Moiré Method

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ME 557

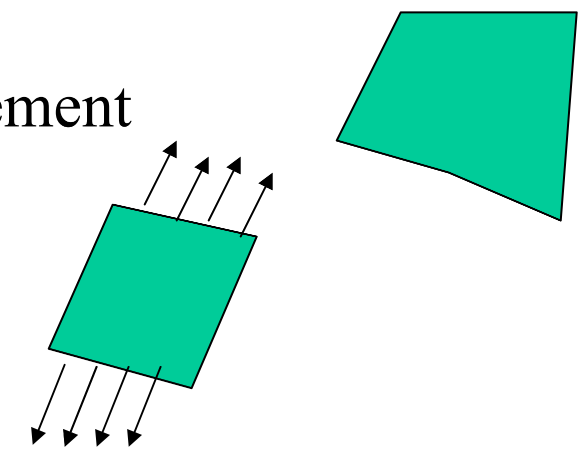
Moiré Methods

Moiré = The French name fro a fabric called “ water silk”, fabric
Exhibits patterns of light/dark bands

Moiré Fringes: or the Moiré effect” refers to light/dark bands seen
By superimposing two nearly identical arrays of lines or dots

In most basic form, Moiré methods are used to measure
Displacement fields; either

- in plane displacement

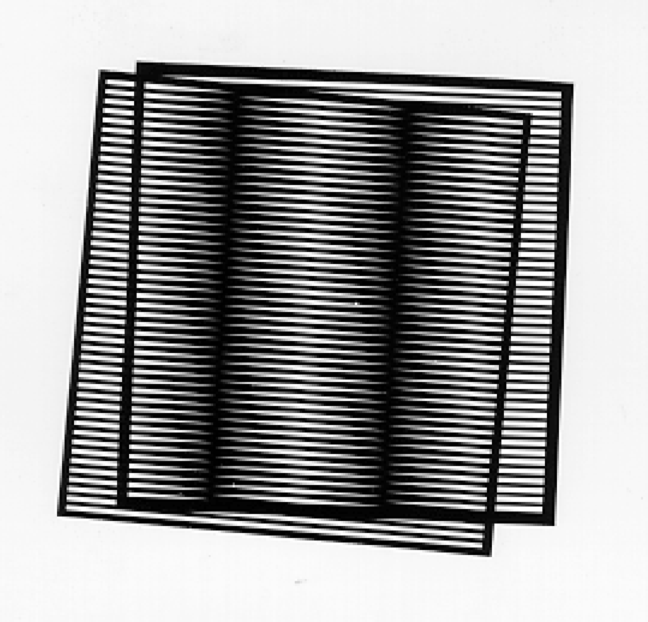


- out of plane

Geometric Moiré

Moiré effect is the mechanical interference of light by superimposed network of lines.

The pattern of broad dark lines that is observed is called a moiré pattern.



Superimposed Gratings

Application of Geometric Moiré

- Specimen attaches to specimen surface (“specimen grating”)
- Specimen viewed through a fixed grating (“reference grating”)
- Specimen loaded, causing deformation/rotation of specimen grating -> Moiré fringe pattern develops.

Fringes are caused by two distinct mechanisms:

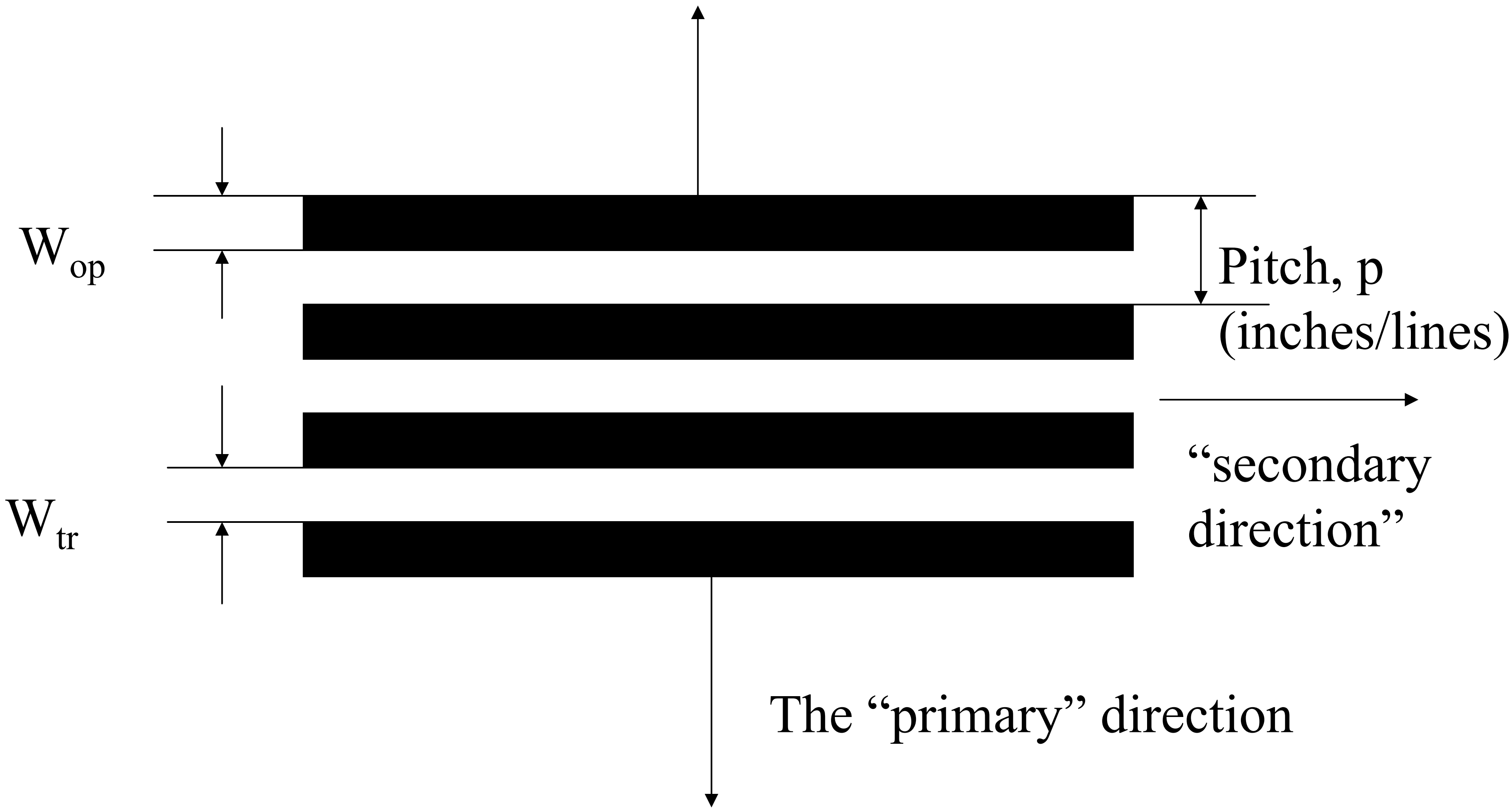
1) Mismatch in pitch between reference and specimen grating (occurs due to deformation of specimen grating in the primary direction)
2) Rotation of specimen grating with respect to reference grating (…rigid body rotations usually not of interest..)

in practice, both deformation and rotation occur simultaneously

Moiré methods can be grouped into two major groups:

1. Geometric Moiré
   Fringes patterns can be interfered based strictly on "geometry"
   - in-plane Moiré – in-plane displacement
   - shadow Moiré – out of plane displacement
   - out of plane Moiré

2. Moiré interferometry
   Interference of fringes seen required consideration of diffraction effects. (grating frequency is higher than geometric moiré)



$p = W_{op} + W_{tr}$, where W_{op} usually ~ $W_{tr} = p/2$
$f = 1/p$ = frequency

For Geometric Moiré a) p= 0.001 -> 0.02 in (50 ->1000 lines/in)
b) p= .025 -> 0.50mm (2 to 40 lines/mm)

Grating and Specimen Preparation

Grating can be formed by holographic interference technique, e-beam writing, X-ray lithography, Phase mask etc.

Grating transferred to metal specimen by lithography using photosensitive coating or photoresist or dichromate gelatin on specimen. (show example on metal specimen)

Gratings are usually 20 to 40 lines/mm (500 to 1000 lines/in) line width is approximately 50% of the pitch (maximum grating strength) (show grating plates)

Sub-micron Lithography

1. Optical Lithography-using lenses to reduce a mask image onto target (smallest feature size~ .25μm)
Phase shift, phase edge or overexpose/overdevelop – 0.1μm

Phase grating mask interference (near-field holography)- minimize 0th order Diffraction and emphasize $\pm$ 1 orders interferences. period of standing wave = ½ of period of phase grating mask

2. X -ray lithography – requires mask made by EBL resolution ~ .25μm

3. Ion beam lithography- damage from ion bombardment limited on Film thickness, advantages no proximity effect so line width control Is good. Possibility of ion beam assisted etching.

4. EBL- 50nm (NTU) 20nm (UW)

Typical EBL system

A commercial electron beam lithography tool.
(courtesy of JEOL Ltd.)

E-beam process

Problem- difficult to achieve accurate pattern placement

Large area patterns are formed by stitching together mosaic of small fields or stripes. Are within each field is accessed by deflecting the focus beam, while successive fields are written by moving the substrate.

Field distortion: Thermal expansion, charging, beam current
and focus drift,
Stiching error

Solution- spatial phase locked e beam lithography, pre exposed film with interference pattern which generates a spatial reference pattern

Lithographic Techniques for gratings

Interference Lithography:

Traditional Ar ion laser (488nm), AZ1350, az1512– long exposure
He-Cd laser(325nm), Shipley Ultra-123

Thin oxide layer and ARC are needed to prevent reflection from the substrate
Honeywell has some ARC materials (free)

Techniques: two arms interference (MachZenhder, Michelson)
Lloyd's mirrors
problem- small spot, incident waves spherical (collimate light to plane wave) reflection from substrate creates secondary standing waves in orthogonal direction leads to poor line width control and ripple edges

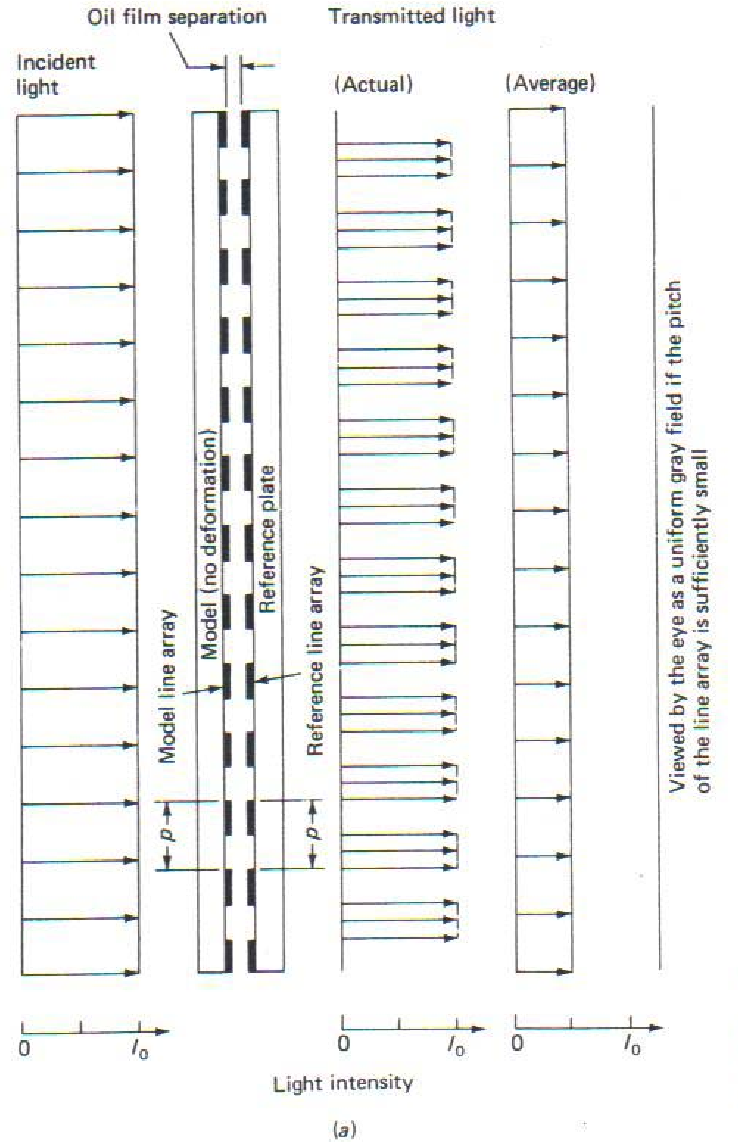
Generating and recording fringe pattern

Direct contact – oil film between specimen and reference gratings

Optical contact- photograph specimen grating before loading, and re-photograph the same specimen grating on the same film creating double exposure. The unloaded specimen grating serve as reference grating. (intensities are different)

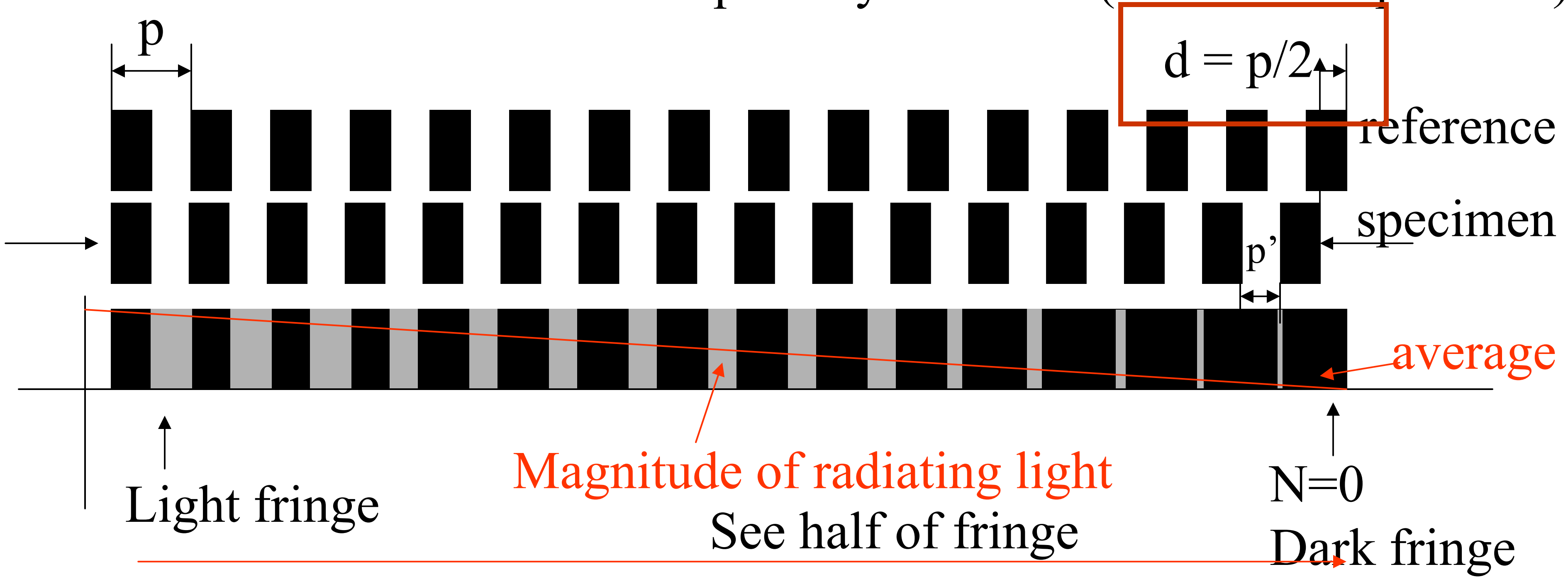
Intensity distribution is due to superposition of two gratings:

* Any region with single line or two superposed lines appears black.
* Any region not covered by either set of lines will appear light.



Moiré Fringe Pattern

Case 1. Uniform deformation in primary direction (assume compression)



In general, total displacement (d) related to fringe number (N),

d= p/2 and we see half of a fringe so

d = pN (no rotation) (N=1/2)

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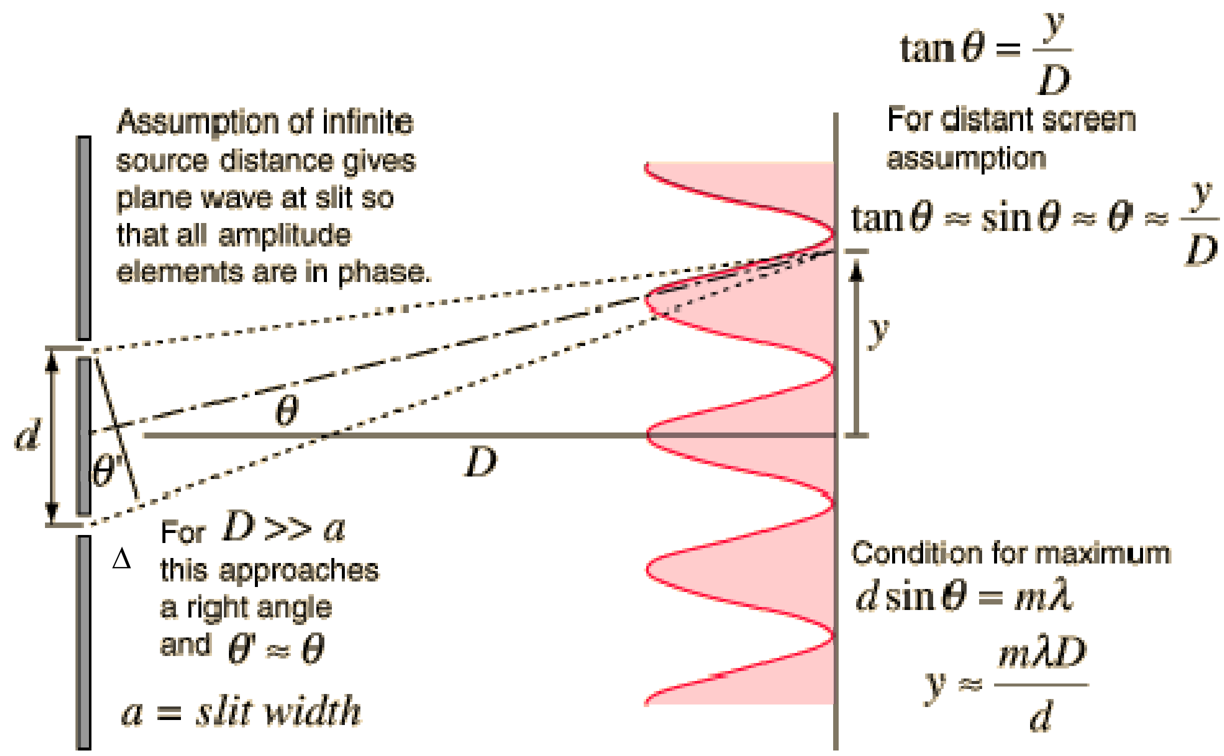
Double Slits Interference



hyperphysics

$d\sin\theta = m\lambda$

$$\boxed{y \sim \bar{m}\lambda D/d}$$

$$I = I_0 \left[\frac{\sin\left(\frac{Nkd}{2}\sin\ \theta\right)}{\sin\left(\frac{kd}{2}\sin\ \theta\right)} \right]^2$$

Where $N = 2$

$$\boxed{d >> a}$$

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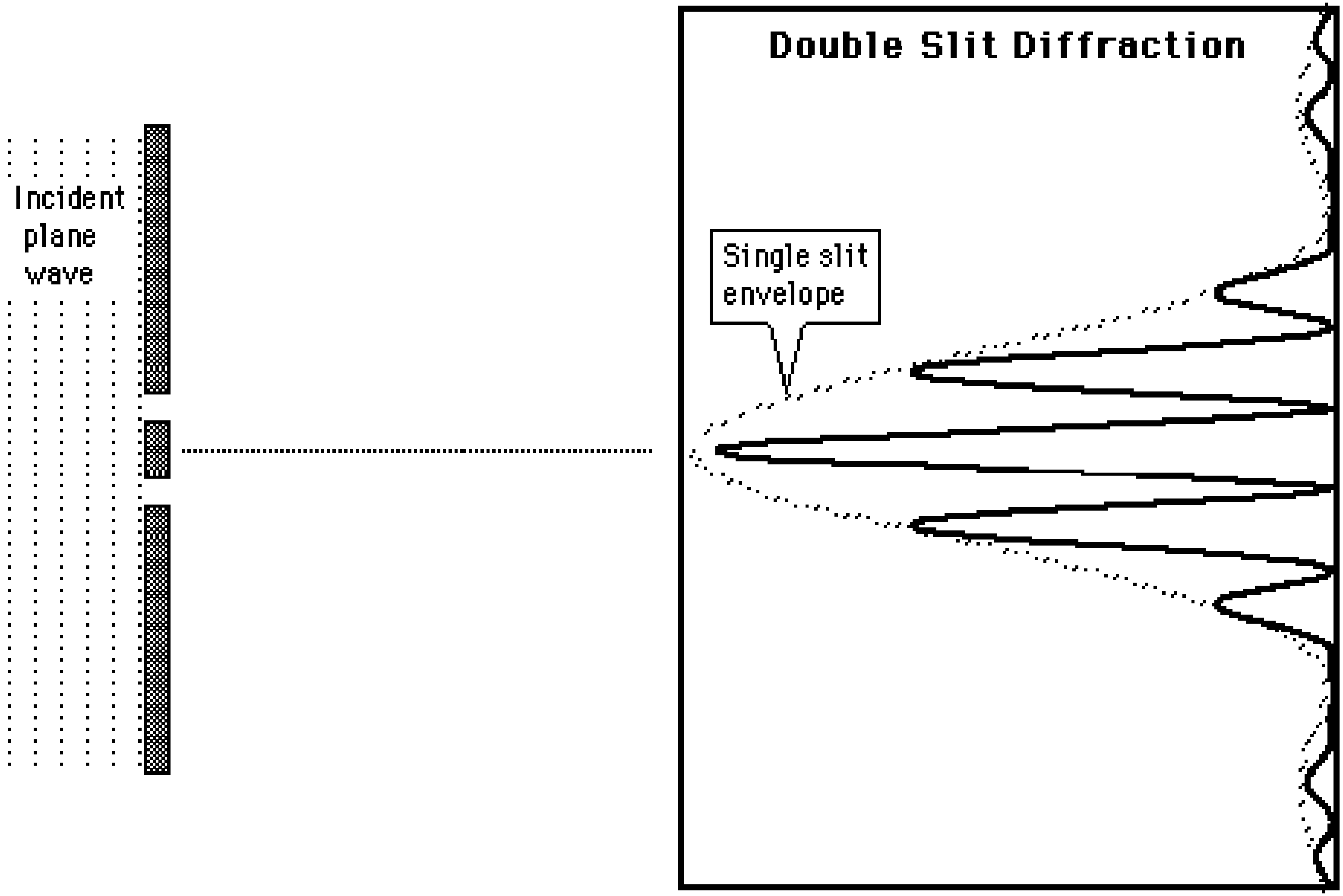

Slit separation ~ slit width

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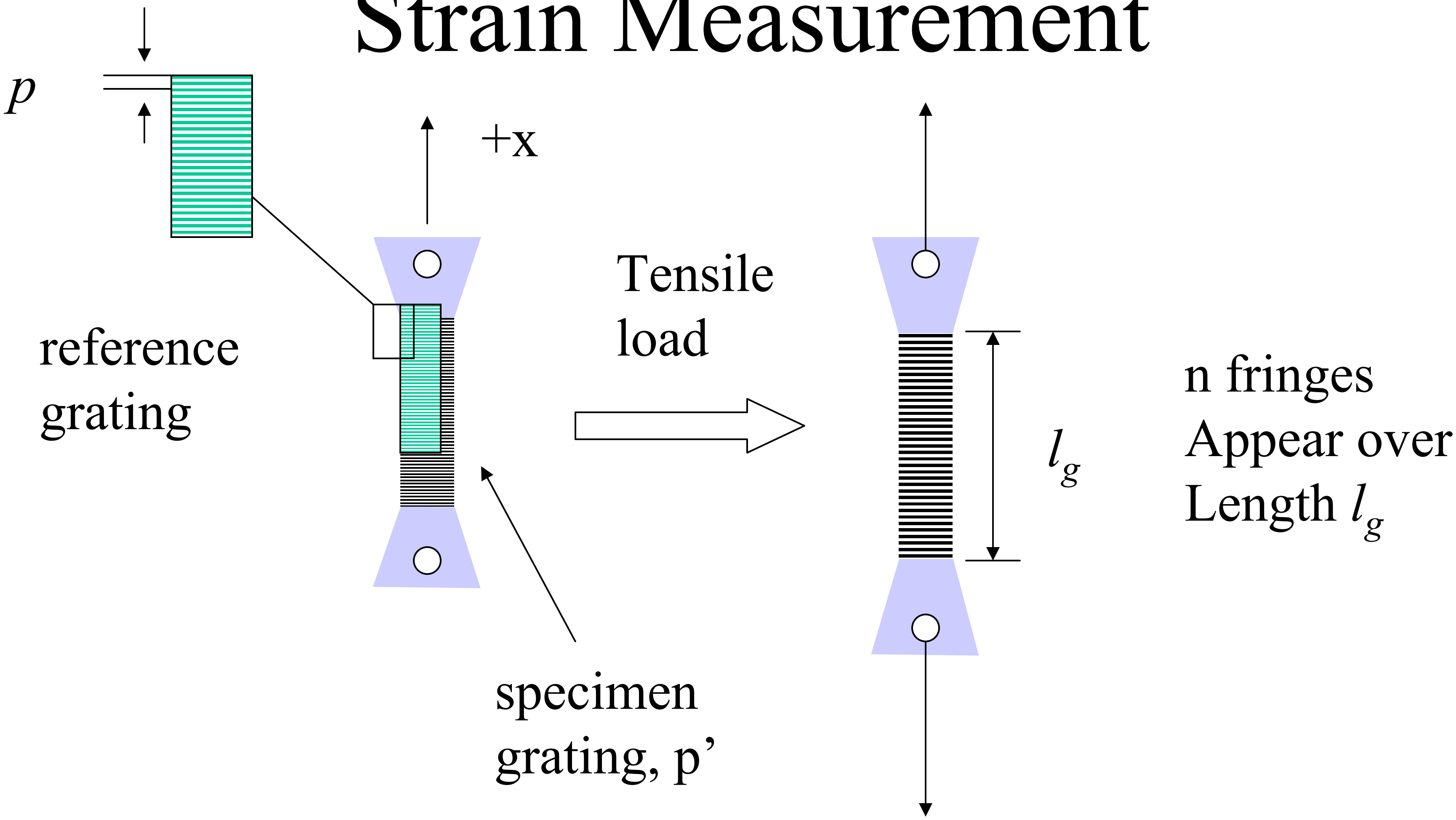
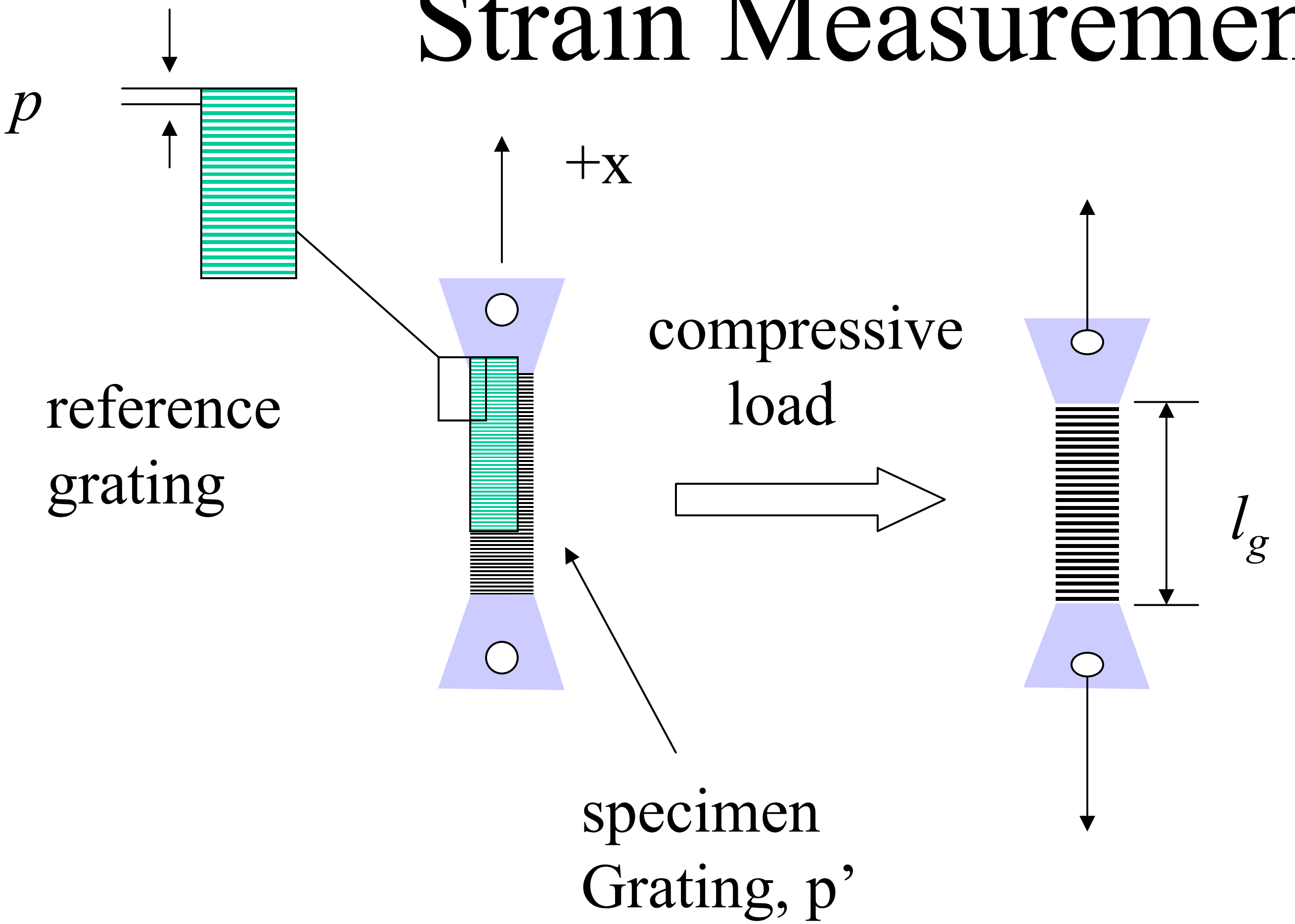
Strain Measurement



Strain $\varepsilon_x = \Delta l / l_o = np / (l_g - np) = (p' - p)/p$

Apply this in point by point fashion

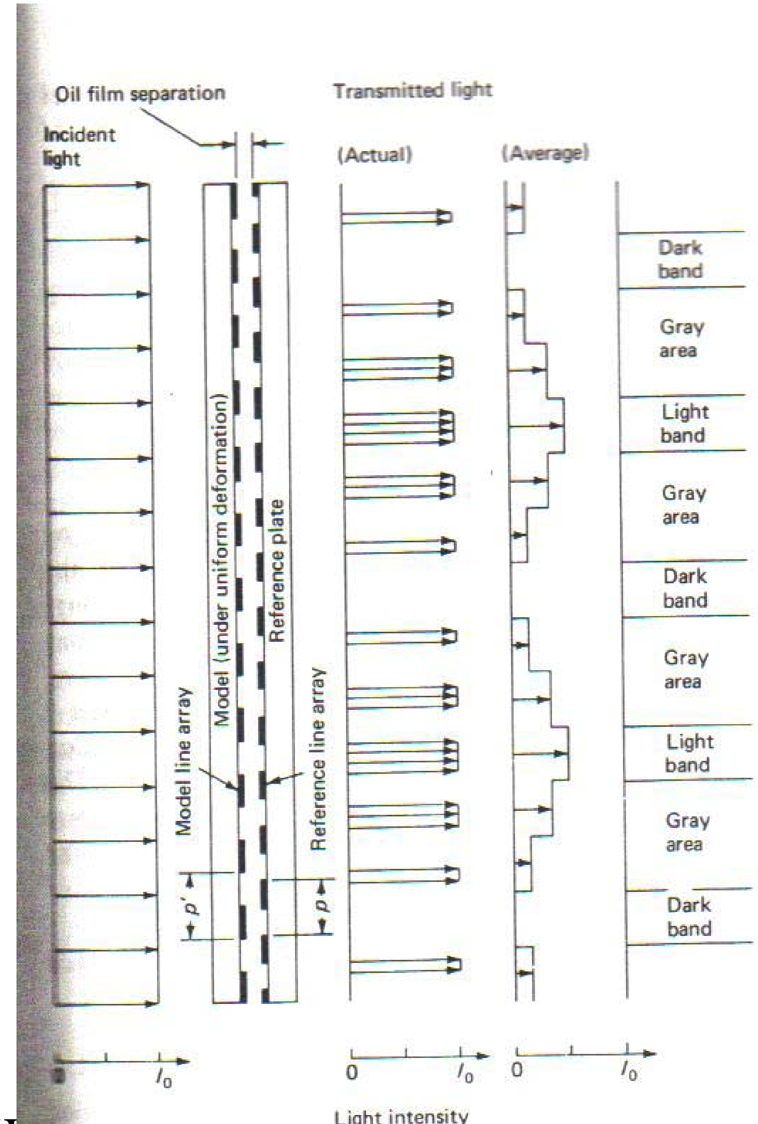
Strain Measurement



n fringes
Appear over
Length l_g

Strain $\varepsilon_x = \Delta l / l_o = -np/(l_g + np)$

Example



p = 0.025mm and 32 fringes have formed in 25mm gage length indicated on the specimen. Thus the change in length of the specimen in 25mm interval is

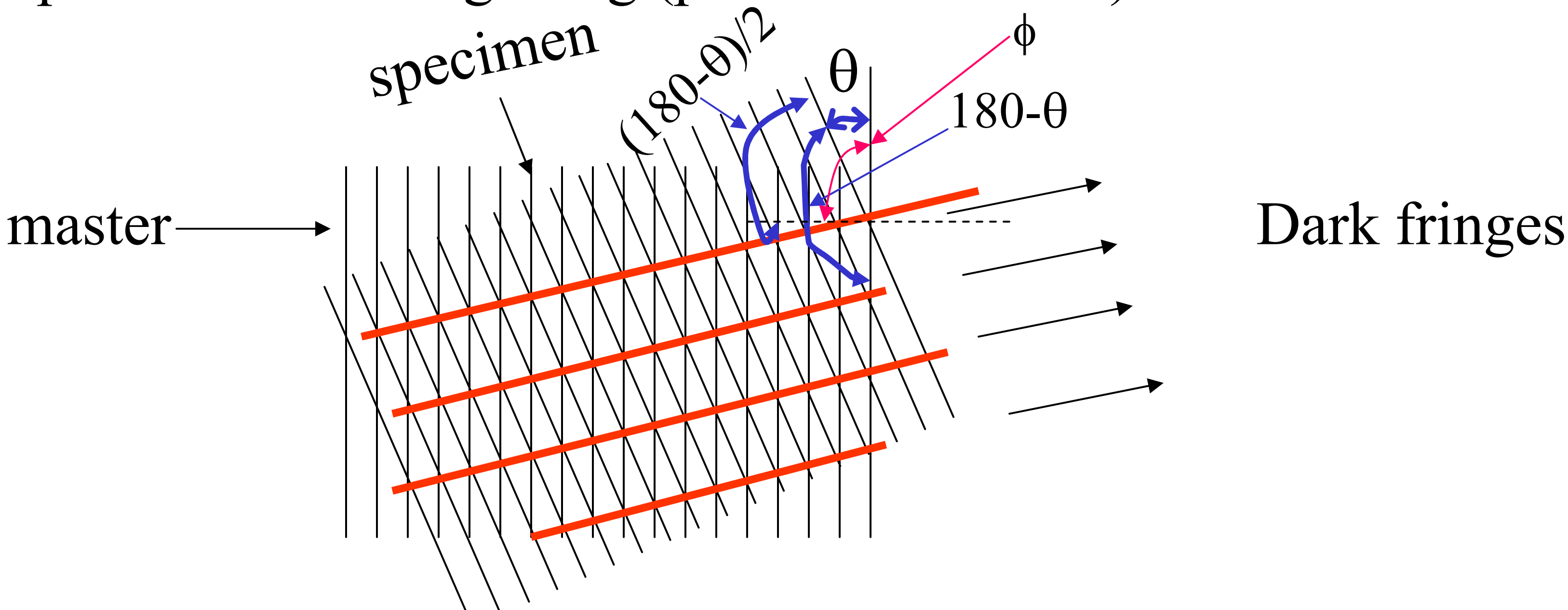
$\Delta l = np = 32\ (0.025) = 0.8\ (mm)$

$\varepsilon = \Delta l/l_o = np/(l_g - np) = 0.8/\ (25-0.8) = 0.033$

Gage length is l_g

Moiré Fringe Pattern

Case 2: Specimen grating experience pure rotation (θ) with Respect to reference grating (p = same for both)



* Fringes bisect the obtuse angle between master and specimen gratings
* Let ϕ = angle between (secondary direction of master grating) and (moiré fringes) $\phi = \theta + (180 - \theta)/2$ ----> $\phi = 90 + \theta/2$

$\theta = 2\phi - 180$ (gives orientation of specimen grating)

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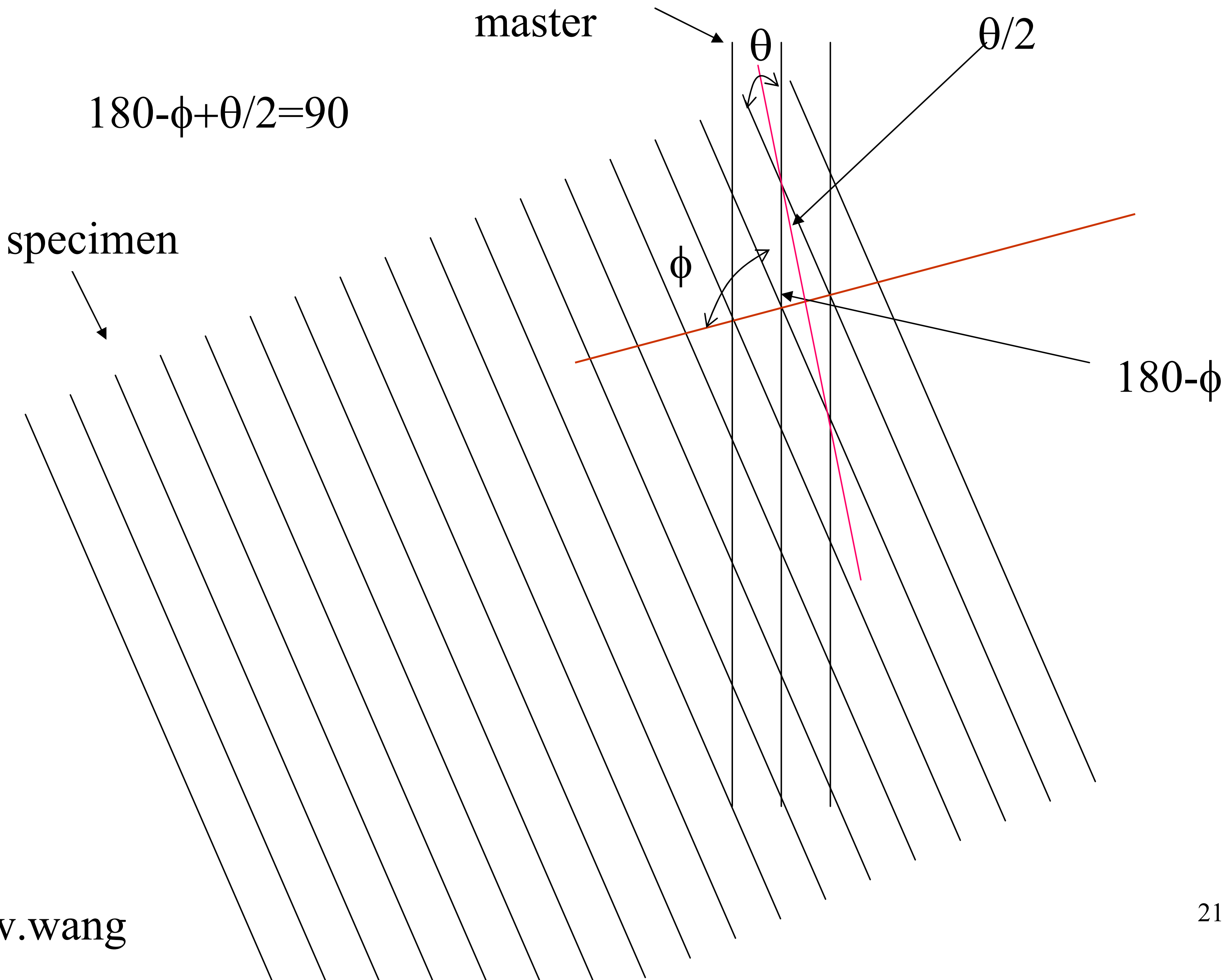

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Example

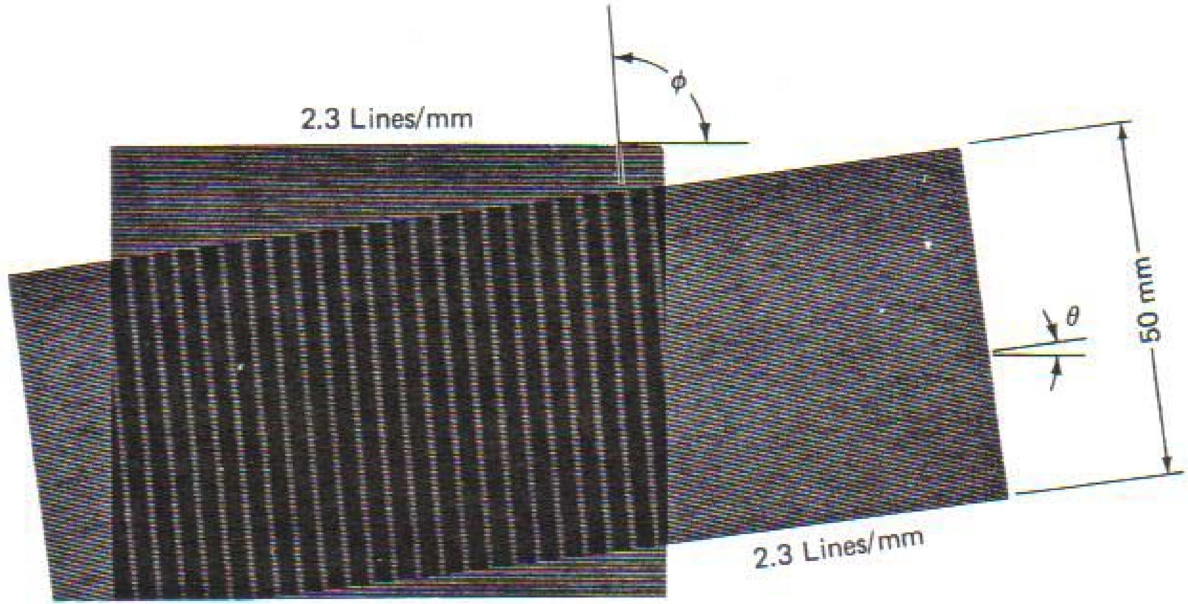

FIGURE 11.3
Moiré fringes formed by rotation of one grating with respect to the other.

pure rotation

Find ϕ solve θ

Moiré Fringe Pattern

Case 3. Specimen grating experiences an unknown deformation
(p -> p’) and unknown rotation (θ) causing fringe pattern

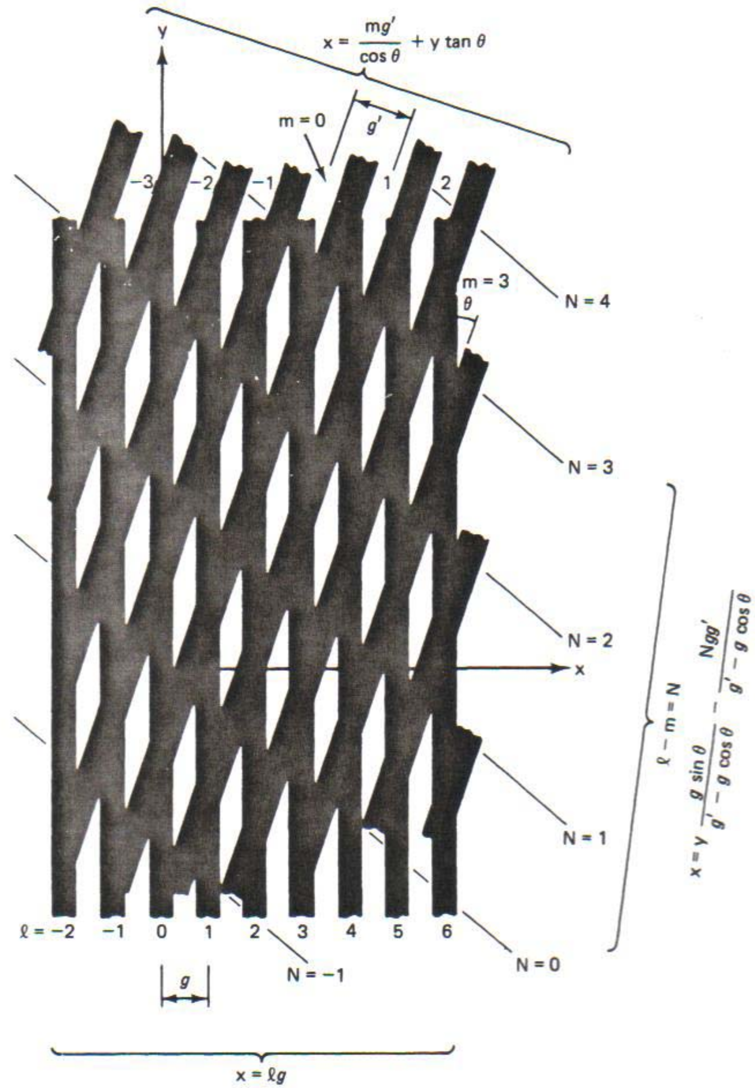

Figure 6-2 Detail of fringe formation between two parallel line gratings set at an angle.

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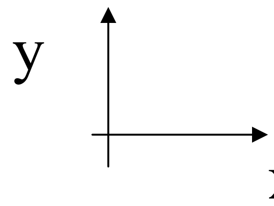
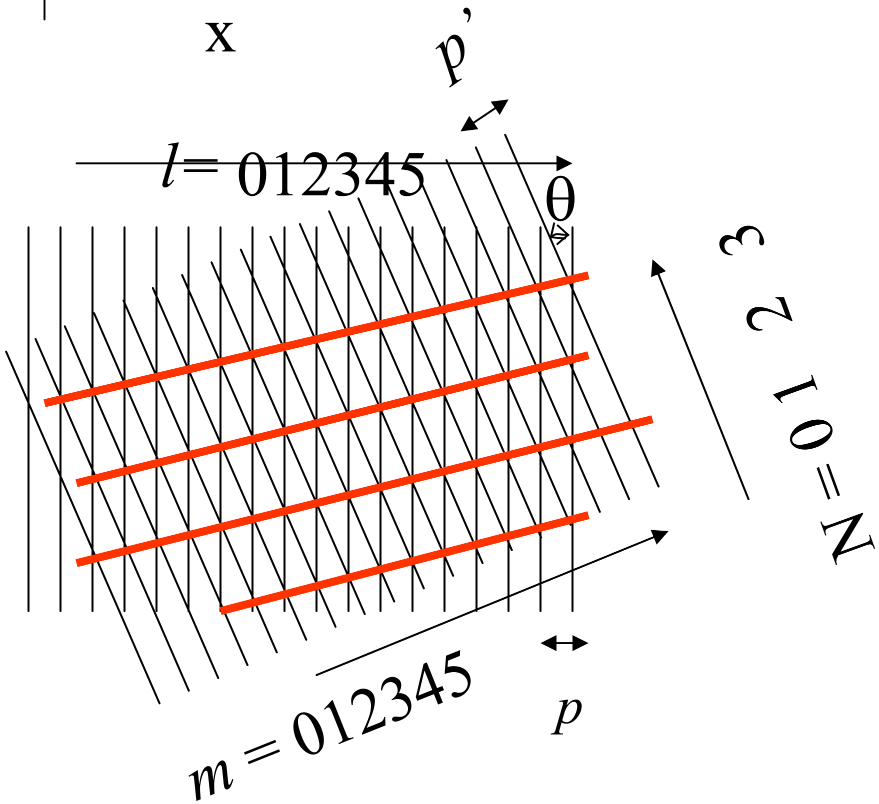
Superimposed gratings
(having initial displacement and rotation)





$m - l =$ N (moiré fringe number)

1st Family of lines along x direction

$$x = lp$$

Where l *= integer*

p*= pitch for specimen grating in x direction*

2nd Family of lines at a small angle θ

$$x\cos\theta = mp' - y\sin\theta$$

Where m *= integer number*

p' *= pitch of the reference grating*

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Small angle rotation and approximation approach

$m - l = \mathrm{N}$ ---> $(x\cos\theta + y\sin\theta)/p' \ \ - x/p \ = N$

---> $((p\cos\theta\, m - p')x + py\sin\theta)/\, pp' \ = N$

If rotation is kept small,

$$(p-p')x + p\theta y = Npp'$$

Or

$$x(p-p')/p + \theta y \ = Np'$$

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$(p-p')/p$ = length change/ original length = ε_x (strain in x direction)

$$\boxed{\varepsilon_x x + \theta y = Np'}$$

Equation implies N depends on the initial pitches of the the gratings, and their initial relative position and orientation.

Fringe shift can be used to measure the change of pitch (strain), change in relative position (translation) and orientation (rotation)

Moiré Fringe Patterns

* Two approaches: "geometric approach" and "calculus approach" (also called "displacement field" approach)

Knowing p, we can measure d (distance between fringes) , ϕ (angle between fringe and x axis) based on θ and find pitch on Specimen p'.

In-plane displacement field using Moiré fringe analysis (exact solution)

Geometrical approach (point by point measurement):
(uses measurement of fringe spacing and inclination at points of interested to determine strain)

1. Define x-y coordinate system, with x-axis = primary direction
2. Reference (master) grating thought as a "family" of lines given by;

$$x = lp' \qquad (1)$$

l is integer increasing in +θ direction, order number of each line

3. Specimen grating thought of as 2nd "family" of straight lines, given by,

$$x = mp/\cos\theta + y\tan\theta \qquad (2)$$

m is integer increasing in +θ direction, p is the pitch of specimen

4. Moiré fringes occur wherever:

$$l\text{-}m = \mathrm{N} \qquad (3)$$

N is integer moiré fringe order

In-plane displacement field using Moiré fringe analysis

Combine equation 1 to 3,

$$(p' - p\cos\theta)x + (p\sin\theta)y - \mathrm{N}pp' = 0 \qquad (4)$$

This is the form of a third family of straight lines, of the general form:

$$[\mathrm{A}]x + [\mathrm{B}]y + [\mathrm{C}] = 0$$

The distance between two moiré fringes, d is

$$\boxed{d = |C| / (A^2 + B^2)^{0.5} = pp' / (p^2\sin^2\theta + (p' - p\cos\theta)^2)^{0.5}} \qquad (5)$$

The slope of the fringe, ϕ is also easily measured, from equation 4:

$$\tan\phi = -p\sin\theta / (p' - p\cos\theta) \qquad (6)$$

In-plane displacement field using Moiré fringe analysis

Equation 5 and 6 can be rearrange to give:

$$p = d \,/\, (1 + (d/p')^2 + 2(d/p')\cos\phi)^{0.5} \qquad (7)$$

$$\theta = \tan^{-1}(\sin\phi \,/\, ((d/p') + \cos\phi)) \qquad (8)$$

Allows calculation of the pitch p and orientation θ of the specimen grating in terms of the reference grating pitch p' and easily measure feature of the moiré fringes distance (d) and angle (ϕ)

Limit is 30o rotation and 30% strain

→ Looking at large area displacement

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Example

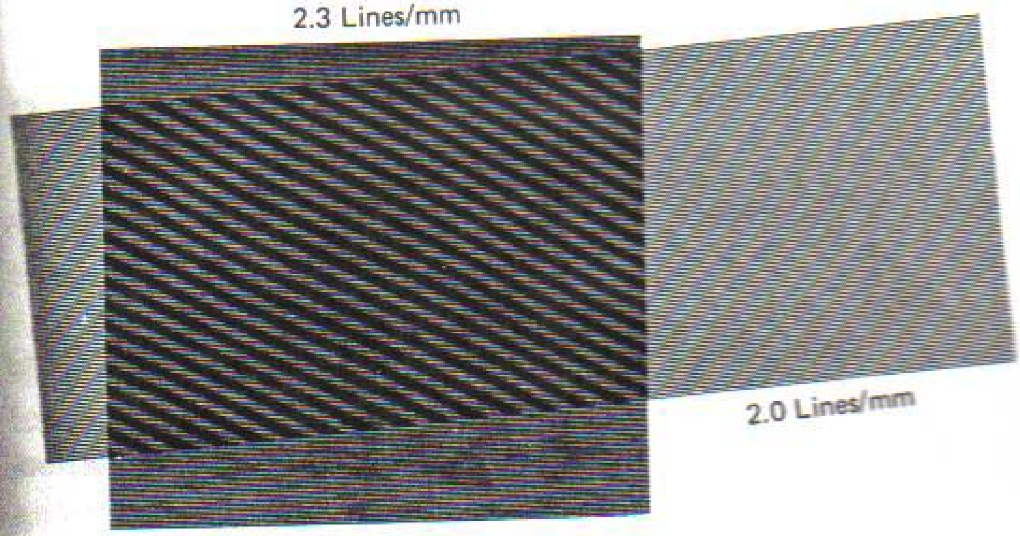

FIGURE 11.4
Moiré fringes formed by a combination of rotation and difference in pitch.

Pitch Mismatch

Difference of pitch between the model and master grills will cause Moiré fringes to form even though the model is unstrained.

Let θ = 0, p = original pitch for the model grating, p' =pitch of master Grating, δp = change in p caused by strain

Then $x(p-p')/p = Np'$

Becomes $$\left(\frac{p+\delta p-p'}{p}\right)x = Np'$$

$$\left[\left(\frac{p-p'}{p}\right)+\left(\frac{\delta p}{p}\right)\right]x = Np'$$

(mismatch ratio, ρ)

ε_x

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$$\rho + \varepsilon_x x = Np'$$

$$\varepsilon_x = \frac{Np'}{x} - \rho$$

This equation eliminates the effects of initial pitch mismatch.

ρ is essentially the pacing of moire fringes in the initial pattern observed before straining the model

$\rho x = N_i$ = initial fringe order

Pitch mismatch
Creating more fringes
For same displacement

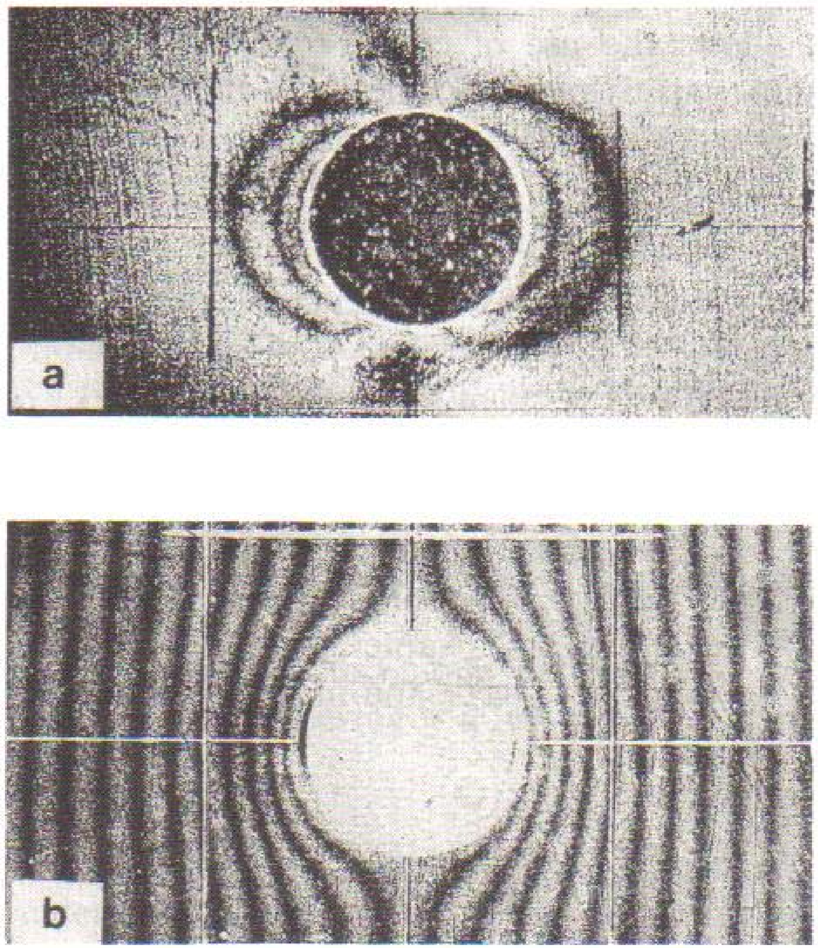

Figure 8.13. Sample geometrical moire pattern for a deformed body taken (a) without mismatch and (b) with a small pitch mismatch (patterns optically filtered) (Cloud 1980).

Whole field analysis

-Whole field as rotations and displacements of a large number of small portions

-moiré fringes will vary in a complicated but continuous way over the extend of the field

-At any point , the fringe order will indicate appropriate mixture of displacement and rotation

-it is not essential the fringes be numbered beginning from zero because <u>**only the spacing of the fringes**</u> (the partial derivative with respect to position) is of any consequence

Moiré basically just another transformation from Cartesian Coordinate into another coordinate. A and B are transformation Factor d is resulting magnitude and f is the resulting angle (i,e. Z t or Fourier transformation…)

Calculus Approach

In theory of elasticity, strain is relate to displacement u_x and u_y as,

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x} \qquad \varepsilon_{yy} = \frac{\partial u_y}{\partial y}$$

(strain in x and y direction)

$$\varepsilon_{xy} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}$$

shear strain

$$\omega_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} - \frac{\partial u_y}{\partial x}\right)$$

(average rotation of the x and y Coordinates)

Above equations can be used

- when load is small and rotations are neglected

or – where Cartesian coordinates need to be reorient where there is rigid body rotation due to the load

General form of strain displacement relation

$$\varepsilon_{xx} = \sqrt{1 + 2\frac{\partial u_x}{\partial x} + (\frac{\partial u_x}{\partial x})^2 + (\frac{\partial u_y}{\partial x})^2} - 1$$

$$\varepsilon_{yy} = \sqrt{1 + 2\frac{\partial u_y}{\partial y} + (\frac{\partial u_y}{\partial y})^2 + (\frac{\partial u_x}{\partial y})^2} - 1$$

$$\varepsilon_{xy} = \arcsin \frac{\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial x}\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\frac{\partial u_y}{\partial y}}{(1+\varepsilon_{xx})(1+\varepsilon_{yy})} \qquad \text{(radians)}$$

Applies to large strain and rotations, no u_z since moiré method is restricted in that direction.

If Cartesian strain ε_{xx}, ε_{yy}, ε_{xy} are small, the principle strains can be obtained from

$$\varepsilon_{1,2} = \frac{1}{2}[\varepsilon_{xx} + \varepsilon_{yy} \pm \sqrt{(\varepsilon_{xx} - \varepsilon_{yy})^2 + \varepsilon_{xy}^2}\,]$$

Moiré pattern is limited to strain ε of 30% and rotation θ of $\pm$ 30°

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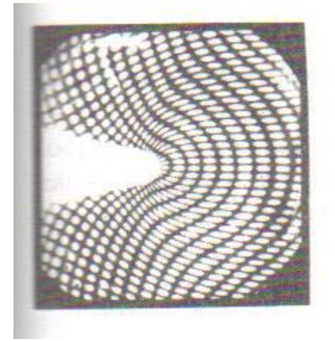
Calculus Approach to Displacement Measurement



To apply ‘calculus approach’:

Load specimen/obtain Moiré patterns representing u_x, u_y

- cross grating used as specimen
- line gratings used as reference grating ()

a) Plot fringe orders (N_x, N_y) along lines originally in the x and y direction,

-> note: coordinate locations must be adjusted:

$$x_{act} = x_{meas} - N_x p$$
$$y_{act} = y_{meas} - N_y p$$

Location maybe insignificant if p is small

b) Plot fringes versus position

c) Take the slope of the plot at each point at which strain is required

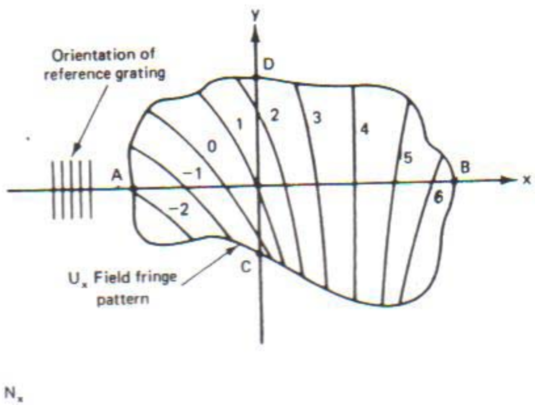
$$\frac{\partial u_x}{\partial x} = p\frac{\partial N_x}{\partial x} \quad \frac{\partial u_x}{\partial y} = p\frac{\partial N_x}{\partial y}$$

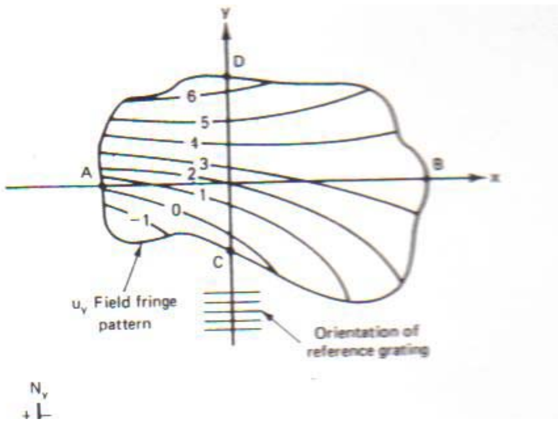
$$\frac{\partial u_y}{\partial y} = p\frac{\partial N_y}{\partial y} \quad \frac{\partial u_y}{\partial x} = p\frac{\partial N_y}{\partial x}$$

d) Calculate strains using general form of strain displacement relation

This approach easily implemented using modern digital image process

a) Record fringe orders along lines originally in the x and y direction





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b) Plot fringes versus position

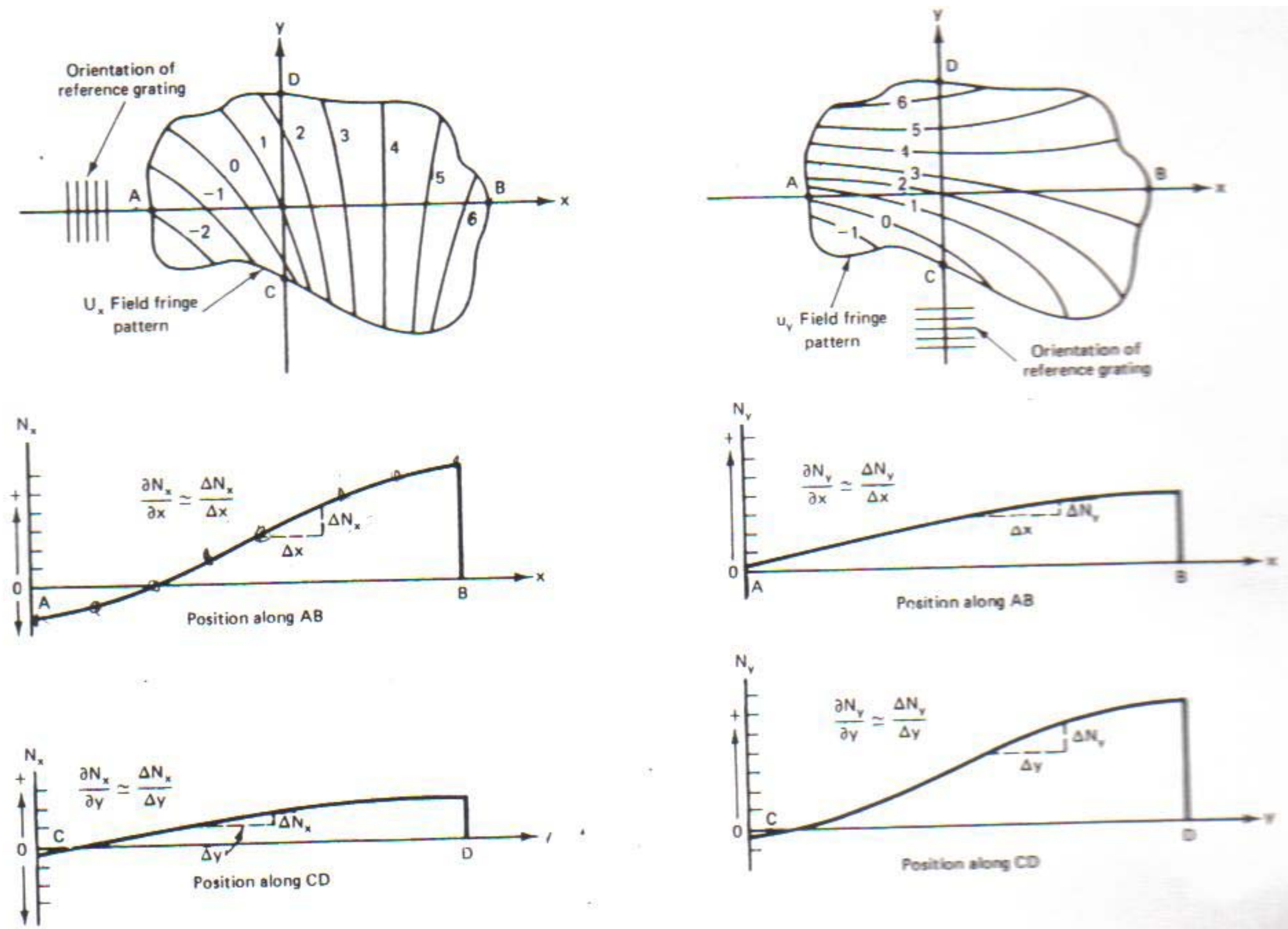

Figure 6-3 Illustration of procedure to obtain the four Cartesian fringe derivatives from the two Cartesian fringe patterns.

c) Take the slope of the plot at each point at which strain is required $\frac{\partial N_x}{\partial x}$ etc…

d) Plot back into general solution for strain displacement

$$\varepsilon_{xx} = \sqrt{1+2\frac{\partial u_x}{\partial x}+(\frac{\partial u_x}{\partial x})^2+(\frac{\partial u_y}{\partial x})^2}-1$$

$$\varepsilon_{yy} = \sqrt{1+2\frac{\partial u_y}{\partial y}+(\frac{\partial u_y}{\partial y})^2+(\frac{\partial u_x}{\partial y})^2}-1$$

$$\varepsilon_{xy} = \arcsin\frac{\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}+\frac{\partial u_x}{\partial x}\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}\frac{\partial u_y}{\partial y}}{(1+\varepsilon_{xx})(1+\varepsilon_{yy})}$$

Problem in displacement field approach

In practice, when u and v moiré fringe patterns are obtained from $\frac{\partial u_x}{\partial x}$ *and* $\frac{\partial u_y}{\partial y}$, but the cross derivatives $\frac{\partial u_x}{\partial y}$ *and* $\frac{\partial u_y}{\partial x}$ can not provide the acceptable accuracy because of the slight errors in alignment of either the specimen or master grating with x and y axes. Misalignment produces a fringe pattern due to rotation in addition to the load-induced pattern

Solutions to shear-strain error

1. To eliminate shear-strain error by using crossed gratings on both the specimen and master to obtain simultaneous displays of the *u* and *v* displacement fields.

(rotation misalignment is then equal for the two fields,
It contribution to the cross derivatives is equal in magnitude
But opposite in sign and thus cancels in the shear-strain
Determination)

Use slightly different pitches on the specimen and master (see Figure on page 36)

2. Using strain-rosette concept employ both perpendicular to x and 45^o with respects to x axis. Then ε_{xx}, ε_n ε_{yy} can be determined. Then use rosset equation to calculate the strain and stress

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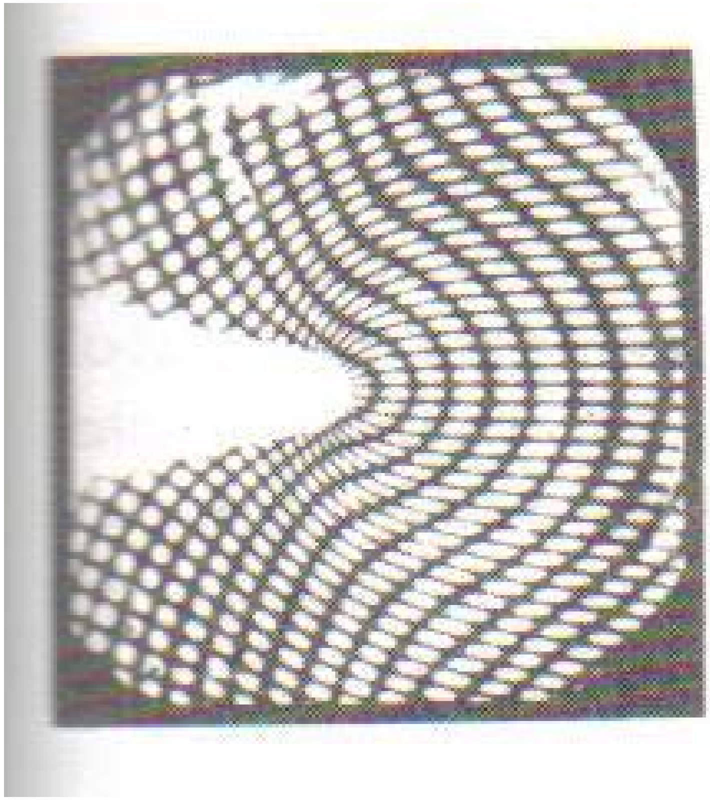


You can
Clearly see
the difference
in fringe pattern
in x and y direction

Moire fringe pattern with crossed gratings of different
Pitch on the mater and specimen

Super Moiré

Superimpose of two identical moiré patterns

To apply ‘moiré of moiré:

a) Load specimen/obtain Moiré patterns representing u_x, u_y

Superimposed two identical moiré patterns (N_{xx}, N_{xy}, N_{yx}, N_{yy} are superfringe orders)

One pattern shifted to a known amount in x or y directions ($s_{xx}, s_{xy}, s_{yx}, s_{yy}$)

b) Plot fringes versus position

c) Take the slope of the plot at each point at which strain is required

$du_x/dx = p\, N_{xx} / s_{xx}$ $\qquad$ $du_x/dy = p\, N_{xy} / s_{xy}$

$du_y/dy = p\, N_{yx} / s_{yx}$ $\qquad$ $du_y/dx = p\, N_{yy} / s_{yy}$

First subscript represents the direction perpendicular to the grating lines and the second to the direction of the shift

d) Calculate strains using general form of strain displacement relation

- *Resolution is higher than other methods*
- *Fringes are harder to see*

Super Moiré

Calculus approach can also be implemented using

Displacement derivatives are obtained using superfringes with the equations,

$$\frac{\partial u_x}{\partial x} = p\frac{N_{xx}}{s_{xx}} \qquad \frac{\partial u_y}{\partial x} = o\frac{N_{yx}}{s_{yx}}$$

$$\frac{\partial u_x}{\partial y} = p\frac{N_{xy}}{s_{xy}} \qquad \frac{\partial u_y}{\partial y} = p\frac{N_{yy}}{s_{yy}}$$

Where N_{xx}, N_{xy}, N_{yx}, N_{yy} are superfringe orders
s_{xx},s_{xy},s_{yx},s_{yy} are respective shifts

Fringe multiplication (increase fringe formation)

by having two different frequency gratings, we can increase the number of the moiré fringes produced by the same loading if original gratings were same period

i.e. If original gratings are both 10lnes/mm and gets 4 fringes after loading, then is one of the gratings increase its frequency to 50lines/mm, we will get 20 fringes. 5 times the original gratings configuration

Sharpening and Multiplication

- Fringe pattern is sharpen by using complementary grating (where opaque part and transparent spaces are not equal width i.e. different intensity in interference)
- Using diffraction effect from the line pattern to increase resolution

In plane Moiré summary

Basically an interferometer…. Using shadow

Counting fringes: Fringes representing the difference of pair of grating lines

Calculus method: Fringe derivative is difference of two fringe orders over a specific distance

Moiré of Moiré method: Super fringe will be proportional to the fringe difference per unit shift and in the direction of the shift

Advantages:
Simple concept, white light (broad band) source
Disadvantages:
Unwanted fringes due to slight mismatch gratings

Applications

Creep
Residual stress
Fracture
Dynamic loading
Thermal deformation in electronic packaging

Direct grating deposition
Laminating grating sheet on specimen

Applications

Figure 6-4 Mismatched moiré pattern of displacement components on a rubber specimen (g = 1/312 in.).

Rubber

Applications

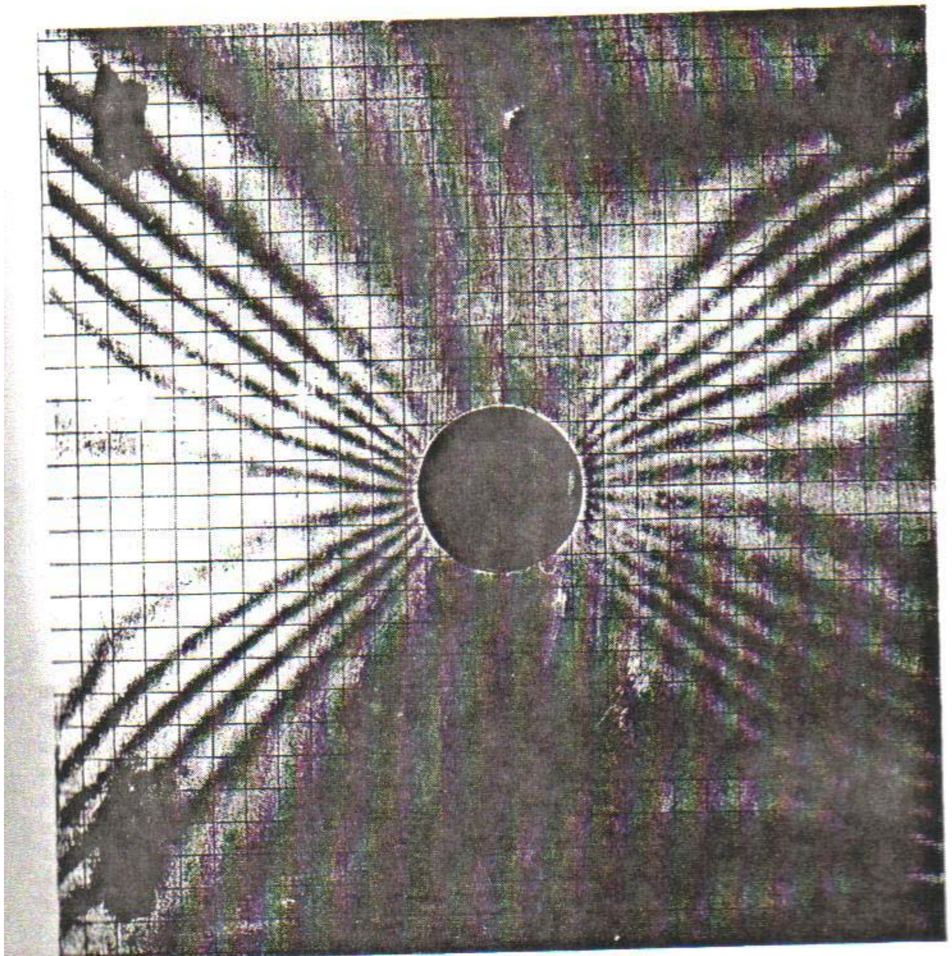


Figure 6-5 Moiré pattern multiplication of vertical displacement components on a stainless steel specimen etched with a 20-line/mm cross grating.

Applications

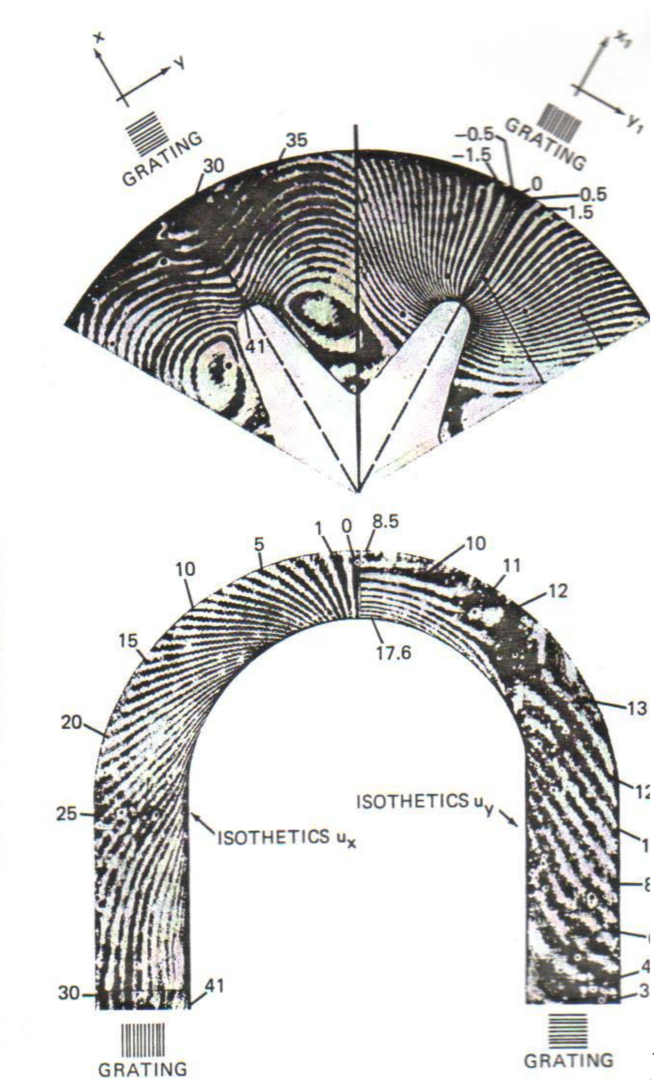


Figure 6-6 Moiré patterns of orthogonal displacement components on (a) vertical and (b) horizont
taken from the interior of a stress-frozen model. (From Ref. 14.)

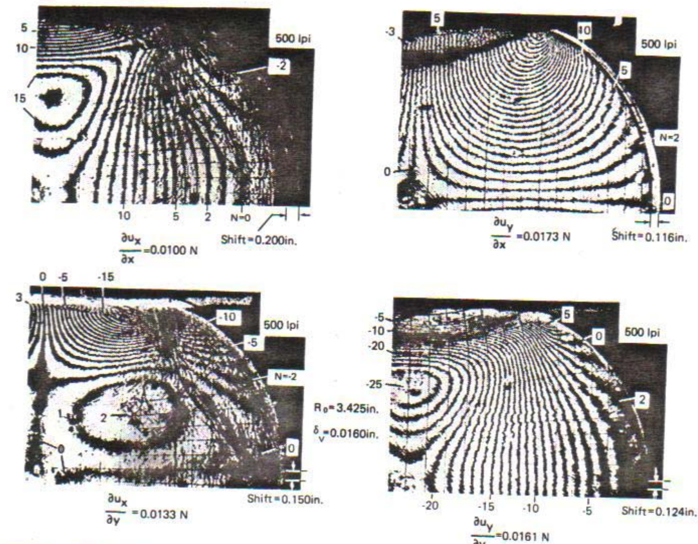


Figure 6-7 Supermoiré fringes corresponding to the four derivatives of displacements in the meridian plane at a sphere subjected to a large compression: lpi = lines/in. (From Ref. 13.)

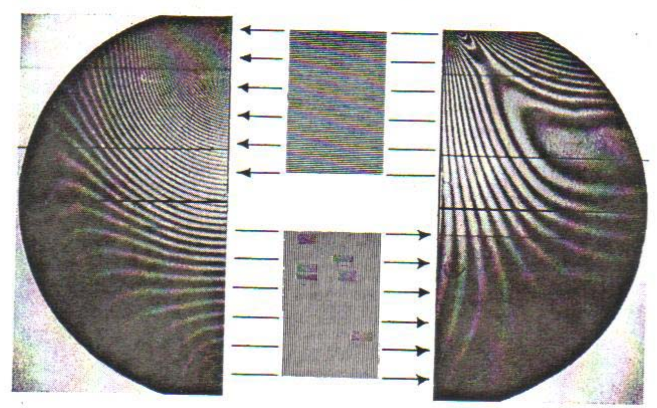


Figure 6-8 Dynamic moiré fringe patterns of vertical and horizontal displacement components on a disk shortly after an impact at the top (g = 0.025 mm). (From Ref. 23.)

Out of plane Moiré

Techniques for using geometric moiré to measure out of plane displacement and slope

Shadow Moiré – utilizes superimposition of a reference grating and its own shadow. the fringes are loci of points of constant out of plane elevation, so they are essentially a contour map of the object being studied.

Method can be used to measure out-of-plane displacements or changes in displacement.

Example of applications: contour mapping of human body with objective of detecting asymmetries that indicates certain infirmities (illness).

Reflection Moiré – Superposition of shift in the reflected grating due to a bend on the specimen, thereby producing a moiré pattern.

Method for measuring <u>slope or rotation or the change of slope</u> of structural components. Approach used in study of plates in bending
(direct measurement of slope)

Example of applications:

Quantify the movement of the scapular (shoulder area)

Flow field mapping

Noncontacting, remote, nontraumatic and nonstressful

Projection Moiré – Out-of-plane displacement measurement or contour mapping involves projection of reference grating on the specimen by means of a slide projector.

Application:

use of moiré fringes to acquire 3D surface shape information

Topography of human scapular mechanism with muscular effort

Shadow Moiré

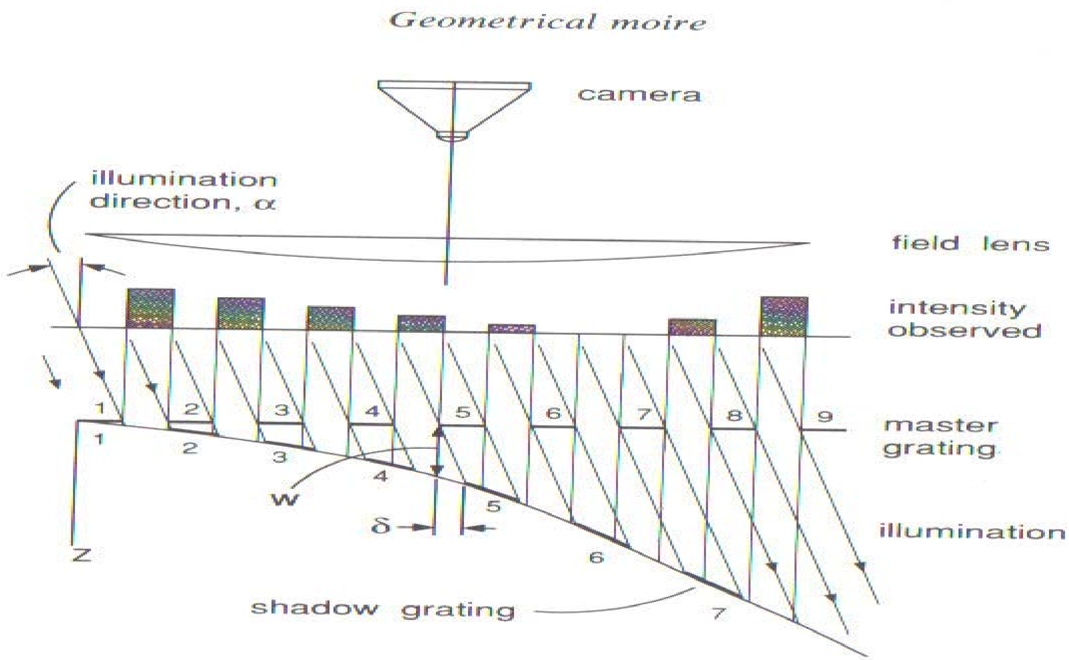


Figure 9.1. Formation of fringes in shadow moire.

- Illumination creates a shadow of grating on the surface of the specimen.
- Grating shadows are elongated on specimen by a fact depends on the inclination of the surface, and they are shifted by the amount depends on the incident angle α and distance ω from the master grating to specimen. Apparent lateral shift of a grating shadow is given by:

$$\delta = \omega \tan \alpha$$

The shadow of m_{th} order lines are spread over the expanse of m+1th lines of grating, Let ω be the z distance between the master and specimen over the same expanse,

$$[(m+1)-m]p = \omega \tan\alpha \to \omega = p/\tan\alpha$$

If there are N moiré fringes between the same expanse, where fringes are observed at normal incidence,

$$w = Np/\tan\alpha$$

A more general form where viewing angle (β) is other than the normal,

$$w = Np/(\tan\alpha - \tan\beta)$$

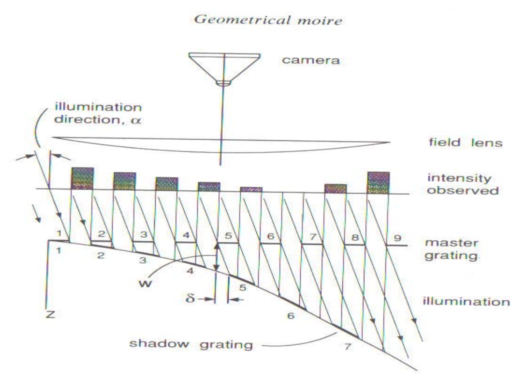


Figure 9.1. Formation of fringes in shadow moire.

where α = *incidence angle*
β = *viewing angle*
p = *grating pitch*
N = *moire fringe order*
w = *axial distance between grating plane to object*

w.wang

Optical Setup for Shadow Moiré

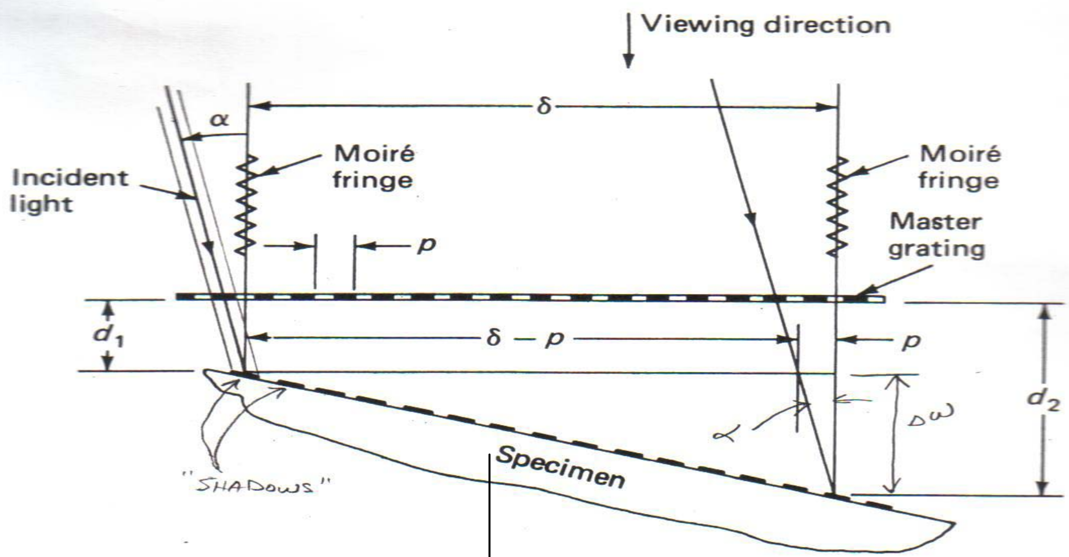
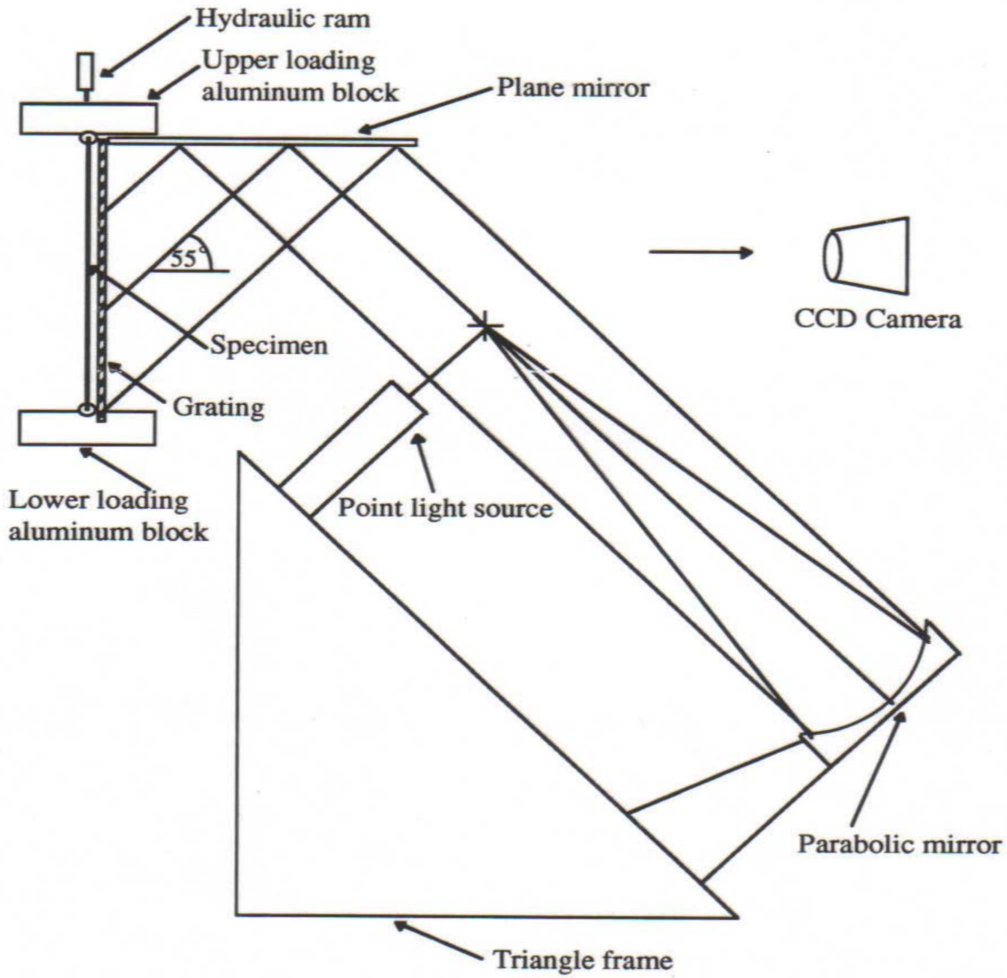


FIGURE 11.10
Moiré method for measuring out-of-plane displacements.

Experimental Methods

Optical Arrangment for Shadow Moire



tuttle

Examples

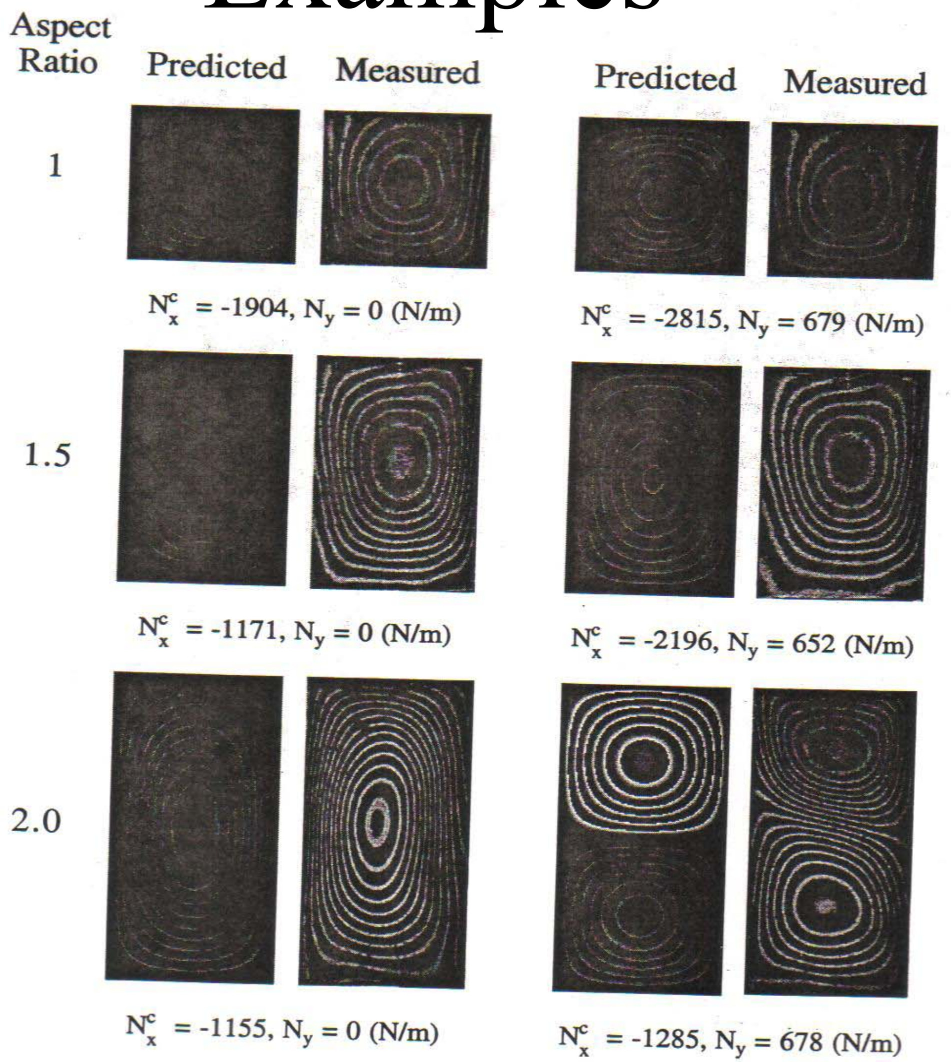


Figure 7: Comparison of Predicted and Measured Buckling Modes Shapes for Specially-Orthotropic $[0]_8$ Panels

Examples

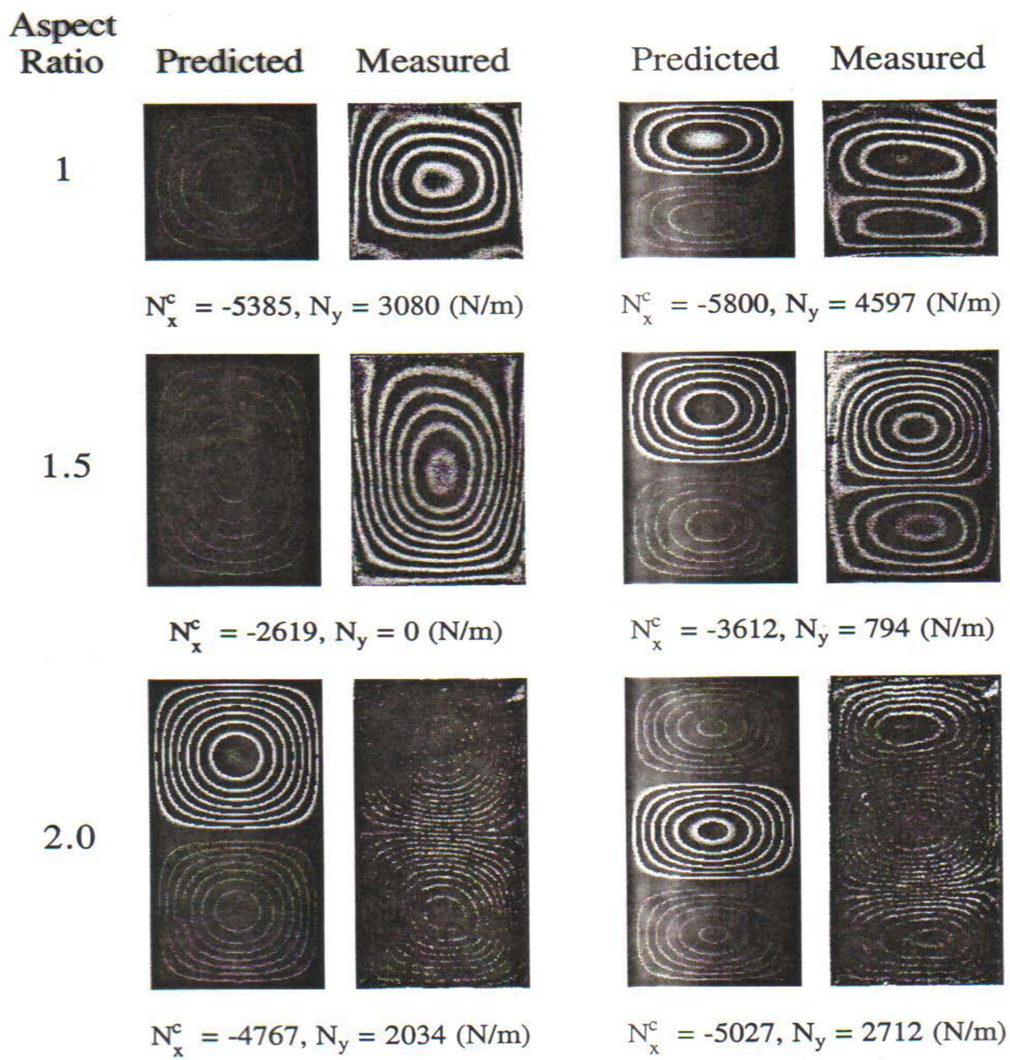


Figure 8: Comparison of Predicted and Measured Buckling Modes Shapes for Specially-Orthotropic $[0/90]_{2s}$ Panels

Examples

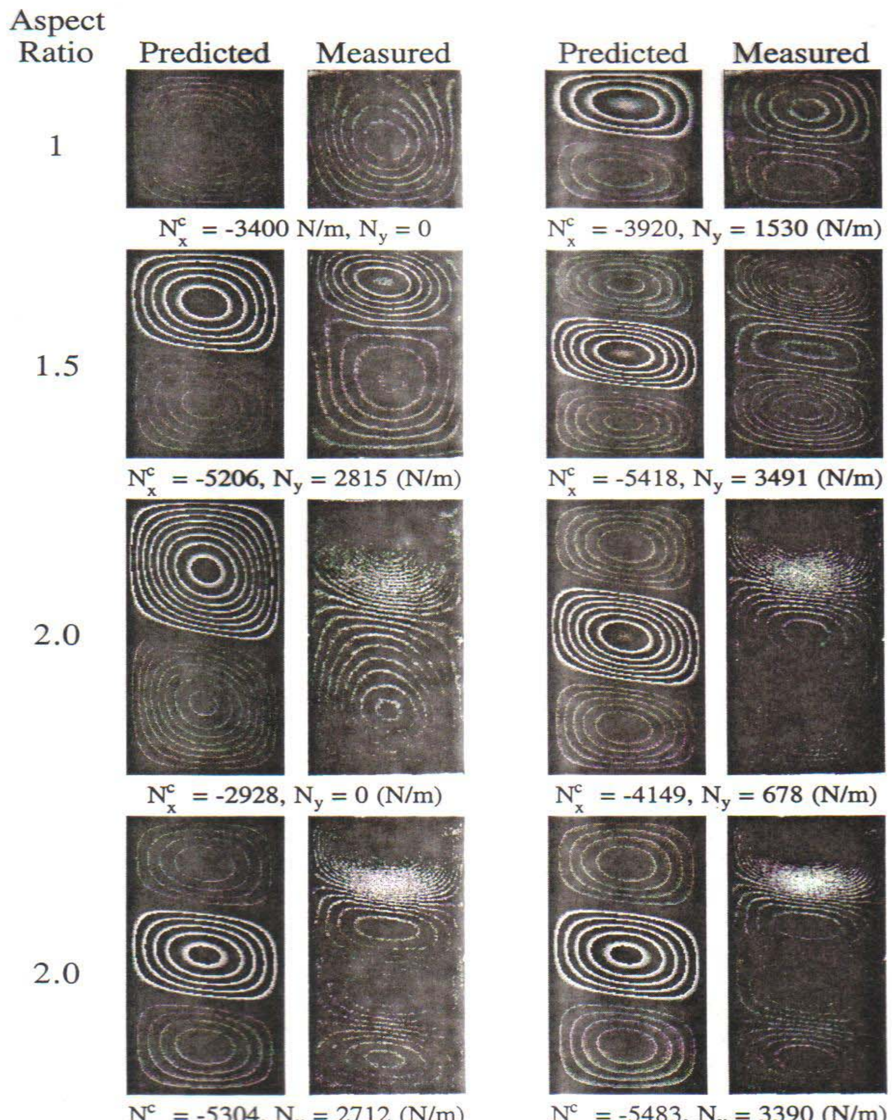


Figure 9: Comparison of Predicted and Measured Buckling Modes Shapes for Generally-Orthotropic $[\pm 45]_{2s}$ Panels

Reflection Moiré

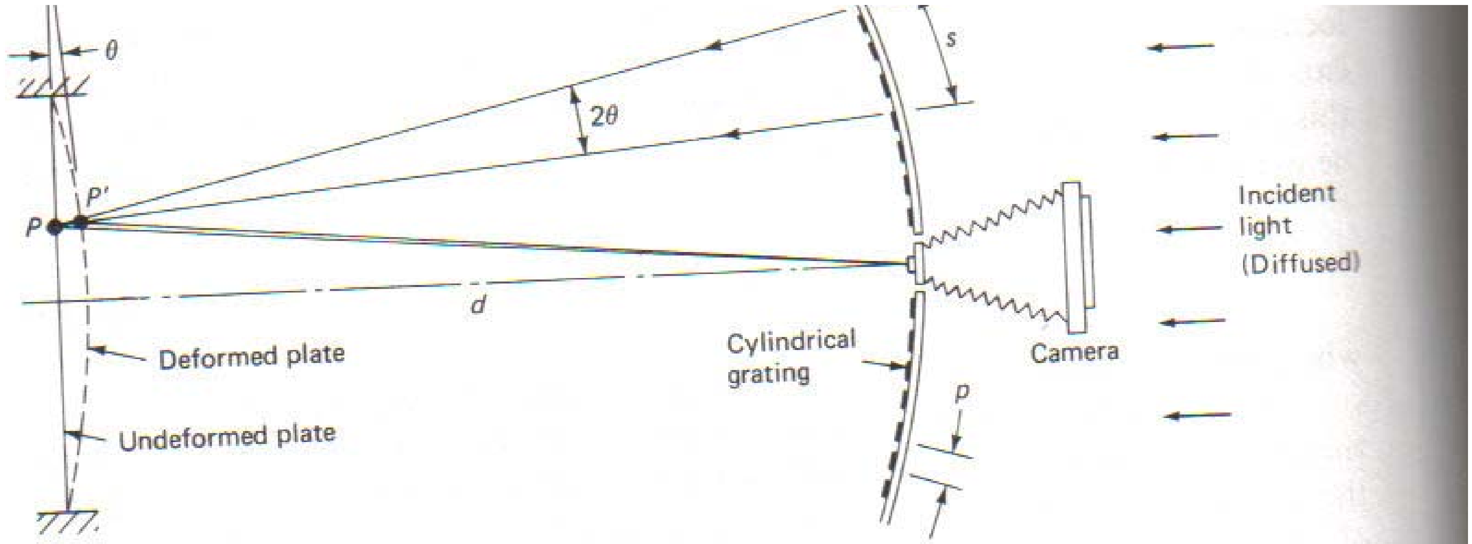


FIGURE 11.11
Moiré method for measuring out-of-plane slopes.

-The plate to be studied is polished on one side so as to act as a mirror.
At some distance d from the plate a moiré master grating is erected.
The cylinder segment with coarse grating is fabricated from a transparency sheet of plastic and the incident light passes through this shell
The grating has a hole at its center and the camera is set up behind the aperture.
Camera is aimed at the plate, but focused on the virtual image of the grating as it is reflected in the polished plate. Double exposure photography is used for the superposition of images

Out-of-plane slope measurement

From the theory of elasticity, it is known that stress at a point in the plate due to bending moments can be expressed in terms of local curvature of plate as

$$\sigma_x = \frac{E_z}{1-\nu^2}\left(\frac{1}{\rho_x} + \nu\frac{1}{\rho_y}\right)$$

$$\sigma_y = \frac{E_z}{1-\nu^2}\left(\frac{1}{\rho_y} + \nu\frac{1}{\rho_x}\right)$$

Where $1/\rho_x$ *and* $1/\rho_y$ are curvatures with respect to x and y axes

The deflections are related to the curvatures by,

$$\frac{1}{\rho_x} = -\frac{\partial^2 w}{\partial x^2}$$

$$\frac{1}{\rho y} = -\frac{\partial^2 w}{\partial y^2}$$

Ligtenberg has developed a moiré method for measuring the partial slopes of $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$ which allows a more accurate approximation of curvature

Moiré Method measuring out-of-plane slopes

1. First exposure is made with specimen at its initial state which taken at unloaded position. Grating Q is reflected from point P

2. Plate is then deformed, now point P on plate has moved to point P’, which approximately the same location on the film as point P.Because of the curvature, grating element Q’ is now superimposed on this image point in the second exposure.

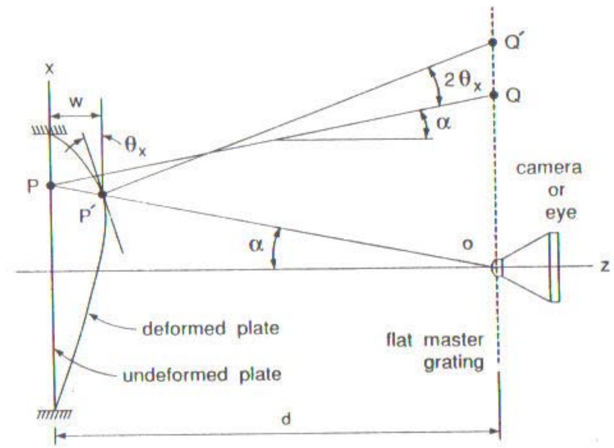
The moiré pattern formed by superposition of the two images provide a measure of the shift. The shift in terms of local slope of the plate is given as,

$$\mathbf{s = 2\theta d}$$

Where s = magnitude of shift

θ = local slope of plate at P’

d= distance between plate and grating

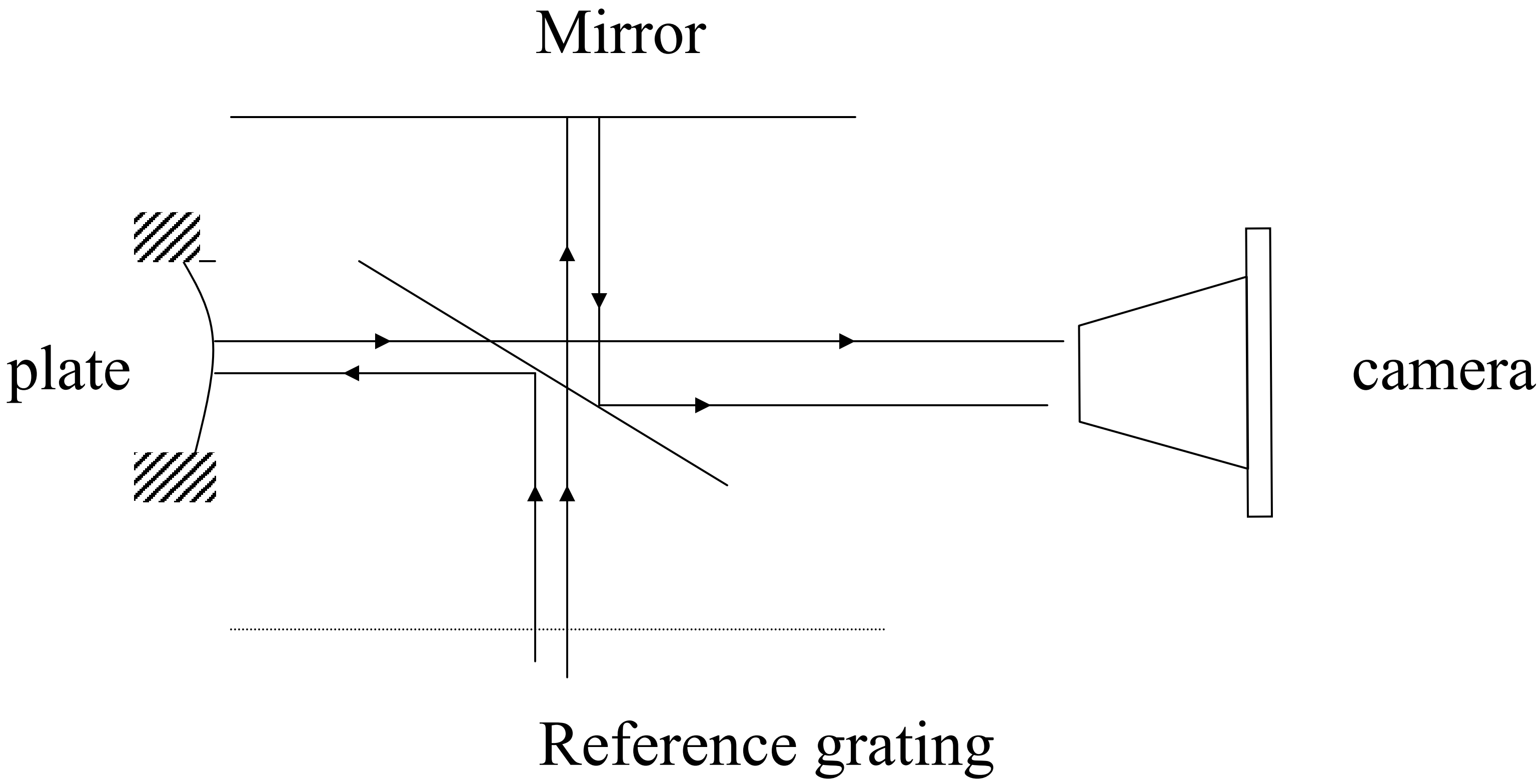


The order of moiré fringe can be expressed as

$$\mathbf{N = 2\theta d/\rho \quad or \quad \theta = Np/2d}$$

d should be large to minimize the effects of out-of-plane displacement w on the shift distance s

Real –time reflection moiré analysis of slope



Double exposure moiré tends to be marginal.
Avoid viewing the plate through gratings.
Allows for angular adjustments (without having to use curve gratings)

Examples

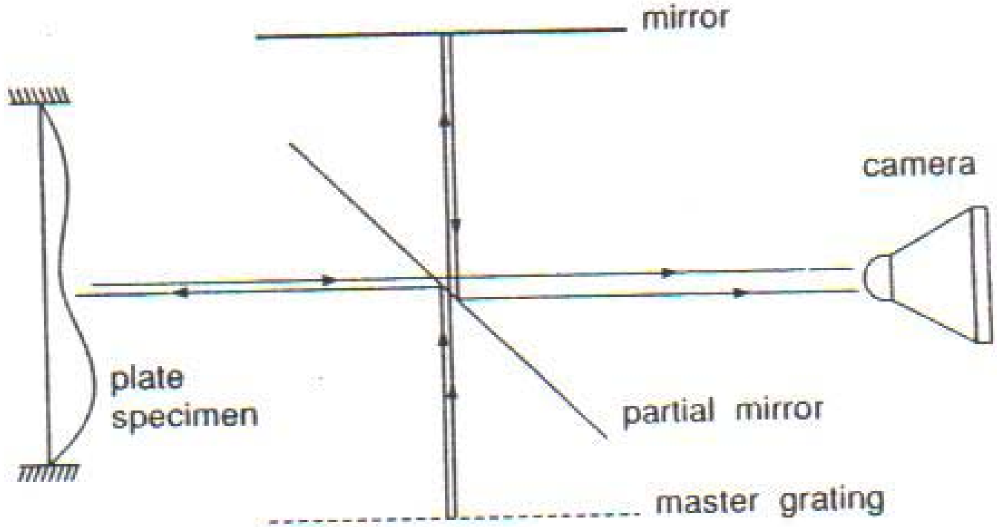


Figure 9.3. Optical arrangement for real-time reflection moire analysis of slope.

Projection Moiré

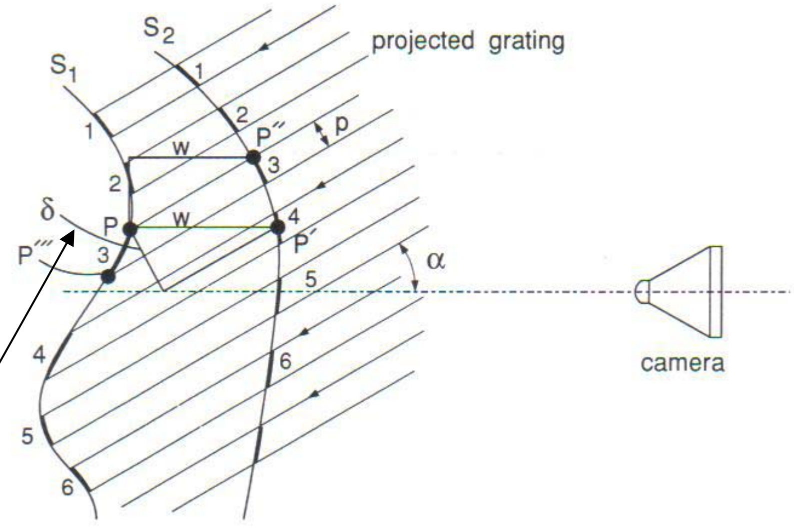


Figure 9.4. Optical arrangement for projection moire analysis of contour or changes of z-displacement.

Similar to Projecting interference Pattern in Moiré interferometer

P undergoes axial movement w, it moves across the projected grating by the amount $\delta = w\sin\alpha$, where α is incident angle of the projected grating,
The fringe order at point P in final moiré will be $N = \delta/p$, where p is grating pitch
The axial displacement w is therefore,

$$w = \frac{Np}{\sin\alpha}$$

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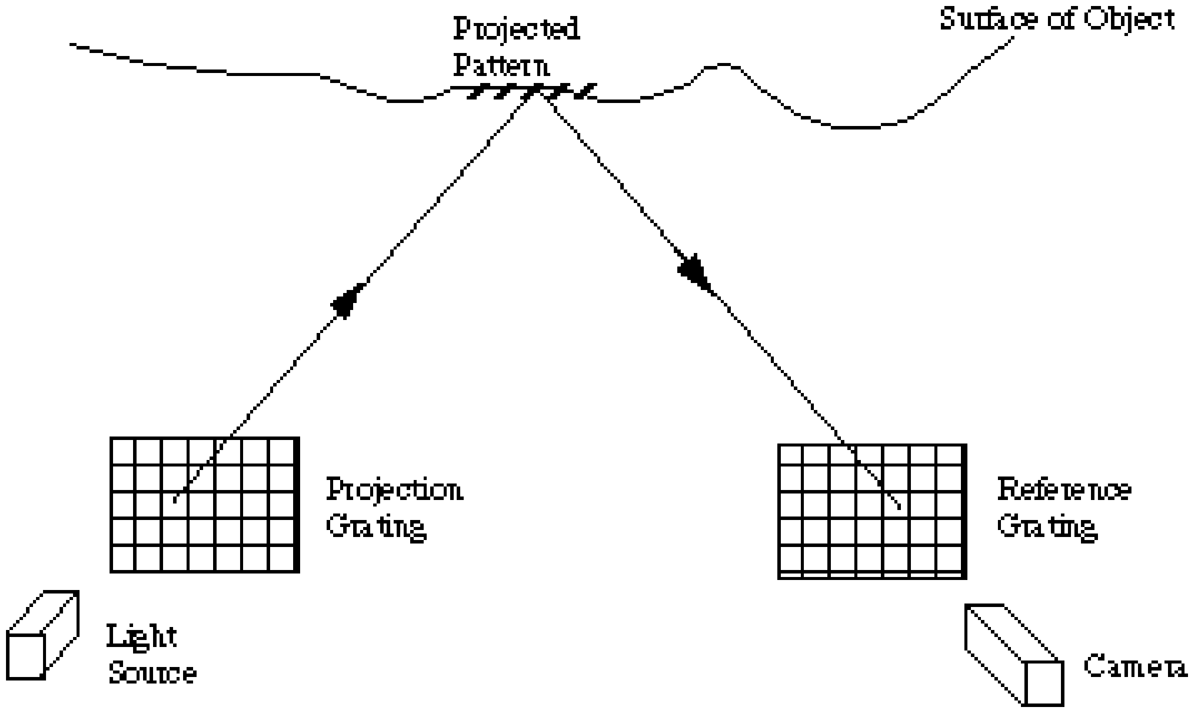
Projection Technique



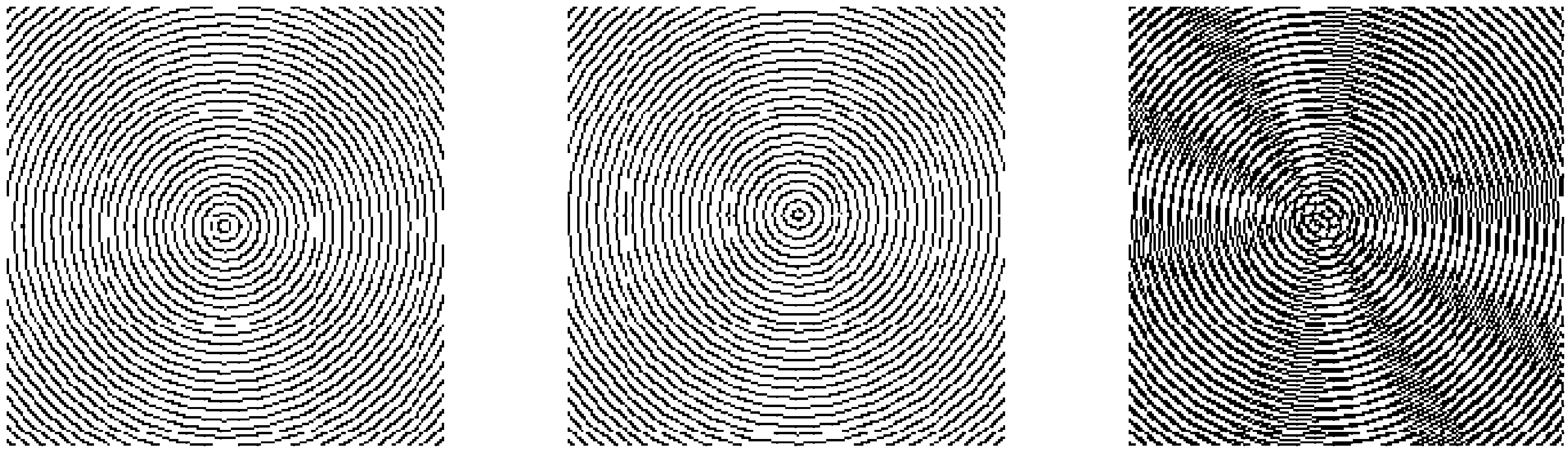
The essence of the method is that a grating is projected onto an object and an image is formed in the plane of some reference grating.

Project Moiré

$$w_d = w_o + \frac{Np}{\sin\alpha}[(1-\frac{2w}{s}\cos\alpha - \frac{Np}{s\tan\alpha}) - \frac{x}{s\sin\alpha}(\frac{s}{d}\cos\alpha - \cos 2a)]$$

Where N = fringe number
P= grating pitch
S= distance from projector to subject reference plane
D=distance from camera to subject reference plane
X=x coorinate of point P observed by camera lnes
A = angle between camera and projector optical axis
Xa = x coordinate of point P'
X_{a0} is x coordinate of point P
X = Xa
N=number of line in cone OSP

Projection Technique

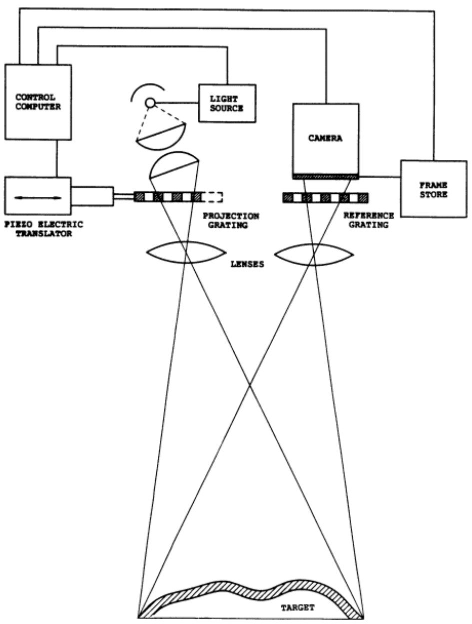


(a) Projection Grating (b) Reference Grating (c) Moire Pattern

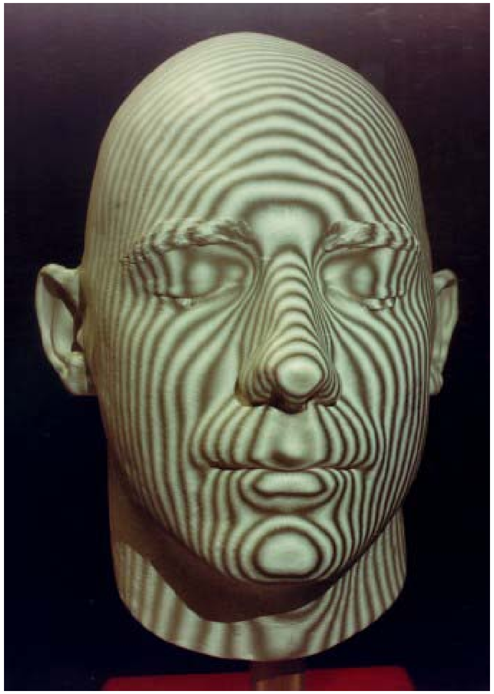
The image then interferes with the reference grating to form Moiré fringe contour patterns which appear as dark and light stripes. Analysis of the patterns then gives accurate descriptions of changes in depth and hence shape.

Moiré Fringe Contouring

The use of moiré fringes to acquire 3D surface shape information is well established. Their application to the measurement of areas of the human body began with the work of Hiroshi Takasaki as early as 1973



Schematic Diagram of Auto-MATE System



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Moiré Fringes Superimposed on Model Head

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Example

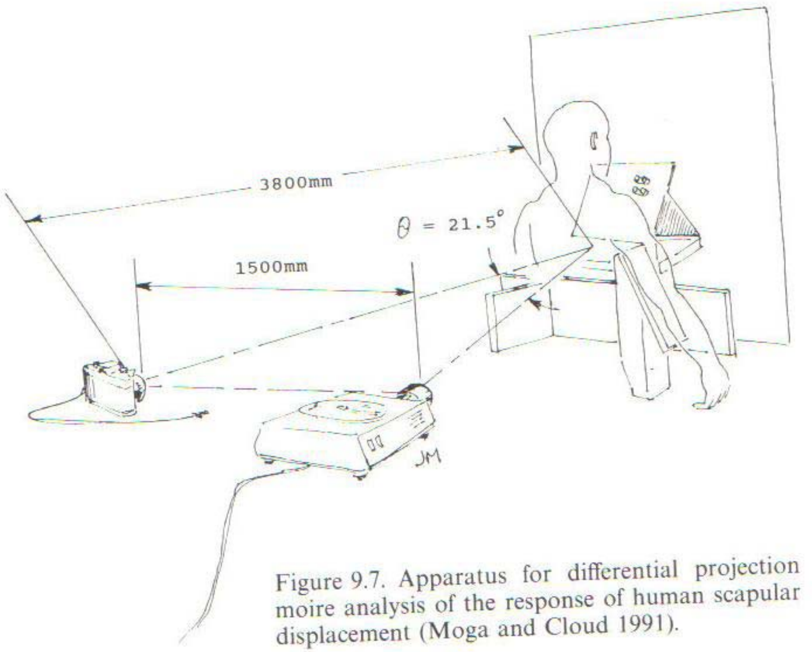

Figure 9.7. Apparatus for differential projection moire analysis of the response of human scapular displacement (Moga and Cloud 1991).

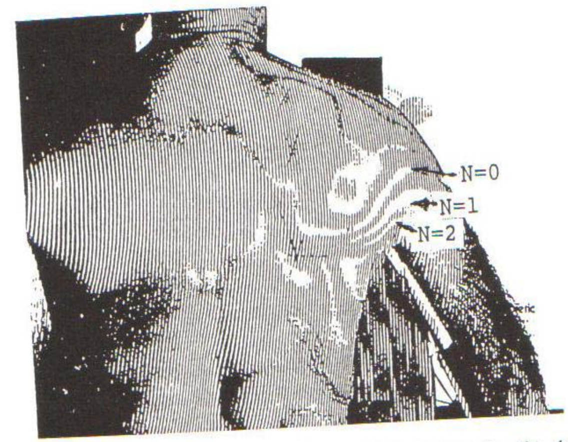

Figure 9.8. Differential projection moire fringe pattern representing the change in topography of the human scapular mechanism with muscular effort; result for 20% adduction of upper arm (Moga and Cloud 1991).