Photoelasticity

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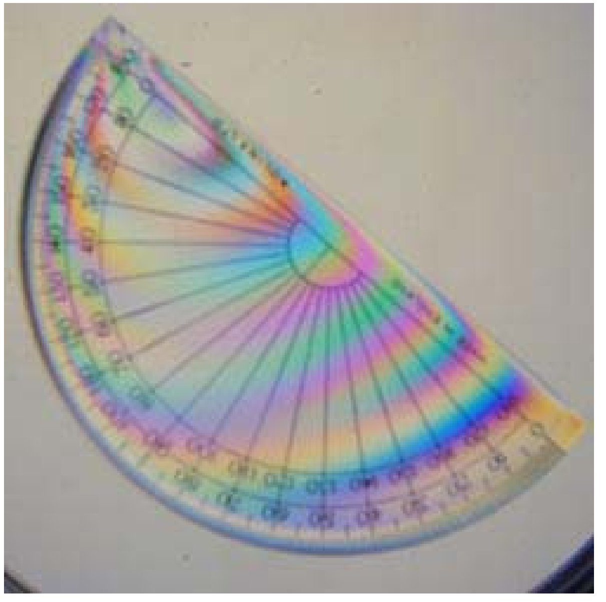
Photoelasticity

Many transparent noncrystalline materials that are optically isotropic when free of stress become optically anisotropic and display characteristics similar to crystals when they are stressed. This behavior is known as *temporary double refraction*.

Theory of photoelasticity



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The effect that an isotropic material can become birefringent (anisotropic), when placed under stress.

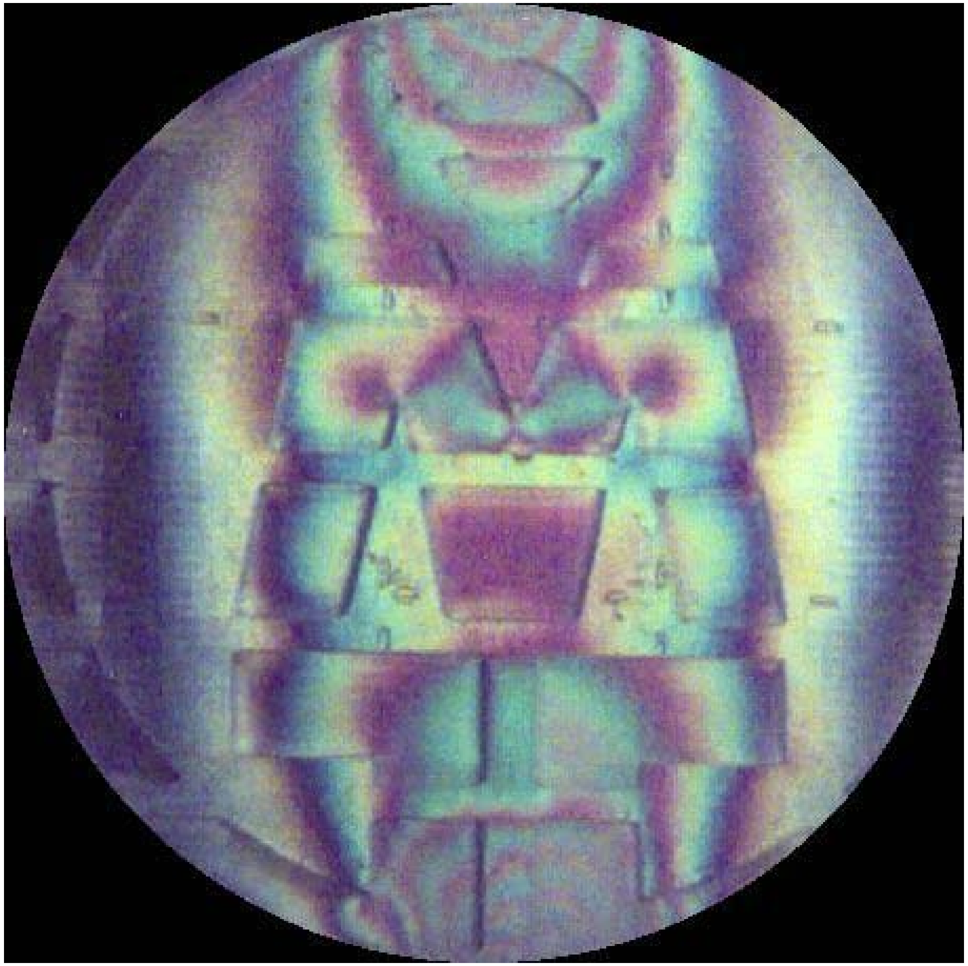
Under compression $\rightarrow$ negative uniaxial crystal.

Under tension $\rightarrow$ positive uniaxial crystal.

Optical axis is in the direction of the stress.

Induced birefringence is proportional to the stress.

Can be used to study stress patterns in complex objects (e.g. bridges) by building a transparent scale model of the device.



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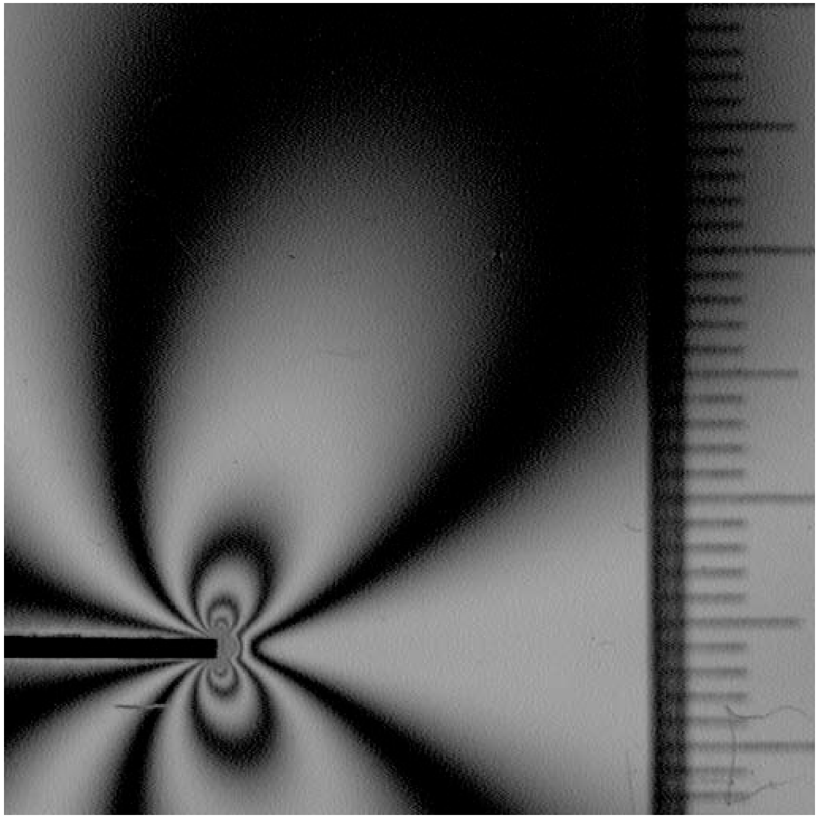
Method utilizes a birefringent model of the actual structure to view the stress contours due to external loading or residual birefringence. When white light is used for illumination, a colourful fringe pattern reveals the stress/strain distribution in the part.



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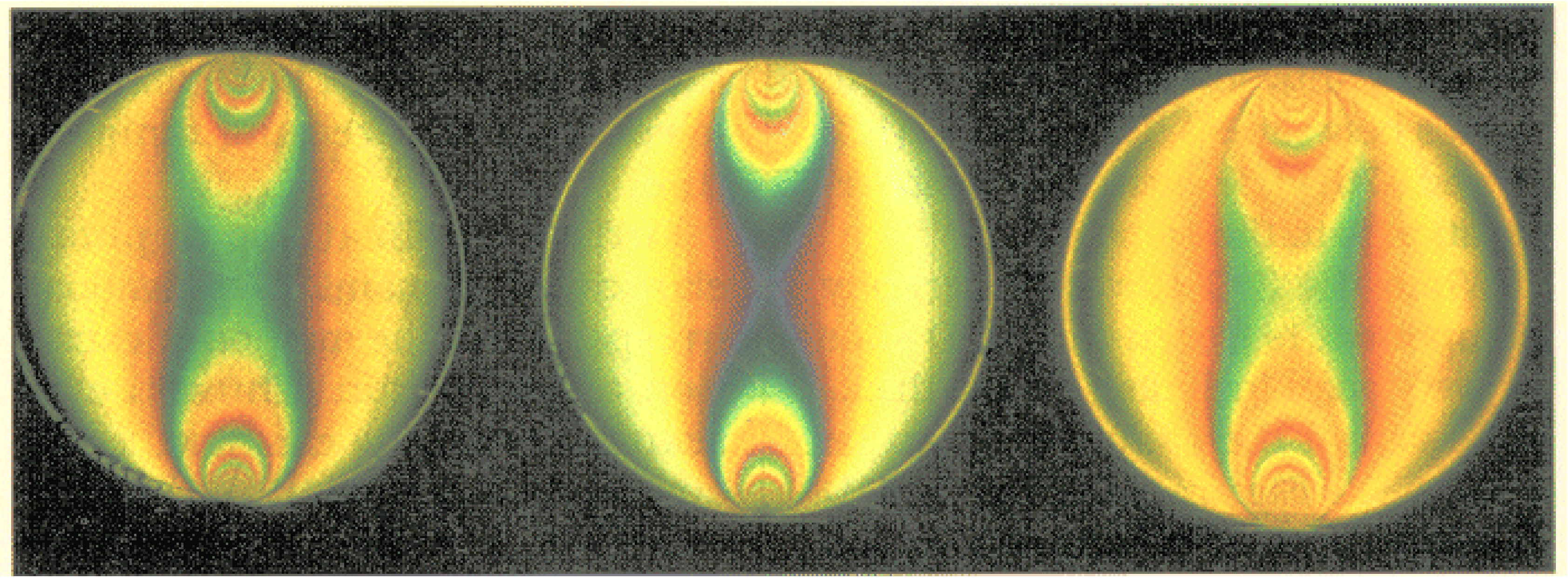
by utilizing a monochromatic light source for illumination. Using monochromatic light enable better definition of fringes especially in areas with dense fringes as at stress concentration points.

Silicon Wafer Stress Analysis



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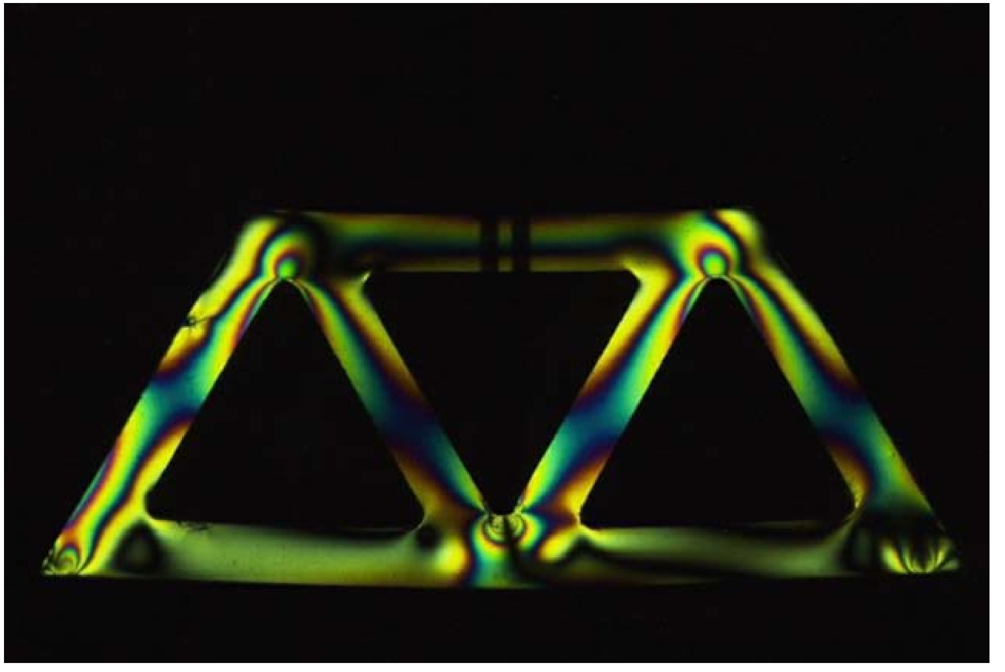
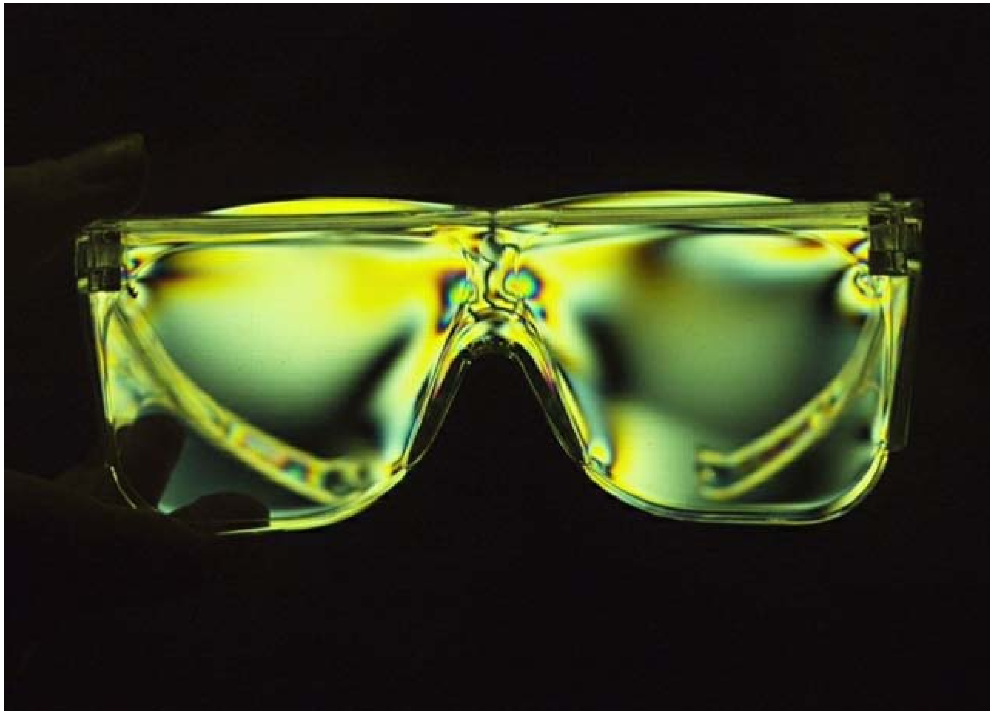
Photoelasticity has staged a revival in the past few years with applications in Silicon Wafer Stress Analysis , Rapid Prototyping and Fiber Optic Sensor Development and Image Processing.



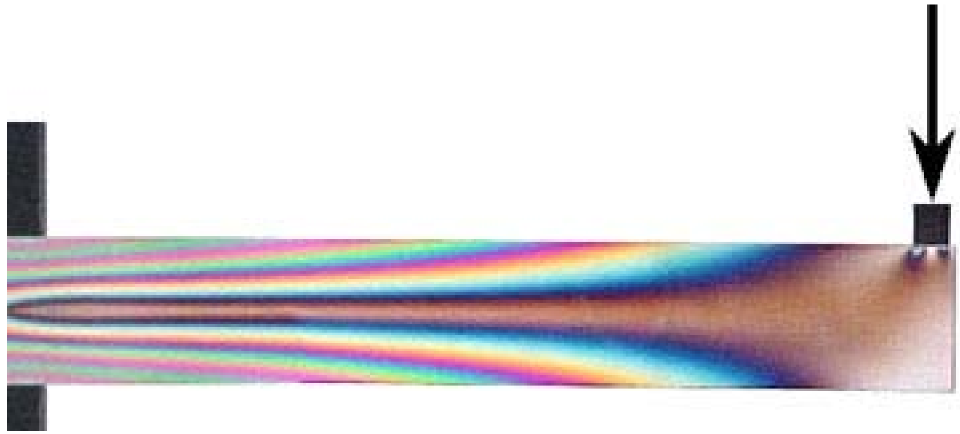
Stress fields (applied and residual) can be exposed using models of structures in photosensitive material placed between polarising filters in the crossed polar position.

Here the stresses in a 7 member model bridge truss, centrally loaded and simply supported are shown.

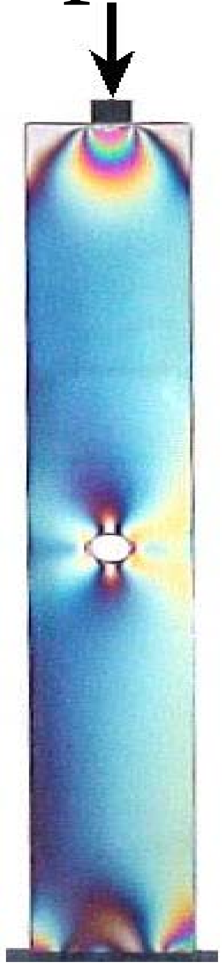


These injection molded safety spectacles contain residual molding stresses shown here using photoelastic viewing techniques.

Photoelastic beam in bending



Photoelastic beam under compressive load



Photoelasticity is a whole-field technique for measuring and visualizing stresses and strains in structures.The method utilizes a birefringent model of the actual structure to view the stress contours due to external loading or residual birefringence. When white light is used for illumination, a colourful fringe pattern reveals the stress/strain distribution in the part. Qualitative analysis such as strain concentration points, uniform stress regions etc. can be identified quite readily. For qunatitative information, a further analysis has to be performed. Upto quite recently this was done by transforming the colour patterns to a black and white picture by utilizing a monochromatic light source for illumination. Using monochromatic light enable better definition of fringes especially in areas with dense fringes as at stress concentration points. Details can be found in numerous books on this topic, with the Experimental Stress Analysis by J.W.Dally and W.F.Riley as a good starter.

Stress-Optic Law

Maxwell reported that indices of refraction were linearly proportional to the loads thus to stresses or strains for a linear elastic material. The relationship can be expressed as,

$$n_1 - n_0 = c_1\sigma_1 + c_2(\sigma_2+\sigma_3)$$
$$n_2 - n_0 = c_1\sigma_2 + c_2(\sigma_3+\sigma_1)$$
$$n_3 - n_0 = c_1\sigma_3 + c_2(\sigma_1+\sigma_2)$$

Where $\sigma_1, \sigma_2, \sigma_3$ *= principal stresses at point*

n_0 *= index refraction of material in unstressed state*

n_1, n_2, n_3 *= principal indices of refraction which coincide with the principal stress directions*

c_1, c_2, c_3 *= stress optic coefficients*

The equation indicates complete state of stress can be determined by measuring the three principal indices of refraction and establishing the directions of the three principal optical axes.

Stress-optic law in terms of relative retardation

The method of photoelasticity make use of relative changes in index of refraction which can be written by eliminating n_o from earlier equations,

$$\Delta n_{12} = n_2 - n_1 = (c_2 - c_1)(\sigma_1 - \sigma_2)$$
$$\Delta n_{23} = n_3 - n_2 = (c_2 - c_1)(\sigma_2 - \sigma_3) \quad (1)$$
$$\Delta n_{31} = n_1 - n_3 = (c_2 - c_1)(\sigma_3 - \sigma_1)$$

Where $c = c_2 - c_1$ *is relative stress-optic coefficient (brewsters)*

1 brewster = $10^{-13} cm^2/dyn = 10^{-12}\ m^2/N = 6.985x10^{-9}\ in^2/lb$

Positive birefringence = velocity of wave associated with the principal stress σ_1 > *Velocity of wave associate with principal stress* σ_2. *So* $\sigma_1 \geq \sigma_2 \geq \sigma_3$ *and* $n_3 \geq n_2 \geq n_1$

$V_p = C/n$

Photorealistic model behave like a temporary wave plate, we can use relative angular phase shift Δ (or relative retardation) to changes in the indices of refraction in the material result from the stresses.

Consider a slice of material (thickness *h*) oriented perpendicular to one of the principal-stress directions at the point of interest in the model. If a linearly polarized light is passing through the slice at normal incidence, the relative retardation Δ accumulated along each of the principal-stress directions can be obtained by substitute earlier relative index change into $\Delta_{12} = 2\pi\Delta n_{12}\delta/\lambda = 2\pi h(n_2-n_1)/\lambda$ *to get*

$$\Delta_{12} = \frac{2\pi hc}{\lambda}(\sigma_1 - \sigma_2)$$

(Relative angular phase shift developed between components of light beam propagating in σ_3 direction)

$$\Delta_{23} = \frac{2\pi hc}{\lambda}(\sigma_2 - \sigma_3) \qquad (2)$$

(Relative angular phase shift developed between components of light beam propagating in σ_1 direction)

$$\Delta_{31} = \frac{2\pi hc}{\lambda}(\sigma_3 - \sigma_1)$$

(Relative angular phase shift developed between components of light beam propagating in σ_2 direction)

Δ is linearly proportional to the difference between the two principal stresses Having directions perpendicular to the path of propagation of the light beam

Stress-optic coefficient c is assume to be material constant independent of the wavelength. A study by Vandaele-Dossche has shown c is a function of wavelengths when model passes from the elastic to the plastic state – dispersion of birefringence

For analysis of the general three-dimensional state of stress at a point and from an analysis of the change in index of refraction with the direction of light propagation in the stressed material, it can be shown that above equations can also be used for secondary principal stresses

$$\Delta'_{12} = \frac{2\pi hc}{\lambda}(\sigma'_1 - \sigma'_2) \qquad (3)$$

Plane stress Measurement

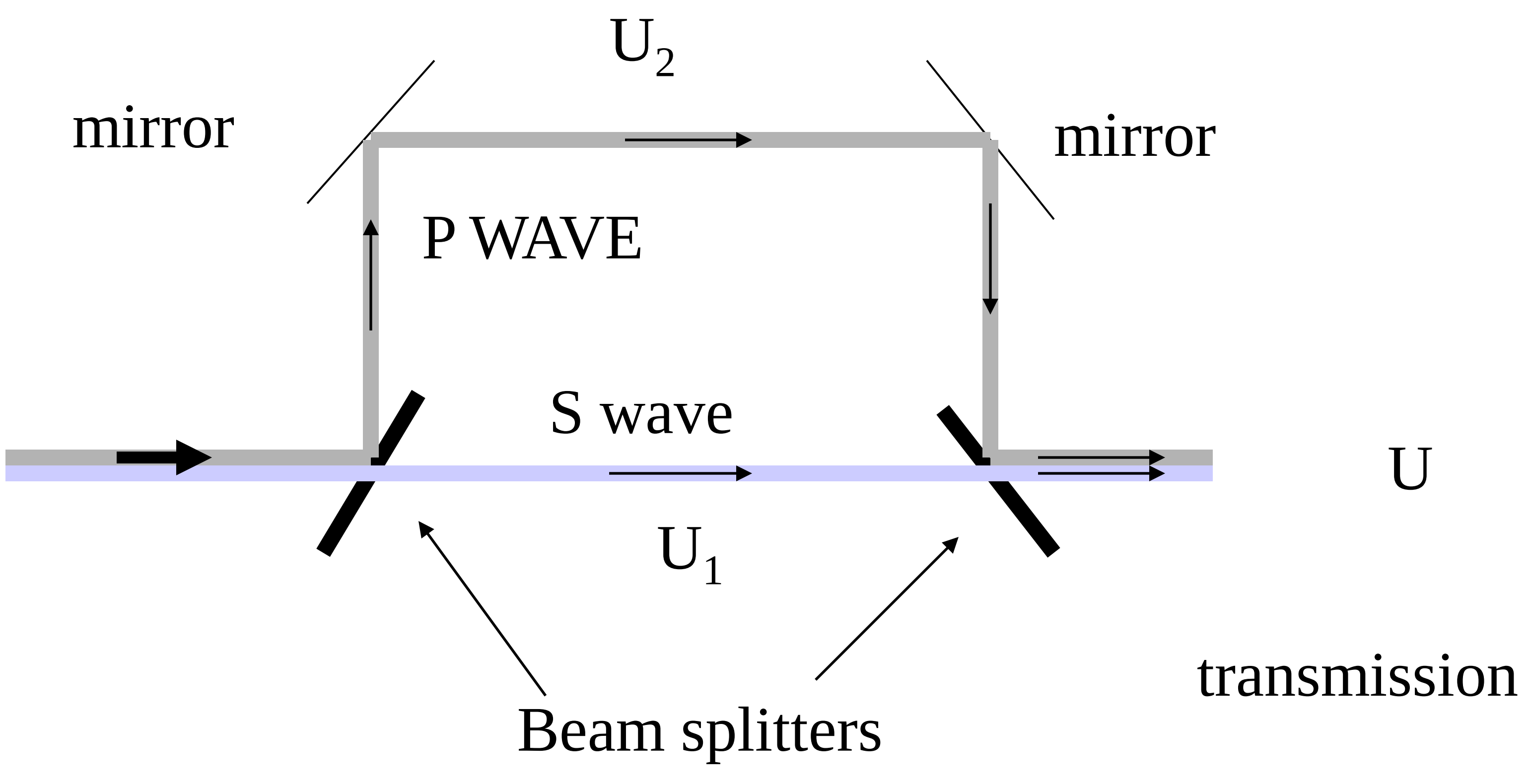
Since the measurement are extremely difficult to make in three dimensional case, practical application has been limited to case of plane stress (σ_3=0), the stress-optic equation (2) reduces
to,

$$n_1 - n_0 = c_1\sigma_1 + c_2\sigma_2$$

$$n_2 - n_0 = c_1\sigma_2 + c_2\sigma_1 \qquad (4)$$

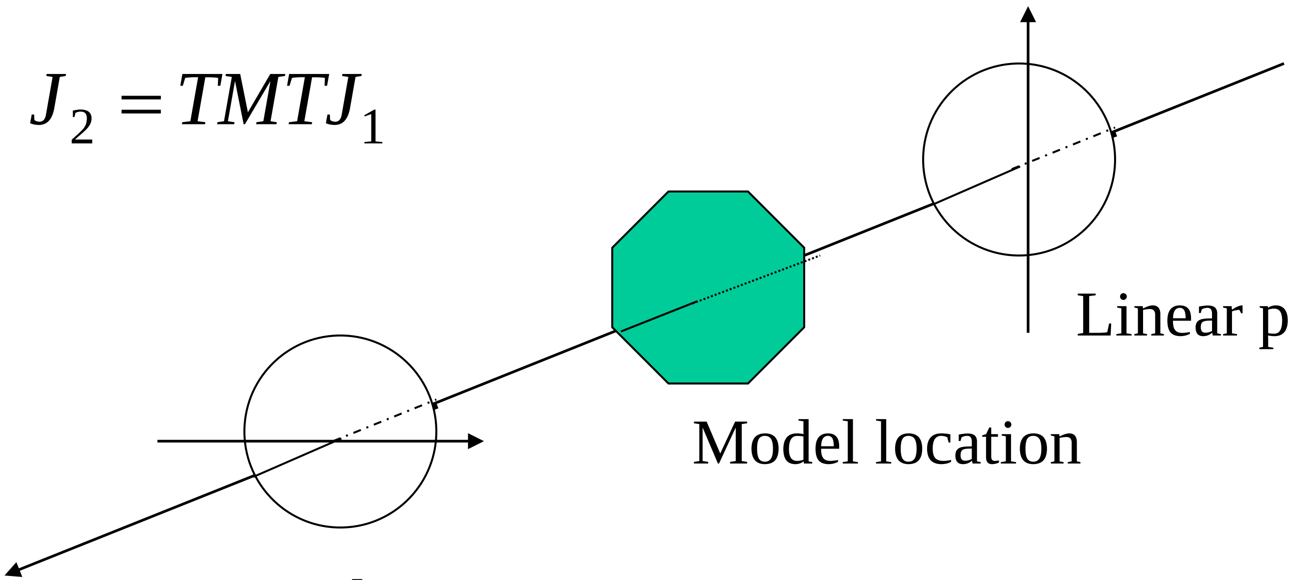
Absolute retardation using Mach-Zhender interferometer has been used to determine the individual principal stress on a loaded <u>two dimensional model</u>. However, a better approach is to use photoelasticity,which measure relative retardation ($n_2 – n_1$), by using <u>simple polariscope</u> which is easy to operate.

Free Space Mach-Zehnder Interferometer



Plane Polariscope

$$J_2 = TMTJ_1$$



$$T = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Plane stress measurement

For two dimensional plane-stress bodies where $\sigma_3 = 0$, the stress-optic for light at normal incident to the plane of the model without the subscript is

$$\boxed{\Delta = \frac{2\pi h c}{\lambda}(\sigma_1 - \sigma_2)} \qquad (5)$$

$$n = \frac{\Delta}{2\pi} = \frac{hc}{\lambda}(\sigma_1 - \sigma_2) = \frac{h}{f_\sigma}(\sigma_1 - \sigma_2) \qquad (6)$$

Where $n = \frac{\Delta}{2\pi}$ is retardation in terms of cycles of retardation, and counted as the fringe order.

$\boxed{f_\sigma = \frac{\lambda}{c}}$ is material fringe value, a property of the model material for a given λ and h

It is clear that relative stress difference in 2-D model can be determined if relative retardation n can be measured and f_σ can be established by calibration. Polariscope is used to determined the value of n each point in the model.

Material with photoelastic effect

Table 5-9 Approximate Properties for a Few Photoelastic Model Materials

Material	Generic Types and Generic Names	Stress Fringe Value, f_σ (green light, $\lambda = 546$ nm) kN/m Fringe	Stress Fringe Value, f_σ lb/in. Fringe	Room-temperature Properties: Young's Modulus, E MPa	Young's Modulus, E ksi	Proportional Limit MPa	Proportional Limit ksi	Poisson's Ratio	Figure of Merit, Q E/f_σ
Glass		−300 +400	−1700 +2200	70,000	10^4	60	8.7	0.25	5600
Plexiglas	Polymethyl methacrylate: Lucite Perspex	−130	−700	2,800	400			0.38	0.57[illegible]
Celluloid	Cellulose nitrate	30–300	170–1700	2,200	300	35	5	0.33	1700–2[illegible]
Homolite 100	Polyester	24	140	3,900	560	48	7	0.35	4000
Homolite 911 (CR-39)	Allyl diglycol carbonate	16	90	1,700	250	21	3	0.4	2800
Epoxy	Araldite, Epon, Bakelite	11	60	3,300	430	55	8	0.37	8000
Polycarbonate	Makrolon PSM-1	7	40	2,600	360	3.5	5	0.28	9000
Polyurethane	Hysol	0.2	1	3	0.5	0.14	0.02	0.46	500
Gelatin		0.09	0.5	0.3	0.04	—	—	0.5	80

If a photoelastic model exhibits a perfectly linear elastic behavior, the difference in the principal strain $\varepsilon_1 - \varepsilon_2$ can be measured by established the fringe order n. The stress-strain relationship for 2-D state of stress are given by

$$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2)$$

$$\varepsilon_2 = \frac{1}{E}(\sigma_2 - \nu\sigma_1) \qquad (7)$$

$$\varepsilon_1 - \varepsilon_2 = \frac{(1+\nu)}{E}(\sigma_1 - \sigma_2)$$

Substitute the above equations into equation 6 $\sigma_1 - \sigma_2 = \frac{nf_\sigma}{h}$ yields

$$\boxed{\frac{nf_\sigma}{h} = \frac{E}{1+\nu}(\varepsilon_1 - \varepsilon_2)} \qquad (8)$$

$$\boxed{\frac{nf_\varepsilon}{h} = \varepsilon_1 - \varepsilon_2}$$ where $f_\varepsilon = \frac{1+\nu}{E} f_\sigma$ (Material fringes in terms of strain) (9)

Conditions

Only work for perfectly linear elastic photoelastic model.

n can be found is when three of the materials properties E, f_σ ν f_ε are known. However, many photoelastic material exhibits viscoelastic properties. So linear elasticity and $f_\varepsilon = \frac{E}{1+\nu} f_\sigma$ are not valid

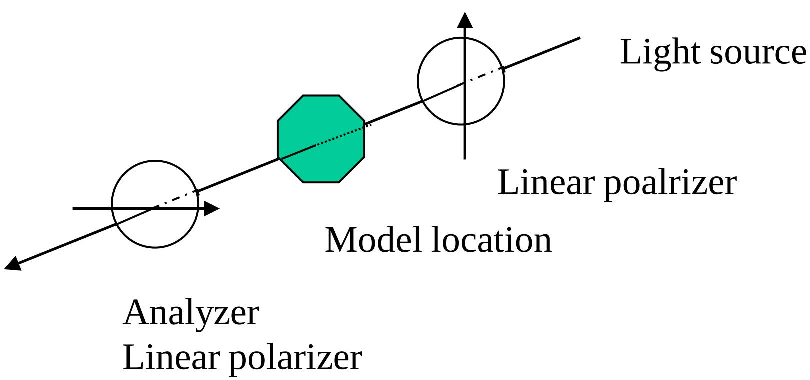
Polariscope

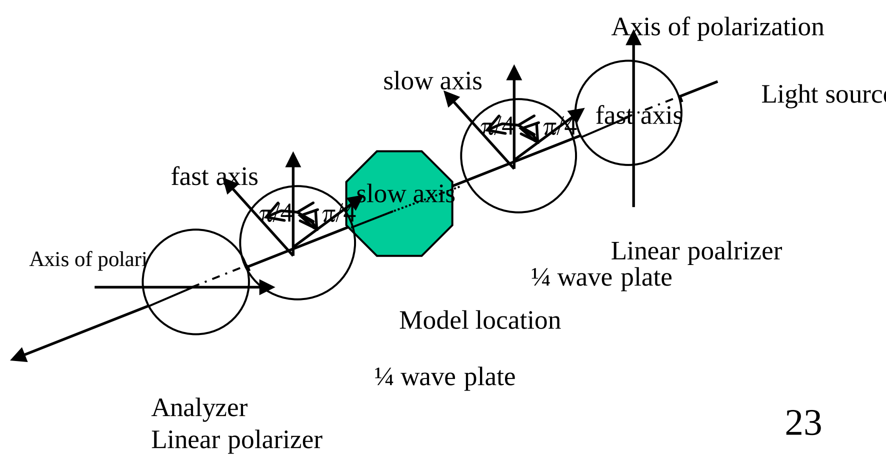
Polariscope is an optical instrument that utilizes the properties of polarized light in its operation.

For stress analysis two types are used:

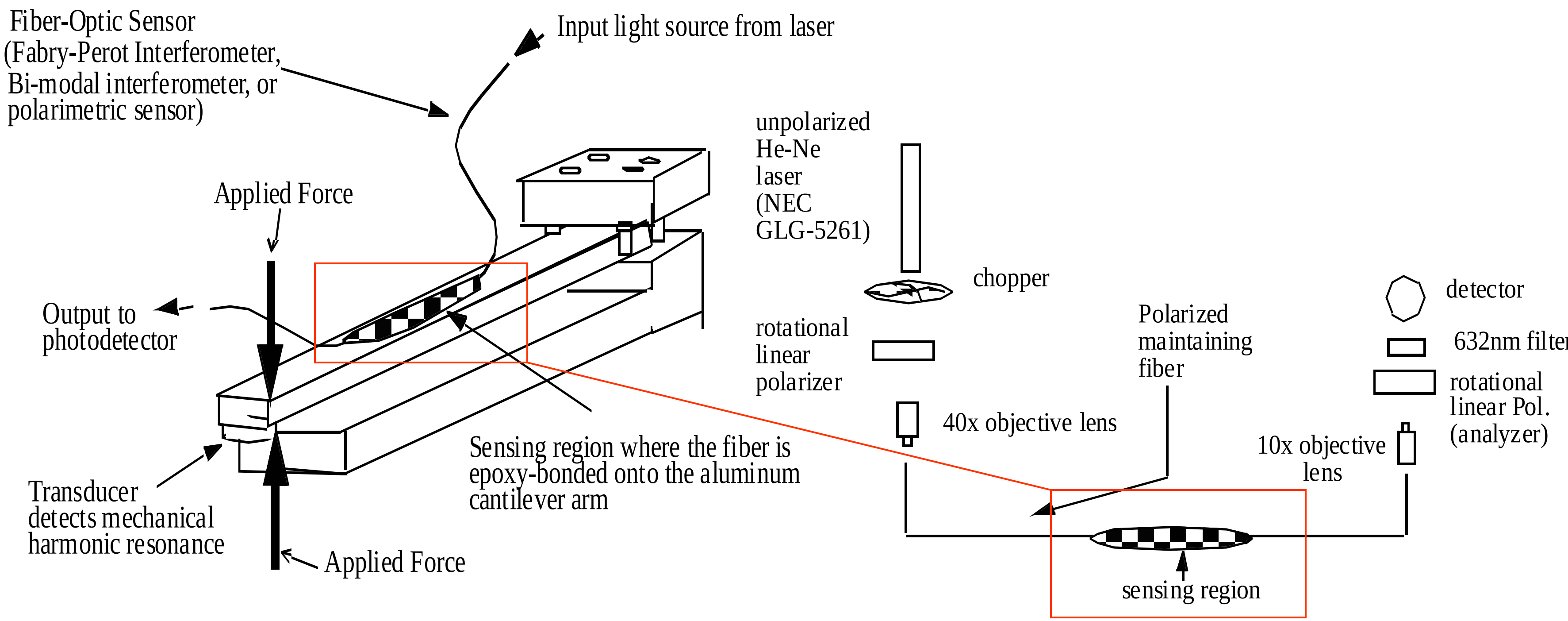
1. Plane (Linear) polariscope
2. Circular polariscope

Optical equipments used to produces circularly elliptically lights requires both <u>linear polarizer and wave plates</u>.





Experiment Setup



W.Wang

Sensor Principle

The construction of polarimetric system can be analyzed by the Jones matrix where it is assumed that the optical fiber can be identified as a retarder without any coupling between the two orthogonal light waves in the fiber

$$E_{out} = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & e^{j\delta\phi} \end{bmatrix} \begin{bmatrix} \cos\theta_1 \\ \sin\theta_1 \end{bmatrix} E_o$$

In the case of $\theta_1 = 45^o$ and $\theta_2 = 45^o$, the output intensity.

$$I \propto 1/2\,(1 + \cos\delta\phi) E_o^2$$

Sensor Principle

The phase change due to the temperature or strain modulation can be expressed as

$$\delta\phi = \frac{2\pi\delta l\Delta B}{\lambda}$$

The birefringences ΔB of PM fibers can be calculated using the stress optic law by determining the stress condition at the center of the fiber core

$$\Delta B_{temp} = -\frac{2CE(T-T_c)}{\pi(1-\nu)}(\alpha_2-\alpha_1)\left(ln\left(\frac{b}{a}\right)-\frac{3}{4}(b^4-a^4)\right)\sin(2\varphi_b)$$

$$\Delta B_{strain} = -\frac{2CE\varepsilon}{\pi(1-\nu)}(\nu_2-\nu_1)\left(ln\left(\frac{b}{a}\right)-\frac{3}{4}(b^4-a^4)\right)\sin(2\varphi_b)$$

Where $\alpha_1-\alpha_2 = -1.14\times10^{-6}/^oC$ are the thermal expansion coefficients of the cladding and the bow-tie material regions , ν_1, ν_2 are the poisson ratios of the cladding and bow-tie region, $\nu = \nu_1$ is the Poisson ratio of the core, $T_c = 900^oC$, is the setting temperature, T is the ambient temperature (variable), C is the stress optic coefficient and $C = -3.36x10^{-6}mm^2/N$, E is Young's Modulus of the fiber (E = 7.83x10^{10} N/m^2), $\varphi_b = 45^o$ angle of the bow-tie, ε is the axial strain (variable), $a = 0.056$ and $b = 0.36$ are the normalized radius from the fiber axis to the beginning of the bow-tie and the radius from the fiber axis to the end of the bow-tie, respectively

Let's assume the real time-space E vector has x and y components:

$$E(z,t) = a\cos(\omega t - kz + \phi_a)\hat{x} + b\cos(\omega t - kz + \phi_b)\hat{y}$$

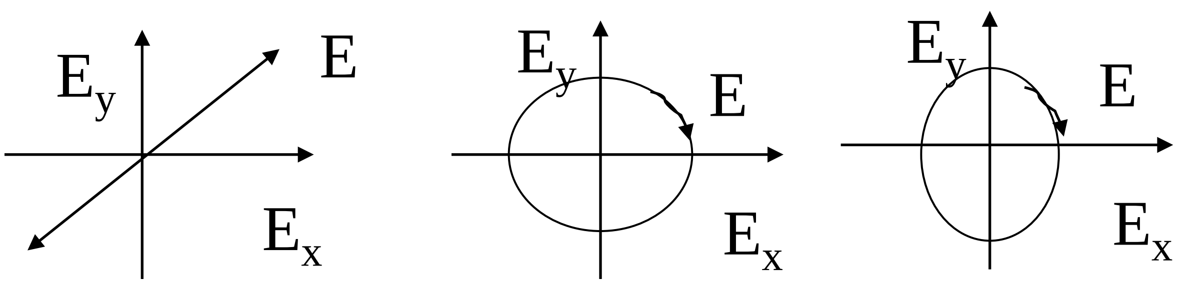
$E_y/E_x = Ae^{j\phi}$

linearly polarized: $\phi_b - \phi_a = 0..or\ \pi$ $\qquad E_y = \pm(\frac{b}{a})E_x$

circularly polarized: $\phi_b - \phi_a = \pm\frac{\pi}{2}$ $\qquad \frac{E_y}{E_x} = \frac{b}{a} = 1$

Elliptically polarized: $\phi_b - \phi_a = anything..except..0, \pi, \pm\frac{\pi}{2}$

$$\frac{E_y}{E_x} = \frac{b}{a} = anything$$



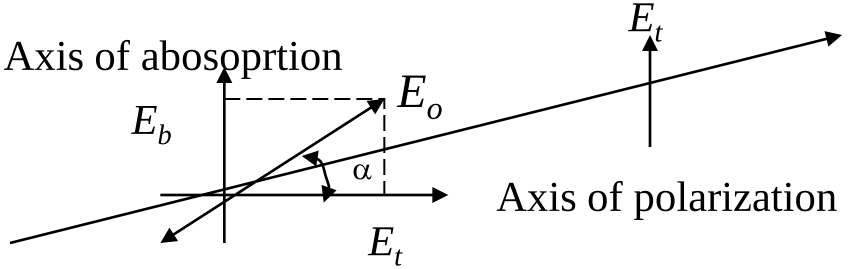
Linear Polarizer

The transmitted components of light vector are

$$E_t(z,t) = E_o \cos(\omega t - kt)\cos\alpha$$ (transmitted component)

$$E_b(z,t) = E_o \cos(\omega t - kt)\sin\alpha$$ (absorbed component)

α = phase difference between axis of polarization and incident wave

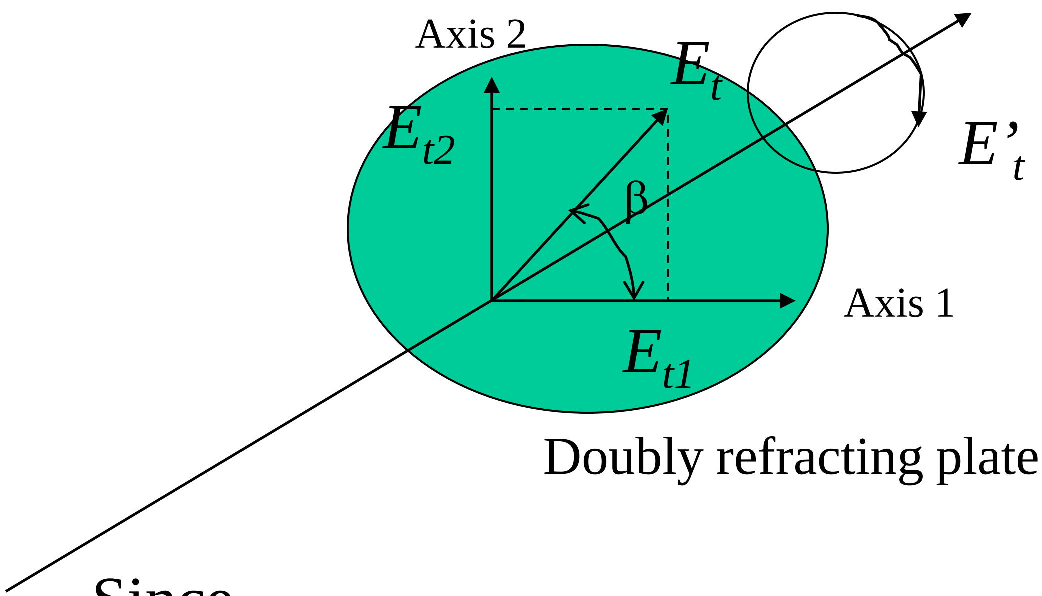


Polaroid filters are almost always used for producing polarized light. Most modern polariscopes containing linear polarizers employed Polaroid H sheet, a trasnparednt matreial with strained and oriented molecules- thin sheet of polyvinyl alcohol is heated, stretched, and immediately bonded to a supporting sheet of cellulose acetate butyrate. The polyvinyl face of the assembly is then strained by a liquid rich in iodine. The amount of iodine diffused into the sheet determines its quality. There are five grades denoted according to their transmittance of light: HN-22, 32, 35, 38. HN-22 has the best transmission.

Wave plate

Optical element which has the ability to resolve a light vector into two orthogonal components and to transmit the components with different velocities. Such a material is called doubly refracting or birefringent.

The birefringent effect can be illustrates in the following figure. Two principal axes labeled 1 and 2 has velocity $c_1 > c_2$, thus axis 1 is called fast axis and axis 2 is called slow axis. The light vector is resolved into two components E_{t1} *and* E_{t2}.



Doubly refracting plate

$$E_{t1} = E_o \cos\alpha \cos\beta$$

$$E_{t2} = E_o \cos\alpha \sin\beta$$

Since

Phase shift between E_{t1} *and* E_{t2} *can be expressed in terms of index change in both axes*

$$\delta_1 = h\,(n_1 - n_o)$$
$$\delta_2 = h\,(n_2 - n_o)$$

n_o is refractive index of air
h = thickness of the plate

Index change between two axes is $\delta = \delta_1 - \delta_2$

The relative phase shift Δ due to relative index change between two axes is

$$\Delta = 2\pi\delta/\lambda = 2\pi h(n_1 - n_2)/\lambda$$

When $\Delta = \pi/2$ *it is called quarter-wave plate*
$\Delta = \pi$ *it is called half-wave plate*
$\Delta = 2\pi$ *it is called full-wave plate*

Upon existing a wave plate exhibiting a retardation Δ*, the component of light are described by*

$$E'_{t1} = E_o \cos\alpha \cos\beta \cos\omega t$$
$$E'_{t2} = E_o \cos\alpha \sin\beta \cos(\omega t - \Delta)$$

We ignore the additional phase as result of passage through waveplate

Amplitude of light after waveplate:

$$E'_t = \sqrt{(E'_{t1})^2 + (E'_{t2})^2} = E_o \cos\alpha \sqrt{\cos^2\beta\cos^2\omega t + \sin^2\beta\cos^2(\omega t - \Delta)}$$

Angle emerge light vector made with axis 1 (fast axis) is

$$\tan\gamma = \frac{E'_{t2}}{E'_{t1}} = \frac{\cos(\omega t - \Delta)}{\cos\omega t} = \phi_{t2} - \phi_{t1}$$

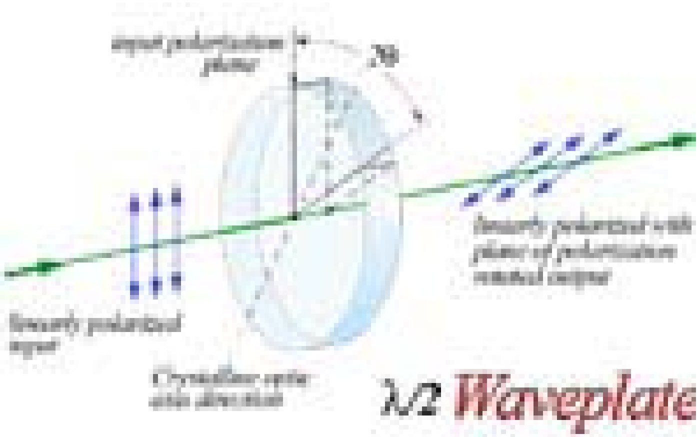
Both amplitude and rotation of emerging light vector can be controlled by the wave plate. Controlling factors are the relative phase difference Δ and the orientation angle β.

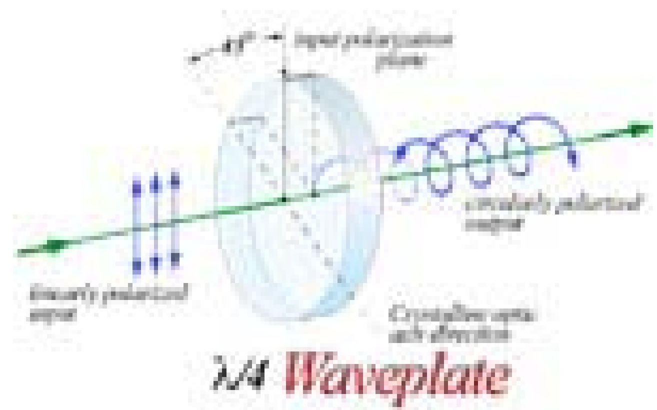
Waveplate is usually consist of a single plate of quartz or calcite cut parallel to the optical Axis or sheet of polyvinyl alcohol. Since most of the sheet is about 20mm for ¼ wave plate, The retarders are usually laminated between two sheets of cellulose acetate byutyrate.

Retardation plates

Retardation plates or phase shifters, including ¼ or ½ wave plates,
Are usually used primarily for synthesis and analysis of light
In various polarization states.

When combine with polarizer, it either rotates the polarization or
change linear polarized light into circularly polarized light.





Conditioning of light by a series combination of linear polarizer and a wave plate

Linearly polarized

$\beta = 0$

$$E_t' = E_o \cos\alpha \sqrt{\cos^2 \omega t}$$

$$\tan\gamma = \frac{E_{t2}'}{E_{t1}'} = 1$$

Circularly polarized

$\beta = \pi/4,\ \Delta = \pi/2$

$$E_t' = \frac{\sqrt{2}}{2} E_o \cos\alpha \sqrt{\cos^2 \omega t + \sin^2 \omega t}$$

$$\gamma = \omega t$$

Jones Vectors

A monochromatic plane wave of frequency of w traveling in z direction is characterized by

$$E_x = a_x\, e^{j\phi x}$$
$$E_y = {}_{ay}\, e^{j\phi y} \qquad (10)$$

of x and y component of the electric fields. It is convenient to write these complex quantities in the form of a column matrix

$$J = \begin{bmatrix} E_x \\ E_y \end{bmatrix} \qquad (11)$$

Total intensity is $I = (|E_x|^2 + |E_y|^2)/2\eta$ Use the ratio a_y / a_x and phase difference $\phi_y - \phi_x$ to determines the orientation and shape of the polarization ellipse

Jones matrix

Consider the transmission of a plane wave of arbitrary polarization through an optical system that maintains the plane-wave nature of the wave, bu alters its polarization, the complex envelopes of the two electric-field components of the input E_{1x}, E_{1y} and those output waves, E_{2x}, E_{2y} can be express by weighted superposition,

$$\begin{bmatrix} E_{2x} \\ E_{2y} \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} E_{1x} \\ E_{1y} \end{bmatrix} \quad \Rightarrow \quad J_2 = TJ_1 \qquad (12)$$

Linearly polarizer

$$T = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Linearly polarized along x axis

Polarization rotator

$$T = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Rotate linearly polarized light By an angle of θ

Wave retarder

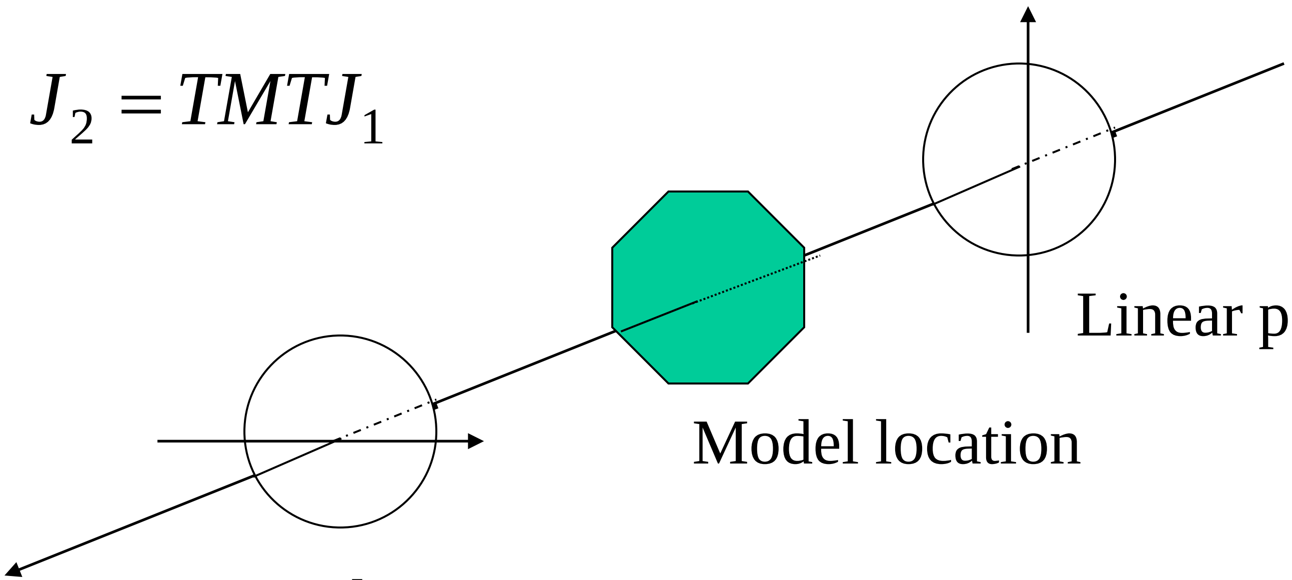
$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{-j\Gamma} \end{bmatrix}$$

Fast axis Along x axis

$\Gamma = \pi/2$ quarter wave retarder
$\Gamma = \pi$ half wave retarder

Plane Polariscope

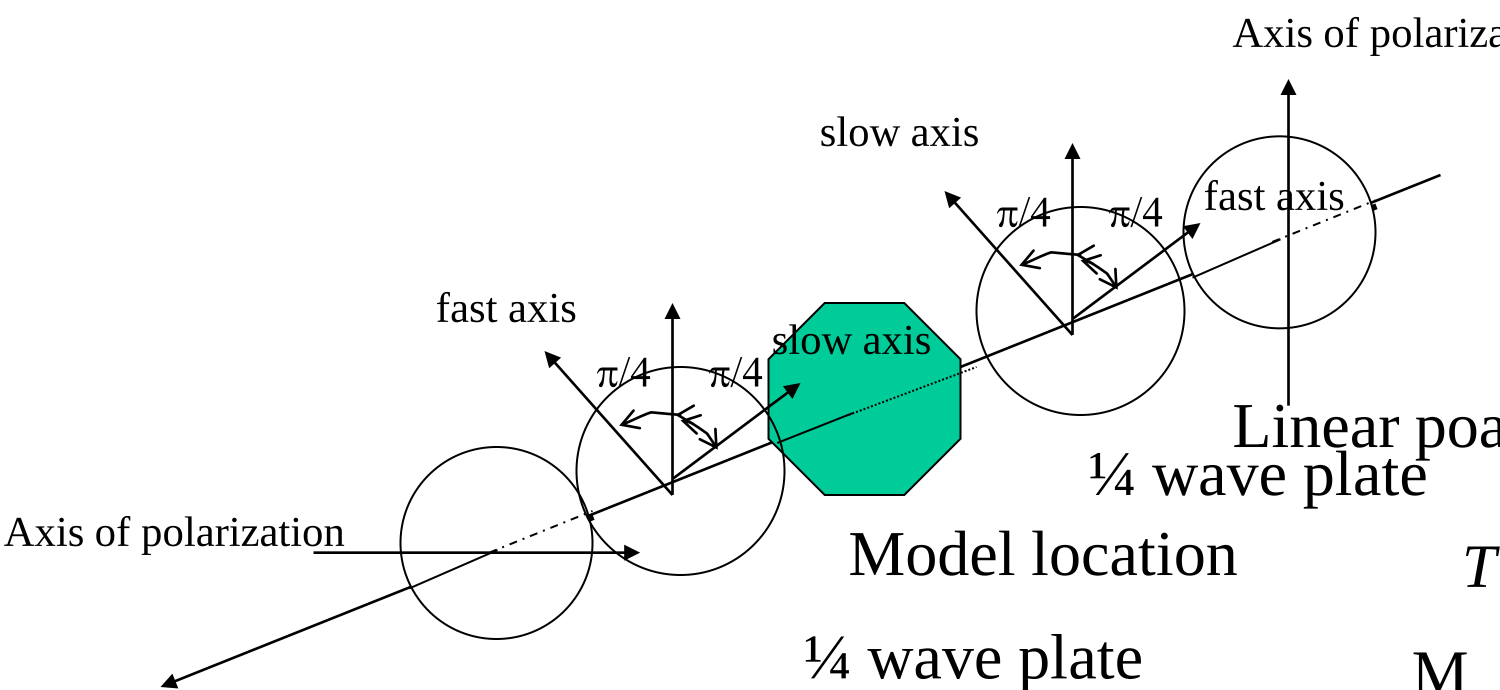
$$J_2 = TMTJ_1$$



$$T = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Circular Polariscope



$$T = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{-j\Gamma} \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{-j\Gamma} \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

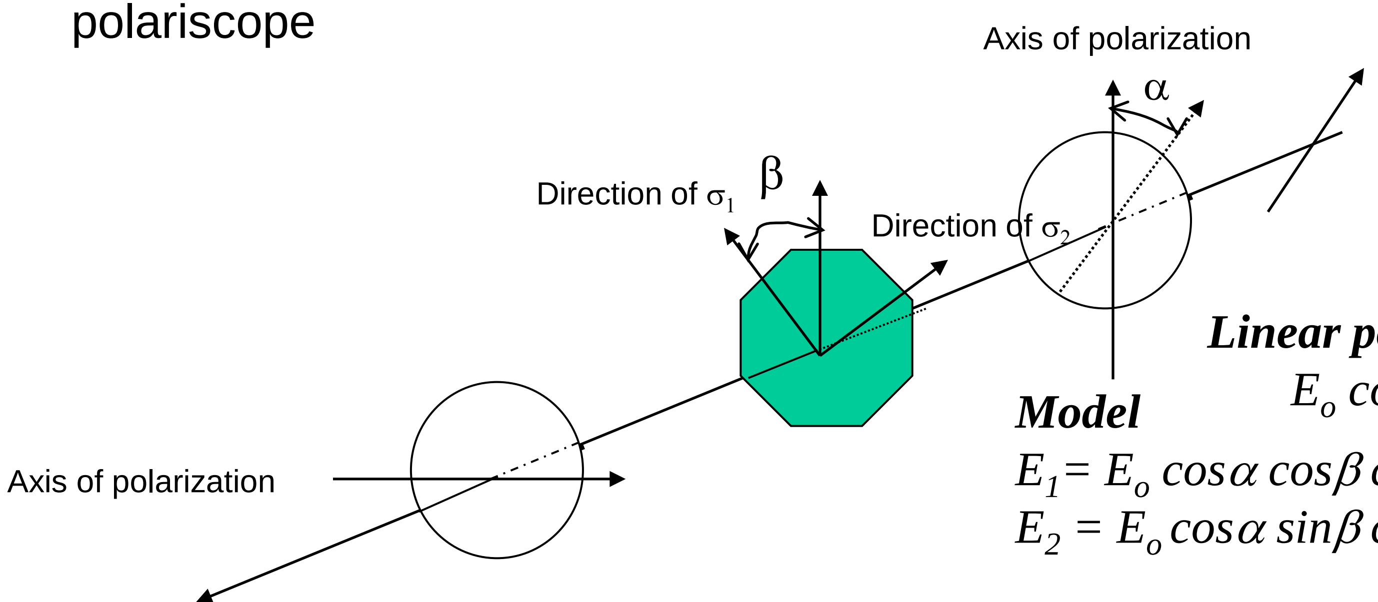
$$J_2 = TTMTTJ_1$$

Effect of stressed model in plane polariscope

<u>It is clear that principal stress difference σ_1-σ_2 can be determined in 2-D model If fringe order N is measured at each point in the model</u>

Optical axes of the model coincide with principal stress directions.

Insert the model into <u>plane polaricopec</u> with its normal coincident with the axis of polariscope



E_o

Linear polarizer

$E_o \cos\alpha$

Model

$E_1 = E_o \cos\alpha \cos\beta \cos\omega t$

$E_2 = E_o \cos\alpha \sin\beta \cos\omega t$

Analyzer

$E'_1 = E_o \cos\alpha \cos\beta \cos(\omega t - \Delta_1)$

$E'_2 = E_o \cos\alpha \sin\beta \cos\omega t - \Delta_2)$

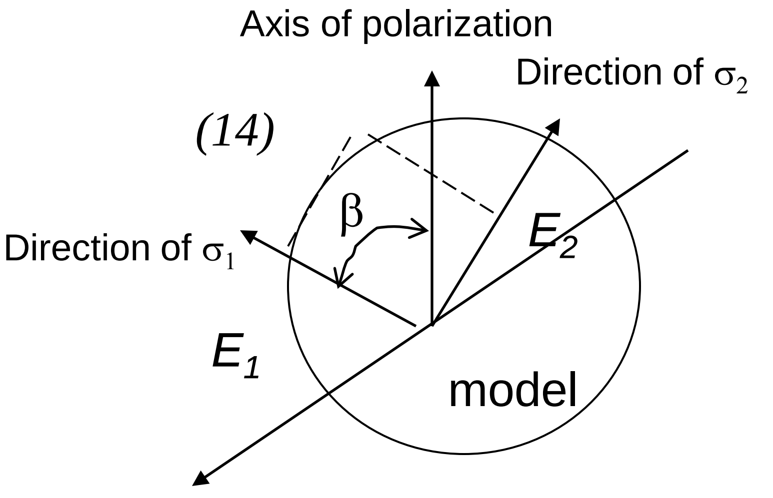
$E_{ax} = E_o \cos\alpha \sin 2\beta \sin[(\Delta_2 - \Delta_1)/2] \sin[wt-(\Delta_2 + \Delta_1)/2]$

Polarized light beam emerged from the linear polarizer can be represented by,

$$E = E_o \cos\alpha \qquad (13)$$

Polarized light enter the model (temporary waveplate), the incident light break Into two components

$$E_1 = E_o \cos\alpha \cos\beta \cos\omega t \qquad (14)$$
$$E_2 = E_o \cos\alpha \sin\beta \cos\omega t$$

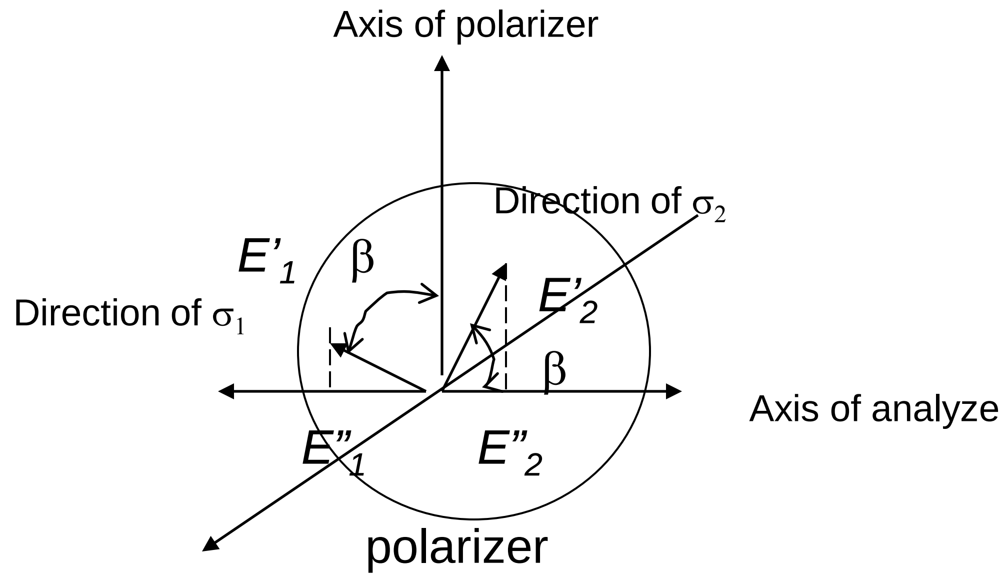


The two components propagates through the model with different velocities, the develop Phase shifts Δ_1 and Δ_2 with respects to a wave in air. The wave upon emerging From the model can be expressed as

$$E'_1 = E_o \cos\alpha \cos\beta \cos(\omega t - \Delta_1) \qquad (15)$$
$$E'_2 = E_o \cos\alpha \sin\beta \cos\omega t - \Delta_2)$$

$$\Delta_1 = 2\pi h/\lambda\, (n_1 - 1)$$
$$\Delta_2 = 2\pi h/\lambda\, (n_2 - 1)$$

Combined horizontal component
Transmitted by the analyzer to
Produce an merging light vector



$$E_{ax} = E''_2 - E''_1 = E'_2\cos\beta - E'_1\cos\beta \qquad (16)$$

$$E_{ax} = E_o \cos\alpha \cos\beta \sin\beta\,[\cos(\omega t - \Delta_2) - \cos(\omega t - \Delta_1)]$$

$$= E_o \cos\alpha \sin 2\beta \sin[(\Delta_2 - \Delta_1)/2]\sin[\omega t - (\Delta_2 + \Delta_1)/2] \qquad (17)$$

1. Average angular phase shift $(\Delta_2 + \Delta_1)/2$ *affects phase shift but not amplitude.*
2. Relative retardation $(\Delta_2 - \Delta_1)/2$ *affect the amplitude and the resulting intensity is given*

$$\boxed{I = (E_o \cos\alpha)^2 \sin^2 2\beta \sin^2(\Delta/2)} \qquad (18)$$

- *where* $\Delta = \Delta_2 - \Delta_1 = 2\pi h\,(n_2 - n_1)/\lambda = 2\pi hc\,(\sigma_1 - \sigma_2)/\lambda$

The intensity I diminishes when either *sin* term goes to zero, and therefore we have two possible fringe patterns of points where the light is extinguished, i.e.,

- **" Isochromatics ", and indicate areas of constant stress magnitudes. Expressed by stress induced phase difference**
- **" Isoclinics ", and indicate principal stress directions expressed by β**

Equation (18) indicates that the intensity will become zero ($I = 0$, **black fringes** occur) in the following two cases:

When $\sin^2(2\beta) = 0$ which is related to the principle stress direction), **Isoclinic fringe patterns** will occur.

When $\sin^2(\Delta/2) = 0$ which is related to the principle stress difference), **Isochromatic fringe patterns** will occur.

Two families of optical fringes are observed through the plane polariscope as a result of the birefringence phenomenon: **Isoclinics and Isochromatics**.

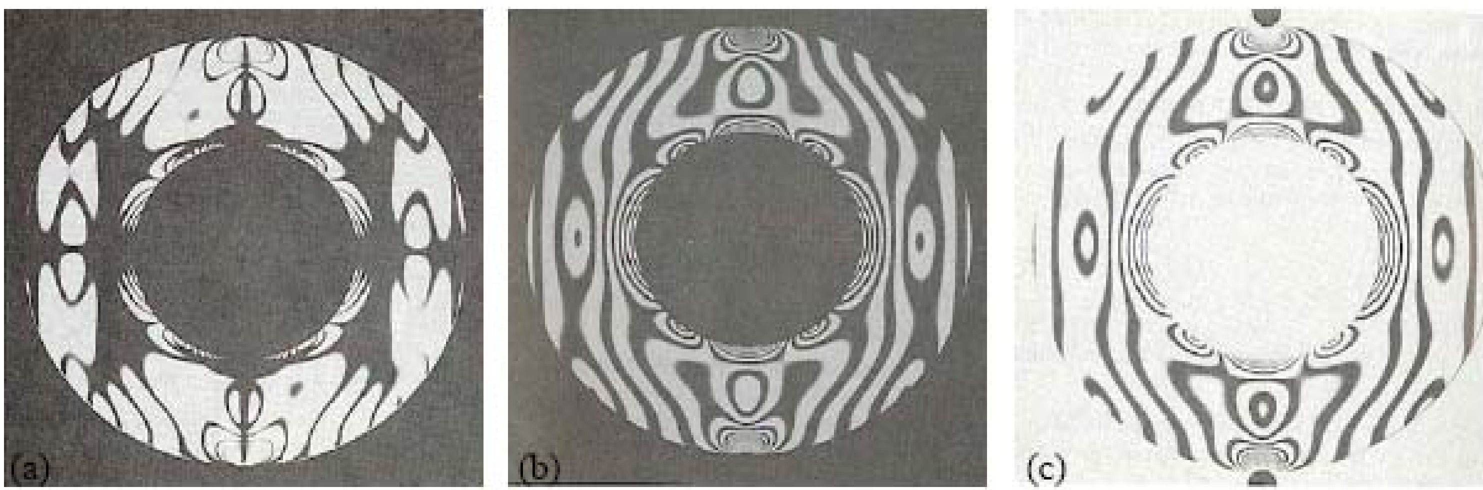


Figure 1: (a) Super-positioning of isochromatic and isoclinic fringes, as seen via a linear polariscope [4]. (b) Isochromatic fringes (lightfield), as seen via a circular polariscope. [5]. (c) Isochromatic fringes (darkfield), as seen via a linear polariscope. [6].

Isoclinics

In order to determine the directions of the principal stress it is necessary to use isoclinic lines as these dark fringes occur whenever the direction of either principal stress aligns parallel to the analyzer or polariser direction. The "isoclinics" are black fringes that describe the loci of constant ***principal directions*,** i.e. the lines joining all the points in the model where the *orientation of principal stresses is the same*. The specific orientation of principal directions corresponding to a certain isoclinic is determined by the specific orientation of the polarizer/analyzer combination, since "β" is the angle between the axis of the polarizer and the principal σ1 direction. Thus by rotating, in increments, the polarizer/analyzer pair of the polariscope in order to reach the conditions of $2\beta = n\pi$, n=0, 1, 2,…, a whole **family of isoclinics** may be obtained.

The principal direction corresponding to a certain isoclinic angle is related by Eq.(5) below to the stress components that define the state of stress at any point along that isoclinic:

$$Tan 2\theta = \frac{2\tau_{xy}}{\sigma_{xx} - \sigma_{yy}}$$

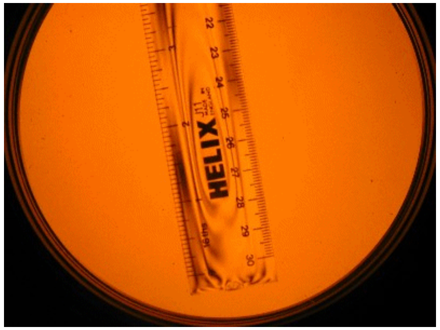
The "isochromatics" are lines of constant color which are obtained when a source of white light is used in the polariscope, and they are related to the *level of loading*. When a monochromatic source is used in the setup, only black fringes are observed, which are labeled by the **fringe order**, N=0,1,2,...., and they are caused by the extinction of the light emerging from the analyzer, as a result of a RELATIVE RETARDATION that meets the condition: $\Delta/2=n\pi$, n=0,1,2,…The fringe order, "N", of any such isochromatic fringe is related to the stress level at any particular point along that fringe by **the stress-optic law**:

$$\boxed{\sigma_1 - \sigma_2 = f_\sigma \frac{N}{h}}$$

where $\boldsymbol{\sigma}$□□ **and** $\boldsymbol{\sigma_2}$ are the principal stresses at that point, $\mathbf{f_\sigma}$ is the material fringe constant, **N** is the fringe order and **h** is the thickness of the plastic model . Contours of constant principal stress difference are therefore observed as σ_1-σ_2 isochromatic lines. It is obvious from Eq.(4) that a large number of fringes (large fringe orders) indicate regions of high stress in the model. In general, the principal-stress difference and the principal-stress directions vary from point to point in a photoelastic model. As a result the isoclinic fringe pattern and the isochromatic fringe pattern are SUPERIMPOSED when the model is viewed through a PLANE polariscope.

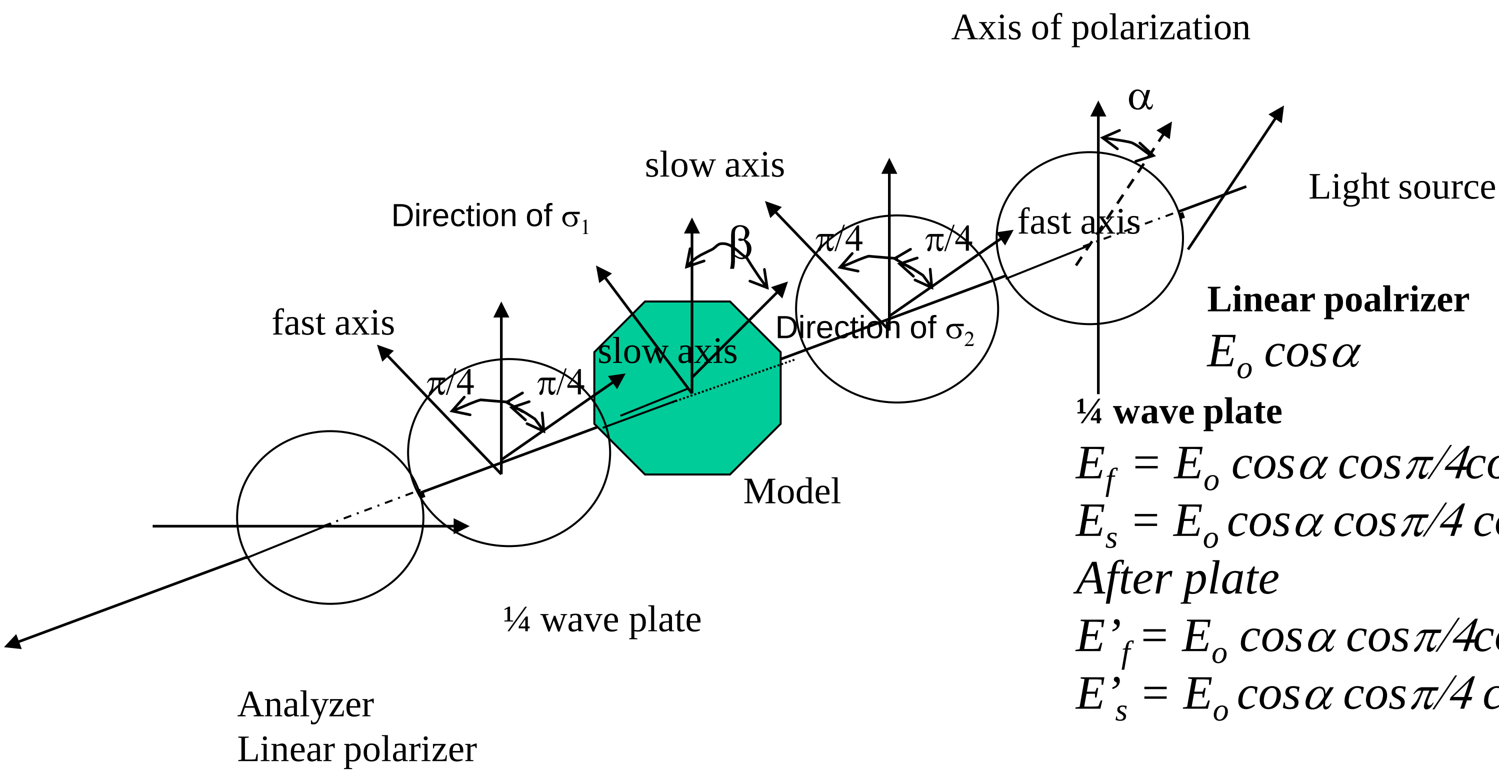
Isoclinic fringes can be removed by:

1. using a circular polariser. $I = \frac{1}{2} E_0^2 \sin^2 \frac{\Delta}{2}$
2. Image capturing digital processing techniques also allow for the separation of the isoclinic and isochromatic fringe patterns.
3. Isoclinic fringes can be observed by reducing the number of isochromatic fringes through either applying a smaller load or by using a material with a high material fringe constant.
4. The two types of fringes can be distinguished by rotating the specimen in a plane polariscope: isoclinic fringes will vary in intensity as they pass through the extinction positions. Isochromatic fringes should be invariant to the position of the specimen with respect to the polariser and analyser.



Effect of A stressed model in a circular polariscope

The use of circular polariscope eliminate the isoclinical fringe pattern while maintain the isochromatic fringe pattern.



Linear poalrizer

$E_o \cos\alpha$

¼ wave plate

$E_f = E_o \cos\alpha \cos\pi/4 \cos\omega t$

$E_s = E_o \cos\alpha \cos\pi/4 \cos\omega t$

After plate

$E'_f = E_o \cos\alpha \cos\pi/4 \cos\omega t$

$E'_s = E_o \cos\alpha \cos\pi/4 \cos(\omega t - \pi/2)$

Dark field arrangement

After linear polarizer, the wave is

$$E = E_o \cos\alpha \qquad (20)$$

Light enter quarter wave plate, it resolved into compoent E_f and E_s with vibration parallel to the fast and slow axes. Since quarter waveplate is oriented 45o to the polarizer Axis:

$$E_f = E_o \cos\alpha \cos\pi/4 \cos\omega t = \frac{\sqrt{2}}{2} E_o \cos\alpha \cos\omega t$$
$$E_s = E_o \cos\alpha \cos\pi/4 \cos\omega t = \frac{\sqrt{2}}{2} E_o \cos\alpha \cos\omega t \qquad (21)$$

Components propagate through the plate, they develop a relative angular pahse shift $\Delta = \pi/2$, And components emerge from plate out of phase by Δ:

$$E'_f = E_o \cos\alpha \cos\pi/4 \cos\omega t = \frac{\sqrt{2}}{2} E_o \cos\alpha \cos\omega t$$
$$E'_s = E_o \cos\alpha \cos\pi/4 \cos(\omega t - \pi/2) = \frac{\sqrt{2}}{2} E_o \cos\alpha \sin\omega t \qquad (22)$$

After leaving ¼ wave plate, the components of light vector enter the model. Since the stressed Model exhibits the characteristics of a temporary wave plate, the components E'_f and E'_s are resolved into component E_1 *and* E_2 which has directions coincident with principal-stress directions In the model:

$$E_1 = E'_f \cos(\frac{\pi}{4} - \beta) + E'_s \sin(\frac{\pi}{4} - \beta)$$

$$E_2 = E'_s \cos(\frac{\pi}{4} - \beta) - E'_f \sin(\frac{\pi}{4} - \beta)$$

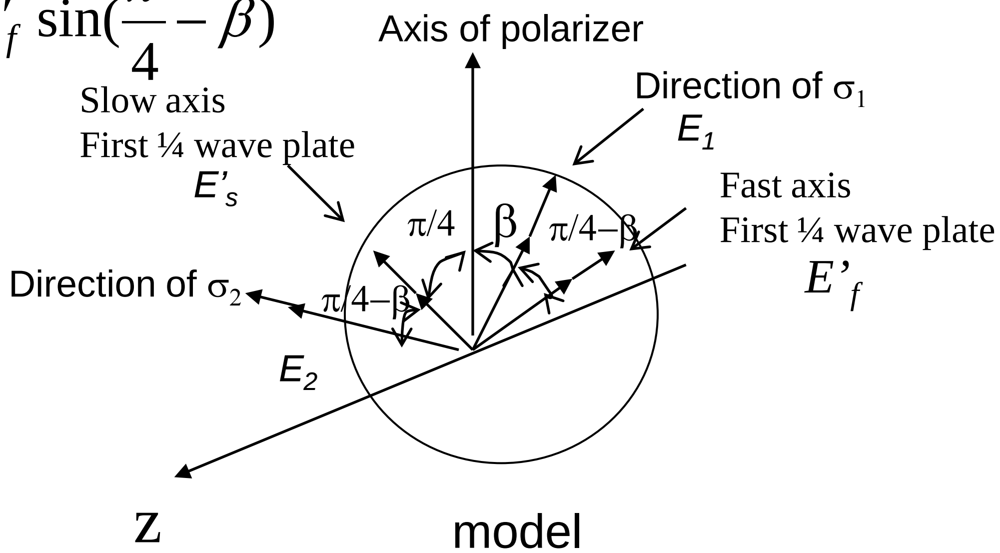
(23)

Substitute equation 22 into 23 we get,

$$E_1 = \frac{\sqrt{2}}{2} E_o \cos(\omega t + \beta - \frac{\pi}{4})$$

$$E_2 = \frac{\sqrt{2}}{2} E_o \sin(\omega t + \beta - \frac{\pi}{4})$$

(24)

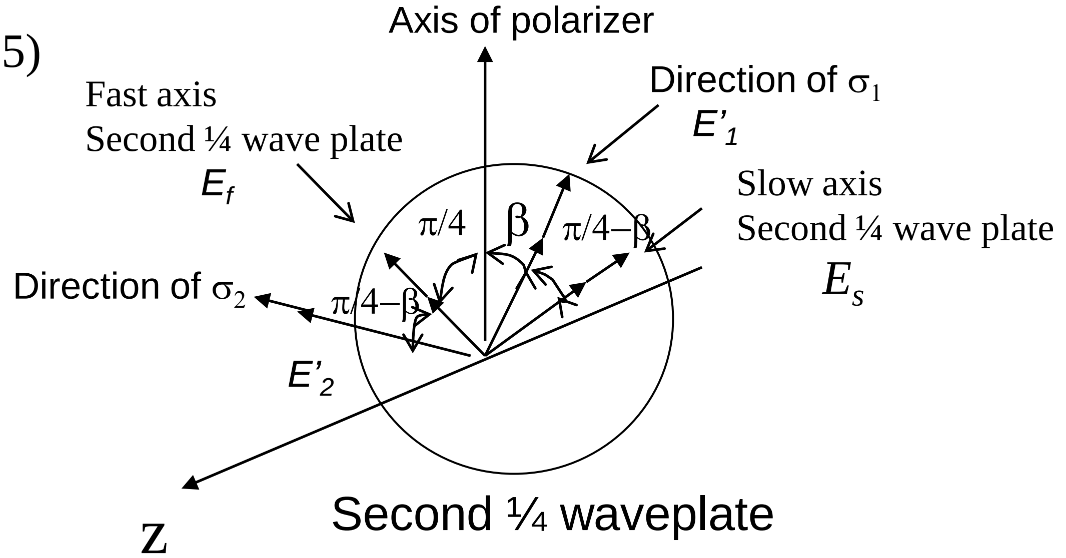


Resolution of the light as they enter the model

Going into the model

When it enter the model, an additional relative retardation Δ accumulated during passage through the model is given by $\Delta = \Delta_2 - \Delta_1 = 2\pi h/\lambda\ (n_2-n_1) = 2\pi hc/\lambda\ (\sigma_1-\sigma_2)$

$$E_1' = \frac{\sqrt{2}}{2} E_o \cos(\omega t + \beta - \frac{\pi}{4})$$
$$E_2' = \frac{\sqrt{2}}{2} E_o \sin(\omega t + \beta - \frac{\pi}{4} - \Delta) \qquad (25)$$



The light emerge from the model propagates to the second ¼ wave plate. The components associate with fast and slow axes of second ¼ wave plate are

$$E_f = E_1' \sin(\frac{\pi}{4} - \beta) + E_2' \cos(\frac{\pi}{4} - \beta)$$
$$E_s = E_1' \cos(\frac{\pi}{4} - \beta) - E_2' \sin(\frac{\pi}{4} - \beta) \qquad (26)$$

Resolution of the light components as they enter second ¼ plate

W.Wang

Substitute equation 25 into 26 yields,

$$E_f = \frac{\sqrt{2}}{2} E_o \cos(\omega t + \beta - \frac{\pi}{4}) \sin(\frac{\pi}{4} - \beta) + \frac{\sqrt{2}}{2} E_o \sin(\omega t + \beta - \frac{\pi}{4} - \Delta) \cos(\frac{\pi}{4} - \beta)$$

$$E_s = \frac{\sqrt{2}}{2} E_o \cos(\omega t + \beta - \frac{\pi}{4}) \cos(\frac{\pi}{4} - \beta) + \frac{\sqrt{2}}{2} E_o \sin(\omega t + \beta - \frac{\pi}{4} - \Delta) \sin(\frac{\pi}{4} - \beta) \qquad (27)$$

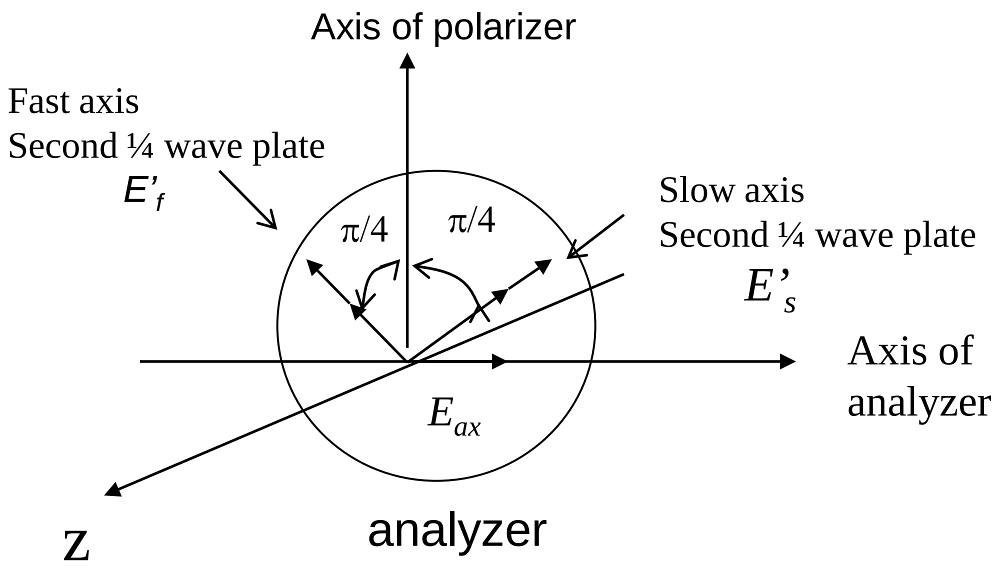
As light pass through ¼ wavelplate, a relative phase shift of $\Delta = \pi/2$ developed between the fast and slow components

$$E'_f = \frac{\sqrt{2}}{2} E_o \cos(\omega t + \beta - \frac{\pi}{4}) \sin(\frac{\pi}{4} - \beta) + \frac{\sqrt{2}}{2} E_o \sin(\omega t + \beta - \frac{\pi}{4} - \Delta) \cos(\frac{\pi}{4} - \beta)$$

$$E'_s = \frac{\sqrt{2}}{2} E_o \sin(\omega t + \beta - \frac{\pi}{4}) \cos(\frac{\pi}{4} - \beta) + \frac{\sqrt{2}}{2} E_o \cos(\omega t + \beta - \frac{\pi}{4} - \Delta) \sin(\frac{\pi}{4} - \beta) \qquad (28)$$

Light vector enter the analyzer vertical components are aborted while horizontal components are transmitted to give

$$E_{ax} = \frac{\sqrt{2}}{2}(E'_s - E'_f) \quad (29)$$



Substitute eq. 28 into 29 to give
An expression for light emerging from
The analyzer (dark field arrangement)

$$E_{ax} = \frac{\sqrt{2}}{2} E_o \sin\frac{\Delta}{2}\sin(\omega t + 2\beta - \frac{\Delta}{2})$$

Components of light vectors which
Are transmitted through the analyzer
(dark field)

Since the intensity of light is proportional to the square of the amplitude of the light wave, the Light emerging from the analyzer of a circular polariscope is given by

$$\boxed{I = \frac{1}{2} E_0^2 \sin^2\frac{\Delta}{2}}$$

Circular Polariscope

- Light beam emerging from the circular polariscope is a function of only principal-stress difference Δ since the angle α does not appear in the expression for the amplitude of the wave.

- Isoclinic fringes have been eliminated

-Extinction of fringes occurs at $\Delta/2 = n\pi$ where $n = 1,2,3,4\ldots.$

$$n = \Delta/2\pi$$

- light field can be observed simply rotate analyzer by 90^o

$$\Delta/2 = (1+2n)\,\pi/2 \text{ where } n = 1,2,3,4\ldots.$$

(extinction of fringe) $N = \Delta/2\pi = ½+n$

How to solve it using polarimetric technique?

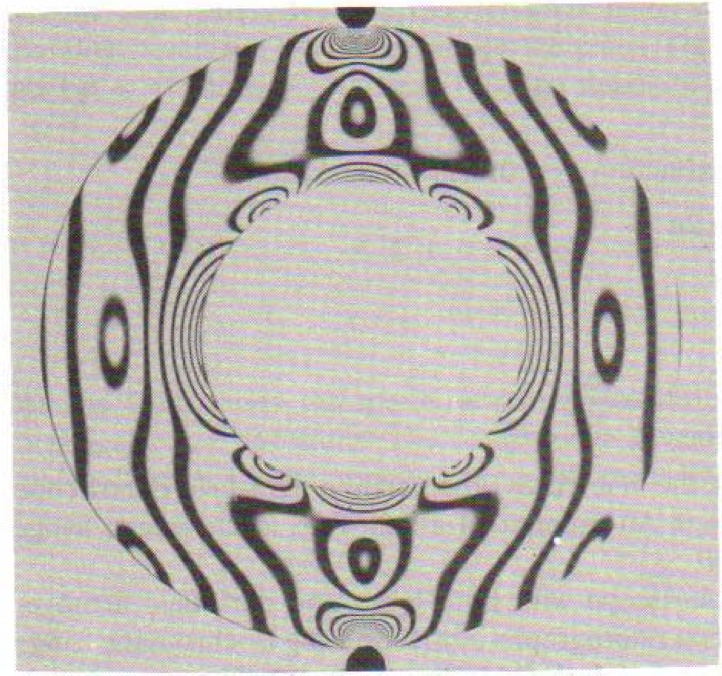
- Need to position principle axes along the two polarizer directions (to eliminate isoclinic fringes)
- Load normal or transverse load so

$$n = \frac{\Delta}{2\pi} = \frac{hc}{\lambda}(\sigma_1 - \sigma_2) = \frac{h}{f_\sigma}(\sigma_1 - \sigma_2) \sim \frac{h}{f_\sigma}\sigma_1 or \frac{h}{f_\sigma}\sigma_2$$

$$n = \frac{h}{f_\varepsilon}\varepsilon_1 - \varepsilon_2 \sim \frac{h}{f_\varepsilon}\varepsilon_1 \, or \quad \frac{h}{f_\varepsilon}\varepsilon_2$$

- When both can’t separated, we use: supplementary data or employing numerical methods:

- Separation methods, analytical separation method, scaling model to prototype stresses

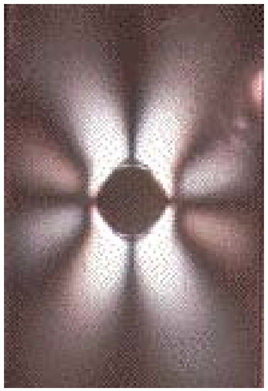


(a) Dark field

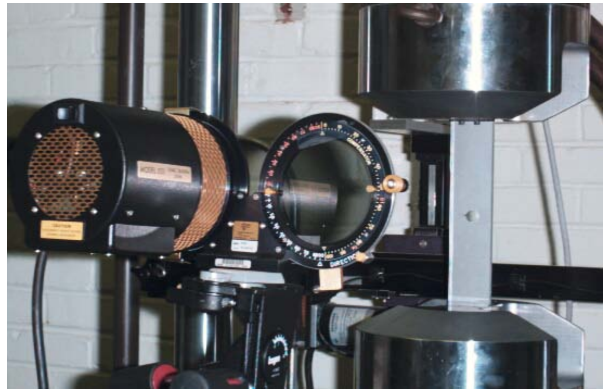
(b) Light field

Isochromatic fringe patterns of a ring loaded in diametral compression

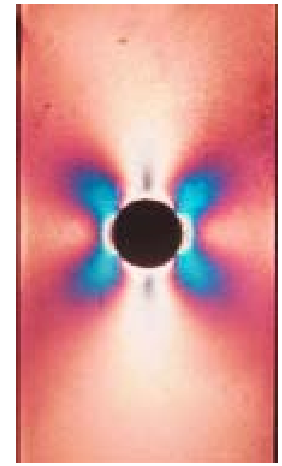
Strain Map around a hole



Zero degree isoclinic image for a hole in plate under vertical tension.



A polariscope positioned in front of a bar with a central hole placed under vertical tension by a hydraulic load frame



Isochromatic fringes for a hole in plate under vertical tension.

Two different patterns are produced with the polariscope -- isochromatics via circular polarization and isoclinics via linear polarization. Isochromatic fringes appear as a series of successive and contiguous different-colored bands each representing a different degree of birefringence corresponding to the underlying strain. The patterns can be read like a topographic map to visualize the stress distribution over the surface of the coated test part. The isoclinic fringes appear as black bands providing the direction of the principal strain.

Tardy Compensation

The analysis for the dark- and light- field arrangements of the circular polariscope can be carried one step further to include rotation of the analyzer through some arbitrary angle. The purpose of such a rotation is to provide a means for <u>determining fractional fringe orders. (accuracy~ 0.02 fringes)</u>

$$n = \Delta/2\pi = n \pm \gamma/\pi$$

Procedure:

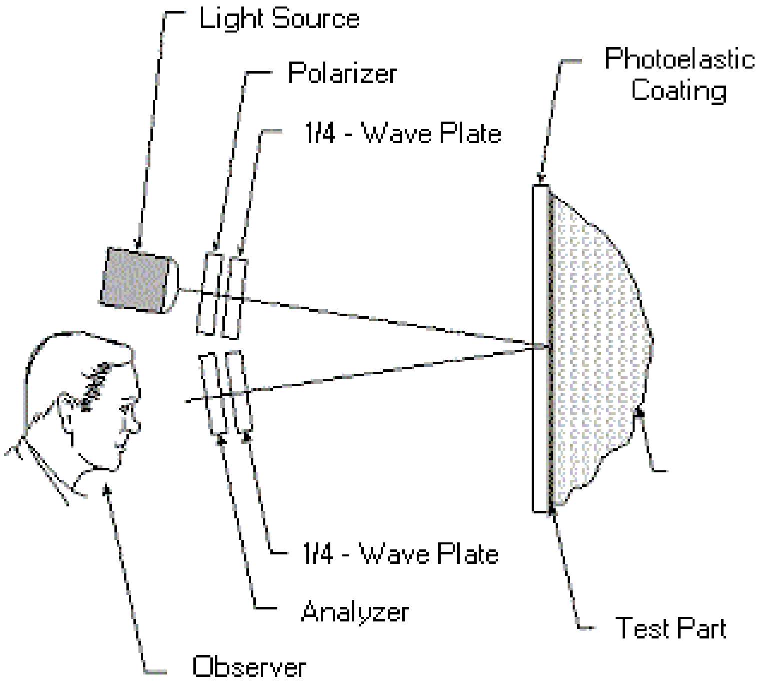
1. Use plane polariscope to find isoclinics
2. Once polarizer is aligned with principal stress direction, other elements of polariscope are oriented to produce a standard dark-filed circular polariscope,
3. Analyzer then rotated until extinction occurs at the point of interest

Reflection Polariscopes

Vishay Measurements Group 030 Series Modular Reflection Polariscope with Model 137 Telemicroscope Accessory

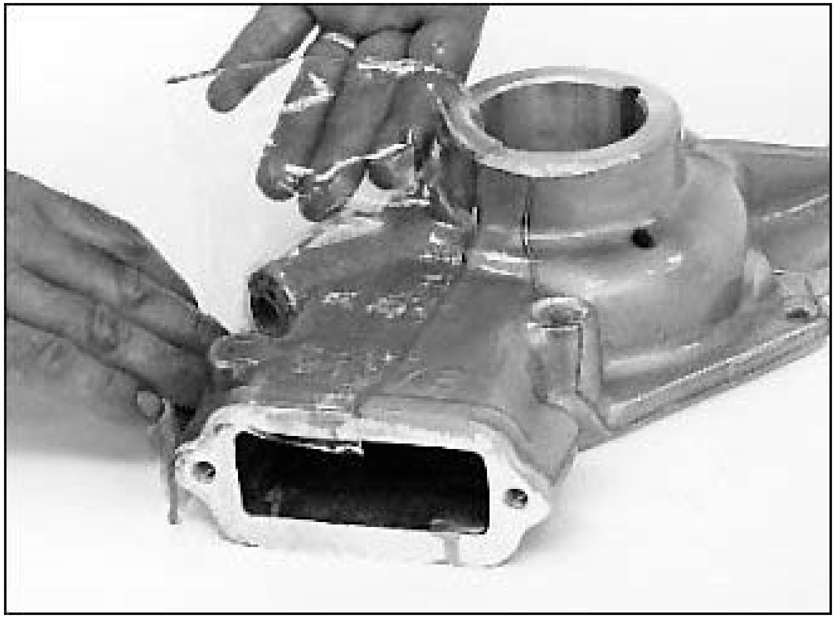
Vishay Measurements Group Model 040 Reflection Polariscope

Reflection Polariscopes



Vishay Measurements Group

For PhotoStress analysis, a reflection polariscope is used to observe and measure the surface profile



Vishay Measurements Group

PhotoStress coating being applied to water pump casting

Coating Materials and Adhesives

PhotoStress coating materials:
flat sheets, liquid plastics for casting contourable sheets, and adhesives for application to metals, concrete, plastics, rubber, and most other materials.
Company: Vishay Measurements Group
http://www.vishay.com/company/brands/measurements-group/guide/pstress/pcoat/pcoat.htm

Problem isolatring isoclinics and isochromatic fringe

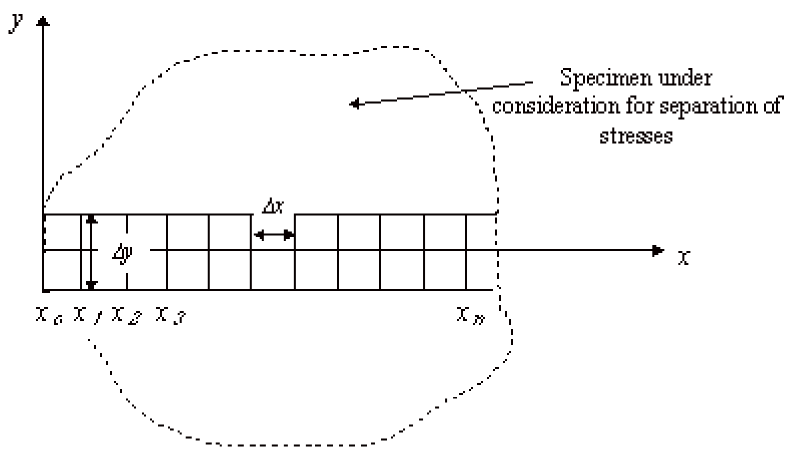
In a plane polarizer, isoclinic and isochromatic fringe patterns are superimposed upon each other. Therefore, in order to obtain the individual values of the stress, we must investigate separation techniques.

1.Methods based on the Equilibrium Equations: Shear Difference Method

The method described here is based solely on the equations of equilibrium (i.e., equilibrium of applied body forces, stresses, and shears), and as a result is independent of the elastic constants of the photoelastic model material. The equations of equilibrium when applied to the plane-stress problem can integrated in approximate form using the following finite difference expressions:

$$\sigma_x = (\sigma_x)_o - \sum \frac{\Delta \tau_{yx}}{\Delta y} \Delta x \quad and \quad \sigma_y = (\sigma_y)_o - \sum \frac{\Delta \tau_{yx}}{\Delta x} \Delta y$$

The benefit of the above method is that is can be readily visualized graphically, and applicable to arbitrary specimen geometry: Since the above procedure implements finite difference techniques, it leads way to the possibility of incorporating the shear difference method to automate the entire separation of isochromatic and isoclinic fringe patterns



2: Methods based on the Compatibility Equations

The compatibility or continuity equations can be expressed in the form of Laplace’s equation, the solution of which is known as a harmonic function. There are many methods for modeling and solving using Laplace’s function, including superposition of analytical harmonic functions, finite element techniques, as well as physical analogy methods such as electrical circuits modeled to suit the geometry in question.

3: Methods based on Hooke’s Law

Separation methods based on Hooke’s law make use of the fact that the sum of principal stresses can be determined if the change in thickness of the model, as a result of the applied loads, can be measured accurately at the point of interest. Instruments developed for the measurements of these changes (which are in the order of a few thousandths of a cm) include lateral extensometers and interferometers.

4: Oblique Incidence Methods

Rather than having the light pass through the model at normal incidence, the model can be rotated in the polariscope so that the light passes through the model at some other angle, producing an oblique incidence fringe pattern. This oblique incidence fringe pattern provides additional data which can be employed to separate the principal stresses.

Summary

Advantages:

No gratings needed, easier to operates

Disadvantages:

Principal axes need to be found
isoclinics and isochromatic fringes complicate the measurement

For model viewed with white light, a series of color bands of fringes will form.

- Intensity is zero and a black fringe appears only when the principal-stress difference is zero and a zero order of extinction occurs for all wavelengths of light.
- For nonzero value of principal-stress difference, only one wavelength can be extinguished from the white light. A complementary color appears as isochromatic fringe.
-Larger stress higher order, different colors at higher orders can disappear at once.

With monochromatic light , the individual fringe in an ischromatic fringe (dark bands) pattern remain sharp and clear to very high orders of extinction.

Number of fringes appearing in an isochromatic fringe pattern is

$$n = N = h(\sigma_1 - \sigma_2)/f_\sigma \tag{19}$$

Principal stress difference and the principal-stress direction vary from point to point in photoelastic model. As a result isoclinic and isochromatic fringes pattern are superimposed Bands should have zero width, but due to eye and photographic film, it appears nonzero.

Two different types of fringes can be observed in photoelasticity: isochromatic and isoclinic fringes.

Isochromatic fringes are lines of constant principal stress difference. If the source light is monochromatic these appear as dark and light fringes, whereas with white light illumination coloured fringes are observed. The difference in principal stresses is related to the birefringence and hence the fringe colour through the Stress-Optic Law.

Isoclinic fringes occur whenever either principal stress direction coincides with the axis of polarisation of the polariser. Isoclinic fringes therefore provide information about the directions of the principal stresses in the model. When combined with the values of from the photoelectric stress pattern, $\Delta = \frac{2\pi hc}{\lambda}(\sigma_1 - \sigma_2)$

isoclinic fringes provide the necessary information for the complete solution of a two-dimensional stress problem.

A standard plane polariscope shows both isochromatic and isoclinic fringes, and this makes quantitative stress analysis difficult.

With monochromatic light , the individual fringe in an ischromatic fringe (dark bands) pattern remain sharp and clear to very high orders of extinction.

Number of fringes appearing in an isochromatic fringe pattern is

$$n = N = h(\sigma_1 - \sigma_2)/f_\sigma \qquad (19)$$

Recall $n = \dfrac{\Delta}{2\pi} = \dfrac{hc}{\lambda}(\sigma_1 - \sigma_2) = \dfrac{h}{f_\sigma}(\sigma_1 - \sigma_2)$ From equation 6

Principal stress difference and the principal-stress direction vary from point to point in photoelastic model. As a result isoclinic and isochromatic fringes pattern are superimposed Bands should have zero width, but due to eye and photographic film, it appears nonzero.