Multilayer Structure

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Reading Materials

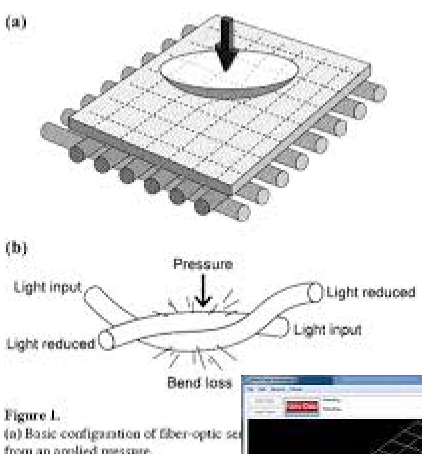
- Lecture notes
- Additional reading provided in class

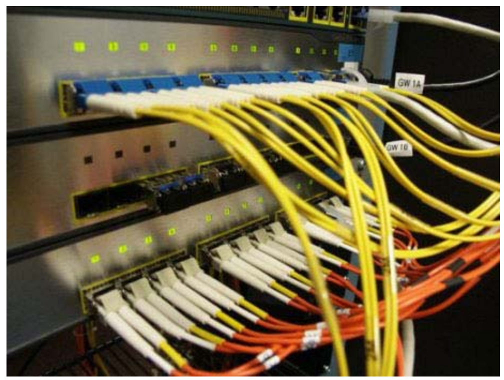
Materials Covered

- Optical Fiber Fundamental (Ray approach)
- Dielectric Waveguide Theory
- Optical Fiber Fundamental (Wave approach)
- Multilayer interference
- Dispersion
- Multi-wave interference

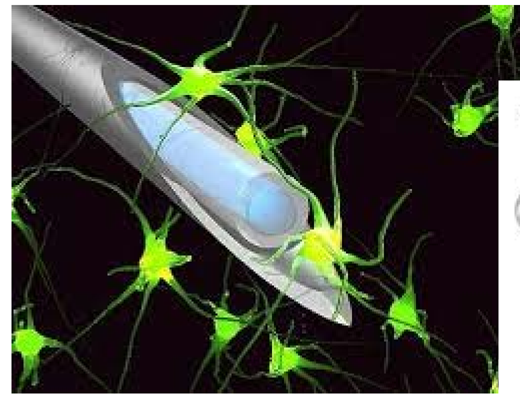
Optical Fiber Application

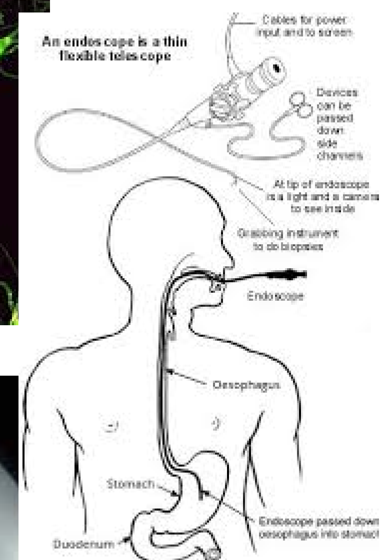
Fiber optic Technology has grown tremendously over the years and toda can be found in many places. Some of applications of fiber optics are shown below

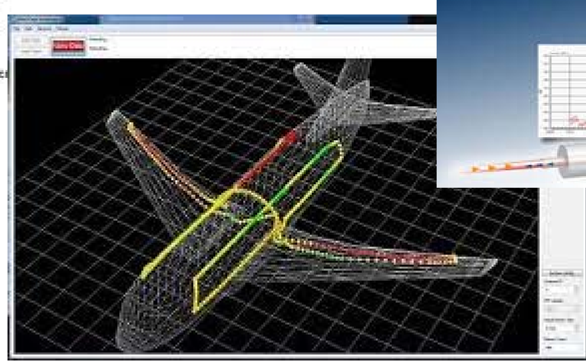


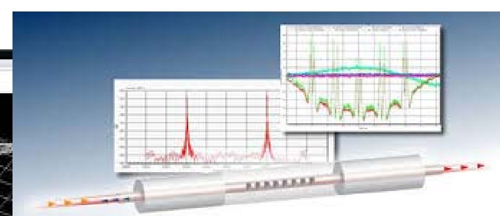


Communication









Medicine

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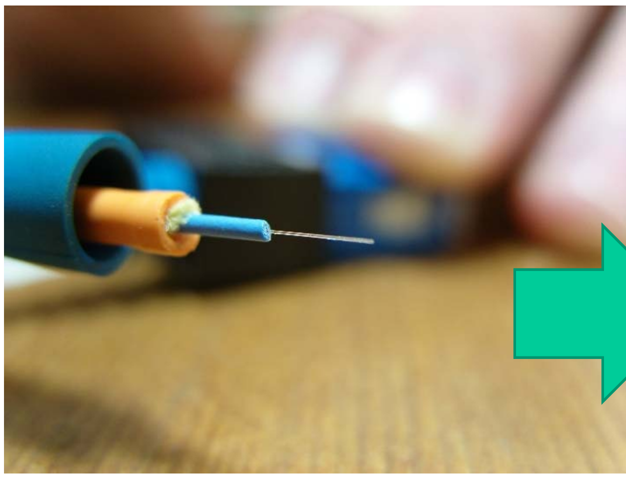
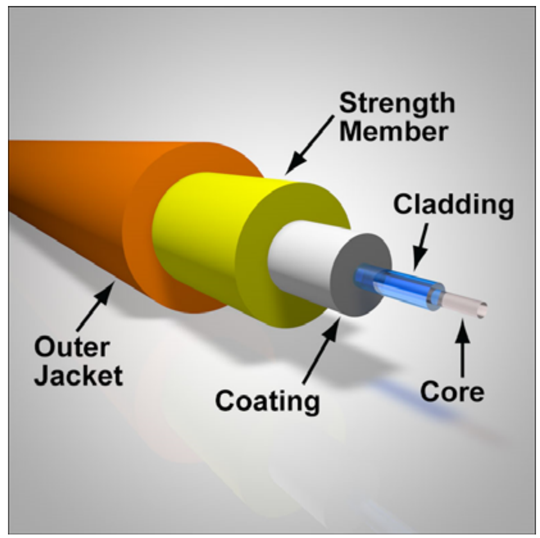
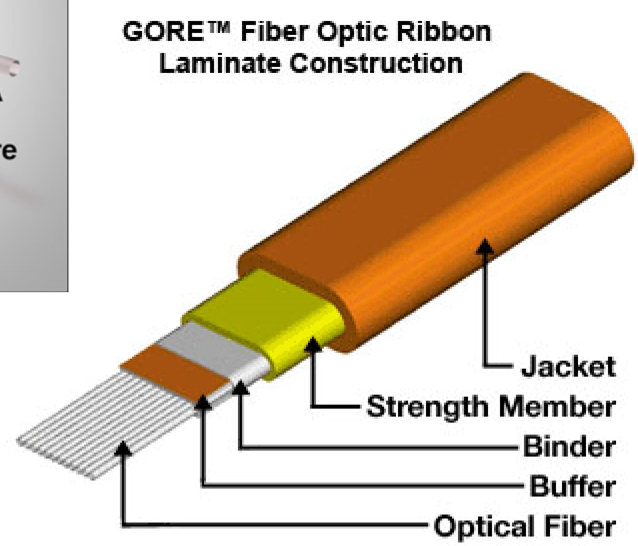
Sensors

Optical Fiber Construction



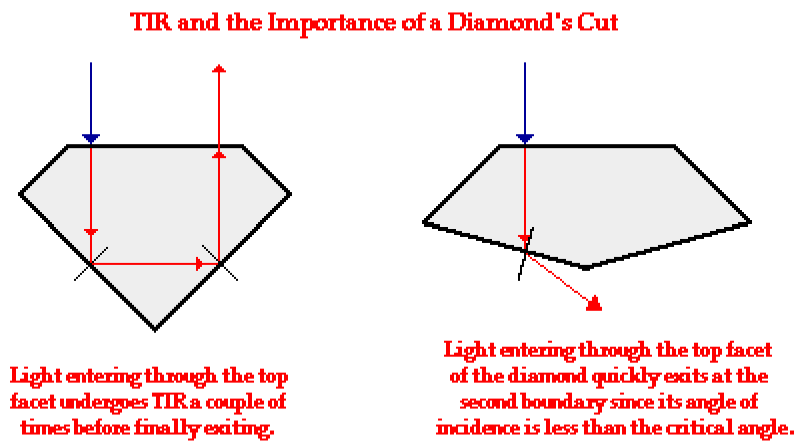


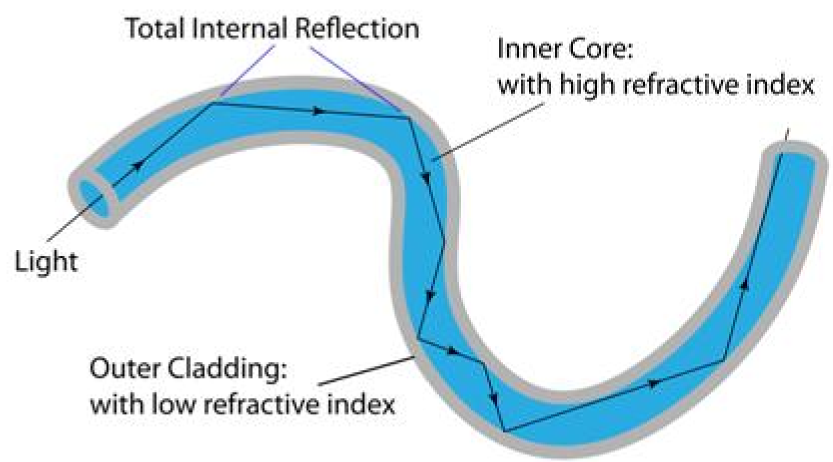
Single fiber construction



Ray Optics Approach

Total Internal Reflection





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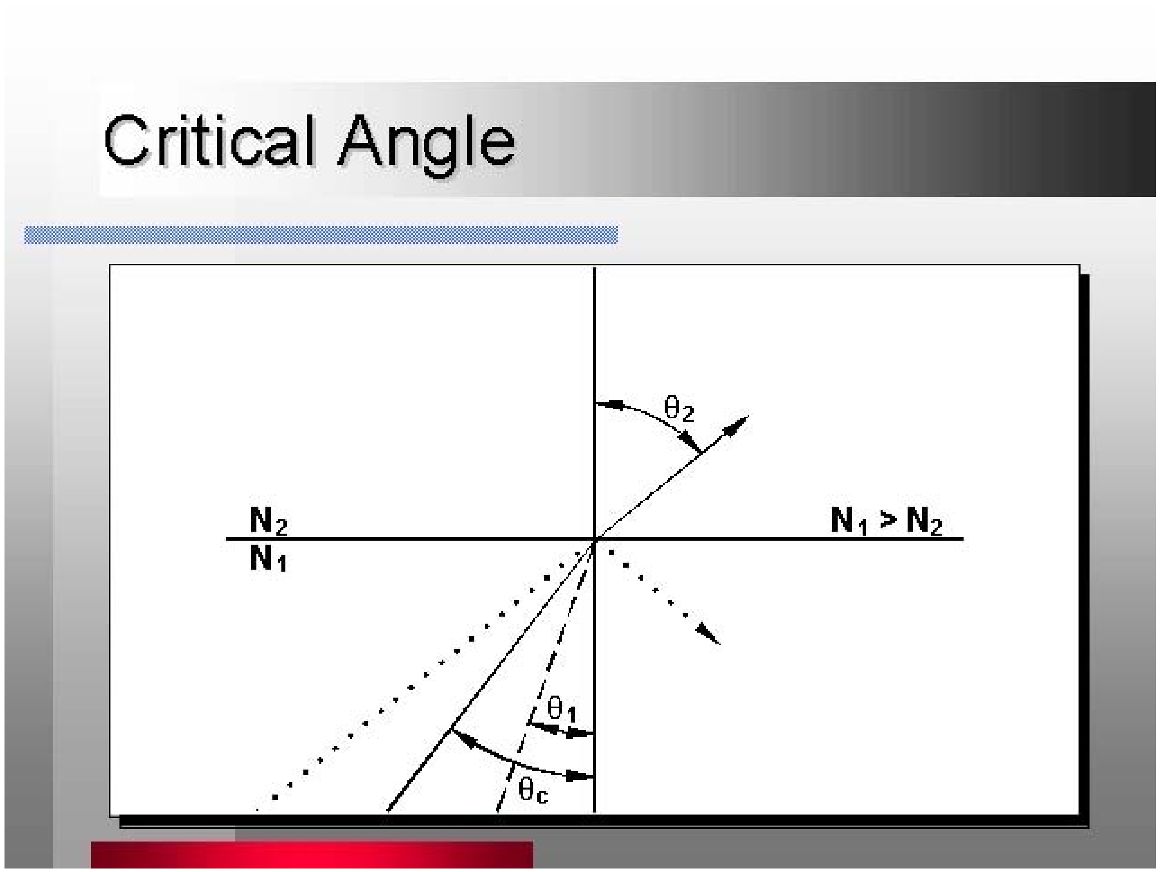
Critical angle:



- Angles 01 and 02 are related by:

$N_1 \sin\theta_1 = N_2 \sin\theta_2$ (Snell's Law)

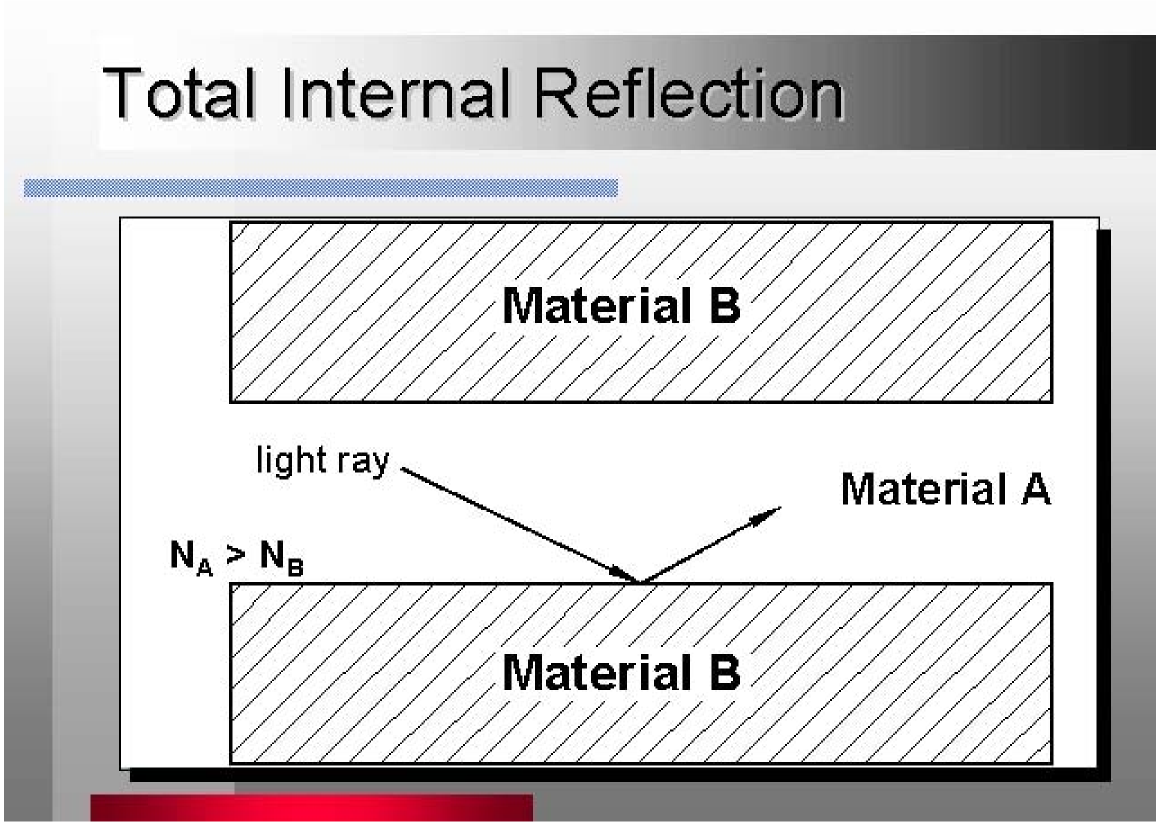
- When $\theta_2 = 90$, 01 is called the Critical Angle

o$N_1 \sin \theta_1 = N_2 \sin 90$, $\sin 90 = 1$

o$\sin \theta c = N_2 / N_1$

- For incident angles greater that the critical angle, total internal reflection occurs

Total reflection



- Total Internal Reflection -- The reflection that occurs when a light ray traveling in one material hits a different material and reflects back into the original material without any loss of light
- Material A = fiber core
- Material B = fiber cladding

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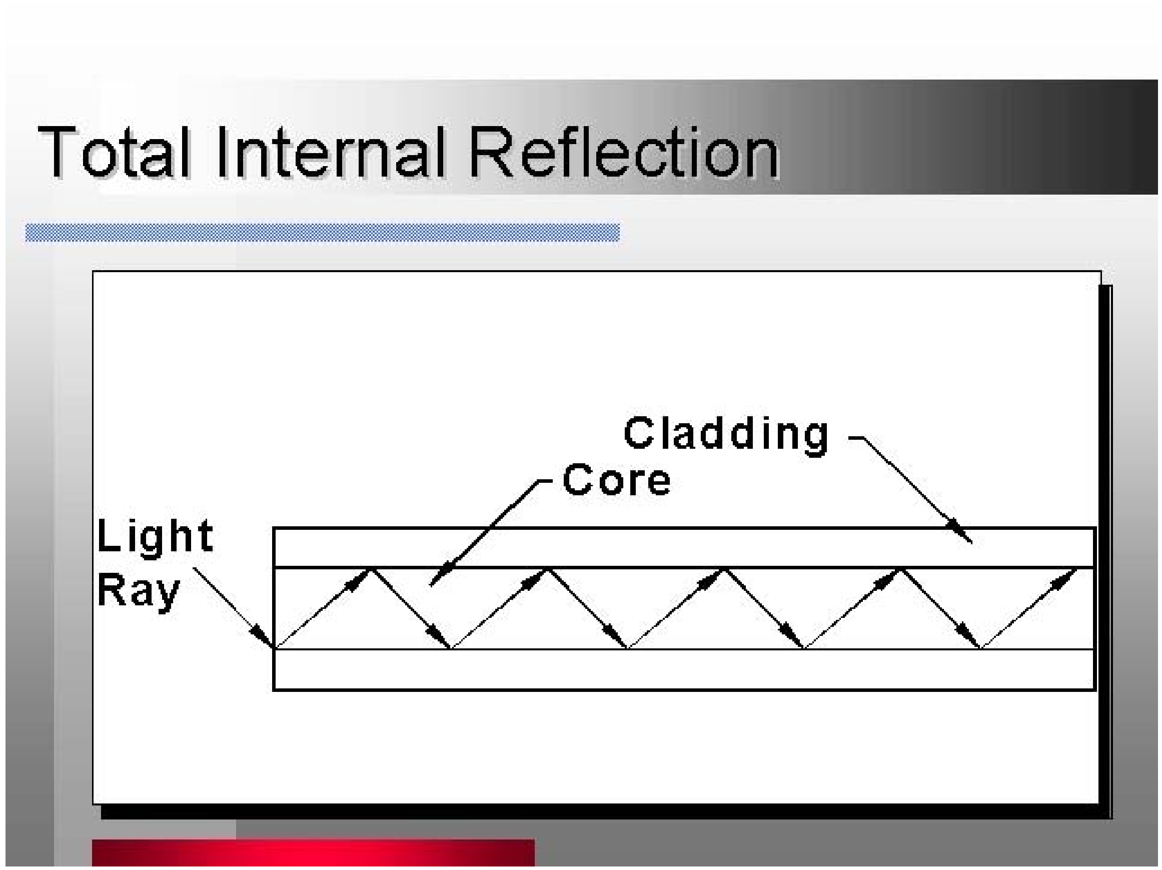
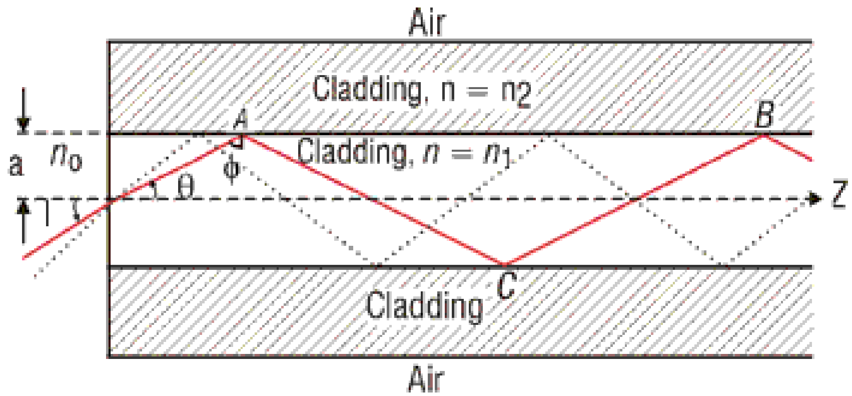


- Light travels down the fiber in a pathway called a light guide

While discussing step-index fibers, we considered light propagation inside the fiber as a set of many rays bouncing back and forth at the core-cladding interface. There the angle θ could take a continuum of values lying between 0 and $\cos^{-1}(n_2/n_1)$, i.e.,

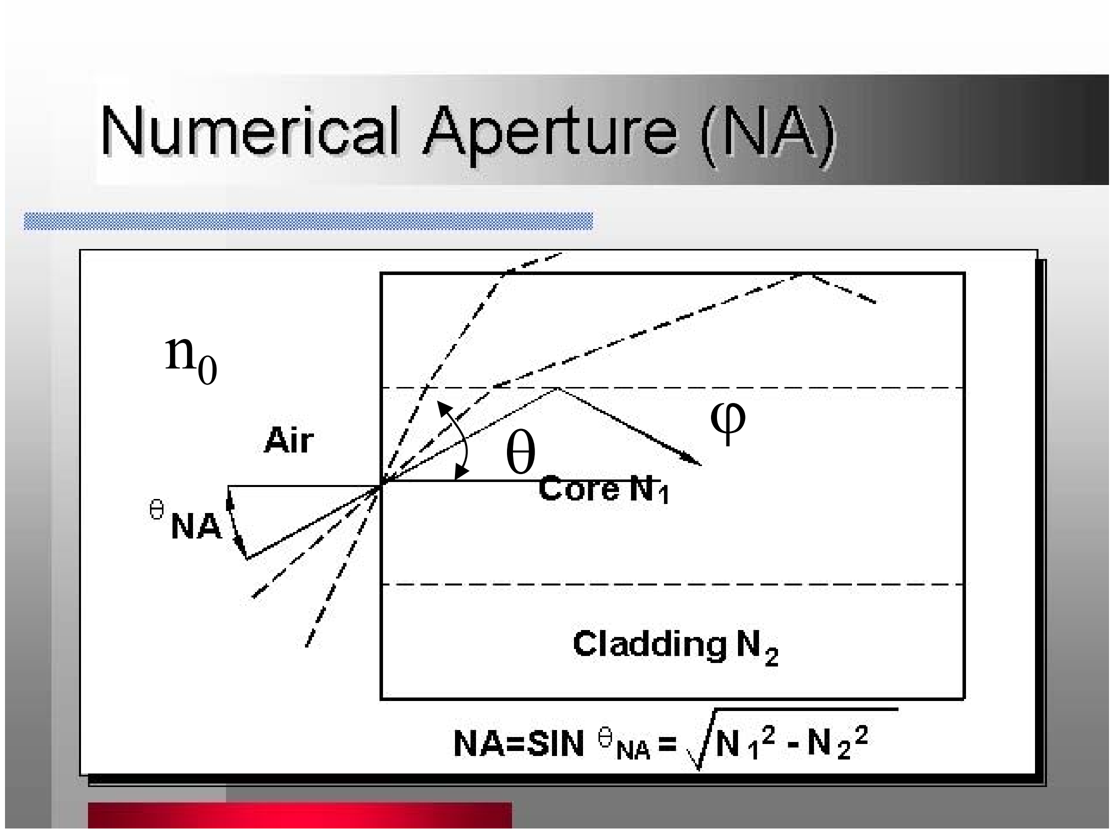


Scientific and Technological Education in Photonics

$$0 < \theta < \cos^{-1}(n_2/n_1)$$

For $n_2 = 1.5$ and $\Delta \approx \frac{n_1 - n_2}{n_1} = 0.01$, we would get $n_2/n_1 \simeq$ and $\cos^{-1}\left(\frac{n_1}{n_2}\right) = 8.1°$, so

$$0 < \theta < 8.1°$$



- The Numerical Aperture (NA) of a fiber is the measure of the maximum angle (θ_{NA}) of the light entering the end that will propagate within the core of the fiber
- Acceptance Cone = 20NA
- Light rays entering the fiber that exceed the angle θ_{NA} will enter the cladding and be lost
- For the best performance the NA of the transmitter should match the NA of the fiber

NA derivation

We know $\dfrac{\sin i}{\sin\theta} = \dfrac{n_1}{n_0}$ and $\sin\phi (= \cos\theta) > \dfrac{n_2}{n_1}$

Since $\sin\theta = \sqrt{1 - \cos^2\ \theta}$ we get $\sin\theta < \left[1 - \left(\dfrac{n_2}{n_1}\right)^2\right]^{1/2}$

Assume the θ_{NA} is the half angle of the acceptance cone,

$$\sin\theta_{NA} = (n_1^2 - n_2^2)^{1/2} = n_1 \mathrm{sqrt}(2\Delta)$$

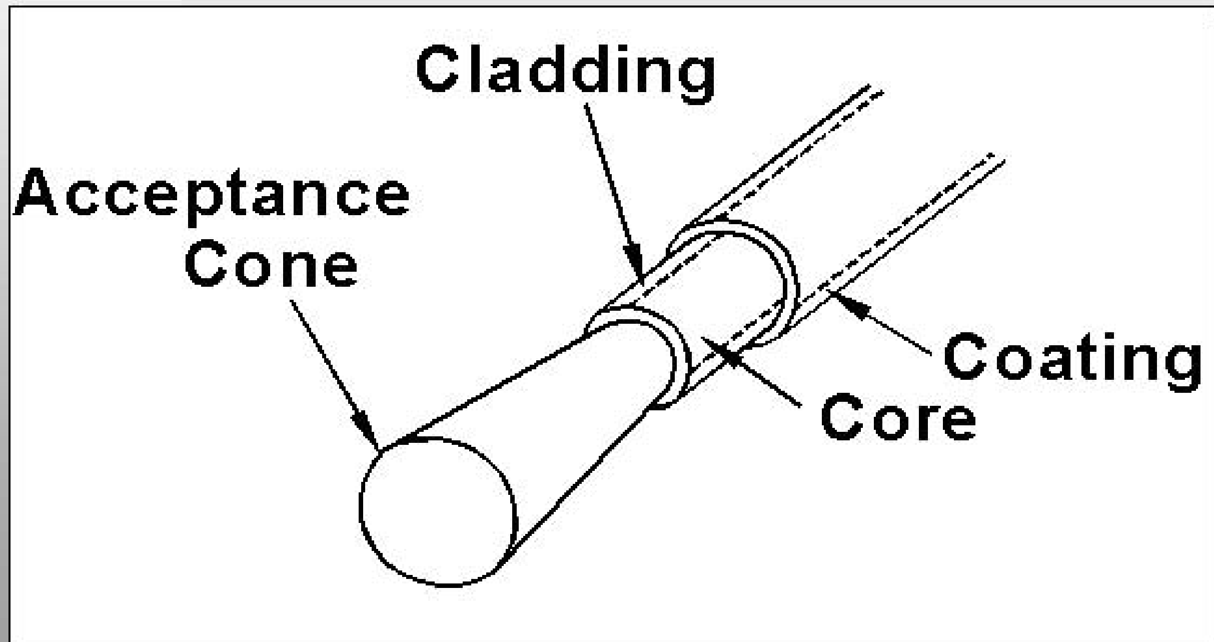
We define a parameter Δ through the following equations.

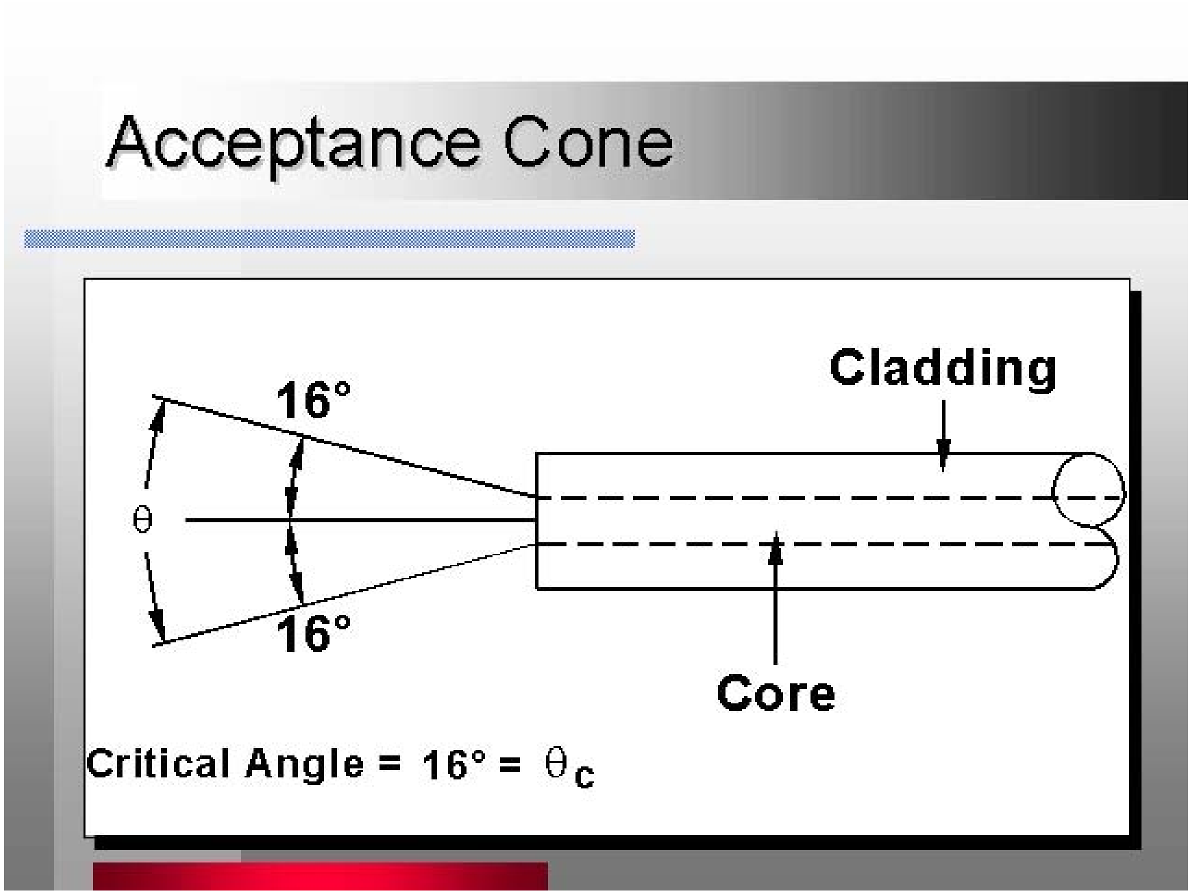
$$\Delta \equiv \frac{n_1^2 - n_2^2}{2n_2^2}$$

When $\Delta \ll 1$ (as is indeed true for silica fibers where n_1 is very nearly equal to n_2) we may write

$$\Delta = \frac{(n_1 + n_2)(n_1 - n_2)}{2n_1^2} \approx \frac{(n_1 - n_2)}{n_1} \approx \frac{(n_1 - n_2)}{n_2}$$

Acceptance Cone





Single mode fiber critical angle <20°
Multimode fiber critical angle <60°

Example

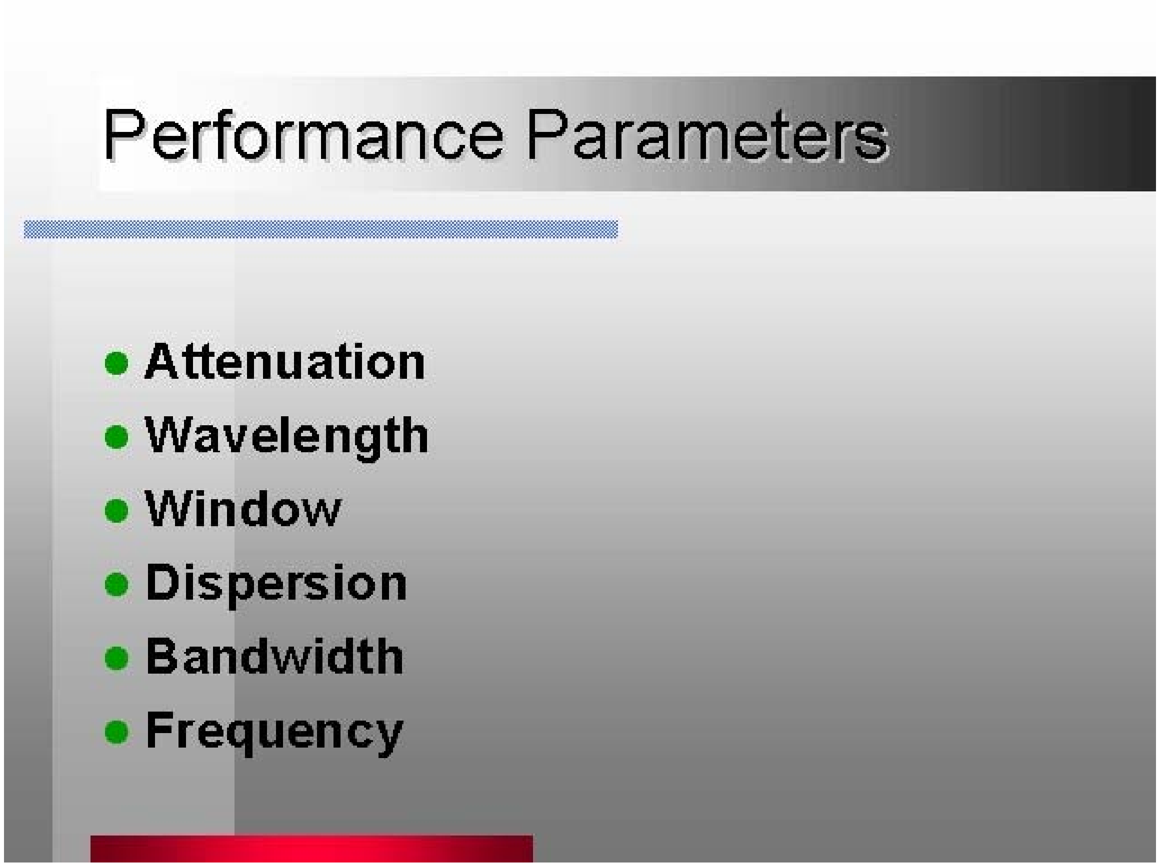
For a typical step-index (multimode) fiber with $n_1 \approx 1.45$ and $\Delta \approx 0.01$, we get

$$\sin i_m = n_1 \sqrt{2\Delta} = 1.45\sqrt{2 \times (0.01)} = 0.205$$

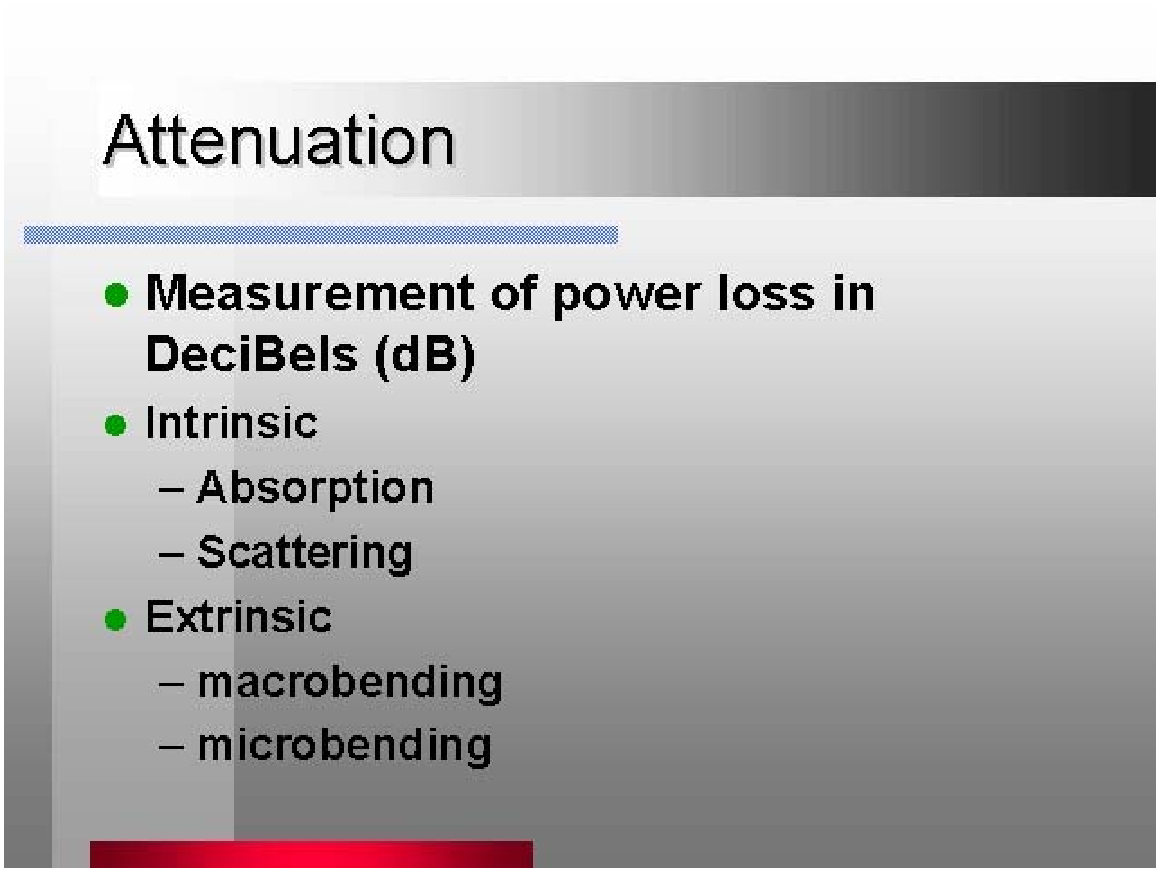
so that $i_m \approx 12°$. Thus, all light entering the fiber must be within a cone of half-angle 12°.

In a short length of an optical fiber, if all rays between $i = 0$ and i_m are launched, the light coming out of the fiber will also appear as a cone of half-angle i_m emanating from the fiber end. If we now allow this beam to fall normally on a white paper and measure its diameter, we can easily calculate the *NA* of the fiber.

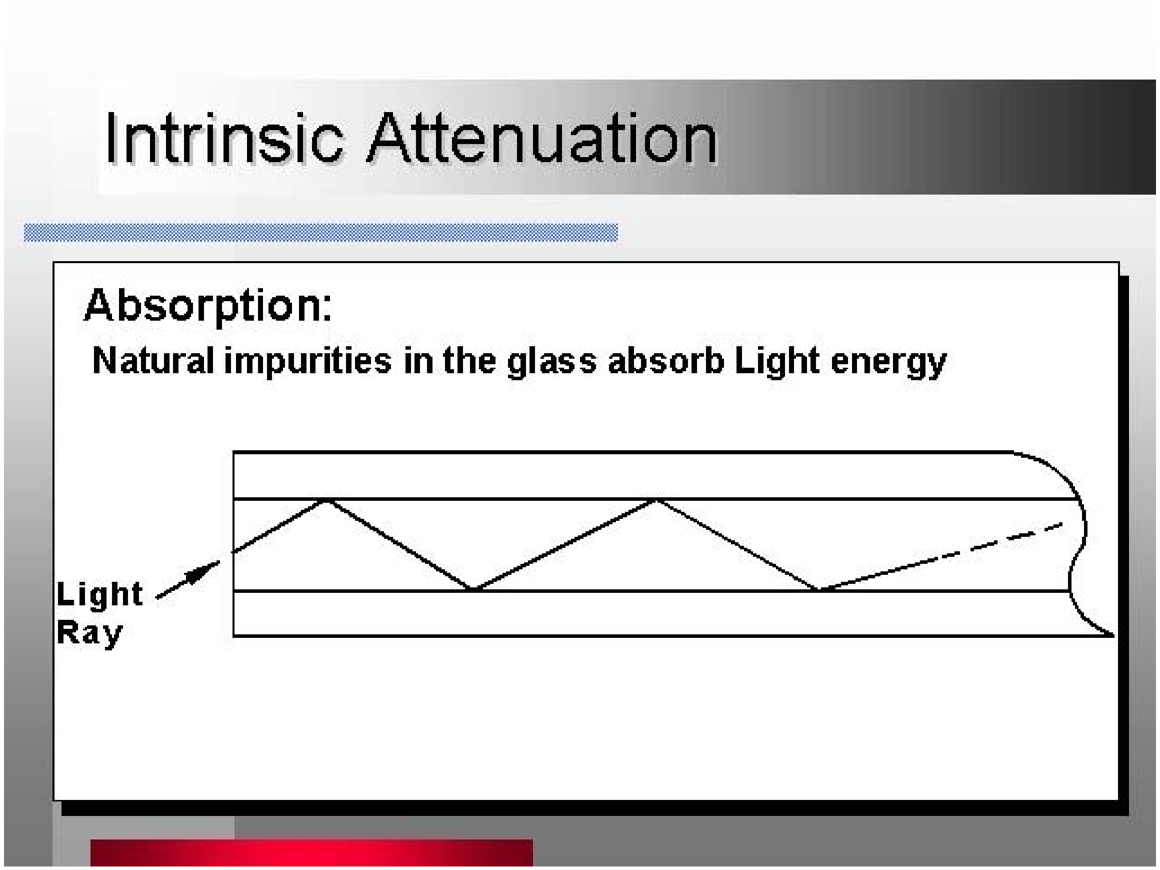
Performance parameters



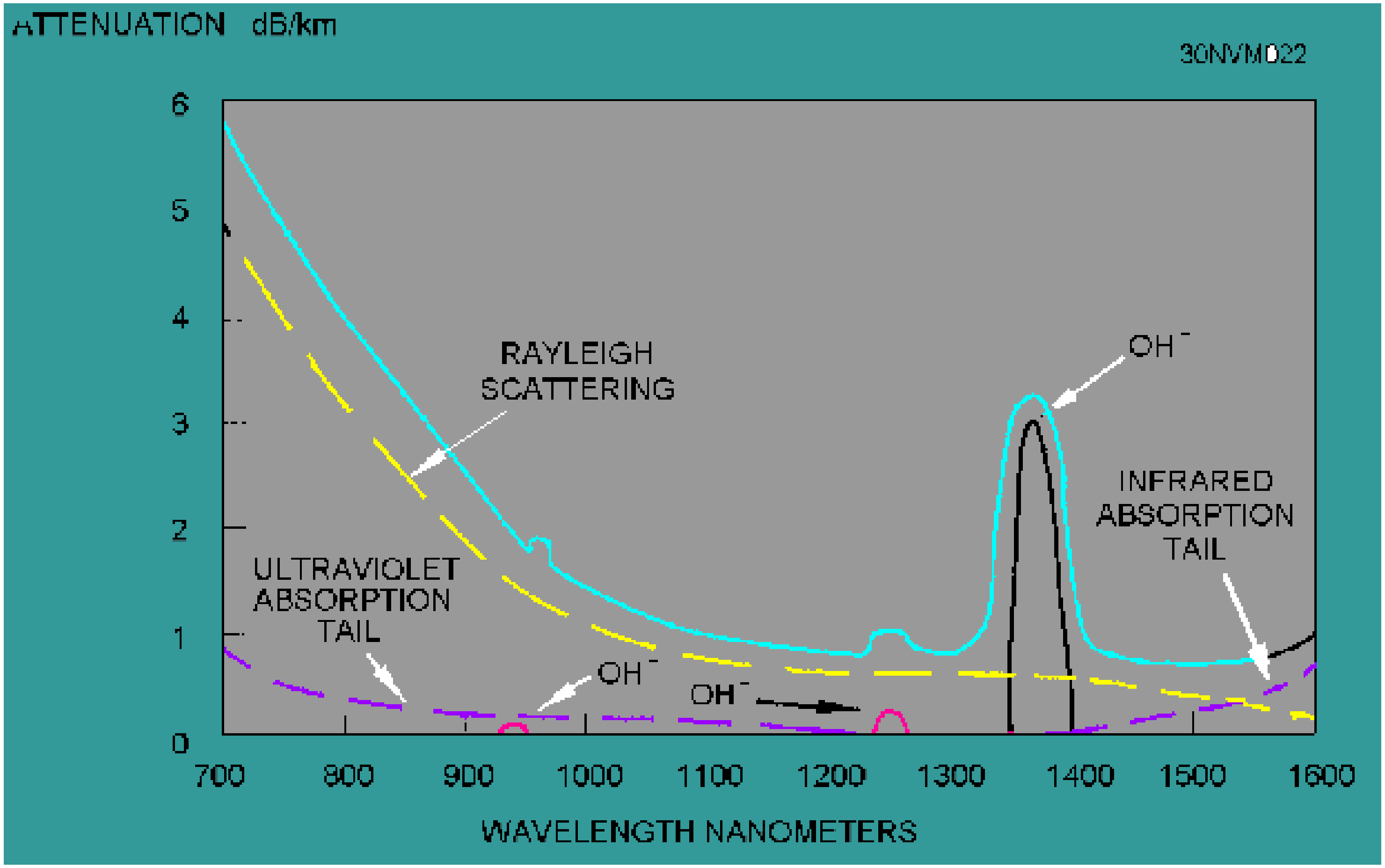
Attenuation



•dB is a ratio of the power received verses the power transmitted
•Loss (dB) = 10log (power transmitted / power received)



- Intrinsic attenuation is controlled by the fiber manufacturer
- Absorption caused by water molecules and other impurities
- Light strikes a molecule at the right angle and light energy is converted into heat
- Absorption accounts for 3-5% of fiber attenuation these is near the theoretical limit



The main cause of **intrinsic absorption** in the infrared region is the characteristic vibration frequency of atomic bonds. In silica glass, absorption is caused by the vibration of silicon-oxygen (Si-O) bonds. The interaction between the vibrating bond and the electromagnetic field of the optical signal causes intrinsic absorption. Light energy is transferred from the electromagnetic field to the bond.

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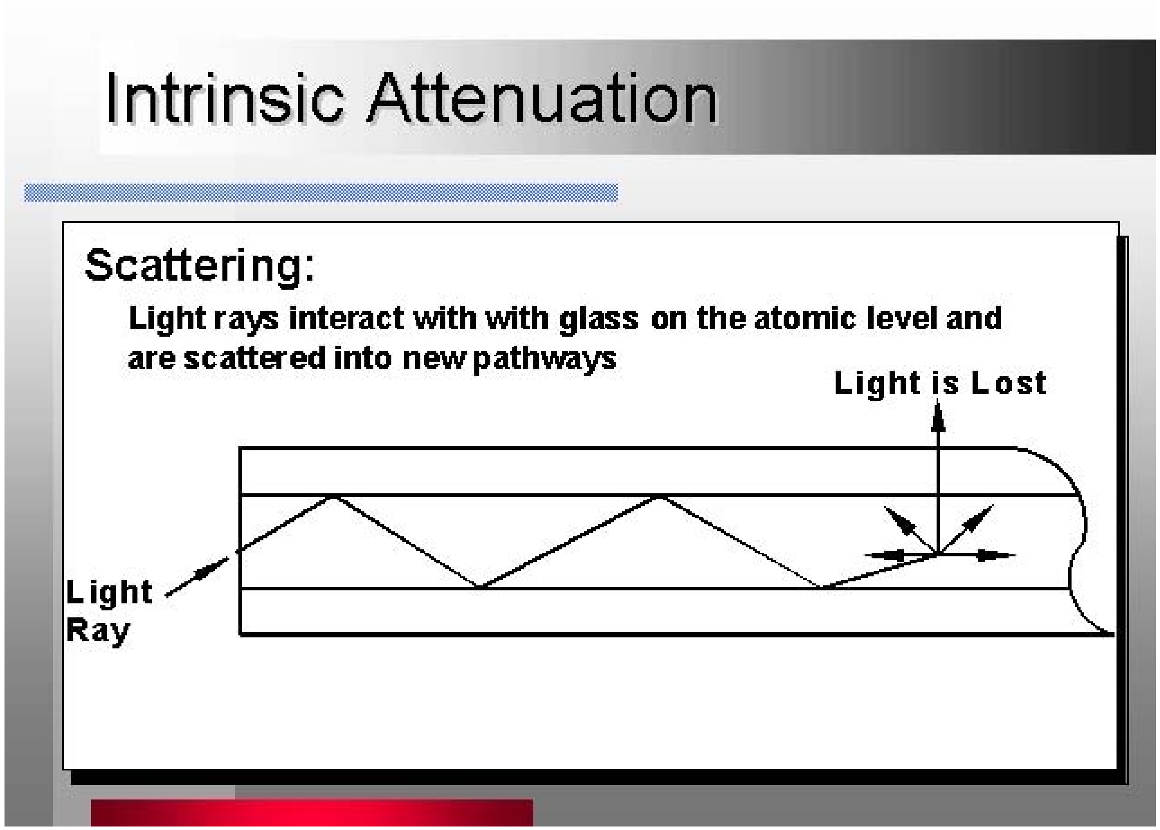
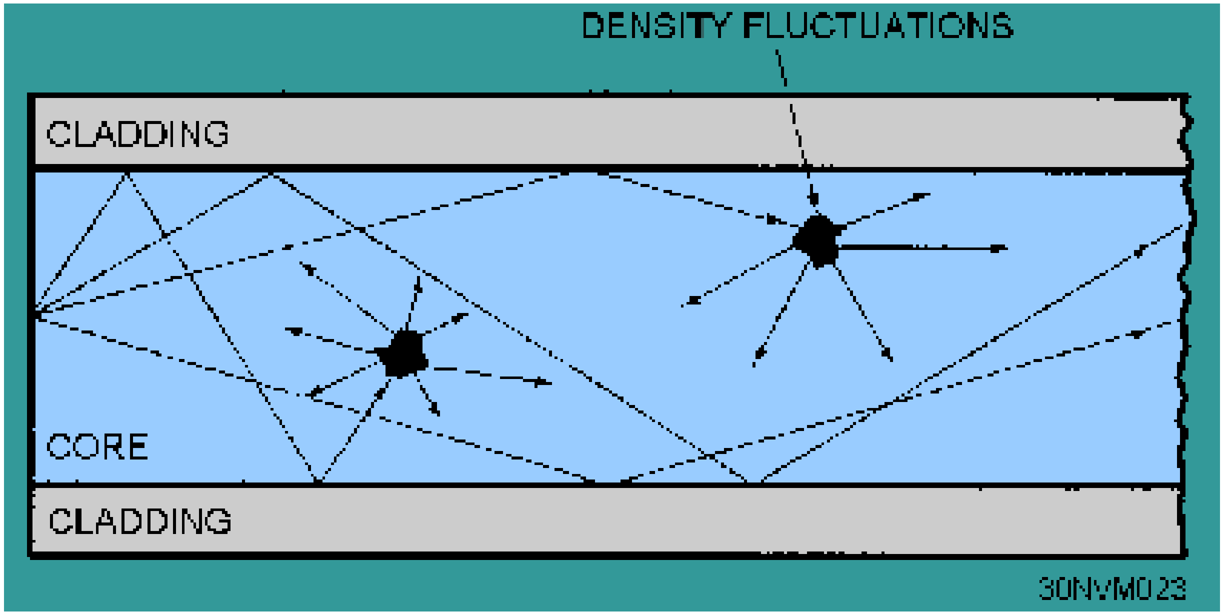
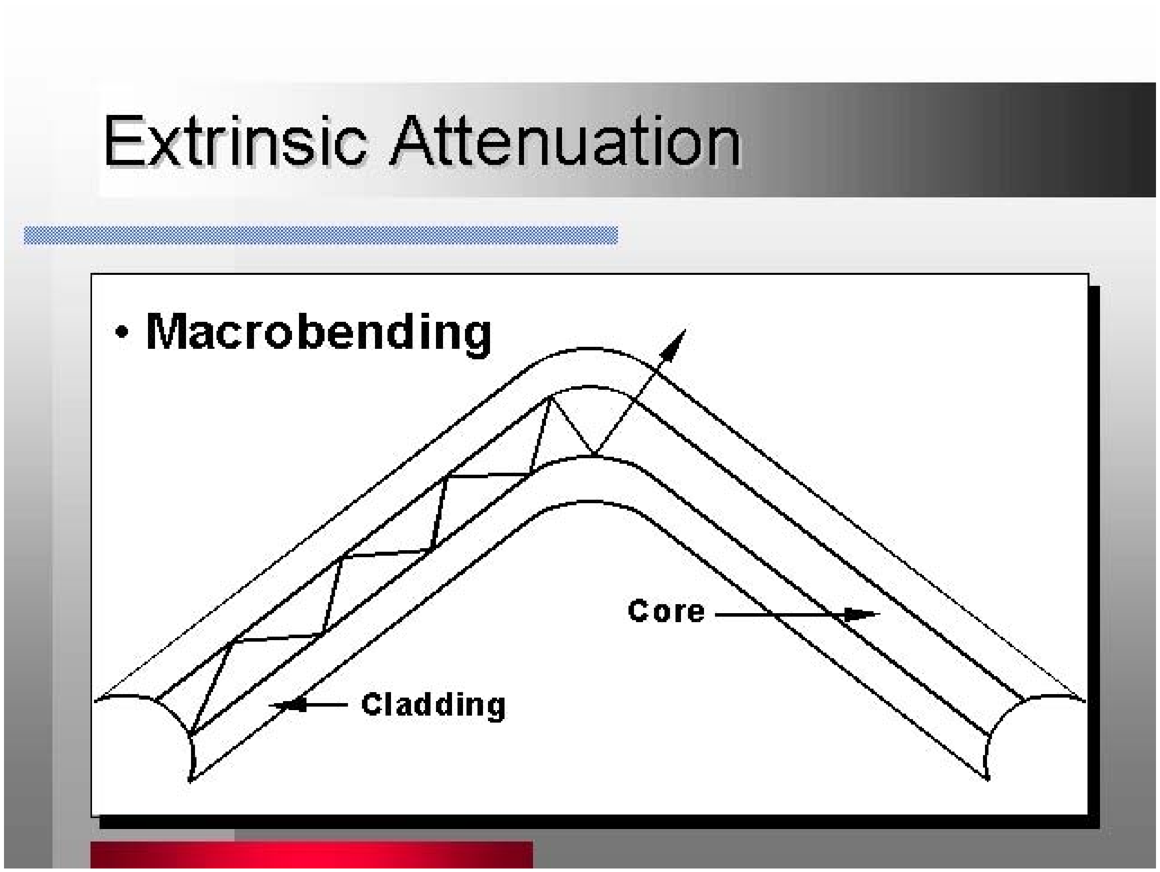


- Light striking the Ge molecules in the core can be scattered into new pathways out of the fiber
- Rayleigh Scattering accounts for 95% of fiber attenuation
- Optical Time Domain Reflectometers (OTDR) use this property to measure loss in a fiber

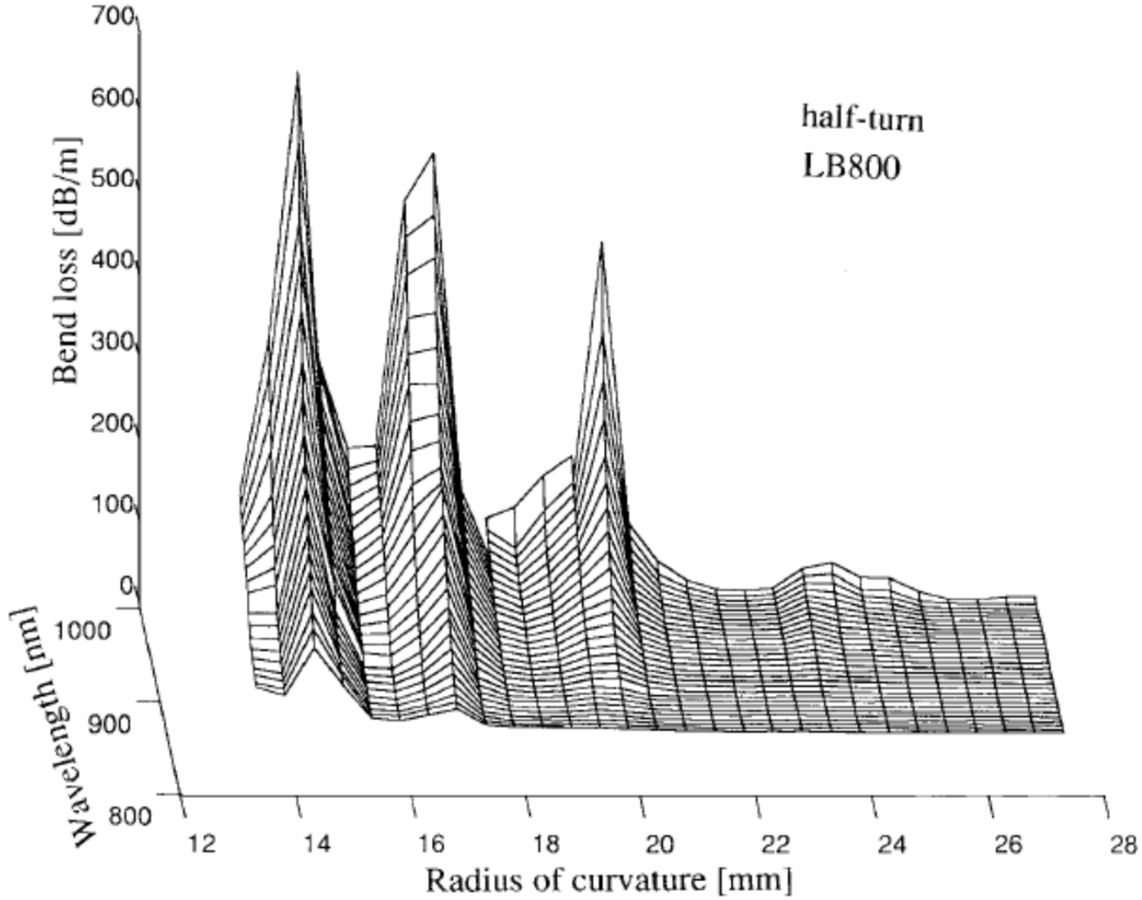


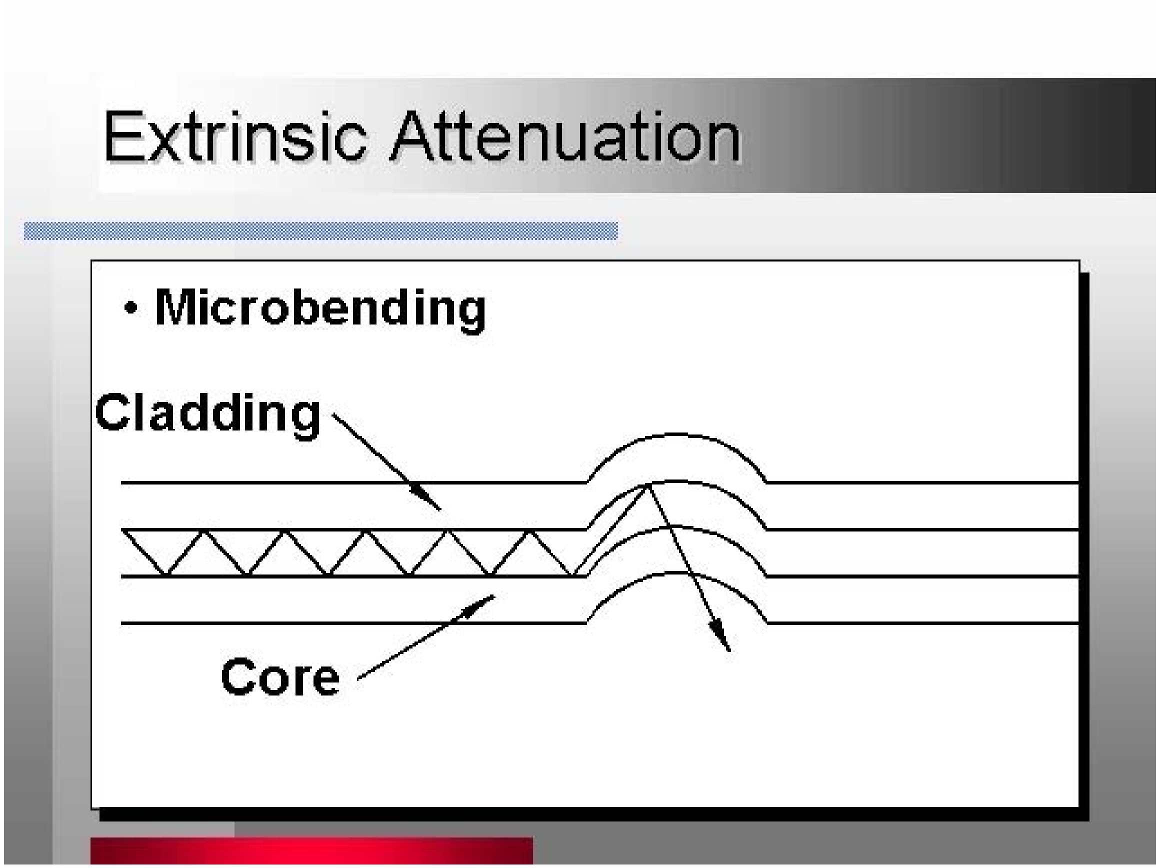
operating between 700-nm and 1600-nm wavelength, the main source of loss is called Rayleigh scattering. Rayleigh scattering is the main loss mechanism between the ultraviolet and infrared regions as shown in absoprtion specturm. **Rayleigh scattering** occurs when the size of the density fluctuation (fiber defect) is less than one-tenth of the operating wavelength of light. Loss caused by Rayleigh scattering is proportional to the fourth power of the wavelength (1/lambda). As the wavelength increases, the loss caused by Rayleigh scattering decreases.

If the size of the defect is greater than one-tenth of the wavelength of light, the scattering mechanism is called **Mie scattering**. Mie scattering, caused by these large defects in the fiber core, scatters light out of the fiber core. However, in commercial fibers, the effects of Mie scattering are insignificant. Optical fibers are manufactured with very few large defects.



Extrinsic attenuation can be controlled by the cable installer
Macrobend losses are observed when a fiber bend's radius of curvature is large compared to the fiber diameter.

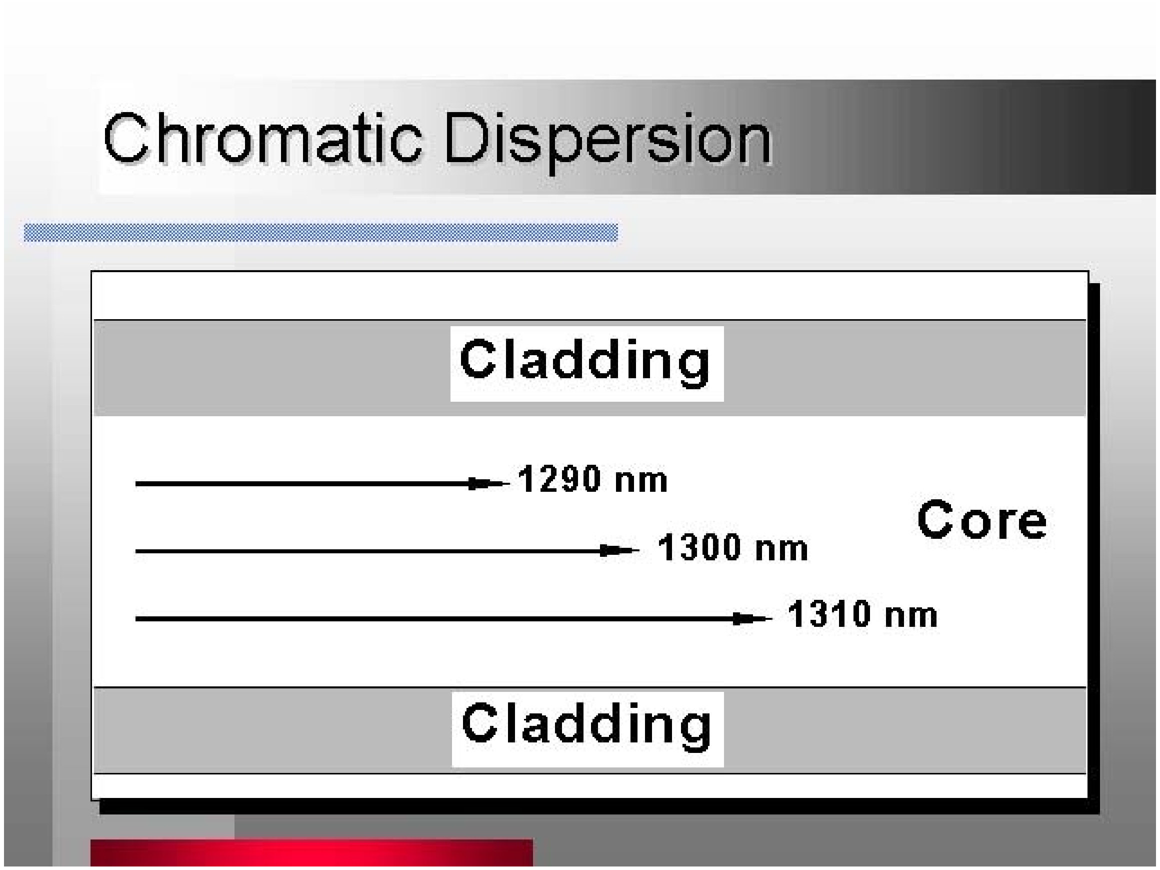

half-turn
LB800
Bend loss [dB/m]
700
600
500
400
300
200
100
0
Wavelength [nm]
1000
900
800
12
14
16
18
20
22
24
26
28
Radius of curvature [mm]



- Microbends may not be visible with the naked eye
- Microbends may be:
  - bend related
  - temperature related
  - tensile related
  - crush related

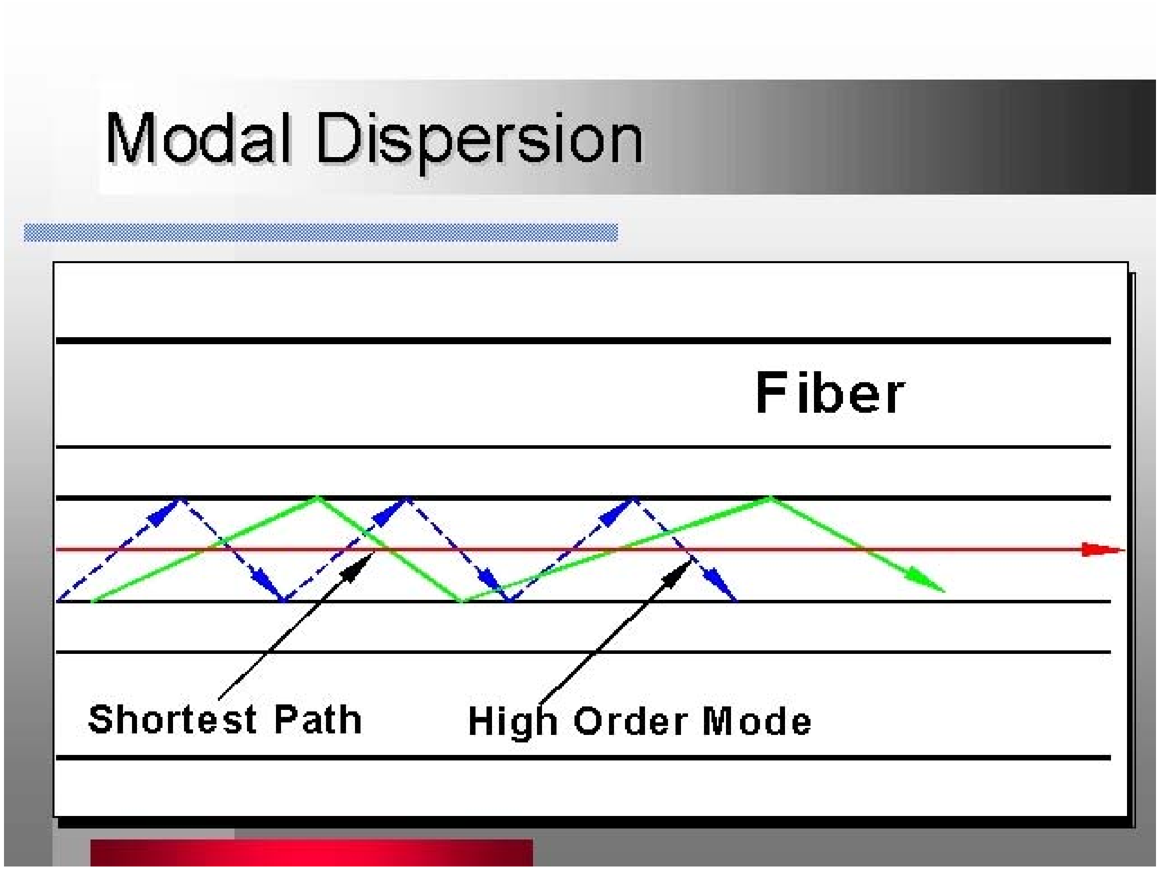
Dispersion

- Dispersion is the variation of light velocities in a fiber
  - Modal
  - Chromatic
- Pulse Spreading

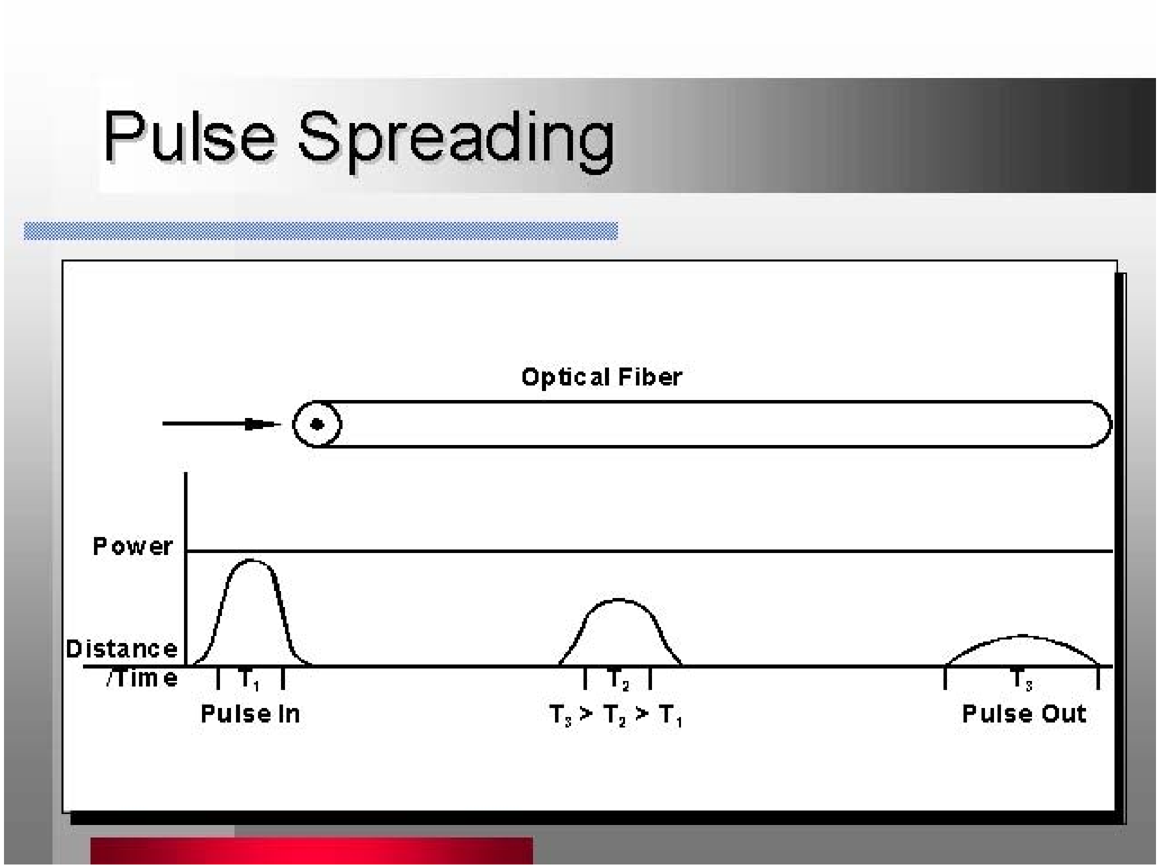


- Index of Refraction is a function of wavelength
- Since light velocity is a function of index of refraction
    - light velocity in a given medium is a function of wavelength

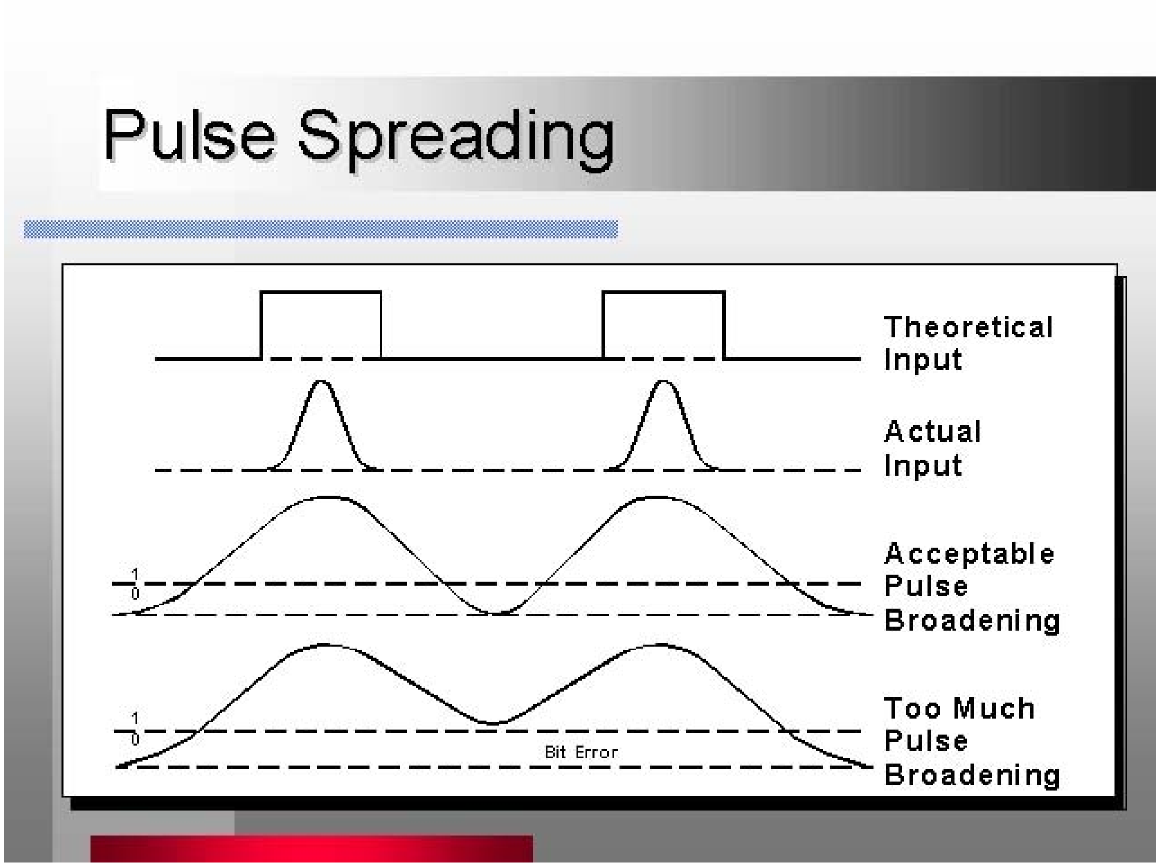
Light pulses at different wavelengths will have different propagation times



Various modes follow different paths causing pulse broadening



- Because a light pulse is made up of different colors and modes of light some portions arrive at the end of the fiber before others causing a spreading effect
- This causes pulses to overlap making them unreadable by the receiver



•The received pulse must be above the receiver threshold to be detected as an on pulse. Likewise an off pulse must be below the receiver threshold to be detected as an off pulse.

•If pulses spread and overlap above the receiver threshold, an off pulse will not be detected and errors in the signal will result

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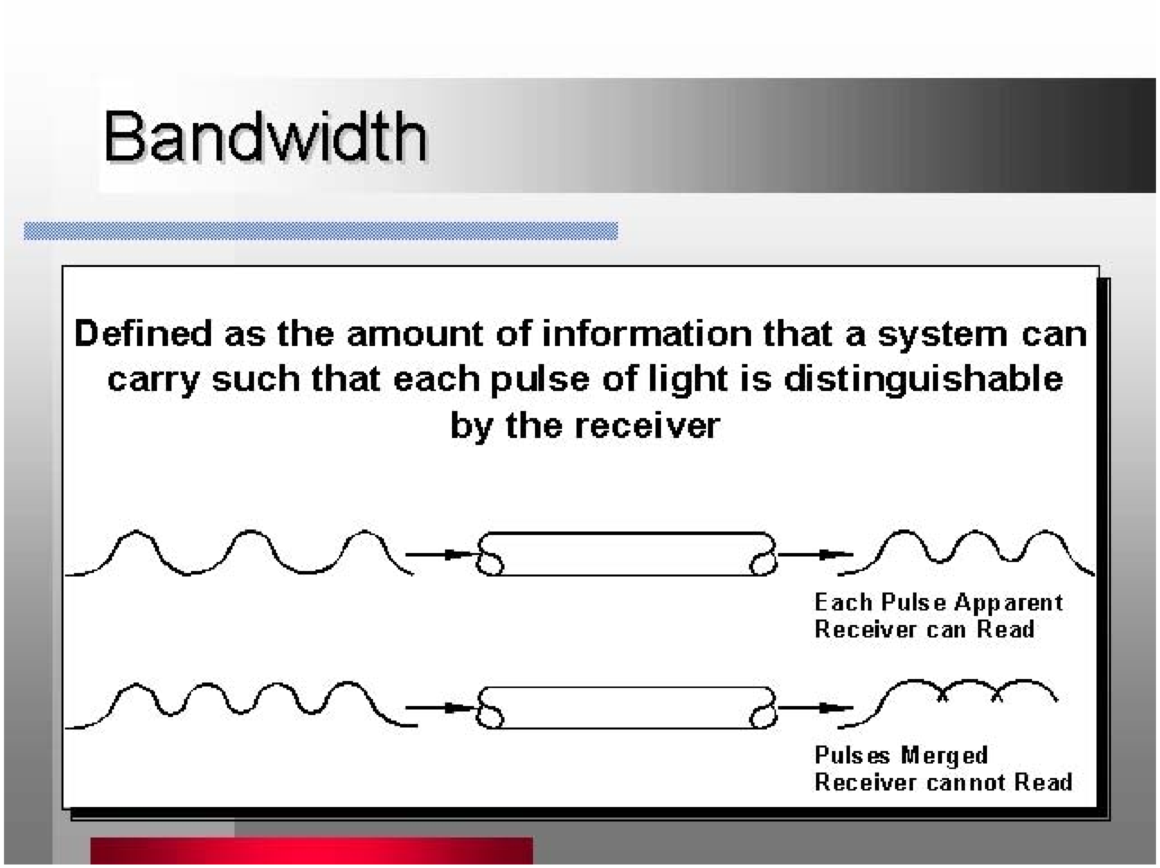
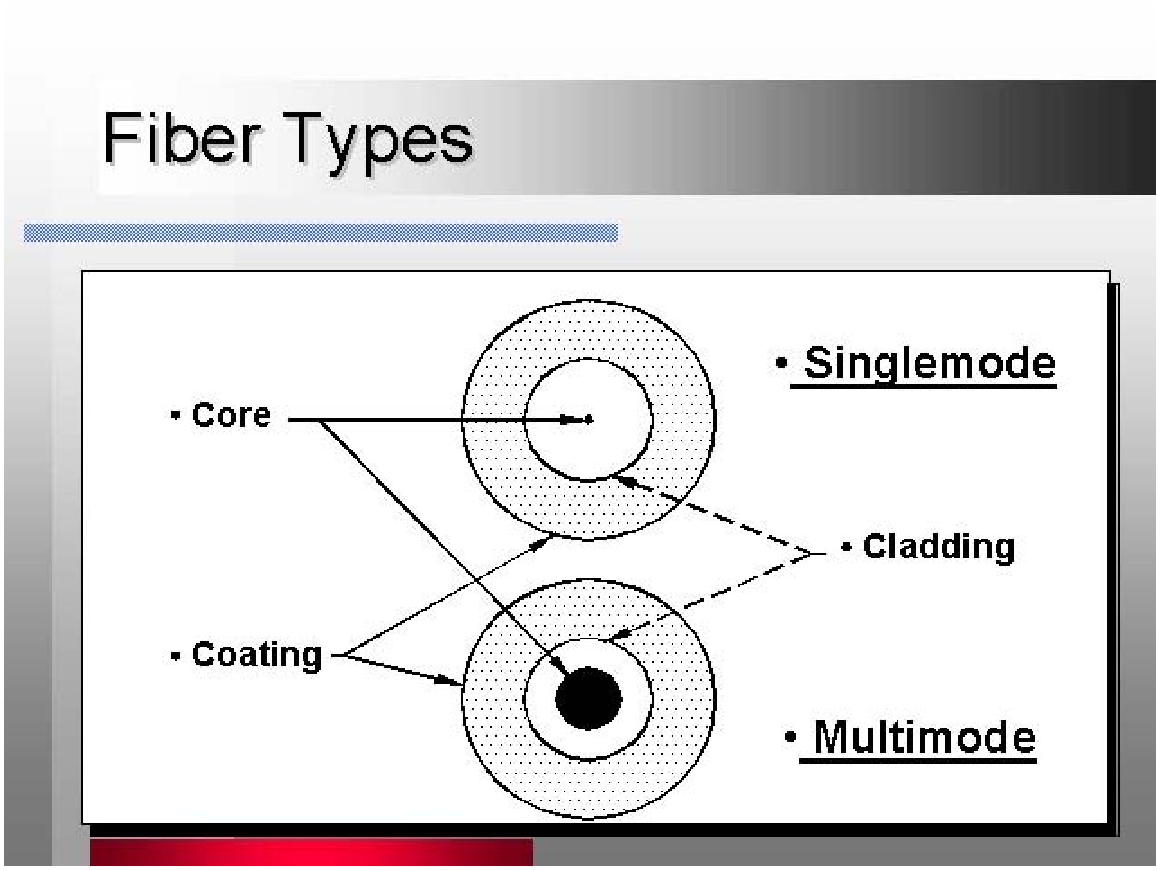
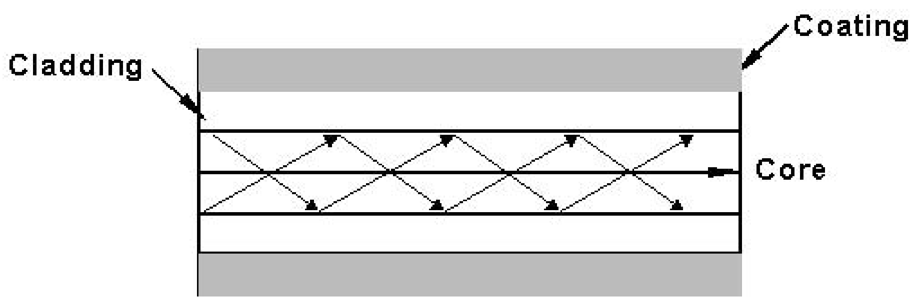


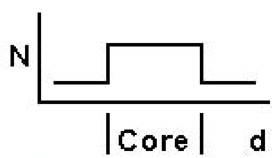
- Fiber bandwidth is measured in MHz x Km. A length of glass is measured for bandwidth. By convention the bandwidth specification for that fiber is the length of that fiber times the measured bandwidth for that fiber.



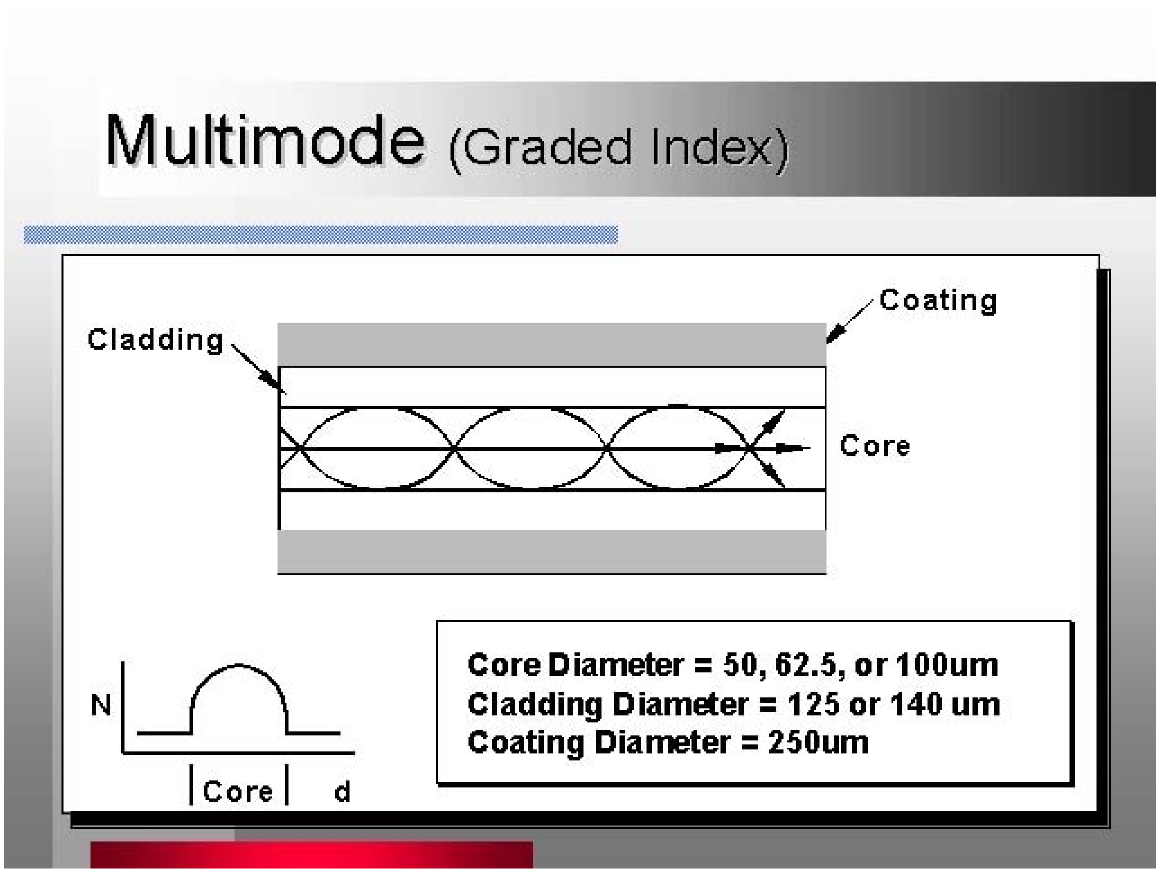
•Multimode fiber allows for more than one pathway or mode of light to travel in the fiber
•Singlemode fiber allows for only one pathway or mode of light to travel within the fiber at a specific operational wavelength
•It is impossible to distinguish between singlemode fiber and multimode with the naked eye

Multimode (Step Index)



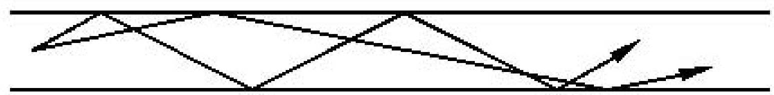


Core Diameter = 50, 62.5, or 100um
Cladding Diameter = 125 or 140 um
Coating Diameter = 250um

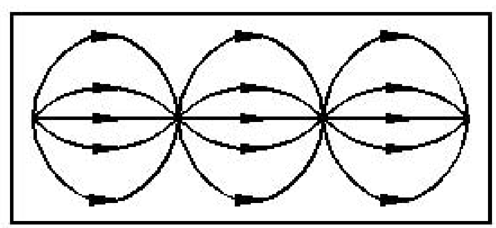


- Most commonly used fiber
- Reduces modal dispersion by equalizing the transit times among the modes
- The core is layered with the index of refraction increasing toward the center of the core

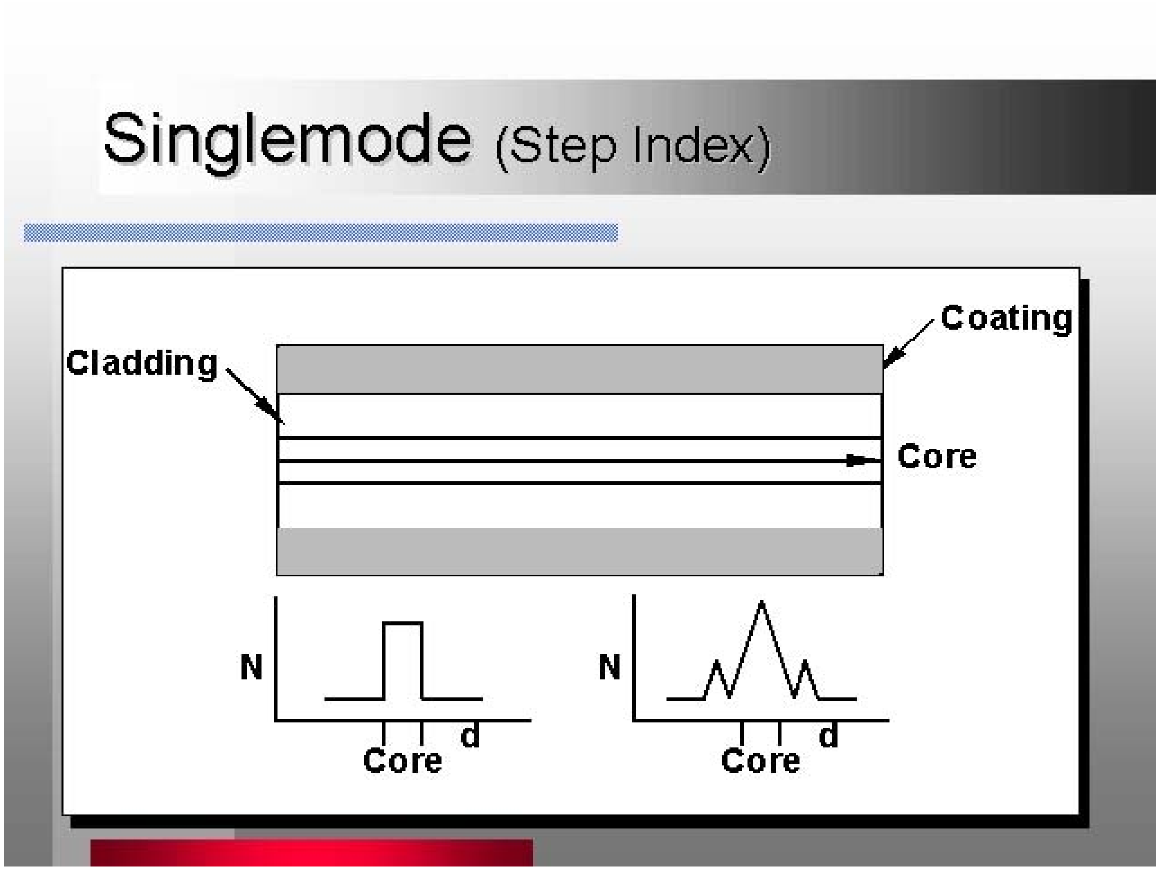
Multimode



Ray Path Step Index Fiber



Ray Path Grin Fiber



•In singlemode fiber, there is only one mode at a typical system wavelength; therefore, there is no modal dispersion. This results in much lower dispersion and more information carrying capacity

Wave Optics Approach

Reflection and Refraction of Plane Waves in Dielectric Media

Consider a uniform dielectric medium with permittivity $\epsilon(\omega)$ and permeability $\mu(\omega)$ and zero conductivity $\sigma = 0$

$$\mathbf{E}(\mathbf{r},t) = \Re\left[\mathbf{E}(\mathbf{r},\omega)e^{-i\omega t}\right] \ , \qquad \mathbf{B}(\mathbf{r},t) = \Re\left[\mathbf{B}(\mathbf{r},\omega)e^{-i\omega t}\right]$$

The harmonic electric and magnetic fields defined by

$$[\nabla^2 + \mu\epsilon\omega^2]\begin{pmatrix}\mathbf{E}(\mathbf{r},\omega)\\ \mathbf{B}(\mathbf{r},\omega)\end{pmatrix} = 0$$

obey the Helmholtz equations x

$$e^{ikx-i\omega t} \ , \qquad k = \sqrt{\mu\epsilon}\,\omega$$

A plane wave propagating in the direction

$$v = \frac{\omega}{k} = \frac{c}{n} \ , \qquad n = \sqrt{\frac{\mu\epsilon}{\mu_0\epsilon_0}}$$

has phase velocity and index of refraction

The index of refraction $n \sim 1.5$ in optical fiber is similar to that of glass

Energy Density and Flux

The energy flux in a mode of frequency ω is given by the real part of the complex Poynting vector

$$\mathbf{S}(\mathbf{r},\omega) = \frac{1}{2}\mathbf{E}\times\mathbf{H}^*$$

and the energy density

$$u(\mathbf{r},\omega) = \frac{1}{4}\left[\epsilon\mathbf{E}\cdot\mathbf{E}^* + \frac{1}{\mu}\mathbf{B}\cdot\mathbf{B}^*\right]$$

Boundary Conditions and Snell's Law

In a dielectric waveguide the most basic confinement mechanism is Total internal reflection from the interface between the core of the guide and the cladding surrounding it.

Consider a pane wave with wvevector k in the xz plane incident on the plane interface in the xy plane at z= 0

$$\mathbf{E} = \mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r} - i\omega t}, \qquad \mathbf{B} = \sqrt{\mu\epsilon}\frac{\mathbf{k}\times\mathbf{E}}{k}$$

The transmitted and reflected waves have wavevectors k' and k'' with magnitudes

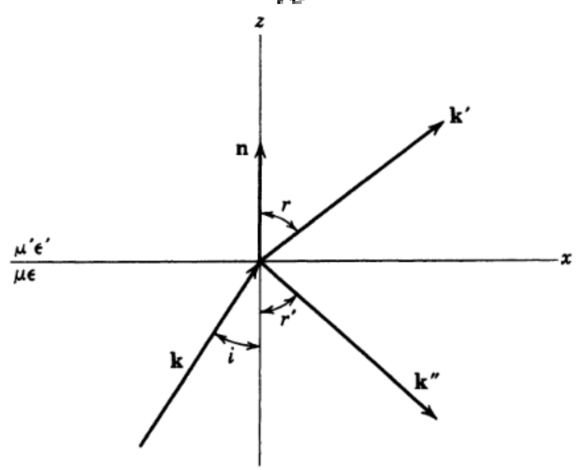


Figure 7.5 Incident wave **k** strikes plane interface between different media, giving rise to a reflected wave **k**″ and a refracted wave **k**′.

Slab Waveguide Theory

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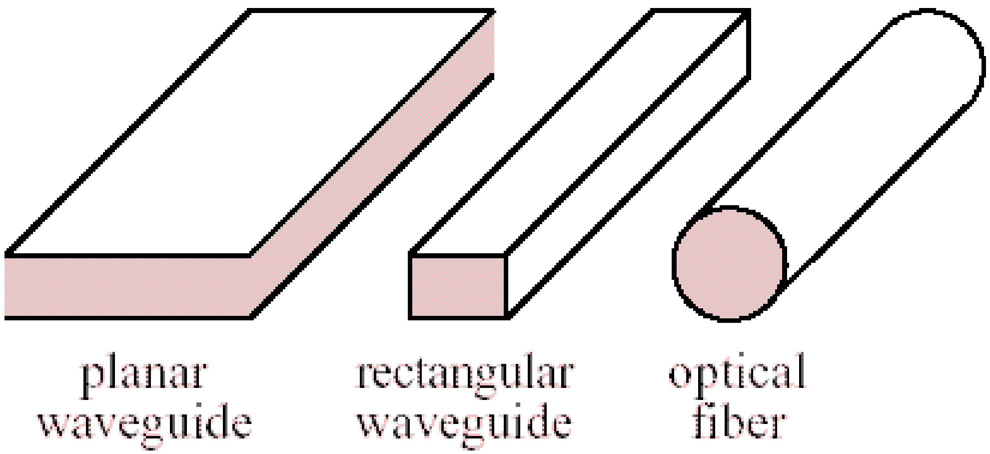
Light can be guided by planar or rectangular wave guides, or by optical fibers.



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E-M Field in a Planar Waveguide

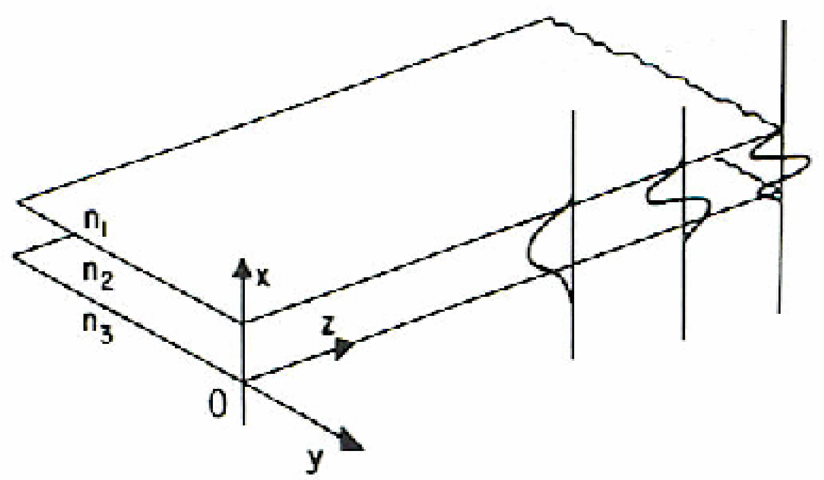


Fig. 2.1. Diagram of the basic three-layer planar waveguide structure. Three mode are shown, representing distributions of electric field in the x direction

Assuming a monochromatic wave propagating in z-direction

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}(\mathbf{r})e^{j\omega t} = \mathbf{E}(x,y)e^{-j\beta z}e^{j\omega t}$$

$$\nabla^2\mathbf{E}(\mathbf{r}) + k^2 n^2(\mathbf{r})\mathbf{E}(\mathbf{r}) = 0$$

Region I: $$\frac{\partial^2}{\partial x^2}E(x,y) + (k^2 n_1^2 - \beta^2)E(x,y) = 0$$

Region II: $$\frac{\partial^2}{\partial x^2}E(x,y) + (k^2 n_2^2 - \beta^2)E(x,y) = 0$$

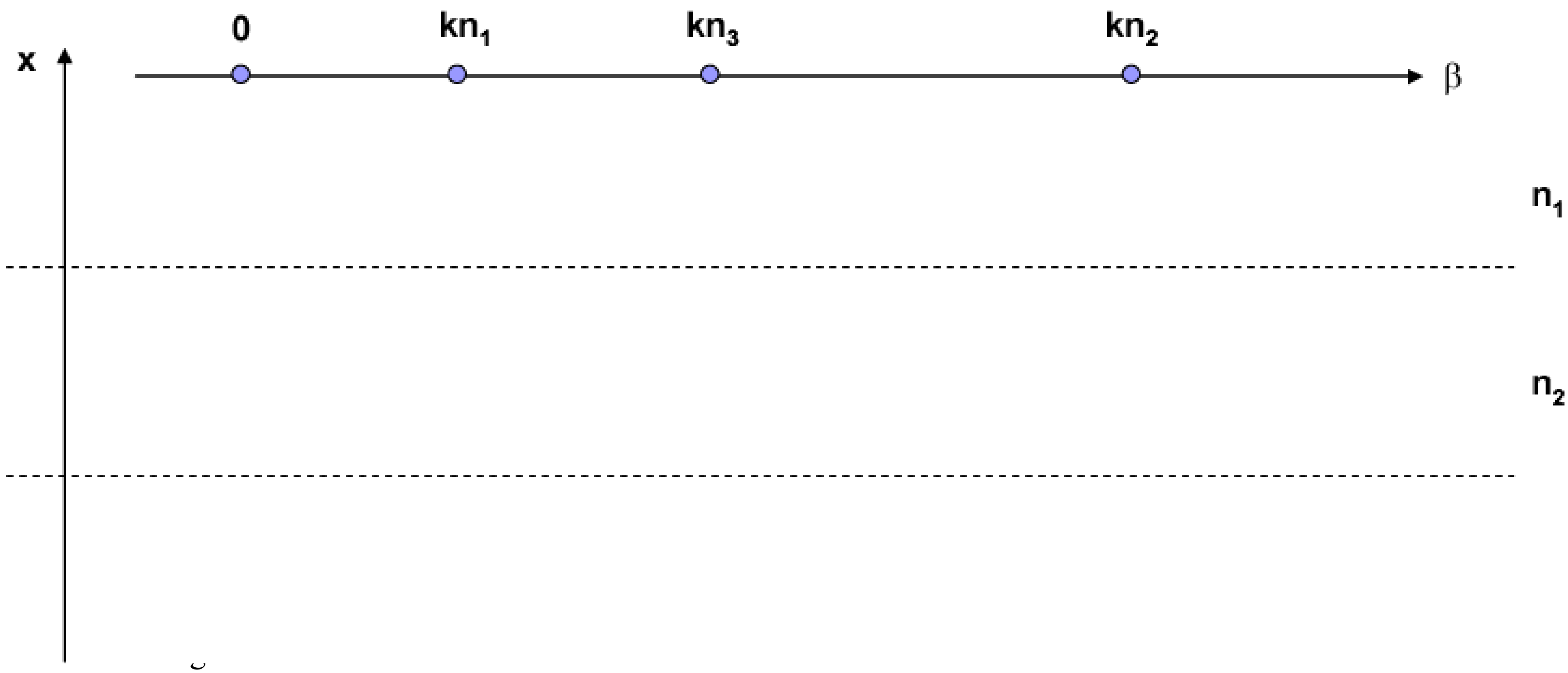
Region III: $$\frac{\partial^2}{\partial x^2}E(x,y) + (k^2 n_3^2 - \beta^2)E(x,y) = 0$$

Modes in a Planar Waveguide

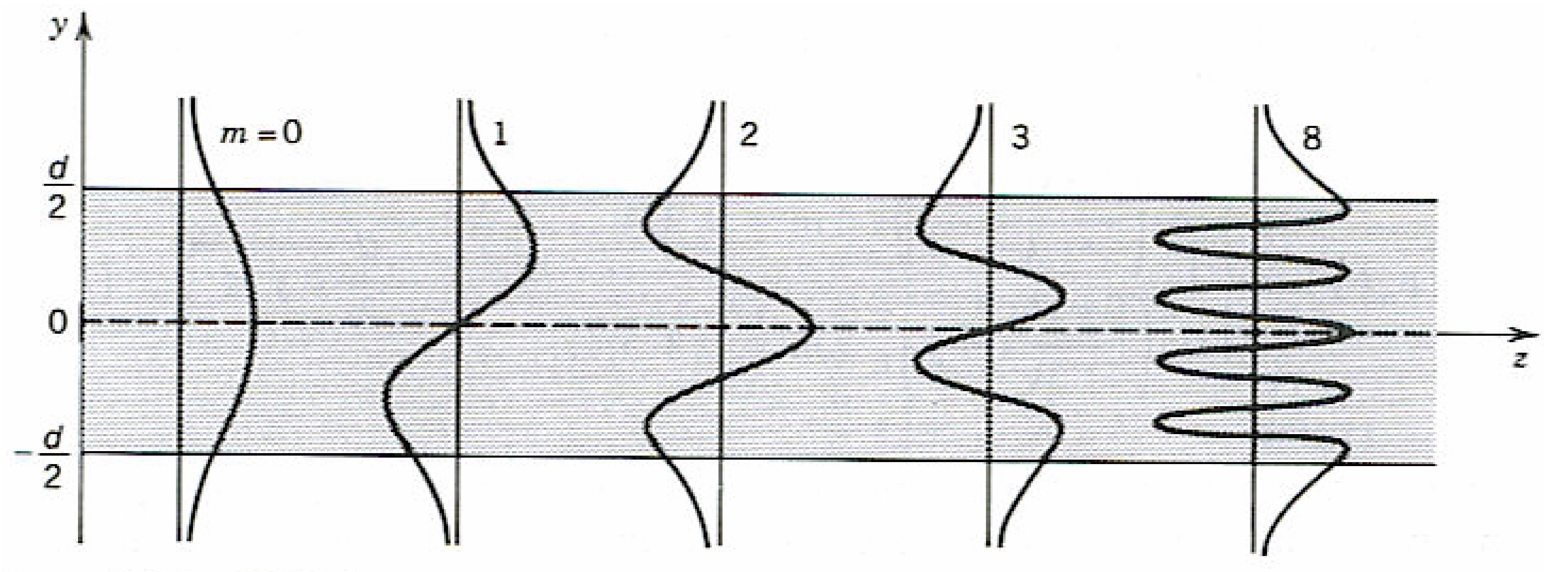
solutions are sinusoidal or exponential, depending on the sign of $(k^2 n_i^2 - \beta^2)$

Boundary conditions: $E(x, y)$ and $\dfrac{\partial E(x, y)}{\delta x}$ must be continuous at the interface between layers.

Assuming $n_2 > n_3 > n_1$, let's draw possible waveguide modes:



Guided Modes in a Planar Waveguide



m: Mode order

Only discrete values of are allowed in a waveguide.

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Experimental Observation of Waveguide Modes

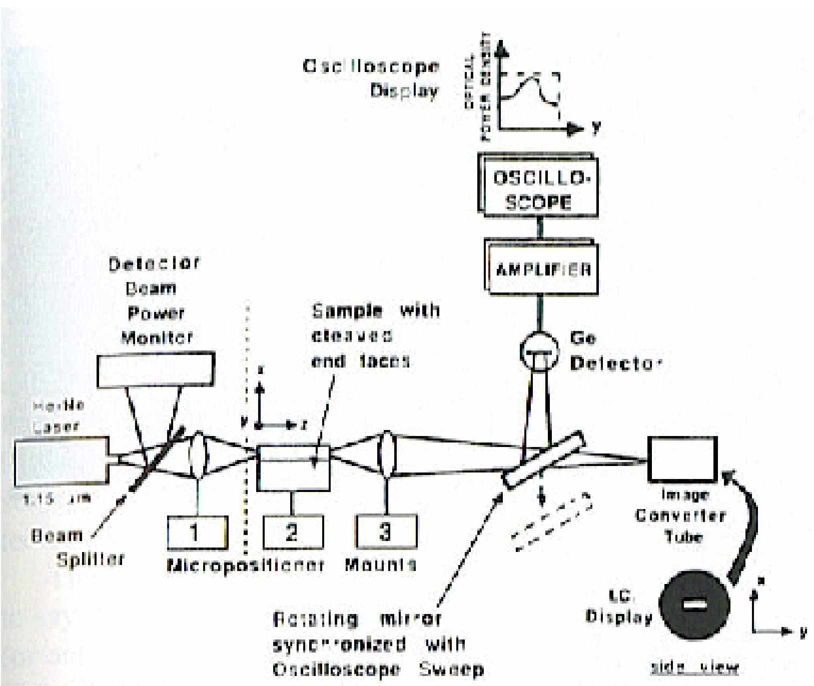
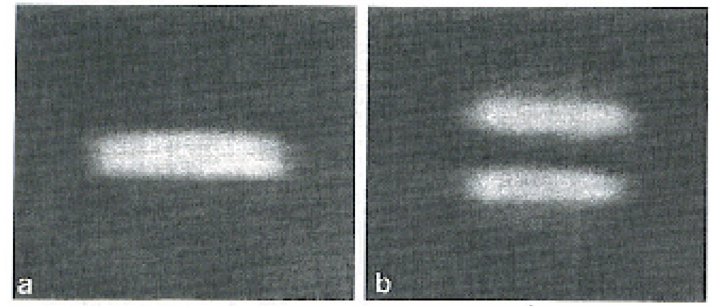


Fig. 2.3. Diagram of an experimental set-up than can be used to measure optical mode shapes [2.9]



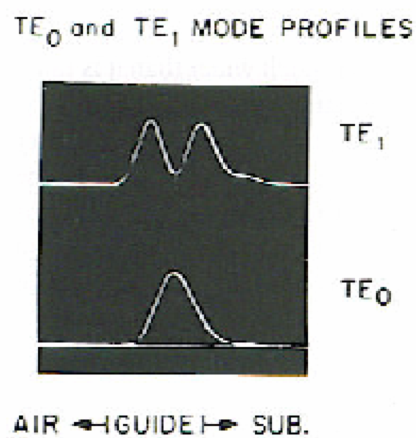


Fig. 2.5. Optical mode shapes are measured using the apparatus of Fig. 2.3. The waveguide in this case was formed by proton implantation into a gallium arsenide substrate to produce a 5 μm thick carrier-compensated layer [2.12]

Ray Patterns in the Three-Layer Planar Waveguide

In the guided region, $E \sim \sin(hx + \gamma)$

$$\beta^2 + h^2 = k^2 n_2^2$$

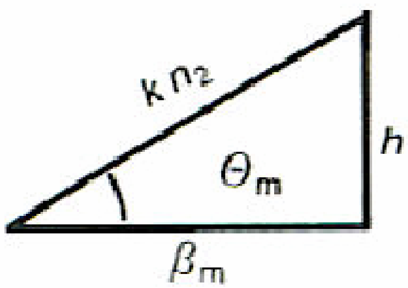


Fig. 2.9. Geometric (vectorial) relationship between the propagation constants of an optical waveguide

For the m-th mode, $\theta_m = \tan^{-1}\dfrac{h}{\beta_m}$

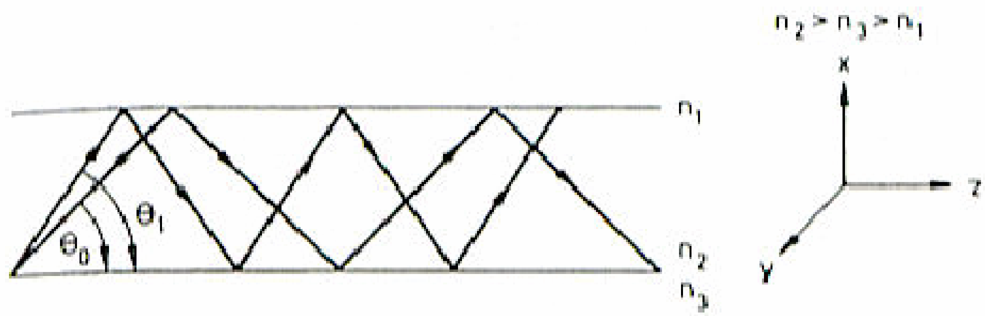
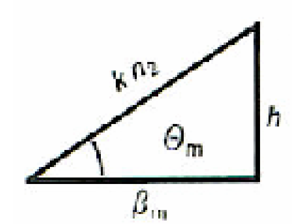


Fig. 2.8. Optical ray pattern within a multimode planar waveguide

Lower-order mode has smaller θ_m and larger β_m (propagating faster!) phasevelocity

Ray Patterns for Different Modes



$$\phi_2 = \sin^{-1}\frac{\beta}{kn_2} < \sin^{\ 1}\frac{n_1}{n_2}$$

$$\frac{n_1}{n_2} < \sin\phi_2 < \frac{n_3}{n_2}$$

$$\phi_2 > \sin^{-1}\frac{n_3}{n_2}$$

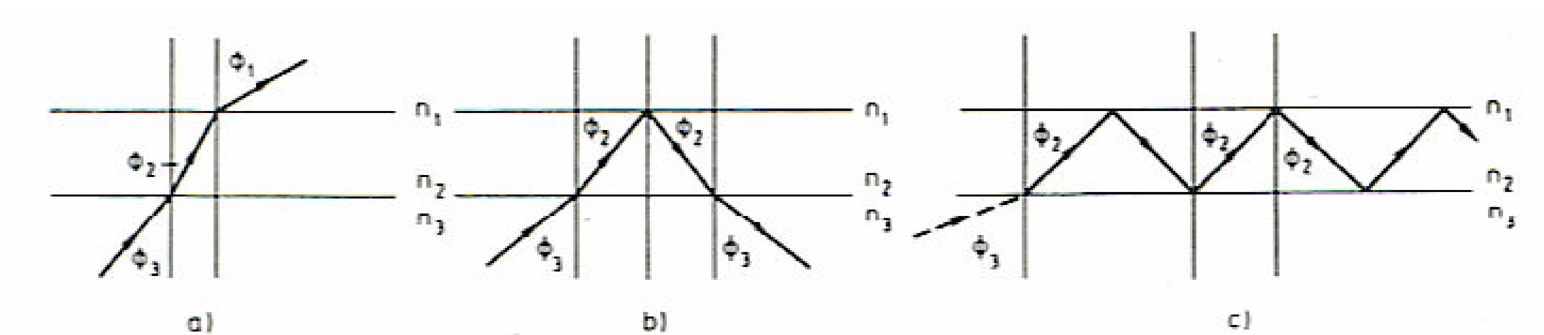


Fig. 2.10a–c. Optical ray patterns for a air radiation modes; **b** substrate radiation modes; **c** guided mode. In each case a portion of the incident light is reflected back into layer 3; however, that ray has been omitted from the diagrams

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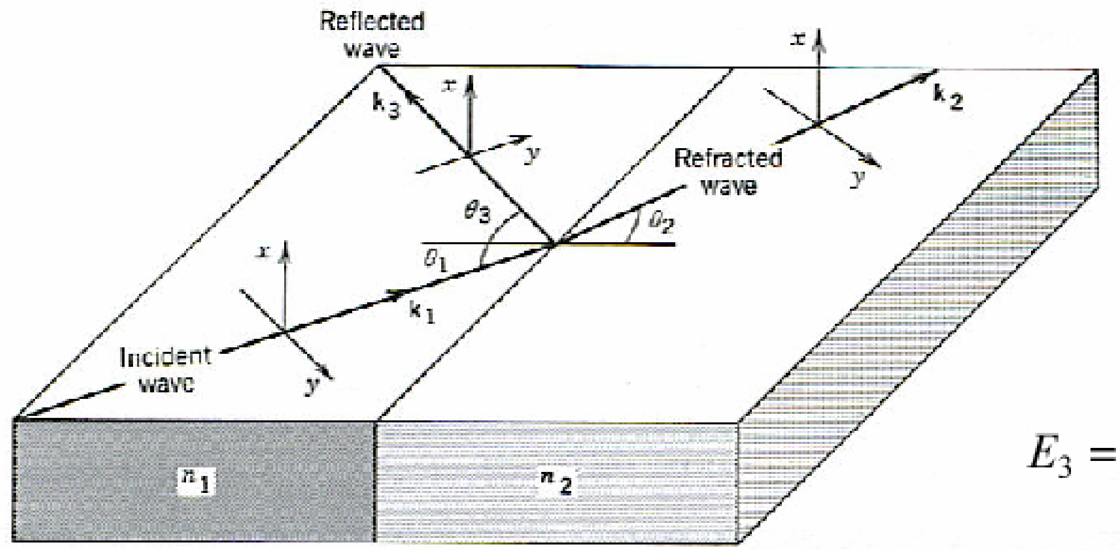
Reflection and Refraction



$$E_3 = rE_1, \quad E_2 = tE_1$$

For TE wave: $r_{TE} \quad \dfrac{n_1 \cos\theta_1 + n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$ $\qquad t_{TE} = 1 + r_{TE}$

For TM wave: $r_{TM} \quad \dfrac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_1 \cos\theta_2}$ $\qquad t_{TM} = \dfrac{n_1}{n_2}(1 + r_{TM})$

$$r_{TE} = |r_{TE}| \exp(j\phi_{TE}), \quad r_{TM} = |r_{TM}| \exp(j\phi_{TM})$$

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Total Internal Reflection for TE Wave

$$\tan\frac{\phi_{TE}}{2} \quad \frac{\sqrt{\sin^2\theta_1 - \sin^2\theta_c}}{\cos\theta_1} \quad \frac{\sqrt{n_1^2\sin^2\theta_1 - n_2^2}}{n_1\cos\theta_1}$$

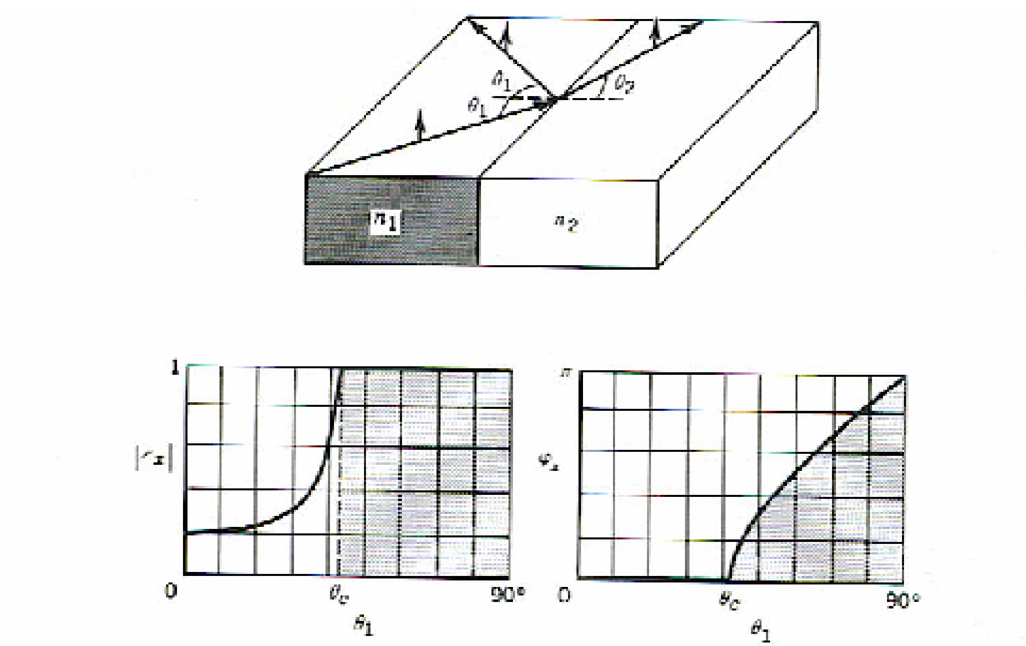

Figure 6.2-3 Magnitude and phase of the reflection coefficient for internal reflection of the TE wave ($n_1/n_2 = 1.5$).

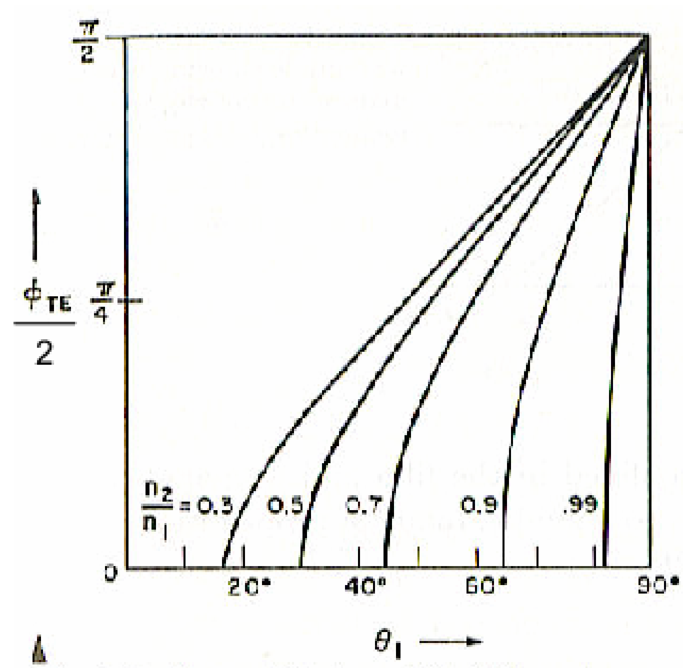

Fig. 2.3. Phase shift ϕ_{TE} of the TE mode as a function of the angle of incidence θ_1

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Total Internal Reflection for TM Wave

$$\tan\frac{\phi_{TM}}{2} = \frac{\sqrt{\sin^2\theta_1 - \sin^2\theta_c}}{\cos\theta_1 \sin^2\theta_c} = \frac{n_1^2}{n_2^2}\frac{\sqrt{n_1^2 \sin^2\theta_1 - n_2^2}}{n_1 \cos\theta_1}$$

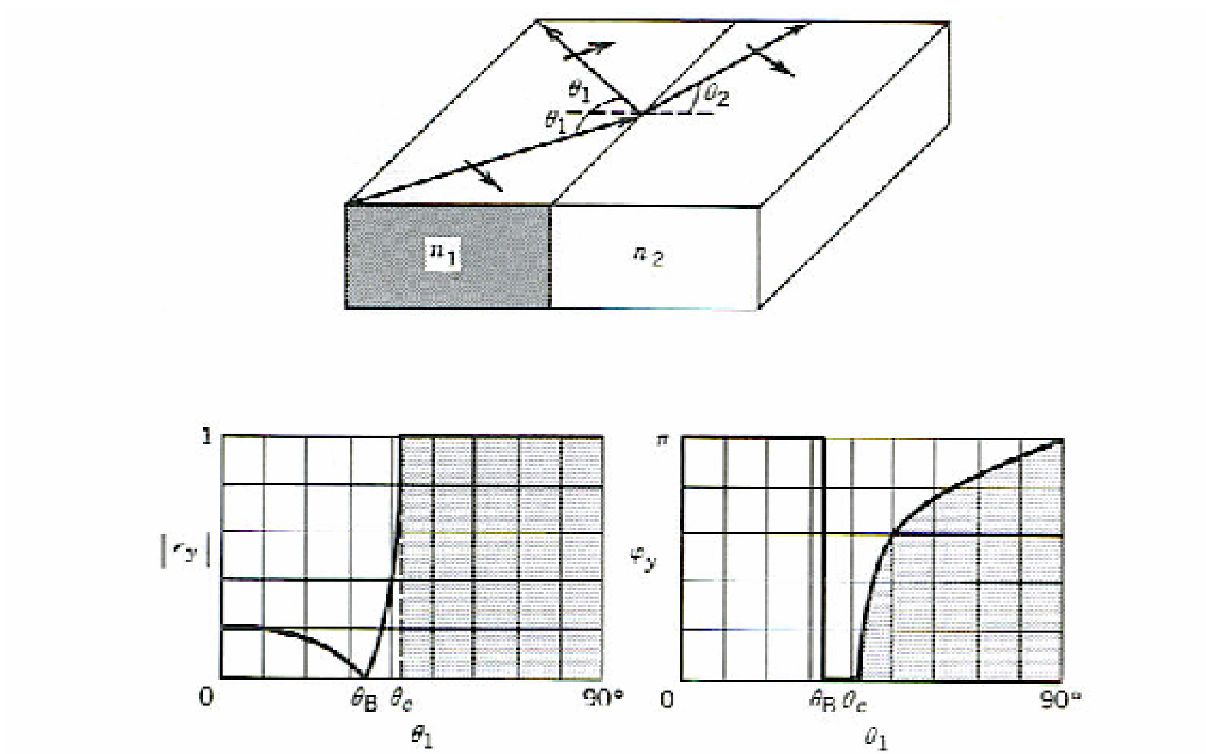

Figure 6.2-5 Magnitude and phase of the reflection coefficient for internal reflection of the TM wave ($n_1/n_2 = 1.5$).

Dispersion Equation

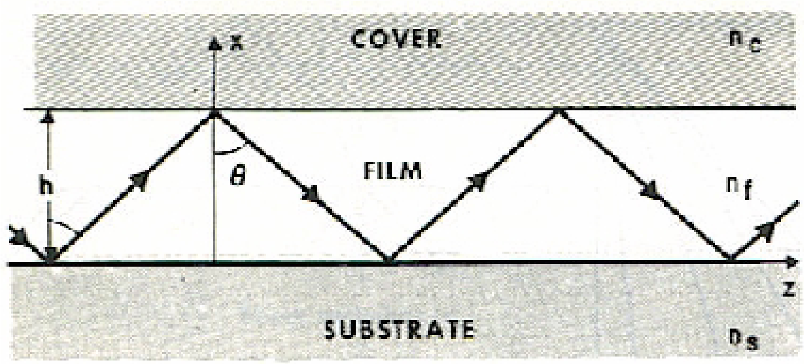

Fig. 2.5. Side-view of a slab waveguide showing wave normals of the zig-zag waves corresponding to a guided mode

Transverse resonance condition:

$$2kn_f h\cos\theta - 2\phi_c - 2\phi_s = 2m\pi$$

m : mode number

$kn_f h\cos$: phase shift for the transverse passage through the film

$2\phi_c (= \phi_{TE,TM})$: phase shift due to total internal reflection from film/cover interface

$2\phi_s (= \phi_{TE,TM})$: phase shift due to total internal reflection from film/substrate interface

Dispersion equation (β vs. ω):

$$kn_f h\cos\theta - \phi_c - \phi_s = m\pi$$

Effective guide index $N = \beta / k = n_f \sin\theta$ $n_s < N < n_f$

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Graphical Solution of the Dispersion Equation

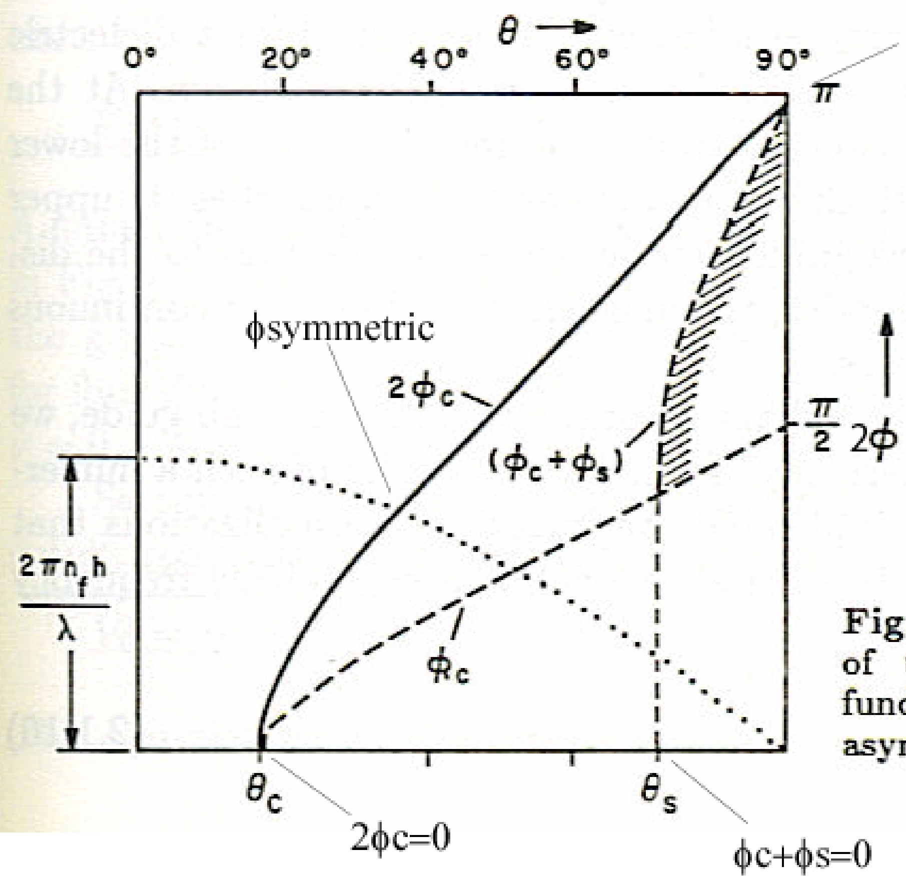
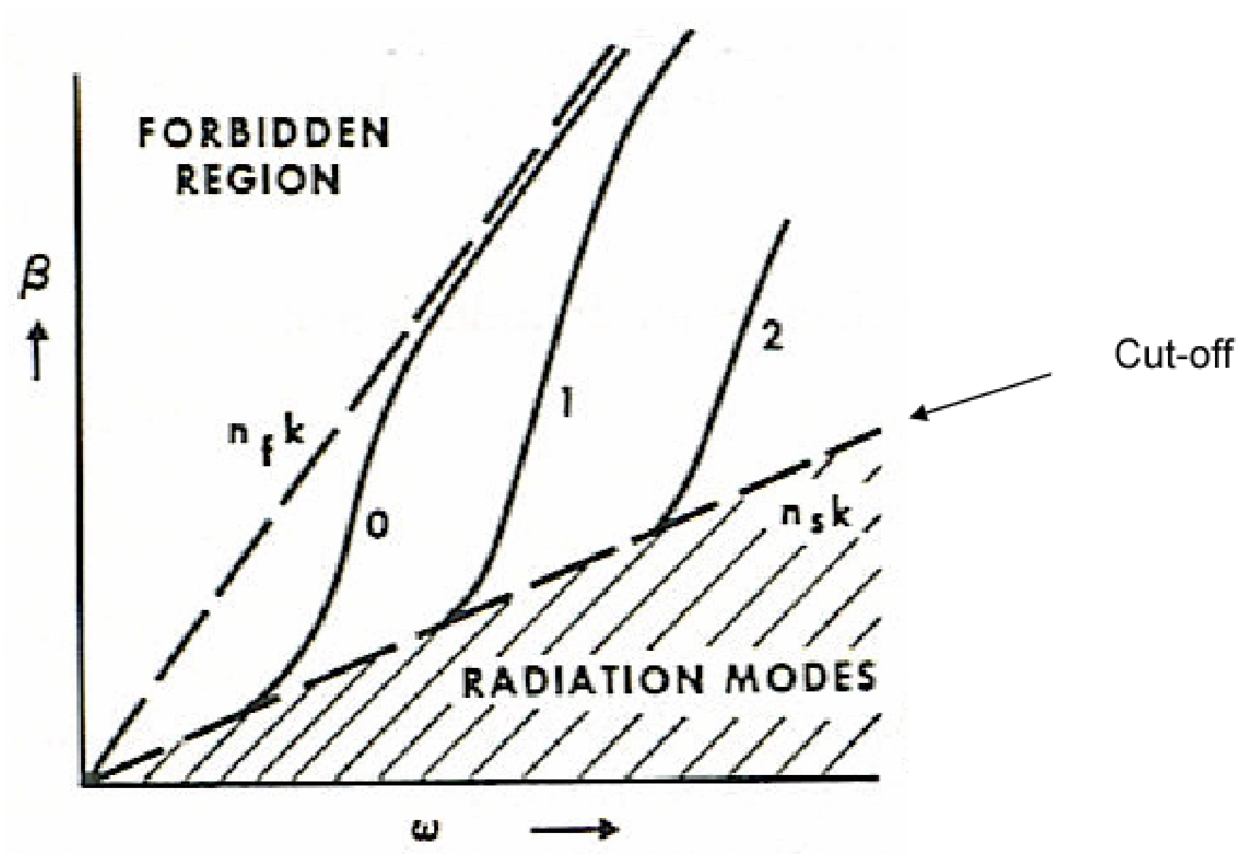


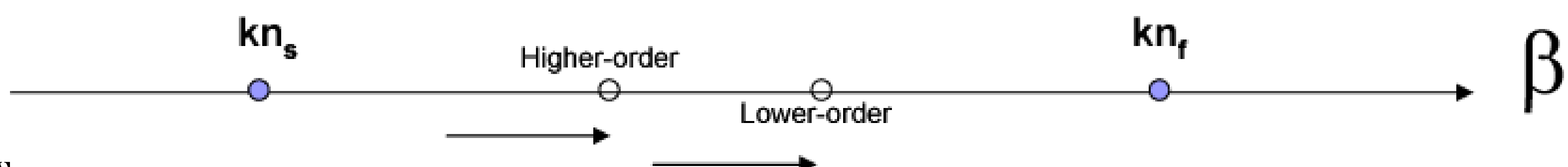
Fig. 2.6. Sketch of graphical solution of the dispersion equation. for the fundamental modes of symmetric and asymmetric slab waveguides

For fundamental mode (m = 0), there is always a solution (no cut-off) for symmetrical waveguide.

Increasing h (and/or decreasing λ) will support more modes.

Typical β – ω diagram





Numerical Solution for Dispersion Relation (I)

Define:

Normalized frequency and film thickness

$$V = kh\sqrt{n_f^2 - n_s^2}$$

Normalized guide index

$$b = \frac{N^2 - n_s^2}{n_f^2 - n_s^2}$$

b = 0 at cut-ooff ($N = n_s$), and approaches 1 as $N \rightarrow n_f$.

Measure for the asymmetry

$$a = \frac{n_s^2 - n_c^2}{n_f^2 - n_s^2} \text{ for TE,} \qquad a = \frac{n_f^4}{n_c^4}\frac{n_s^2 - n_c^2}{n_f^2 - n_s^2} \text{ for TM}$$

a = 0 for perfect symmetry ($n_s = n_c$), and a approaches infinity for strong asymmetry ($n_s \neq n_c$, $n_s \sim n_f$).

Table 2.2. Asymmetry measures for the TE modes (a_E) and the TM modes (a_M) of slab waveguides

Waveguide	n_s	n_f	n_c	a_E	a_M
GaAlAs, double heterostructure	3.55	3.6	3.55	0	0
Sputtered glass	1.515	1.62	1	3.9	27.1
Ti-diffused $LiNbO_3$	2.214	2.234	1	43.9	1093
Outdiffused $LiNbO_3$	2.214	2.215	1	881	21206

Numerical Solution for Dispersion Relation (II)

For TE modes, dispersion relation

$$kn_f h\cos\theta - \phi_c - \phi_s = m\pi \;\Rightarrow\; V\sqrt{1-b} = m\pi + \tan^{-1}\sqrt{\frac{b}{1-b}} \quad \tan^{-1}\sqrt{\frac{b+a}{1-b}}$$

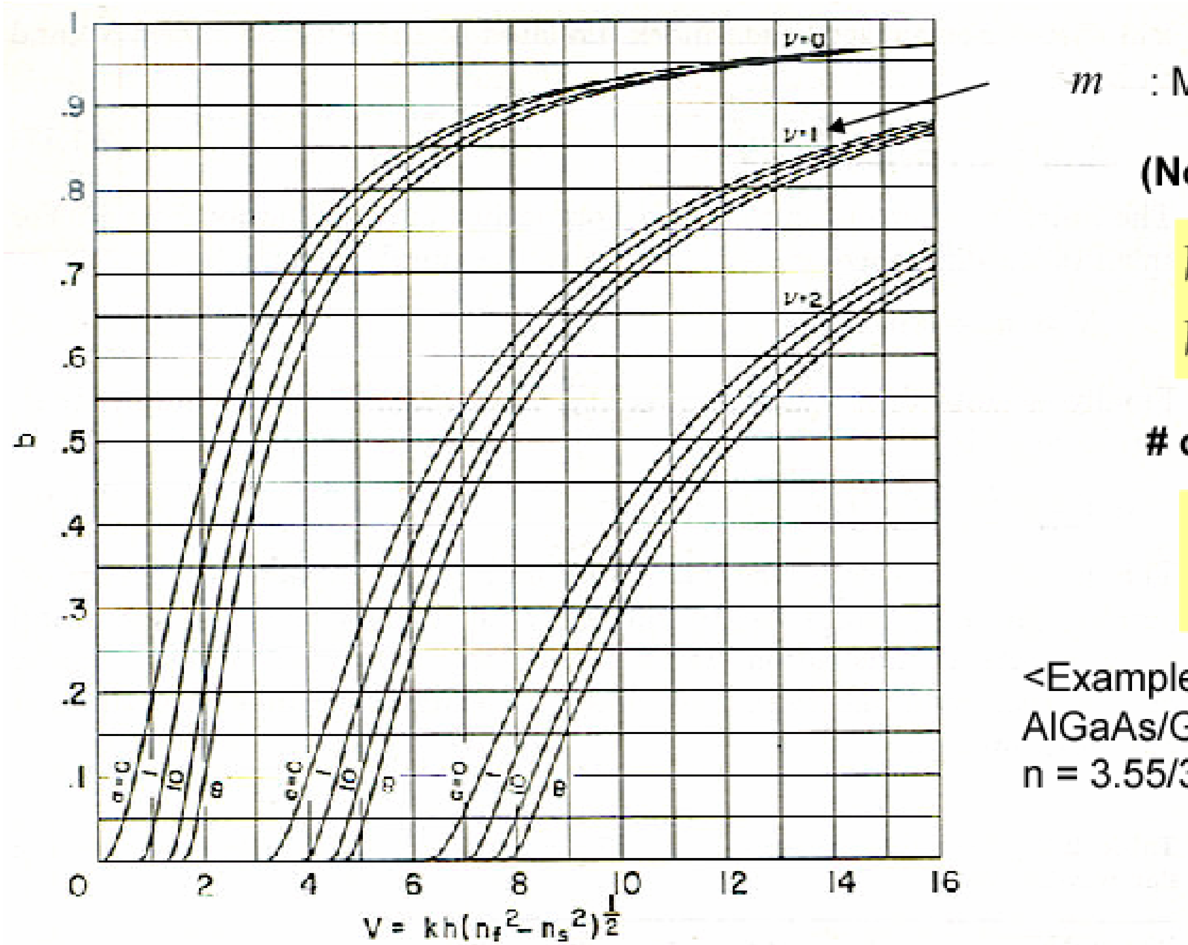


Fig. 2.8. Normalized ω-β diagram of a planar slab waveguide showing the guide index b as a function of the normalized thickness V for various degrees of asymmetry [2.20]

m : Mode number

(Normalized) cut-off frequency:

$$V_0 = \tan^{-1}\sqrt{a}$$
$$V_m = V_0 + m\pi$$

# of guided modes allowed:

$$m = \frac{2h}{\lambda}\sqrt{n_f^2 - n_s^2}$$

<Example>
AlGaAs/GaAs/AlGaAs double heterostructure
n = 3.55/3.6/3.55

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The Goos-Hänchen Shift

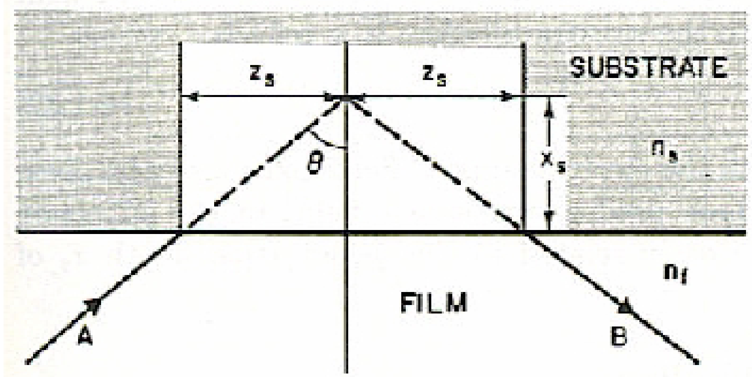

Fig. 2.9. Ray picture of total reflection at the interface between two dielectric media showing a lateral shift of the reflected ray (Goos-Hänchen shift)

For TE modes $$kz_s = (N^2 - n_s^2)^{-1/2} \tan\theta$$

For TM modes $$kz_s = \frac{(N^2 - n_s^2)^{-1/2} \tan\theta}{\frac{N^2}{n_s^2} + \frac{N^2}{n_f^2} - 1}$$

The lateral ray shift indicates a penetration depth:

$$x_s = \frac{z_s}{\tan\theta} \qquad z_s = \frac{d\phi_s}{d\beta}$$

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Effective Waveguide Thickness

Effective thickness

$$h_{eff} = h + x_s + x_c$$

Normalized effective thickness

$$H = kh_{eff}\sqrt{n_f^2 - n_s^2}$$

For TE modes

$$H = V + \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{b+a}}$$

Minimum H - > Maximum confinement

<Example> Sputtered glass, n_s = 1.515, n_f = 1.62, n_c = 1, a = 3.9

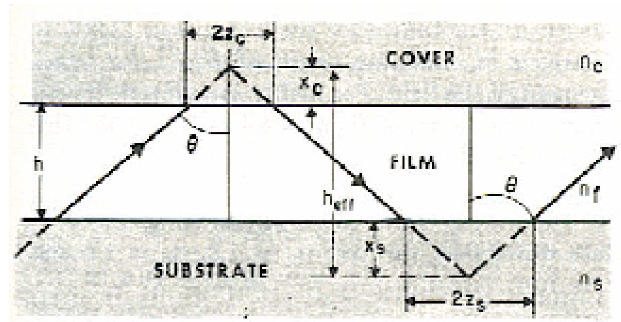

Fig. 2.10. Ray picture of zig-zag light propagation in a slab waveguide. Goos Hänchen shifts are incorporated in the model, and the effective guide thickness h_{eff} is indicated

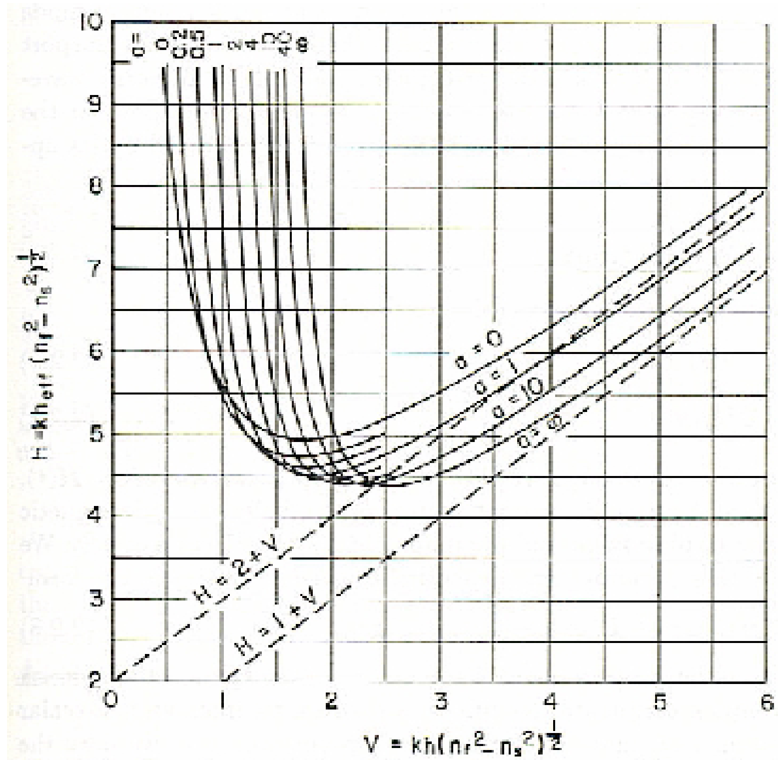

Fig. 2.11. Normalized effective thickness of a slab waveguide as a function of the normalized film thickness V for various degrees of asymmetry (after [2.20])

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Guided E-M Wave in a Planar Waveguide

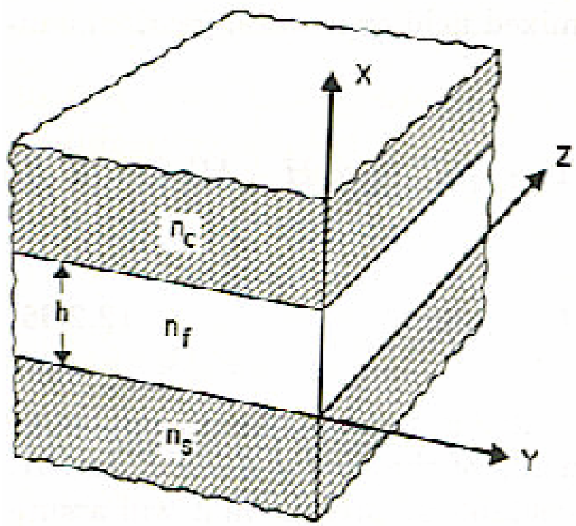

Fig. 2.14. Sketch of an "asymmetric" slab waveguide and the choice of the coordinate system. Note that the z-axis lies in the film-substrate interface

Define:

$$\kappa_c^2 = n_c^2 k^2 - \beta^2 = -\gamma_c^2$$

$$\kappa_f^2 = n_f^2 k^2 - \beta^2$$

$$\kappa_s^2 = n_s^2 k^2 - \beta^2 = -\gamma_s^2$$

Cover: $$\frac{\partial^2}{\partial x^2} E(x,y) + (n_c^2 k^2 - \beta^2) E(x,y) = 0 \Rightarrow \frac{\partial^2}{\partial x^2} E - \gamma_c^2 E = 0$$

Film: $$\frac{\partial^2}{\partial x^2} E(x,y) + (n_f^2 k^2 - \beta^2) E(x,y) = 0 \Rightarrow \frac{\partial^2}{\partial x^2} E - \gamma_f^2 E = 0$$

Substrate: $$\frac{\partial^2}{\partial x^2} E(x,y) + (n_s^2 k^2 - \beta^2) E(x,y) = 0 \Rightarrow \frac{\partial^2}{\partial x^2} E - \gamma_s^2 E = 0$$

TE Modes (I)

Modal solutions are sinusoidal or exponential, depending on the sign of $(k^2 n_i^2 - \beta^2)$

Boundary conditions: The tangential components of **E** and **H** are continuous at the interface between layers. # E_y and $\partial E_y / x$ continuous at the interface.

For guided modes:

Cover: $$\frac{\partial^2}{\partial x^2} E_y - \gamma_c^2 E_y = 0 \Rightarrow E_y = E_c \exp[-\gamma_c (x - h)]$$

Film: $$\frac{\partial^2}{\partial x^2} E_y - \kappa_f^2 E_y = 0 \Rightarrow E_y = E_f \cos(\kappa_f x - \phi_s)$$

Substrate: $$\frac{\partial^2}{\partial x^2} E_y - \gamma_s^2 E_y = 0 \Rightarrow E_y = E_c \exp(\gamma_s x)$$

Applying boundary conditions, we obtain:

$$\tan\phi_s = \frac{\gamma_s}{\kappa_f}, \quad \tan\phi_c = \frac{\gamma_c}{\kappa_f}$$

$$\kappa_f h - \phi_s - \phi_c = m\pi$$ -> Dispersion relation

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TE Modes (II)

Relation between the peak fields:

$$E_f^2(n_f^2 - N^2) = E_s^2(n_f^2 - n_s^2) = E_c^2(n_f^2 - n_c^2)$$

E_c, E_f, and E_s can be determined by,

Optical power

$$P = \frac{1}{2}\int \mathrm{Re}\{\mathbf{E} \quad \mathbf{H}^*\} \; \hat{\mathbf{z}} dx$$

Optical confinement factor

$$\Gamma = \frac{\frac{1}{2}\int_0^h \mathrm{Re}\{\mathbf{E} \quad \mathbf{H}^*\} \cdot \hat{\mathbf{z}} dx}{\frac{1}{2}\int_{-\infty}^{\infty} \mathrm{Re}\{\mathbf{E} \quad \mathbf{H}^*\} \cdot \hat{\mathbf{z}} dx}$$

TM Modes

Cover: $\frac{\partial^2}{\partial x^2} H_y - \gamma_c^2 H_y = 0 \Rightarrow \; H_y = H_c \exp[-\gamma_c (x - h)]$

Film: $\frac{\partial^2}{\partial x^2} H_y + \kappa_f^2 H_y = 0 \Rightarrow \; H_y = H_f \cos(\kappa_f x - \phi_s)$

Substrate: $\frac{\partial^2}{\partial x^2} H_y - \gamma_s^2 H_y = 0 \Rightarrow \; H_y = H_c \exp(\gamma_s x)$

Boundary conditions: H_y and E_z continuous at the interface between the layers

$\Rightarrow \; H_y$ and $\frac{1}{n^2}\frac{\mathrm{d}H_y}{\mathrm{d}x}$ continuous at the interface between the layers

Applying boundary conditions, we obtain:

$$\tan\phi_s = \left[\frac{n_f}{n_s}\right]^2 \frac{\gamma_s}{\kappa_f}, \quad \tan\phi_c = \left[\frac{n_f}{n_c}\right]^2 \frac{\gamma_c}{\kappa_f}$$

$\kappa_f h - \phi_s - \phi_c = m\pi$ -> Dispersion relation

Relation between the peak fields: $H_f^2 \frac{(n_f^2 - N^2)}{n_f^2} = H_s^2 (n_f^2 - n_s^2)\frac{q_s}{n_s^2} = H_c^2 (n_f^2 - n_c^2)\frac{q_c}{n_c^2}$

$$q_s = \left[\frac{N}{n_f}\right]^2 + \left[\frac{N}{n_s}\right]^2 - 1, \quad q_c = \left[\frac{N}{n_f}\right]^2 + \left[\frac{N}{n_c}\right]^2 - 1$$

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Multilayer Stack Theory

Focusing on TE modes first,

$$U = E_y, \quad V = \omega \mu H_z$$

$$U = A\exp(-j\kappa x) + B\exp(j\kappa x)$$

$$V = \kappa\,[A\exp(-j\kappa x) - B\exp(j\kappa x)]$$

At x = 0,

$$U_0 = U(0), \quad V_0 = V(0)$$

$$\begin{bmatrix} U \\ V \end{bmatrix} = \begin{bmatrix} \cos(\kappa x) & \frac{j}{\kappa}\sin(\kappa x) \\ j\kappa\sin(\kappa x) & \cos(\kappa x) \end{bmatrix} \begin{bmatrix} U \\ V \end{bmatrix}$$

$$= \mathbf{M}\begin{bmatrix} U \\ V \end{bmatrix}$$

M: Characteristic matrix of the layer

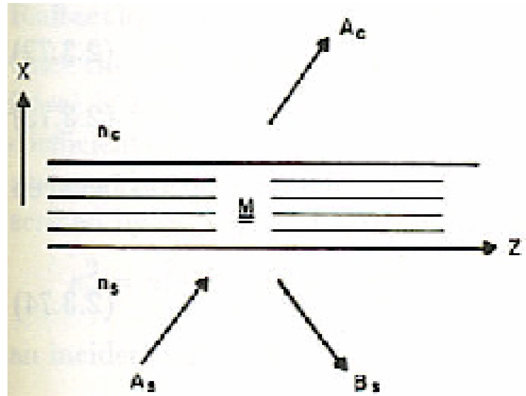

Fig. 2.15. Sketch of a multilayer stack waveguide with substrate index n_s and cover index n_c. The z-axis indicates the direction of mode propagation

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$$\mathbf{M}_i = \begin{bmatrix} \cos(\kappa_i h_i) & \frac{j}{_i}\sin(\kappa_i h_i) \\ j\kappa_i\sin(\kappa_i h_i) & \cos(\kappa_i h_i) \end{bmatrix}$$

$$\begin{bmatrix} U_0 \\ V_0 \end{bmatrix} = \mathbf{M}\begin{bmatrix} U_n \\ V_n \end{bmatrix}$$

$$\mathbf{M} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} = \mathbf{M}_1\ \mathbf{M}_2\ .\ \ \mathbf{M}_n$$

Dispersion Relation for Multilayer Slab Waveguide

Consider guided mode. For substrate and cover,

$$U = A\exp(\gamma x) + B\exp(-\gamma x)$$

$$V = j\gamma[A\exp(\gamma x) - B\exp(-\gamma x)]$$

In the substrate,

$$U_0 = A_s, \quad V_0 = j\gamma_s A_s$$

In the cover,

$$U_n = A_c, \quad V_n = -j\gamma_c A_c$$

Using the multilayer stack matrix theory, we obtain:

$$j(\gamma_s m_{11} + \gamma_c m_{22}) = m_{21} - \gamma_s \gamma_c m_{12}$$

-> ***Dispersion relation for multilayer slab waveguide***

<Example> Four-layer waveguides

Multilayer Stack Theory for TM Modes

$$U = H_y, \quad V = \omega\mu_0 E_z$$

$$U = A\exp(-j\kappa x) + B\exp(j\kappa x)$$

$$V = -\frac{\kappa}{n^2}[A\exp(-j\kappa x) - B\exp(j\kappa x)]$$

Therefore,

$$TE \geqslant TM \qquad \kappa \ \text{->} -[\ \frac{\kappa}{n^2}\ ]$$

Dispersion relation:

$$-j\left(m_{11}\frac{\gamma_s}{n_s^2} + m_{22}\frac{\gamma_c}{n_c^2}\right) = m_{21} - \frac{\gamma_s\gamma_c}{n_s^2 n_c^2}m_{12}$$

Characteristic matrix of the i-th layer:

$$\mathbf{M}_i = \begin{bmatrix} \cos(\kappa_i h_i) & -j\frac{n_i^2}{{}_i}\sin(\kappa_i h_i) \\ -j\frac{\kappa_i}{n_i^2}\sin(\kappa_i h_i) & \cos(\kappa_i h_i) \end{bmatrix}$$

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Rectangular Waveguide Geometries

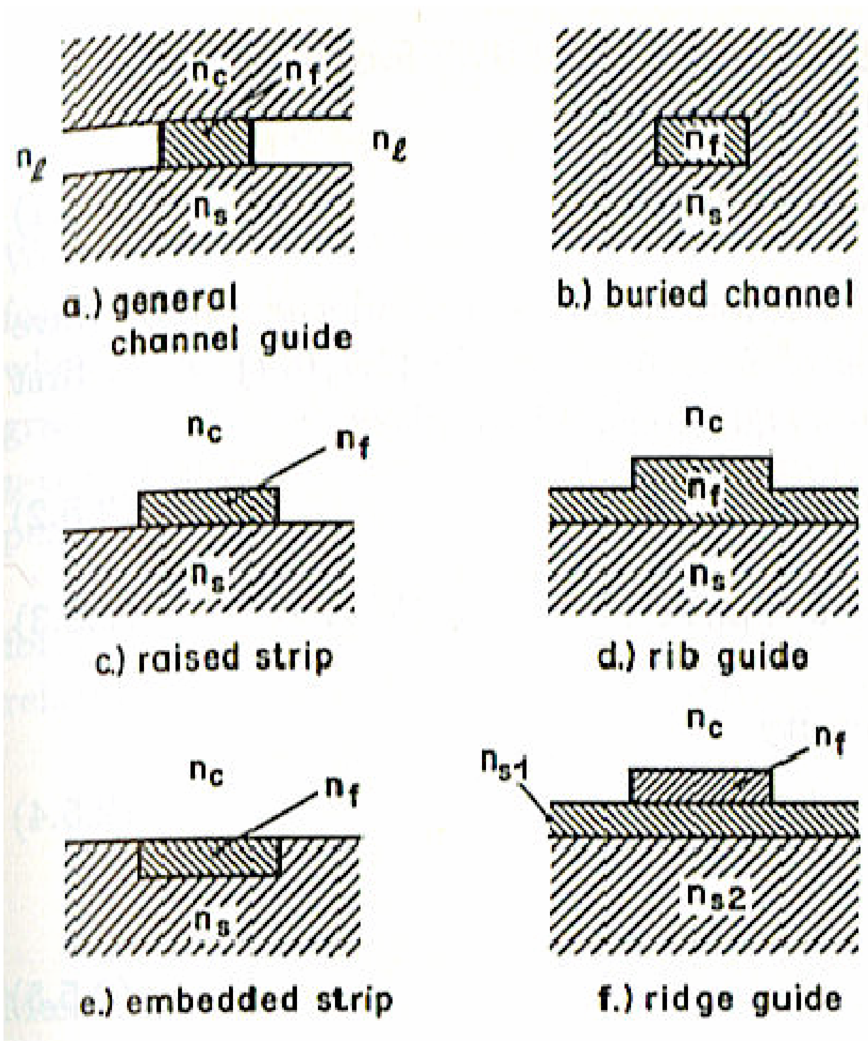

Fig. 2.24a–f. Cross-sections of six channel guide structures

The Method of Field Shadows (I)

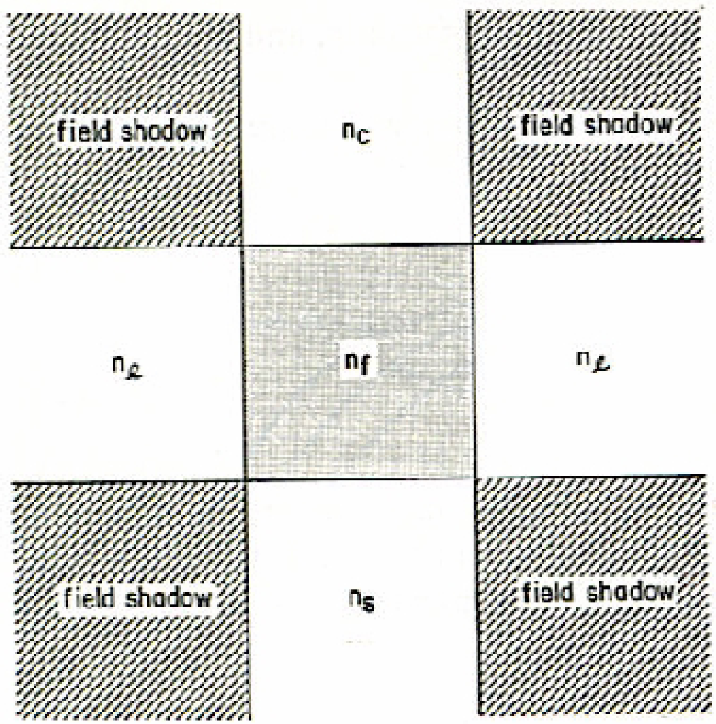

Fig. 2.26. Illustration of the method of field shadows showing the cross-section of a buried channel guide. The method ignores the fields in the shaded "shadow" areas

Ignore the fields and refractive indices in the shaded field shadow regions.

-> Results in separable index profiles.

Works well as long as the fields are well confined in the high index (n_f) region of the waveguide.

-> Not applicable at cut-off.

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The Method of Field Shadows (II)

Assuming a buried channel waveguide structure.

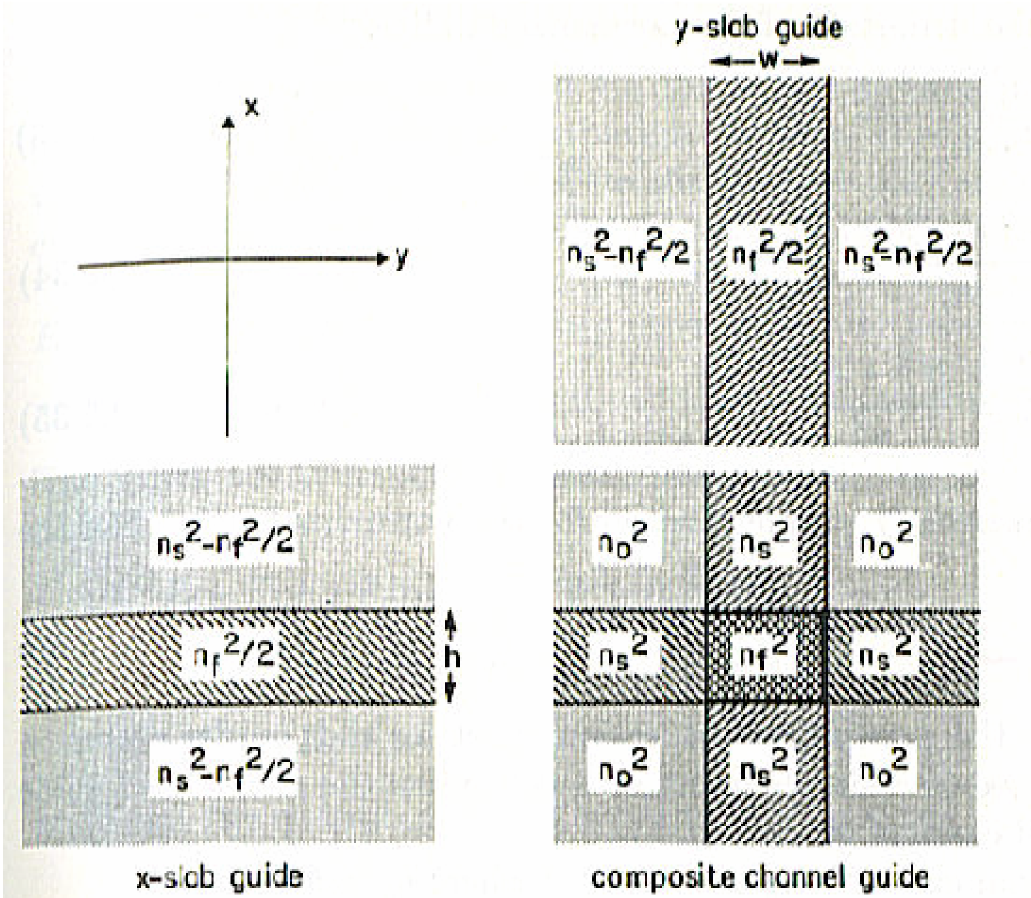


Fig. 2.27. Method of field shadows. The sketch shows the x-y cross-section of a composite guide made up by summing the permittivities (n^2) of an x-slab guide of height h and a y-slab guide of height w. The various n^2 values are indicated

$$E(x, y) = X(x)Y(y)$$

$$\beta^2 = \beta_x^2 + \beta_y^2 \qquad N^2 = N_x^2 + N_y^2$$

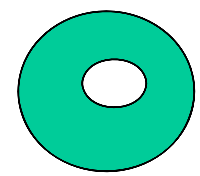
$$V_x = kh\sqrt{n_f^2 - n_s^2}$$

$$V_y = kw\sqrt{n_f^2 - n_s^2}$$

Obtain N_x and N_y, therefore N, by using the dispersion relation chart and

$$b_x = \frac{N_x^2 - n_s^2 + n_f^2/2}{n_f^2 \quad n_s^2}$$

$$b_y = \frac{N_y^2 - n_s^2 + n_f^2/2}{n_f^2 \quad n_s^2}$$



Or instead of solving for N_x and N_y, we can use

$$b = \frac{N^2 - n_s^2}{n_f^2 - n_s^2} = b_x + b_y - 1$$

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The Effective-Index Method

(1) Determine the normalized thickness of the channel and lateral guides.

$$V_f = kh\sqrt{n_f^2 - n_s^2}, \quad V_l = kl\sqrt{n_f^2 - n_s^2}$$

(2) Use the dispersion relation chart to determine the normalized guide indices b_f and b_l.

Determine the corresponding effective indices by referring to the Table on **Effective index for rectangular waveguide**

$$N_{f,l}^2 = n_s^2 + b_{f,l}(n_f^2 - n_s^2)$$

(3) Determine the normalized width. $V_{eq} = kw\sqrt{N_f^2 - N_l^2}$

Then determine the normalized guide index b_{eq} using the dispersion relation chart.

(4) The effective index of the waveguide can be determined from

$$b_{eq} = \frac{N^2 - N_l^2}{N_f^2 - N_l^2}$$

$$\Rightarrow \quad N^2 = N_l^2 + b_{eq}(N_f^2 - N_l^2)$$

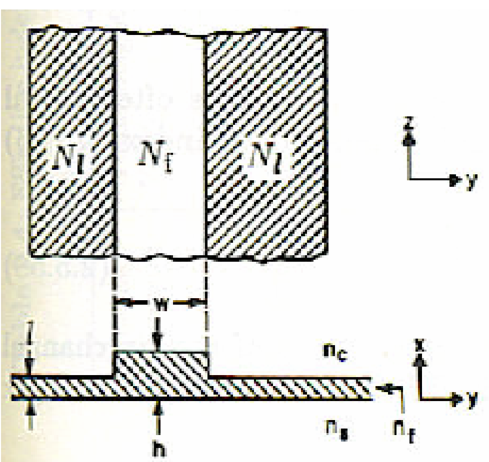

Fig. 2.28. Illustration of the effective-index method showing the top view and the cross section of a rib guide

Note: For multi-layer waveguide structure, such as ridge waveguides, use the matrix method to determine N_f and N_l, then continue on (3).

<Example> Ti:LiNbO$_3$, λ = 0.8 µm,
n_f = 2.234, n_s = 2.2 14n_c = 1,
h = 1 .8µm, l = " 1µm,

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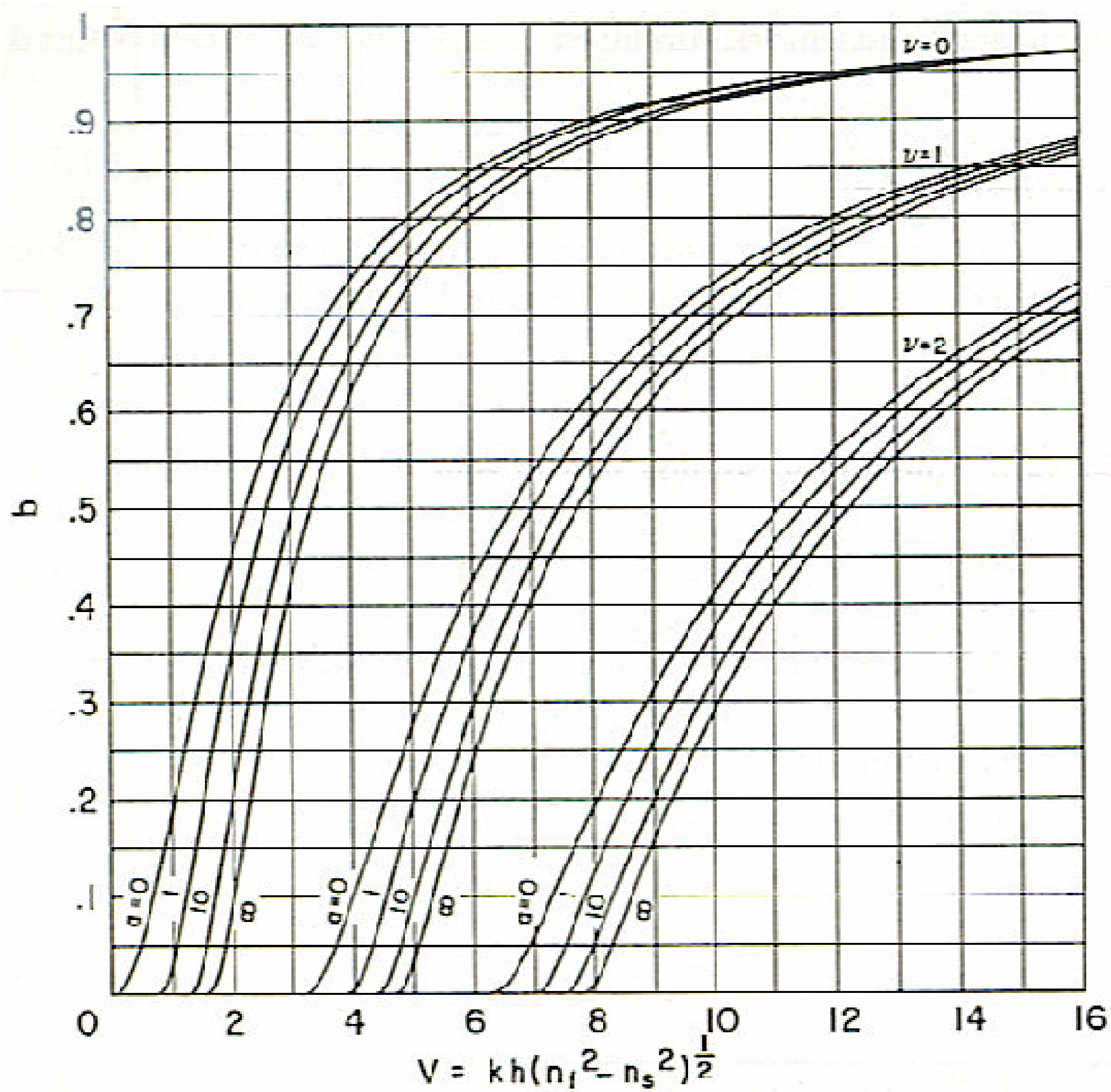
The Dispersion Relation Chart



Effective Index Parameters for Rectangular Waveguides

Channel structure	Guide height V_f, V_l	Eff. index N_f, N_l	$N_f^2 - N_l^2$	Channel guide index b
a) General	$V_f = kh\sqrt{n_f^2 - n_s^2}$ $V_l = kh\sqrt{n_l^2 - n_f^2}$	$N_f^2 = n_s^2 + b_f(n_f^2 - n_s^2)$ $N_l^2 = n_s^2 + b_l(n_l^2 - n_s^2)$	$b_f(n_f^2 - n_s^2) - b_l(n_l^2 - n_s^2)$	$b_f b_{eq} + b_l(1 - b_{eq})a_{ch}$
b) Buried	$V_f = kh\sqrt{n_f^2 - n_s^2}$	$N_f^2 = n_s^2 + b_f(n_f^2 - n_s^2)$ $N_l = n_s$	$b_f(n_f^2 - n_s^2)$	$b_f b_{eq}$
c) Raised	$V_f = kh\sqrt{n_f^2 - n_s^2}$	$N_f^2 = n_s^2 + b_f(n_f^2 - n_s^2)$ $N_l = n_c$	$(n_s^2 - n_c^2) + b_f(n_f^2 - n_s^2)$	$b_f b_{eq} - (1 - b_{eq})a$
d) Rib	$V_f = kh\sqrt{n_f^2 - n_s^2}$ $V_l = kl\sqrt{n_f^2 - n_s^2}$	$N_f^2 = n_s^2 + b_f(n_f^2 - n_s^2)$ $N_l^2 = n_s^2 + b_l(n_f^2 - n_s^2)$	$(b_f - b_l)(n_f^2 - n_s^2)$	$b_f b_{eq} + b_l(1 - b_{eq})$
e) Embedded	$V_f = kh\sqrt{n_f^2 - n_s^2}$	$N_f^2 = n_s^2 + b_f(n_f^2 - n_s^2)$ $N_l = n_s$	$b_f(n_f^2 - n_s^2)$	$b_f b_{eq}$
f) Ridge	$V_f = kh\sqrt{n_f^2 - n_s^2}$ $V_l = kl\sqrt{n_f^2 - n_s^2}$	$N_f^2 = n_{s1}^2 + b_f(n_f^2 - n_{s1}^2)$ $N_l^2 = n_{s2}^2 + b_l(n_{s1}^2 - n_{s2}^2)$	$(1 - b_l)(n_{s1}^2 - n_{s2}^2) + b_f(n_f^2 - n_{s1}^2)$	$b_{eq}(1 + b_f \cdot a_{ridge}) + b_l(1 - b_{eq})$

Numerical Comparisons Between Different Methods

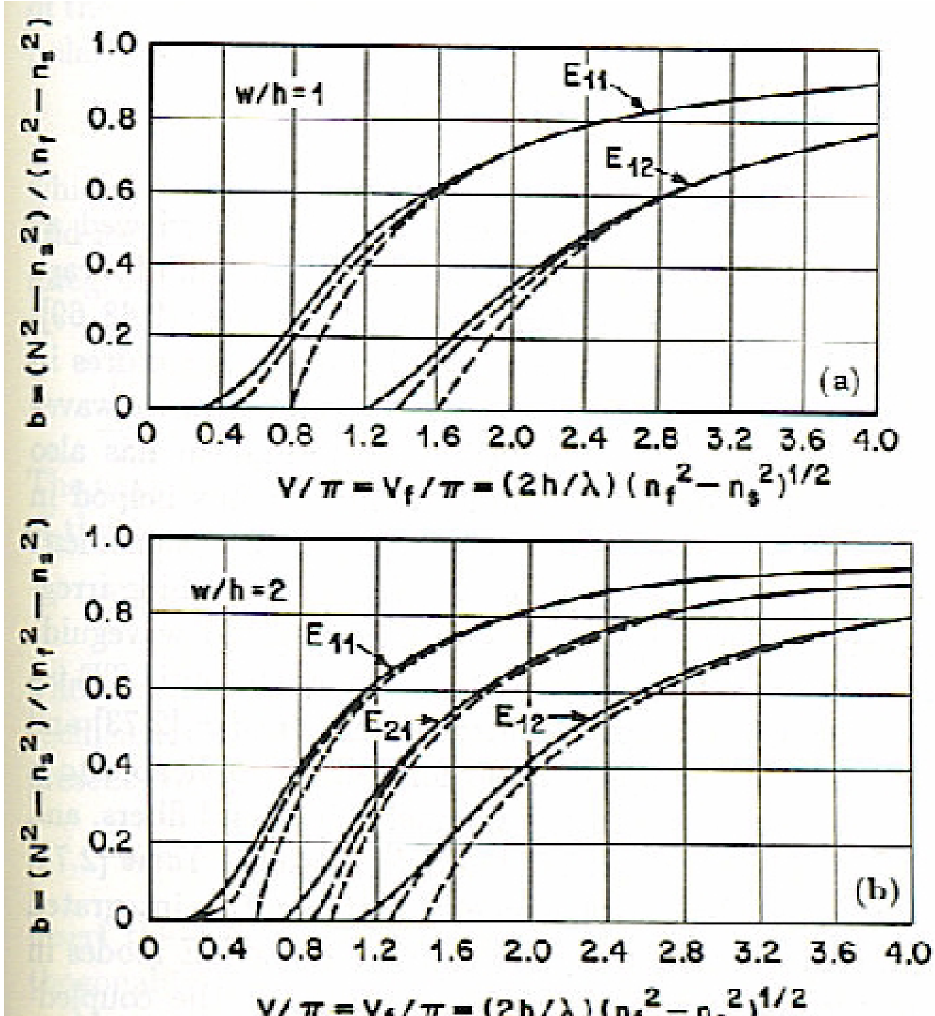


Fig. 2.29a,b. Normalized dispersion curves for a buried channel guide comparing the predictions of the numerical calculations (*dot-dashed lines*), of the effective index method (*solid lines*), and of the field-shadow method (*dashed lines*). Comparisons are shown for the aspect ratios of $w/h = 1$ and $w/h = 2$. (After [2.66])

Effective index method provides good approximation even near cut-off.

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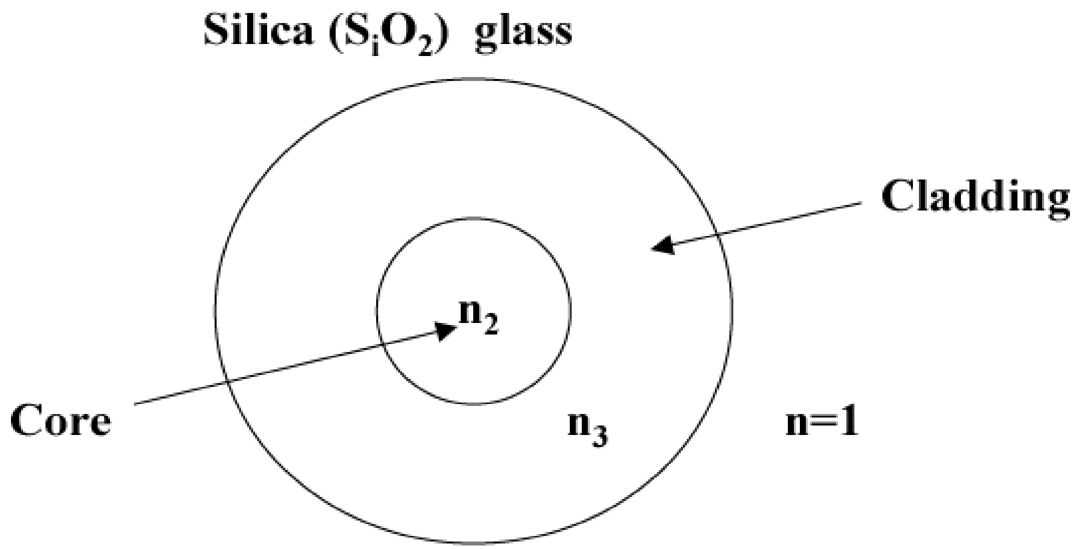
Optical Fiber



Impurities

a) Increase n of core

OR

b) Decrease n of cladding

Increase	Decrease
Ge	B
F	F
Na-B	

Wave Analysis:

Cylindrical dielectric waveguide
(step fiber)

assume all fields proportional to $e^{j(\omega t-\beta z)}$

$$E = (E_r, E_\phi, E_z)$$
$$H = (H_r, H_\phi, H_z)$$

E_i and H_i are function of (r, ϕ)

$$\nabla^2 \begin{pmatrix} E \\ H \end{pmatrix} = \omega^* \mu\varepsilon \begin{pmatrix} E \\ H \end{pmatrix}$$

But now need to use cylindrical coordinates:

$$d^2E_z/dr^2 + 1/r\ dE_z/dr + 1/r^2\ d^2E_z/d\phi^2 + (n_1{}^2k^2 - \beta^2)E_z = 0$$

Assume E_z proportional to $E(r)\,h(\phi)$ separation of variables

Since $h(2p + \phi) = h(\phi)$ =>try $h(\phi) =$ $\sin l\,\phi$

$\cos(l\,\phi)$

$e^{jl\phi}$

where l= integers

Substitute back into

$d^2E_z/dr^2 + 1/r\, dE_z/dr + [(n_1^2k^2 - \beta^2) - l^2/r^2]E_z = 0$ => Bessel function

Solutions closer to match physical situation.

For guided solutions:

In core, solutions must be finite
In cladding, solutions must approach 0 as r --> ∞

For r < a: $E(r) \propto J_l\,(UR)$ “ Bessel function of 1st kind”

For r > a: $E(r) \propto K_l\,(WR)$ “ modified Bessel function of 2nd kind”

$$UR = (n_1^2k^2 - \beta^2)^{0.5}\, r = \underset{U}{a(n_1^2k^2 - \beta^2)^{0.5}}\ \underset{R}{r/a}$$

$$WR = (\beta^2 - n_1^2k^2)^{0.5}\, a$$

Let $V^2 = U^2 + W^2 = a^2\,[n_1^2k^2 - \beta^2 + \beta^2 - n_2^2k^2] = a^2k^2[n_1^2 - n_2^2]$

$$\therefore\ V = a \cdot (2\pi/\lambda)\,[n_1^2 - n_2^2]^{0.5} \quad \text{(Normalized frequency)}$$
$$= a \cdot (2\pi/\lambda) \cdot NA$$

Solution procedure for step-index fiber modes:

1.

$$\left\{\begin{array}{l} E_z \\ H_z \end{array}\right. = A\, J_l\,(UR)\, e^{jl\phi} e^{j(\omega t-\beta z)} \qquad r < a$$

$$= B\, K_l\,(WR)\, e^{jl\phi} e^{j(\omega t-\beta z)} \qquad r > a$$

2. Match E_z and H_z at $r = a$

3. Use Maxwell's curl equations to find E_θ and H_θ E_z and H_z and E_θ and H_θ must match for $r = +a$ and $-a$. Solve all four equations simultaneously to yield eigenvalues

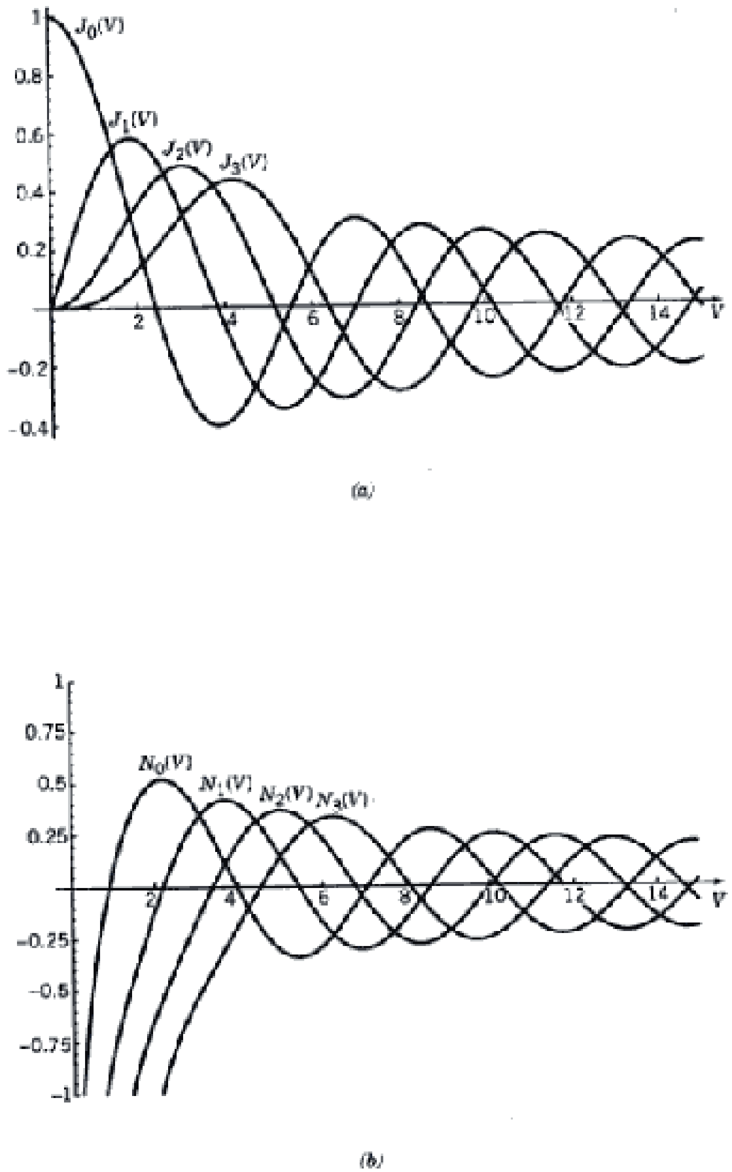


Figure 3.3. Ordinary Bessel functions.

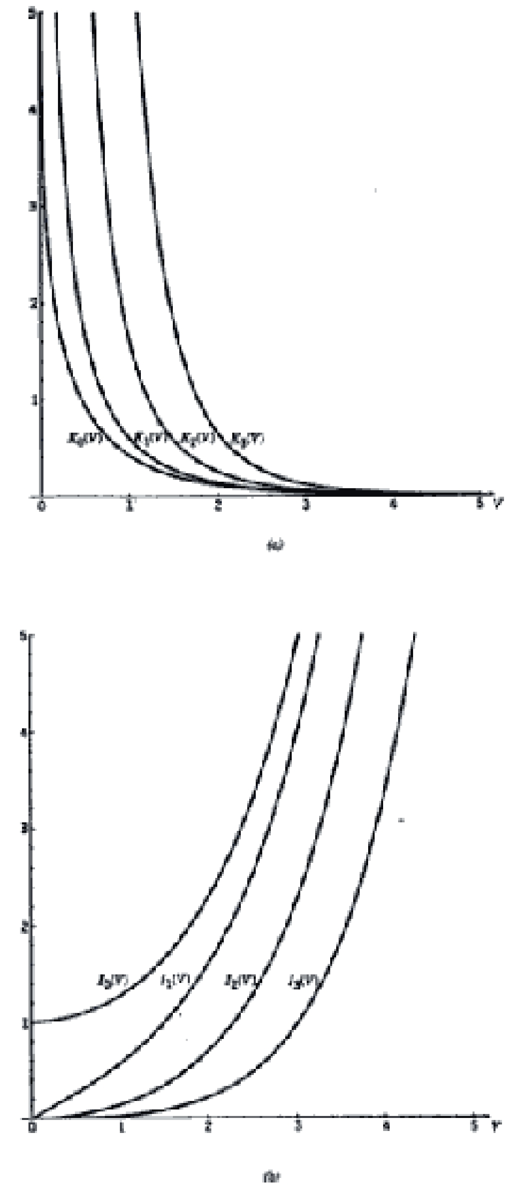


Figure 3.4. Modified Bessel functions.

A major simplification in math results if $(n_1 - n_2)/n_1 \ll 1$
(weakly-guiding approximation $\Delta \ll 1$)

The eigenequations reduces to

$$J_{l\pm 1}(U) / J_l(U) = +(W/U)\,(K_{l\pm 1}(W) / K_l(W) \quad (+ \text{ only for } l = 0)$$

There are *m* possible solutions for each value of *l*

$\therefore$ U_{lm} are solutions

From definition of U, knowing U_{lm} permits calculation of β

$$\beta_{lm} = (n_1^{\,2} k^2 - U_{lm})^{0.5}$$

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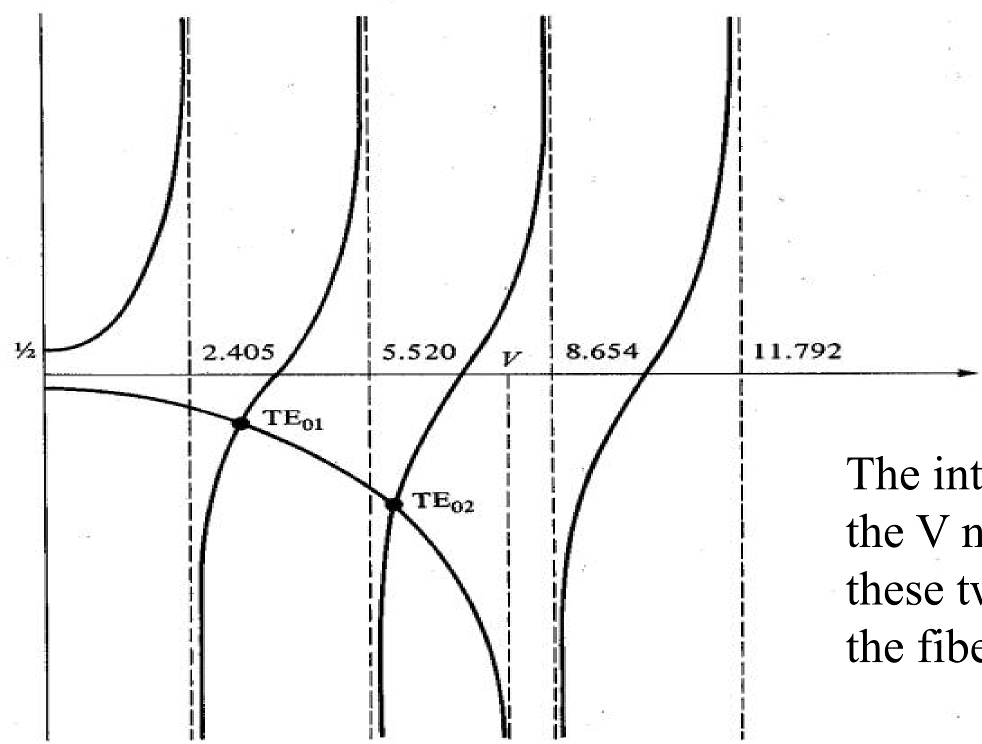
The resulting system of equations can only be solved graphically. The graphical solutions represent the mode cutoffs for the different modes that can propagate in the fiber for any given V, where V is a convenient parameter determined by the properties of the fiber and wavelength of incident light.

$$V = 2*\pi/\lambda*a*NA$$



The intersections represent the V numbers at which these two modes turn on in the fiber.

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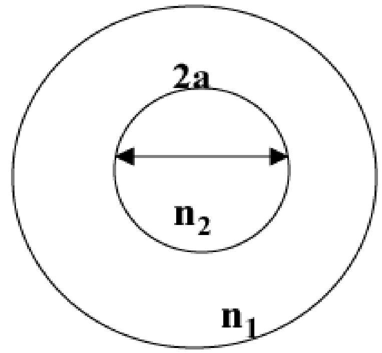
Normalization Parameters for Cylindrical Waveguides

Normalized frequency



$$V=\frac{2\pi a}{\lambda}\sqrt{n_2^2 - n_1^2}$$

$$\Delta = \frac{n_2^2 - n_1^2}{2n_2^2} \cong \frac{n_2 - n_1}{n_2}$$

$$V \cong \frac{2\pi a}{\lambda} n_2 \sqrt{2\Delta}$$

Normalized propagation constant

$$b = \frac{\left(\beta / k_0\right)^2 - n_1^2}{n_2^2 - n_1^2}$$

$$b = \frac{\beta / k_0 - n_1}{n_2 - n_1}$$

$$b\ \max = 1$$

$$b\ \min = 0$$

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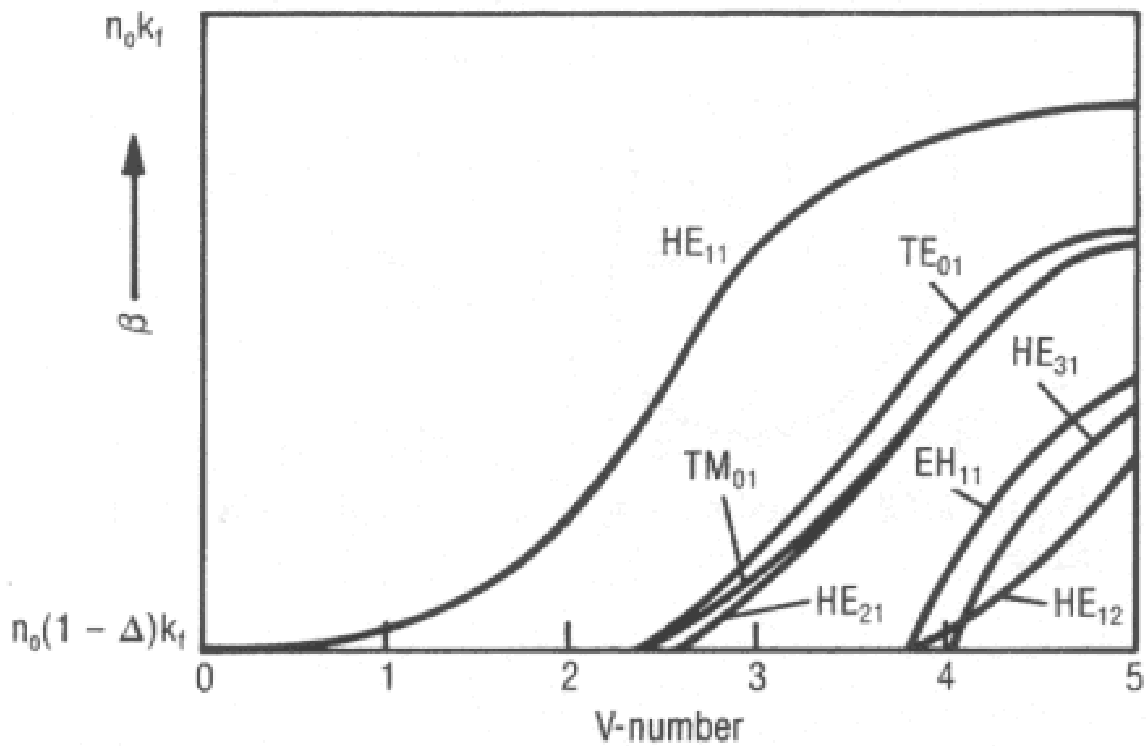
Same as slab waveguide except a=0 (almost always)



The normalized wave number, or V-number of a fiber is defined as $V = k_f a$ NA. Here k_f, is the free space wave number, $2\pi/\lambda_0$, a is the radius of the core, and NA is the numerical aperture of the fiber, NA = $(n_{core}^2 - n_{cladding}^2)^{1/2} \approx n_{core}(2\Delta)^{1/2}$, with $\Delta = (n_{core}-n_{cladding})/n_{core}$. Many fiber parameters can be expressed in terms of V. The TE and TM modes have non-vanishing cut-off frequencies. The cutoff frequency is found from $V = a\omega(2\Delta)^{1/2}/c = 2.405$. Only the lowest HE mode, HE_{11}, has no cutoff frequency. For $0 < V < 2.405$ it is the only mode that propagates in the fiber.

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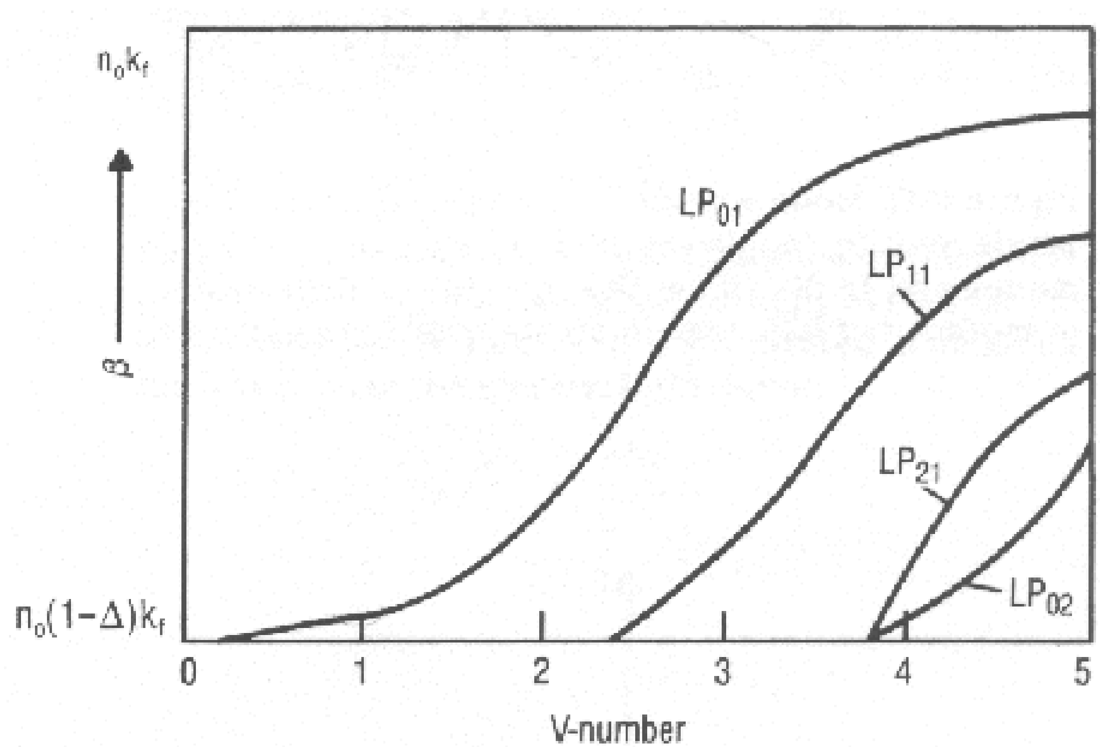
In the weakly-guiding approximation (Δ << 1), the modes propagating in the fiber are linearly polarized (LP) modes characterized by two subscripts, m and n. (The longitudinal components of the fields are small when Δ << 1.) The LP modes are combinations of the modes found from the exact theory of the wave guide. The HE_{11} mode becomes the LP_{01} mode in the weakly-guiding approximation.



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The following table presents the first ten cutoff frequencies in a step-index fiber, as well as their fundamental modes.

V_c	Bessel function	q	Modes	LP desig.
0	–	0	HE_{11}	LP_{01}
2.405	J_0	1	$TE_{01}, TM_{01}, HE_{21}$	LP_{11}
3.832	J_1	2	EH_{11}, HE_{31}	LP_{21}
3.832	J_{-1}	0	HE_{12}	LP_{02}
5.136	J_2	3	EH_{21}, HE_{41}	LP_{31}
5.520	J_0	1	$TE_{02}, TM_{02}, HE_{22}$	LP_{12}
6.380	J_3	4	EH_{31}, HE_{51}	LP_{41}
7.016	J_1	2	EH_{12}, HE_{32}	LP_{22}
7.016	J_{-1}	0	HE_{13}	LP_{03}
7.588	J_4	5	EH_{41}, HE_{61}	LP_{51}

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Single mode fiber

Single mode (SM) fiber is designed such that all the higher order waveguide modes are cut-off by a proper choice of the waveguide parameters as given below.

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2}$$

where, λ is the wavelength, *a* is the core radius, and n_1 *and* n_2 *are the core and cladding refractive indices, respectively. When* $V <$ *2.405 single mode condition is ensured. SM fiber is an essential requirement for interferometric sensors. Due to the small core size (~4* μ *m) alignment becomes a critical factor.*

Figure 3.10 shows plots of b, calculated using (3.76) and (3.77) in (3.78), for several LP modes as functions of V. As plotted on this scale, these results are essentially indistinguishable from those given by the exact numerical solution [5].

When considering single-mode fibers, the accuracy of the LP_{01} curve in Fig. 3.10 is of increased importance. It was in fact found to be accurate to within 5% over the range of V between 2.0 and 3.0, with the error increasing to around 10% as V decreases to 1.5 [7]. Numerous other approximate formulas exist as alternatives to (3.77) for determining b. The best of these was found by Rudolf and Neumann [8], who recognized that w is a nearly linear function of V over the range $1.3 < V < 3.5$. They were thus able to approximate w over this range by the simple function

$$w \approx 1.1428V - 0.9960 \tag{3.80}$$

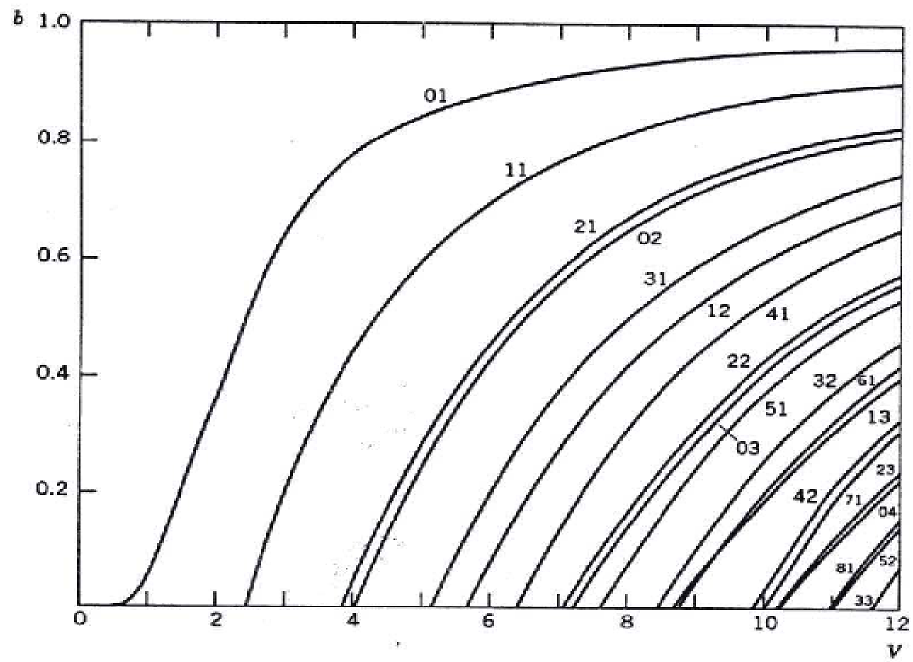
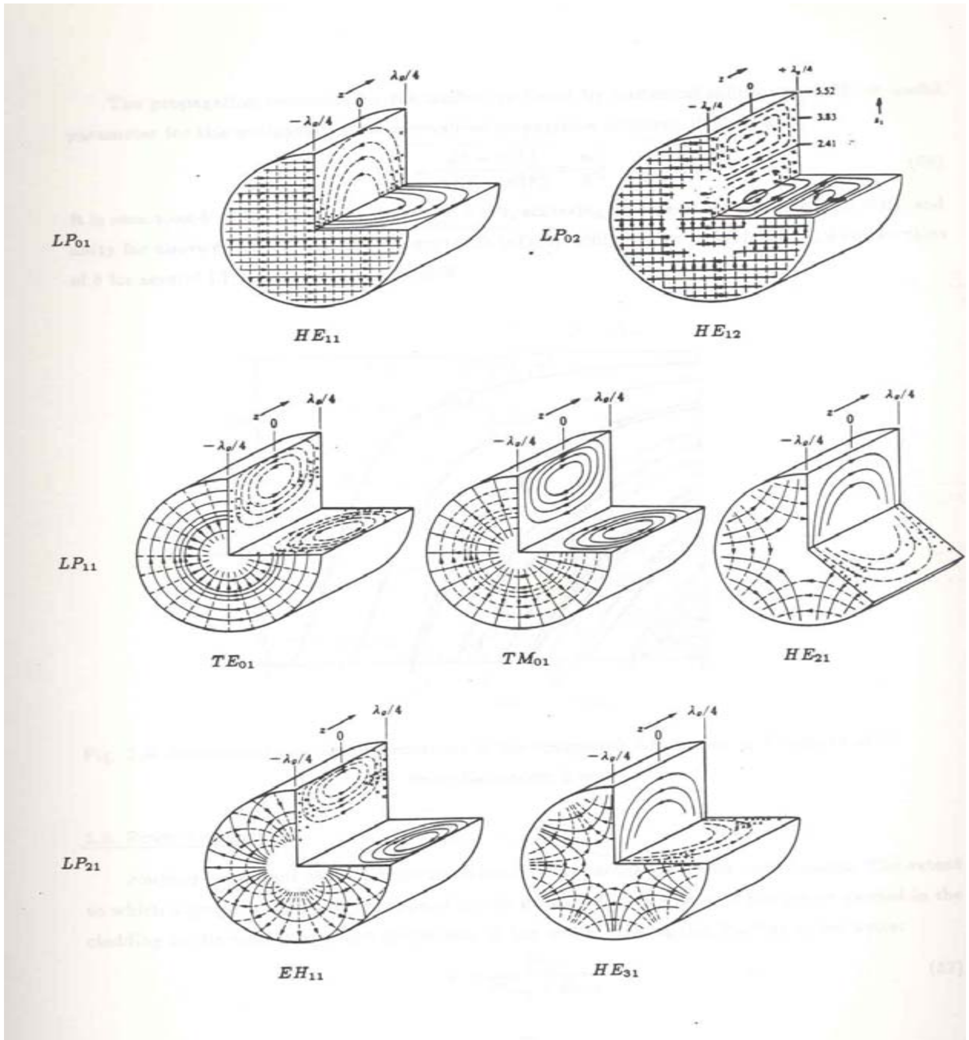
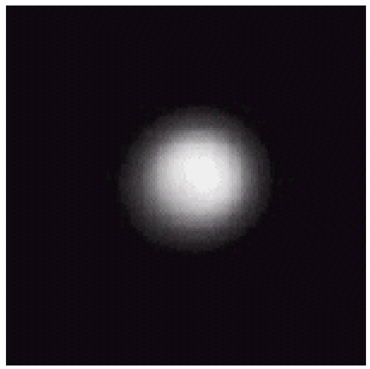


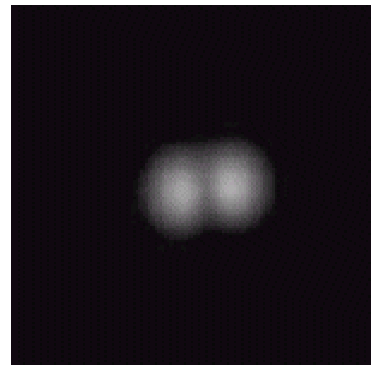
Figure 3.10. Normalized propagation constant, b, for designated LP modes as functions of V. (Adapted from ref. 5.)



Electric and magnetic fields for eight fundamental modes.



LP01



LP11

When the V number is less than 2.405 only the LP_{01} mode propagates. When the V number is greater than 2.405 the next linearly-polarized mode can be supported by the fiber, so that both the LP_{01} and LP_{11}, modes will propagate.

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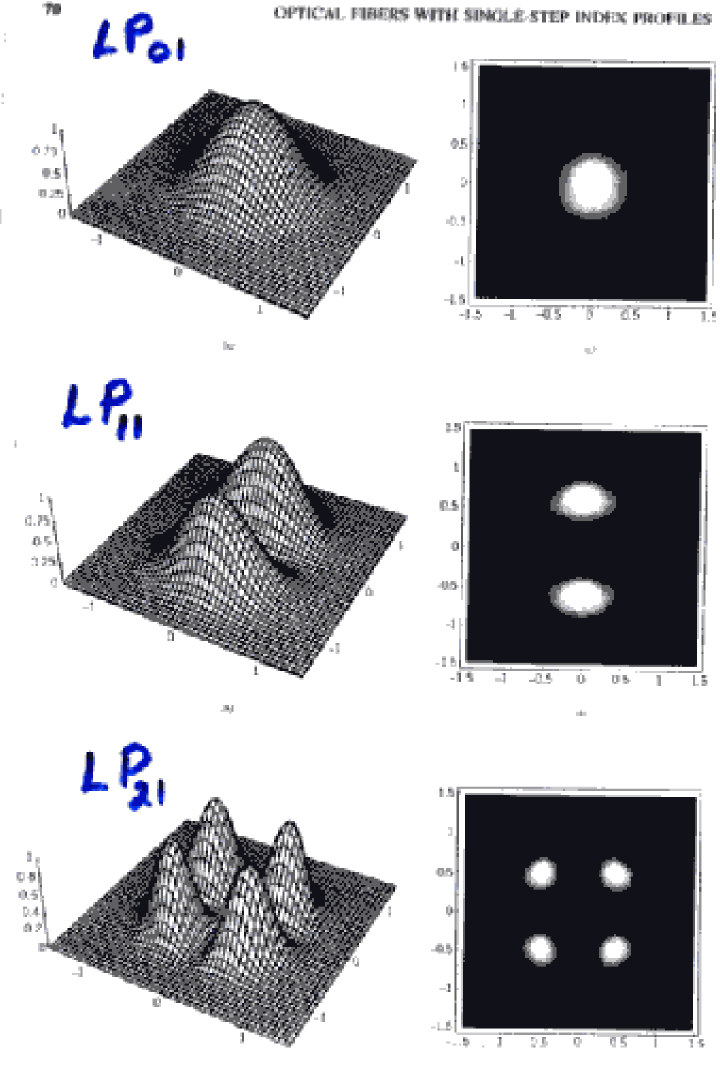

Figure 3.9. Intensity plots for the six LP modes, with $a = 1$. (a) LP_{01}: $v = 2$. (b) LP_{11}: $v = 3$. (c) LP_{21}: $v = 4.5$. (d) LP_{02}: $v = 4.5$. (e) LP_{31}: $v = 5.5$. (f) LP_{12}: $v = 6.3$.

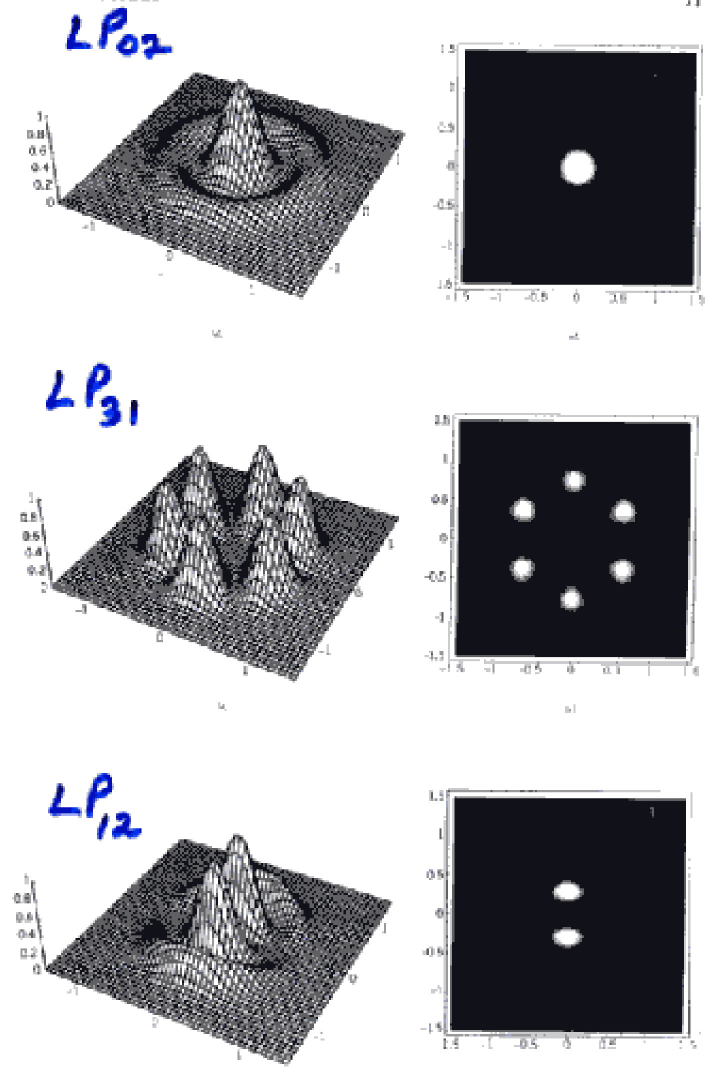

Figure 3.9. (*Continued*)

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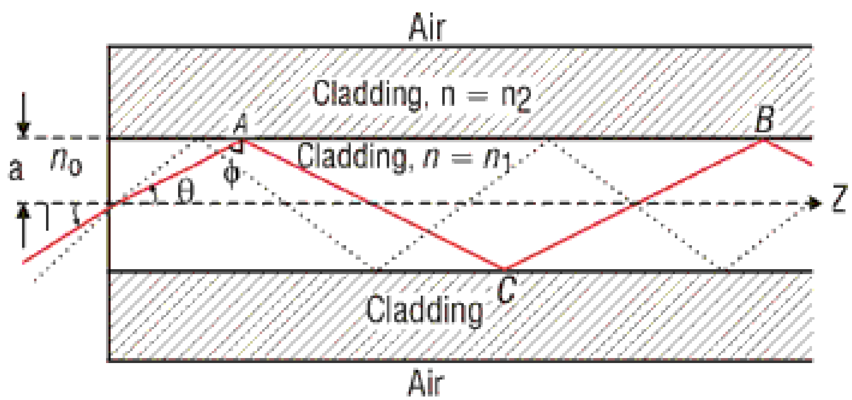
The SM fiber mentioned above is not truly single mode in that two modes with degenerate polarization states can propagate in the fiber. This can lead to signal interference and noise in the measurement. The degeneracy can be removed and a single mode polarization preserving fiber can be obtained by the use of an elliptical core fiber of very small size or with built in stress. In either case light launched along the major axis of the fiber is preserved in its state of polarization. It is also possible to make a polarizing fiber in which only one state of polarization is propagated. Polarimetric sensors make use of polarization preserving fibers. Thus, multimode fiber, single mode fiber and polarization preserving fiber are the three classes of fibers which are used in the intensity type, the interferometric type and the polarimetric type of sensors, respectively.

While discussing step-index fibers, we considered light propagation inside the fiber as a set of many rays bouncing back and forth at the core-cladding interface. There the angle θ could take a continuum of values lying between 0 and $\cos^{-1}(n_2/n_1)$, i.e.,



Scientific and Technological Education in Photonics

$$0 < \theta < \cos^{-1}(n_2/n_1)$$

For $n_2 = 1.5$ and $\Delta \approx \frac{n_1 - n_2}{n_1} = 0.01$, we would get $n_2/n_1 \simeq$ and $\cos^{-1}\left(\frac{n_1}{n_2}\right) = 8.1°$, so

$$0 < \theta < 8.1°$$

Now, when the core radius (or the quantity Δ) becomes very small, ray optics does not remain valid and one has to use the more accurate wave theory based on Maxwell's equations.

In wave theory, one introduces the parameter

$$V = \frac{2\pi}{\lambda_0} a \sqrt{n_1^2 - n_2^2} = \frac{2\pi}{\lambda_0} a n_1 \sqrt{2\Delta} \approx \frac{2\pi}{\lambda_0} a n_2 \sqrt{2\Delta}$$

where Δ has been defined earlier and $n_1 \simeq n_2$. The quantity V is often referred to as the "*V-number*" or the "*waveguide parameter*" of the fiber. It can be shown that, if

$$V < 2.4045$$

only one guided mode (as if there is only one discrete value of θ) is possible and the fiber is known as a *single-mode fiber*. Further, for a step-index single-mode fiber, the corresponding (discrete) value of θ is approximately given by the following empirical formula

$$\cos\theta \approx 1 - \Delta\left[1 - \left(1.1428 - \frac{0.996}{V}\right)^2\right]$$

We may mention here that because of practical considerations the value Δ of ranges from about 0.002 to about 0.008

Assignment

Consider a step-index fiber (operating at 1300 nm) with $n_2 = 1.447$, $\Delta = 0.003$, and $a = 4.2$ μm. Thus,

$$V = \frac{2\pi}{\lambda_0(\mu m)} \times 4.2(\mu m) \times 1.447 \times \sqrt{0.006} \approx \frac{2.958}{\lambda_0(\mu m)}$$

Thus the fiber will be single moded and the corresponding value of θ—will be about θ = 3.1º. It may be mentioned that for the given fiber we may write

$$V = \frac{2\pi}{1.3(\mu m)} \times 4.2(\mu m) \times 1.447 \times \sqrt{0.006} \approx 2.275$$

Thus, for $\lambda_0 > 2.958/2.4045 = 1.23$ μm

which guarantees that $V < 2.4045$, the fiber will be single moded. The wavelength for which $V = 2.4045$ is known as the *cutoff wavelength* and is denoted by λ_c. In this example, $\lambda_c = 1.23$ μm and the fiber will be single moded for $\lambda_0 > 1.23$ μm.

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Assignment

For reasons that will be discussed later, the fibers used in current optical communication systems (operating at 1.55 μm) have a small value of core radius and a large value of Δ. A typical fiber (operating at $\lambda_0 \approx 1.55$ μm) has $n_2 = 1.444$, $\Delta = 0.0075$, and $a = 2.3$ μm. Thus, at $\lambda_0 = 1.55$ μm, the *V*-number is,

$$V = \frac{2\pi}{1.55(\mu m)} \times 2.3(\mu m) \times 1.444 \times \sqrt{0.015} \approx 1.649$$

The fiber will be single moded (at 1.55 μm) with θ = 5.9°. Further, for the given fiber we may write

$$V = \frac{2\pi}{\lambda_0(\mu m)} \times 2.3(\mu m) \times 1.444 \times \sqrt{0.015} \approx \frac{2.556}{\lambda_0(\mu m)}$$

and therefore the cutoff wavelength will be $\lambda_c = 2.556/2.4045 = 1.06$ μm.

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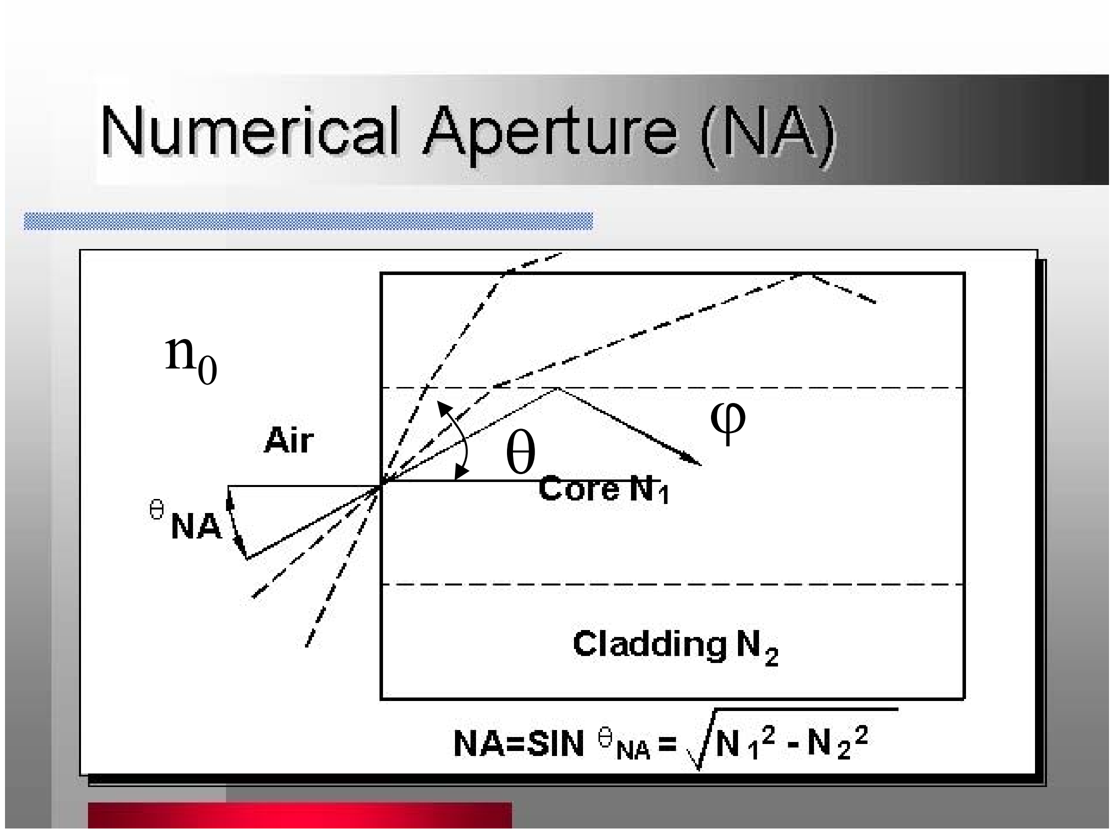


- The Numerical Aperture (NA) of a fiber is the measure of the maximum angle (θ_{NA}) of the light entering the end that will propagate within the core of the fiber
- Acceptance Cone = $2\theta NA$
- Light rays entering the fiber that exceed the angle θ_{NA} will enter the cladding and be lost
- For the best performance the NA of the transmitter should match the NA of the fiber

NA derivation

We know $\dfrac{\sin i}{\sin \theta} = \dfrac{n_1}{n_0}$ and $\sin \phi (= \cos \theta) > \dfrac{n_2}{n_1}$

Since $\sin \theta = \sqrt{1 - \cos^2 \theta}$ we get $\sin \theta < \left[1 - \left(\dfrac{n_2}{n_1}\right)^2\right]^{1/2}$

Assume the θ_{NA} is the half angle of the acceptance cone,

$$\sin\theta_{NA} = (n_1^2 - n_2^2)^{1/2} = n_1 \mathrm{sqrt}(2\Delta)$$

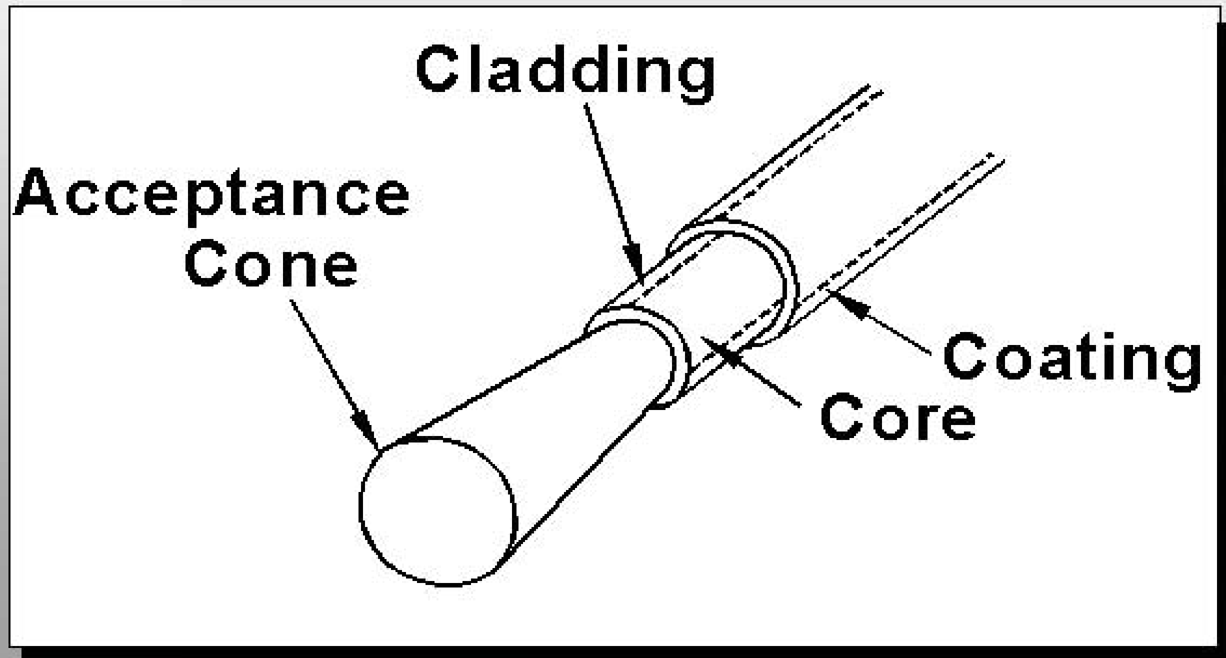
We define a parameter Δ through the following equations.

$$\Delta \equiv \frac{n_1^2 - n_2^2}{2n_2^2}$$

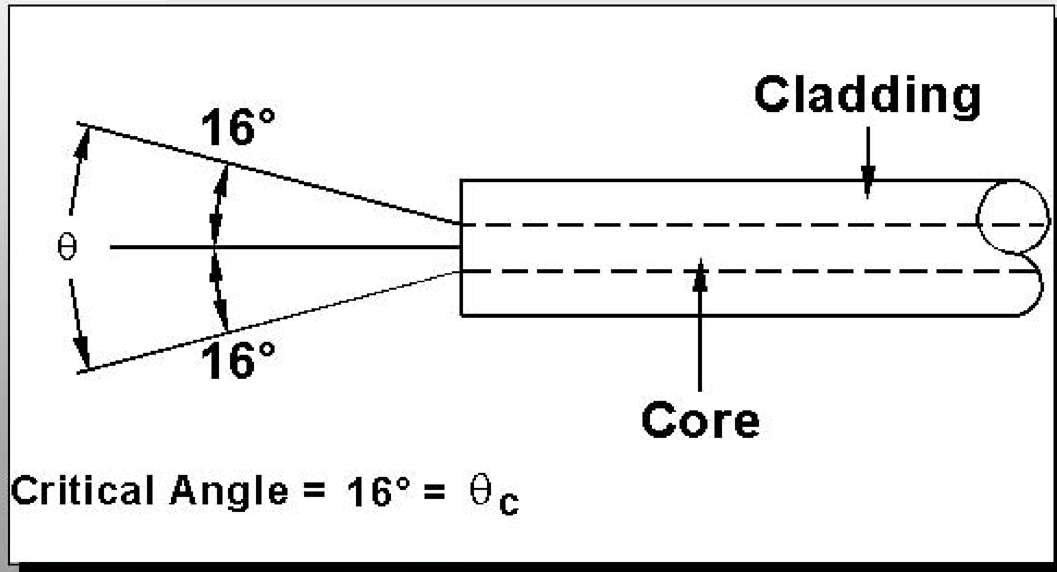
When $\Delta << 1$ (as is indeed true for silica fibers where n_1 is very nearly equal to n_2) we may write

$$\Delta = \frac{(n_1+n_2)(n_1-n_2)}{2n_1^2} \approx \frac{(n_1-n_2)}{n_1} \approx \frac{(n_1-n_2)}{n_2}$$

Acceptance Cone



Acceptance Cone



Single mode fiber critical angle $<20^{o}$
Multimode fiber critical angle $<60^{o}$

Example

For a typical step-index (multimode) fiber with $n_1 \approx 1.45$ and $\Delta \approx 0.01$, we get

$$\sin i_m = n_1 \sqrt{2\Delta} = 1.45\sqrt{2 \times (0.01)} = 0.205$$

so that $i_m \approx 12°$. Thus, all light entering the fiber must be within a cone of half-angle 12°.

In a short length of an optical fiber, if all rays between $i = 0$ and i_m are launched, the light coming out of the fiber will also appear as a cone of half-angle i_m emanating from the fiber end. If we now allow this beam to fall normally on a white paper and measure its diameter, we can easily calculate the *NA* of the fiber.

Assignment

For a typical step-index (multimode) fiber with $n_1 \approx 1.45$ and $\Delta \approx 0.01$, we get

$$\sin i_m = n_1 \sqrt{2\Delta} = 1.45\sqrt{2 \times (0.01)} = 0.205$$

so that $i_m \approx 12°$. Thus, all light entering the fiber must be within a cone of half-angle 12°.

Spot size of the fundamental mode

A single-mode fiber supports only one mode that propagates through the fiber. This mode is also referred to as the *fundamental mode* of the fiber. The transverse field distribution associated with the fundamental mode of a single-mode fiber is an extremely important quantity. It determines various important parameters like splice loss at joints, launching efficiencies, bending loss, etc. For most single-mode fibers, the fundamental mode-field distributions can be approximated by a Gaussian function, which may be written in the form

$$\psi(r) = A \quad e^{-r^2/w^2}$$

where *w* is referred to as the spot size of the mode-field pattern.

When $r = w$, the value of ψ is equal to $1/e$ of the value A at $r = 0$. For a step-index (single-mode) fiber, one has the following empirical expression for w [Marcuse]:

$$\frac{w}{a} \approx \left(0.65 + \frac{1.619}{V^{3/2}} + \frac{2.879}{V^6}\right); \qquad 0.8 < V < 2.5$$

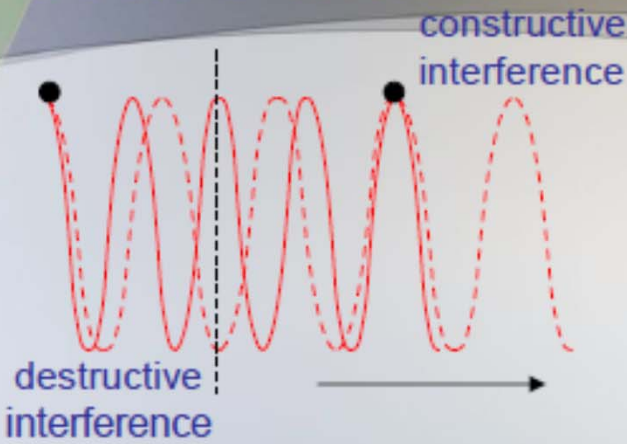
where a is the core radius and V is the V-number given by Equation 7-28. We may mention here that the light coming from a HeNe laser (or from a laser pointer) has a transverse intensity distribution very similar to that coming from a single-mode fiber except that the spot size for the HeNe laser is much larger. The quantity $2w$ is also referred to as the mode-field diameter (MFD) of the fiber and is a very important property of single-mode fibers. In fact, MFD is a more important property than core diameter in the case of single-mode fibers, since it determines the splice loss across a joint, bending loss, dispersion, etc. of single-mode fibers.

Assignment

Consider a step-index fiber (operating at 1300 nm) with n_2 = 1.447, Δ = 0.003, and a = 4.2 μm. For this fiber (see Example 7-12), $V \approx 2.28$. Using Equation 7-31, with $V = 2.28$ and a = 4.2 mm, one obtains $w \approx 4.8$ μm. The same fiber will have a V-value of 1.908 at λ_0 = 1550 nm, giving a value of the spot size $\approx$ 5.5 μm. Thus the spot size increases with wavelength.

For a step-index fiber (operating at 1550 nm) with n_2 = 1.444, Δ = 0.0075, and a = 2.3 μm (see Example 7-13), $V \approx 1.65$, giving $w \approx 3.6$ μm. The same fiber will have a V-value of 1.97 at λ_0 = 1300 nm, giving a value of the spot size $\approx$ 3.0 μm.

Two-Wave Interference — Different f and λ



$$E_1 = E_0 \cos(k_1 x - \omega_1 t)$$

$$E_2 = E_0 \cos(k_2 x - \omega_2 t)$$

$$E = E_1 + E_2 = 2E_0 \cos(k_p x - \omega_p t)\cos(k_g x - \omega_g t)$$

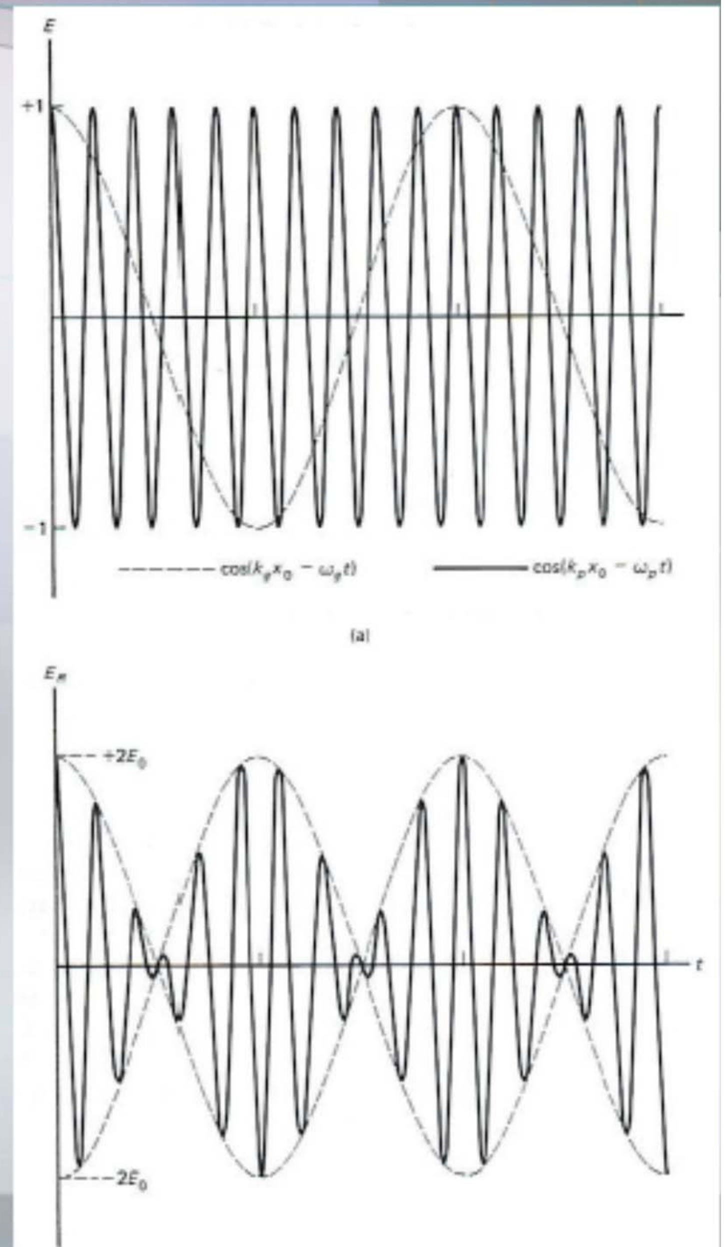
$$k_p = \frac{k_1 + k_2}{2}, \quad \omega_p = \frac{\omega_1 + \omega_2}{2}$$

$$k_g = \frac{|k_1 - k_2|}{2}, \quad \omega_g = \frac{|\omega_1 - \omega_2|}{2}$$

$\cos(k_p x - \omega_p t)$: Average

$\cos(k_g x - \omega_g t)$: Modulation, beating

Beat frequency $\omega_b = 2\omega_g = |\omega_1 - \omega_2|$



Phase Velocity, Group Velocity, and Dispersion

Assume the two waves are close in ω and k.

Phase velocity: $$v_p = \frac{\omega_p}{k_p} = \frac{\omega_1+\omega_2}{k_1+k_2} \cong \frac{\omega}{k} = \frac{c}{n}$$

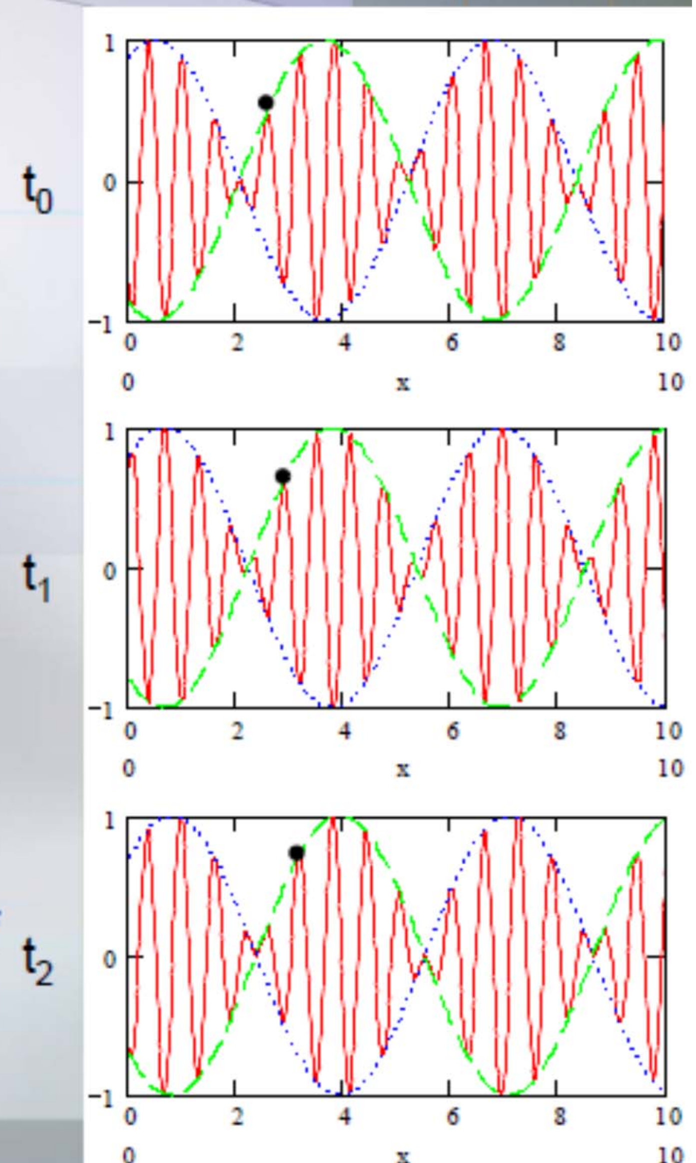
Group velocity: $$v_g = \frac{\omega_g}{k_g} = \frac{\omega_1-\omega_2}{k_1-k_2} \cong \frac{d\omega}{dk}$$

$$v_g = v_p\left[1+\frac{\lambda}{n}\left(\frac{dn}{d\lambda}\right)\right]$$

Dispersion. n = n(λ). Light with different wavelength travels with different velocity in a medium.

Normal dispersion: $dn/d\lambda < 0$, $v_g < v_p$.

v_g determines the speed with which energy is transmitted. It is the directly measurable speed of the wave.



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Dispersion in an Optical Fiber

Group index: $v_g \equiv \dfrac{c}{N}$

$$N = n - \lambda \frac{dn}{d\lambda}$$

Dispersion coefficient:

$$D_\lambda = -\frac{\lambda}{c}\frac{d^2 n}{d\lambda^2} \quad \text{(s/m-nm)}$$

→ A measure of time delay per wavelength (nm) after certain distance.

Bandwidth limited by dispersion:

$$\nu_{\max} L = \frac{0.5}{\delta(\tau / L)}$$

If the original pulse width cannot be neglected compared to the broadening,

$$\tau_f^{\,2} \approx \tau_0^{\,2} + (\delta\tau)^2$$

$$\nu_{\max} = 0.5 / \tau_f$$

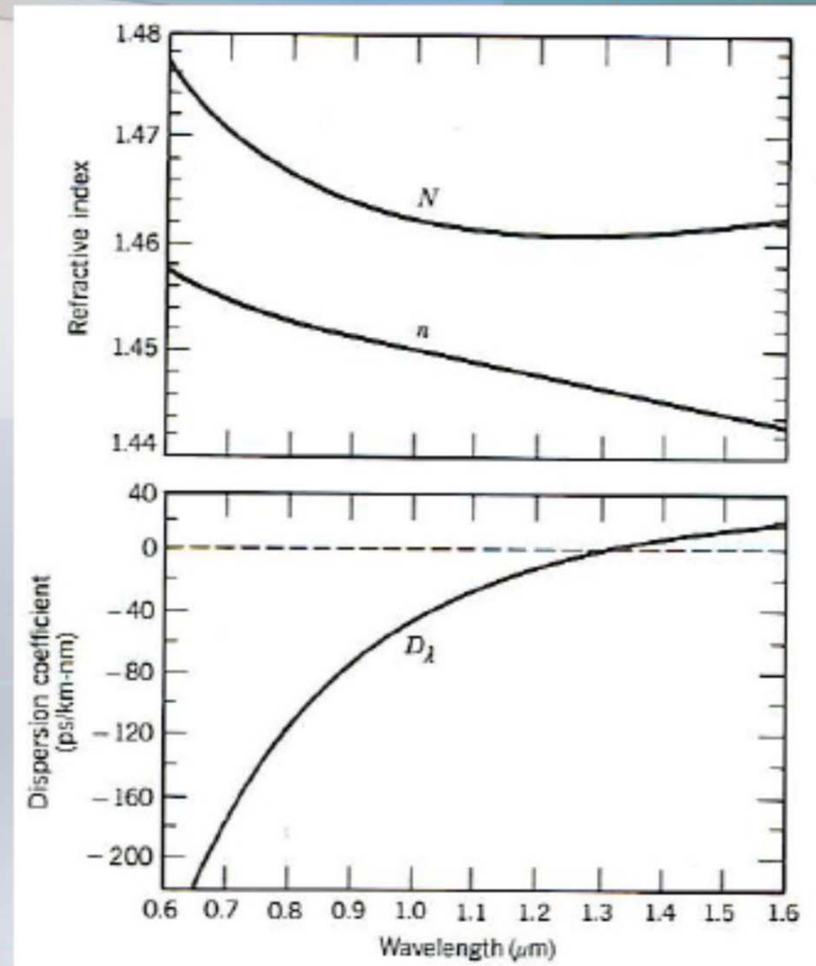
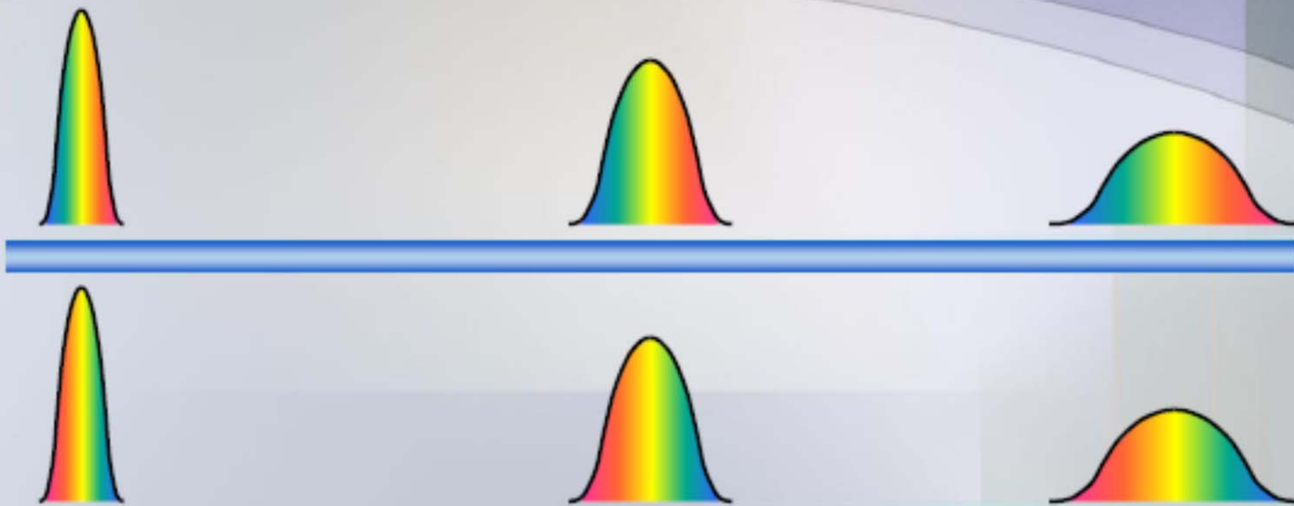
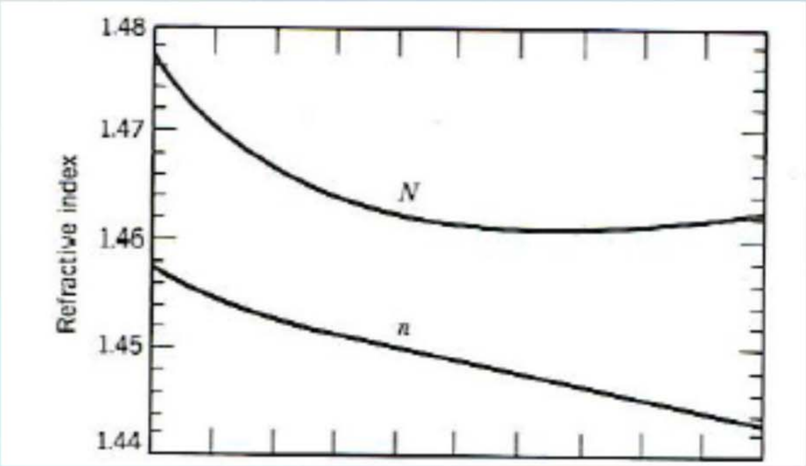


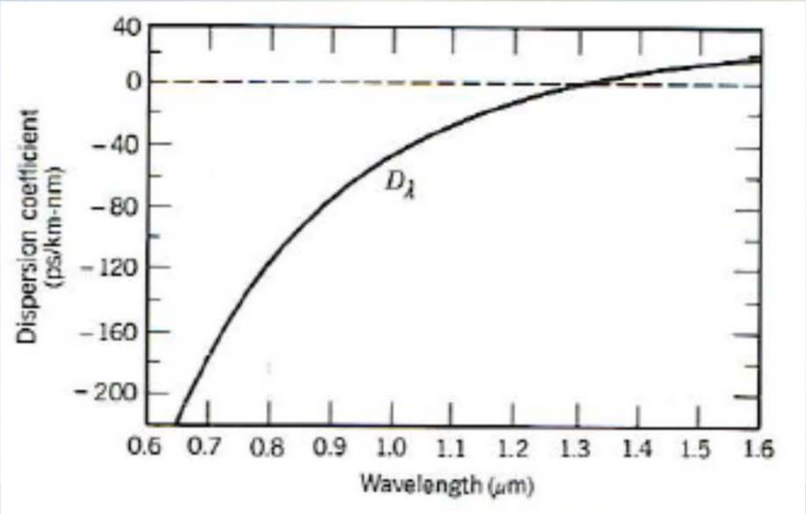
Figure 5.6-5 Wavelength dependence of optical parameters of fused silica: the refractive index n, the group index $N = c_o/v$, and the dispersion coefficient D_λ. At $\lambda_o = 1.312$ μm, n has a point of inflection, the group velocity v is maximum, the group index N is minimum, and the dispersion coefficient D_λ vanishes. At this wavelength the pulse broadening is minimal.

Pulse Broadening in an Optical Fiber



or,





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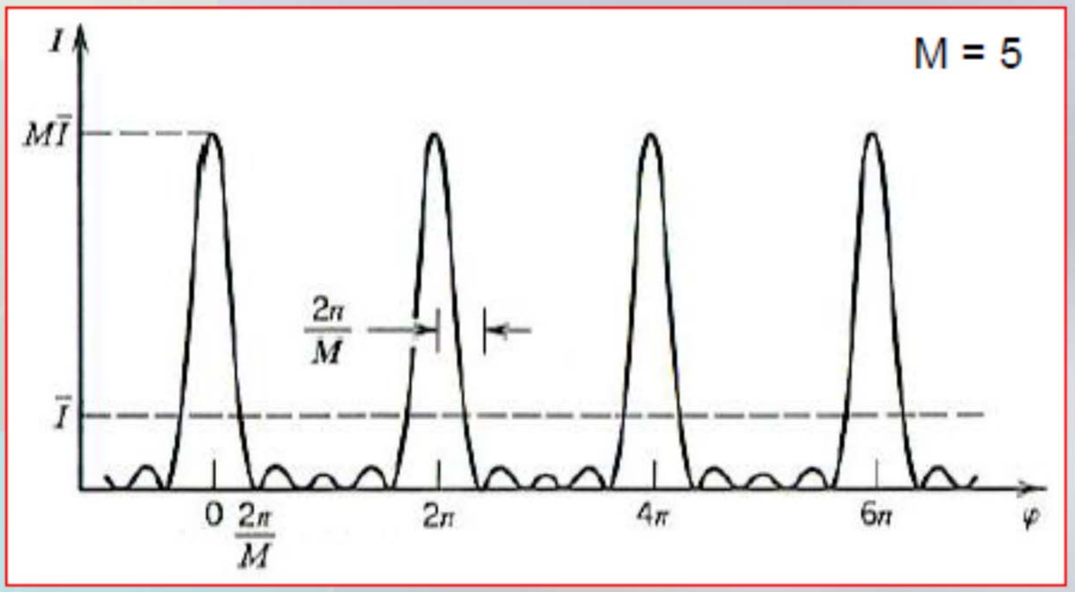
Multi-Wave Interference
— Equal Amplitude and Equal Phase Difference

$$U_m = \sqrt{I_o}\exp[i(m-1)\phi], \qquad m = 1, 2, \ldots, M$$

$$U = \sum_{m=1}^{M} U_m = \sqrt{I_o}\,\frac{1-\exp(iM\phi)}{1-\exp(i\phi)}$$

$$I = |U|^2 = I_0 \frac{\sin^2(M\phi/2)}{\sin^2(\phi/2)}$$



Interesting features:

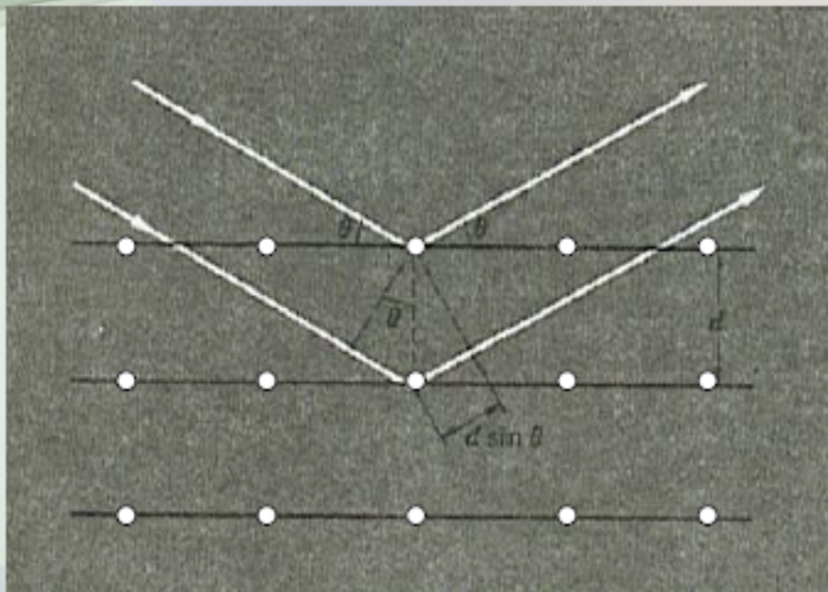
- Mean intensity

$$\bar{I} = \frac{1}{2\pi}\int_0^{2\pi} I\,d\phi = MI_0$$

- Peak intensity = $M^2 I_0$
- Intensity drops to zero at ϕ = $2\pi/M$ from peak intensity.
- Sensitivity to ϕ increases with M.
- (M - 2) minor peaks between major peaks.

Multi-Wave Interference

Example: Bragg Reflection

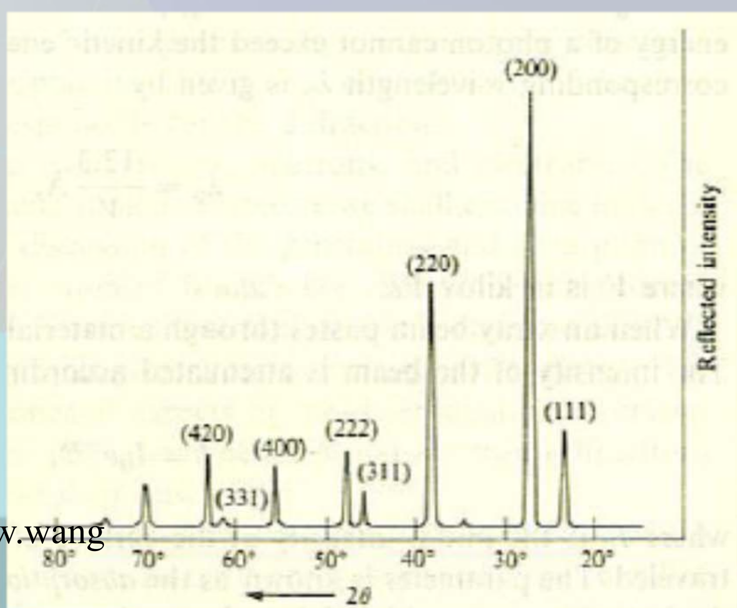


Intensity of the reflected light maximizes at

$$\sin\theta = \frac{n\lambda}{2d}$$

One of the applications:

Characterizing the lattice constant of a crystal.



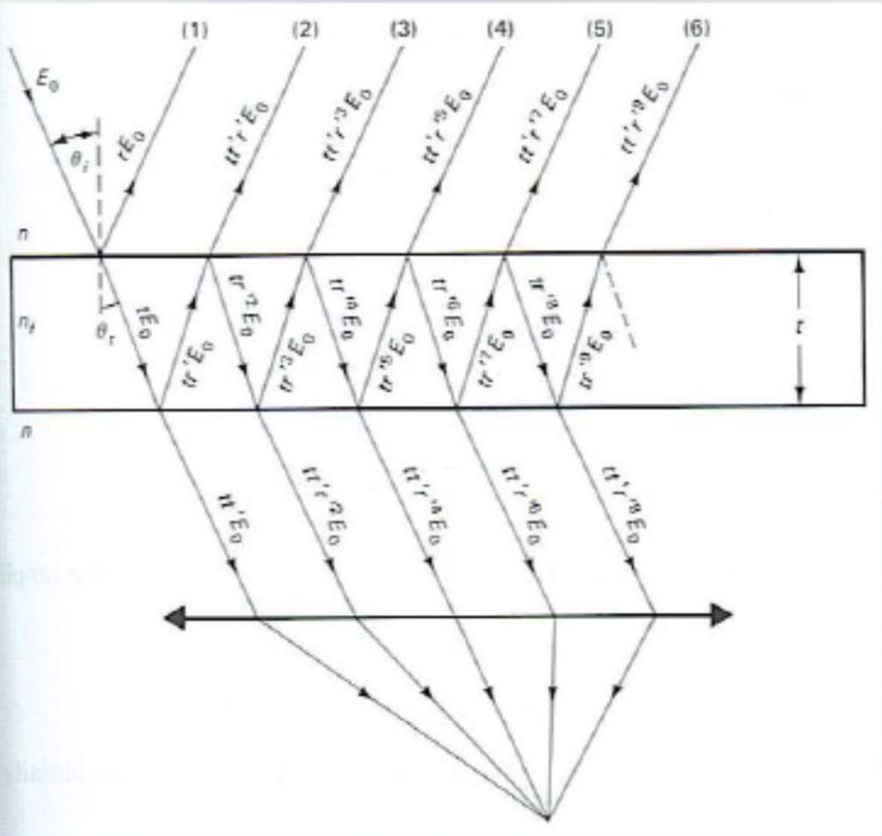
λ = 1.16 Å

For (200) direction,

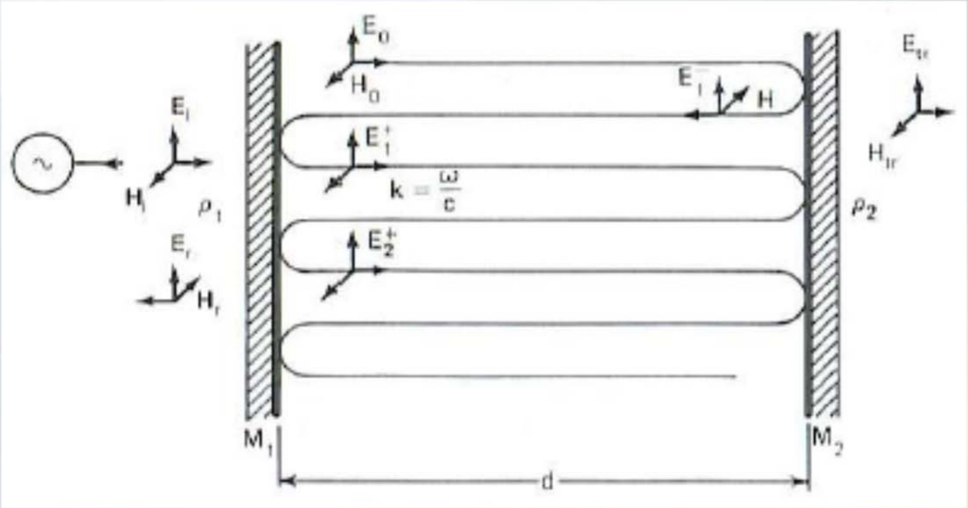
$$d = \frac{1.16}{2\sin(27^\circ/2)} = 2.48 \text{ Å}$$

Multi-Wave Interference
— Progressively Smaller Amplitude and Equal Phase Difference

Parallel plate



Fabry-Perot Interferometer



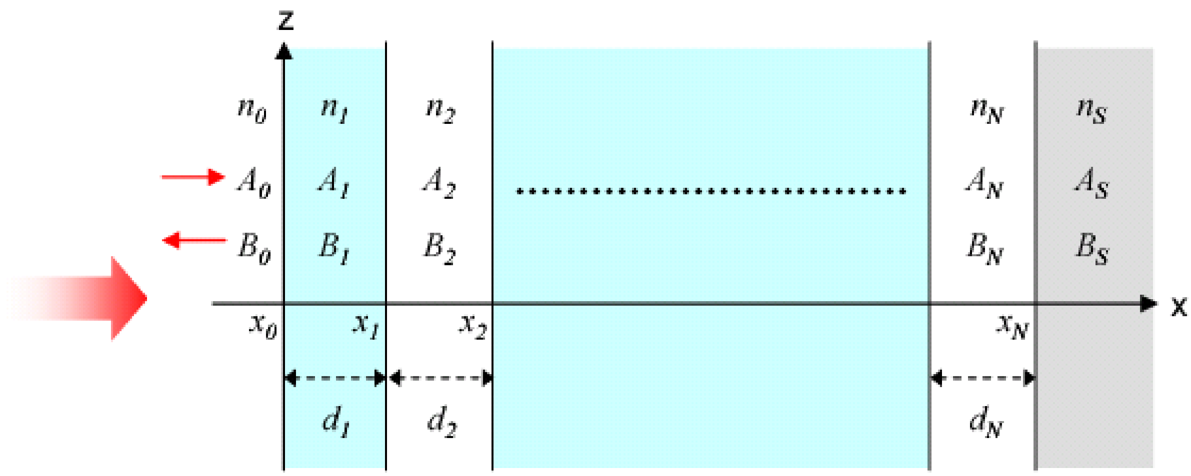
Two high-reflection plates separated by a distance d.

d is often tunable.

r, r': Reflection coefficient

t, t': Transmission coefficient

Principles of Dielectric Mirror



$$E = E(x)e^{i(\omega t - \beta z)}$$

Electric field of a general plane-wave

$$E(x) = \begin{cases} A_0 e^{-ik_{0x}(x-x_0)} + B_0 e^{ik_{0x}(x-x_0)}, & x < x_0 \\ A_l e^{-ik_{lx}(x-x_{l-1})} + B_l e^{ik_{lx}(x-x_{l-1})}, & x_{l-1} < x < x_l \\ A_S e^{-ik_{sx}(x-x_N)} + B_S e^{ik_{sx}(x-x_N)}, & x_N < x \end{cases}$$

$$k_{lx} = n_l \frac{\omega}{c} \cos\theta_l$$

x component of the wave vectors (θ_l: ray angle)

Ref: P. Yeh, *Optical Waves in Layered Media*, Wiley.

Principles of Dielectric Mirror

2x2 matrix formulation for multi-layer system

$$\begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = D_0^{-1} D_1 \begin{pmatrix} A_1 \\ B_1 \end{pmatrix}$$

$$\begin{pmatrix} A_l \\ B_l \end{pmatrix} = P_l D_l^{-1} D_{l+1} \begin{pmatrix} A_{l+1} \\ B_{l+1} \end{pmatrix}, \quad l = 1, 2, ..., N$$

$$D_l = \begin{pmatrix} 1 & 1 \\ n_l \cos\theta_l & -n_l \cos\theta_l \end{pmatrix} \text{ for TE wave}$$

$$D_l = \begin{pmatrix} \cos\theta_l & \cos\theta_l \\ n_l & -n_l \end{pmatrix} \text{ for TM wave}$$

$$P_l = \begin{pmatrix} e^{i\phi_l} & 0 \\ 0 & e^{-i\phi_l} \end{pmatrix}, \quad \phi_l = k_{lx} d_l$$

Transmission and reflection coefficients can be determined from:

$$\begin{matrix} A_0 \\ B_0 \end{matrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} A_S \\ B_S \end{pmatrix}$$

$$\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} = D_0^{-1} \left(\prod_{l=1}^{N} D_l P_l D_l^{-1} \right) D_S$$

Dependent on wavelength and thickness of the dielectric layers