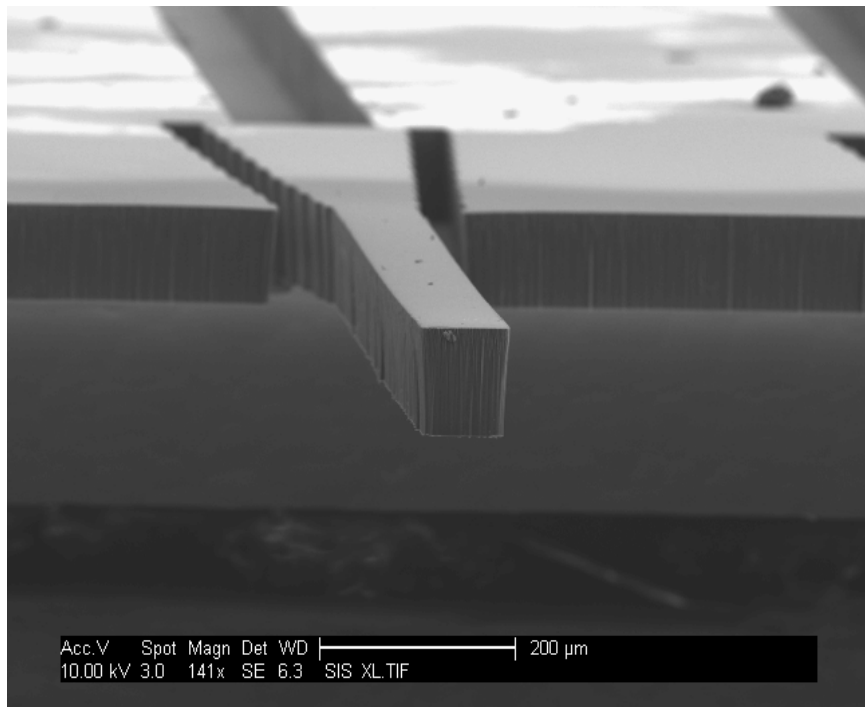
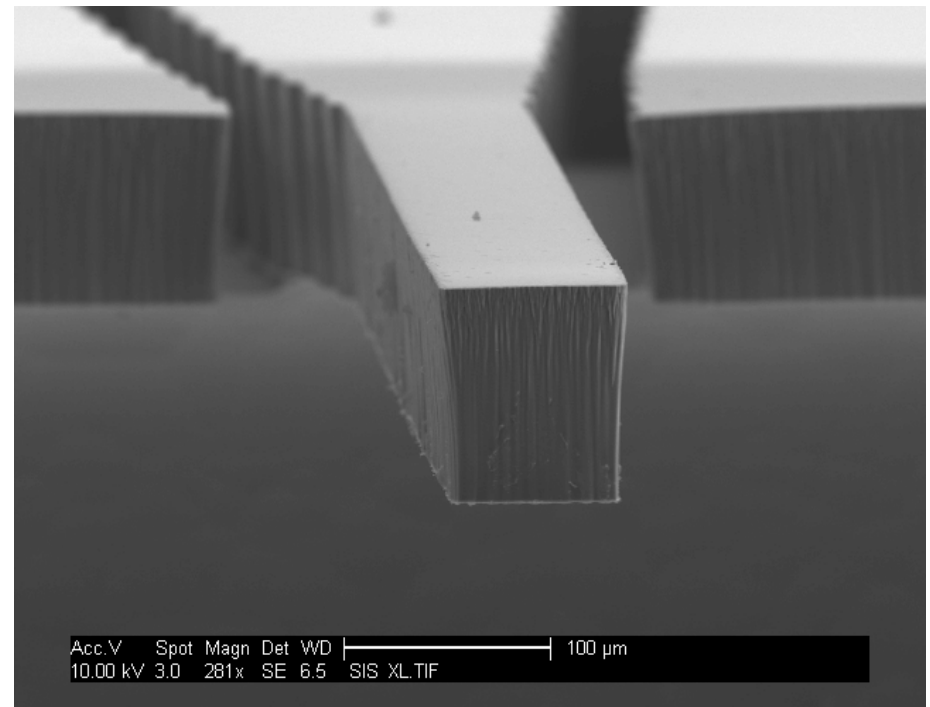


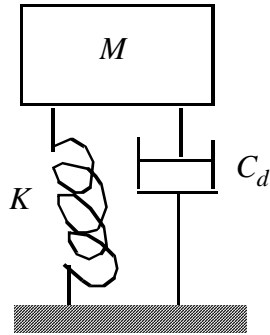
Microfabricated Beams



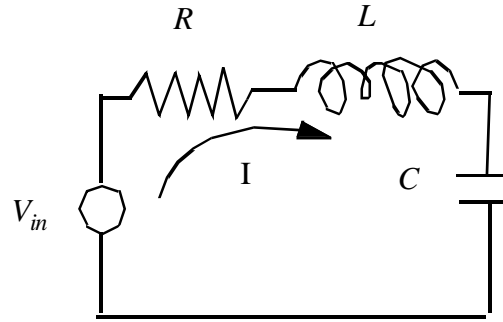
SU8 Beams



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Simplified Mechanical Model



Equivalent Circuit

$$\dot{x} \Leftrightarrow I$$

$$F \Leftrightarrow V$$

$$C_d \Leftrightarrow R$$

$$K \Leftrightarrow \frac{1}{C}$$

$$M \Leftrightarrow L$$

$$V_{in} = IR + (1/C) \int Idt + L \frac{dI}{dt}$$

$$I = \frac{V}{R}$$

$$F(t) = C_d \dot{x} + M\ddot{x} + Kx$$

$$x_{\max} = \frac{F}{(C_d \omega_o)}$$

$$\omega_o = \sqrt{\frac{k}{m}}$$

$$\omega_o = \sqrt{\frac{1}{LC}}$$

$$\xi = \frac{1}{2Q} = \frac{C_d}{2\omega_o M} = \frac{C_d}{2\sqrt{KM}}$$

$$\xi = \frac{1}{2Q} = \frac{R}{2\omega_o L} = \frac{R}{2} \sqrt{\frac{C}{L}}$$

Dynamics

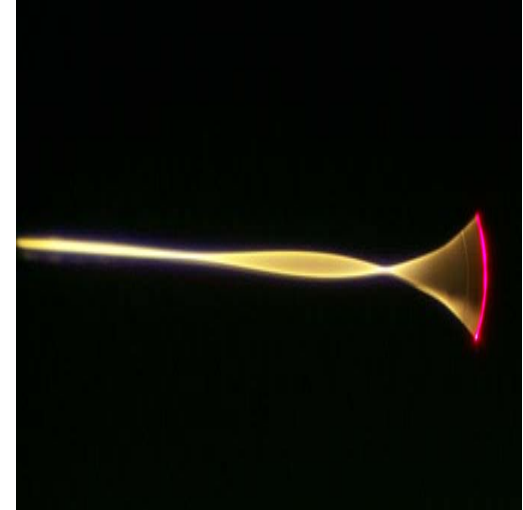
Resonant frequency of a cantilever beam

$$f_o = \frac{1}{2\pi} \left(\frac{1.875}{l} \right)^2 \left(\frac{E_{eq} I_{eq}}{\rho_{eq} A_{eq}} \right)^{0.5}$$

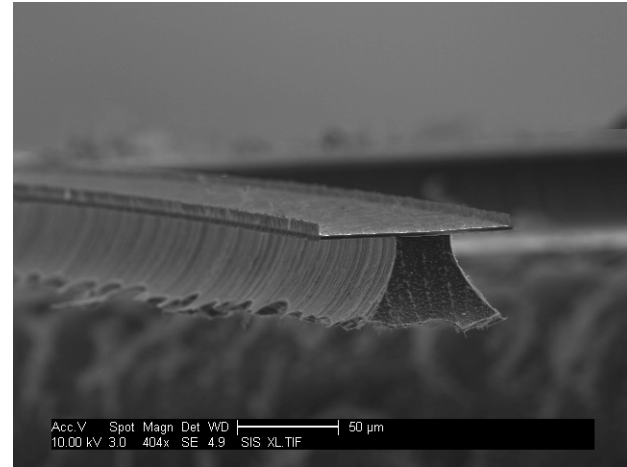
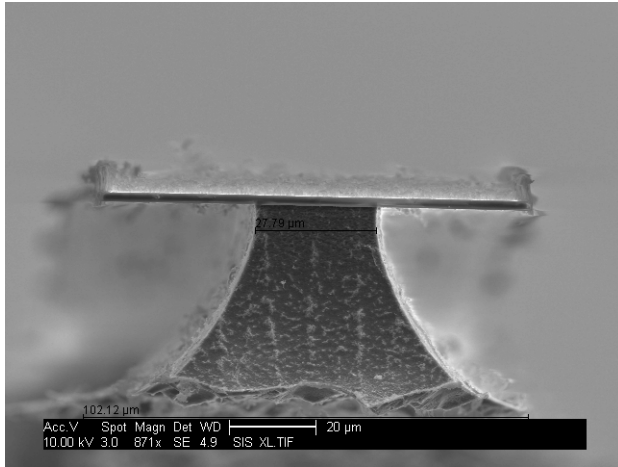
Where the equivalents of E, I, ρ , and A are Young's modulus, moment of inertia ($bh^3/12$), mass density and cross sectional area (bh) of a rectangular beam

b = base of the rectangular cross section and

h = height of the rectangular cross section



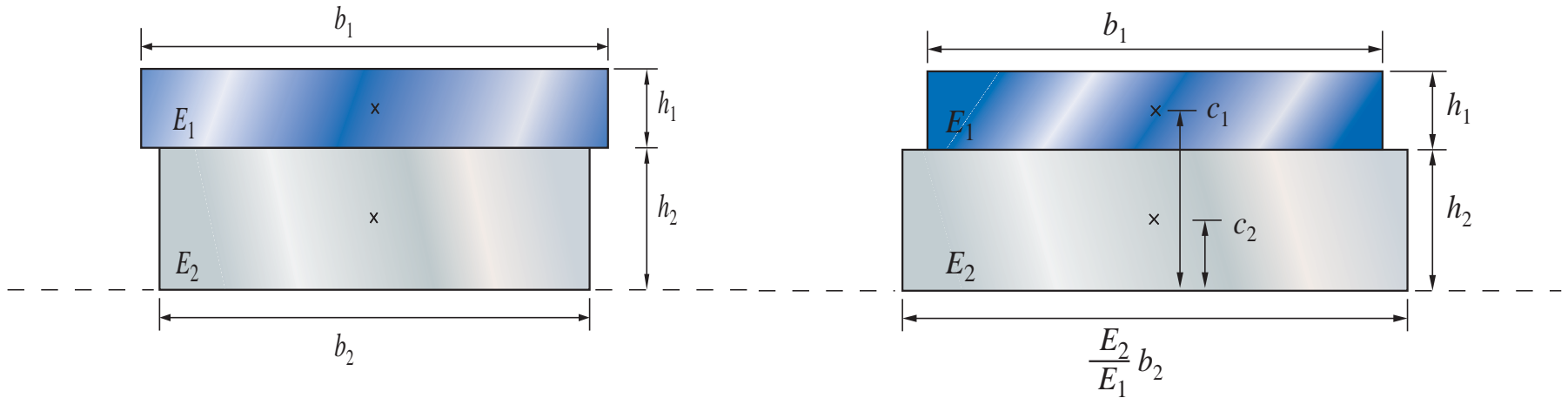
Composite structure



SiO₂/Si composite beam

UWMictech

Composite beam



For a composite beam bending along its horizontal axis, the stiffness of different materials can be handled by selecting one layer as a reference material and then adjusting the widths of the other layers relative to the reference material.

Example: the top layer $E = 70$ GPa and the bottom is 140 GPa & assuming both originally have the same width, if the top layer is selected, the bottom layer needs to be twice as wide to keep the same bending stiffness if the whole beam was made out of just the material of the top layer

Where the equivalents of E , I , ρ , and A are found by using the method of composite beams [13]. To compensate for stiffer or more flexible layers of a composite, this method adjusts the geometry (width only) of each of the layers of the composite beam, as illustrated in Figure 8. This allows one to use the one reference value of Young's modulus (labeled E_1 for this case) for the entire beam by adjusting the beam's geometry to compensate for having a second Young's modulus (E_2). After adjusting the geometry of the beam, the equivalent variables in eq (1) are found by the following formulas: (assuming all layers have rectangular cross sections)

$$I_{eq} = \frac{1}{E_1} \sum_{i=1}^n E_i I_i + \frac{1}{E_1} \sum_{i=1}^n E_i A_i (\bar{c} - c_i)^2$$

where E_i = Young's modulus of layer i
 I_i = moment of inertia of layer i
 A_i = original unadjusted area of layer i
 c_i =centroid of layer i

where centroid of the adjusted beam is found by:

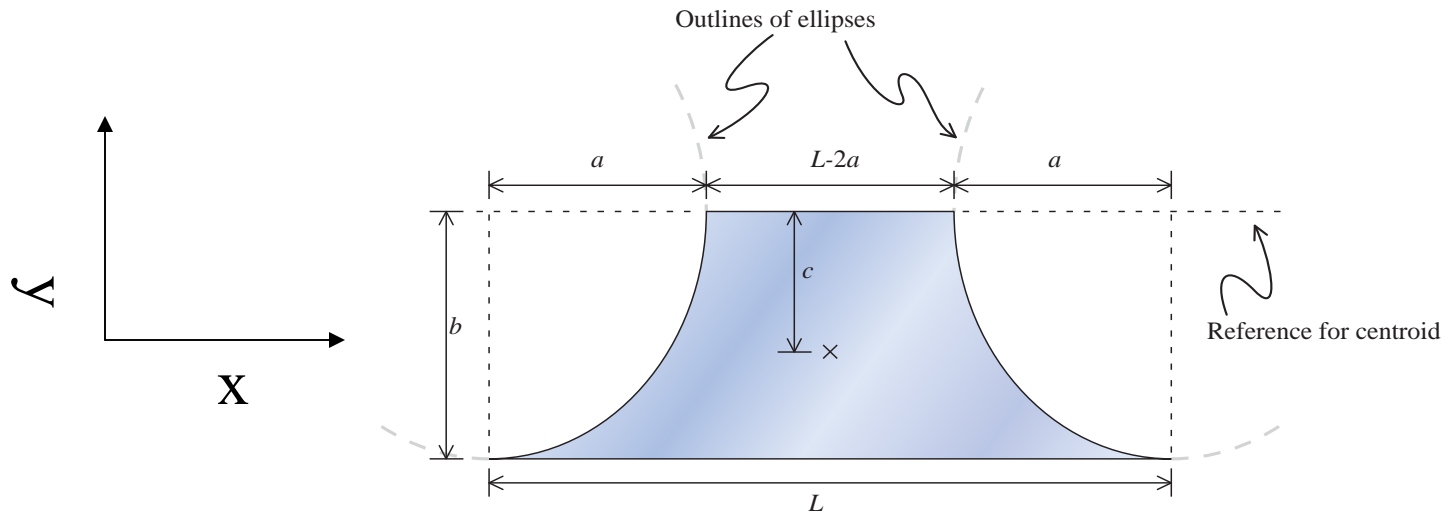
$$\bar{c} = \frac{\sum_{i=1}^n c_i A_i}{\sum_{i=1}^n A_i}$$

The adjusted cross-sectional area is:

$$A_{eq} = \frac{1}{E_1} \sum A_i E_i$$

The mass density is adjusted to maintain the same mass per unit length:

$$\rho_{eq} = \frac{E_i \sum_{i=1}^n \rho_i A_i}{\sum_{i=1}^n E_i A_i}$$



Based on the actual etching profile, an elliptical shape curve (fig 9) is used to approximate the silicon section of the beam. The centroid of the silicon/silicon oxide composite beam is therefore equal to,

$$c_{silicon} = \frac{\int_0^b y(L - 2a\sqrt{1 - \frac{y^2}{b^2}})dy}{(Lb - \frac{\pi ab}{2})} = \frac{b(4a - 3L)}{3(\pi a - 2L)}$$

And the moment of inertia of the composite beam with respect to x axis is,

$$I_x = \int_0^b y^2 dA = \int_0^b y^2 [L - 2(a\sqrt{1 - \frac{y^2}{b^2}})]dy = \frac{1}{24}b^3(8L - 3a\pi)$$

Development of an Optical Waveguide Cantilever Scanner

UWME Dynamics and Vibration Laboratory



Lever #	Thickness/width/length ($\mu\text{m}/\mu\text{m}/\mu\text{m}$)	Observed resonant frequency (kHz)	Calculated resonant frequency (kHz)	Observed Q factor
1	2.2 / 93 / 1500 (SiO ₂) 30 / 52 / 1500 (Si)	16.90	16.84	260
2	2.2 / 97 / 1500 (SiO ₂) 31 / 50 / 1500 (Si)	17.77	17.36	296
3	2.2 / 97 / 1000 (SiO ₂) 30 / 84 / 1000 (Si)	39.62	37.89	514
4	2.2 / 92 / 1000 (SiO ₂) 33 / 56 / 1000 (Si)	43.10	41.42	540
5	2.2 / 96 / 490 (SiO ₂) 35 / 59 / 490 (Si)	N/A	172.50	N/A

Table1. Resonance frequencies of the SiO₂/Si composite levers.