Fabry-Perot Interferomter

The Fabry - Perot interferometer and related instruments make use of the multiple reflections between two plane parallel surfaces. They are all characterized by the same function, the Airy function $A(\phi)$, a function of increment of phase ϕ between successive beams. Suppose two parallel plates separated by an air gap of thickness d. Let the incident angle be θ. For simplicity, refraction and absorption of light is disregarded. As shown, a fraction of the light R_1 will reflect at the first face and a fraction T_1 will be transmitted and this will then meet the second reflected beam where the amount R_1T_1 will be reflected with a phase shift of $\phi/2$; (ϕ is the phase shift caused by 2 times the separation of plates) and $T_1T_1e^{j\phi/2}$ will be transmitted. The whole operation will repeat itself and net result is a series of diminishing geometric intensity. The total amplitude for m waves is:

$$E_t = t_1t_2e^{j\frac{\phi}{2}} + t_1t'_2r_1r_2e^{j\frac{3\phi}{2}} + t_1t'_2r_1^2r_2^2e^{j\frac{5\phi}{2}} + \dots + t_1t'_2r_1^{(m-1)}r_2^{(m-1)}$$

$$= t_1t_2e^{j\frac{\phi}{2}}(1 + r_1r_2e^{j\phi} + r_1^2r_2^2e^{2j\phi} + \dots)$$

$$E_t = t_1t_2e^{j\phi}\left(\frac{1}{1 - r_1r_2e^{j\phi}}\right)$$

Normalized transmission intensity therefore:

$$I_t(\phi) = E_{t(total)} \times E_{t(total)}^*$$

$$= \frac{|t_1 t_2|^2}{1 - |r'_1||r'_2|e^{-j\phi} - |r'_1||r'_2|e^{j\phi} + r'_1 r'_2}$$

$$I_t = \frac{T_1 T_2}{1 - 2(R_1 R_2)^{0.5}\cos(\phi) + R_1 R_2}$$

Where

$T_1 = t_1^2$;

$T_2 = t_2^2$;

$R_1 = r^2{}_{1;}$

$R_2 = r^2{}_2$;

$t_1 = 1 - r_1$

$r_1 = -r_1'$ (giving either direction from one mirror to the other is the same);

$r^2{}_1 + t'_1 t_1 = 1$ (Stoke's relation);

$e^{j\phi} + e^{-j\phi} = 2\cos(\phi)$;

In the same matter, there is a series of reflection therefore, the sum of these reflection will be

$$E_r = r_1 + (1 - r_1) \times (1 - r'_1) \times r'_2 \times e^{j\phi} + (1 - r_1) \times (1 - r'_1) \times r'_1 \times r'^2_2 \times e^{2j\phi} + \ldots$$

Since $t_1 = 1 - r_1$ and use above identities:

$$= r_1 + (1 - r_1)^2 \times r_2 \times e^{j\phi} \times (1 + r_1 r_2 e^{j\phi} + \ldots)$$

$$= r_1 + (1 - r_1)^2 r_2 e^{j\phi} \times \frac{1}{1 + (-r_1) r_2 e^{j\phi}}$$

Therefore

$$E_{r(total)} = \frac{-r_1 + r_2 e^{j\phi}}{1 - r_1 r_2 e^{j\phi}}$$

$$I_{r(total)} = \frac{|r_1|^2 - |r_1 r_2| e^{j\phi} - |r_1 r_2| e^{(-j)\phi} + |r_2|^2}{1 - |r_1 r_2| e^{j\phi} - |r_1 r_2| e^{(-j)\phi} + |r_1 r_2|^2}$$

$$I_r = \frac{R_1 + R_2 - 2(R_1 R_2)^{0.5} \cos\phi}{1 + R_1 R_2 - 2(R_1 R_2)^{0.5} \cos\phi}$$

where

$$\phi = 2 \text{ x } (2\pi\, yn)/\lambda \text{ x } \cos(\theta) ;$$

Finesse ξ

Finesse ξ ιs defined as the ratio of the half power bandwidth vs. the peak to peak full bandwidth. in the transmission intensity curve.

(half of the transmission power) $$\frac{I_t}{2} = \frac{T_1 T_2}{(1 + R_1 R_2 - 2(R_1 R_2)^{0.5})\cos\left(\frac{\delta}{2}\right)}$$

where

$$\delta = \text{half power bandwidth;}$$

$$F = 4\frac{(R_1 R_2)^{0.5}}{(1 - \sqrt{R_2 R_1})^2}$$

$$\frac{1}{2} = \frac{1}{1 + F\sin\left(\frac{\delta}{2}\right)^2}$$

$$F\sin^2\left(\frac{\delta}{2}\right) = 1$$

Since δ ιs small therefore sin(δ/2) = δ/2

$$\frac{\delta}{2} = \frac{1}{\sqrt{F}}$$

in terms of angles:

$$\xi = \frac{2\pi\sqrt{F}}{2}$$