Electromagnetic fields induced in a uniaxial multiferroic material by a point source or an ellipsoidal inclusion

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Multiferroic materials possess two or more types of orders simultaneously that couple the electric and magnetic fields, rendering them a rich variety of microstructural phenomena and macroscopic properties. In this paper, we derive explicit closed-form expressions of magnetoelectric Green's functions for uniaxial multiferroic materials induced by a point electric or magnetic charge, and use them to determine the electromagnetic fields in an ellipsoidal inclusion with spontaneous polarization and magnetization embedded in a multiferroic material. Numerical results show that for a typical multiferroic composite, it is easier to induce magnetic field by electric charge or spontaneous polarization, suggesting that it is probably easier to manipulate the electric polarization by a magnetic field in those composites. We expect our solutions to have a wide range of applications in mesoscopic analysis of multiferroic materials.

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I. INTRODUCTION

Multiferroic materials possess two or more types of orders simultaneously that couple the electric and magnetic fields, rendering them a rich variety of microstructural phenomena and macroscopic properties. For example, it is possible to manipulate the ferroelectric state of a multiferroic material through the magnetic field or vice versa, which is not only appealing scientifically, but also makes it promising for a wide range of applications, including electrically controlled microwave phase shifters or ferromagnetic resonance devices, magnetically controlled electro-optic or piezoelectric devices, broadband magnetic field sensors, and magnetoelectric memory devices. As a result, there has been renewed interest in magnetoelectric effect of multiferroic materials over the last few years, and significant progress has been made in the experimental studies of multiferroic materials in both single-phase and multi-phase forms. For single-phase multiferroic materials, the coupled magnetic and electric domains have recently been observed,[1,2] and the magnetic control of polarization and electric control of magnetization have been demonstrated.[3–7] For multi-phase multiferroic materials, a variety of magnetoelectric composites consisting of ferroelectric and ferromagnetic phases have been fabricated,[8–15] with the magnetoelectric coupling much higher than that of single-phase materials.

The rapid experimental progress demands a better theoretical analysis and understanding of multiferroic materials. In particular, a fundamental solution of electromagnetic fields in a multiferroic material in the presence of both electric and magnetic ordering is needed. Such a solution is not only essential for the analysis of magnetoelectric domains in a single-phase multiferroic material, such as switching of the polarization by a magnetic field or vice versa, but is also necessary for the predictions of the effective magnetoelectric coefficients of multiferroic composites. And thus in this paper, we seek to determine the distribution of electric and magnetic fields in a multiferroic material induced by a point source or an ellipsoidal inclusion using the Green's function method, which complements recent first-principle studies of single-phase multiferroic materials[16] and micromechanical analysis of multiferroic composites.[17–24] Such a solution can serve as the cornerstone for the mesoscopic analysis of multiferroic materials.

As the core of singular integral methods, Green's functions are essential to many problems in the mechanics and physics of solids,[25,26] for example, in the analysis of heterogeneous piezoelectric materials.[27–29] In the past few years, there have been attempts to derive Green's functions for solids with full couplings between elastic, electric, and magnetic fields.[30–32] However, there is rarely a material that possesses all the couplings simultaneously, and the full magneto-electro-elastic coupling makes the analysis inevitably complicated and an explicit solution of Green's functions virtually impossible, rendering the analysis difficult to use. And thus in this paper we will focus on magnetoelectric coupling only that is relevant to many multiferroic materials, especially single-phase one. In particular, we seek to derive explicit closed-form expressions of magnetoelectric Green's functions for uniaxial multiferroic materials in the presence of point electric or magnetic charge, and use them to determine the electromagnetic fields in an ellipsoidal inclusion with spontaneous polarization and magnetization embedded in a multiferroic material. Such a solution will find a wide range of applications in the analysis of multiferroic materials, for example the evolution of domain configurations in a single-phase multiferroic material, and the effective properties of multiferroic composites.

II. MAGNETOELECTRIC GOVERNING EQUATIONS

For a multiferroic material that possesses electric and magnetic orderings simultaneously, namely spontaneous polarization P_i^s and magnetization M_i^s, the electric field E_i and magnetic field H_i are coupled together through the linear magnetoelectric coefficient a_{il}. For example, the electric displacement in the material cannot only be induced by an electric field, but may also be induced by a magnetic field. As such, the behavior of the multiferroic materials is governed by the following constitutive equations:

$$\begin{bmatrix} D_i \\ B_i \end{bmatrix} = \begin{bmatrix} \kappa_{il} & a_{il} \\ a_{il} & \mu_{il} \end{bmatrix} \begin{bmatrix} E_l \\ H_l \end{bmatrix} + \begin{bmatrix} P_i^s \\ M_i^s \end{bmatrix}, \tag{1}$$

where κ_{il} and μ_{il} are dielectric permittivity and magnetic permeability, and D_i and B_i are electric displacement and magnetic flux, which satisfy Gauss's equation in the presence of free electric and magnetic charges ρ_e and ρ_m,

$$\begin{bmatrix} D_{i,i} \\ B_{i,i} \end{bmatrix} = \begin{bmatrix} \rho_e \\ \rho_m \end{bmatrix}. \tag{2}$$

The electric and magnetic fields, on the other hand, can be derived from electric potential ϕ and magnetic potential ψ,

$$\begin{bmatrix} E_i \\ H_i \end{bmatrix} = -\begin{bmatrix} \phi_{,i} \\ \psi_{,i} \end{bmatrix}, \tag{3}$$

where the electric potential cannot only be induced by an electric charge, but may also be induced by a magnetic monopole, due to the magnetoelectric coupling. In these equations, Latin subscripts range from 1 to 3 and repeated Latin subscripts are summed from 1 to 3; the subscript comma is used to denote partial differentiation with respect to x_i. Also notice that the magnetic charge is introduced for mathematical convenience, as we elaborate later. Combining Eqs. (1)–(3), we obain the governing equation for the multiferroic magnetoelectric material in the absence of free electric and magnetic charges,

$$\begin{bmatrix} \kappa_{il} & a_{il} \\ a_{il} & \mu_{il} \end{bmatrix} \begin{bmatrix} \phi_{,li} \\ \psi_{,li} \end{bmatrix} = \begin{bmatrix} P_{i,i}^s \\ M_{i,i}^s \end{bmatrix}, \tag{4}$$

from which it is clear that the divergences of spontaneous polarization and magnetization function as electric charge $-\rho_e$ and magnetic charge $-\rho_m$. As such, although magnetic charge or monopole does not exist, the distribution of magnetization may lead to a nonzero divergence, which produces a magnetic field that is equivalent to that of magnetic charge. The problem then is to determine the electric and magnetic fields in the material for a given distribution of electric and magnetic charges, and Green's function method can be used for this purpose.

We are particularly interested in a uniaxial multiferroic material having the unique axis along x_3, with the constitutive moduli given by

$$\begin{bmatrix} \kappa_{11} & 0 & 0 & a_{11} & 0 & 0 \\ 0 & \kappa_{11} & 0 & 0 & a_{11} & 0 \\ 0 & 0 & \kappa_{33} & 0 & 0 & a_{33} \\ a_{11} & 0 & 0 & \mu_{11} & 0 & 0 \\ 0 & a_{11} & 0 & 0 & \mu_{11} & 0 \\ 0 & 0 & a_{33} & 0 & 0 & \mu_{33} \end{bmatrix}, \tag{5}$$

which can be used to represent a wide range of multiferroic materials, including tetragonal and hexagonal multiferroic crystals, and transversely isotropic multiferroic composites. As a result, the governing equation can be rewritten as

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \phi \\ \psi \end{bmatrix} = -\begin{bmatrix} \rho_e \\ \rho_m \end{bmatrix}, \tag{6}$$

where

$$A_{11} = A_{22} = a_{11}\left(\frac{\partial^2}{\partial^2 x_1} + \frac{\partial^2}{\partial^2 x_2}\right) + a_{33}\frac{\partial^2}{\partial^2 x_3},$$

$$A_{12} = \mu_{11}\left(\frac{\partial^2}{\partial^2 x_1} + \frac{\partial^2}{\partial^2 x_2}\right) + \mu_{33}\frac{\partial^2}{\partial^2 x_3},$$

$$A_{21} = \kappa_{11}\left(\frac{\partial^2}{\partial^2 x_1} + \frac{\partial^2}{\partial^2 x_2}\right) + \kappa_{33}\frac{\partial^2}{\partial^2 x_3}.$$

This is the equation we seek to solve for electric potential ϕ and magnetic potential ψ with given distribution of electric and magnetic charges.

III. MAGNETOELECTRIC GREEN'S FUNCTIONS

To be specific, we consider a unit point electric or magnetic charge in the material locating at the source point $\mathbf{x}'$, and seek to determine the electric and magnetic potentials at observing point $\mathbf{x}$. Due to the magnetoelectric coupling in the material, the electric charge will not only induce an electric field, but will also induce a magnetic field, while the magnetic charge will not only induce a magnetic field, but will also induce an electric field. As such, four magnetoelectric Green's functions $G_{\alpha\beta}(\mathbf{x}-\mathbf{x}')$, where the Greek subscripts range from 1 to 2, can be introduced for this purpose, which have the following physical interpretations,

$G_{11}(\mathbf{x}-\mathbf{x}')$: the electric potential at $\mathbf{x}$ due to a unit point electric charge at $\mathbf{x}'$;

$G_{21}(\mathbf{x}-\mathbf{x}')$: the magnetic potential at $\mathbf{x}$ due to a unit point electric charge at $\mathbf{x}'$;

$G_{12}(\mathbf{x}-\mathbf{x}')$: the electric potential at $\mathbf{x}$ due to a unit point magnetic charge at $\mathbf{x}'$;

$G_{22}(\mathbf{x}-\mathbf{x}')$: the magnetic potential at $\mathbf{x}$ due to a unit point magnetic charge at $\mathbf{x}'$.

In addition, the following boundary conditions have to be satisfied: (1) ϕ and ψ vary as $1/r$ for $r \to 0$ where $r = |\mathbf{x}-\mathbf{x}'|$; (2) ϕ and ψ vanish when $r \to \infty$; (3) the resultant electric displacement acting on the surface of an infinitesimal region centered at the source point is equivalent to the applied point electric charge; and (4) the resultant magnetic flux acting on the surface of an infinitesimal region centered at the source point is equivalent to the applied point magnetic charge. Without loss of generality, we assume that the source point is located at the origin in the following derivation.

Following similar ideas in elasticity[33] and piezoelectricity,[27] a magnetoelectric potential function g is introduced, such that the electric potential ϕ and the magnetic potential ψ can be expressed as

$$\begin{bmatrix} \phi \\ \psi \end{bmatrix} = \begin{bmatrix} A_{11} \\ -A_{21} \end{bmatrix} g. \tag{7}$$

Substituting Eq. (7) into Eq. (6), we obtain a differential equation governing g,

$$\left[\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}\right)^2+\frac{a}{c}\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}\right)\frac{\partial^2}{\partial x_3^2}+\frac{b}{c}\frac{\partial^4}{\partial x_3^4}\right]g=0, \quad (8)$$

where

$$a=2a_{11}a_{33}-\kappa_{11}\mu_{33}-\kappa_{33}\mu_{11},$$

$$b=a_{33}^2-\kappa_{33}\mu_{33},$$

$$c=a_{11}^2-\kappa_{11}\mu_{11}$$

are constants that depend on the constitutive moduli. Equation (8) can then be rewritten as a z-weighted dual-harmonic function,

$$\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}+\frac{1}{v_1^2}\frac{\partial^2}{\partial x_3^2}\right)\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}+\frac{1}{v_2^2}\frac{\partial^2}{\partial x_3^2}\right)g=0, \quad (9)$$

where $-1/v_1^2$ and $-1/v_2^2$ are roots of the quadratic equation

$$s^2+\frac{a}{c}s+\frac{b}{c}=0, \quad (10)$$

and thus they only depend on the constitutive moduli of the material. Considering the axial symmetry of the problem, and taking into account the first two boundary conditions, we propose the following form of g,

$$g=\sum_{\alpha=1}^{2}[A_\alpha v_\alpha x_3\ln(R_\alpha+v_\alpha x_3)+B_\alpha R_\alpha], \quad (11)$$

where $R_\alpha=\sqrt{x_1^2+x_2^2+v_\alpha^2x_3^2}$, and A_α and B_α are constants to be determined. Notice that no summation is implied for the repeated Greek subscript. Since g satisfies Eq. (9), we have

$$\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}+\frac{1}{v_\alpha^2}\frac{\partial^2}{\partial x_3^2}\right)f_\alpha=0, \quad (12)$$

where

$$f_\alpha=[A_\alpha v_\alpha x_3\ln(R_\alpha+v_\alpha x_3)+B_\alpha R_\alpha].$$

After considerable manipulation, it is established that

$$A_\alpha=-B_\alpha, \quad (13)$$

which leads to

$$\phi=\sum_{\alpha=1}^{2}\frac{(-a_{11}+a_{33}v_\alpha^2)A_\alpha}{R_\alpha}, \quad \psi=\sum_{\alpha=1}^{2}\frac{(\kappa_{11}-\kappa_{33}v_\alpha^2)A_\alpha}{R_\alpha}. \quad (14)$$

In order to determine A_α, we recall the last two boundary conditions. First let us consider a unit point electric charge such that $\rho_e=1$ and $\rho_m=0$ at the origin, which requires

$$\oint_S D_in_idS=1, \quad \oint_S B_in_idS=0, \quad (15)$$

where n_i is the outward surface normal. The corresponding constants, denoted by A_α^e, are then determined as

$$A_1^e=\frac{C_4}{J}, \quad A_2^e=\frac{-C_3}{J}.$$

In a similar manner, we can determine constants A_α^m when there is only a unit magnetic charge at the origin,

$$A_1^m=\frac{-C_2}{J}, \quad A_2^m=\frac{C_1}{J},$$

with the constants given in the following:

$$J=C_1C_4-C_2C_3,$$

$$C_1=4\pi v_1(a_{33}\kappa_{11}-\kappa_{33}a_{11}),$$

$$C_2=4\pi v_2(a_{33}\kappa_{11}-\kappa_{33}a_{11}),$$

$$C_3=2\pi[h_1(\kappa_{11}\mu_{11}-a_{11}^2+2a_{11}a_{33}v_1^2-\mu_{11}\kappa_{33}v_1^2-\mu_{33}\kappa_{11}v_1^2+a_{33}^2v_1^4+\mu_{33}\kappa_{33}v_1^4)+2v_1(\mu_{33}\kappa_{11}-a_{33}a_{11}+a_{33}^2v_1^2-\mu_{33}\kappa_{33}v_1^2)],$$

$$C_4=2\pi[h_2(\kappa_{11}\mu_{11}-a_{11}^2+2a_{11}a_{33}v_2^2-\mu_{11}\kappa_{33}v_2^2-\mu_{33}\kappa_{11}v_2^2+a_{33}^2v_2^4+\mu_{33}\kappa_{33}v_2^4)+2v_2(\mu_{33}\kappa_{11}-a_{33}a_{11}+a_{33}^2v_2^2-\mu_{33}\kappa_{33}v_2^2)],$$

$$h_\alpha=\frac{2\arctan\sqrt{\dfrac{1-v_\alpha^2}{v_\alpha^2}}}{(1-v_\alpha^2)^{3/2}}-\frac{2v_\alpha}{1-v_\alpha^2}.$$

The explicit, closed-form expressions of the magnetoelectric Green's functions can then be obtained from our solutions,

$$G_{11}(\mathbf{x})=\sum_{\alpha=1}^{2}\frac{(-a_{11}+a_{33}v_\alpha^2)A_\alpha^e}{R_\alpha},$$

$$G_{21}(\mathbf{x})=\sum_{\alpha=1}^{2}\frac{(\kappa_{11}-\kappa_{33}v_\alpha^2)A_\alpha^e}{R_\alpha},$$

$$G_{12}(\mathbf{x})=\sum_{\alpha=1}^{2}\frac{(-a_{11}+a_{33}v_\alpha^2)A_\alpha^m}{R_\alpha},$$

$$G_{22}(\mathbf{x})=\sum_{\alpha=1}^{2}\frac{(\kappa_{11}-\kappa_{33}v_\alpha^2)A_\alpha^m}{R_\alpha}.$$

Although it is not obvious, the Green's functions are symmetric in the sense that $G_{12}=G_{21}$. Notice that in an infinite body, only the relative position to the source point matters.

From the explicit expressions of Green's functions, it appears that they could be zero when the material is uncoupled, i.e., when the magnetoelectric coefficients are zero. This is not true, since A_i^e and A_i^m will approach infinity under such circumstances, and the limit of $G_{\alpha\beta}$ exists when $a_{ij}\rightarrow 0$. In fact, the limits of these expressions when $a_{ij}\rightarrow 0$ do recover the well-known solutions for a single-ferroic material, dielectric or magnetic, in which the electric and magnetic fields are uncoupled. Similar conclusion can also be drawn for cubic or isotropic materials where $a_{11}=a_{33}$. Such degen-

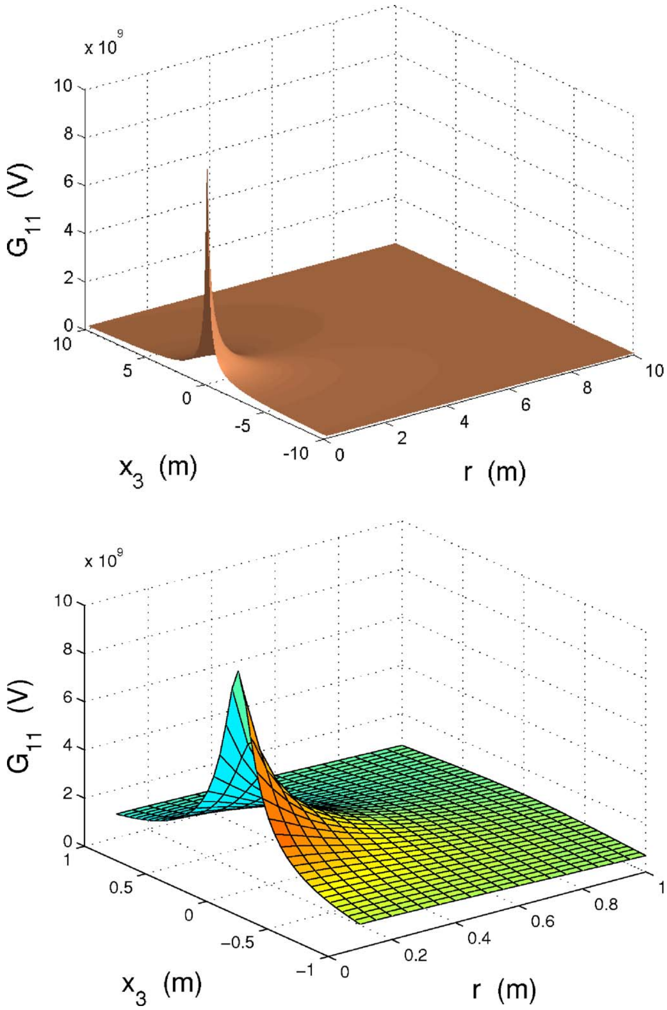

FIG. 1. (Color online) The distribution of the Green's function G_{11} in space.

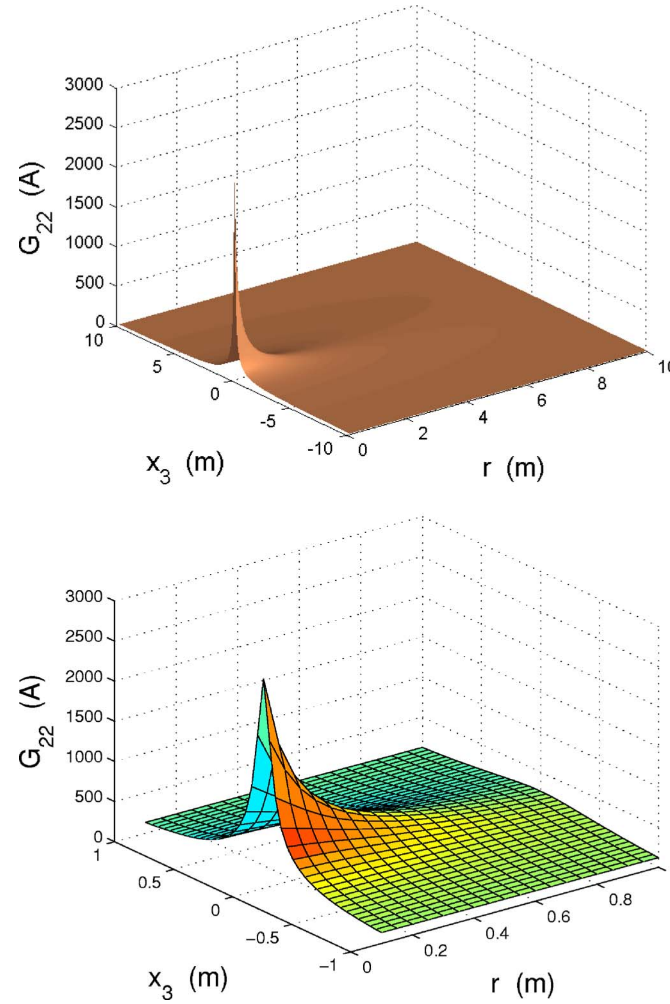

FIG. 2. (Color online) The distribution of the Green's function G_{22} in space.

eracies do not present an obstacle to the application of our solutions, because a valid numerical solution can be obtained by slightly perturbing one or several moduli in a degenerate set of material constants to eliminate the degeneracy.

To demonstrate our solutions, we present a numerical result of Green's functions as shown in Figs. 1–3 for a multiferroic material, with the material constants listed in Table I. Since no constitutive moduli for any single-phase multiferroic material is completely determined, we use the values that are typical for a multiferroic composite consisting of ferroelectric and ferromagnetic phases. Because all the Green's functions have axial-symmetry along x_3, we plot them using two-dimensional cylindrical coordinates r and x_3. The figures on the top provide the distribution of the Green's functions in the whole space while the figures at the bottom show more details about the Green's functions near the source point. From these figures, it is clear that the distributions of different Green's functions are very similar, while the magnitudes vary significantly. In particular, the electric potentials induced by a unit electric and magnetic charges are different by eight orders of magnitude, while the difference between magnetic potentials are different by only two orders of magnitude. This suggests that the manipulation of an electric charge by a magnetic field might be easier than the manipulation of the magnetic charge by an electric field for a typical multiferroic composite.

IV. MAGNETOELECTRIC INCLUSION PROBLEM

Now instead of point charges, let us consider an inclusion Ω embedded in an infinite multiferroic material. The inclusion has the same magnetoelectric moduli κ_{il}, μ_{il}, and a_{il} as the matrix, but has spontaneous polarization P_i^s and magnetization M_i^s that are absent in the matrix. As such, it could be a domain in a multiferroic crystal, or a particle in a multiferroic composite. There is no electric or magnetic field applied at the boundary of the infinite multiferroic matrix, yet the electric and magnetic fields can still be induced in the material since the divergences of spontaneous polarization and magnetization function as electric and magnetic charges. In order to determine the electric and magnetic potentials induced by the spontaneous polarization and magnetization in the inclusion, we resort to the Green's functions we derived,

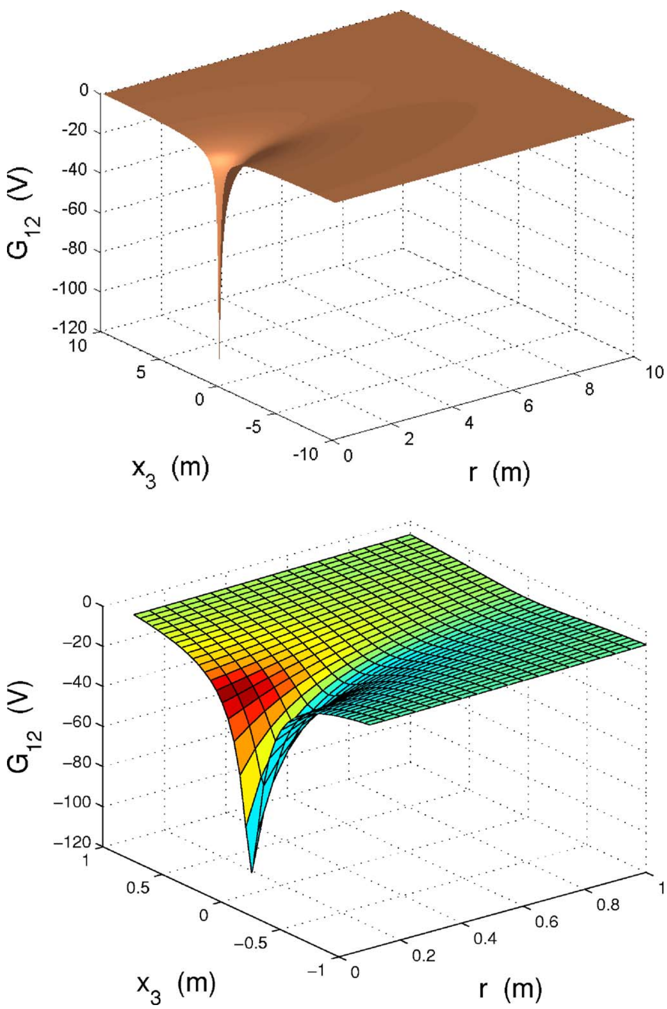

FIG. 3. (Color online) The distribution of the Green's function G_{12} in space.

$$\begin{bmatrix}\phi(\mathbf{x})\\ \psi(\mathbf{x})\end{bmatrix} = -\int_{\Omega}\begin{bmatrix}G_{11} & G_{12}\\ G_{21} & G_{22}\end{bmatrix}\begin{bmatrix}P^{s}_{i,i'}\\ M^{s}_{i,i'}\end{bmatrix}dV(\mathbf{x}') + \int_{\partial\Omega}\begin{bmatrix}G_{11} & G_{12}\\ G_{21} & G_{22}\end{bmatrix}\begin{bmatrix}P^{s}_{i}n_i\\ M^{s}_{i}n_i\end{bmatrix}dS(\mathbf{x}'), \quad (16)$$

where the first term is due to the volume charges in Ω, with the differentiation taken with respect to x_i', and the second term is due to the interface charge at $\partial\Omega$, induced by the discontinuity of polarization and magnetization at the boundary of Ω. Applying the divergence theorem to the equation, we obtain

TABLE I. The constitutive moduli of a multiferroic material.[a]

a_{11}	a_{33}	κ_{11}	κ_{33}	μ_{11}	μ_{33}
5	3	8	9.3	5.9	1.57

[a]Units: dielectric permittivity 10^{-11} $C^2/N\,m^2$; magnetic permeability 10^{-4} $N\,s^2/C^2$; magnetoelectric coefficient 10^{-12} N s/VC.

$$\begin{bmatrix}\phi(\mathbf{x})\\ \psi(\mathbf{x})\end{bmatrix} = -\int_{\Omega}\begin{bmatrix}G_{11,i} & G_{12,i}\\ G_{21,i} & G_{22,i}\end{bmatrix}\begin{bmatrix}P^{s}_{i}\\ M^{s}_{i}\end{bmatrix}dV(\mathbf{x}'), \quad (17)$$

where the differentiation is taken with respect to x_i.

We are particularly interested in the ellipsoidal inclusions given by

$$\left(\frac{x_1'}{a_1}\right)^2 + \left(\frac{x_2'}{a_2}\right)^2 + \left(\frac{x_3'}{a_3}\right)^2 \leqslant 1, \quad (18)$$

where a_i are the dimensions of the inclusion along the x_i axis. The ellipsoidal inclusions can be used to represent a wide range of micro-geometries, including fibers, particles, and laminates, commonly observed in heterogeneous materials. For such an ellipsoidal inclusion with uniform polarization and magnetization, the volume integral in Eq. (17) can be expressed in terms of the spherical coordinates for an arbitrary point $\mathbf{x}$ inside the inclusion,

$$\begin{bmatrix}\phi(\mathbf{x})\\ \psi(\mathbf{x})\end{bmatrix} = \int_{\Omega}\begin{bmatrix}g_{11i}(\mathbf{l}) & g_{12i}(\mathbf{l})\\ g_{21i}(\mathbf{l}) & g_{22i}(\mathbf{l})\end{bmatrix}\begin{bmatrix}P^{s}_{i}\\ M^{s}_{i}\end{bmatrix}\sin\theta d\theta d\varphi dr, \quad (19)$$

with

$$g_{MJi}(\mathbf{l}) = -r^2 G_{MJ,i}(\mathbf{x}-\mathbf{x}'), \quad \mathbf{l} = \frac{\mathbf{x}'-\mathbf{x}}{r}, \quad r = |\mathbf{x}'-\mathbf{x}|.$$

After integrating Eq. (19) with respect to r, we obtain

$$\begin{bmatrix}\phi(\mathbf{x})\\ \psi(\mathbf{x})\end{bmatrix} = \int_{\partial\Omega}\begin{bmatrix}g_{11i}(\mathbf{l}) & g_{12i}(\mathbf{l})\\ g_{21i}(\mathbf{l}) & g_{22i}(\mathbf{l})\end{bmatrix}\begin{bmatrix}P^{s}_{i}\\ M^{s}_{i}\end{bmatrix}r(\mathbf{l})\sin\theta d\theta d\varphi, \quad (20)$$

where $r(\mathbf{l})$ is the boundary of the inclusion, satisfying

$$\left(\frac{x_1 + rl_1}{a_1}\right)^2 + \left(\frac{x_2 + rl_2}{a_2}\right)^2 + \left(\frac{x_3 + rl_3}{a_3}\right)^2 = 1. \quad (21)$$

We thus have

$$\mathbf{l} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$$

on the boundary of the inclusion, which allows us to derive

$$\begin{bmatrix}\phi(\mathbf{x})\\ \psi(\mathbf{x})\end{bmatrix} = x_k\int_{\partial\Omega}\frac{\lambda_k}{m}\begin{bmatrix}g_{11i}(\mathbf{l}) & g_{12i}(\mathbf{l})\\ g_{21i}(\mathbf{l}) & g_{22i}(\mathbf{l})\end{bmatrix}\begin{bmatrix}P^{s}_{i}\\ M^{s}_{i}\end{bmatrix}\sin\theta d\theta d\varphi, \quad (22)$$

where

$$\boldsymbol{\lambda} = \left(-\frac{\sin\theta\cos\varphi}{a_1^2}, -\frac{\sin\theta\sin\varphi}{a_2^2}, -\frac{\cos\theta}{a_3^2}\right),$$

$$m = \frac{\sin^2\theta\cos^2\varphi}{a_1^2} + \frac{\sin^2\theta\sin^2\varphi}{a_2^2} + \frac{\cos^2\theta}{a_3^2}. \quad (23)$$

Differentiating Eq. (22) with respect $\mathbf{x}$ then yields

$$\begin{bmatrix}E_i\\ H_i\end{bmatrix} = \begin{bmatrix}Q^{11}_{ij} & Q^{12}_{ij}\\ Q^{21}_{ij} & Q^{22}_{ij}\end{bmatrix}\begin{bmatrix}P^{s}_{j}\\ M^{s}_{j}\end{bmatrix}, \quad (24)$$

where

$$\begin{bmatrix} Q_{ij}^{11} & Q_{ij}^{12} \\ Q_{ij}^{21} & Q_{ij}^{22} \end{bmatrix} = -\int_0^{2\pi}\int_0^{\pi} \frac{\lambda_i}{m}\begin{bmatrix} g_{11j}(\mathbf{l}) & g_{12j}(\mathbf{l}) \\ g_{21j}(\mathbf{l}) & g_{22j}(\mathbf{l}) \end{bmatrix} \sin\theta d\theta d\varphi. \tag{25}$$

The integral in Eq. (25) can be evaluated when $a_1 = a_2 = \alpha a_3$, resulting in 4 second-rank tensors $Q_{ij}^{\alpha\beta}$ that relate the electric and magnetic fields inside the ellipsoidal inclusion to its spontaneous polarization and magnetization,

$$Q_{11}^{11} = Q_{22}^{11} = \sum_{\beta=1}^{2} (a_{11} - a_{33}v_\beta^2) A_\beta^e K_1(v_\beta),$$

$$Q_{33}^{11} = \sum_{\beta=1}^{2} (a_{11} - a_{33}v_\beta^2) A_\beta^e K_2(v_\beta),$$

$$Q_{11}^{12} = Q_{22}^{12} = \sum_{\beta=1}^{2} (a_{11} - a_{33}v_\beta^2) A_\beta^m K_1(v_\beta),$$

$$Q_{33}^{12} = \sum_{\beta=1}^{2} (a_{11} - a_{33}v_\beta^2) A_\beta^m K_2(v_\beta),$$

$$Q_{11}^{21} = Q_{22}^{21} = \sum_{\beta=1}^{2} (-\kappa_{11} + \kappa_{33}v_\beta^2) A_\beta^e K_1(v_\beta),$$

$$Q_{33}^{21} = \sum_{\beta=1}^{2} (-\kappa_{11} + \kappa_{33}v_\beta^2) A_\beta^e K_2(v_\beta),$$

$$Q_{11}^{22} = Q_{22}^{22} = \sum_{\beta=1}^{2} (-\kappa_{11} + \kappa_{33}v_\beta^2) A_\beta^m K_1(v_\beta),$$

$$Q_{33}^{22} = \sum_{\beta=1}^{2} (-\kappa_{11} + \kappa_{33}v_\beta^2) A_\beta^m K_2(v_\beta), \tag{26}$$

where $K_1(v_\beta)$ and $K_2(v_\beta)$ are given by

$$K_1(v_\beta) = 2\pi(\alpha^2 - v_\beta^2)^{-3/2}[-v_\beta\sqrt{\alpha^2 - v_\beta^2} + \alpha^2 \arctan(\sqrt{\alpha^2 - v_\beta^2}/v_\beta)],$$

$$K_2(v_\beta) = 4\pi\alpha^2 v_\beta(\alpha^2 - v_\beta^2)^{-3/2}[\sqrt{\alpha^2 - v_\beta^2} - v_\beta \arctan(\sqrt{\alpha^2 - v_\beta^2}/v_\beta)].$$

Although K_1 and K_2 are generally complex, $Q_{ij}^{\alpha\beta}$ are always real. Notice that Q_{ij}^{11} relates the electric field and spontaneous polarization in the inclusion, Q_{ij}^{22} relates the magnetic field and spontaneous magnetization in the inclusion, and $Q_{ij}^{12} = Q_{ij}^{21}$ are additional factors due to the magnetoelectric coupling, relating electric field to magnetization, and magnetic field to polarization. When the magnetoelectric coefficient $a_{il} \to 0$, it is easy to show that the limit of Q_{ij}^{11} recovers the well-known depolarization factor for an uncoupled dielectrics, while the limit of Q_{ij}^{22} recovers the well-known demagnetization factor for an uncoupled magnets.

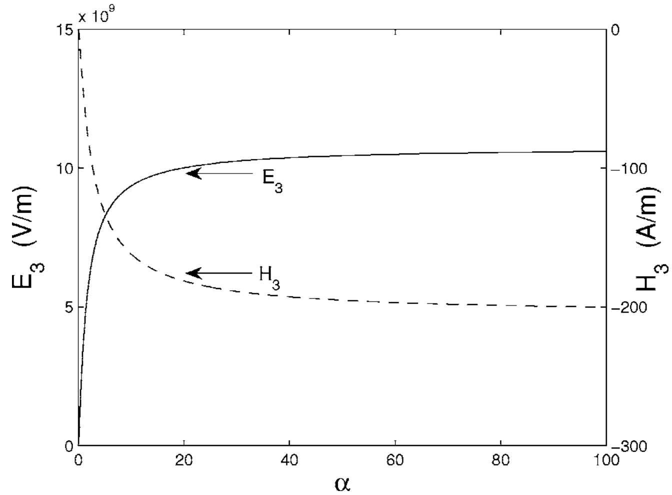

FIG. 4. The electric and magnetic fields E_3 and H_3 inside an ellipsoidal inclusion with unit polarization $-P_3^s$ versus the shape aspect ratio α of the inclusion in a multiferroic material.

To demonstrate our results, we calculated the electric field E_i and the magnetic field H_i inside an ellipsoidal inclusion as a function of the shape aspect ratio of the inclusion $\alpha = a_1/a_3$, induced by a unit spontaneous polarization or magnetization along x_3, shown in Figs. 4 and 5. The constitutive moduli used in the calculation is listed in Table I, which is typical for a multiferroic composite. As expected, no electric or magnetic field is induced when the inclusion is a cylinder with $\alpha \to 0$, and the electric and magnetic fields are maximum for the lamellar inclusion when $1/\alpha \to 0$. Also, it is noticed that the difference between the magnetic field induced by a unit magnetization and polarization is much

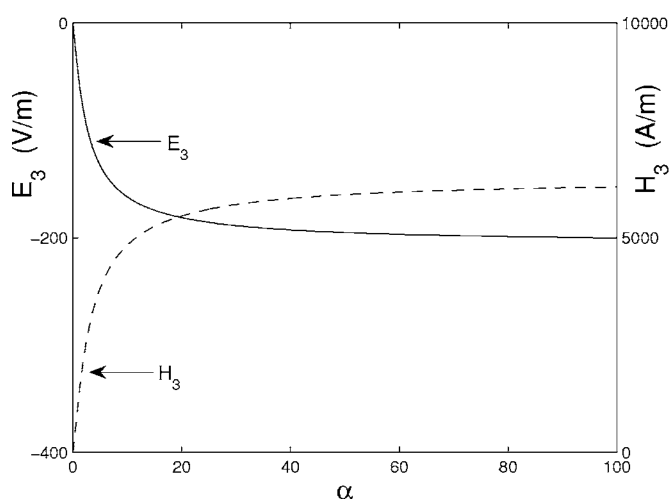

FIG. 5. The electric and magnetic fields E_3 and H_3 inside an ellipsoidal inclusion with unit magnetization $-M_3^s$ versus the shape aspect ratio α of the inclusion in a multiferroic material.

smaller than the difference between the electric field induced by a unit polarization and magnetization, consistent with our calculations for Green's functions. This again suggests that for a typical multiferroic composite, it is probably easier to manipulate the polarization by a magnetic field than manipulating the magnetization by an electric field, which may explain that while the polarization has been reversed by a magnetic field in multiferroic materials,[5] no magnetization reversal by an electric field has been reported yet. Instead, it has been observed that an electric field can change the magnetic structure of multiferroic materials.[6]

V. CONCLUDING REMARKS

In summary, we have derived the magnetoelectric Green's functions explicitly for uniaxial multiferroic materials, and used them to determine the electric and magnetic fields in a ellipsoidal inclusion with spontaneous polarization and magnetization. Numerical results using constitutive moduli typical for a multiferroic composite show that it is easier to induce magnetic field by electric charge or spontaneous polarization, suggesting that it is probably easier to manipulate the electric polarization by a magnetic field for a typical multiferroic composite.

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[1] M. Fiebig, T. Lottermoser, D. Fröhlich, A. V. Goltsev, and R. V. Pisarev, Nature (London) **419**, 818 (2002).
[2] S. Lee, A. Pirogov, J. H. Han, J.-G. Park, A. Hoshikawa, and T. Kamiyama, Phys. Rev. B **71**, 180413(R) (2005).
[3] T. Kimura, T. Goto, H. Shintani, K. Ishizaka, T. Arima, and Y. Tokura, Nature (London) **426**, 55 (2003).
[4] J. Wang, J. B. Neaton, H. Zheng, V. Nagarajan, S. B. Ogale, B. Liu, D. Viehland, V. Vaithyanathan, D. G. Schlom, U. V. Waghmare *et al.*, Science **299**, 1719 (2003).
[5] N. Hur, S. Park, P. A. Sharma, J. S. Ahn, S. Guha, and S.-W. Cheong, Nature (London) **429**, 392 (2004).
[6] T. Lottermoser, T. Lonkai, U. Amann, D. Hohlwein, J. Ihringer, and M. Fiebig, Nature (London) **430**, 541 (2004).
[7] B. Ruette, S. Zvyagin, A. P. Pyatakov, A. Bush, J. F. Li, V. I. Belotelov, A. K. Zvezdin, and D. Viehland, Phys. Rev. B **69**, 064114 (2004).
[8] G. Harshe, J. P. Dougherty, and R. E. Newnham, Int. J. Appl. Electromagn. Mater. **4**, 161 (1993).
[9] J. Ryu, S. Priya, K. Uchino, and H. E. Kim, J. Electroceram. **8**, 107 (2002).
[10] K. Mori and M. Wutting, Appl. Phys. Lett. **81**, 100 (2002).
[11] K. S. Chang, M. A. Aronova, C. L. Lin, M. Murakami, M. H. Yu, J. Hattrick-Simpers, O. O. Famodu, S. Y. Lee, R. Ramesh, M. Wuttig *et al.*, Appl. Phys. Lett. **84**, 3091 (2004).
[12] H. Zheng, J. Wang, S. E. Lofland, Z. Ma, L. Mohaddes-Ardabili, T. Zhao, L. Salamanca-Riba, S. R. Shinde, S. B. Ogale, F. Bai *et al.*, Science **303**, 661 (2004).
[13] Y. Lin, N. Cai, J. Zhai, G. Liu, and C. W. Nan, Phys. Rev. B **72**, 012405 (2005).
[14] J. G. Wan, Z. Y. Li, Y. Wang, M. Zeng, G. H. Wang, and J. M. Liu, Appl. Phys. Lett. **86**, 202504 (2005).
[15] S. Dong, J. Zhai, F. Bai, J. F. Li, D. Viehland, and T. A. Lograsso, J. Appl. Phys. **97**, 103902 (2005).
[16] J. B. Neaton, C. Ederer, U. V. Waghmare, N. A. Spaldin, and K. M. Rabe, Phys. Rev. B **71**, 014113 (2005).
[17] C. W. Nan, Phys. Rev. B **50**, 6082 (1994).
[18] M. Avellaneda and G. Harshe, J. Intell. Mater. Syst. Struct. **5**, 501 (1994).
[19] Y. Benveniste, Phys. Rev. B **51**, 16424 (1995).
[20] J. H. Huang and W. S. Kuo, J. Appl. Phys. **81**, 1378 (1997).
[21] J. Y. Li and M. L. Dunn, J. Intell. Mater. Syst. Struct. **9**, 404 (1998).
[22] M. I. Bichurin, V. M. Petrov, and G. Srinivasan, Phys. Rev. B **68**, 054402 (2003).
[23] S. Srinivas and J. Y. Li, Acta Mater. **53**, 4135 (2005).
[24] C. W. Nan, G. Liu, Y. Lin, and H. Chen, Phys. Rev. Lett. **94**, 197203 (2005).
[25] D. J. Bacon, D. M. Barnet, and R. O. Scattergood, Prog. Mater. Sci. **23**, 51 (1978).
[26] T. Mura, *Micromechanics of Defects in Solids*, 2nd ed. (Martinus Nijhoff, Dordrecht, 1987).
[27] M. L. Dunn and H. A. Wienecke, Int. J. Solids Struct. **33**, 4571 (1996).
[28] M. L. Dunn and H. A. Wienecke, Int. J. Solids Struct. **34**, 3571 (1997).
[29] T. Michelitsch, Z. Phys. B: Condens. Matter **104**, 497 (1997).
[30] J. Y. Li and M. L. Dunn, Philos. Mag. A **77**, 1341 (1998).
[31] J. Y. Li, Int. J. Solids Struct. **39**, 4201 (2002).
[32] E. Pan, ZAMP **53**, 815 (2002).
[33] H. C. Hu, Acta Sci. Sin. **2**, 145 (1953).