Moiré Interferometry

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Interference

When two or more optical waves are present simultaneously in the same region of space, the total wave function is the sum of the individual wave functions

Optical Interferometer

Criteria for interferometer:

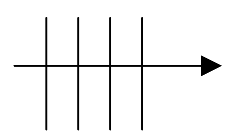
(1) Mono Chromatic light source- narrow bandwidth

(2) Coherent length: Most sources of light keep the same phase for only a few oscillations. After a few oscillations, the phase will skip randomly. The distance between these skips is called the *coherence length.*

normally, coherent length for a regular 5mW HeNe laser is around few cm,

$$l_c = \lambda^2/\Delta\lambda \,,$$

(3) Collimate- Collimated light is unidirectional and originates from a single source optically located at an infinite distance so call plane wave



(4) Polarization dependent

Interference of two waves

When two monochromatic waves of complex amplitudes U1(r) and U2(r) are superposed, the result is a monochromatic wave of the Same frequency and complex amplitude,

$$U(r) = U_1(r) + U_2 (r) \qquad (1)$$

Let Intensity $I_1= |U_1|^2$ and $I_2=| |U_2|^2$ then the intensity of total waves is

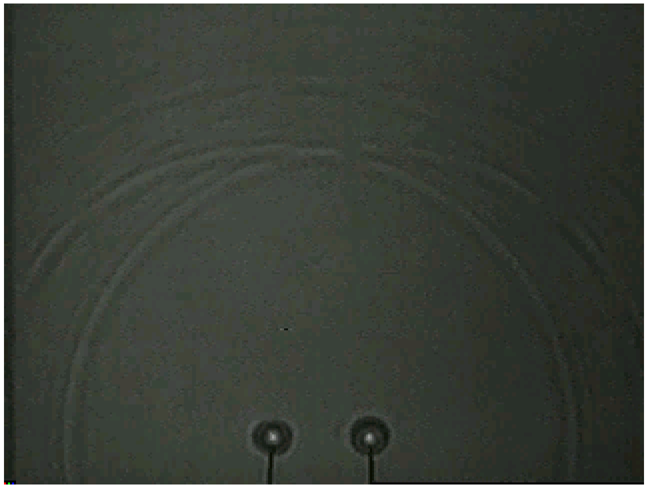
$$I=|U|^2 = |U_1+U_2|^2 = |U_1|^2 +|U_2|^2 + U_1^* U_2 + U_1 U_2^* \qquad (2)$$

Basic Interference Equation

Let $U_1 = I_1^{\ 0.5}\, e^{\,j\phi_1}$ and $U_2 = I_2^{\ 0.5}\, e^{\,j\phi_2}$ Then

$$\boxed{I = I_1 + I_2 + 2(I_1 I_2)^{0.5} \mathrm{COS}\phi} \qquad (3)$$

Where $\phi = \phi_2 - \phi_1$ (phase difference between two wave)



Ripple tank
Interference pattern
created by phase
Difference ϕ

Interferometers

•Mach-Zehnder

•Michelson

•Sagnac Interferometer

•Fabry-Perot Interferometer

Interferometers is an optical instrument that splits a wave into two waves using a beam splitter and delays them by unequal distances, redirect them using mirrors, recombine them using another beam splitter and detect the intensity of their superposition

Intensity sensitive to phase change

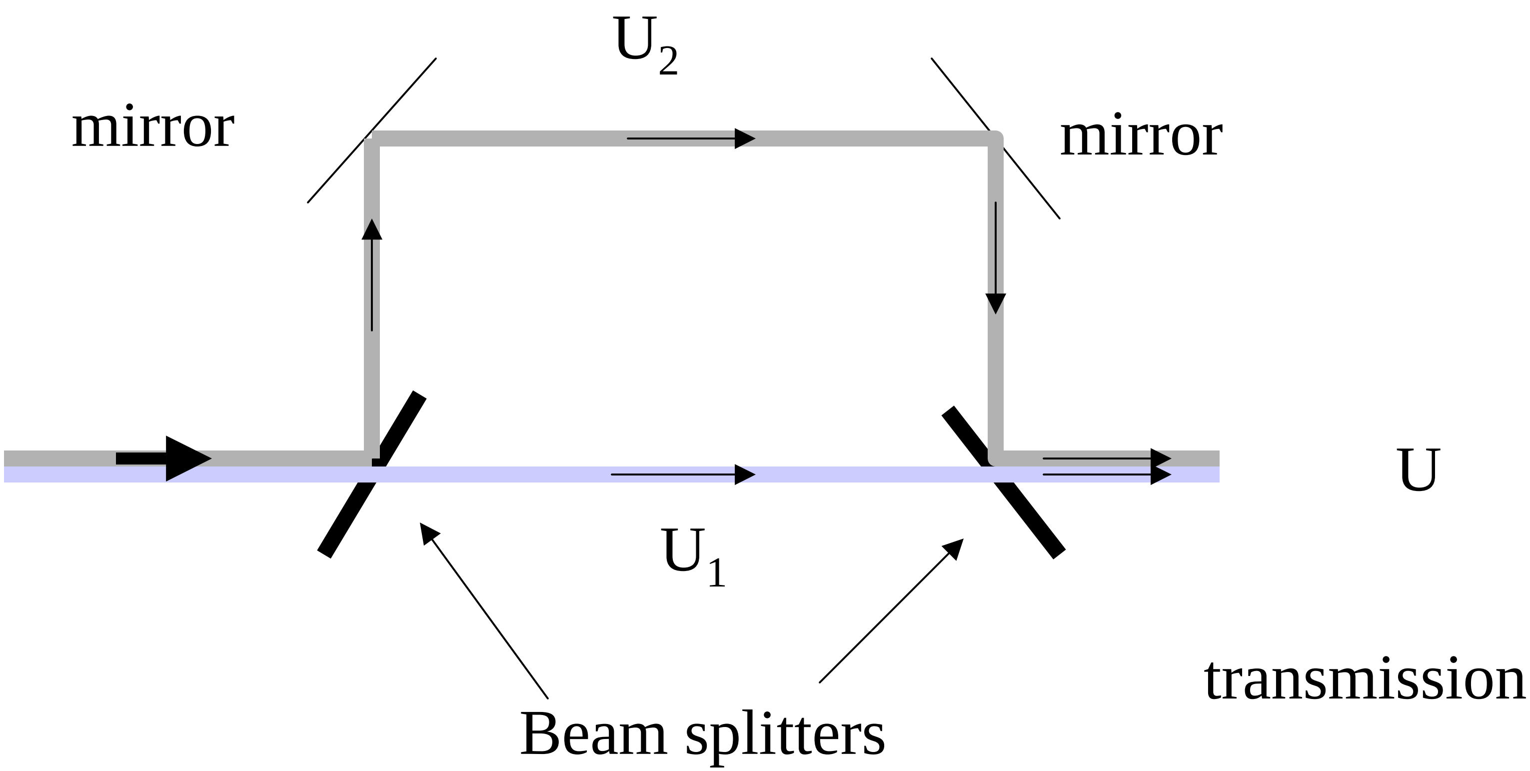
$$\phi = 2\pi nd/\lambda$$

Where n = index of refraction of medium wave travels
λ = operating wavelength
d = optical path length

Intensity change with n, d and λ

The phase change is converted into an intensity change using interferometric schemes (Mach-Zehnder, Michelson, Fabry-Perot or Sagnac forms).

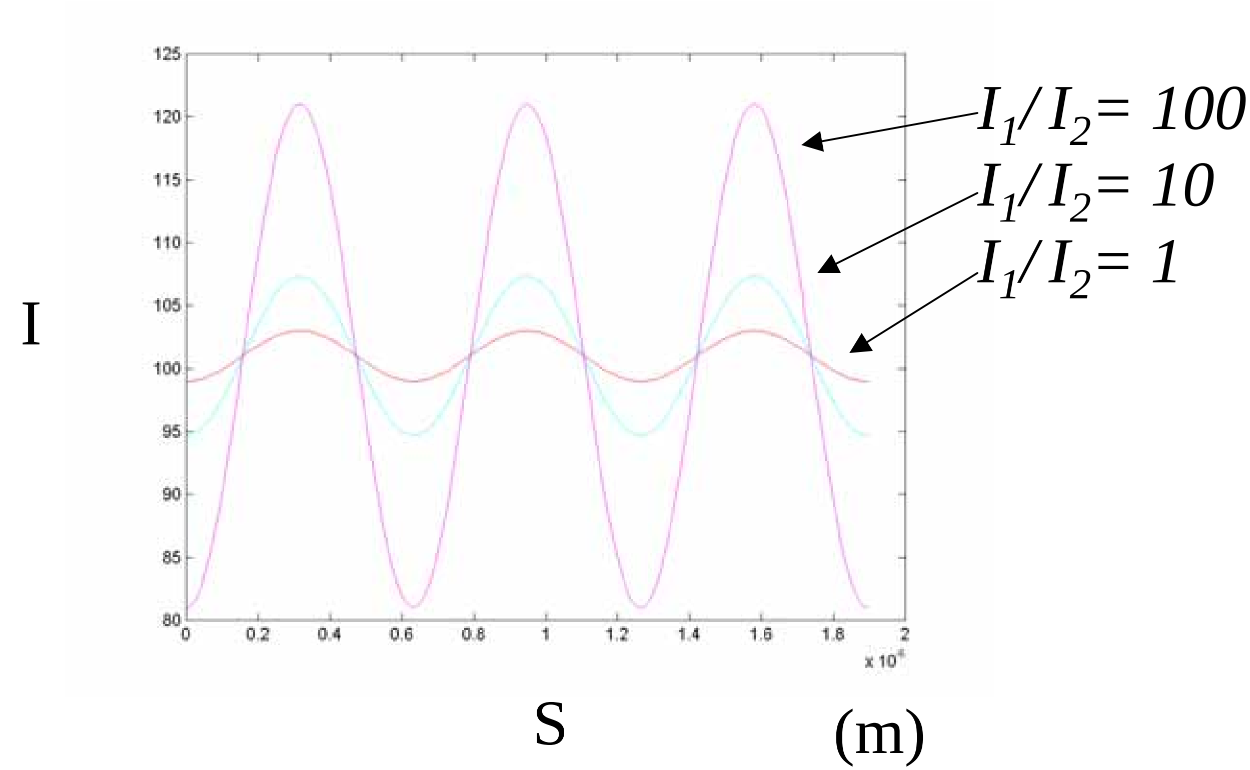
Free Space Mach-Zehnder Interferometer



Impure Two-beam interference

Suppose input beams 1 and 2 have unequal intensities I_1 and I_2 and Phase difference due to a small path difference S

$$I = I_{1\_} + I_2 + 2(I_1 I_2)^{0.5} COS(2\pi S/\lambda)$$



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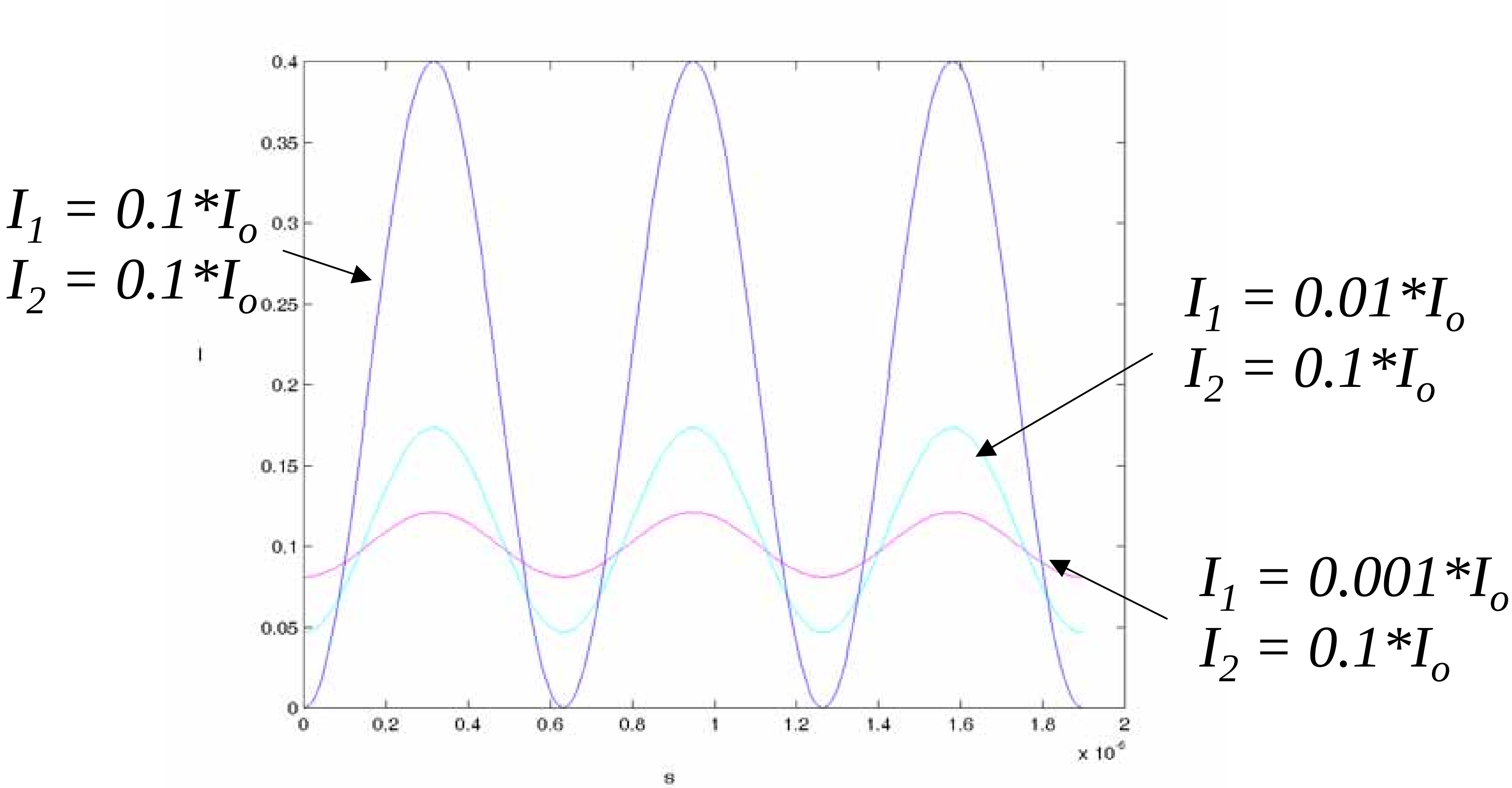
Impure Two-beam interference with fix input intensity



Contrast: $\% = \dfrac{I_{\max} - I_{\min}}{I_{\max}}$

Where $I_{\max} = I_1 + I_2 + 2(I_1 I_2)^{0.5}$

$I_{\min} = I_1 + I_2$

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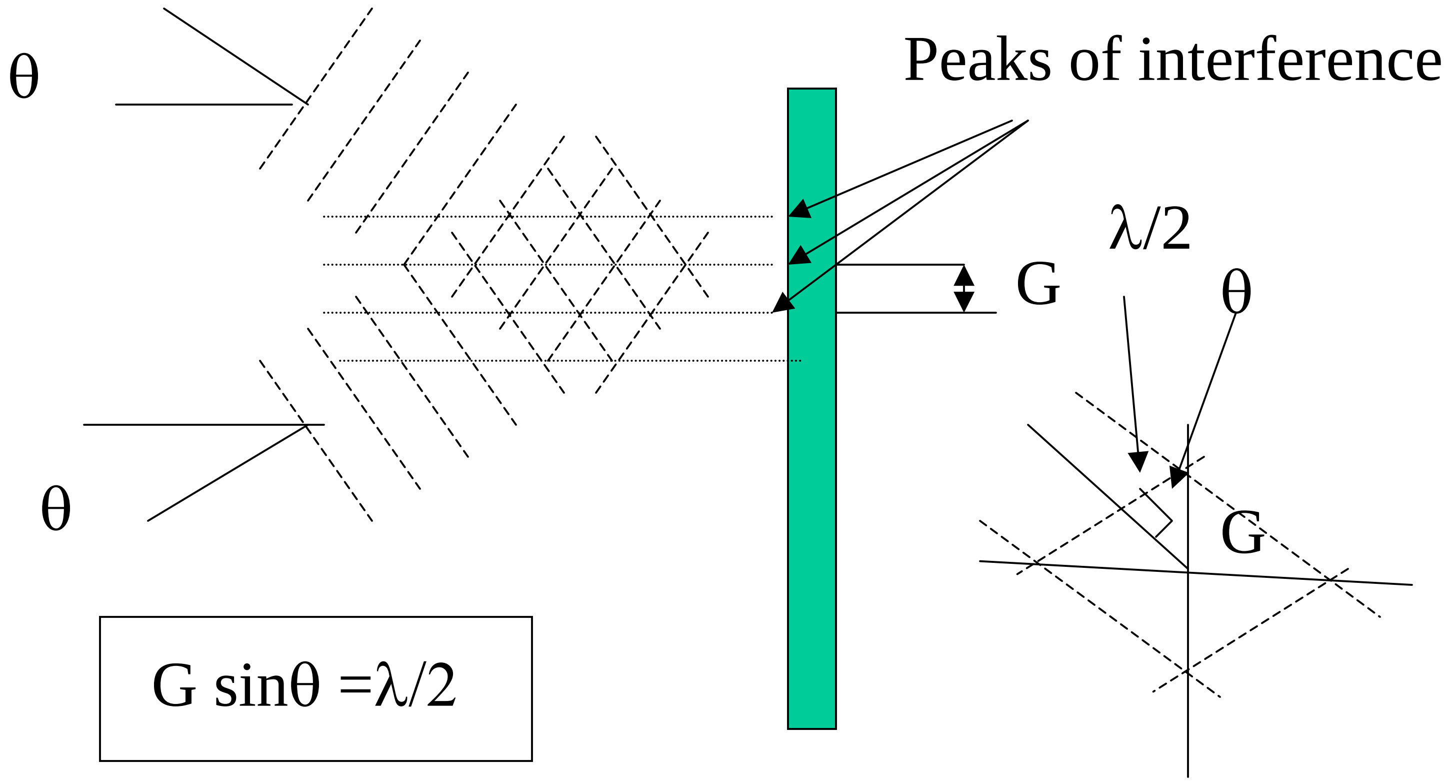
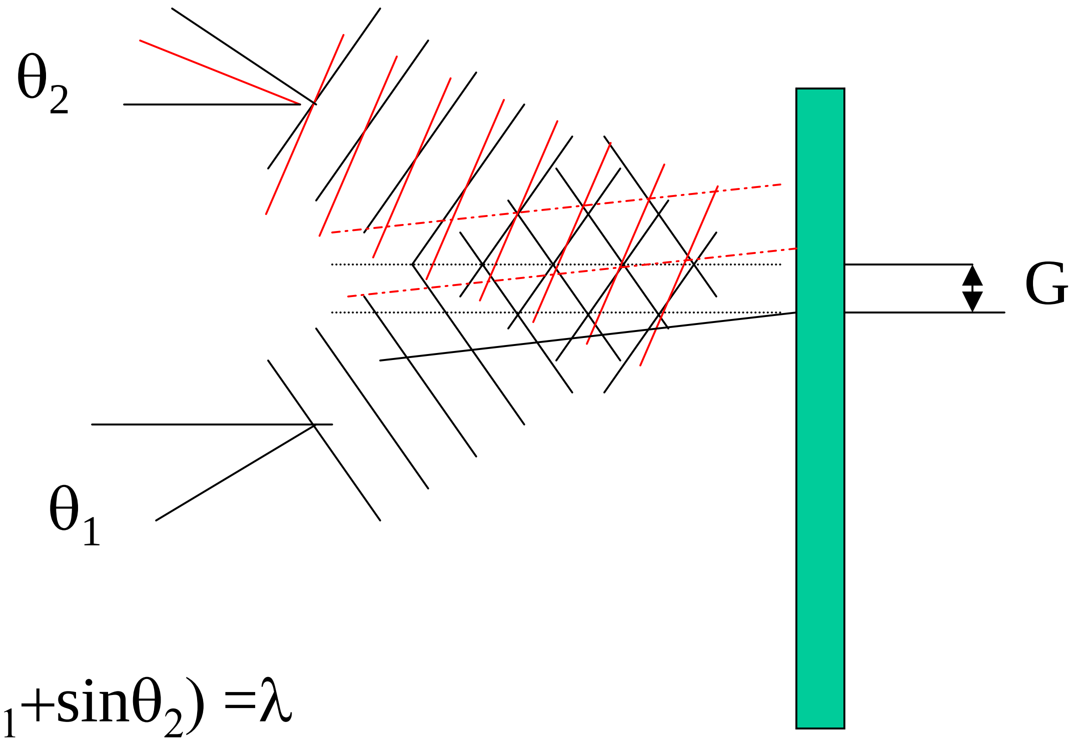
Interference by two beams at an angle θ



$$G \sin\theta = \lambda/2$$

When two coherent beams of collimated light intersects at an Angle of 2θ, a volume of interference fringe planes is created.

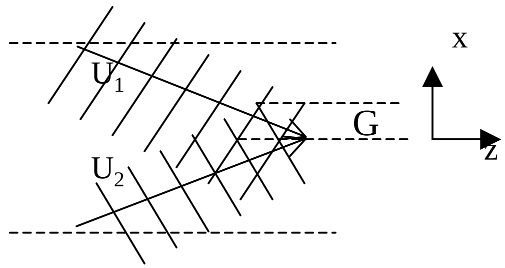
Interference by two beams at angles θ_1 and θ_2



$G(\sin\theta_1 + \sin\theta_2) = \lambda$

When two coherent beams of collimated light intersects at an Angle of $\theta_1+\theta_2$, a volume of interference fringe planes is created.

Interference by two beams at angles θ_1 and θ_2



Complex amplitude:

$$U(r) = U_1(r) + U_2(r)$$

$$I = |U_1|^2 + |U_2|^2 + U_1 U_2^* + U_2 U_1^*$$

Assume two waves have the same intensities

$$U_1 = I_0^{\frac{1}{2}} \exp(-j(k\cos\theta_1 z - k\sin\theta_1 x))$$

$$U_2 = I_0^{\frac{1}{2}} \exp(-j(k\cos\theta_2 z + k\sin\theta_2 x))\exp(j\phi_0)$$

Here ϕ_0 is the phase difference between two waves
at z=0;

$$U_1 = I_0^{\frac{1}{2}} \exp(jk\sin\theta_1 x)$$

$$U_2 = I_0^{\frac{1}{2}} \exp(-jk\sin\theta_2 x)\exp(j\phi_0)$$

The pattern varies sinusoidal with x.

$$k(\sin\theta_1 + \sin\theta_2)x = 2\pi$$

$$G = x = \frac{2\pi}{k(\sin\theta_1 + \sin\theta_2)}$$

Grating period G:

$$G = \frac{\lambda}{\sin\theta_1 + \sin\theta_2}$$

$$I = I_0 + I_0 + I_0 \exp(jk\sin\theta_1 x)\exp(jk\sin\theta_2 x)\exp(-j\phi_0) + I_0 \exp(-jk\sin\theta_1 x)\exp(-jk\sin\theta_2 x)\exp(j\phi_0)$$

$$I = 2I_0 + I_0[\exp(j(k(\sin\theta_1 + \sin\theta_2)x - \phi_0)) + \exp(-j(k(\sin\theta_1 + \sin\theta_2)x - \phi_0))]$$

$$I = 2I_0 + I_0 2\cos(k(\sin\theta_1 + \sin\theta_2)x - \phi_0)$$

$$I = 2I_0[1 + \cos(k(\sin\theta_1 + \sin\theta_2)x - \phi_0)]$$

Frequency limit

Theoretical limit of the frequency of the interference pattern produced with the two intersecting beams is given by

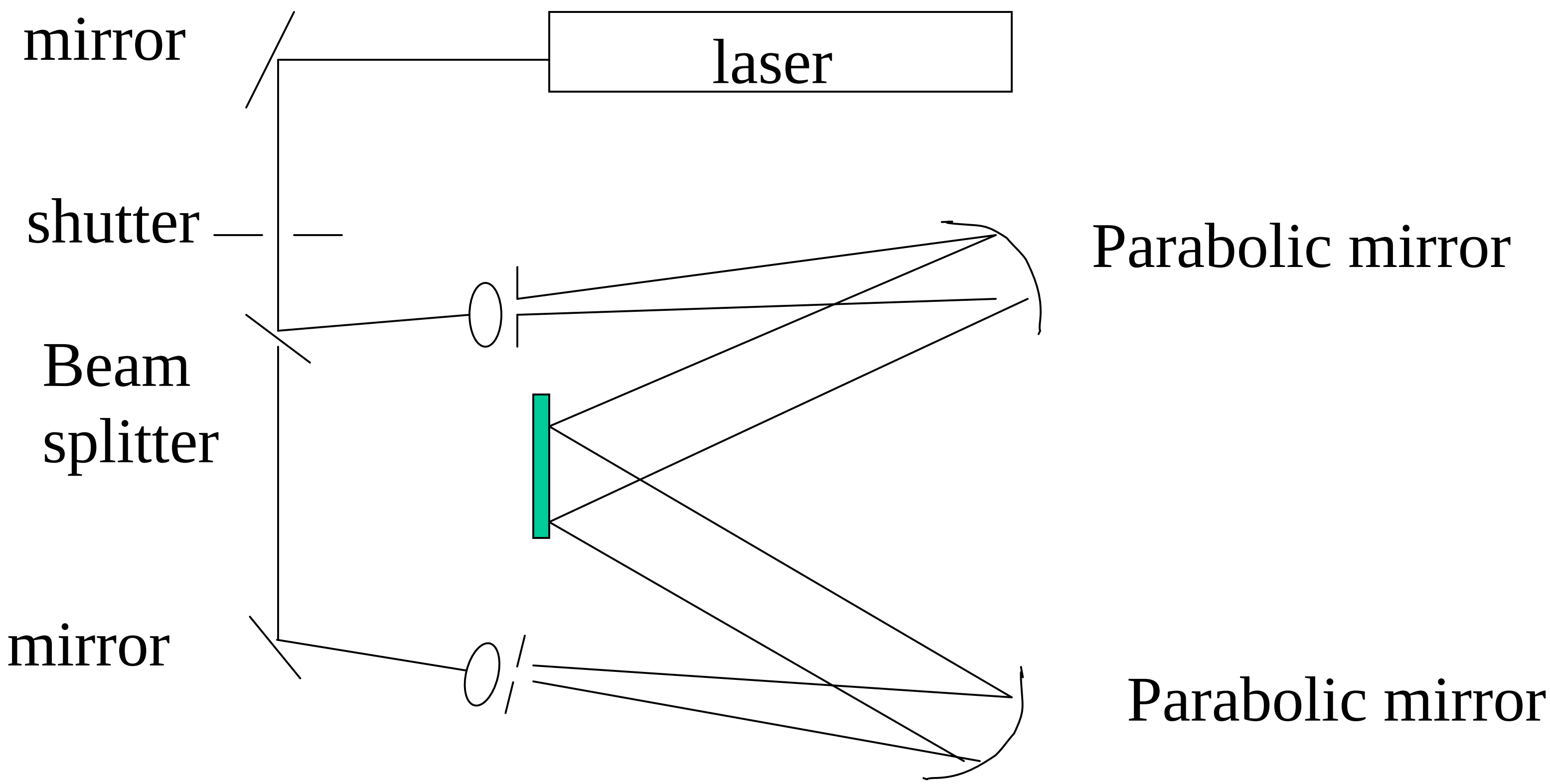
$$\boxed{f_{max} = 1/G_{min} = 2/\lambda}$$

Thus, the frequency depends on wavelength of the light source

HeNe (632nm) f_{max} = 3160 lines/mm
Argon (488nm) f_{max} = 4098 lines/mm
HeCd (325nm) f_{max} = 6154 lines/mm

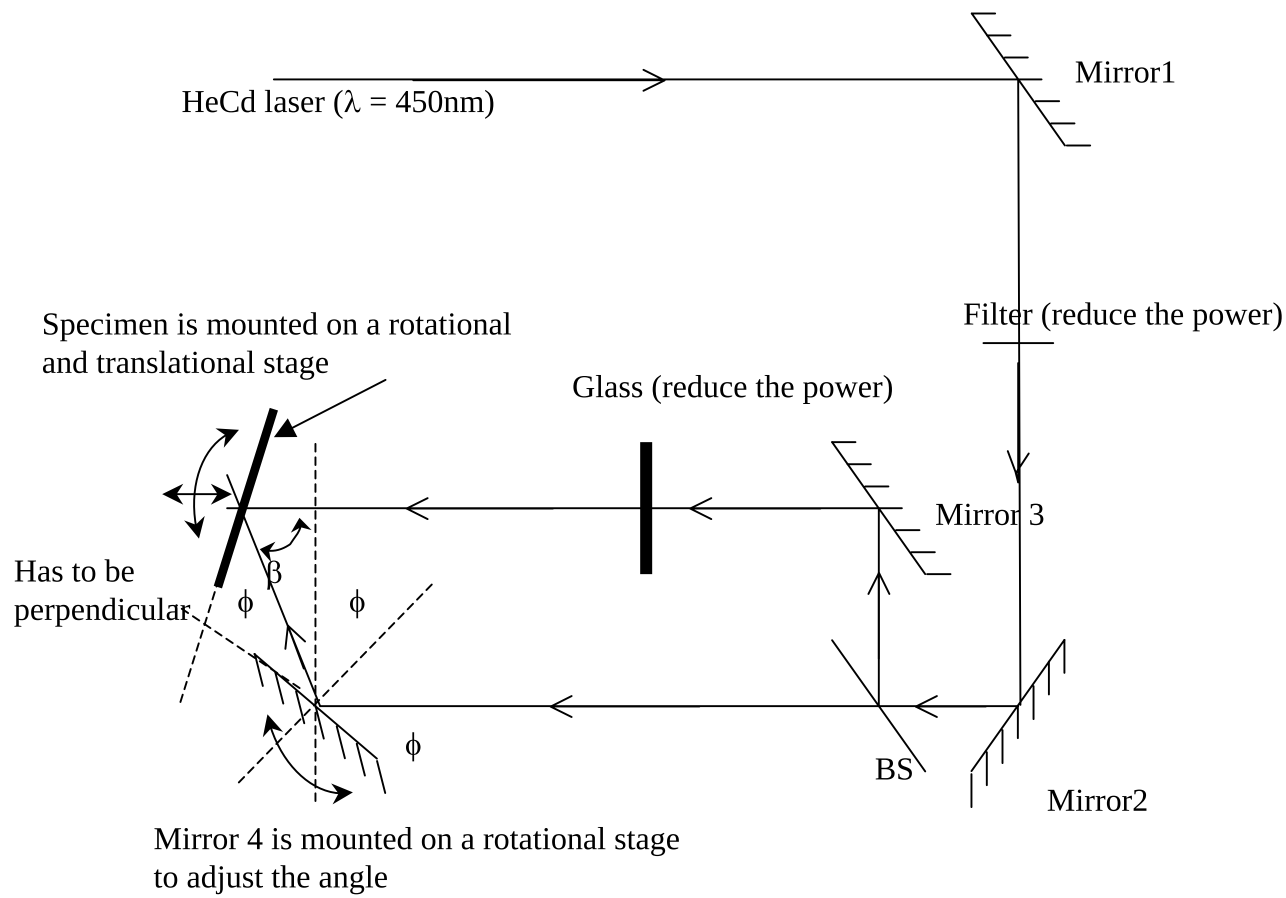
However, it is impossible to form these patterns because $\theta = \pi/2$!

Optical Arrangement for grating construction



Two intersected beams are used to produce high frequency
Grating which are employed as the specimen gratings
In Moiré interferometry.

Optical Setup for Two Beams Interference



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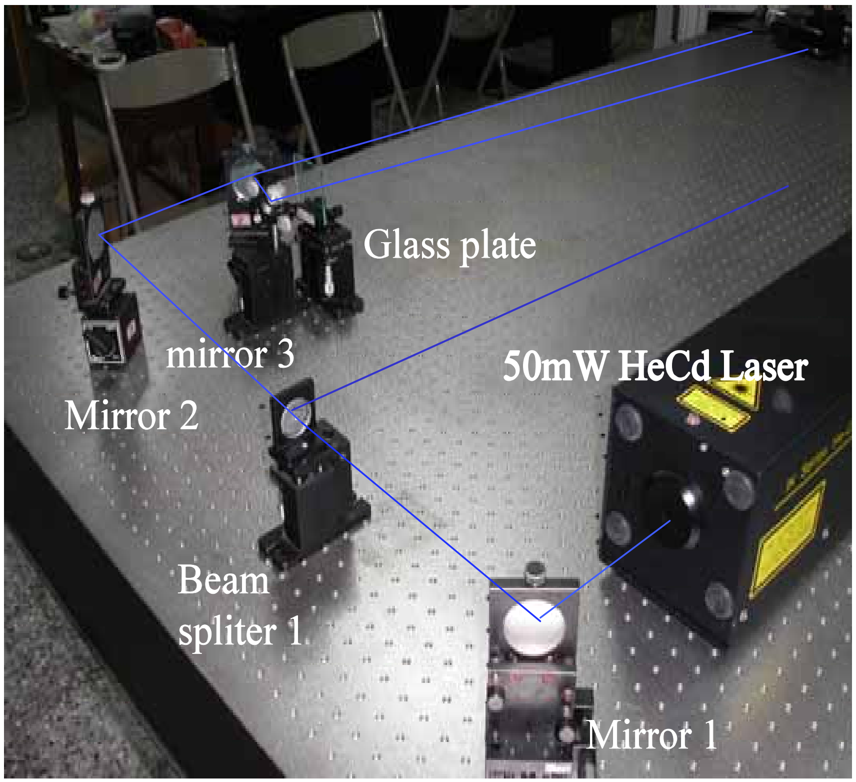
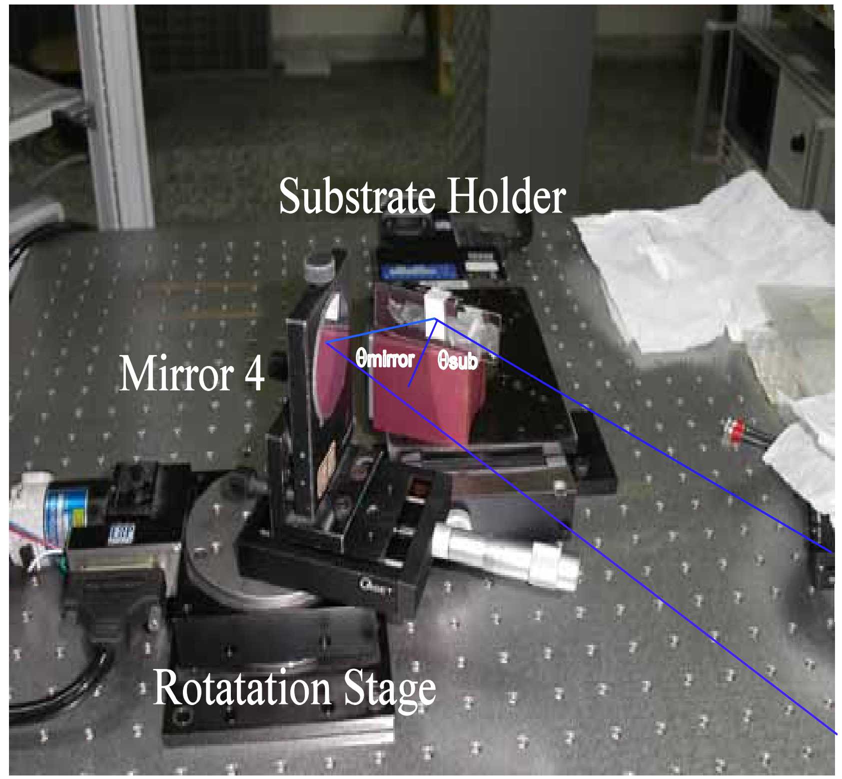
Interference Lithography Setup



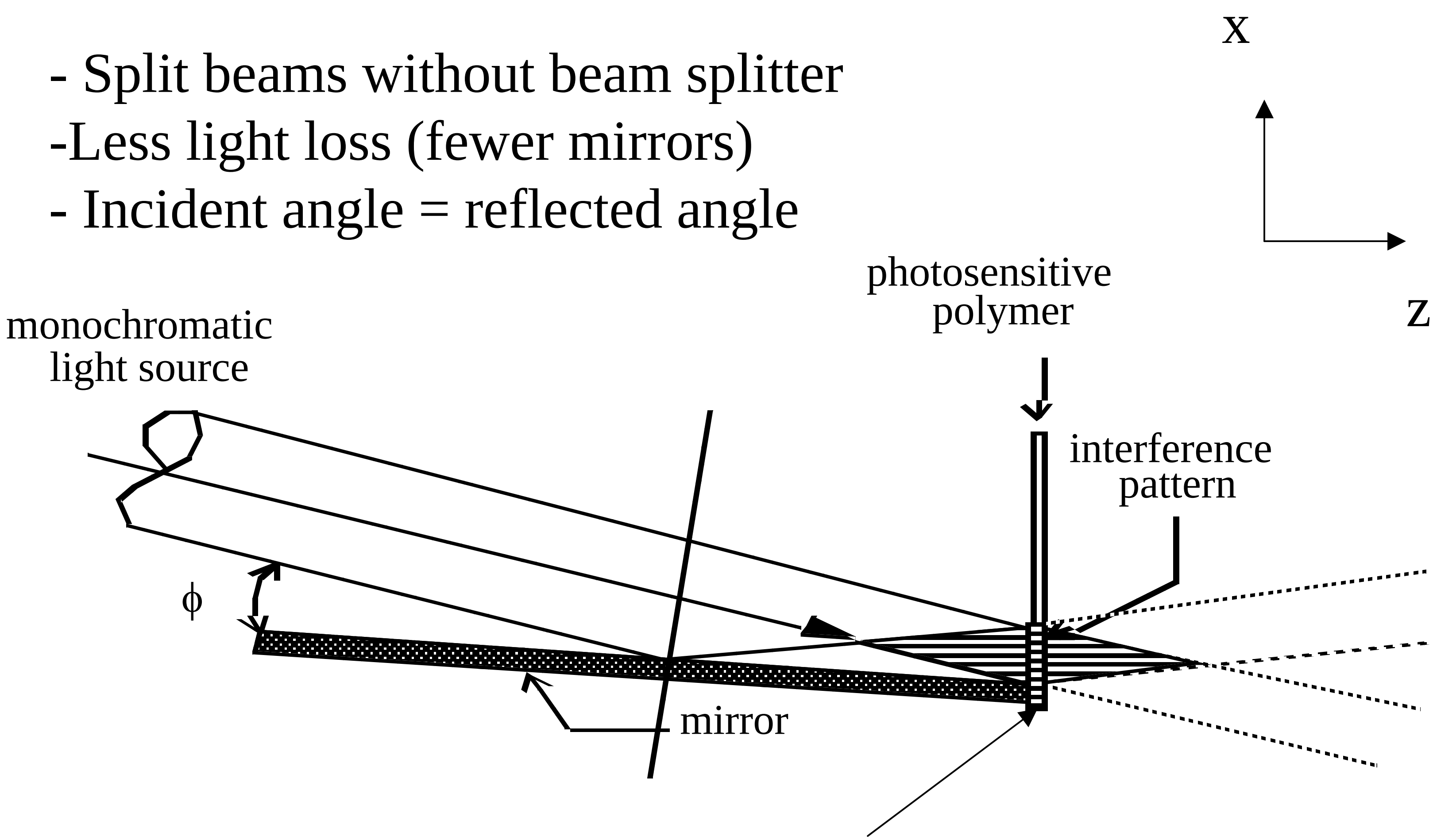
Light branching region



Exposing region

Lloyd's Mirror

- Split beams without beam splitter
- Less light loss (fewer mirrors)
- Incident angle = reflected angle



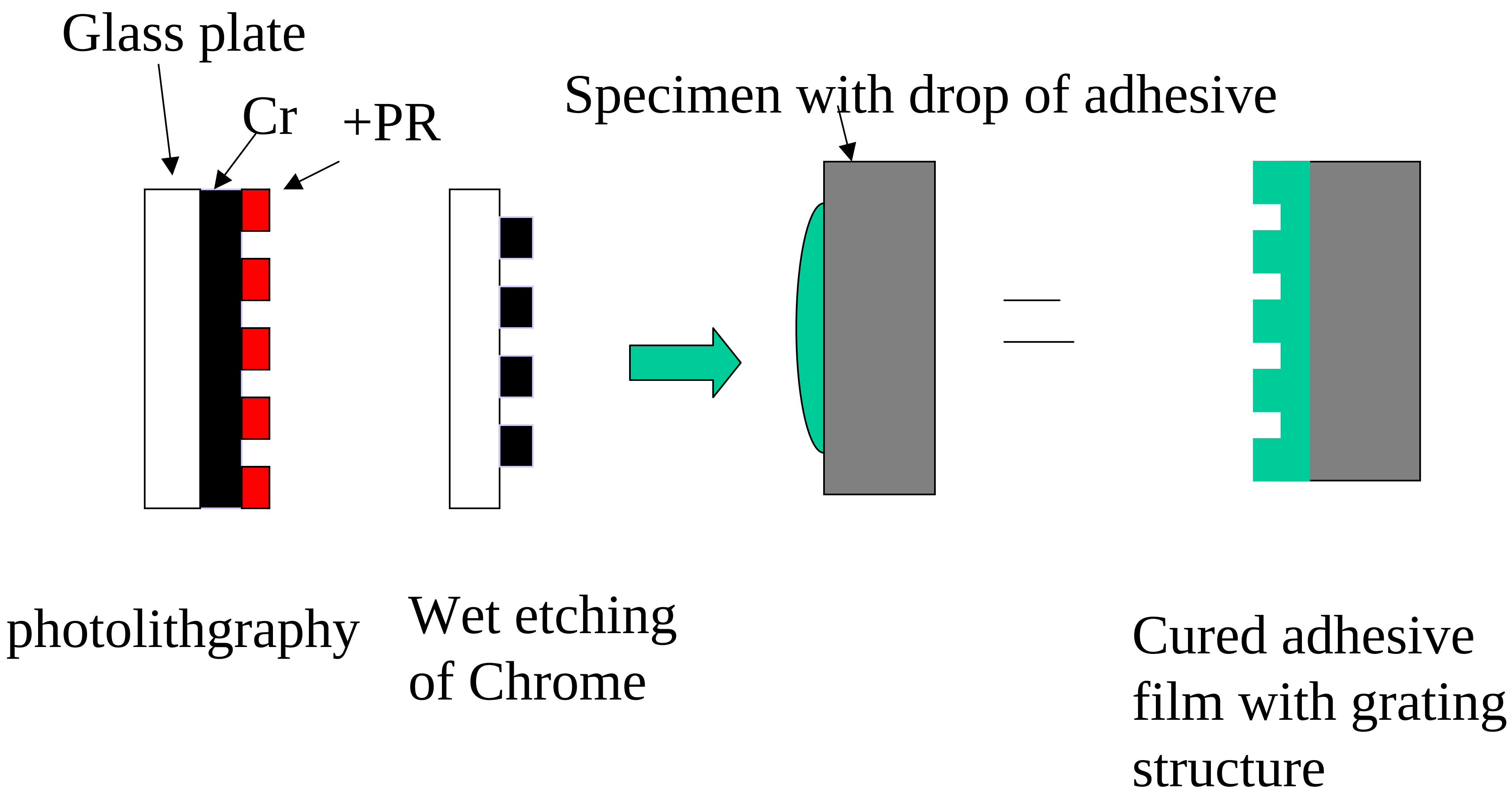
G = $\lambda/(2\sin\phi)$ (grating spacing)

Specimen Gratings

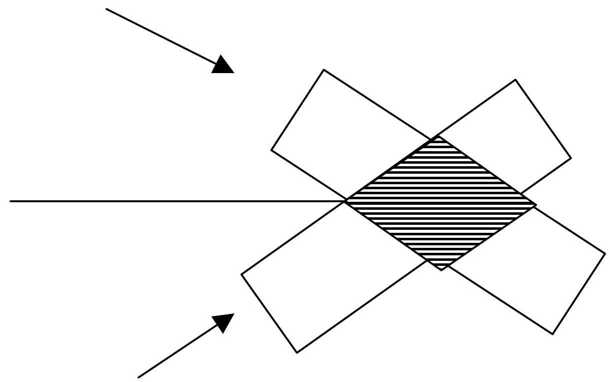
Frequency of gratings for moiré interference usually around 1000 to 2000 lines/mm (photoresist, molding, e-beam, etc.)

Frequency of gratings for geometric Moire usually rarely exceed 80 lines/mm (cutting with fine knife)

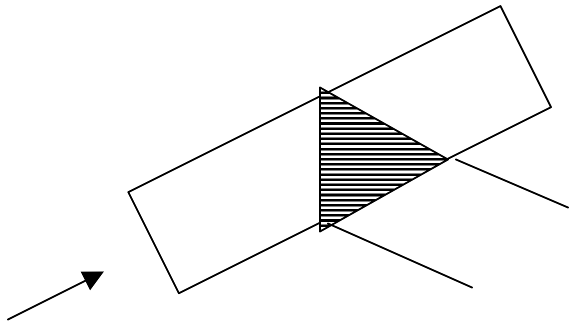
Transferring grating using molding process



Virtual Grating



Two coherent beams



Diffraction by real grating

Virtual grating can be formed either by diffraction grating lines
Of a real grating or interference pattern from two coherent beams.

Warped Wavefronts

When stress applied to initially flat surface, the resulting nonhomogenous deformation creates a warped wavefronts.
A consequence of wavefront changes, each wall of interference has a continuous but warped shape, with its warpage systematically different from that of the neighboring wall. A photographic plate that cuts the walls at one location might record the fringe pattern but different location.That fringe pattern would be somewhat different.

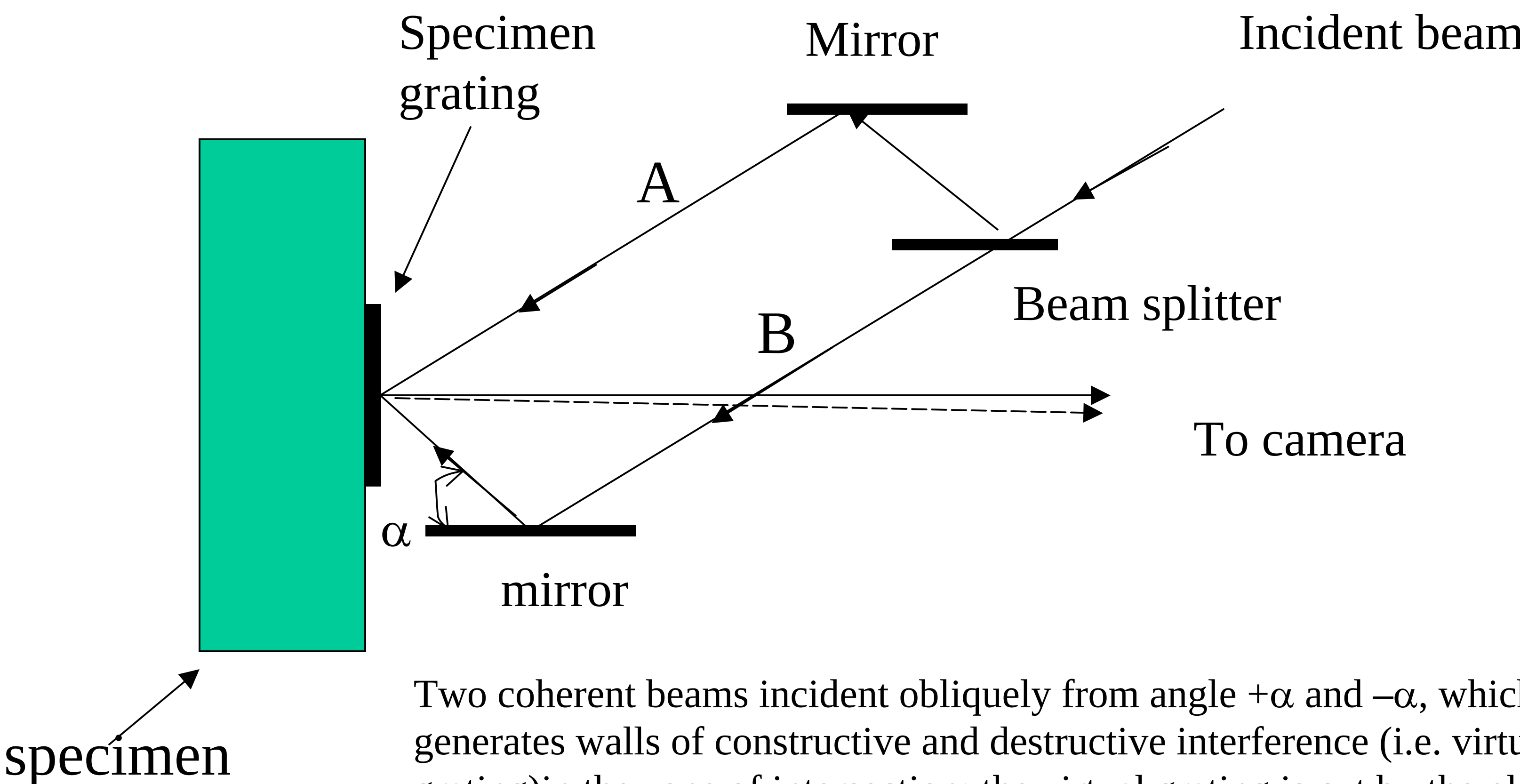
Moiré Interference

Moiré interference combines concept of general geometric Moiré and optical interferometry.

Moiré interferometer is identical to mechanical moiré in the sense that a moiré image is formed by light passing through two gratings- one on the specimen and the other a reference grating.

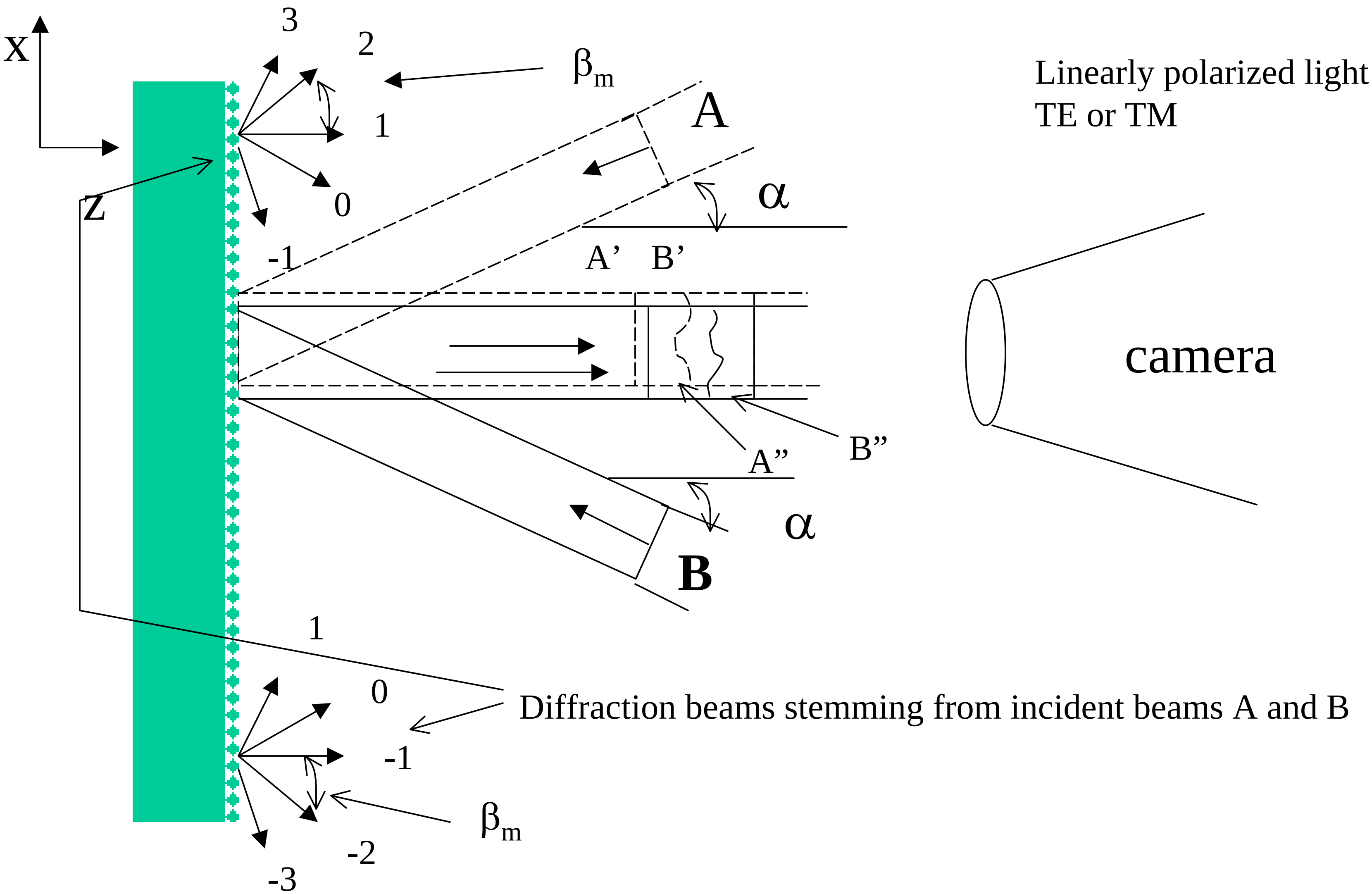
For mechanical Moiré, the reference grating is real, with moiré interfereometry, the grating is imaginary, consists of virtual image of an interference pattern produced with mirrors.

Optical arrangement for Moiré Interferometry



Two coherent beams incident obliquely from angle +α and –α, which generates walls of constructive and destructive interference (i.e. virtual grating)in the zone of intersection; the virtual grating is cut by the plane of the specimen surface, where an array of parallel and very closely space fringes are formed. These fringes act like reference grating in geometric Moiré. The specimen grating and virtual reference grating interact to form a moiré pattern which is viewed by the camera.

Specimen grating diffracts incident light into rays A', B' for the initial no-load condition of the specimen and A" and B" from nonhomogeneous deformation of the specimen, where these two wavefronts interfere and generate an interference pattern on the film plan of camera



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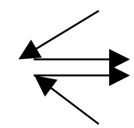
Beams A and B intersects in the region of space immediately in front of the specimen grating. Beams A and B form a <u>virtual grating</u> whose pitch (G) is given by

$$G = \lambda/2\,\sin\alpha$$

In the case of moiré interference, the angle of intersection (2α) is not arbitrary. Rather α is selected such that the we observe the m = $\pm$ 1 diffracted beams emerge from the specimen grating and the corresponding diffracting angle ($\beta_{\pm 1} = 0$)
General grating equation is

$$\sin\beta_m = m\lambda/g - \sin\alpha \;\; \Rightarrow \;\; \sin\alpha = \pm\lambda/g \quad (\beta_{\pm 1} = 0)$$



Therefore the virtual grating which formed by the moiré interferometry has fringe spacing (G) given by,

$$\boxed{G = \lambda/(2\lambda/g) = g/2}$$

Frequency of virtual grating *F* is twice the pitch of the specimen gratings *f*

$$\boxed{F = 1/G = 2f} \quad \textit{where } f = 1/g = \textit{frequency of specimen grating (lines/mm)}$$

When $F = 2f$ and lines of specimen and reference gratings are parallel, light from A, diffracted in the first order of the specimen grating , emerge as $\beta = 0$ with plane wavefront A' and light from B, diffracted in the -1 order also emerge as $\beta = 0$ with wavefront B'. If they are coherent and complete in phase, then we see a pattern of zero freuquency (0 Lines/mm) or so called *null filed*.

When specimen subject to forces that stretch it uniformly in the x direction such that the Uniform normal strain ε_{xx} is a constant, the frequency of the specimen grating is thereby Decrease to

$$f' = \frac{f}{1+\varepsilon_{xx}}$$

Corresponding frequency in virtual grating is $F' = \dfrac{F}{1+\varepsilon_{xx}}$

With new frequency, New diffraction angle at m = $\pm$1 and $\sin\alpha = F\lambda/2$,

$$\begin{aligned} \sin\beta'_{\pm 1} &= \pm\lambda f' - \sin\alpha \\ &= \pm\lambda F'/2 - \sin\alpha \\ &= \pm\lambda F'/2 - F\lambda/2 \\ &= \pm\lambda\,(F/(1+\varepsilon_{xx})) - F\lambda/2 \end{aligned}$$

If $\varepsilon_{xx} \sim small$, $\boxed{\sin\beta'_{\pm 1} = \mp\lambda F\varepsilon_{xx}/2}$

Now $\beta_{\pm 1}$ not zero anymore and because of the angle difference, we create interference

If β’ ~ small, sinβ’ ~ β’, then $\boxed{\beta'_{\pm} = \mp\lambda F\varepsilon_{xx}/2}$

This angle is similar to the interference pattern with two beams separated by angle of θ.

The fringe gradient $\partial N / \partial x$ is same as the frequency F_m of this interference (moiré) pattern that is given as

$$\frac{\partial N}{\partial x} = F_m = \frac{2\sin\beta'}{\lambda}$$

Since β’ ~ small, sinβ’ ~ β’, $\frac{\partial N}{\partial x} = \frac{2\beta'}{\lambda}$ Sub $\beta'_{\pm} = +\lambda F\varepsilon_{xx}/2$

We get $\frac{1}{F}\frac{\partial N}{\partial x} = \boxed{\varepsilon_{xx} = \frac{F_m}{F} = \frac{G}{G_m}}$ Where G = pitch of the virtual grating

G_m = distance between two adjacent fringes with orders m and m$\pm$1

Strain along x direction proportional to frequency change or pitch change

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During the development of moiré interferometry, it was empirically established that fringe order and displacement *u* in primary diffraction direction (m=$\pm$1) were related according to:

$$u = Ng$$

Above equation is similar to the Geometric Moiré's displacement equation:

$$\delta = p\,N$$

where N = *fringe number*
p = *pitch of periodic referemce gating*

Moiré interferometer is identical to mechanical moiré in the sense that a moiré image is formed by light passing through two gratings- one on the specimen and the other a reference grating. (Post, chap 7 handbook of experiential mechanics, p 314 1993)

Example

In practice, angle β is very small. For example, with λ = 488nm, g = 2000lines/mm, ε_{xx}=0.002, based on $\beta'_{\pm} = +\bar{\lambda} F \varepsilon_{xx}/2$,

It gives $|\beta'_{\pm}|$ = 0.0559^{o}

The correspnding fringe pattern frequency is 4 lines/mm

Optical arrangement for Moiré Interferometry

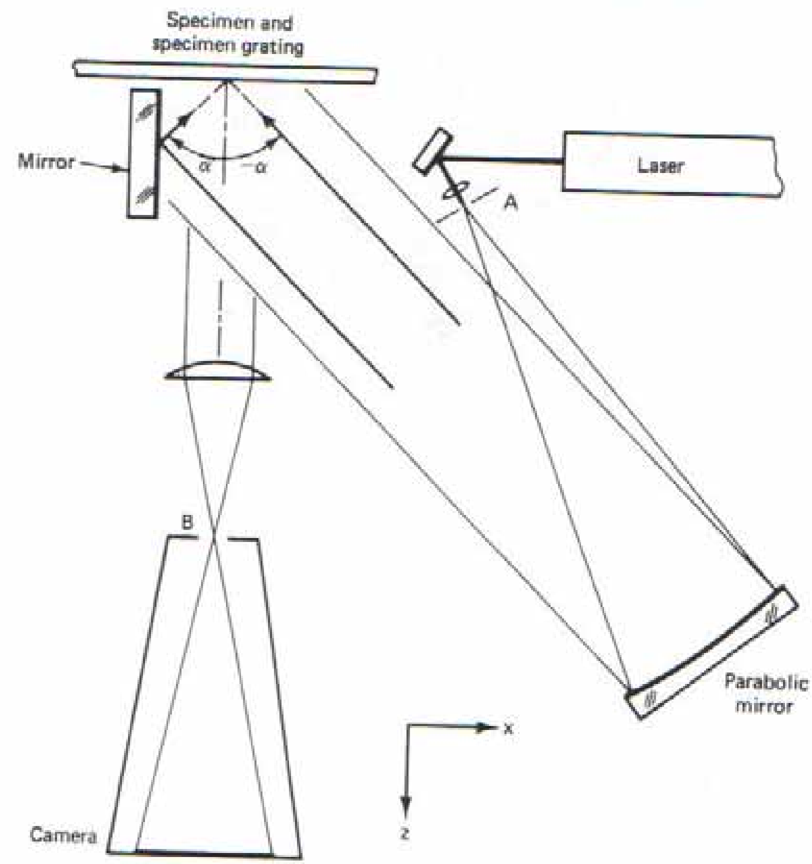


Figure 7-20 Optical arrangement for moiré interfermetry.

Two dimensional displacement field

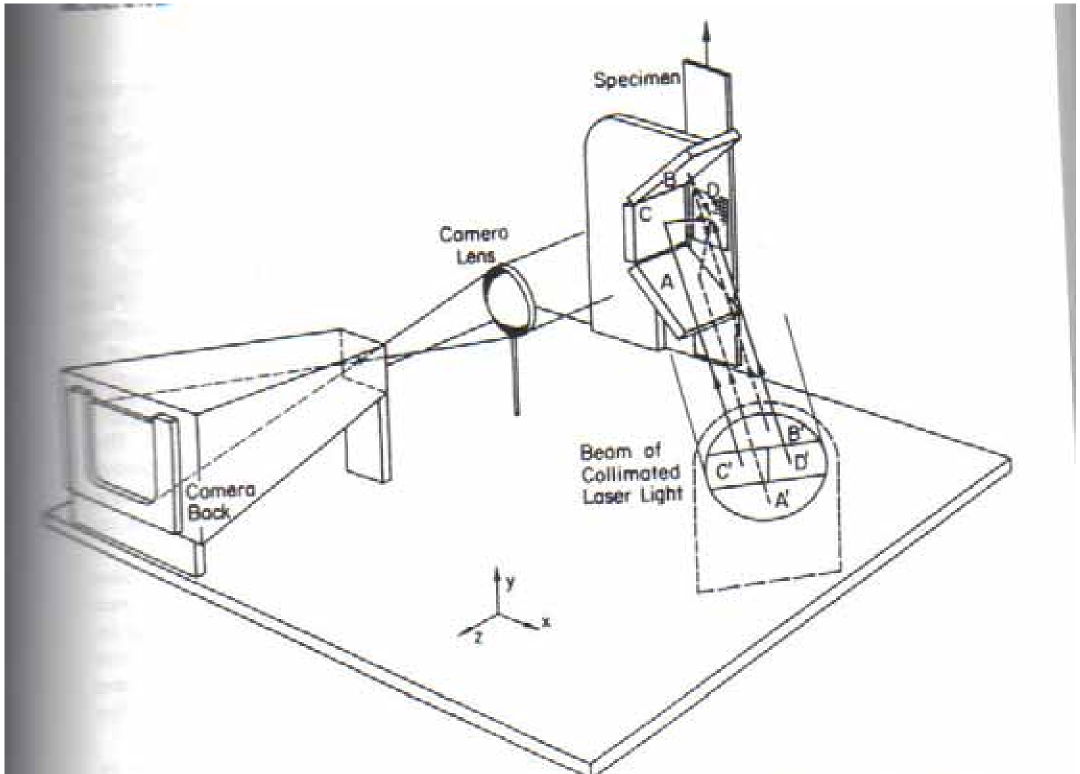
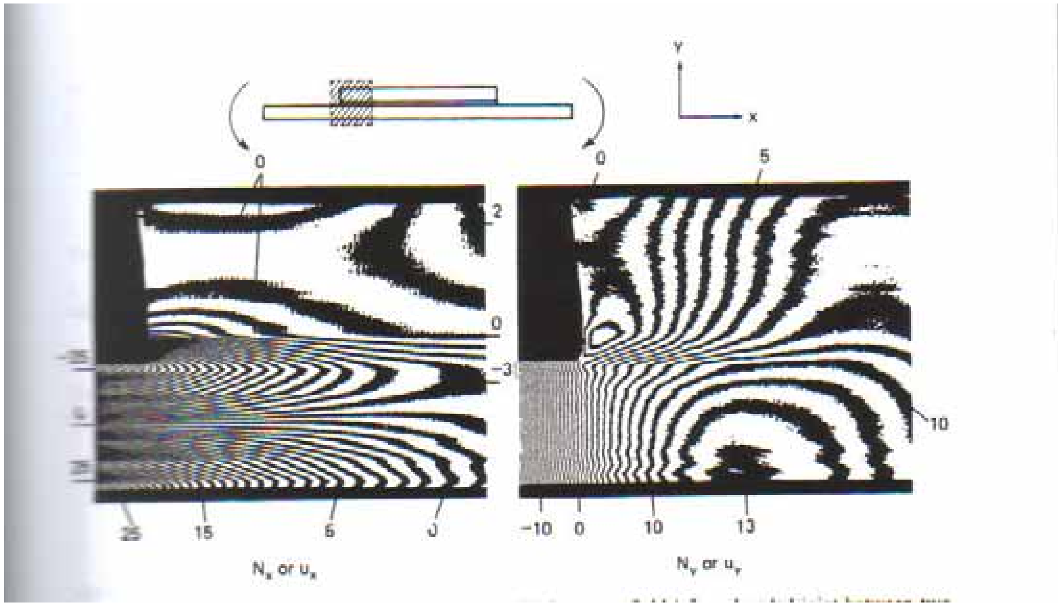


Figure 7-21 Four-beam optical arrangement to produce the N_x pattern with beams C' and D' and the N_y pattern with beams A' and B'.

Both u and v dispplacement fields are needed to determine the complete strain fields $\varepsilon_{xx}(x,y)$, $\varepsilon_{yy}(x,y)$ and $\gamma_{xy}(x,y)$

Example of Two Dimensional Displacement Measurement



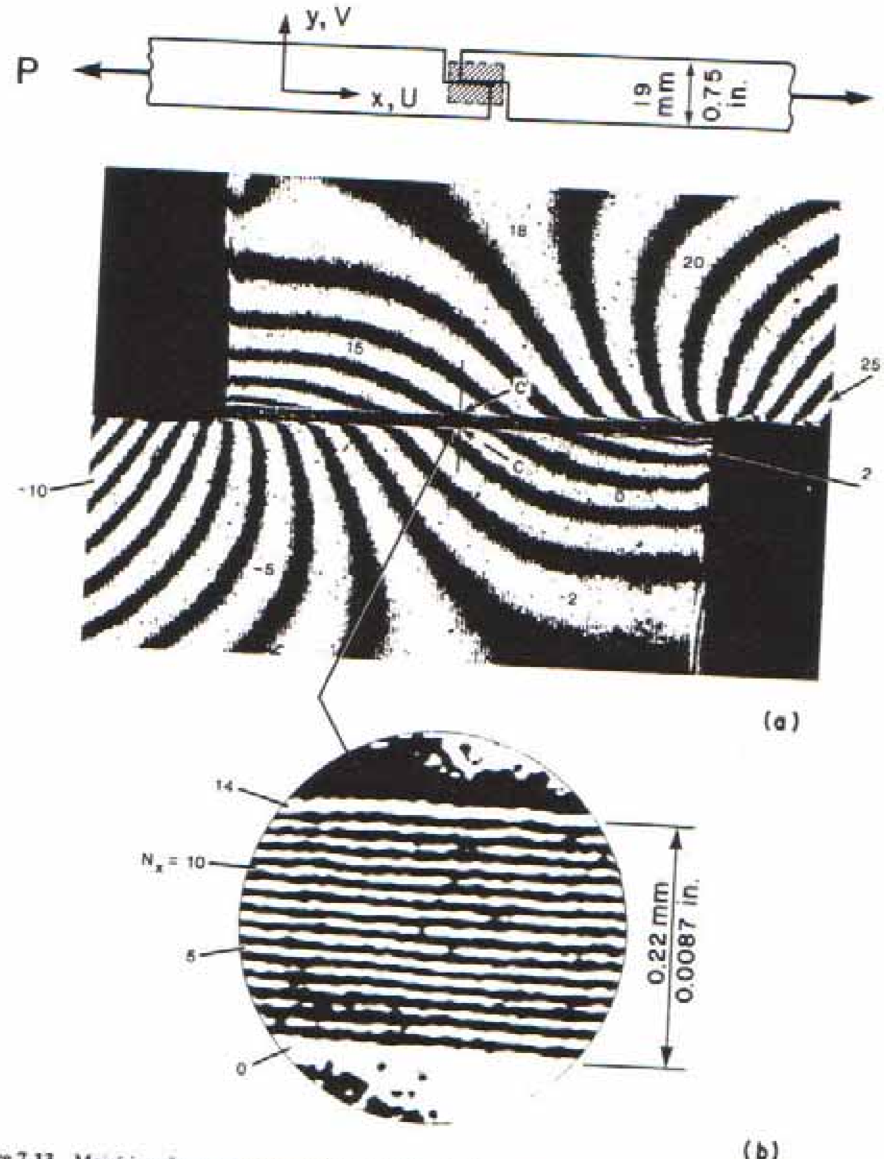
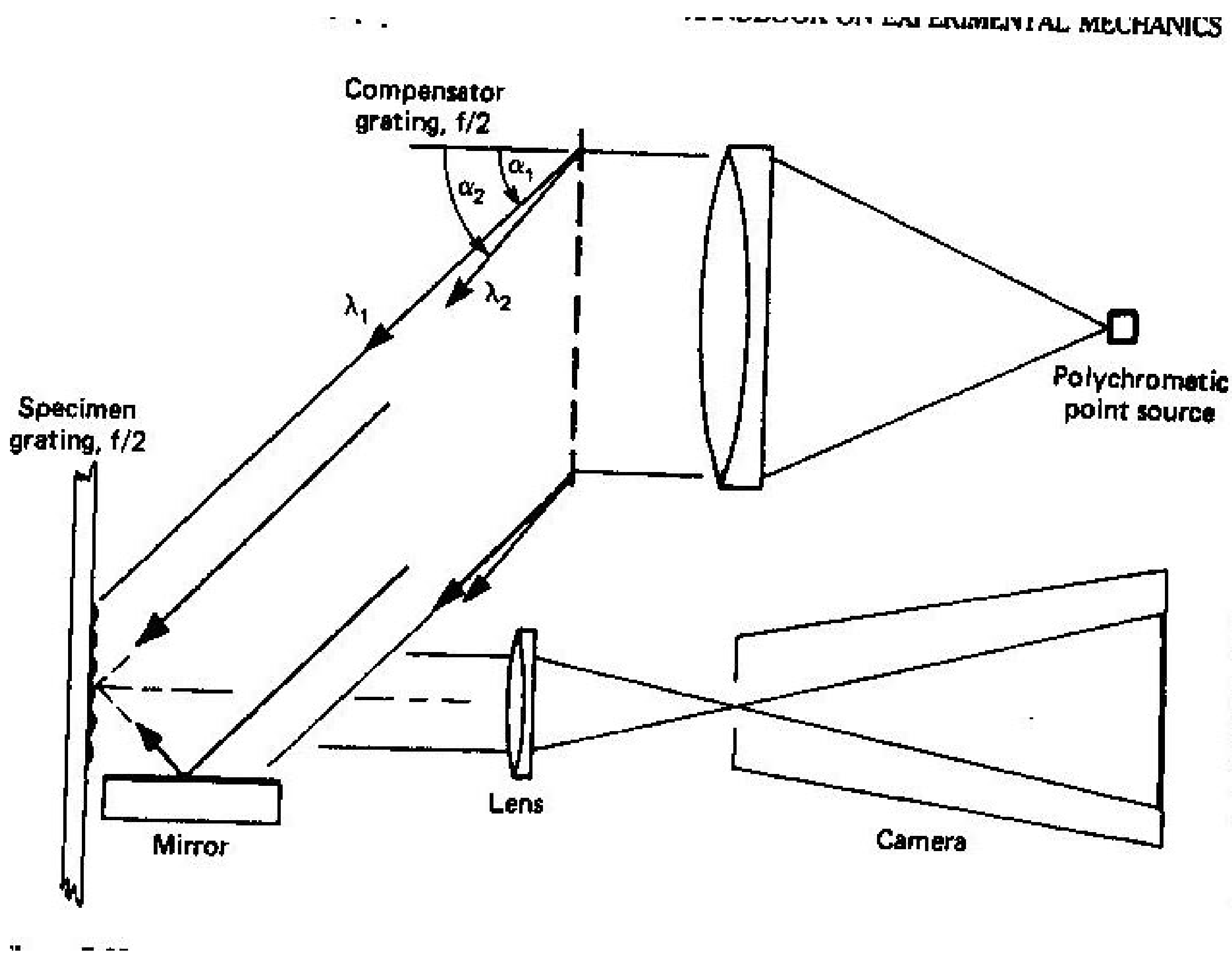


Figure 7-13 Moiré interferometry pattern of N_x, or the u_x displacement field, for a lap joint. The adherends are aluminum and the adhesive is a rubber-modified epoxy. (b) An enlargement of the zone surrounding line cc' in (a). f = 2400 lines/mm (60,960 lines/in.).

Achromatic System



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Optical Setup for two beams and four beams

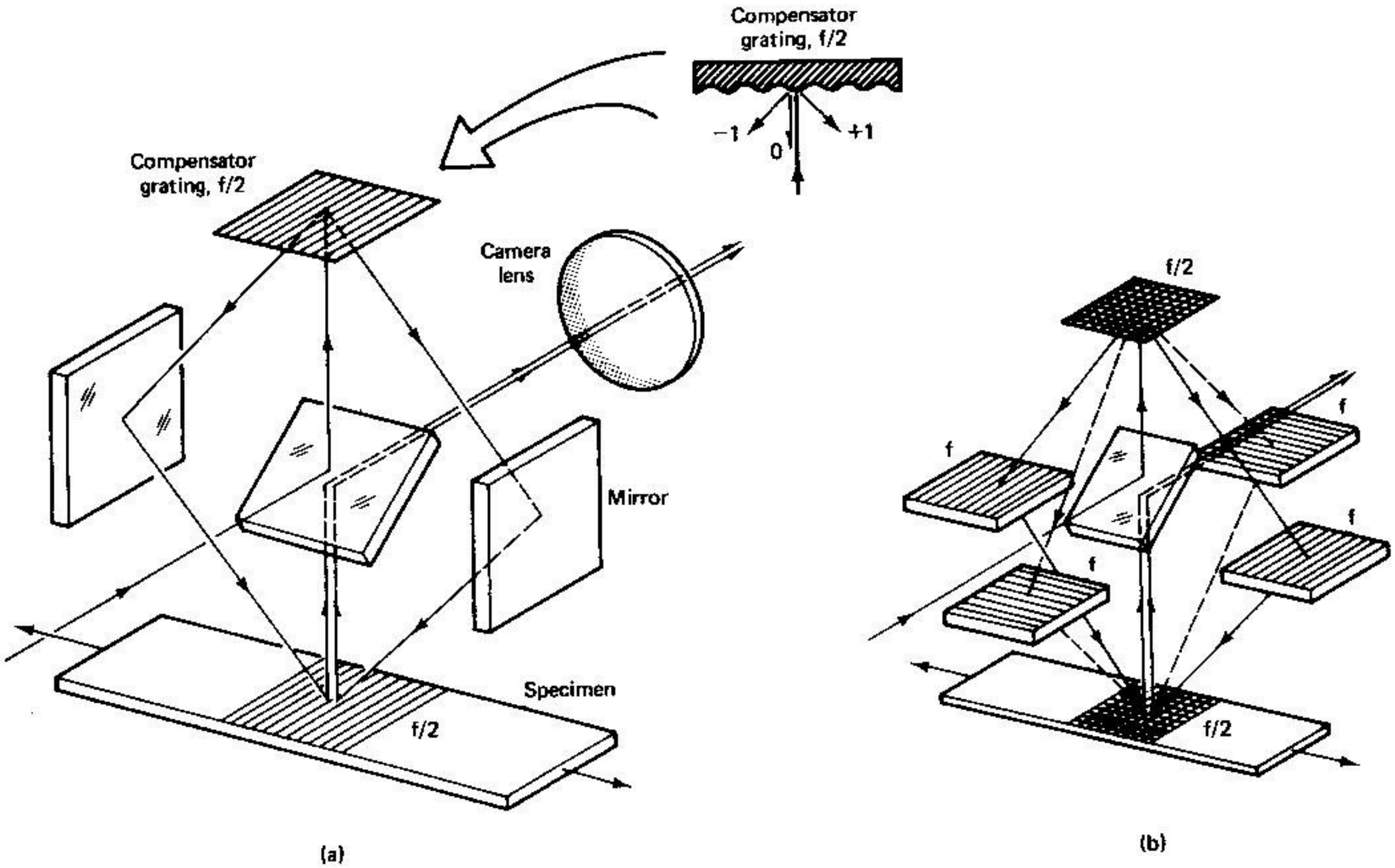
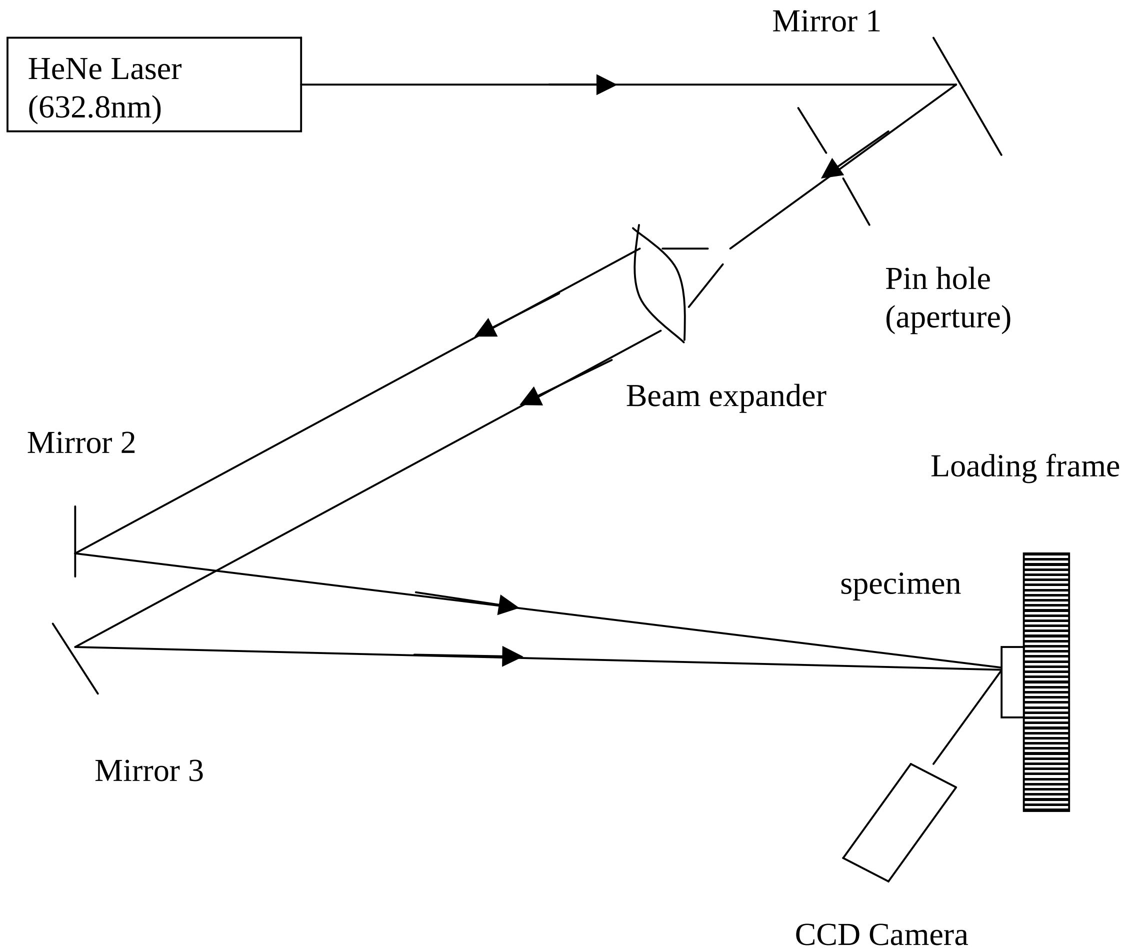


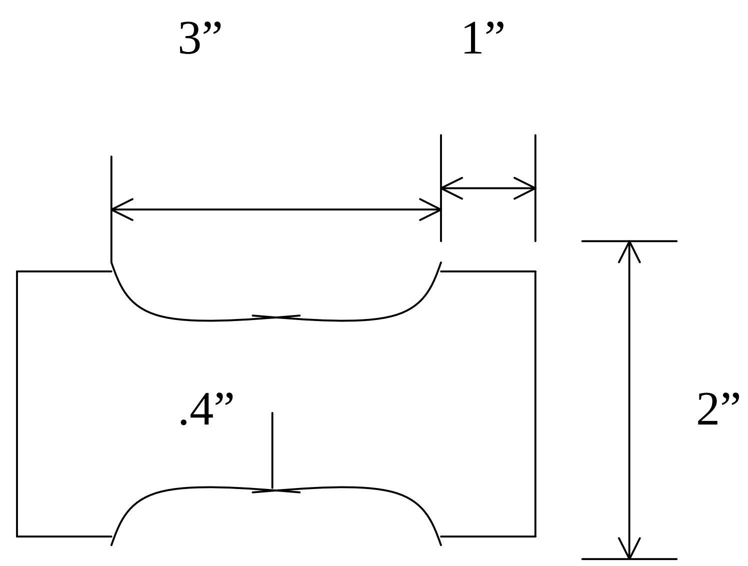
Figure 7-30 Optical arrangements for compact, achromatic moiré interferometers; the compensator allows use of noncoherent light: (a) two-beam system; (b) four-beam system. Both (a) and (b) allow temporally noncoherent light and (b) also allows spatially noncoherent light.

Lab 3 Moiré Interferometery



The experiment is to use moiré interferometry technique to generate a stress-strain curve from the tensile test of a dog bone specimen. The dog bone specimen is made of 76075-T7351 aluminum.

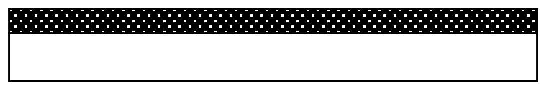
Metal Specimen



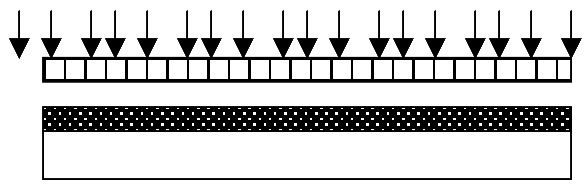
Dog bone specimen of 7075-T7351 aluminum with a sharp notch

Grating Replication Process

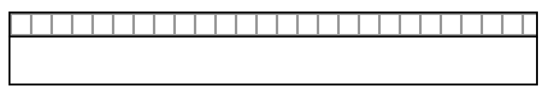
Polish from 9 to ¼ microns to mirror quality



Spin coat AZ1400 series photoresist (1 to 5μm)



Expose grating pattern 40lines/mm using UV light

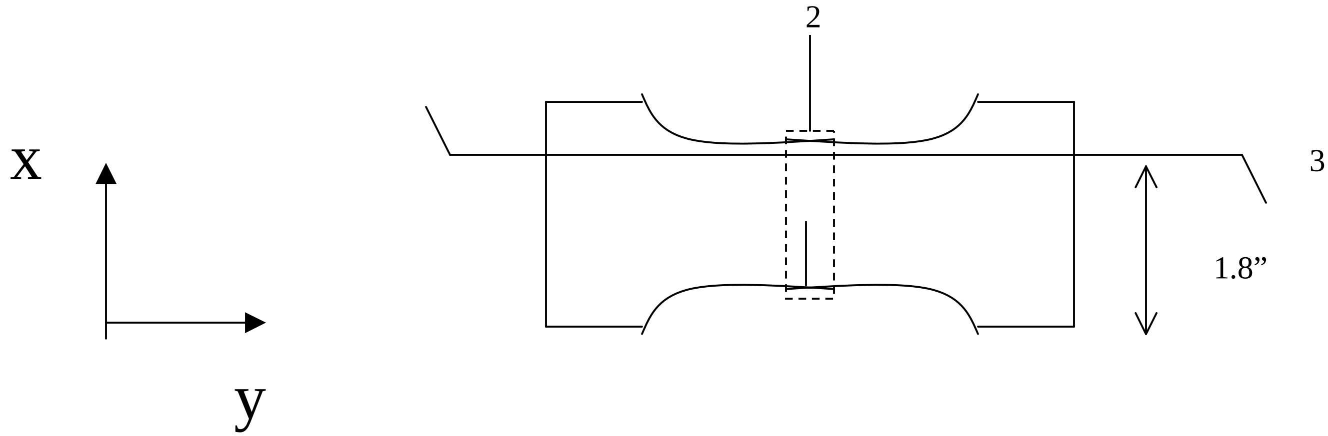


After developing, cross grating pattern
Formed on the +PR

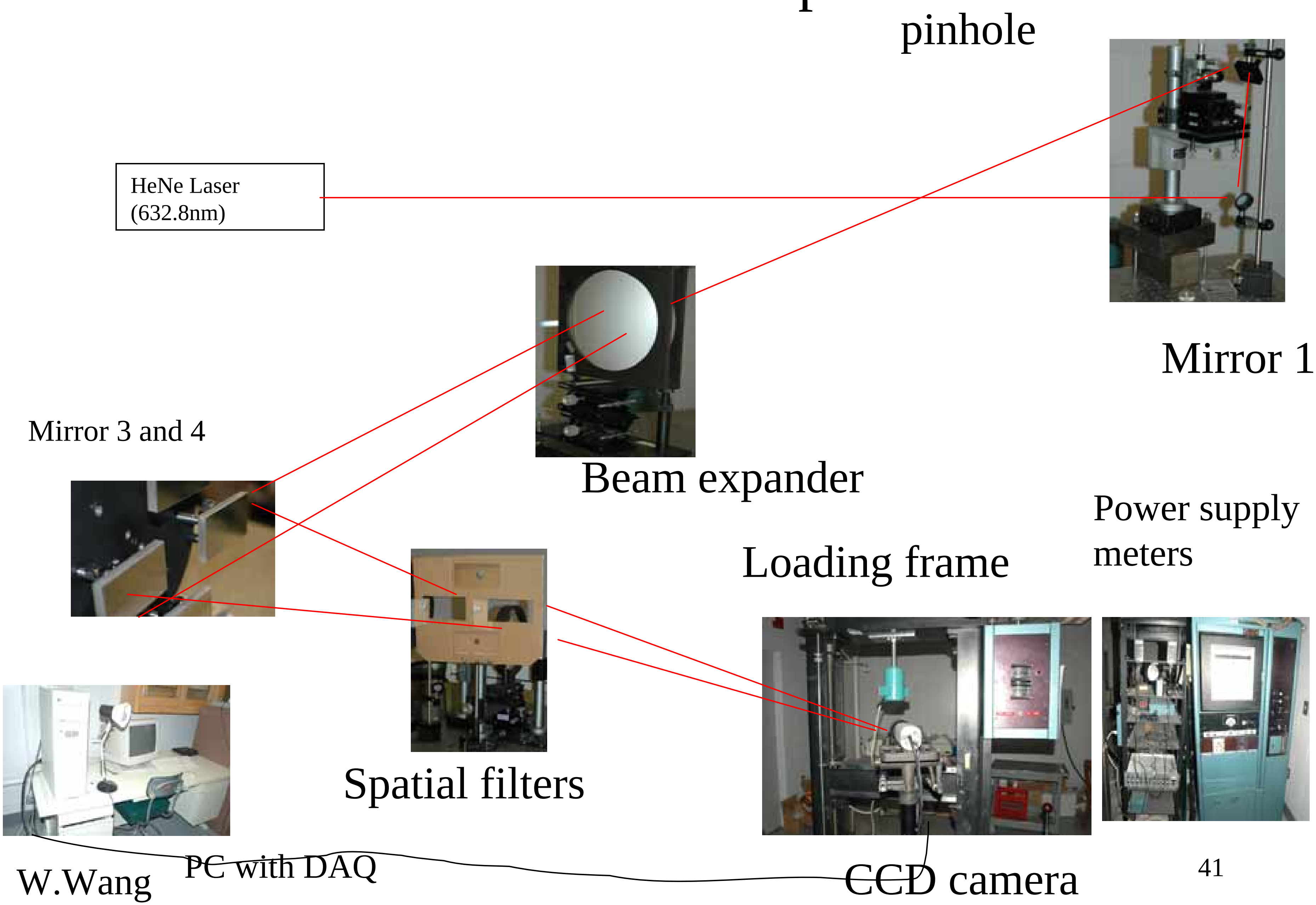
Lab 3

Things to turn in:

1. Moire interference pattern at load = 100lbs
2. Generates the displacement field (y) as a function of distance (x) behind the notch at 400lbs (take data every 0.2")
3. Generate a stress-strain curve based on the observed moiré interference pattern (take at every 100lbs until the sample breaks)



Actual Setup



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