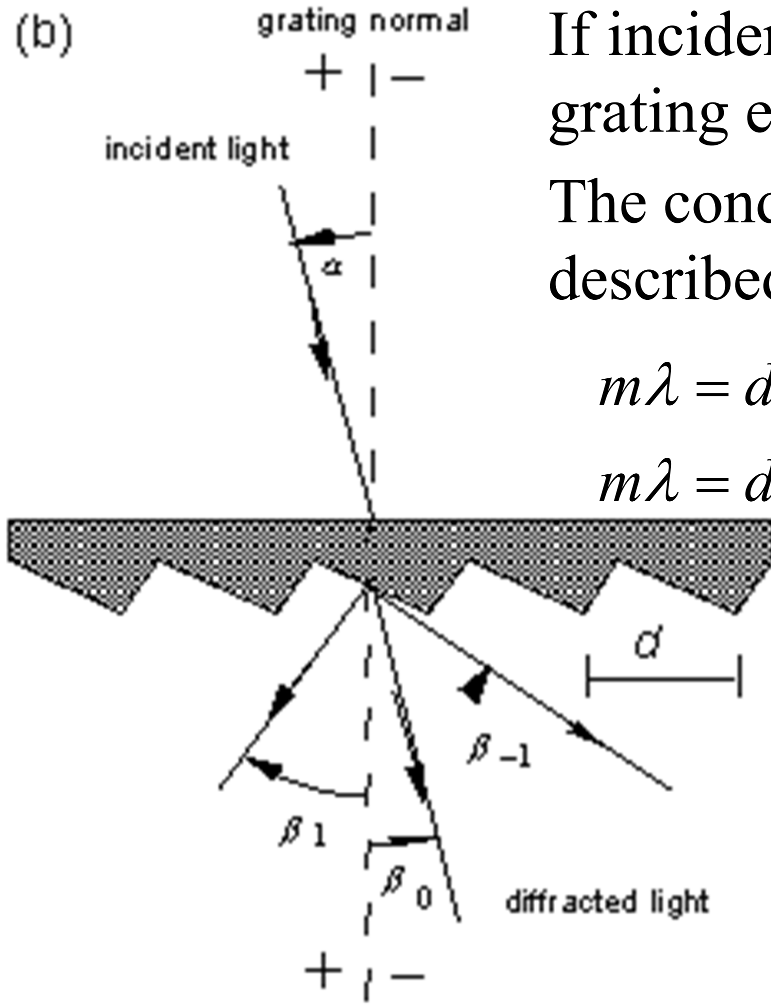
Refractive Diffraction Grating



The light is incident on the grating along the grating normal ($\alpha \neq 0$), the grating equation,

$$d(\sin\alpha + \sin\theta_m) = m\lambda \text{ where } m = 0, \pm 1, \pm 2 \ldots\ldots$$

Diffractive Grating



If incident angle is not normal, the grating equation $m\lambda = d\sin(\beta_m)$

The conditions of diffraction are described by two equations:

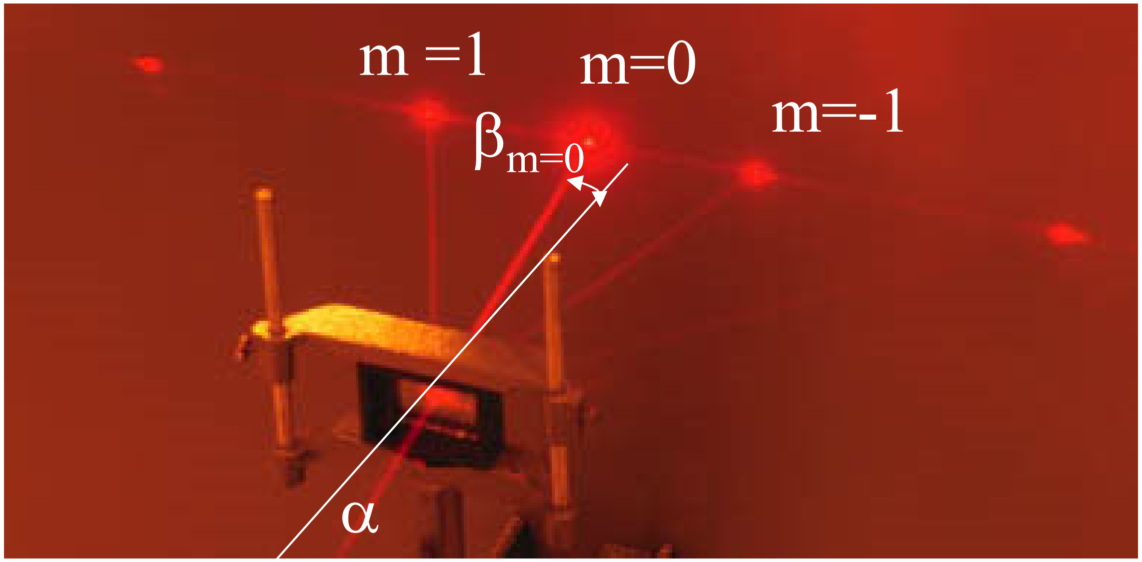
$$m\lambda = d[\sin(\beta_m + \alpha) - \sin\alpha] \qquad m = +$$

$$m\lambda = d[\sin(\beta_m - \alpha) + \sin\alpha] \qquad m = -$$

The condition for maximum intensity is the same as that for the double slit or multiple slits, but with a large number of slits the intensity maximum is very sharp and narrow, providing the high resolution for spectroscopic applications. The peak intensities are also much higher for the grating than for the double slit.

When light of a single wavelength , like the 632.8nm red light from a helium-neon laser at left, strikes a diffraction grating it is diffracted to each side in multiple orders. Orders 1 and 2 are shown to each side of the direct beam. Different wavelengths are diffracted at different angles, according to the grating relationship.

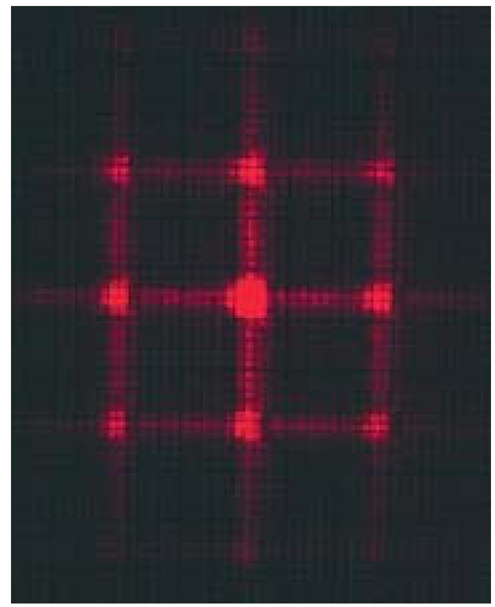
Diffraction Grating and Helium-Neon Laser



While directing the 632.8 nm red beam of a helium-neon laser through a 600 lines/mm diffraction grating, a cloud was formed using liquid nitrogen. You can see the direct beam plus the first and second orders of the diffraction.

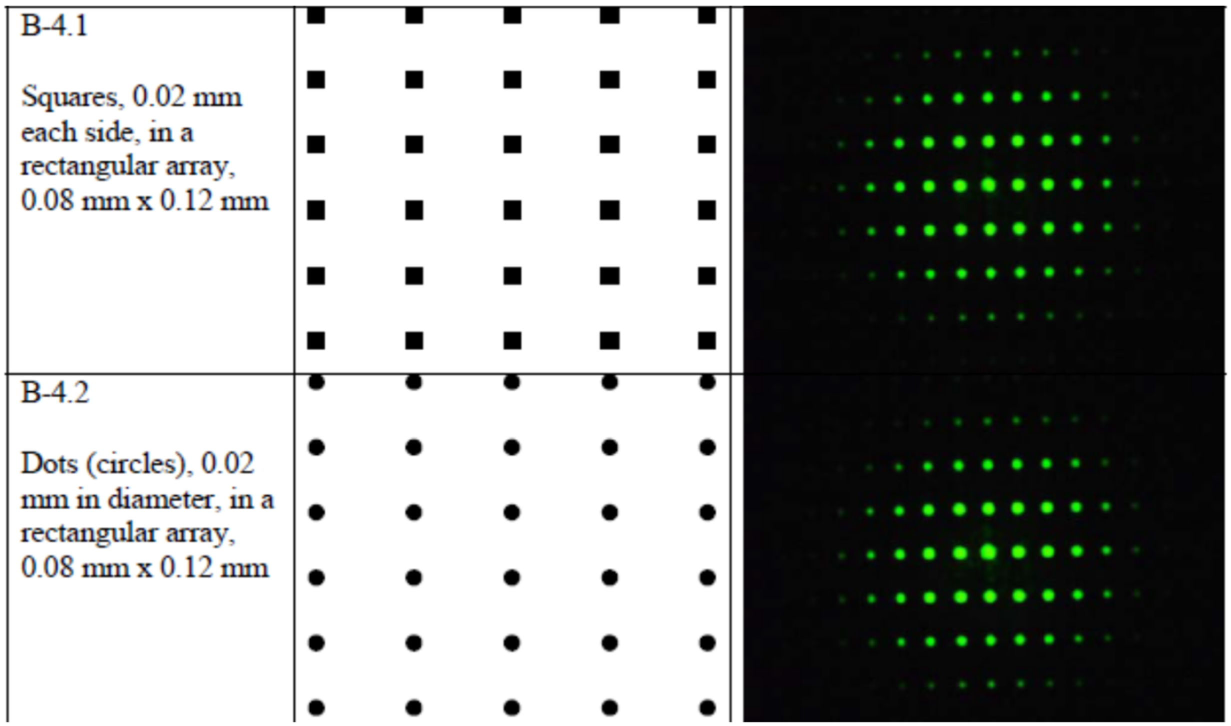
Diffraction from Crossed Slits

Appearance of object.



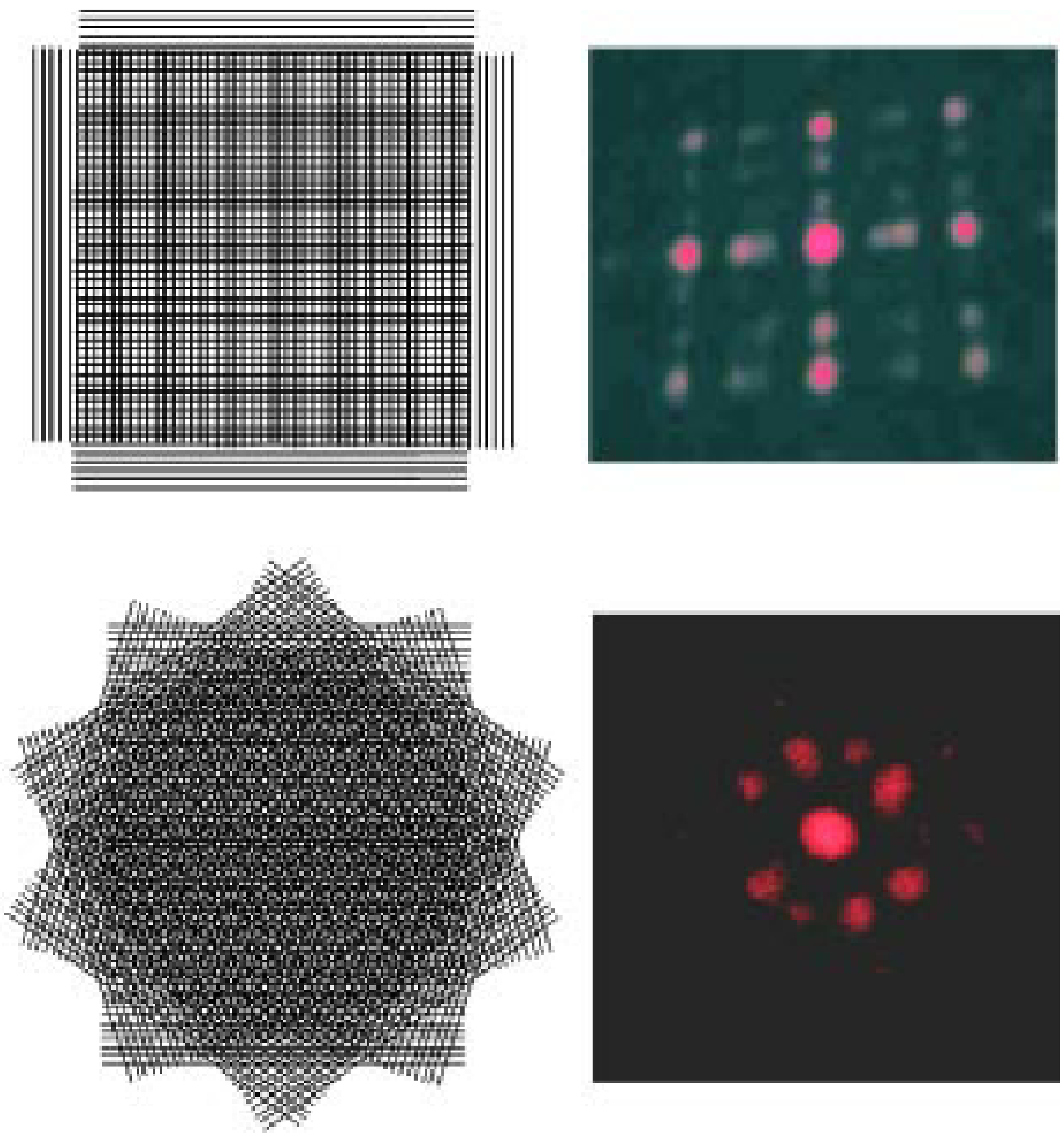
Example of 2D diffraction grating

Diffraction from 2D dots Pattern



Soresen UW physics

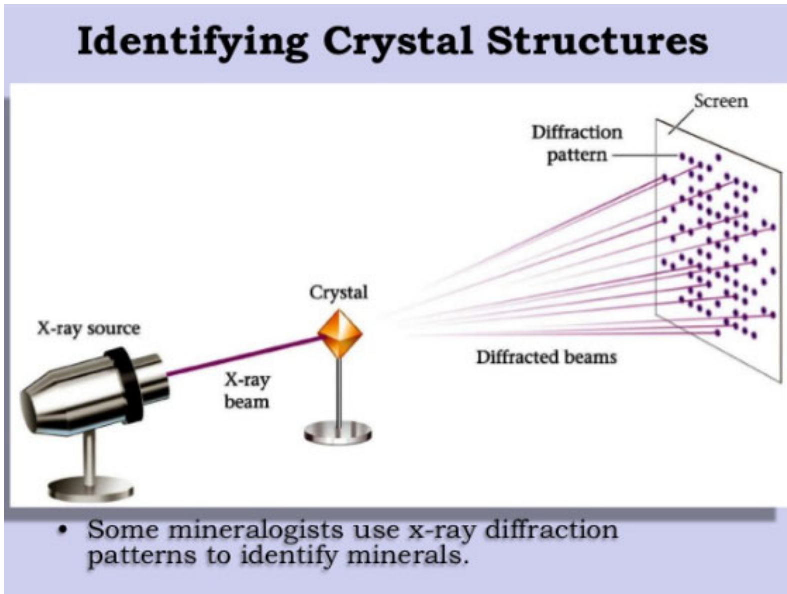
You will see the similar pattern in your Lab 1 grating experiment!

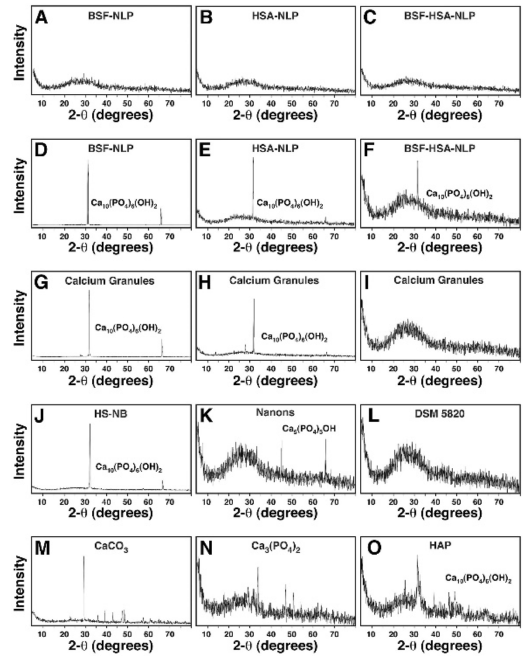


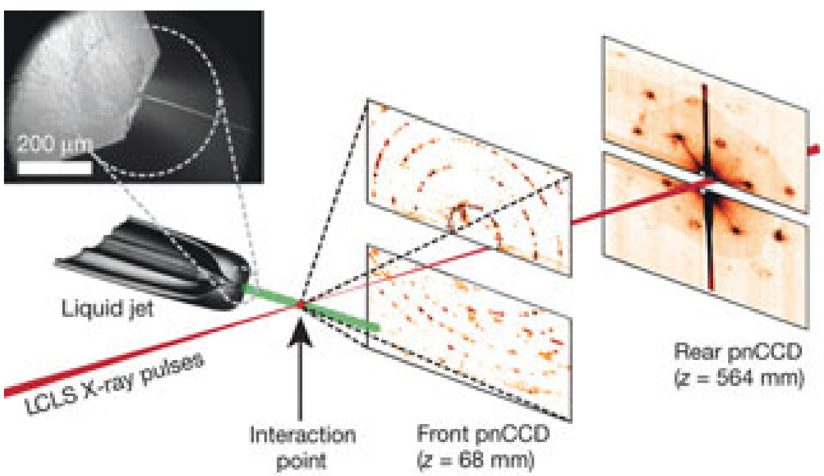
a

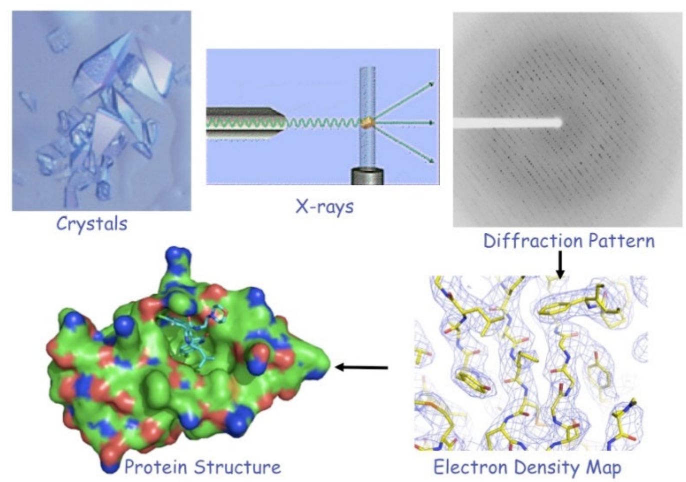
b

X Ray Diffraction



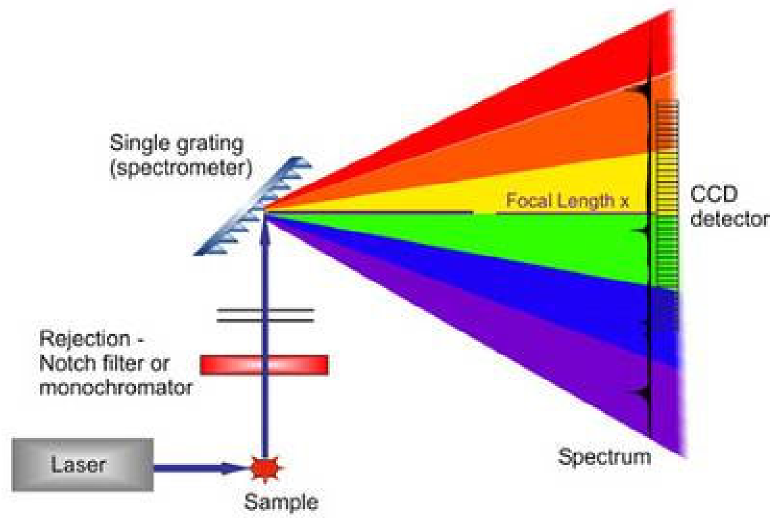


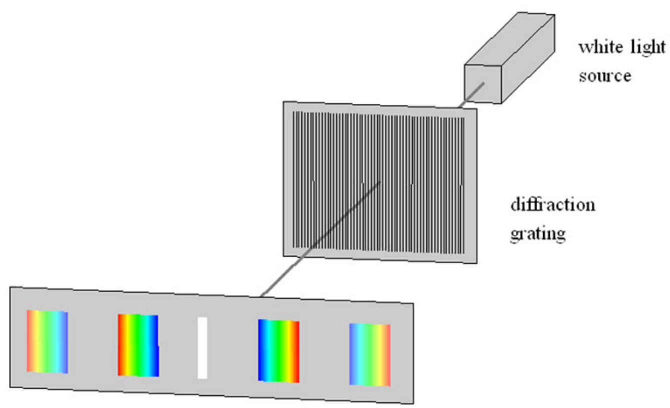




W. Wang

Dispersive Grating Spirometer



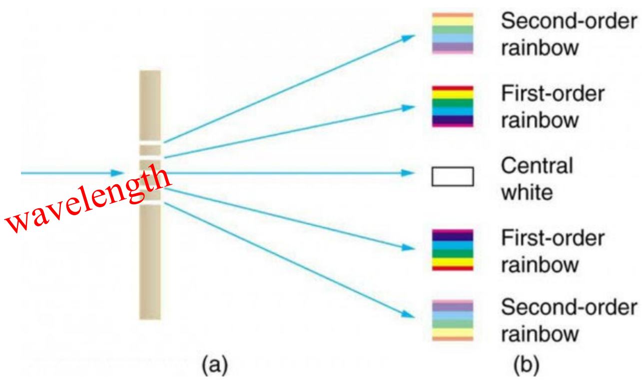


Resolvance and wavelength resolution

To distinguish light waves whose wavelengths are close together, the maxima of these wavelengths formed by the grating should be as narrow as possible. Express otherwise, resolvance or "chromatic resolving power" for a device used to separate the wavelengths of light is defined as

$$R = \lambda/\Delta\lambda = mN$$

where $\Delta\lambda$ = smallest resolvable wavelength difference
m = order number
N = grating frequency

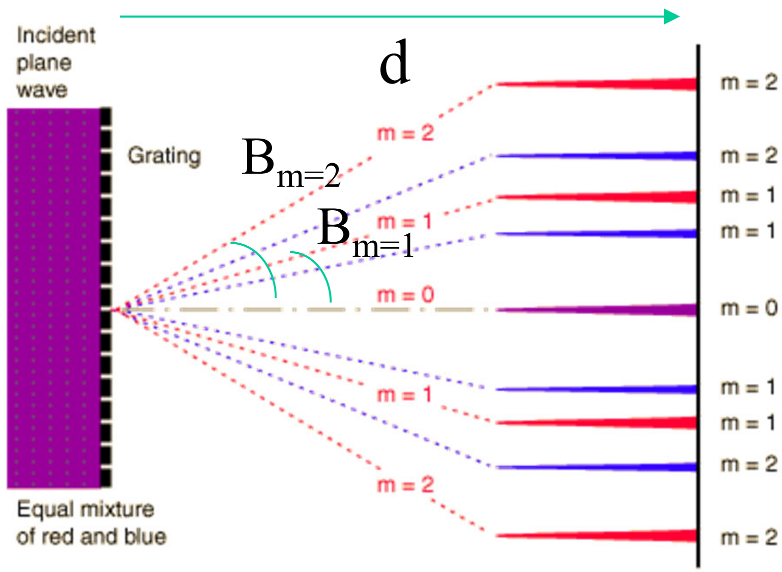


How well grating can resolve the wavelength

Using the limit of resolution is determined by the Raleigh criterion as applied to the diffraction maxima, i.e., two wavelengths are just resolved when the maximum of one lies at the first minimum of the other, the above $R = mN$ can be derived.

The resolvance of such a grating depends upon how many slits are actually covered by the incident light source; i.e., if you can cover more slits, you get a higher resolution in the projected spectrum **(e.g. useful in dispersion spectrum analysis)**

Angular Dispersion



How well grating can disperse wavelength as a function of angle

hperphysics

A diffraction grating is the tool of choice for separating the colors in incident light. This is dispersion effect similar to prism. The angular dispersion is the amount of change of diffraction angle per unit change of the wavelength. It is a measure of the angular separation between beams of adjacent wavelengths. An expression for the angular dispersion can be derived from earlier equation by differentiating, keeping the angle fixed.

$$D = \frac{d\beta_m}{d\lambda} = \frac{-m}{d\cos\beta_m}$$

D is measure of the angular separation produced between two incident monochromatic waves whose wavelengths differ by a small wavelength interval

Two Ways to solve this

We did this before to find out the diffraction limit!

Grating Intensity

The intensity is given by the interference intensity expression

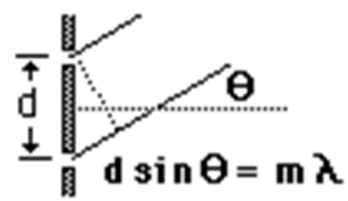
$$I = I_0 \left[\frac{\sin\left(\frac{Nkd}{2}\sin\theta\right)}{\sin\left(\frac{kd}{2}\sin\theta\right)} \right]^2$$

Modulated by the single slit diffraction envelope for the slits which make up the grating:

$$I = I_0 \left[\frac{\sin\left(\frac{ka}{2}\sin\theta\right)}{\frac{ka}{2}\sin\theta} \right]^2$$

The given total intensity expression,

$$I = I_0 \left[\frac{\sin\left(\frac{ka}{2}\sin\theta\right)}{\frac{ka}{2}\sin\theta} \right]^2 \left[\frac{\sin\left(\frac{Nkd}{2}\sin\theta\right)}{\sin\left(\frac{kd}{2}\sin\theta\right)} \right]^2$$



Method 1

Remember we have N number of gratings

N= grating frequency
m = order number
D = angular dispersion

phase diff to 1st min

$\Delta\phi = \frac{\pi}{N}$

Look at next page

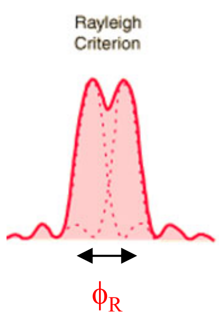
$\phi = (m+\frac{1}{2})\pi = \boxed{\frac{\pi d \sin\theta}{\lambda}}$ (with min for double slits)

(phase diff between 2 minimum) $d\phi = \frac{\pi d \cos\theta}{\lambda} d\theta$

since $d\sin\theta = m\lambda \Rightarrow d\cos\theta d\theta = m d\lambda$

$$\boxed{\begin{aligned} d\phi &= \frac{\pi d \cos\theta}{\lambda} d\theta \\ &= \frac{\pi m d\lambda}{\lambda} \end{aligned}}$$

$$\boxed{D = \frac{d\theta}{d\lambda} = \frac{m}{d\cos\theta}}$$



Combine the two equations

w wang

Method 2

page ...

$\Delta\phi$ = phase diff between 2 adjacent slits on grating

$= \frac{2\pi}{N}$ (phase diff between 2 max)

(phase diff between 2 adjacent slits in term of path length diff $d\sin\theta$) $\phi = \frac{2\pi}{\lambda} \underbrace{d\sin\theta}_{m\lambda}$ (from double slits)

$d\sin\theta = m\lambda$ ($\Rightarrow d\cos\theta d\theta = m d\lambda$) its differential is

For principle max

$$\boxed{d\phi = \frac{2\pi}{\lambda} d\cos\theta d\theta}$$

(phase between two maximums as a function of d, θ and λ)

$$\boxed{= \frac{2\pi}{\lambda} m d\lambda}$$

$\frac{2\pi}{N} = \frac{2\pi}{\lambda} m d\lambda$

(phase between two grating)

$$\boxed{R = \frac{\lambda}{\Delta\lambda} = mN}$$

$\boxed{d\phi/d\lambda}$

$\boxed{d\phi/d\theta}$

$\frac{\pi}{N} = \frac{\pi m d\lambda}{\lambda}$ $\boxed{\frac{\lambda}{\Delta\lambda} = mN}$

should be same $\Delta\phi_{max} = \Delta\phi_{min}$

Examples of Resolvance

A standard benchmark for the resolvance of a grating or other spectroscopic instrument is the resolution of the sodium doublet. The two sodium "D-lines" are at 589.00 nm and 589.59 nm. Resolving them corresponds to resolvance

$$R=\lambda/\Delta\lambda = 0.589/.59 = 1000$$

Use R and assume a M you want to use and find out what N is needed to resolve these two wavelengths

$$R = NM = 1000$$

If grating number is 1000 then first order will be able to resolve it.

Higher the grating frequency and higher the grating order, the higher the resolvance

Diffraction grating based Spectrometer

Lab 2 spectrometer is basically a diffraction grating based spectrometer and if $\Delta\lambda = 0.2$ nm @ λ=632,8nm , what is N?

$$R=\lambda/\Delta\lambda = 632.8/0.2 = 3169$$

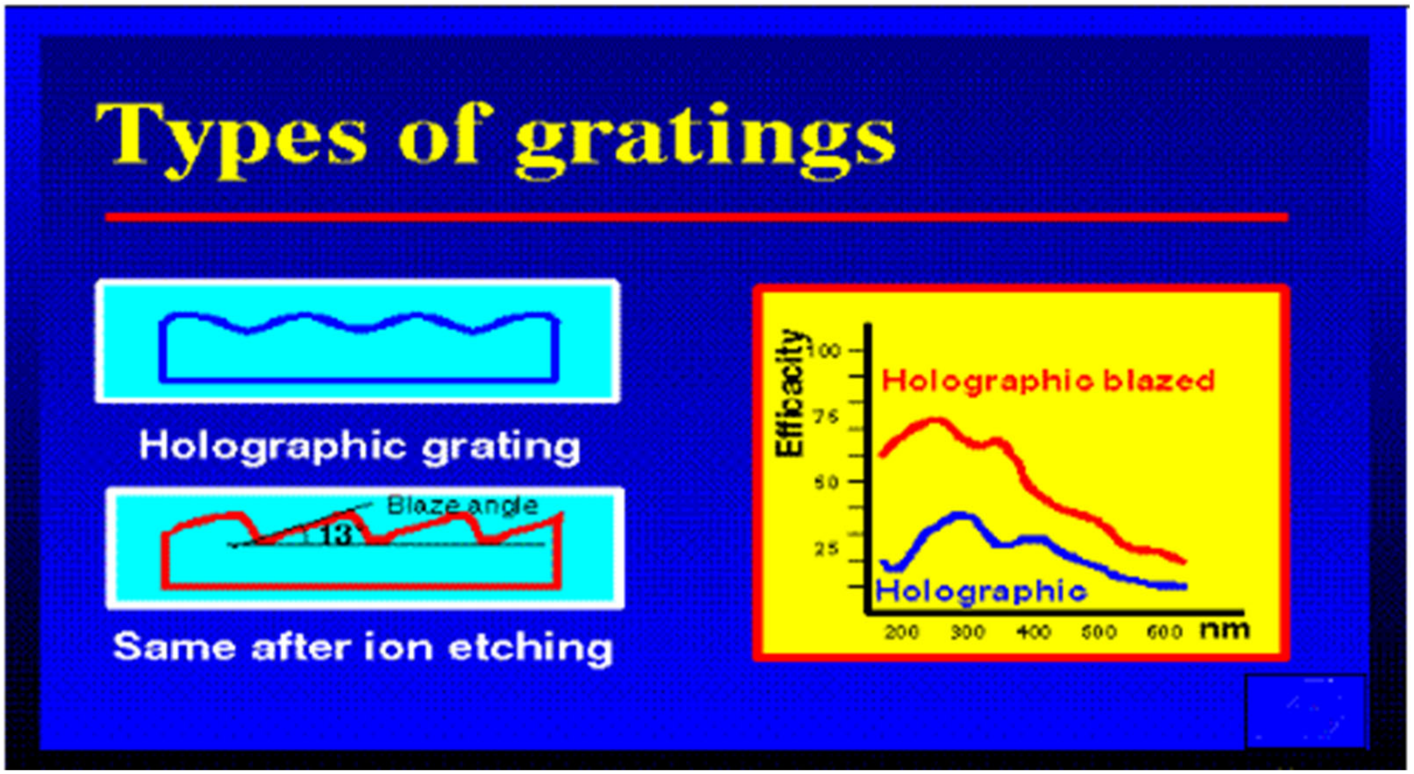
Use R and assume a M you want to use and find out what N is needed to resolve these two wavelengths

$$N = R/M = 3169$$

If first order is used, the grating number is 3169 will be able to resolve it.

Higher the grating frequency and higher the grating order, the higher the resolvance

Blazed versus Sinusoidal

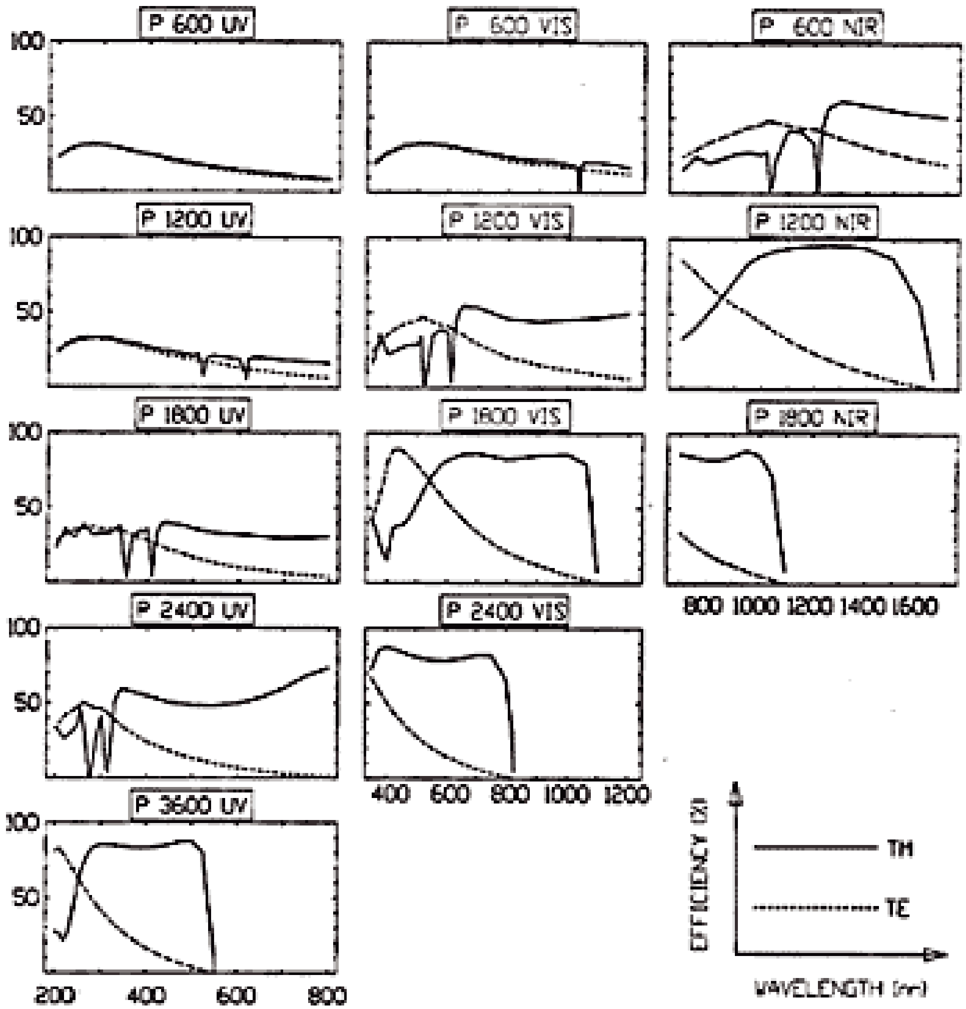


Surface Analytical

Why Sinusoidal gratings ?

- Holographically manufactured
- Gratings of standard type have a sinusoidal groove profile.
- The efficiency curve is rather smooth and flatter than for ruled gratings. The efficiency is optimized for specific spectral regions by varying the groove depth, and it may still be high, especially for gratings with high frequency.
- When the groove spacing is less than about 1.25 times the wavelength, only the -1 and 0 orders exist, and if the grating has an appropriate groove depth, most of the diffracted light goes into the -1 order. In this region, holographically recorded gratings give well over 50 % absolute efficiency.

Efficiency Curve



http://www.spectrogon.com/gratpropert.html

The absolute efficiency is defined as the amount of the incident flux that is diffracted into a given diffraction order. The relative efficiency is related to the reflectance of a mirror, coated with the same material as the grating, and it should be noted that the relative efficiency is always higher than the absolute efficiency.

Efficiency curves for the most common holographic grating types. Each grating is denoted P XXXX YY, where P stands for Plane holographic grating, XXXX is the groove frequency, and YY is the spectral range where the efficiency is highest.

Littrow Condition

Blazed grating groove profiles are calculated for the Littrow condition where the incident and diffracted rays are in auto collimation (i.e., $\alpha = \beta$). The input and output rays, therefore, propagate along the same axis. In this case at the "blaze" wavelength λ_B.

$$\sin\alpha + \sin\beta = mN\lambda_B$$

$$\omega = \alpha = \beta\,,\ \omega = \text{blazed angle}$$

$$2\sin\omega = mN\lambda_B$$

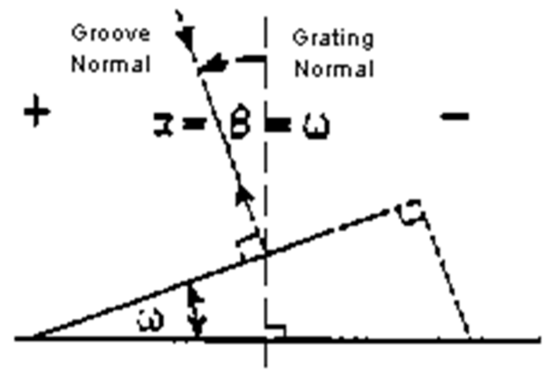


Figure 4 - Littrow Condition for a Single Groove of a Blazed Grating

For example, the blaze angle (ω) for a 1200 g/mm grating blazed at 250 nm is 8.63° in first order (m = 1).

Has to do with phase and also incident angle

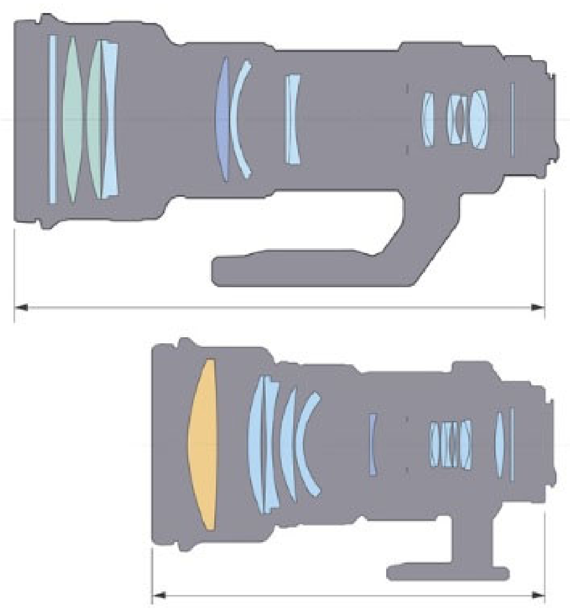
Blaze: The concentration of a limited region of the spectrum into any order other than the zero order. Blazed gratings are manufactured to produce maximum efficiency at designated wavelengths. A grating may, therefore, be described as "blazed at 250 nm" or "blazed at 1 micron" etc. by appropriate selection of groove geometry.

A blazed grating is one in which the grooves of the diffraction grating are controlled to form right triangles with a "blaze angle, ω," as shown in Fig. 4. However, apex angles up to 110° may be present especially in blazed holographic gratings. The selection of the peak angle of the triangular groove offers opportunity to optimize the overall efficiency profile of the grating.

Blazed grating usually formed by dry etching (Reactive ion etching) with a tilted bottom electrode.

Diffractive Lens

- Using Diffractive grating lens instead of refractive lens
- Size reduction
- Diffraction grating can be used to introduce corrections, rather than create aberrations

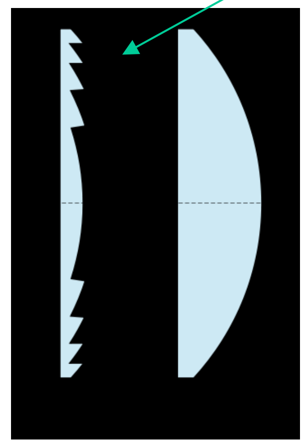


Connon DO Diffractive Optics Lens
(shown here 70-300mm DO lens)
Photokina 2000 exhibition in Cologne

Fresnel Lens

Notice prims is used again here for refracting light into different directions

- Using grating to simulate the refractive lens effect (e.g. flash light, slde projector lens)
- The Fresnel lens reduces the amount of material required compared to a conventional lens by dividing the lens into a set of concentric annular sections. An ideal Fresnel lens would have an infinite number of sections. In each section, the overall thickness is decreased compared to an equivalent simple lens. This effectively divides the continuous surface of a standard lens into a set of surfaces of the same curvature, with stepwise discontinuities between them.
- Fresnel lens design allows a substantial reduction in thickness (and thus mass and volume of the materials)

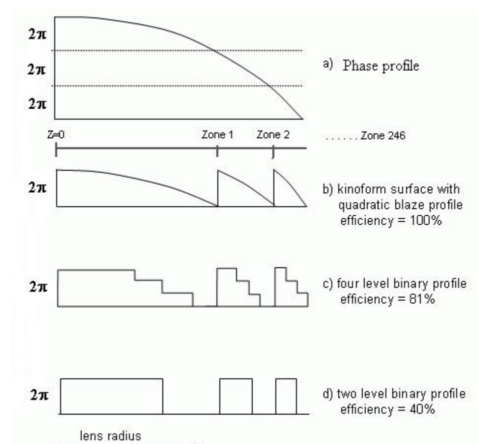


Cross section of a conventional spherical plano-convex lens of equivalent power

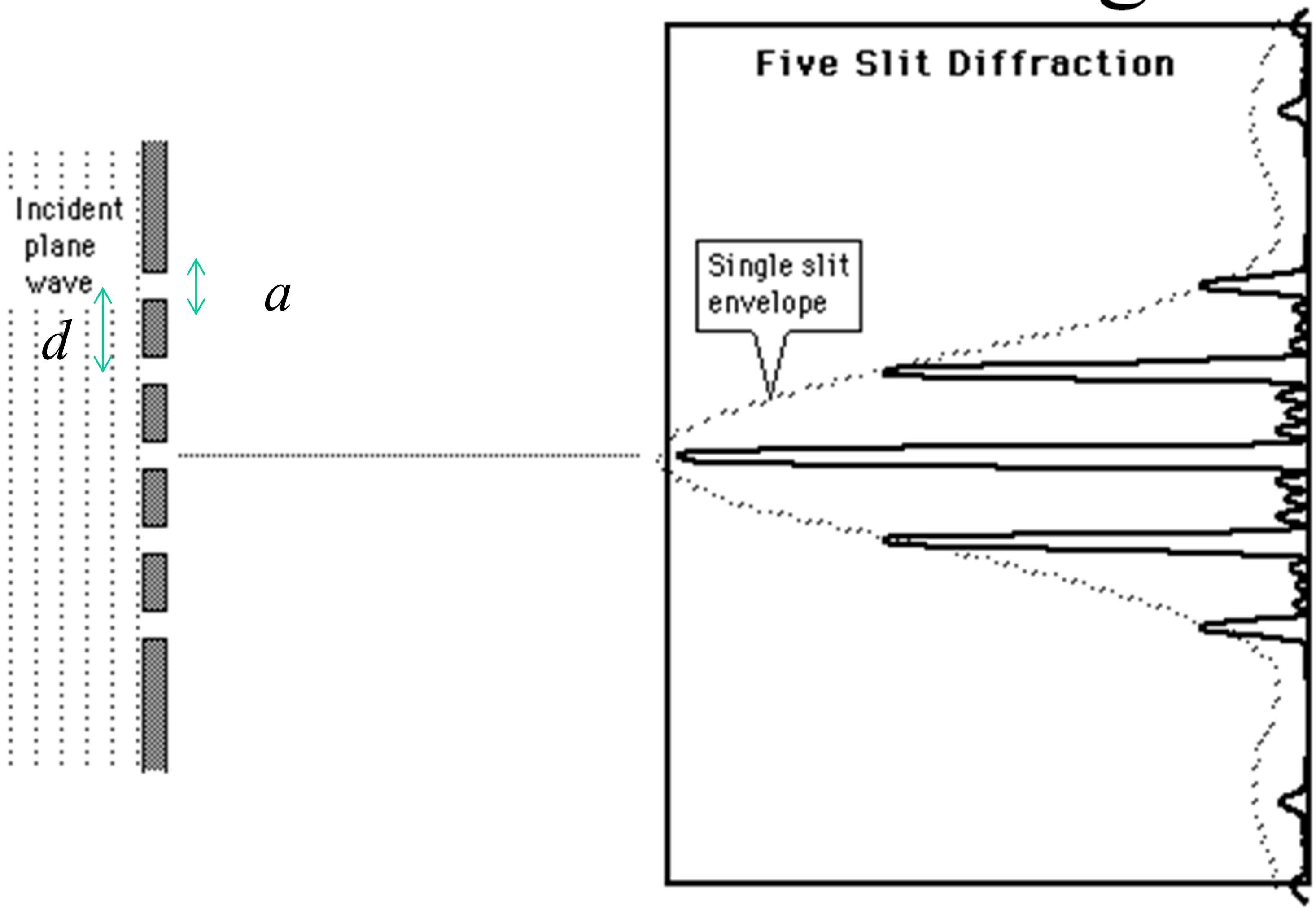
Close-up view of a flat Fresnel lens shows concentric circles on the surface

wikipedia

Close-up view of a flat Fresnel lens shows concentric circles on the surface

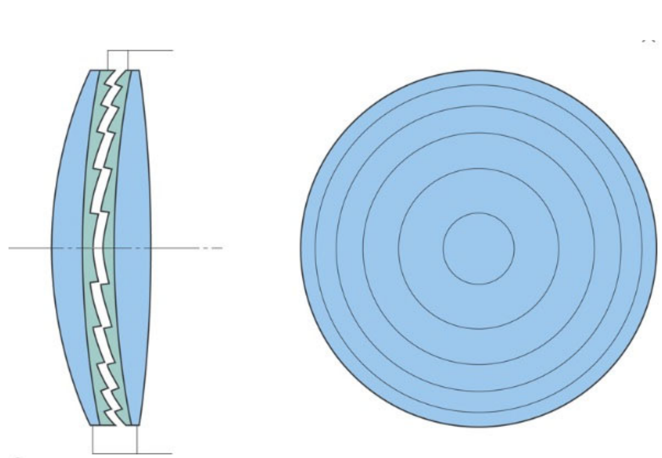


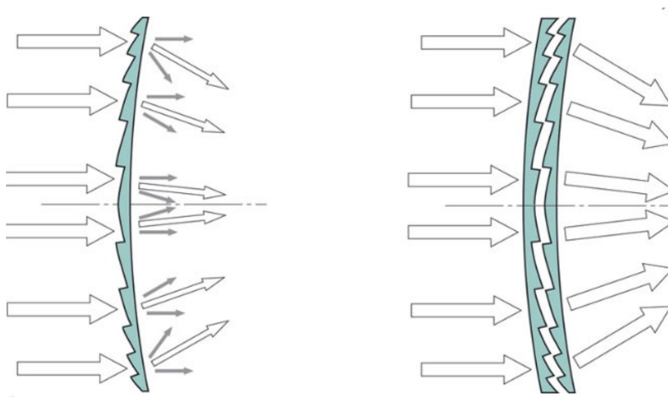
1 D Diffractive Grating

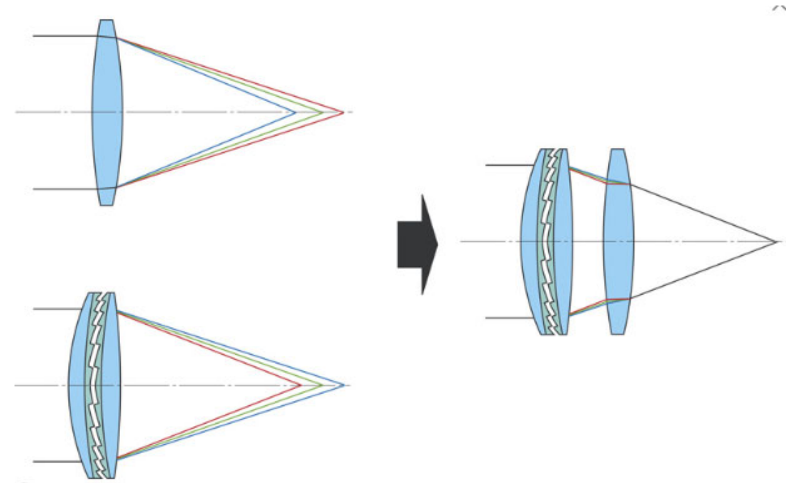


Slit separation d ~ slit width a

Cannon Diffractive Lens



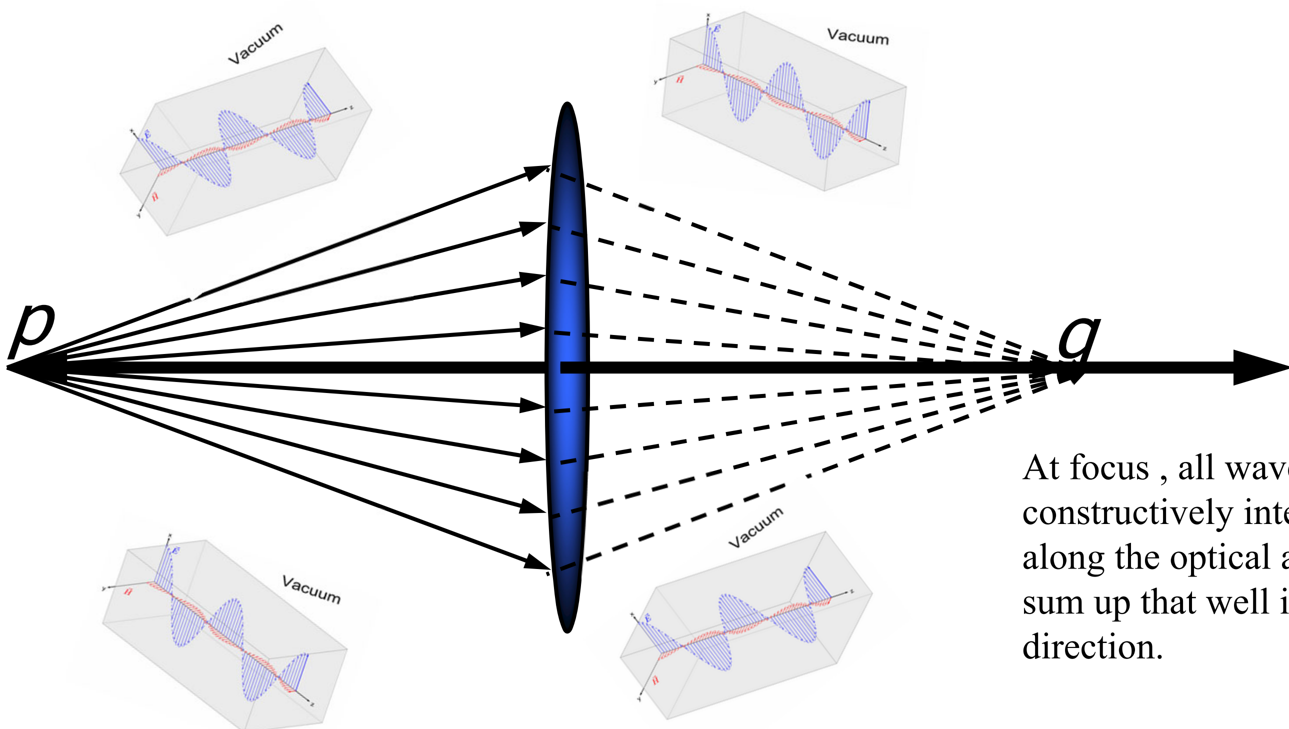




Chromatic aberration, where light of different wavelengths comes to a focus at different positions on the optical axis, is a characteristic of both conventional glass elements (left top) and the Multi-layer Diffractive Optical (DO) Element (left bottom). However, the DO element focuses the wavelengths in a reverse order to conventional optical elements. By combining a DO element with a conventional element (right), chromatic aberration can be eliminated.

A single diffraction grating (left) creates a lot of superfluous light which degrades the final image. By combining two gratings (right), Canon has overcome this problem.

Lens focusing using wave theory



At focus , all waves arrive in phase so constructively interfere. Other location along the optical axis, waves don't sum up that well in the traverse direction.

Phase from each ray is not the same going into the lens and so are those coming out of the lens