Phase Modulation

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Interference

When two or more optical waves are present simultaneously in the same region of space, the total wave function is the sum of the individual wave functions

Interferometer

Criteria for waveguide or fiber optic based interferometer:

Single mode excitation
polarization dependent

Interference of two waves

When two monochromatic waves of complex amplitudes U1(r) and U2(r) are superposed, the result is a monochromatic wave of the Same frequency and complex amplitude,

$$U(r) = U_1(r) + U_2\,(r)$$

Let Intensity $I_1 = |U_1|^2$ and $I_2 = |\,|U_2|^2$ then the intensity of total waves is

$$I = |U|^2 = |U_1 + U_2|^2 = |U_1|^2 + |U_2|^2 + U_1^*\,U_2 + U_1 U_2^{\,*}$$

Interference of two waves

Let $U_1 = I_1^{\ 0.5}\ e^{\ j\phi_1}$ and $U_2 = I_2^{\ 0.5}\ e^{\ j\phi_2}$ Then

$$I = I_1 + I_2 + 2(I_1 I_2)^{0.5} COS\phi$$

Where $\phi = \phi_2 - \phi_1$

Interferometers

- Mach-Zehnder
- Michelson
- Sagnac Interferometer
- Fabry-Perot Interferometer

Interferometers is an optical instrument that splits a wave into two waves using a beam splitter and delays them by unequal distances, redirect them using mirrors, recombine them using another beam splitter and detect the intensity of
their superposition

Intensity sensitive to phase change

$$\phi = 2\pi nd/\lambda$$

Where n = index of refraction of medium wave travels

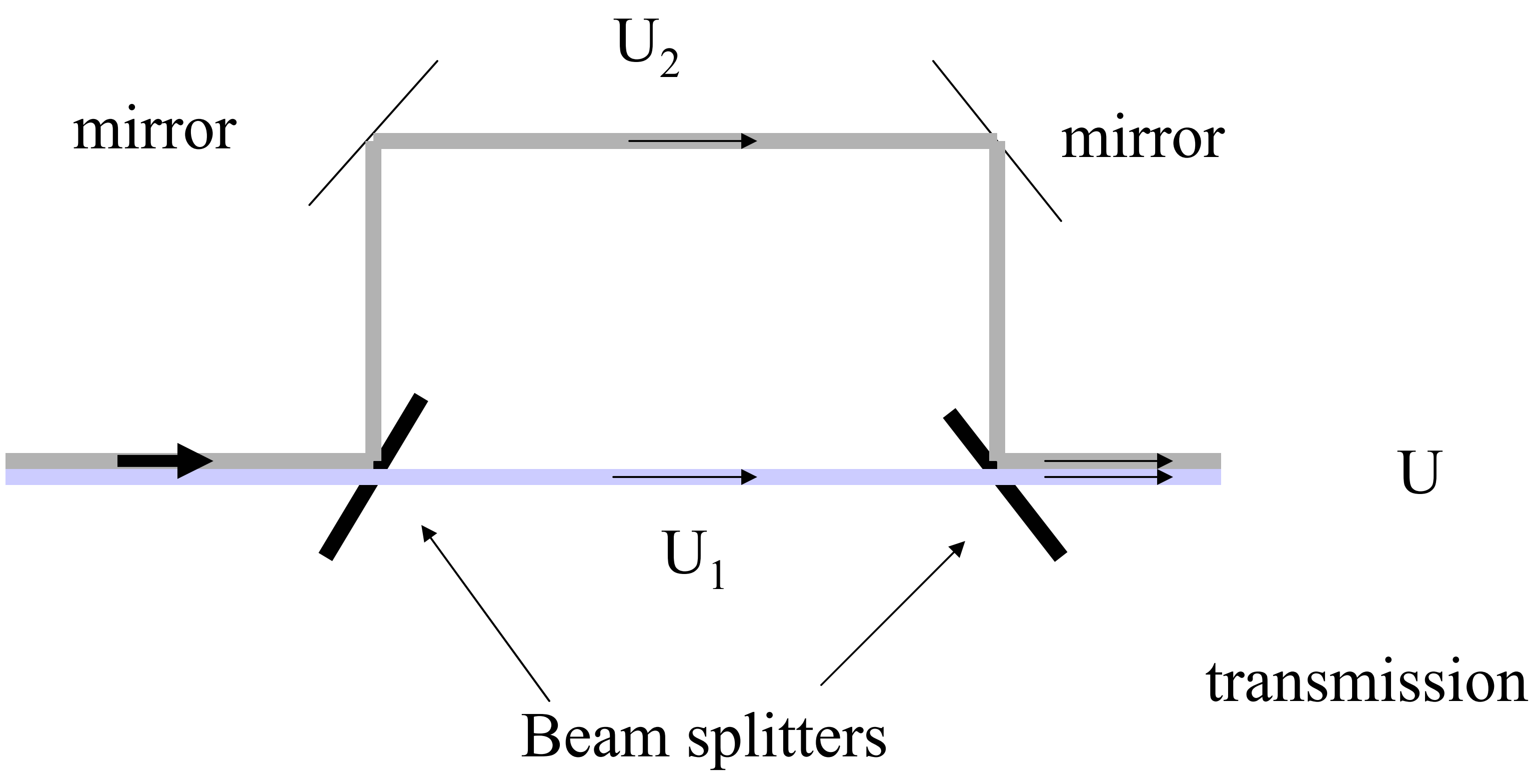
λ = operating wavelength

d = optical path length

Intensity change with n, d and λ

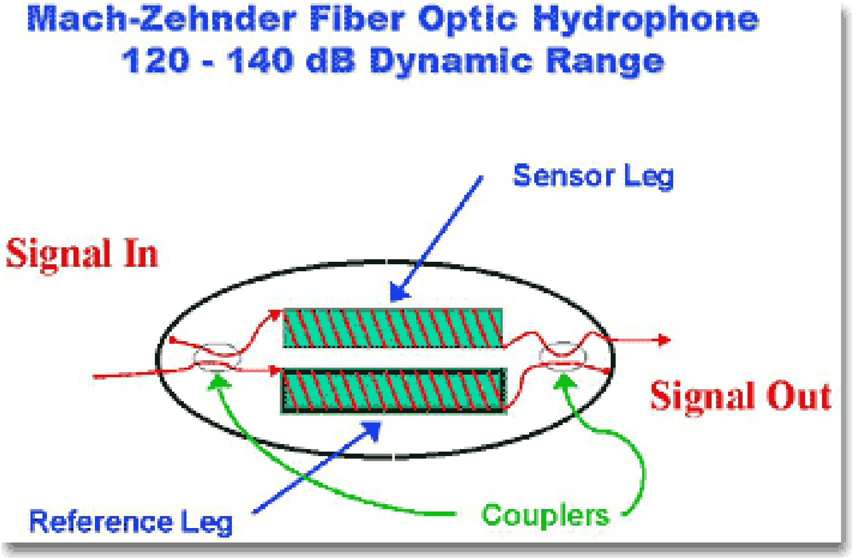
The phase change is converted into an intensity change using interferometric schemes (Mach-Zehnder, Michelson, Fabry-Perot or Sagnac forms).

Mach-Zehnder Interferometer



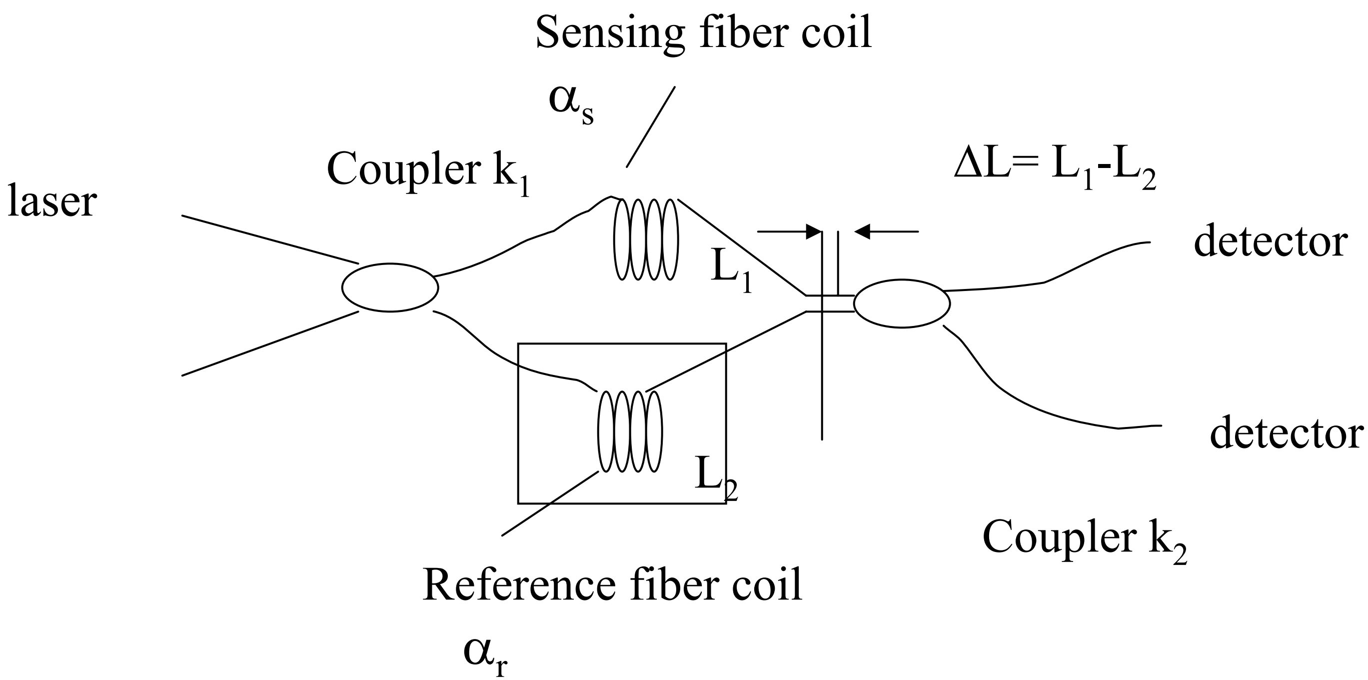
Fiber-optic hydrophone

(Mach-Zehnder Interferometer)



Two arms Interferometer- Sensor and reference arms

Mach-Zehnder interferometer



Mach-Zehnder interferometer

Let output fields of the signal and reference arms to be,

$$E_r = E_o\sqrt{\alpha_r k_1 k_2}\cos(\omega_o t + \phi_r)$$

$$E_s = E_o\sqrt{\alpha_s (1-k_1)(1-k_2)}\cos(\omega_o t + \phi_s)$$

The output intensity of the interferometer:

$$I = <E_r^2> + <E_s^2> + 2<E_r E_s>$$

$$= I_o[\alpha_r k_1 k_2 + \alpha_s (1-k_1)(1-k_2)$$

$$+ 2\sqrt{\alpha_s \alpha_r k_1 k_2 (1-k_1)(1-k_2)}\cos(\phi_r - \phi_s)]$$

Where < > denote a time average over a period > $2\pi/\omega_o$

α_r, α_s are optical loss associate with reference and signal paths

Mach-Zehnder interferometer

Fringe visibility is given by,

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

$$= \frac{2\sqrt{\alpha_s \alpha_r k_1 k_2 (1-k_1)(1-k_2)}}{\alpha_r k_1 k_2 + \alpha_s (1-k_1)(1-k_2)}$$

Polarization and coherence effects are ignored.
Assumes Lorentzian line shape, self-coherence function
$\gamma(\tau) = \exp[-|\tau|/\tau_c]$ where τ is delay between tow arms, τ_c is source coherence time, make $\tau < \tau_c$ -> $\gamma(\tau) \sim 1$

Mach-Zehnder interferometer

Complementary output of the interferometer,

$$I' = I_o[\alpha_r k_1(1-k_2) + \alpha_s(1-k_1)k_2 + 2\sqrt{\alpha_s\alpha_r k_1 k_2(1-k_1)(1-k_2)}\cos(\phi_s - \phi_r)]$$

The fringe visibility of the output:

$$V' = \frac{2\sqrt{\alpha_s\alpha_r k_1 k_2(1-k_1)(1-k_2)}}{\alpha_r k_1(1-k_2) + \alpha_s(1-k_1)k_2}$$

Mach-Zehnder interferometer

Output intensities in simplified forms,

$$I = I_o \alpha (A + B \cos \Delta\phi)$$

$$I' = I_o \alpha (C - B \cos \Delta\phi)$$

$$where\ \alpha_r = \alpha_s = \alpha$$

$$A = k_1 k_2 + (1 - k_1)(1 - k_2)$$

$$B = 2\sqrt{k_1 k_2 (1 - k_1)(1 - k_2)}$$

$$C = k_1 (1 - k_2) + (1 - k_1) k_2$$

$$\Delta\phi = \phi_r - \phi_s$$

Mach-Zehnder interferometer

Let us assume differential phase shift in interferometer is separated into $\Delta\phi$ of amplitude ϕ_s and frequency ω and a slowly varying phase shift ϕ_d

$$I = \frac{I_o\alpha}{2}(1+\cos(\phi_d + \phi_s \sin\omega t))$$

$$I' = \frac{I_o\alpha}{2}(1-\cos(\phi_d + \phi_s \sin\omega t))$$

Different current of these two output intensities is

$$i = \varepsilon I_o \alpha \cos(\phi_d + \phi_s \sin\omega t)$$

Mach-Zehnder interferometer

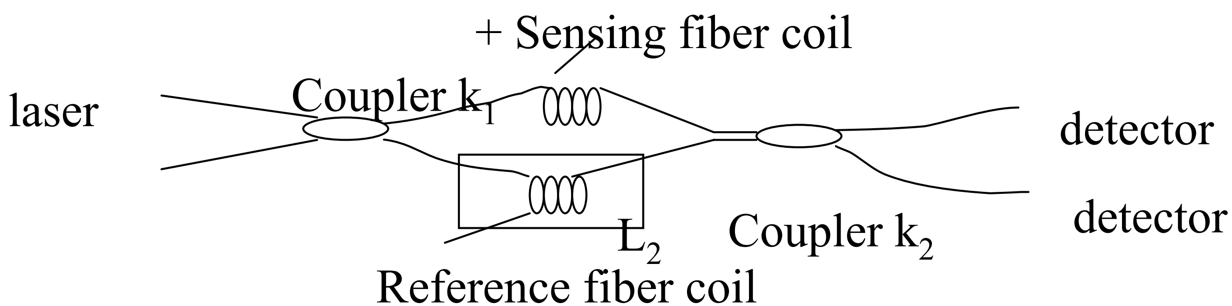
Quadrature point

$$\phi_d = (2m+1)\pi/2$$

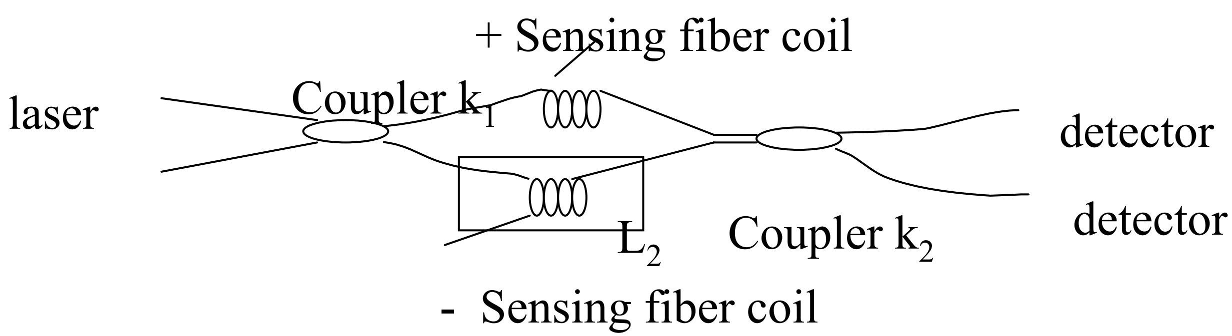
Where signal is maximized due to the fact the operating
Point is along the slope of the fringe

Various configurations

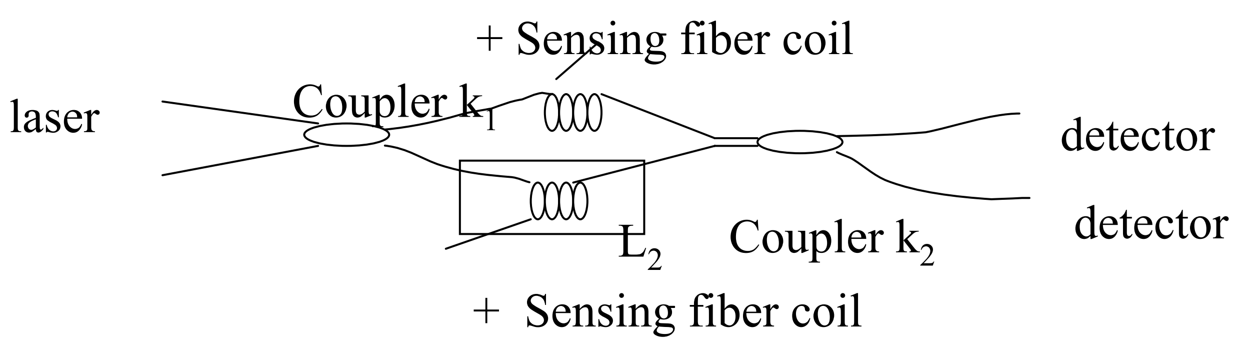
(Push)



(Push-pull)



(Push-push)



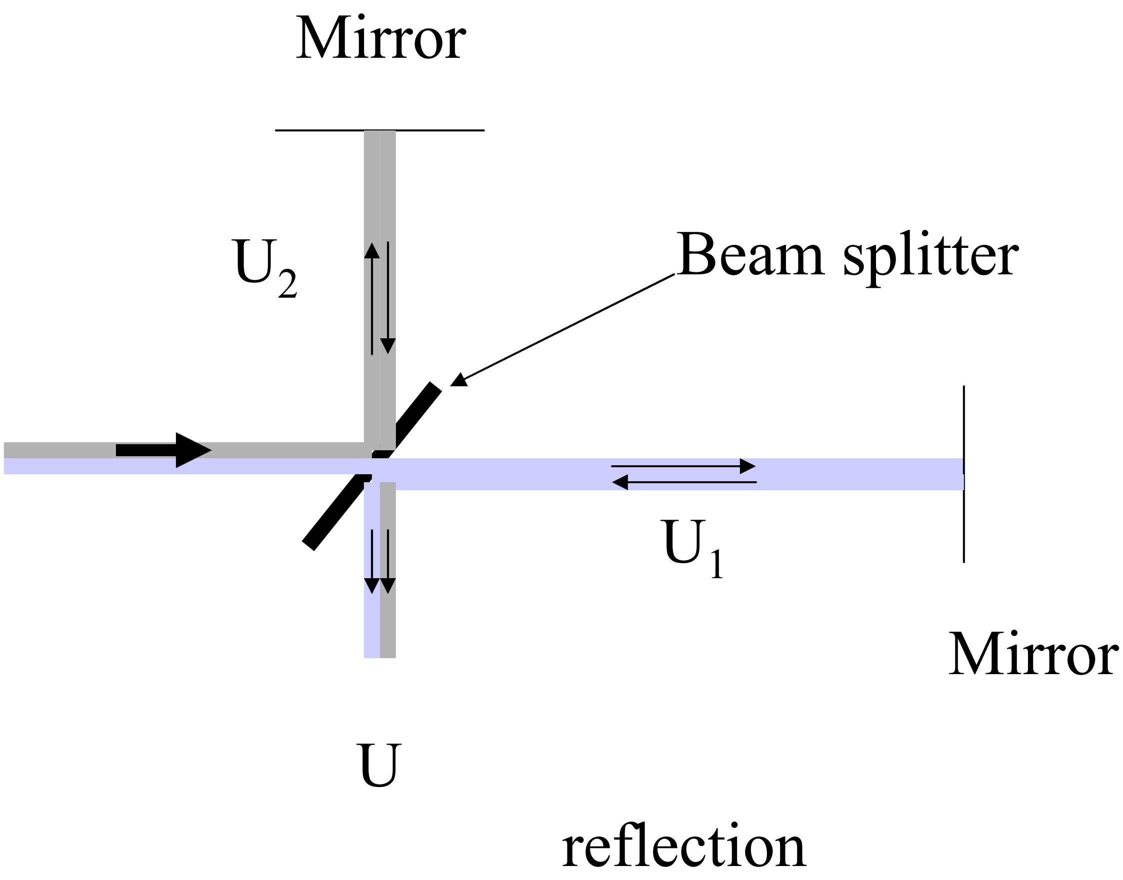
Assignment

What would be the output intensities and fringe visibility
From both outputs?

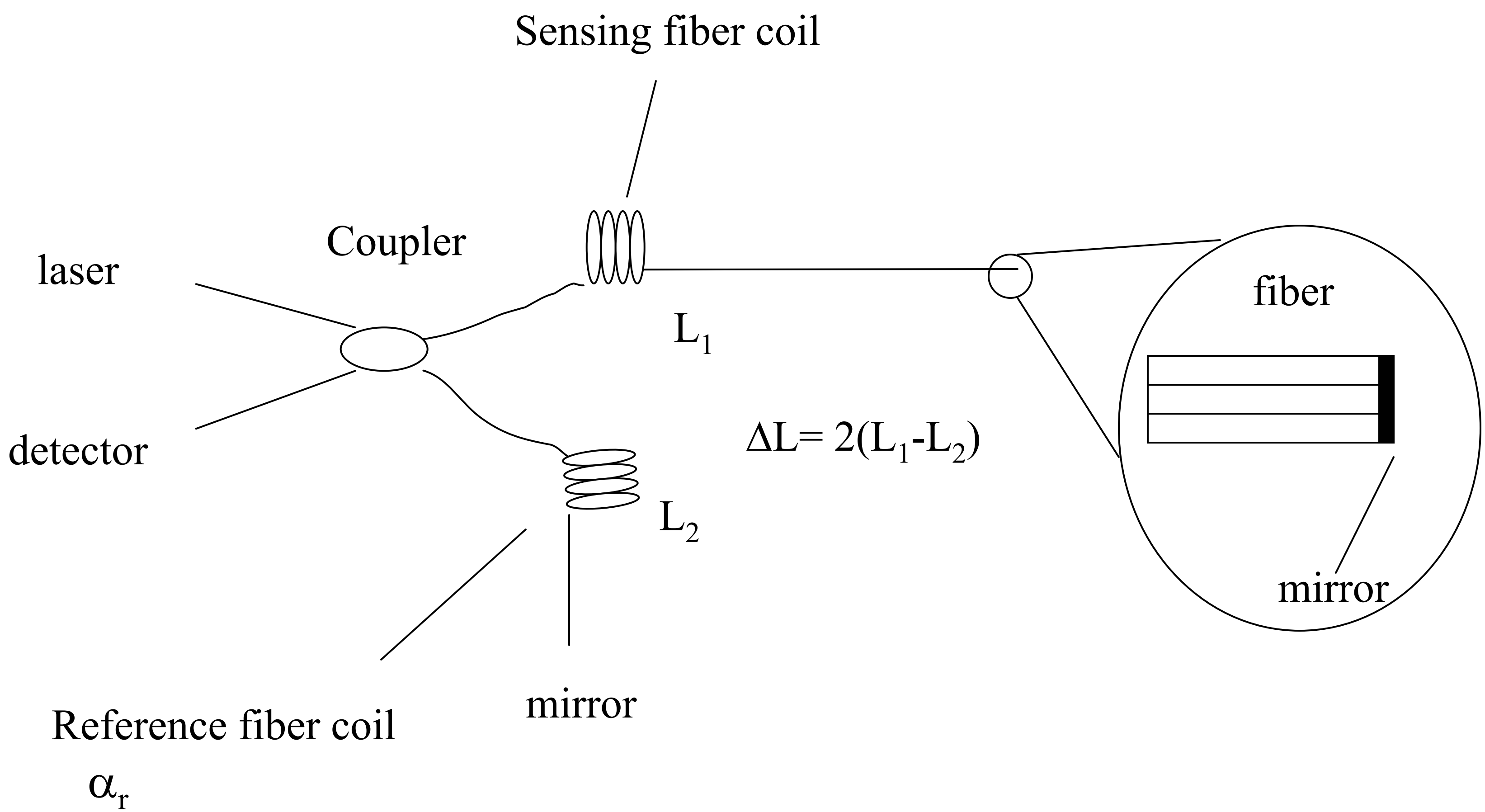
$$I = (I_o / 2)\alpha(1 + \cos\Delta\phi)$$

V=1

Michelson Interferometer



Michelson Interferometer

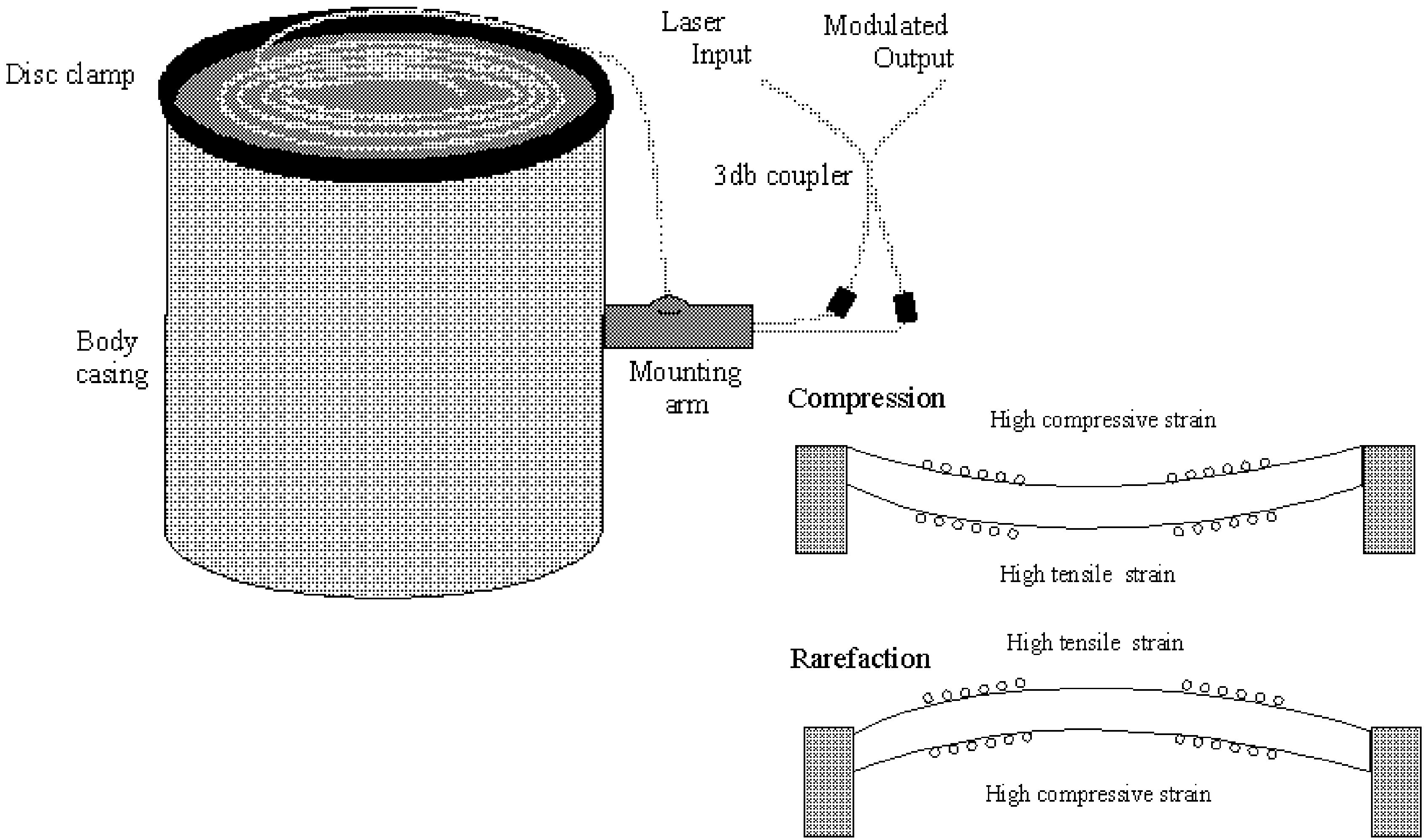


Michelson Interferometer

Differences between Michelson and Mach-Zehnder:

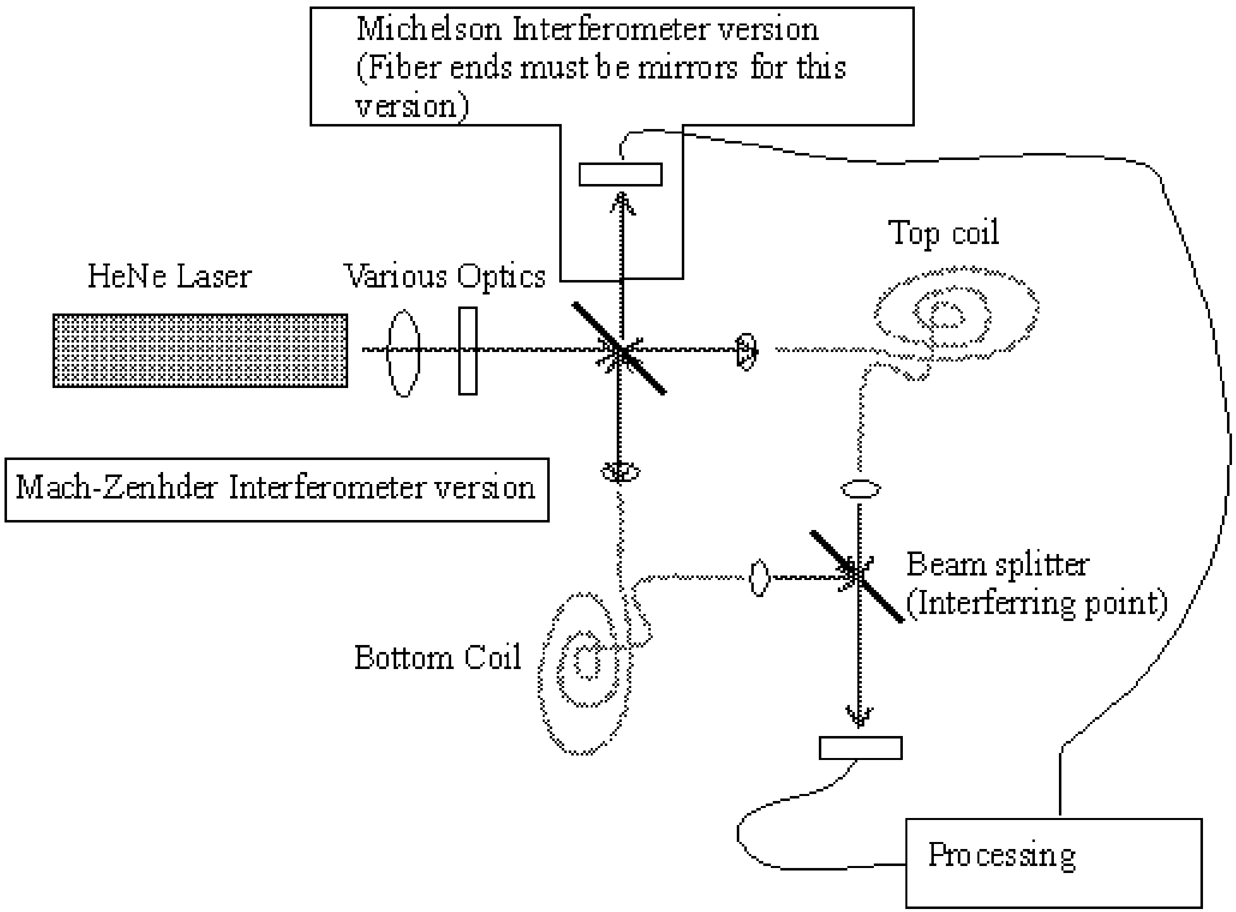
1. Single fiber coupler.
2. Pass through reference and signal fibers twice, phase shift per unit length doubled.
3. Interrogated with only single fiber between source/detector and sensor.

Fiber-optic hydrophone

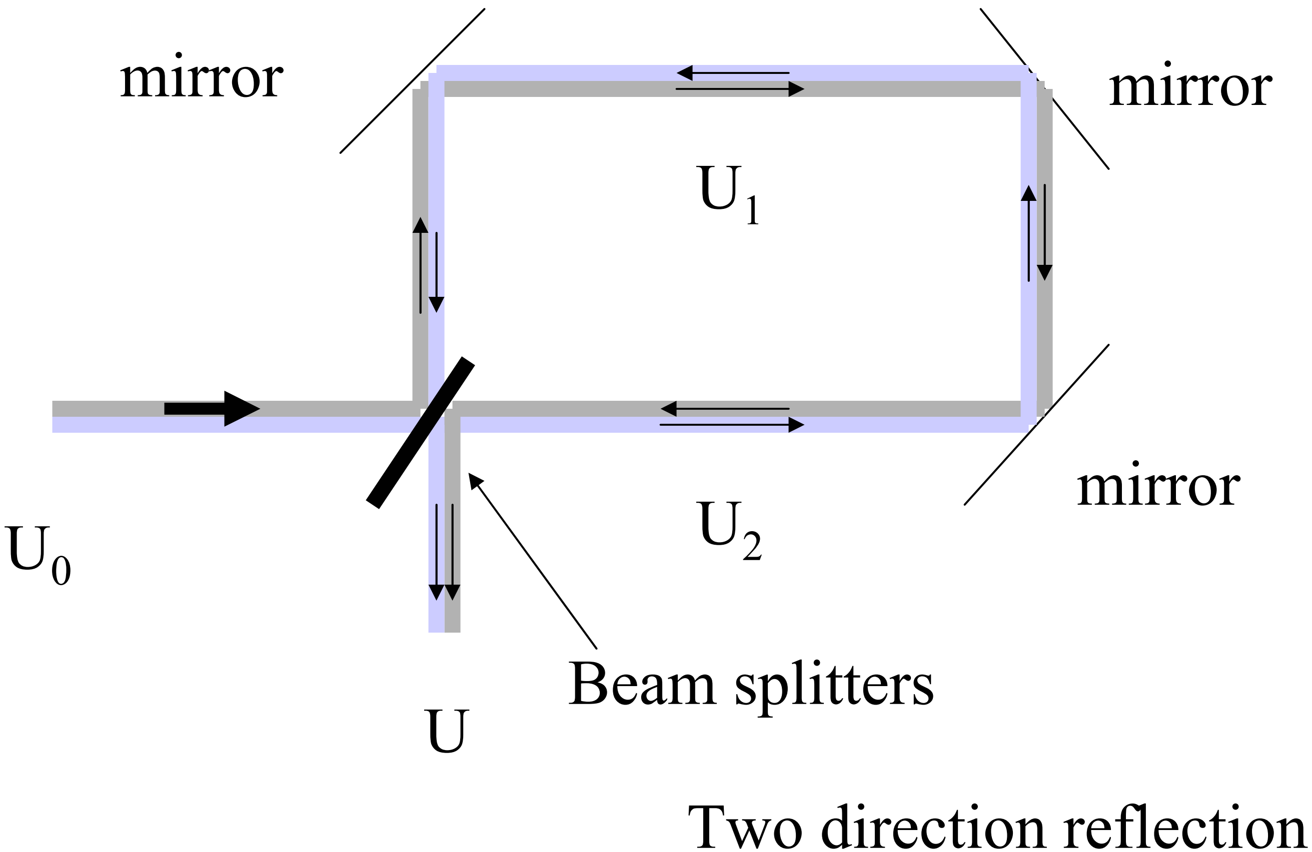


Fiber-optic hydrophone

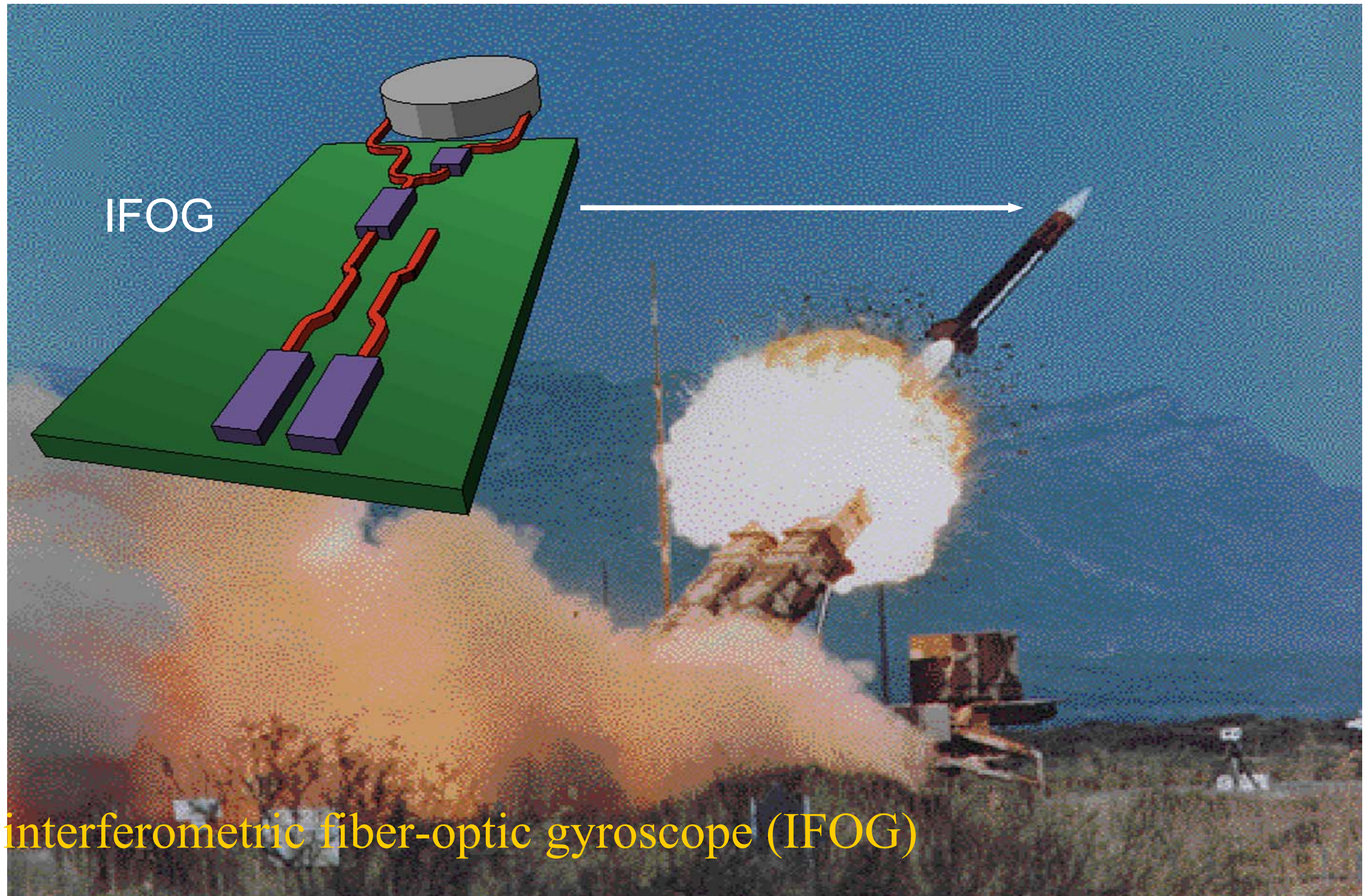
(Michelson Interferometer)



Sagnac Interferometer

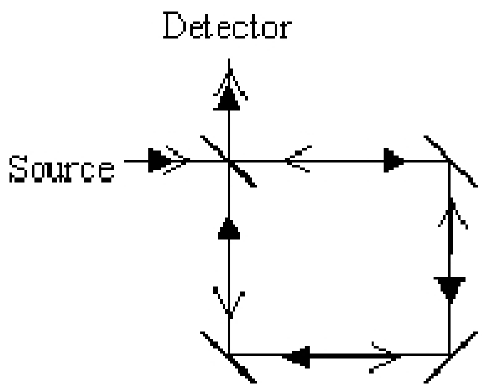


Sagnac Interferometric Fiber-Optic Gyroscope



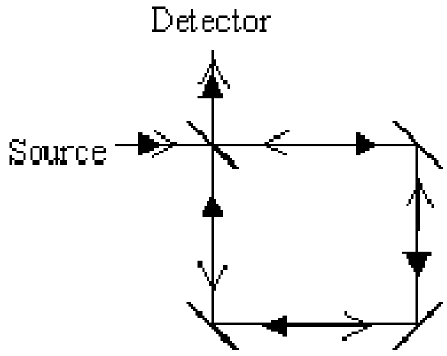
- interferometric fiber-optic gyroscope (IFOG)

The Sagnac Effect



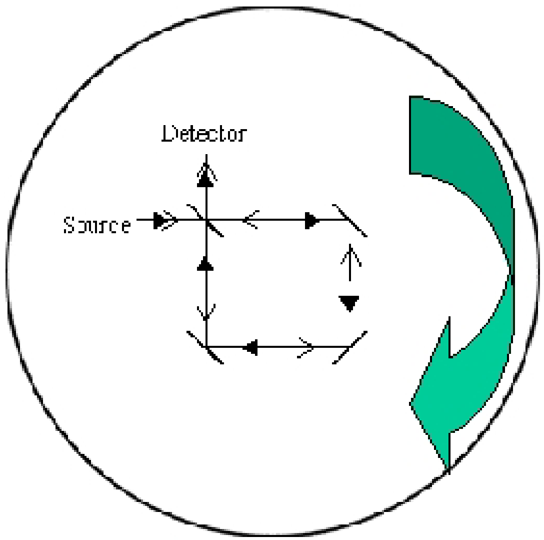
Suppose that a beam of light is split by a half-silvered mirror into two beams, and those beams are directed around a loop of mirrors in opposite directions (as shown)

The Sagnac Effect (2 of 3)



If the apparatus is stationary, the two beams of light will travel equal distances around the loop, and arrive at the detector simultaneously and in phase.

The Sagnac Effect (3 of 3)

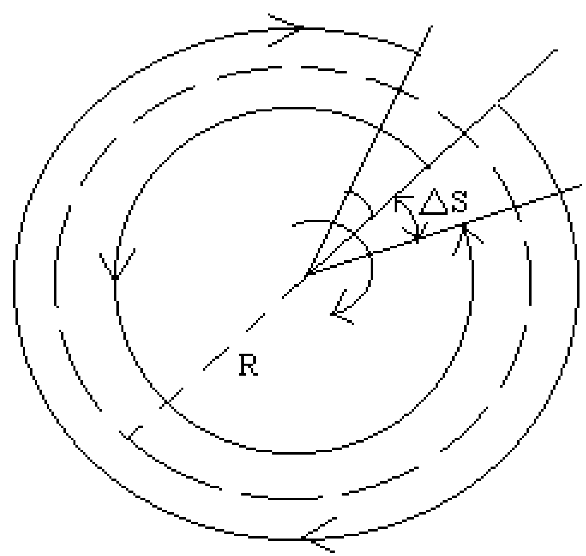


Clockwise Rotation

However, when the device is rotating, the beam traveling around the loop in the direction of rotation will have farther to travel than the beam traveling counter to the direction of rotation.

$$\sin\alpha + \sin\beta = 2\sin(0.5(\alpha+\beta))\cos(0.5(\alpha-\beta))$$

Two counter propagating beams, (one clockwise, CW, and another counterclockwise, CCW) arising from the same source, propagate inside an interferometer along the same closed path. At the output of the interferometer the CW and CCW beams interfere to produce a fringe pattern which shifts if a rotation rate is applied along an axis perpendicular to the plane of the path of the beam. Thus, the CW and CCW beams experience a relative phase difference which is proportional to the rotation rate. Consider a hypothetical interferometer, with a circular path of radius R as shown in fig.



When the interferometer is stationary, the CW and CCW propagating beams recombine after a time period given by,

$$T = \frac{2\pi R}{c}$$

where R is the radius of the closed path and c is the velocity of light. But, if the interferometer is set into rotation with an angular velocity, Ω rad/sec about an axis passing through the centre and normal to the plane of the interferometer, the beams re-encounter the beam splitter at different times.

The CW propagating beam traverses a path length slightly greater (by Δ s) than 2π R to complete one round trip. The CCW propagating beam traverses a path length slightly lesser than 2π R in one round trip. If the time taken for CW and CCW trips are designated as $T+$ and $T-$, then,

$$\Delta T = (T_{+} - T_{-}) = \frac{4\pi R^2 \Omega}{c^2 - (R\Omega)^2}$$

The difference yields

$$\Delta T = \frac{4\pi R^2 \Omega}{c^2}$$

With the consideration that, $c^2 >> (R^2 \Omega)$,

The round trip optical path difference is given by

$$\Delta L = \frac{4\pi R^2 \Omega}{c}$$

and the phase difference is given by

$$\Delta \phi = \frac{8\pi^2 R^2 \Omega}{c\lambda}$$

If the closed path consists of many turns of fiber, $\Delta\,\phi$ is given by,

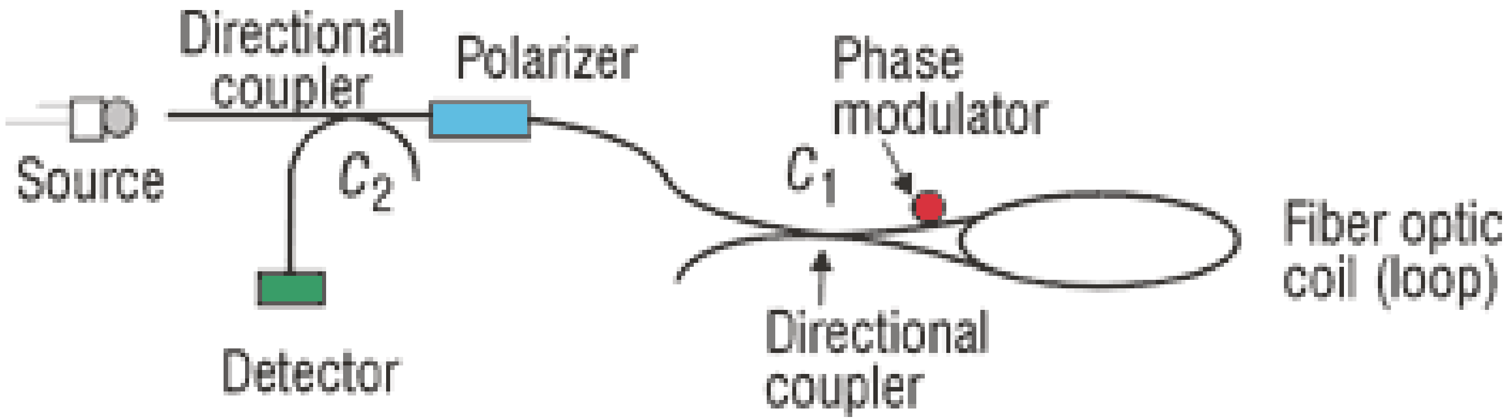
$$\Delta\phi = \frac{4\pi L R \Omega}{c\lambda} = \frac{8\pi^2 R^2 N \Omega}{c\lambda}$$

where A = area of the enclosed loop, N = number of turns of fiber, each of radius R, and L = total length of the fiber.

As a general case, the Sagnac frequency shift is given by,

$$\Delta f = \frac{4A\Omega}{P\lambda}$$

Sagnac Interferometer



if the loop rotates clockwise, by the time the beams traverse the loop the starting point will have moved and the clockwise beam will take a slightly longer time than the counterclockwise beam to come back to the starting point. This difference of time or phase will result in a change of intensity at the output light beam propagating toward C_2.

If the entire loop arrangement rotates with an angular velocity Ω, the phase difference between the two beams is given by

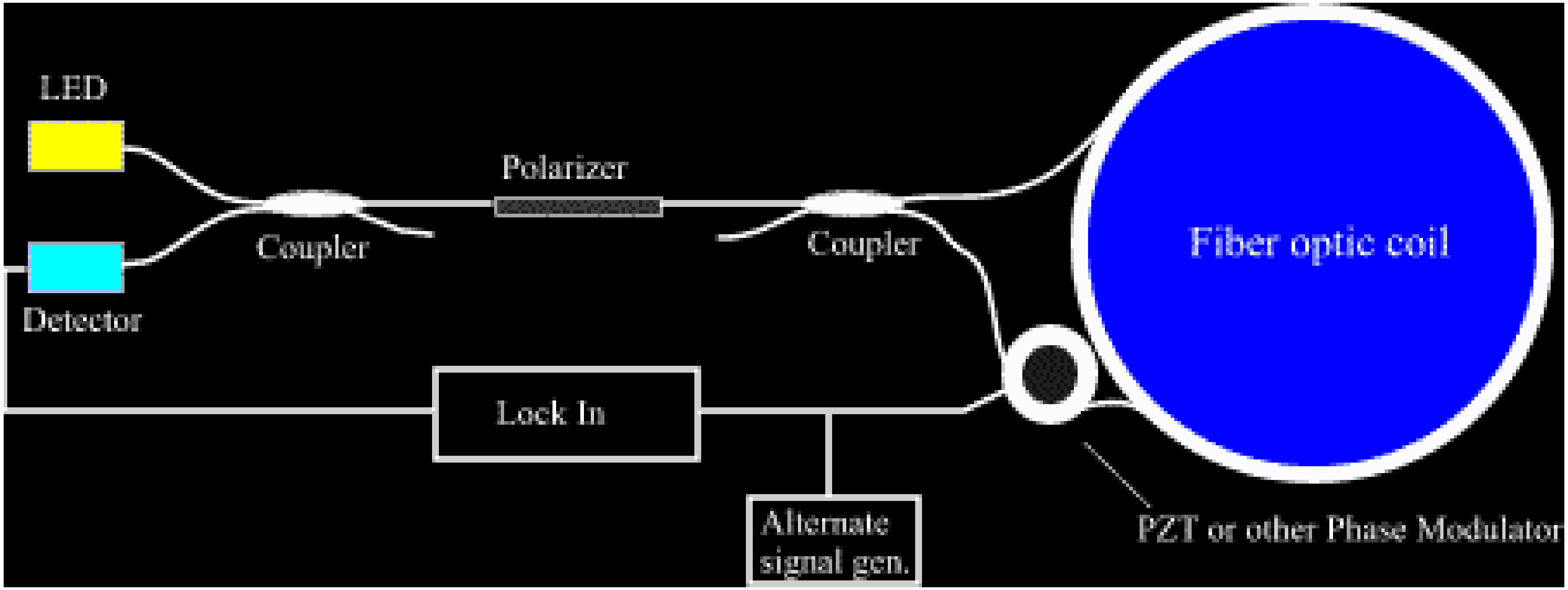
$$\Delta\phi = \frac{8\pi N A \Omega}{c\lambda_0}$$

where N is the number of fiber turns in the loop

A is the area enclosed by one turn (which need not be circular)

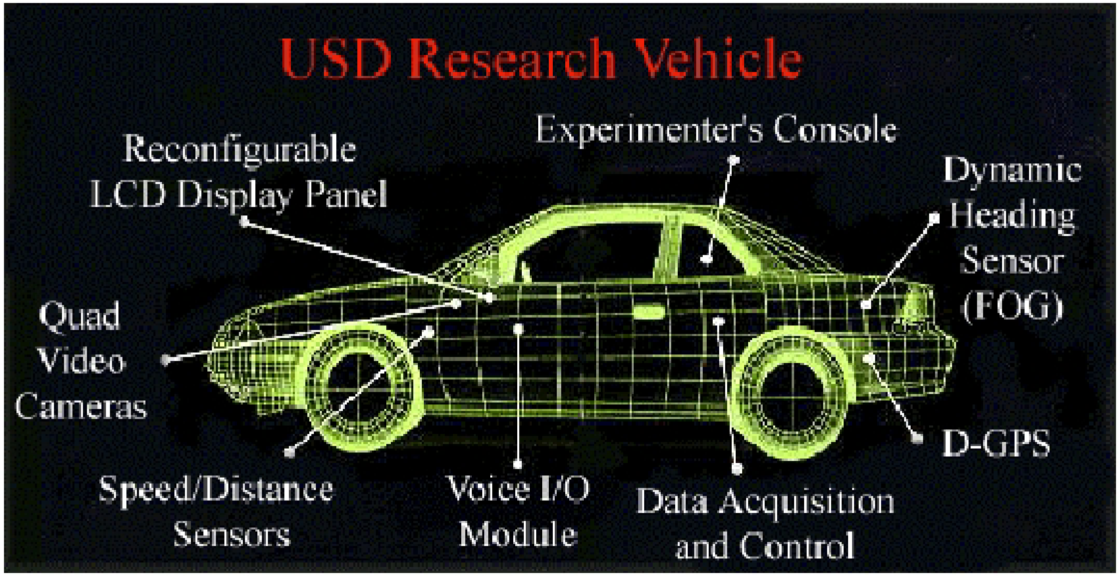
λ_0 is the free space wavelength of light

Minimum configuration of fiber-optic gyroscope



Automobile Yaw Rate Sensor for Assessing the Intrusiveness of Secondary Tasks

Test Platform



Special Thanks to Toyota USA

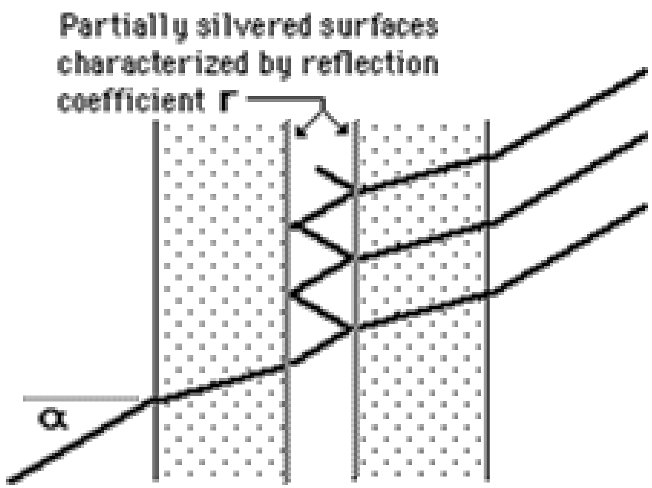
KVH autoGYRO fiberoptic gyroscope case study video

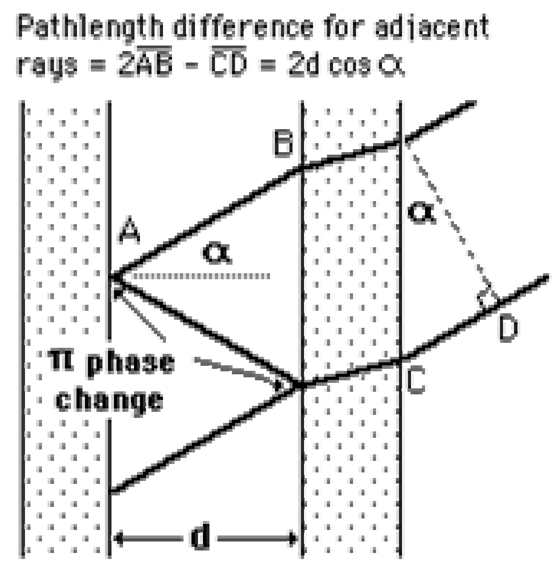
Case Study Results

Driving Scenario	**Steering Instability Factor**
Baseline (Straightaway)	1.0
Adjust Climate Control	1.5
Tune Radio	2.0
Dial Cell Phone	3.0
Interactive Text Display	6.0

Fabry-Perot Interferometer

Interference of an infinite number of waves progressively smaller amplitude and equal phase difference.





Fabry-Perot Interferometer

$$I_r(\phi) = \frac{(R_1 + R_2 - 2x\sqrt{R_1 x R_2}\cos(\phi))}{1 + R_1 x R_2 - 2x\sqrt{R_1 x R_2}\, x\cos(\phi)}$$

$$\phi = \frac{2 \times 2 \times \pi \times y \times n}{\lambda}\cos(\theta)$$

where $\cos(\theta) = 1$ normal incident;

y = distance separation of mirror and fiber end;

n = index of refraction of the air gap;

λ= wavelength of the incoming He-Ne laser = 632.8 nm;

R_1 = intensity reflection coefficient of fiber;

R_2 = intensity reflection coefficient of mirror;

Transmission Intensity

$$I_r(\phi) = \frac{T_1 T_2}{1 + R_1 x R_2 - 2x\sqrt{R_1 x R_2}\, x \cos(\phi)}$$

$$\phi = \frac{2 \times 2 \times \pi \times y \times n}{\lambda} \cos(\theta)$$

where cos(θ) = 1 normal incident;

y = distance separation of mirror and fiber end;

n = index of refraction of the air gap;

λ= wavelength of the incoming He-Ne laser = 632.8 nm;

T_1 = intensity transmission coefficient of fiber;

T_2 = intensity transmission coefficient of mirror;

Finesse ξ

$$\xi = \frac{2\pi\sqrt{f}}{2}$$

$$f = \frac{4\times\sqrt{R_1\times R_2}}{(1-\sqrt{R_1\times R_2})^2}$$

$$\sqrt{f} = \frac{2}{\delta}$$

Where δ= half power bandwidth

This parameter is defined as the ratio of the half power bandwidth over the peak to peak full bandwidth. It’s a way to measure the sharpness of the curve.

Transmission Spectrum

The frequency of each line is given by

$$f = p\ C_o/(2ny\cos\theta) \text{ where } p = \underline{+}1, \underline{+}2, \underline{+}3,\ldots$$

The lines are separated in frequencies by

$$\Delta f = Co/(2ny\cos\theta)$$

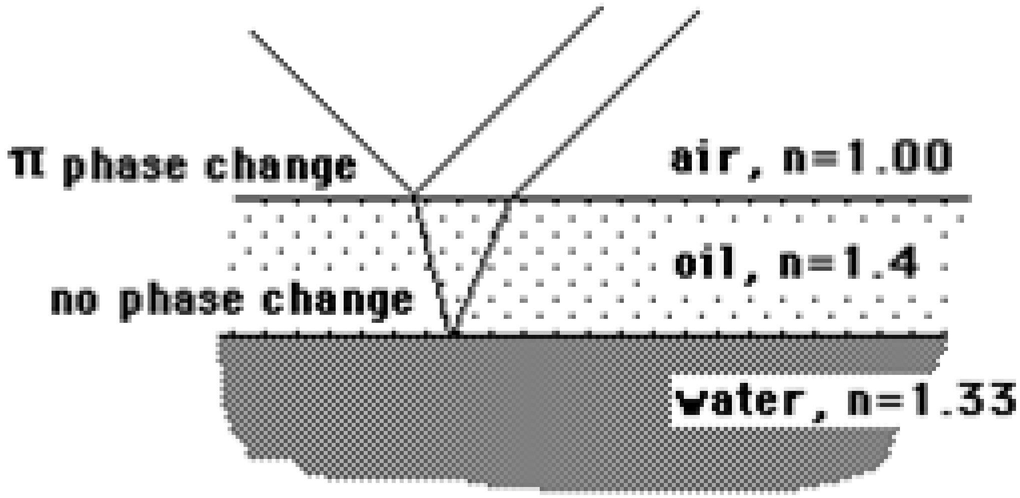
The spacing between etalon modes is

$$\Delta\lambda = \Delta f\, \lambda^2/C_o$$

The mode number of the etalon is

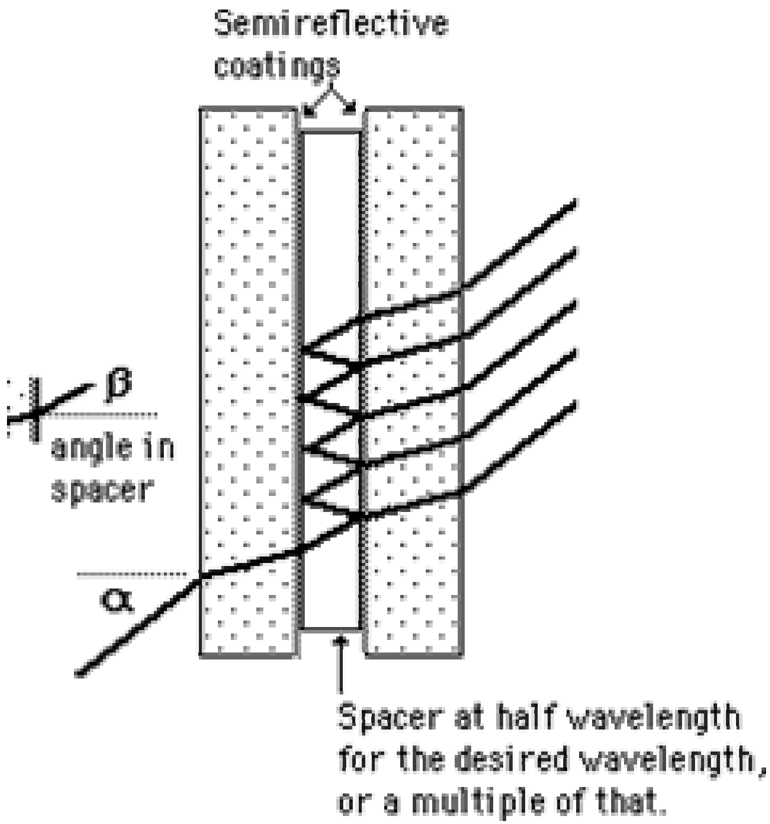
$$p = f/\Delta f$$

Film thickness Measurement



This phase change is important in the interference which occurs in thin films, the design of anti-reflection coatings, interference filters, and thin film mirrors.

Interference Filters



Thickness calculated from the interference condition:

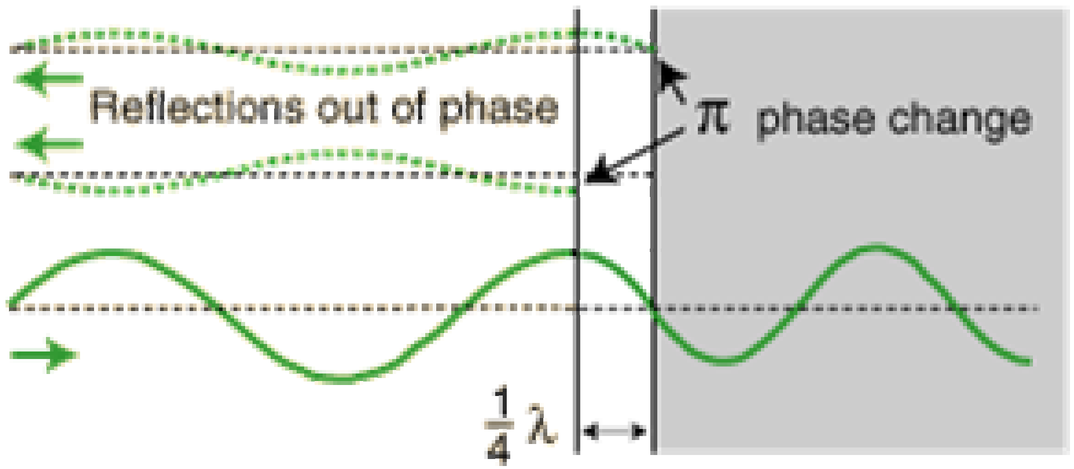
$$d = \frac{\lambda}{2n\cos\beta}$$

The passed wavelength is given by

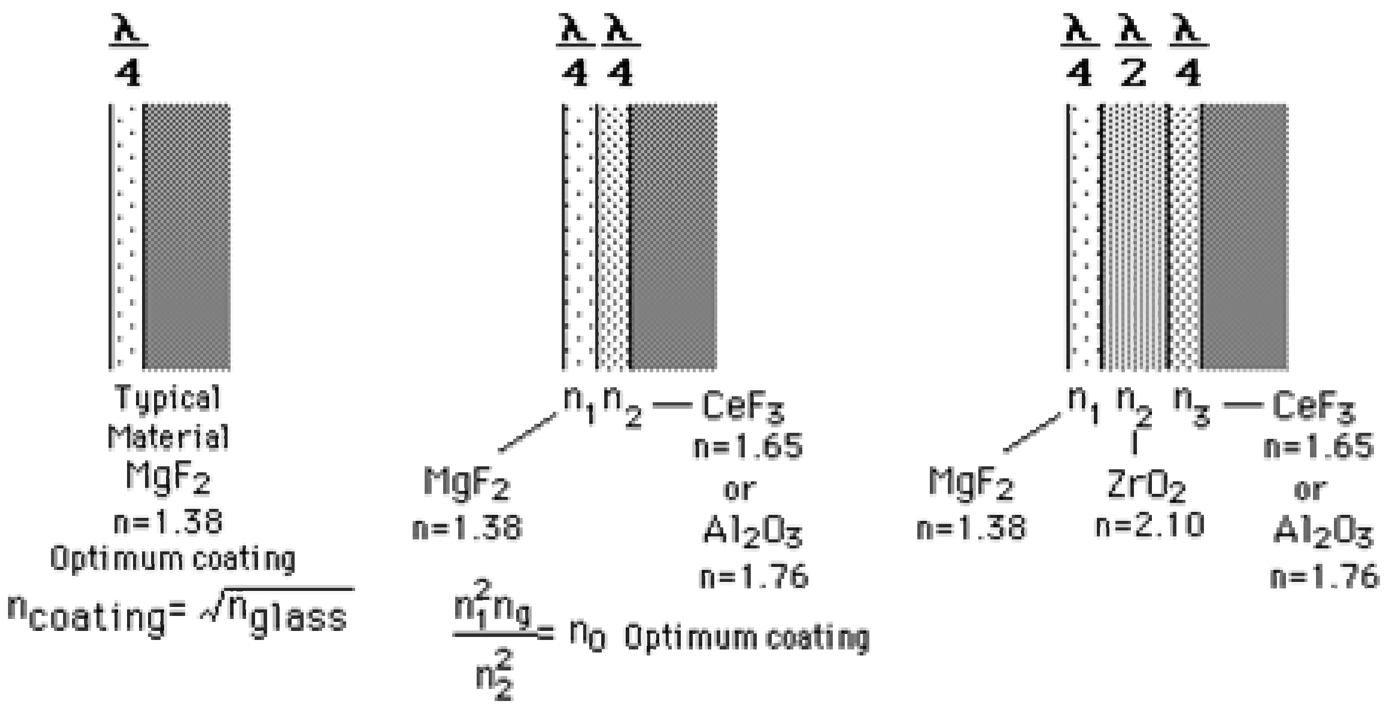
$$\lambda = \lambda_0 \sqrt{1 - \frac{\sin^2\alpha}{n^2}}$$

Anti-Reflection Coatings

Anti-reflection coatings work by producing two reflections which interfere destructively with each other.



Multi-Layer Anti-Reflection Coatings



Temperature Strain and Pressure Sensing

$$\phi = \frac{2\times 2\times \pi \times y\times n}{\lambda}\cos(\theta)$$

Strain response due to

- Physical change corresponding to optical path y change
- index n change due to photoelastic effect

$$\frac{\Delta\phi}{\phi} = \varepsilon_z - \frac{n^2}{2}\left[(p_{11} + p_{12})\varepsilon_r + p_{12}\varepsilon_z\right]$$

Thermal response arise from

- Internal thermal expansion
- temperature dependent index change

The change in phase due to a unit perturbation such as pressure change is given by,

$$\Delta\phi = \beta\Delta l + l\Delta\beta = \beta\Delta l + l\left[k_o \Delta n + \frac{\partial\beta}{\partial a}\Delta a\right]$$

where n = refractive index, and a = radius of the fiber. The change in β , due to radius variations is very small and can be neglected. The change in refractive index can be obtained from the the index variation due to photoelastic effect as,

$$\Delta\left(\frac{1}{n^2}\right)_{ij} = \sum_{i,j} p_{ijhl}\ \varepsilon_{hl}$$

where p_{ijhl} is the photoelastic tensor and ε_{hl} is the strain. In the case of an optical fiber made of isotropic glass there are only two independent photoelastic constants p_{11} and p_{12}.

Let $\varepsilon_z = \frac{\Delta l}{l}$ and $\varepsilon_x = \varepsilon_y = \frac{\Delta r}{r} = \varepsilon_r$

Combining the above,

$$\frac{\Delta \phi}{\phi} = \varepsilon_z - \frac{n^2}{2}\left[(p_{11} + p_{12})\,\varepsilon_r + p_{12}\,\varepsilon_z\right]$$

The above analysis can be generalized and extended to obtain the induced phase changes in an optical fiber due to pressure, temperature or strain variations. The normalized phase changes are as given below.

$$\frac{\Delta \phi}{L} = \frac{\pi}{\lambda_0}\left[\frac{\lambda_0 a}{\pi}\,\frac{\partial \beta}{\partial a} - n^2(p_{11} - p_{12})\right]\left[\frac{1 - \nu - 2\nu^2}{E}\right]\Delta P$$

$$\frac{\Delta \phi}{L} = \frac{2\pi}{\lambda_0}\left[\left(n + \frac{\lambda_0 a}{2\pi}\,\frac{\partial \beta}{\partial a}\right)\alpha + \frac{\partial n}{\partial T}\right]\Delta T$$

where, L= length of the fiber, ΔP = change in hydrostatic pressure; p_{11}, , p_{12} = *photoelastic constants;* ν *= Poisson's ratio; E = Young's modulus;* α *= linear expansion coefficient; S = strain;* λ *= wavelength of light in free space; n = refractive index; a = core radius of the fiber;* $\frac{\partial \beta}{\partial a}$ *= rate of change of propagation constant with core radius;* ΔT *= change in temperature.*

In an optical interferometer the reference and phase modulated light are combined and detected using a photodetector. One obtains an interference equation which has a sinusoidal dependence. A fixed phase bias of π /2 is introduced in the reference arm with the help of a piezoelectric modulator so that the output variation is linear. The current output from the detector is given by,

$$i_s = I_o \frac{qe}{h\nu} \Delta\phi = \left(\frac{I_o qe}{h\nu}\right)\left(\frac{d\phi}{dP}\right)(\Delta P)$$

The photon noise current associated with this detection is

$$i_N^2 = 2e\left(\frac{I_o qe}{h\nu}\right)B$$

Signal to noise ratio,

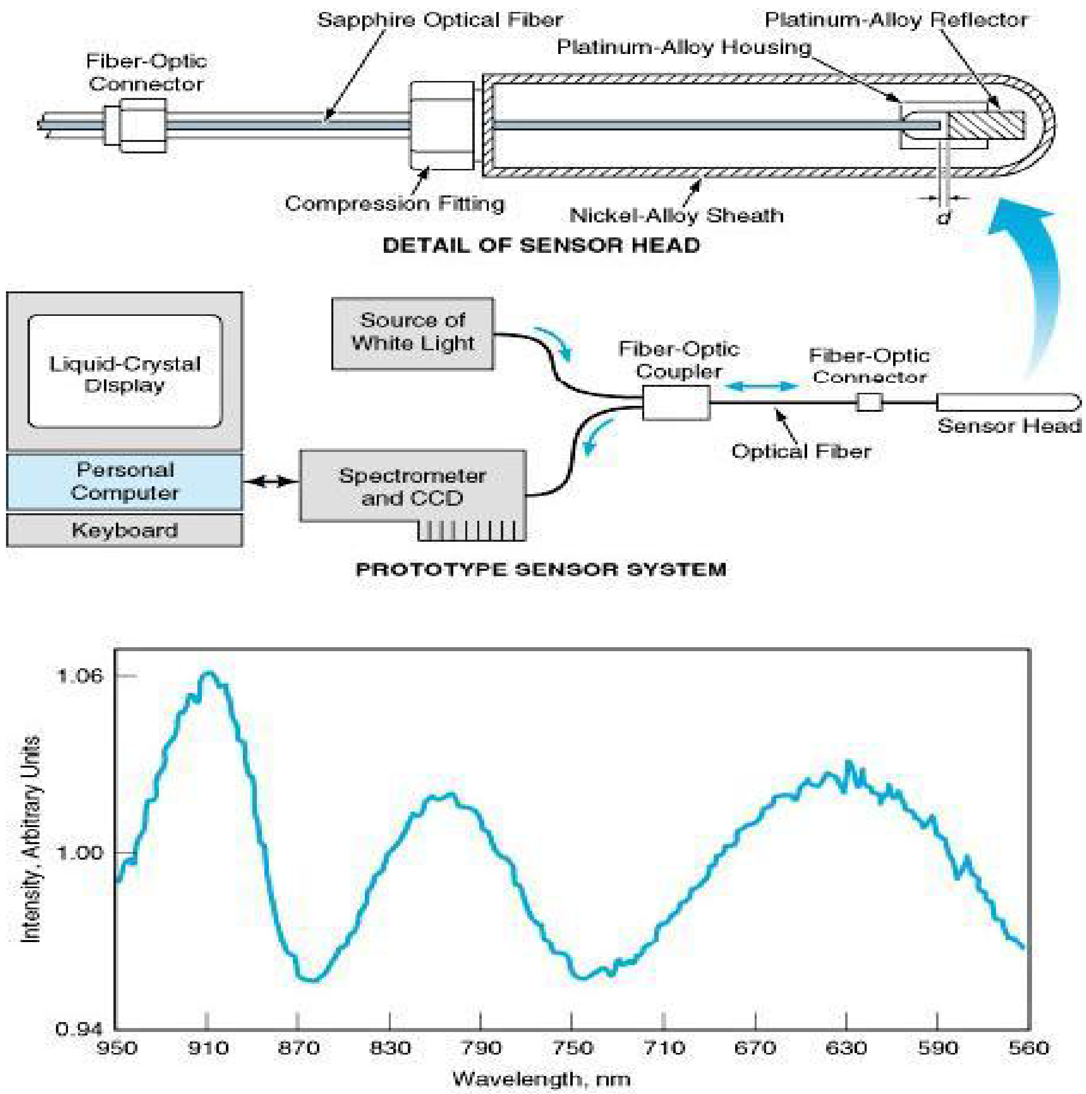
$$\mathrm{SNR} = \frac{i_s^2}{i_N^2}$$

The minimum detectable pressure is found by setting SNR = 1. Hence *Pmin* is obtained as

$$P_{\min} = \left(\frac{2h\nu B}{I_o q}\right)^{1/2}\left(\frac{d\phi}{dP}\right)^{-1}$$

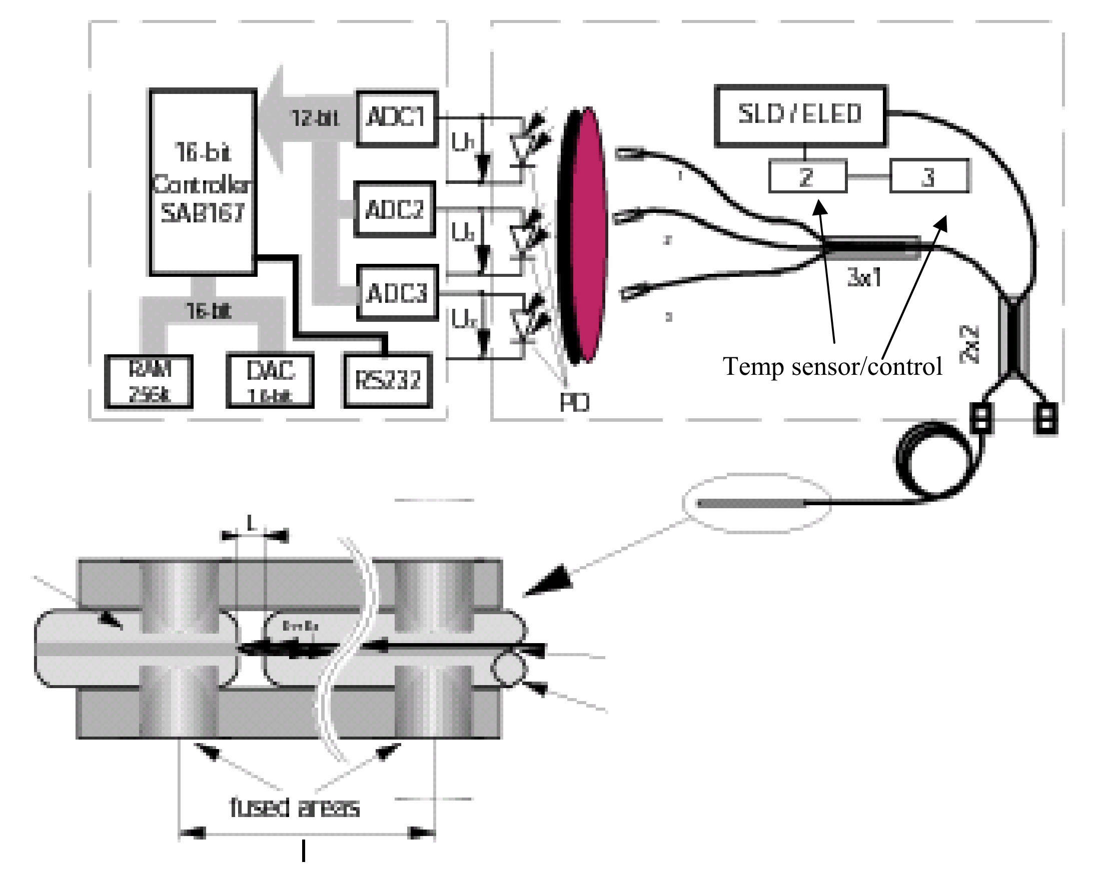
where *h* = Plank's constant, *n* = optical frequency, B = detection bandwidth and *q* = quantum efficiency.

Fabry-Perot Fiber-Optic Temperature Sensor



EXAMPLE OF SPECTRUM AT A TEMPERATURE NEAR UPPER END OF RANGE

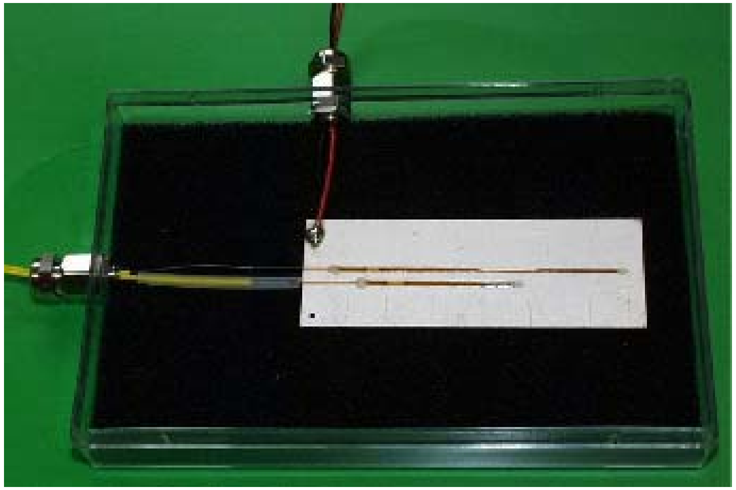
Extrinsic Fabry-Perot Interferometer Strain Sensor



3-λ demodulation EEPI

M. Schmidt, et al., OSA, 2001, vol.8 No. 8, p475-480

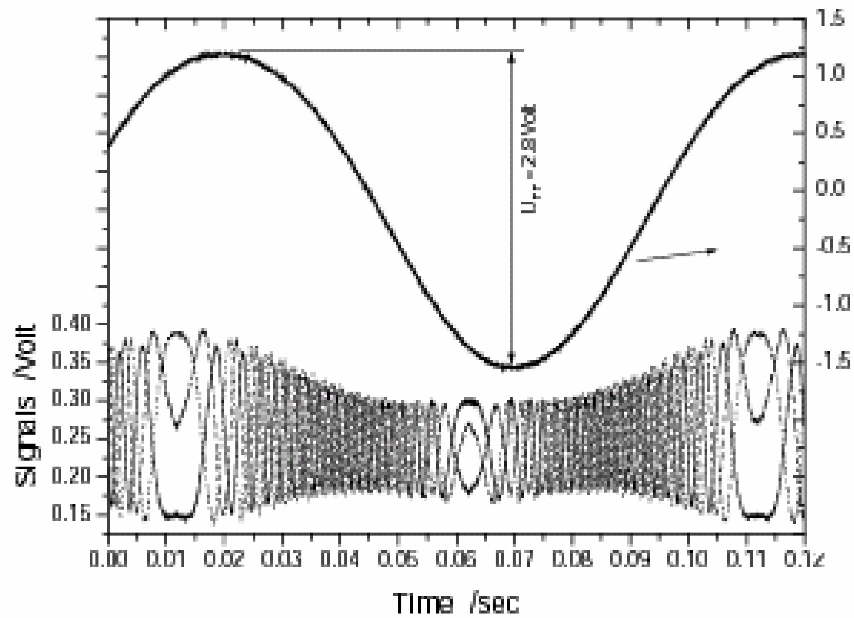
Extrinsic Fabry-Perot Interferometer



Two EFPI's epoxied to the top Electrodes of a 1mm thick PZT-Sheet actuator.

- 50 pm displacement resolution
- 2nm/m strain

Extrinsic Fabry-Perot Interferometer



Phase demodulated signal

3λ output signals

3λ outpu signals with 1800V PZT excitation at 10Hz

Microring Resonator

Resonant wavelength: $$\lambda_m = \frac{2\pi N_{eff} R_{eff}}{m}$$

N_{eff}: Effective index
R_{eff}: Effective ring radius, defined as the radial distance to the centroid of the radial function.

$$\lambda_{FSR} = 2\pi R_{eff} \left[\frac{N_{eff}(\lambda_m)}{m} - \frac{N_{eff}(\lambda_{m+1})}{m+1} \right]$$

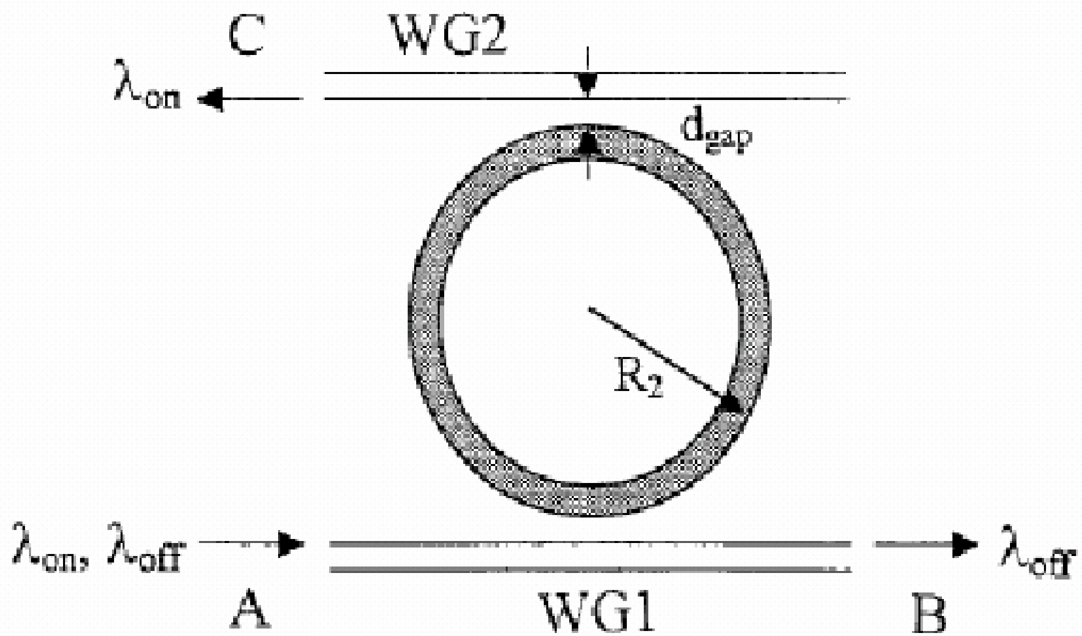
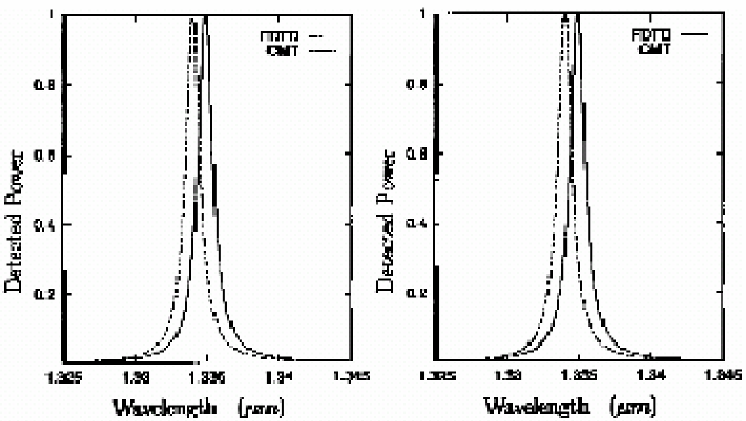


Fig. 1. A schematic of the waveguide-coupled microcavity resonator, showing a microring resonator coupled to straight waveguides.

Lorentzian Filter Response



Half bandwidth of the detected signal power:

$$\Delta\lambda = \frac{2\kappa_T^2 \lambda_m^2}{(2\pi)^2 R_{eff} N_{eff}}$$

$$\kappa_T = \int \kappa(z) e^{-j\Delta\beta z}$$

$\kappa(z)$: Coupling coefficient between the two waveguides

κ_T^2 : Fraction of power coupled out of the ring over the interaction distance

Q: Time-averaged stored energy per optical cycle, divided by power coupled out.

$$Q = \frac{2\pi^2 R_{eff} N_{eff}}{\lambda_m \kappa_T^2}$$

Principles of Fabry-Perot Etalon

Resonant condition: $$\frac{2nd}{\lambda} = q$$

n: Index of refraction of the cavity media

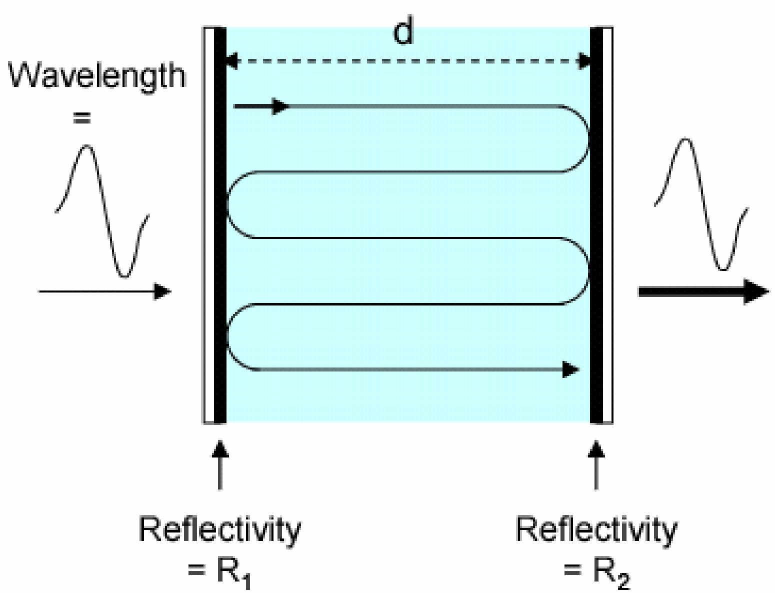
Power transmission coefficient:

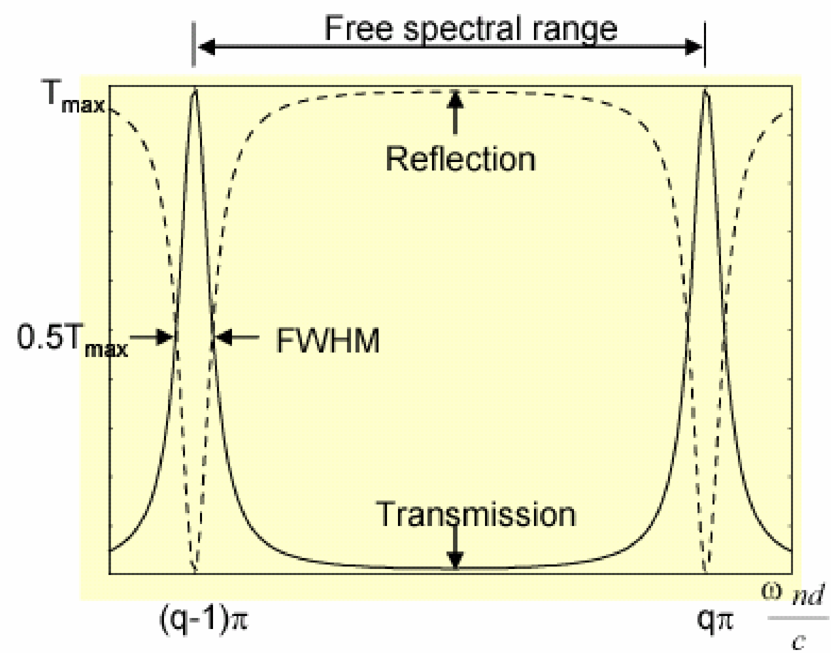
$$T = \frac{1 \quad R_1 \quad 1 \quad R_2}{1 \quad \sqrt{R_1R_2}^{\;2} \quad 4\sqrt{R_1R_2}\,\sin^2\frac{\omega nd}{c}}$$

Power reflection coefficient:

$$R = \frac{\sqrt{R_1} \quad \sqrt{R_2}^{\;2} \quad 4\sqrt{R_1R_2}\,\sin^2\frac{\omega nd}{c}}{1 \quad \sqrt{R_1R_2}^{\;2} \quad 4\sqrt{R_1R_2}\,\sin^2\frac{\omega nd}{c}}$$

$$\omega \quad = \quad 2 \quad c/\lambda$$





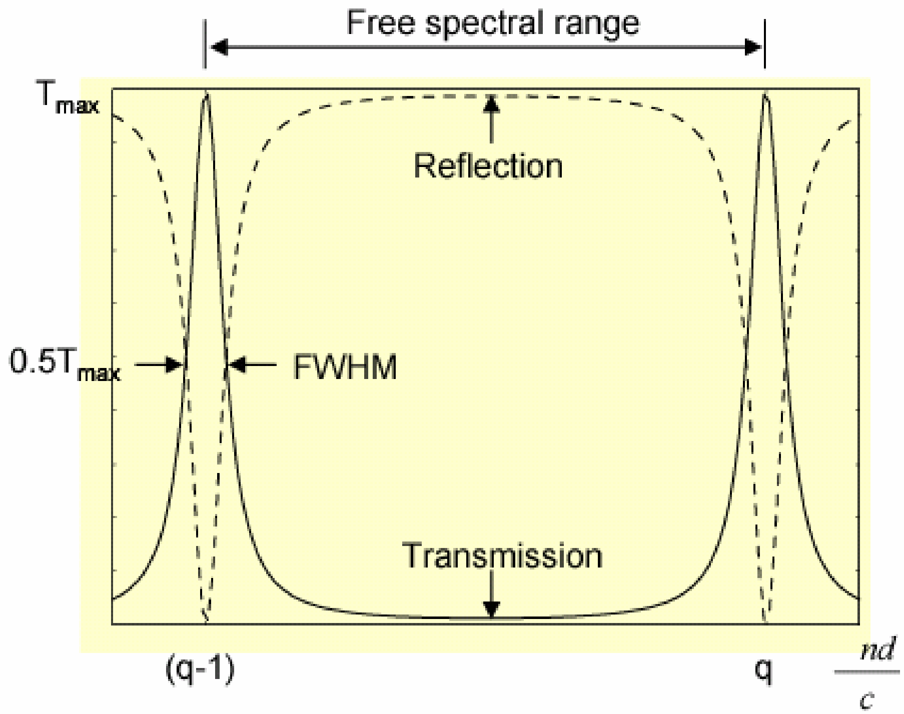
Principles of Fabry-Perot Etalon

Free-spectra range: $FSR \quad {}^{c}\!/_{2nd}$

Full-width at half maximum: $$\frac{c}{2nd}\;\frac{1 \quad R_1R_2{}^{1/2}}{R_1R_2{}^{1/4}}$$

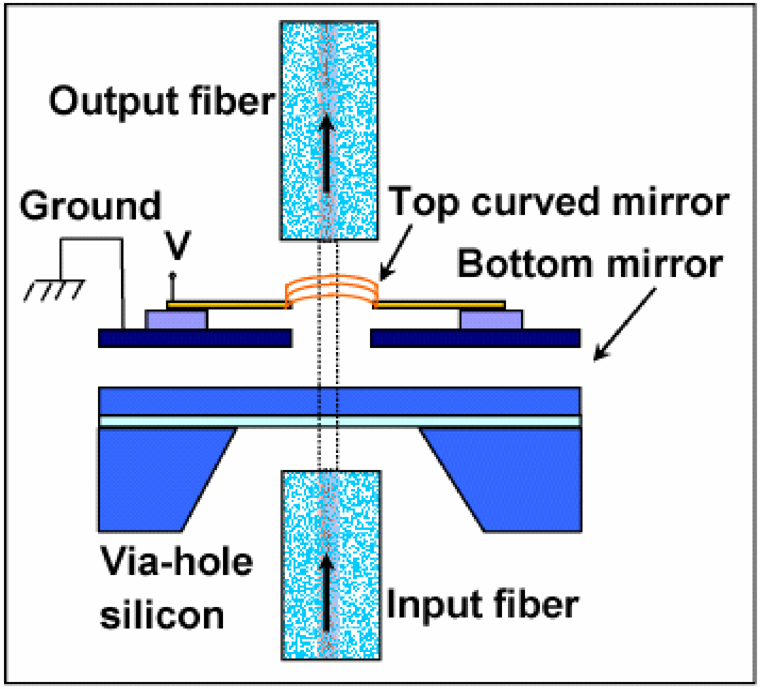
Finesse:

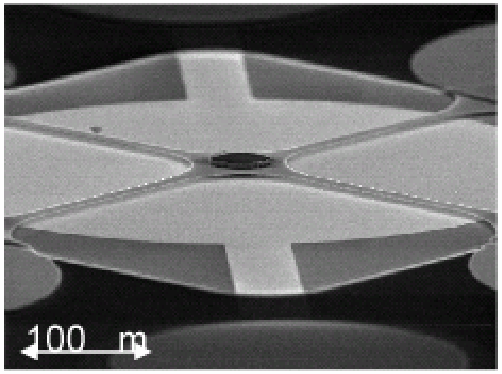
$$F = \frac{FSR}{FWHM} = \frac{R_1R_2{}^{1/4}}{1 \quad R_1R_2{}^{1/2}}$$

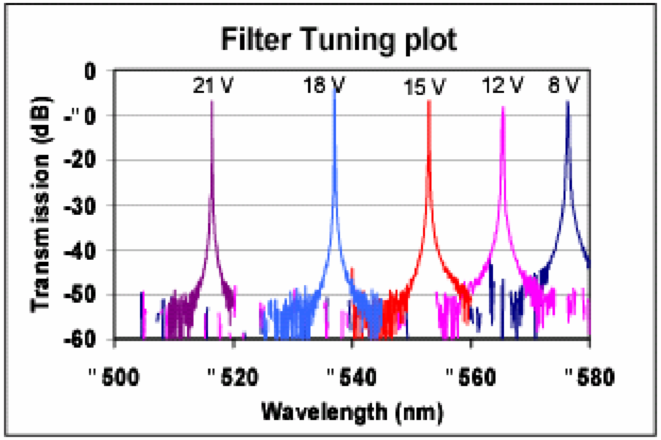


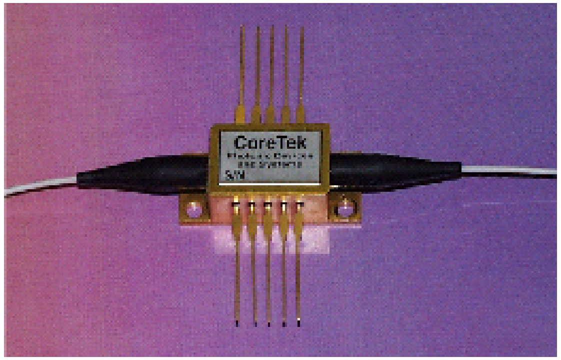
Tunable Filter with Curved Mirror Cavity

- Silicon processing:
  - Half-symmetric curved mirror cavity
  - 3dB linewidth: < 2 GHz
  - Finesse: > " 2,000
- Curved micro-mirrors:
  - Matching cavity mode to fiber mode
  - Low-cost lens-free packaging
  - Fiber insertion loss: 3 - 7dB
- Tuning speed: " 0nm /msec









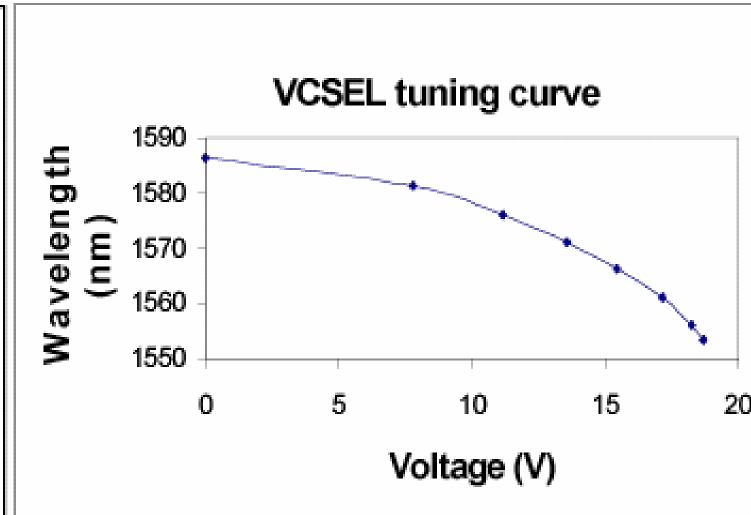
Lih Y. Lin

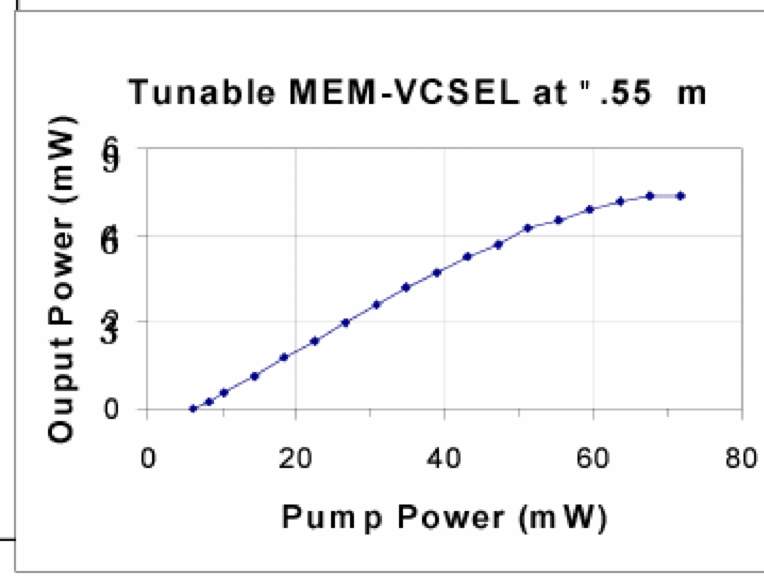
Developed by CoreTek, Inc.

High-power Tunable " 550-nm VCSEL

- MEM-based half-symmetric curved mirror:
  - Single spatial mode
  - Designed mode matches to fiber mode

- Integration with pump laser:
  - Side mode suppression > 50dB
  - Power in single mode fiber > 7mW



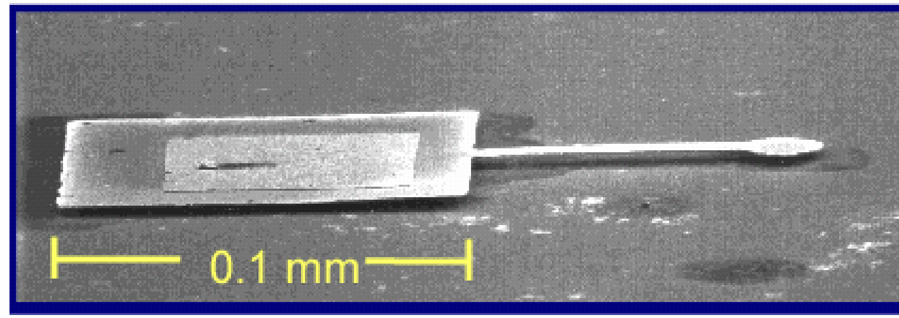


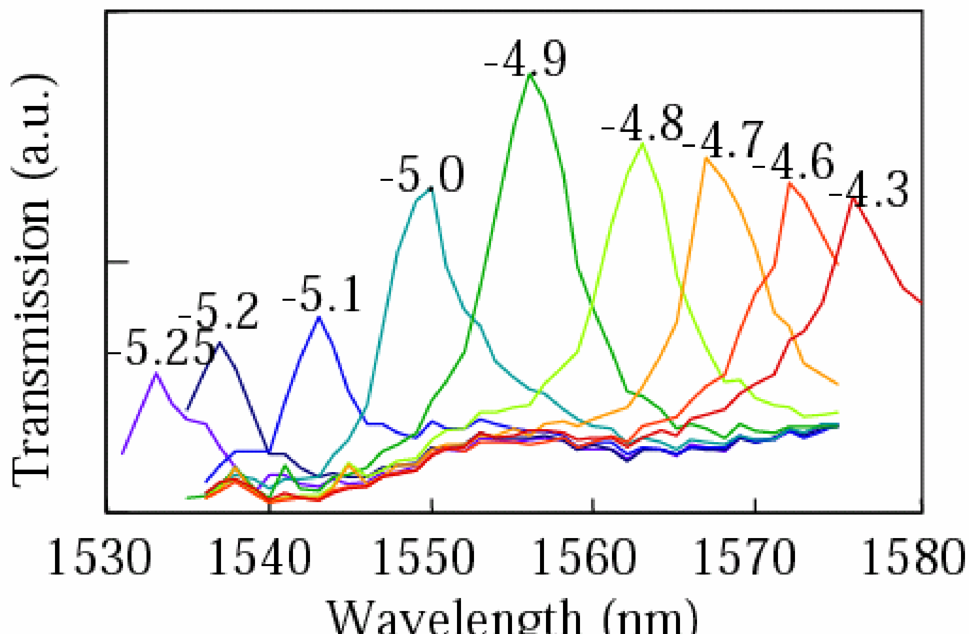
Developed by CoreTek, Inc.

Lih Y. Lin

Tunable DBR " .55- m Filter

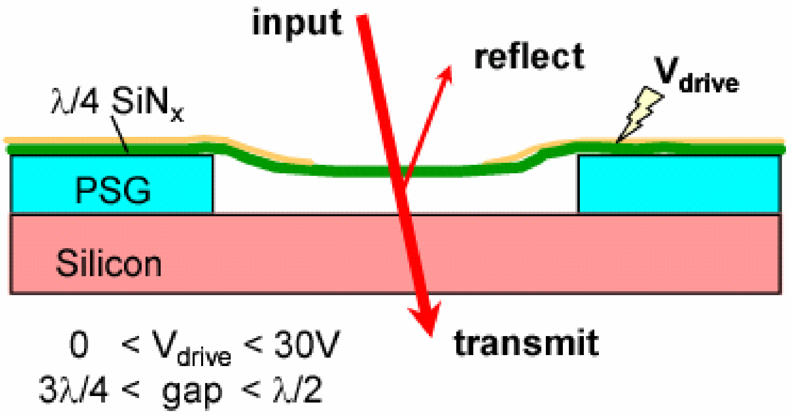
- Using wide-band AlO_x/GaAs DBRs (distributed Bragg reflectors)
- Wide tuning range and efficiency: 50 nm/V

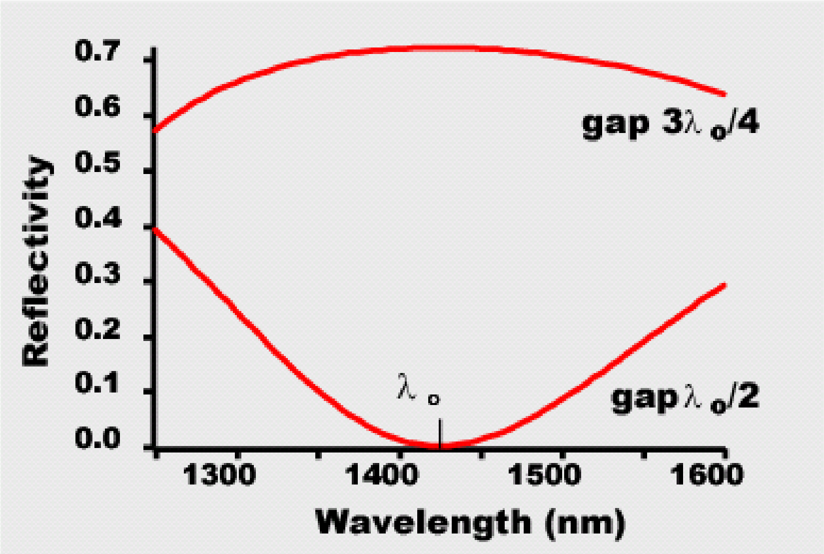




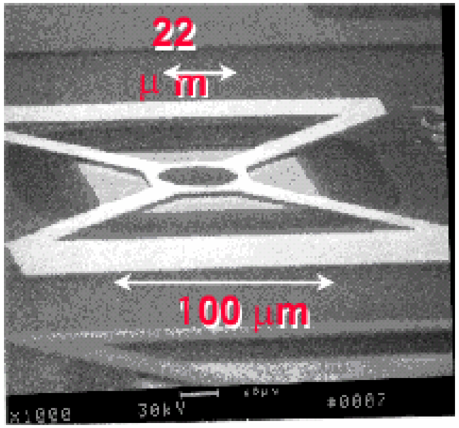
"MARS" Micromechanical Modulator

(Mechanical Anti-Reflection Switch)

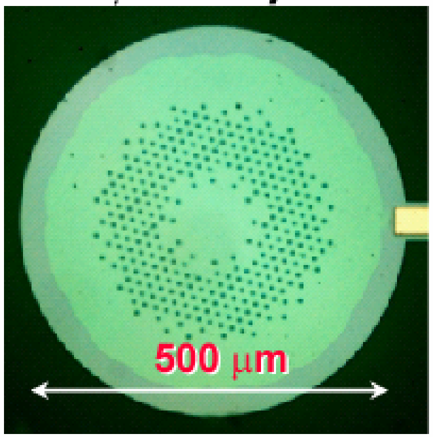




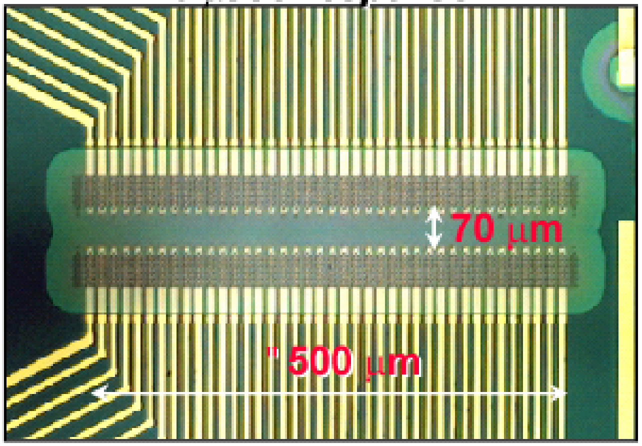
Data transmitter
85 ns response



Variable attenuator
1.1 µsec response

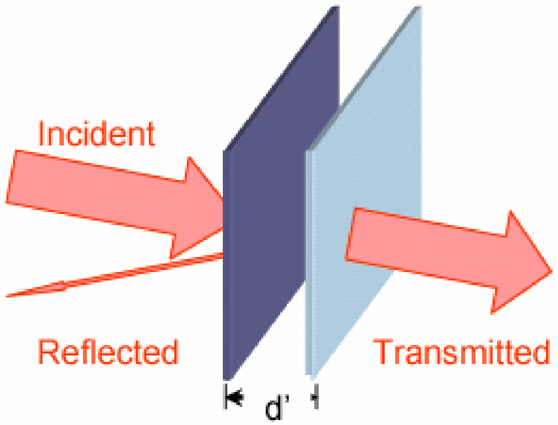


Equalizer mirror stripe
10 µsec response



Ford, Walker, Greywall & Goossen, *IEEE J. Lightwave Tech.* 16, 1998

Fabry-Perot Etalon



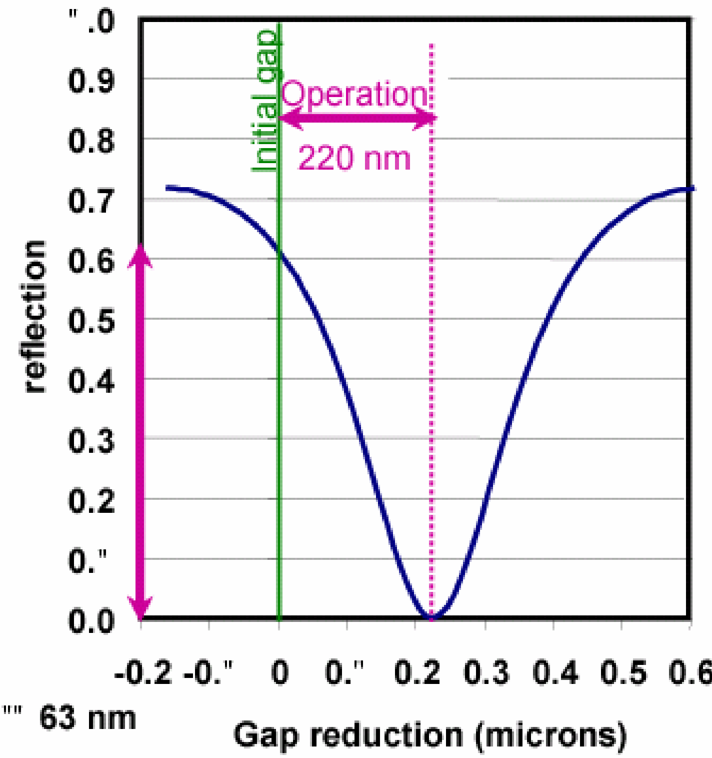
$$\text{Reflectivity} = \frac{F \sin^2(\pi d/d_o)}{1 + F \sin^2(\pi d/d_o)}$$

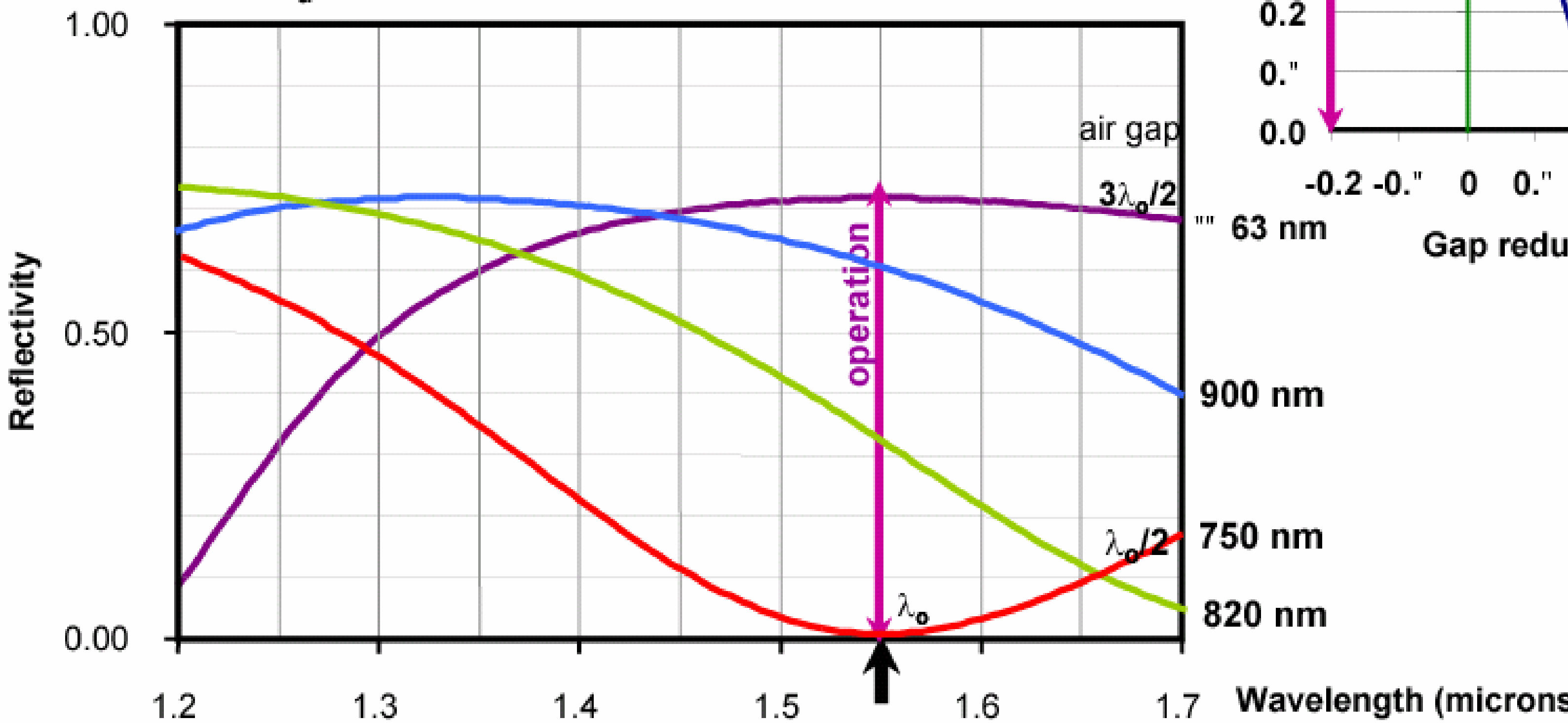
$$F = 4R_s/(1-R_s)^2$$

R_s = top interface reflectivity = 30.6%

d = gap between plates

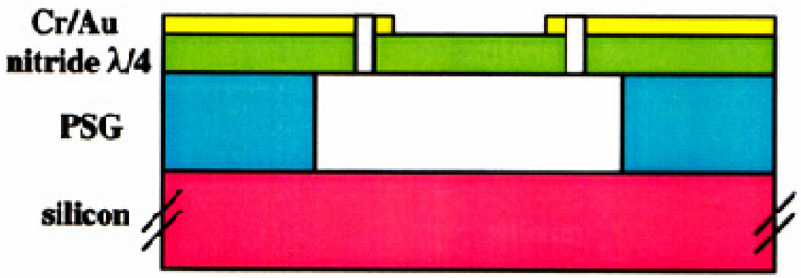
d_o = gap @ minimum reflectivity (λ/2)

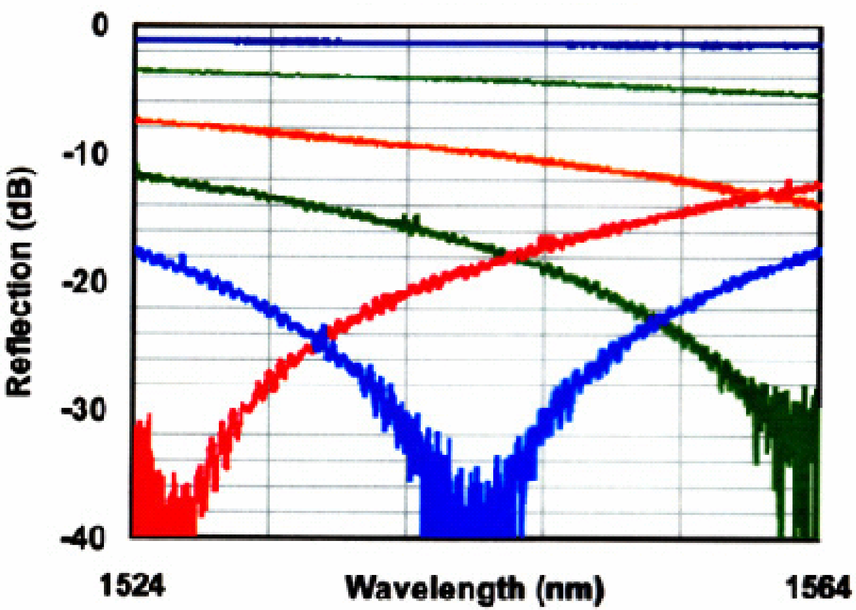




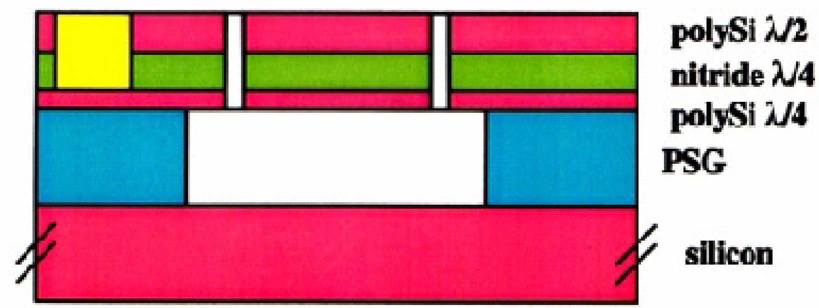
Dielectric Multilayer Structures

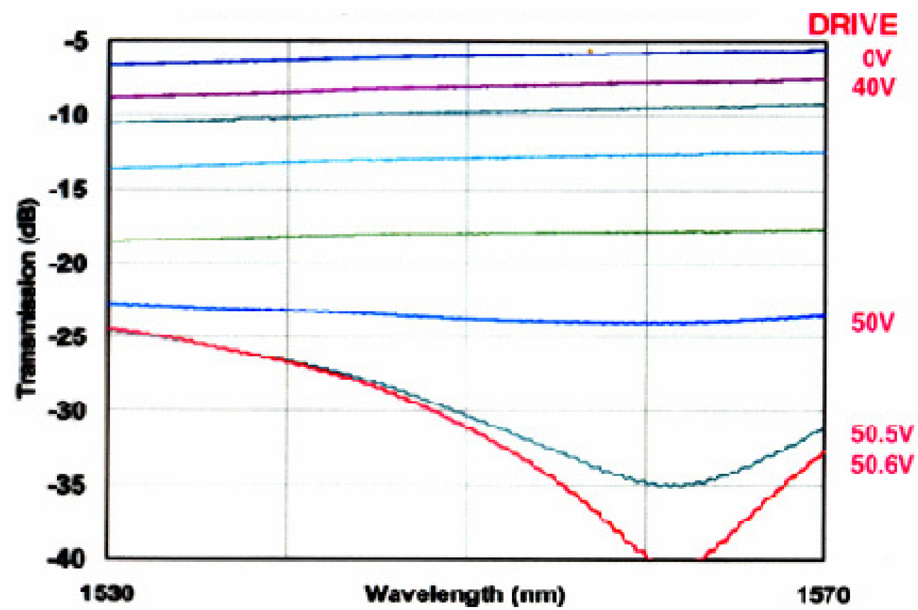
Dielectric Silicon Nitride





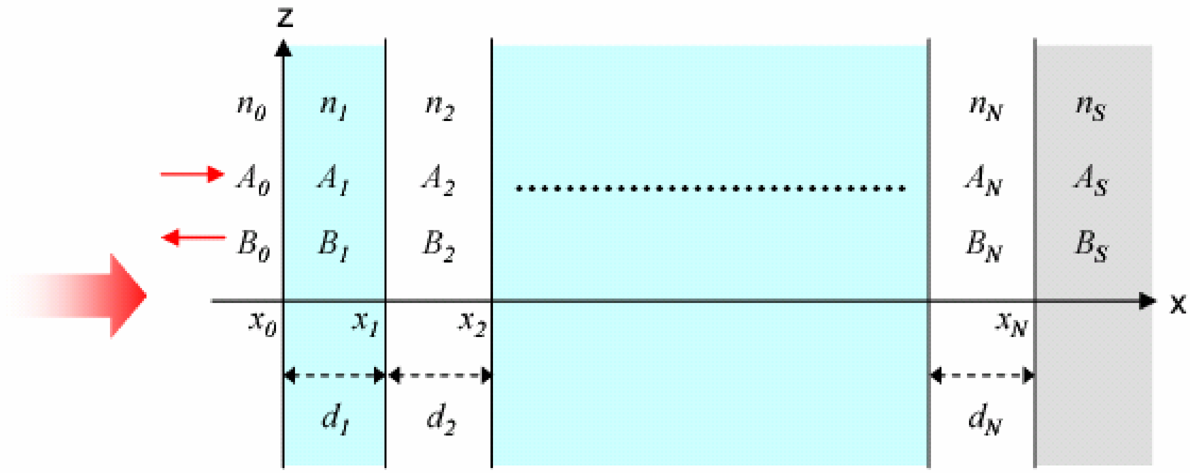
Conductive Polysilicon + Nitride





Ford, Walker, Greywall & Goossen, *J. Lightwave Technol.* 16, 1998

Principles of Dielectric Mirror



$$E = E(x)e^{i(\omega t - \beta z)}$$

Electric field of a general plane-wave

$$E(x) = \begin{cases} A_0 e^{-ik_{0x}(x - x_0)} + B_0 e^{ik_{0x}(x - x_0)}, & x < x_0 \\ A_l e^{-ik_{lx}(x - x_{l-1})} + B_l e^{ik_{lx}(x - x_{l-1})}, & x_{l-1} < x < x_l \\ A_S e^{-ik_{sx}(x - x_N)} + B_S e^{ik_{sx}(x - x_N)}, & x_N < x \end{cases}$$

$$k_{lx} = n_l \frac{\omega}{c} \cos\theta_l$$

x component of the wave vectors (θ_l: ray angle)

Ref: P. Yeh, *Optical Waves in Layered Media*, Wiley.

Principles of Dielectric Mirror

2x2 matrix formulation for multi-layer system

$$\begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = D_0^{-1} D_1 \begin{pmatrix} A_1 \\ B_1 \end{pmatrix}$$

$$\begin{pmatrix} A_l \\ B_l \end{pmatrix} = P_l D_l^{-1} D_{l+1} \begin{pmatrix} A_{l+1} \\ B_{l+1} \end{pmatrix}, \quad l = 1, 2, \ldots, N$$

$$D_l = \begin{pmatrix} 1 & 1 \\ n_l \cos\theta_l & -n_l \cos\theta_l \end{pmatrix} \text{ for TE wave}$$

$$D_l = \begin{pmatrix} \cos\theta_l & \cos\theta_l \\ n_l & -n_l \end{pmatrix} \text{ for TM wave}$$

$$P_l = \begin{pmatrix} e^{i\phi_l} & 0 \\ 0 & e^{-i\phi_l} \end{pmatrix}, \quad \phi_l = k_{lx} d_l$$

Transmission and reflection coefficients can be determined from:

$$\begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} A_S \\ B_S \end{pmatrix}$$

$$\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} = D_0^{-1} \left(\prod_{l=1}^{N} D_l P_l D_l^{-1} \right) D_S$$

Dependent on wavelength and thickness of the dielectric layers