Optical Waveguides, Devices and Applications

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Southern Taiwan University of Technology

Class Information

- Time: T/Th 10:00-12:00 change to M/T 10:00 to 12:00?
- Instructor: Wei-Chih Wang

  office: S606

  office hour: Mon and Tues
- Textbooks:

  - Guide-wave Optoeletronics, T. Tamir, ed., Springer Verlag
  - Optoelectronics and Photonics: Principles and Practices, S. O. Kasap, Prentice Hall.
  - Integrated Optics: Theory and Technology, R. G. Hunsperger, Springer-Verlag
  - Optical Fiber Communications, J. Senior, Prentice- Hall
  - Fundamentals of Photonics, B. Saleh, John Wiley& Sons
  - Fiber optic Sensors, E. Udd, John Wiley& Sons
  - Selected papers in optical sensors, optical MEMS devices and integrated Optical devices.

Class information

- Grading

  Homework assignments 60%

  Final Project 40%

- Final Project:

  - Choose topics related to fiberopic sensors, waveguide sensors or integrated optics devices and optical MEMS system.

  - Details of the project will be announced in mid quarter

  - Two people can work as a team on a project, but each person needs to turn in his/her own final report.

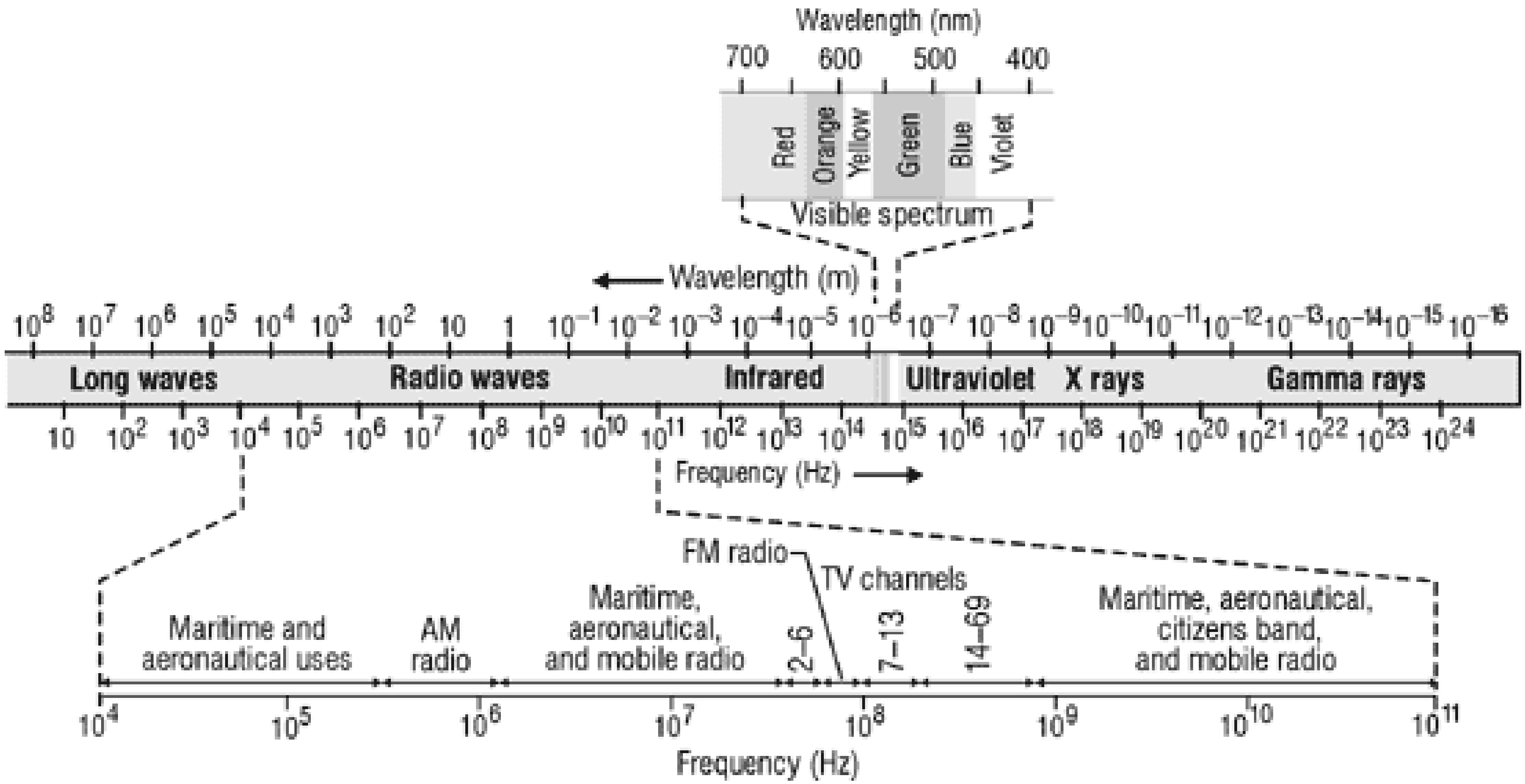
  - Poster presentation will be held in the end of the quarter on your final project

Course Outline

GOALS: To develop student understanding of

Week 1 Theory of Waveguides : Ray-Optics Approach
Week 2 Theory of Waveguides : Electromagnetic-Wave Approach
Week 3 Theory of Waveguides : Modes in Rectangular Waveguides, Losses in Waveguides
Week 4 Theory of waveguides : Waveguide coupling
Week 5 Optical sources and detectors
Week 6 Intensity modulation sensors
Week 7 Interferometric sensors
Week 8 Biosensor, bio-optics and biomedical Applications
Week 9 Final project presentation

Electromagnetic spectrum



Photonics is defined as the generation, manipulation, transport, detection, and use of light information and energy whose quantum unit is the photon.

Photonics is based on the science of optics and electronics. The origins of optical technology (photonics) date back to the remote past. Exodus 38:8 (ca 1200 BCE) tells of "the looking glasses of the women." In the coming century, photonics promises to be a vital part of the information revolution.

To enable us to understand and apply photonics, it is necessary to have a basic understanding of the behavior and properties of light. This course focuses on these fundamentals of photonics

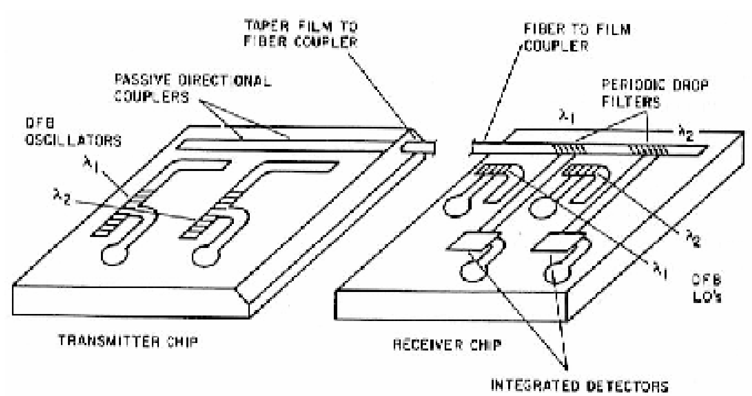
Photonics Opportunities

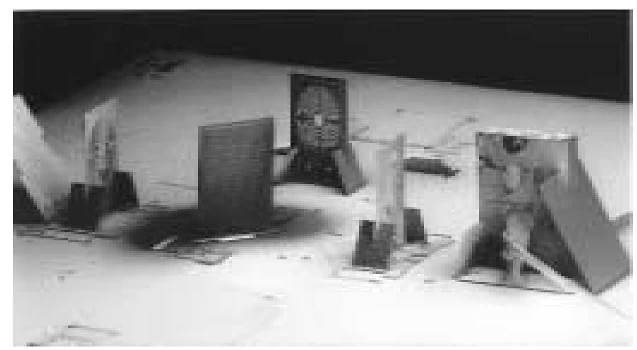
•Medicine-biomedical (laser surgery and in noninvasive diagnostic tools)
•Environmental (measure the pollutants in air and water)
•Energy (harness solar energy)
•Transportation (, provides guidance, collision avoidance, and continuous tuning of engines based on driving conditions)
•Defense (weapon guidance, remote sensing, image processing, and high-energy laser operation)
•Computers and Communication and information technology (gathering, manipulating, storing, routing, and displaying information)
•Manufacturing with photonics and test and analysis (industrial lasers that cut, weld, trim, drill holes, and heat-treat products. inspection is performed using spectroscopy, interferometry, machine vision, and image processing)

Optical MEMS and Waveguide Integrated Optics

Waveguide Integrated Optics
(what's known as integrated optics in earlier day)

Optical MEMS
(can be free space or waveguide)





Photonic integrated circuit include optical MEMS and waveguide integrated optics

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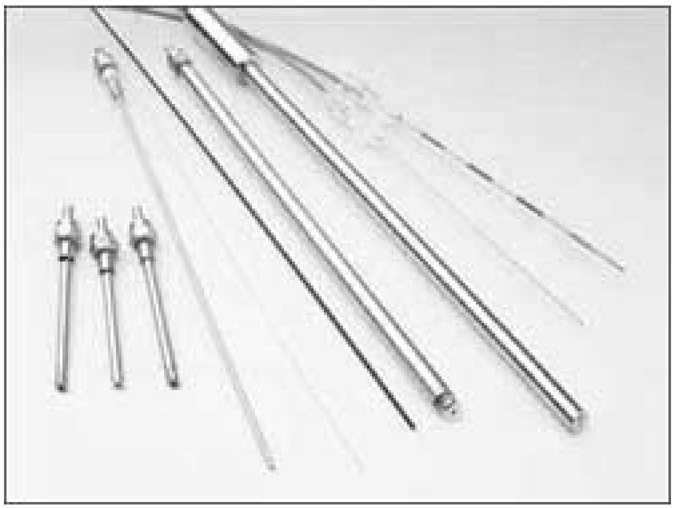
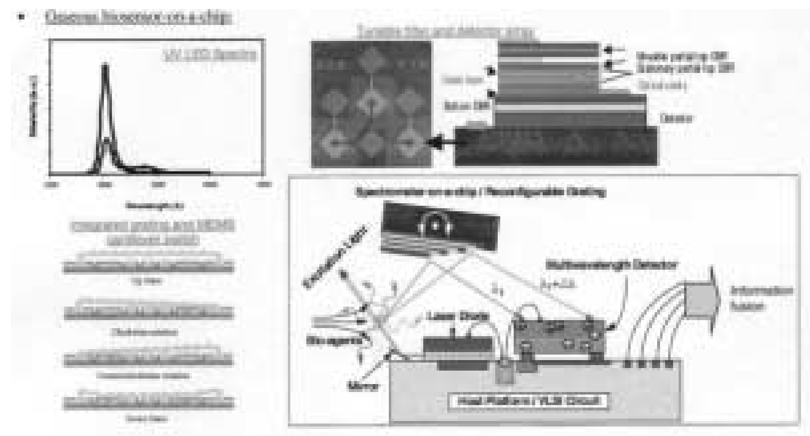
Fiber optic and Waveguide Sensors

Fiber optic Sensors



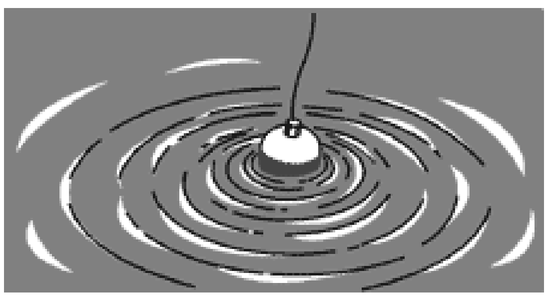
Ocenan optics

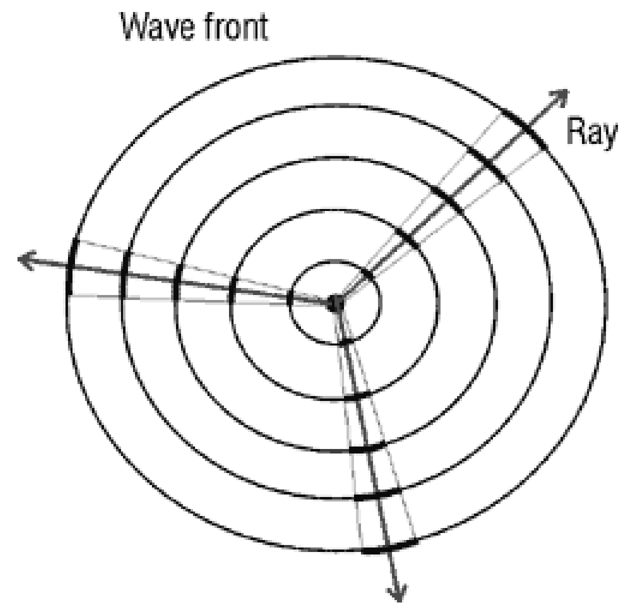
waveguide based optoelectronic bio-sensor systems



Center for Bio-Optoelectronic Sensor SystemsUniversity of Illinois ,

Light rays and light waves

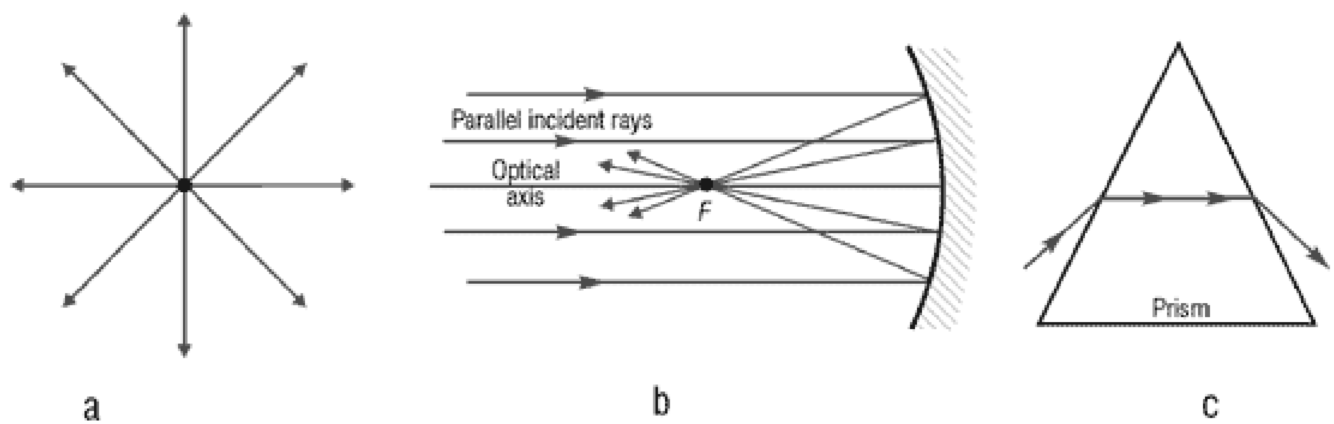




Wave from the bubble

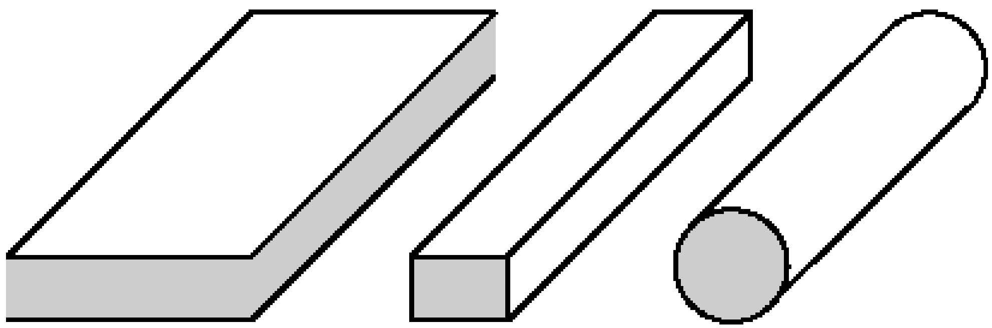
Light rays and wavefronts

Geometric construct of a light ray we can illustrate propagation, reflection, and refraction of light



Typical light rays in (a) *propagation*, (b) *reflection*, *and* (c) *refraction*

Light can be guided by planar or rectangular wave guides, or by optical fibers.



E-M Field in a Planar Waveguide

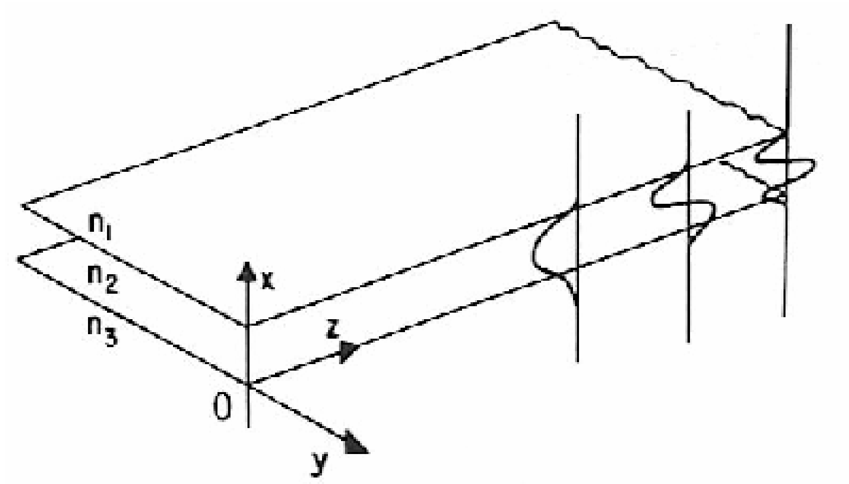


Fig. 2.1. Diagram of the basic three-layer planar waveguide structure. Three mode are shown, representing distributions of electric field in the x direction

Assuming a monochromatic wave propagating in z-direction

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}(\mathbf{r})e^{j\omega t} = \mathbf{E}(x,y)e^{-j\beta z}e^{j\omega t}$$

$$\nabla^2\mathbf{E}(\mathbf{r}) + k^2 n^2(\mathbf{r})\mathbf{E}(\mathbf{r}) = 0$$

Region I: $$\frac{\partial^2}{\partial x^2}E(x,y) + (k^2 n_1^2 - \beta^2)E(x,y) = 0$$

Region II: $$\frac{\partial^2}{\partial x^2}E(x,y) + (k^2 n_2^2 - \beta^2)E(x,y) = 0$$

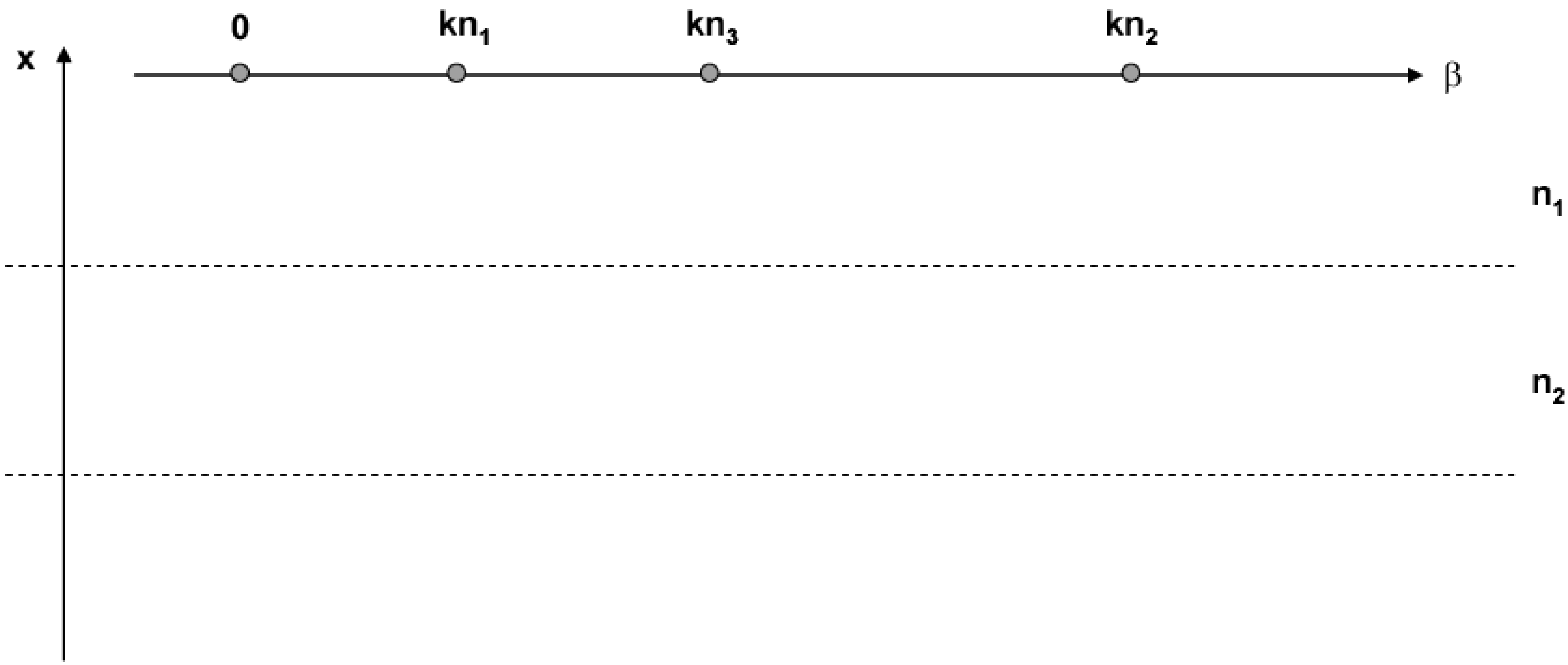
Region III: $$\frac{\partial^2}{\partial x^2}E(x,y) + (k^2 n_3^2 - \beta^2)E(x,y) = 0$$

Modes in a Planar Waveguide

solutions are sinusoidal or exponential, depending on the sign of $(k^2 n_i^2 - \beta^2)$

Boundary conditions: $E(x,y)$ and $\frac{\partial E(x,y)}{\delta x}$ must be continuous at the interface between layers.

Assuming $n_2 > n_3 > n_1$, let's draw possible waveguide modes:



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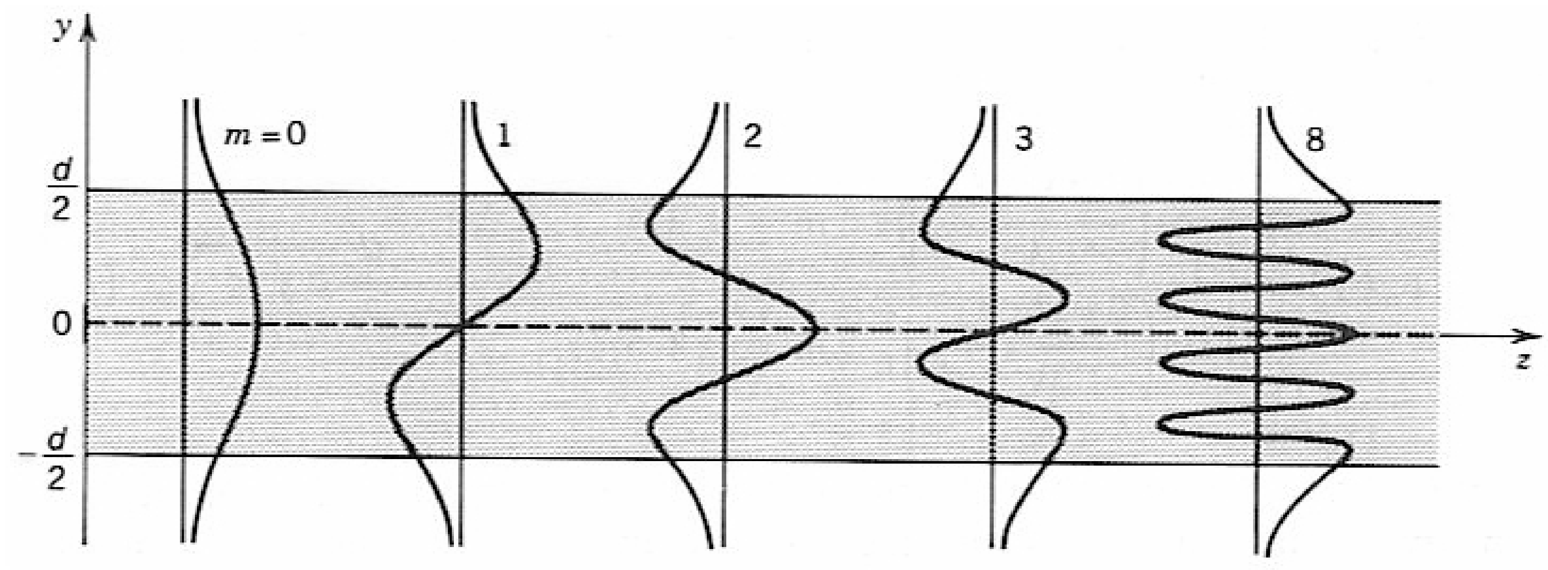
Guided Modes in a Planar Waveguide



m: Mode order

Only discrete values of   are allowed in a waveguide.

Experimental Observation of Waveguide Modes

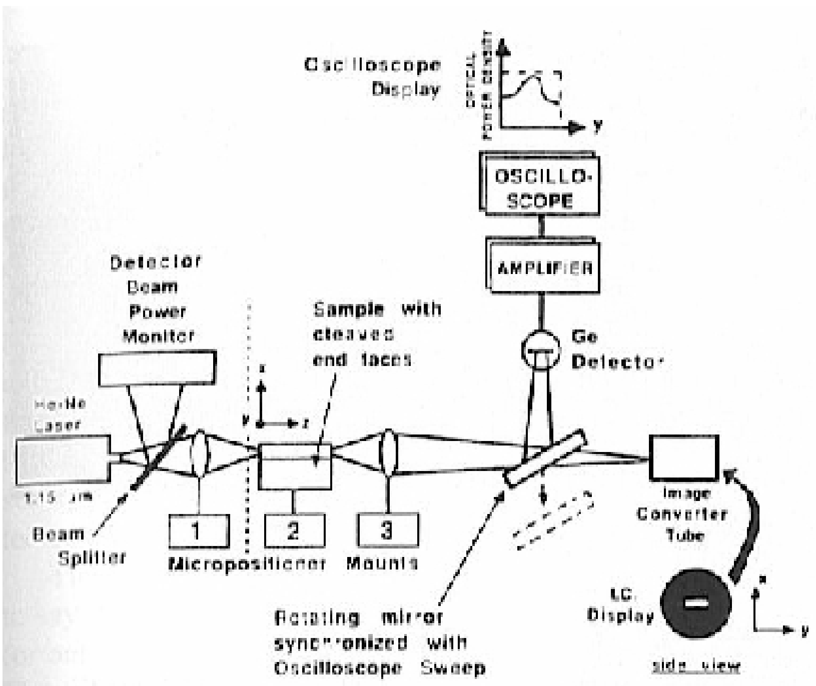
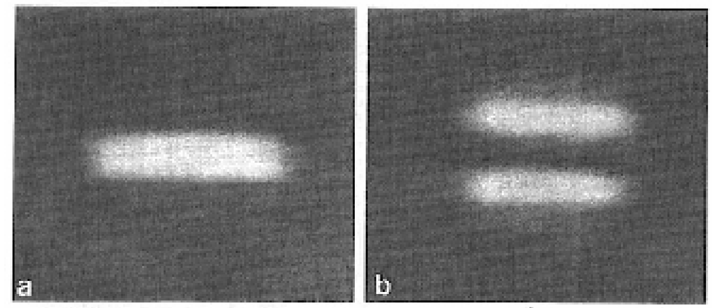


Fig. 2.3. Diagram of an experimental set-up than can be used to measure optical mode shapes [2.9]



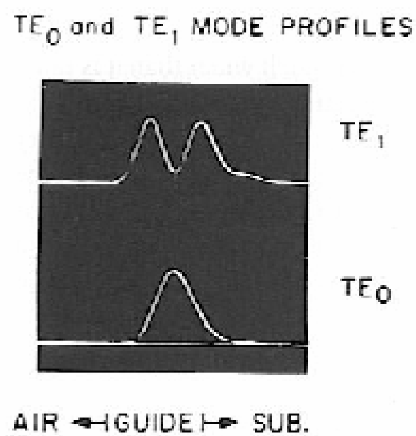


Fig. 2.5. Optical mode shapes are measured using the apparatus of Fig. 2.3. The waveguide in this case was formed by proton implantation into a gallium arsenide substrate to produce a 5 μm thick carrier-compensated layer [2.12]

Ray Patterns in the Three-Layer Planar Waveguide

In the guided region, $E \sim \sin(hx + \gamma)$

$$\beta^2 + h^2 = k^2 n_2^2$$

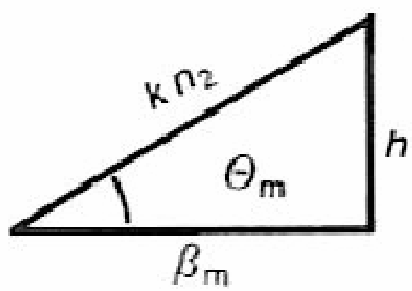


Fig. 2.9. Geometric (vectorial) relationship between the propagation constants of an optical waveguide

For the m-th mode, $\theta_m = \tan^{-1} \frac{h}{\beta_m}$

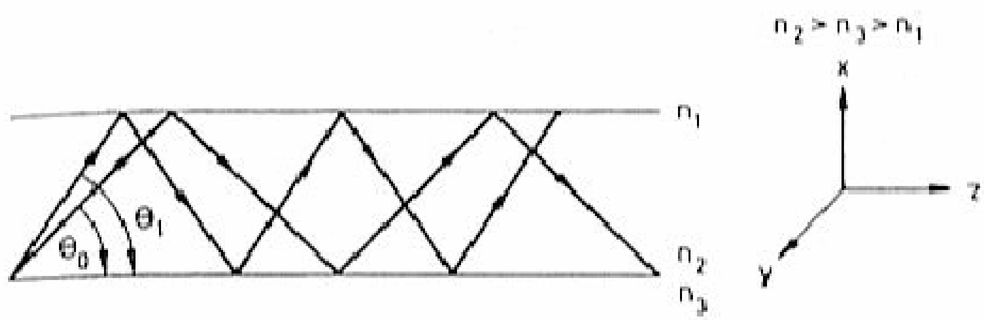


Fig. 2.8. Optical ray pattern within a multimode planar waveguide

Lower-order mode has smaller θ_m and larger β_m (propagating faster!) phasevelocity

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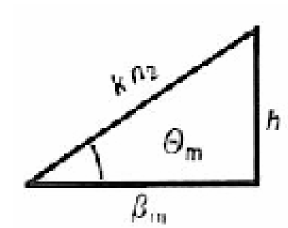
Ray Patterns for Different Modes



$$\phi_2 = \sin^{-1}\frac{\beta}{kn_2} < \sin^{-1}\frac{n_1}{n_2}$$

$$\frac{n_1}{n_2} < \sin\phi_2 < \frac{n_3}{n_2}$$

$$\phi_2 > \sin^{-1}\frac{n_3}{n_2}$$

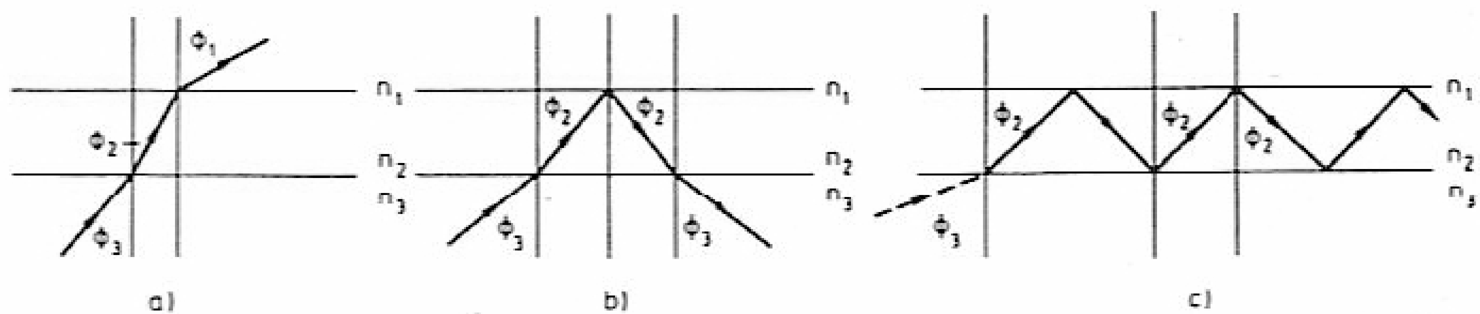


Fig. 2.10a–c. Optical ray patterns for **a** air radiation modes; **b** substrate radiation modes; **c** guided mode. In each case a portion of the incident light is reflected back into layer 3; however, that ray has been omitted from the diagrams

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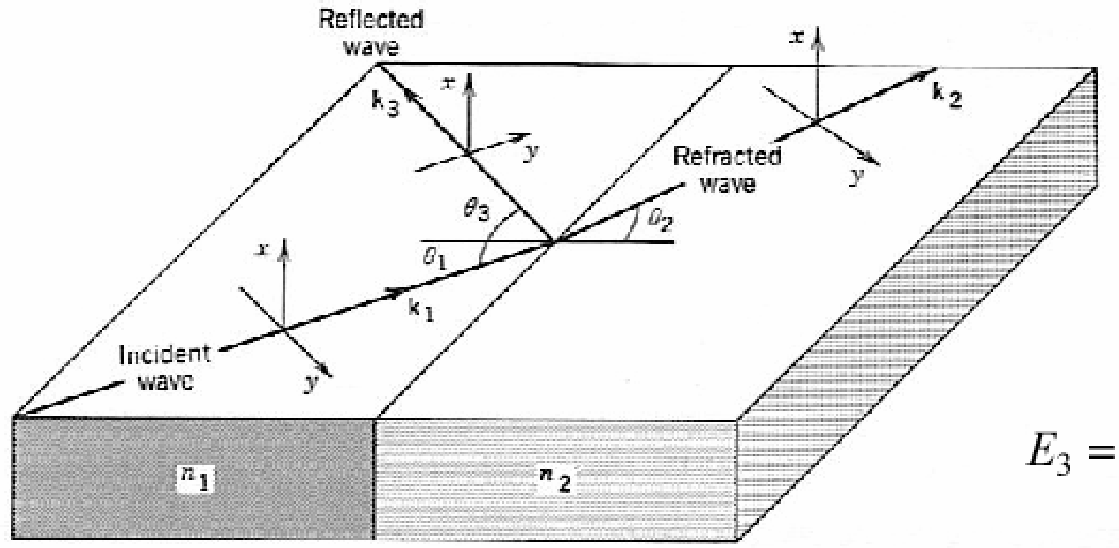
Reflection and Refraction



$$E_3 = rE_1, \quad E_2 = tE_1$$

For TE wave: $r_{TE} \quad \dfrac{n_1 \cos\theta_1 + n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$ $\qquad t_{TE} = 1 + r_{TE}$

For TM wave: $r_{TM} \quad \dfrac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_1 \cos\theta_2}$ $\qquad t_{TM} = \dfrac{n_1}{n_2}(1 + r_{TM})$

$$r_{TE} = |r_{TE}| \exp(j\phi_{TE}), \quad r_{TM} = |r_{TM}| \exp(j\phi_{TM})$$

Total Internal Reflection for TE Wave

$$\tan\frac{\phi_{TE}}{2} \quad \frac{\sqrt{\sin^2\theta_1 - \sin^2\theta_c}}{\cos\theta_1} \quad \frac{\sqrt{n_1^2\sin^2\theta_1 - n_2^2}}{n_1\cos\theta_1}$$

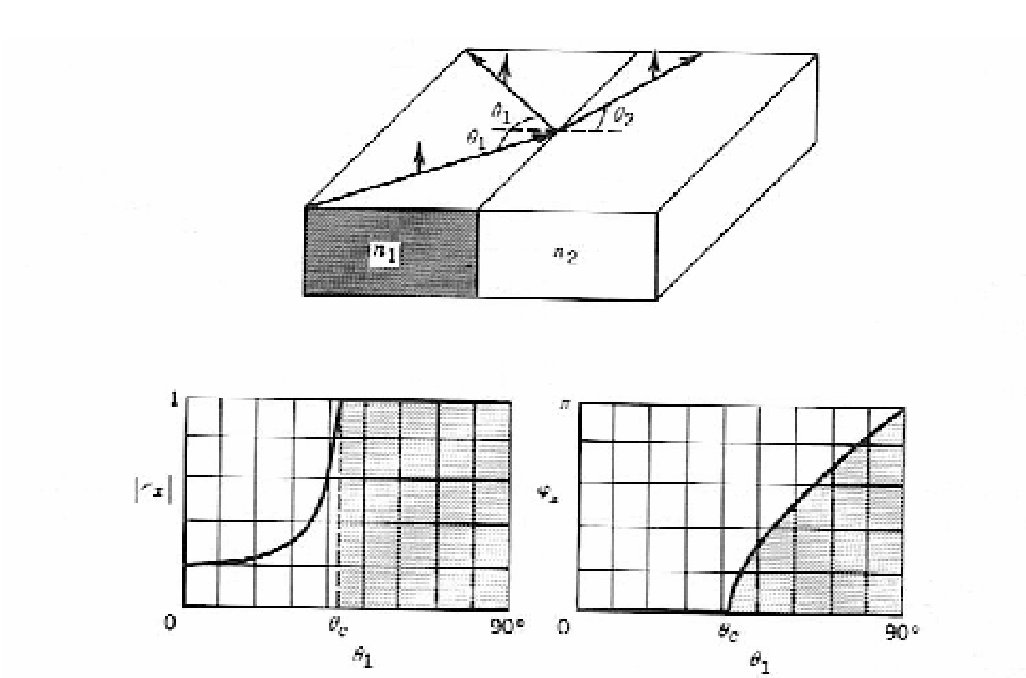


Figure 6.2-3 Magnitude and phase of the reflection coefficient for internal reflection of the TE wave ($n_1/n_2 = 1.5$).

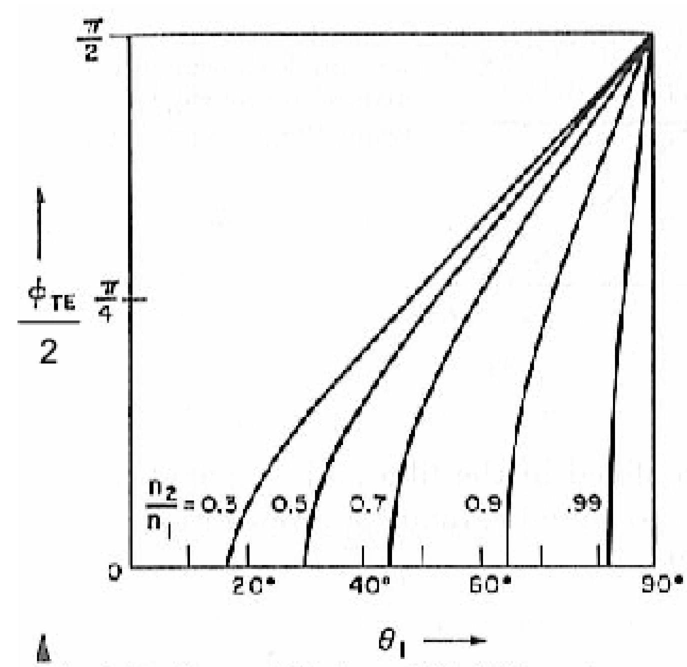


Fig. 2.3. Phase shift ϕ_{TE} of the TE mode as a function of the angle of incidence θ_1

Total Internal Reflection for TM Wave

$$\tan\frac{\phi_{TM}}{2} = \frac{\sqrt{\sin^2\theta_1 - \sin^2\theta_c}}{\cos\theta_1 \sin^2\theta_c} = \frac{n_1^2}{n_2^2}\,\frac{\sqrt{n_1^2\sin^2\theta_1 - n_2^2}}{n_1\cos\theta_1}$$

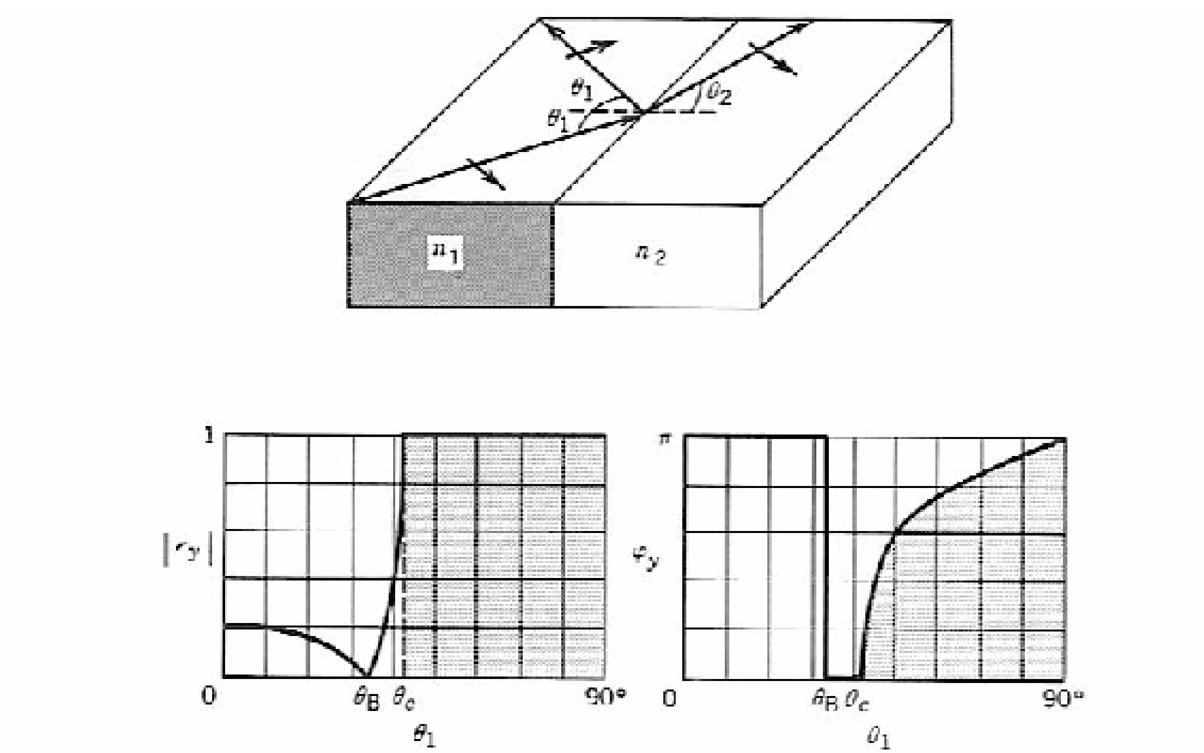


Figure 6.2-5 Magnitude and phase of the reflection coefficient for internal reflection of the TM wave $(n_1/n_2 - 1.5)$.

Dispersion Equation

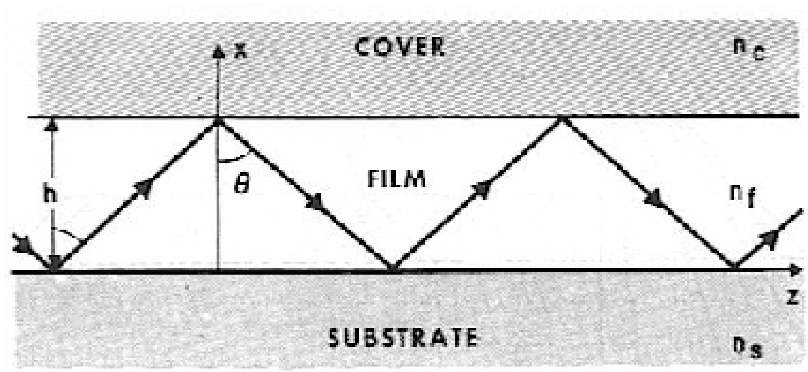


Fig. 2.5. Side-view of a slab waveguide showing wave normals of the zig-zag waves corresponding to a guided mode

Transverse resonance condition:

$$2kn_f h\cos\theta - 2\phi_c - 2\phi_s = 2m\pi$$

m : mode number

$kn_f h\cos$: phase shift for the transverse passage through the film

$2\phi_c (=\phi_{TE,TM})$: phase shift due to total internal reflection from film/cover interface

$2\phi_s (=\phi_{TE,TM})$: phase shift due to total internal reflection from film/substrate interface

Dispersion equation (β vs. ω):

$$kn_f h\cos\theta - \phi_c - \phi_s = m\pi$$

Effective guide index $N = \beta/k = n_f \sin\theta$ $n_s < N < n_f$

Graphical Solution of the Dispersion Equation

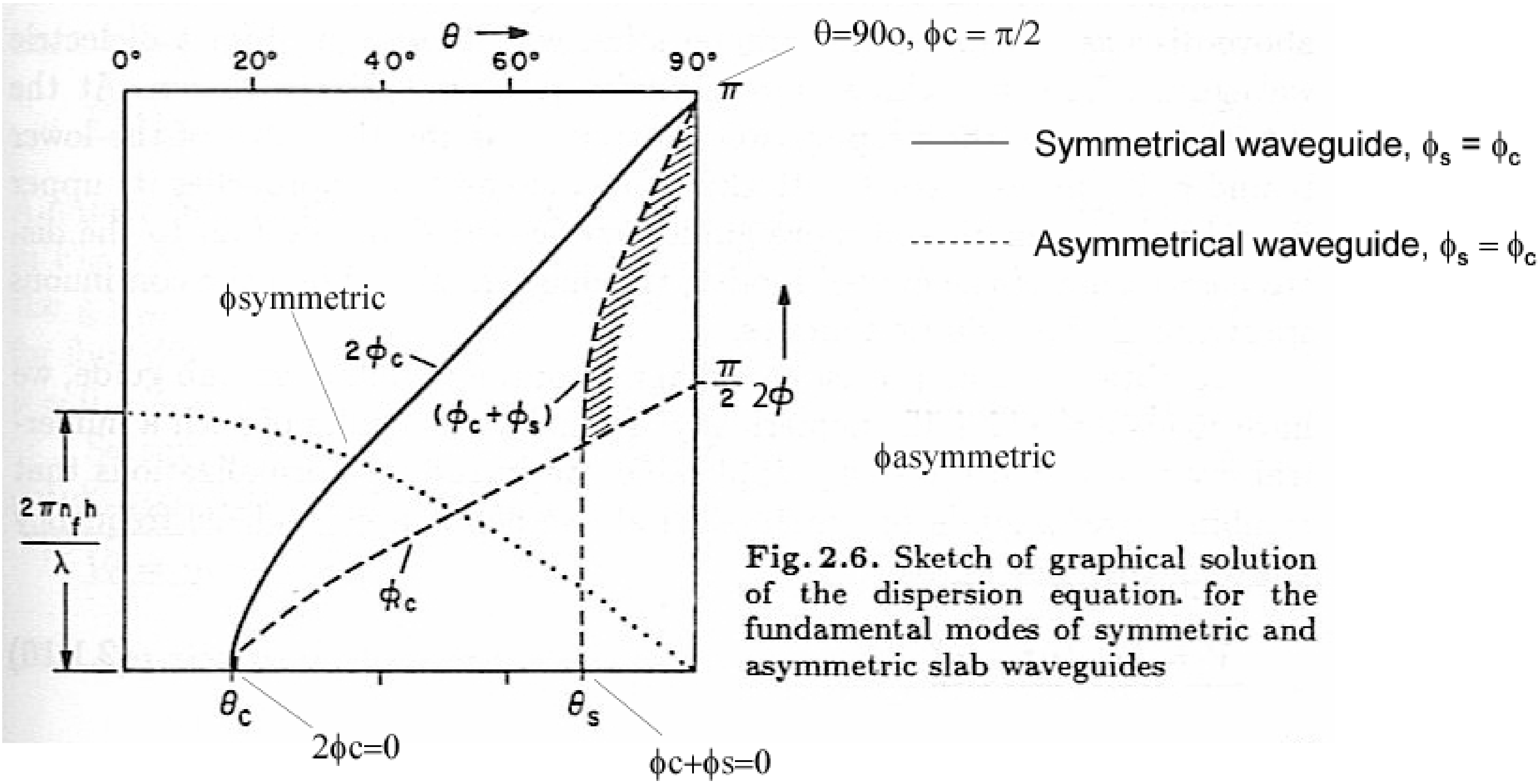
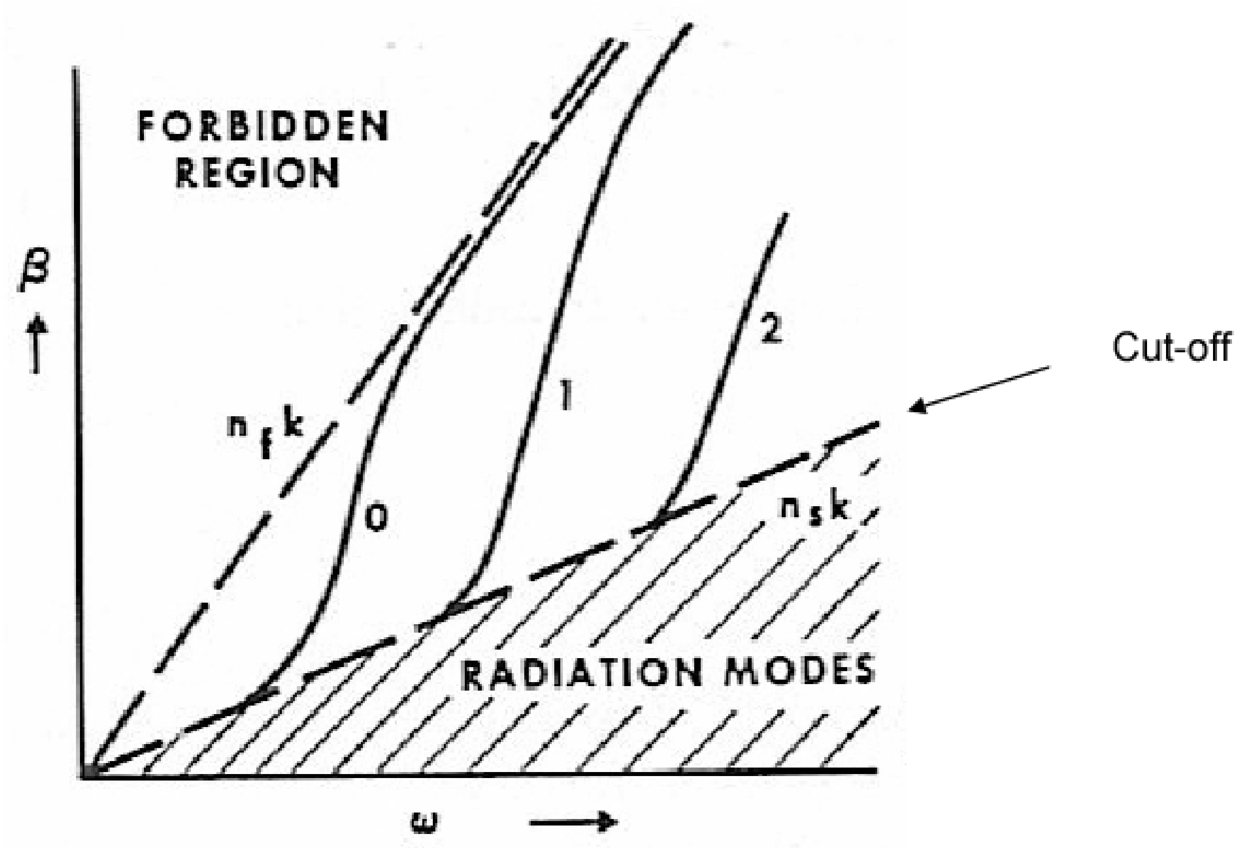


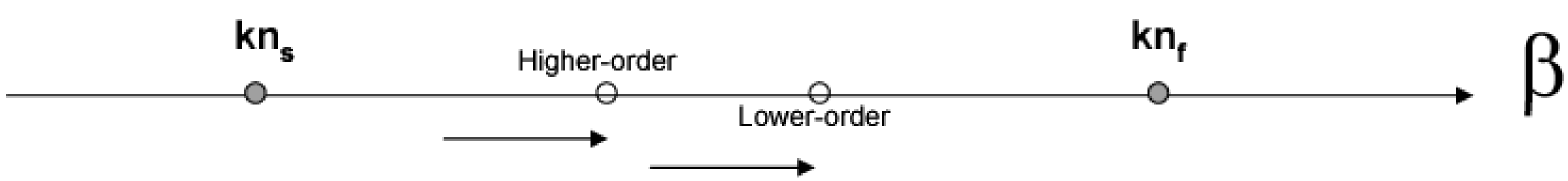
Fig. 2.6. Sketch of graphical solution of the dispersion equation. for the fundamental modes of symmetric and asymmetric slab waveguides

For fundamental mode (m = 0), there is always a solution (no cut-off) for symmetrical waveguide.

Increasing h (and/or decreasing λ) will support more modes.

Typical β – ω diagram





Numerical Solution for Dispersion Relation (I)

Define:

Normalized frequency and film thickness

$$V = kh\sqrt{n_f^2 - n_s^2}$$

Normalized guide index

$$b = \frac{N^2 - n_s^2}{n_f^2 - n_s^2}$$

b = 0 at cut-ooff (N = n_s), and approaches 1 as N -> n_f.

Measure for the asymmetry

$$a = \frac{n_s^2 - n_c^2}{n_f^2 - n_s^2} \text{ for TE,} \quad a = \frac{n_f^4}{n_c^4}\frac{n_s^2 - n_c^2}{n_f^2 - n_s^2} \text{ for TM}$$

a = 0 for perfect symmetry ($n_s = n_c$), and a approaches infinity for strong asymmetry ($n_s \neq n_c$, $n_s \sim n_f$).

Table 2.2. Asymmetry measures for the TE modes (a_E) and the TM modes (a_M) of slab waveguides

Waveguide	n_s	n_f	n_c	a_E	a_M
GaAlAs, double heterostructure	3.55	3.6	3.55	0	0
Sputtered glass	1.515	1.62	1	3.9	27.1
Ti-diffused $LiNbO_3$	2.214	2.234	1	43.9	1093
Outdiffused $LiNbO_3$	2.214	2.215	1	881	21206

Numerical Solution for Dispersion Relation (II)

For TE modes, dispersion relation

$$kn_f h\cos\theta - \phi_c - \phi_s = m\pi \;\; > \quad V\sqrt{1-b} = m\pi + \tan^{-1}\sqrt{\frac{b}{1-b}} \quad \tan^{-1}\sqrt{\frac{b+a}{1-b}}$$

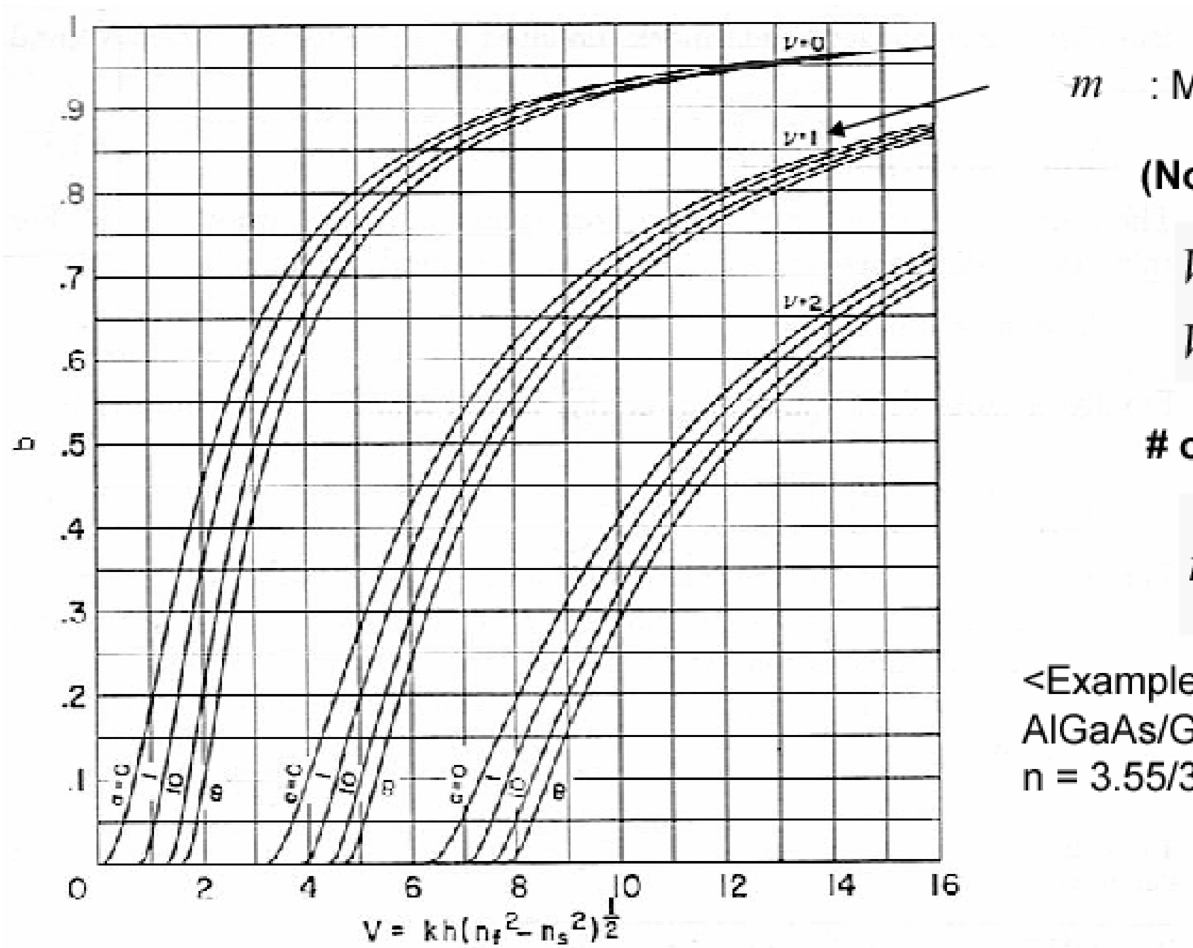


Fig. 2.8. Normalized ω-β diagram of a planar slab waveguide showing the guide index b as a function of the normalized thickness V for various degrees of asymmetry [2.20]

m : Mode number

(Normalized) cut-off frequency:

$$V_0 = \tan^{-1}\sqrt{a}$$
$$V_m = V_0 + m\pi$$

# of guided modes allowed:

$$m = \frac{2h}{\lambda}\sqrt{n_f^2 - n_s^2}$$

<Example>
AlGaAs/GaAs/AlGaAs double heterostructure
n = 3.55/3.6/3.55

The Goos-Hänchen Shift

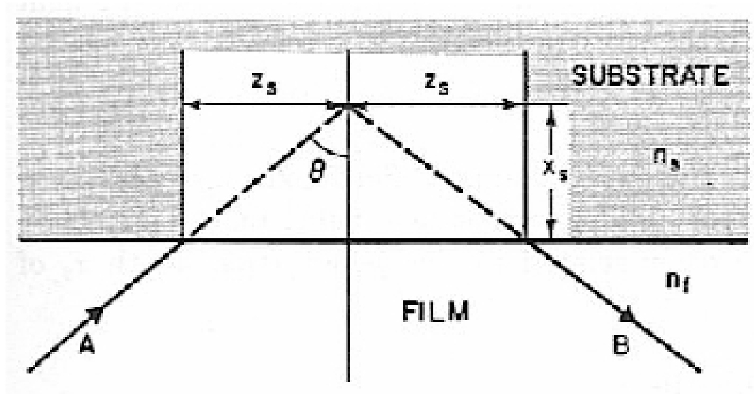


Fig. 2.9. Ray picture of total reflection at the interface between two dielectric media showing a lateral shift of the reflected ray (Goos-Hänchen shift)

For TE modes

$$kz_s = (N^2 - n_s^2)^{-1/2} \tan\theta$$

For TM modes

$$kz_s = \frac{(N^2 - n_s^2)^{-1/2} \tan\theta}{\frac{N^2}{n_s^2} + \frac{N^2}{n_f^2} - 1}$$

The lateral ray shift indicates a penetration depth:

$$x_s = \frac{z_s}{\tan\theta} \qquad z_s = \frac{d\phi_s}{d\beta}$$

Effective Waveguide Thickness

Effective thickness

$$h_{eff} = h + x_s + x_c$$

Normalized effective thickness

$$H = kh_{eff}\sqrt{n_f^2 - n_s^2}$$

For TE modes

$$H = V + \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{b+a}}$$

Minimum H - > Maximum confinement

<Example> Sputtered glass, n_s = 1.515, n_f = 1.62, n_c = 1, a = 3.9

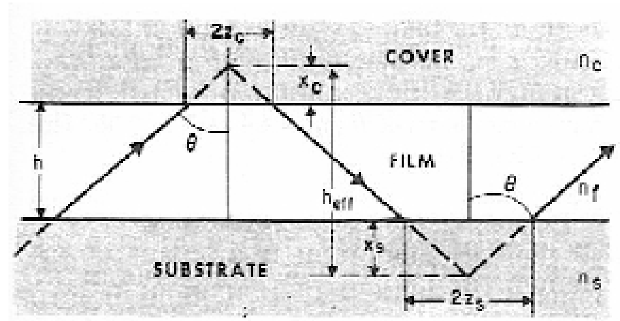


Fig. 2.10. Ray picture of zig-zag light propagation in a slab waveguide. Goos Hänchen shifts are incorporated in the model, and the effective guide thickness h_{eff} is indicated

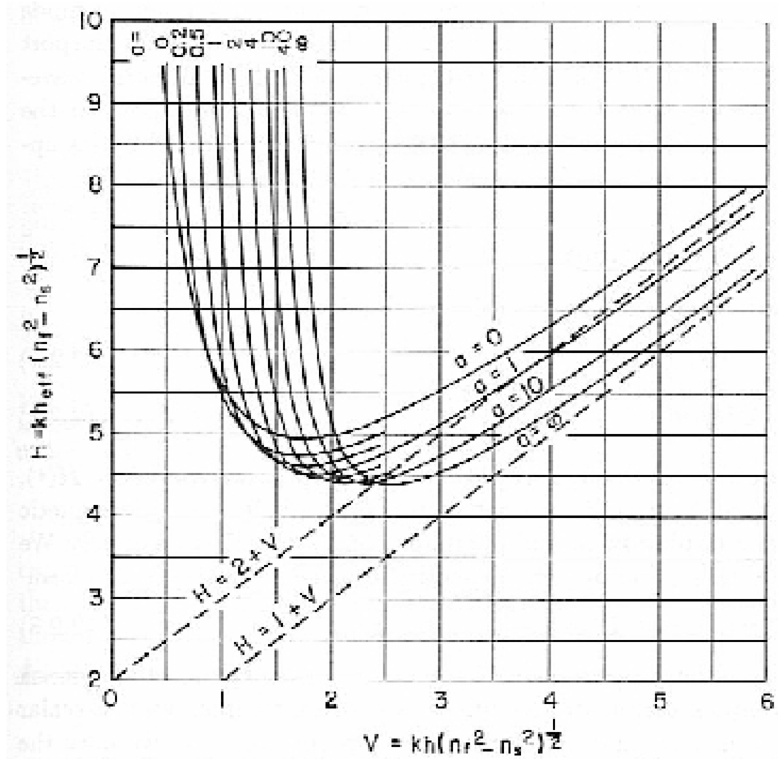


Fig. 2.11. Normalized effective thickness of a slab waveguide as a function of the normalized film thickness V for various degrees of asymmetry (after [2.20])

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Guided E-M Wave in a Planar Waveguide

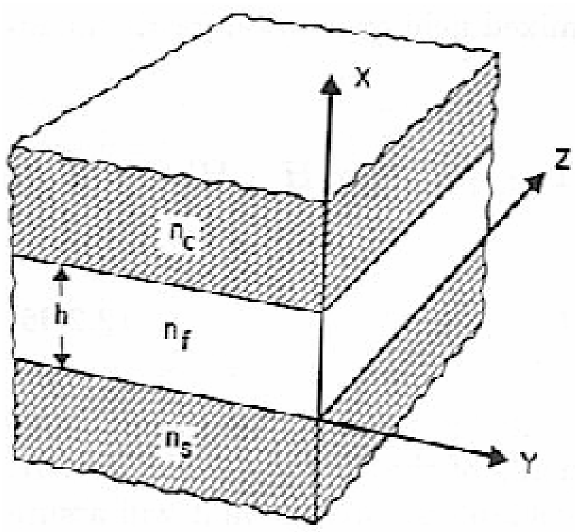

Fig. 2.14. Sketch of an "asymmetric" slab waveguide and the choice of the coordinate system. Note that the z-axis lies in the film-substrate interface

Define:

$$\kappa_c^2 = n_c^2 k^2 - \beta^2 = -\gamma_c^2$$

$$\kappa_f^2 = n_f^2 k^2 - \beta^2$$

$$\kappa_s^2 = n_s^2 k^2 - \beta^2 = -\gamma_s^2$$

Cover: $$\frac{\partial^2}{\partial x^2} E(x,y) + (n_c^2 k^2 - \beta^2) E(x,y) = 0 \Rightarrow \frac{\partial^2}{\partial x^2} E - \gamma_c^2 E = 0$$

Film: $$\frac{\partial^2}{\partial x^2} E(x,y) + (n_f^2 k^2 - \beta^2) E(x,y) = 0 \Rightarrow \frac{\partial^2}{\partial x^2} E - \gamma_f^2 E = 0$$

Substrate: $$\frac{\partial^2}{\partial x^2} E(x,y) + (n_s^2 k^2 - \beta^2) E(x,y) = 0 \Rightarrow \frac{\partial^2}{\partial x^2} E - \gamma_s^2 E = 0$$

TE Modes (I)

Modal solutions are sinusoidal or exponential, depending on the sign of $(k^2 n_i^2 - \beta^2)$

Boundary conditions: The tangential components of **E** and **H** are continuous at the interface between layers. # E_y and $\partial E_y / \partial x$ continuous at the interface.

For guided modes:

Cover: $$\frac{\partial^2}{\partial x^2} E_y - \gamma_c^2 E_y = 0 \Rightarrow E_y = E_c \exp[-\gamma_c (x - h)]$$

Film: $$\frac{\partial^2}{\partial x^2} E_y - \kappa_f^2 E_y = 0 \Rightarrow E_y = E_f \cos(\kappa_f x - \phi_s)$$

Substrate: $$\frac{\partial^2}{\partial x^2} E_y - \gamma_s^2 E_y = 0 \Rightarrow E_y = E_c \exp(\gamma_s x)$$

Applying boundary conditions, we obtain:

$$\tan\phi_s = \frac{\gamma_s}{\kappa_f}, \quad \tan\phi_c = \frac{\gamma_c}{\kappa_f}$$

$$\kappa_f h - \phi_s - \phi_c = m\pi$$ -> Dispersion relation

TE Modes (II)

Relation between the peak fields:

$$E_f^2(n_f^2 - N^2) = E_s^2(n_f^2 - n_s^2) = E_c^2(n_f^2 - n_c^2)$$

E_c, E_f, and E_s can be determined by,

Optical power

$$P = \frac{1}{2}\int \mathrm{Re}\{\mathbf{E} \quad \mathbf{H}^*\} \ \hat{\mathbf{z}}dx$$

Optical confinement factor

$$\Gamma = \frac{\frac{1}{2}\int_0^h \mathrm{Re}\{\mathbf{E} \quad \mathbf{H}^*\}\cdot\hat{\mathbf{z}}dx}{\frac{1}{2}\int_{-\infty}^{\infty} \mathrm{Re}\{\mathbf{E} \quad \mathbf{H}^*\}\cdot\hat{\mathbf{z}}dx}$$

TM Modes

Cover: $\frac{\partial^2}{\partial x^2} H_y - \gamma_c^2 H_y = 0 \Rightarrow \quad H_y = H_c \exp[-\gamma_c (x - h)]$

Film: $\frac{\partial^2}{\partial x^2} H_y + \kappa_f^2 H_y = 0 \Rightarrow \quad H_y = H_f \cos(\kappa_f x - \phi_s)$

Substrate: $\frac{\partial^2}{\partial x^2} H_y - \gamma_s^2 H_y = 0 \Rightarrow \quad H_y = H_c \exp(\gamma_s x)$

Boundary conditions: H_y and E_z continuous at the interface between the layers

$\Rightarrow \quad H_y$ and $\frac{1}{n^2}\frac{dH_y}{dx}$ continuous at the interface between the layers

Applying boundary conditions, we obtain:

$$\tan\phi_s = \left[\frac{n_f}{n_s}\right]^2 \frac{\gamma_s}{\kappa_f}, \quad \tan\phi_c = \left[\frac{n_f}{n_c}\right]^2 \frac{\gamma_c}{\kappa_f}$$

$\kappa_f h - \phi_s - \phi_c = m\pi$ -> Dispersion relation

Relation between the peak fields: $H_f^2 \frac{(n_f^2 - N^2)}{n_f^2} = H_s^2 (n_f^2 - n_s^2) \frac{q_s}{n_s^2} = H_c^2 (n_f^2 - n_c^2) \frac{q_c}{n_c^2}$

$$q_s = \left[\frac{N}{n_f}\right]^2 + \left[\frac{N}{n_s}\right]^2 - 1, \quad q_c = \left[\frac{N}{n_f}\right]^2 + \left[\frac{N}{n_c}\right]^2 - 1$$

Multilayer Stack Theory

Focusing on TE modes first,

$$U = E_y, \quad V = \omega\, \mu H_z$$

$$U = A\exp(-\ j\kappa x) + B\exp(j\kappa x)$$

$$V = \kappa\, [A\exp(-\ j\kappa x) -\ B\exp(j\kappa x)]$$

At x = 0,

$$U_0 = U(0), \quad V_0 = V(0)$$

$$\begin{bmatrix} U \\ V \end{bmatrix} = \begin{bmatrix} \cos(\kappa x) & \frac{j}{\kappa}\sin(\kappa x) \\ j\kappa\sin(\kappa x) & \cos(\kappa x) \end{bmatrix} \begin{bmatrix} U \\ V \end{bmatrix}$$

$$= \mathbf{M} \begin{bmatrix} U \\ V \end{bmatrix}$$

M: Characteristic matrix of the layer

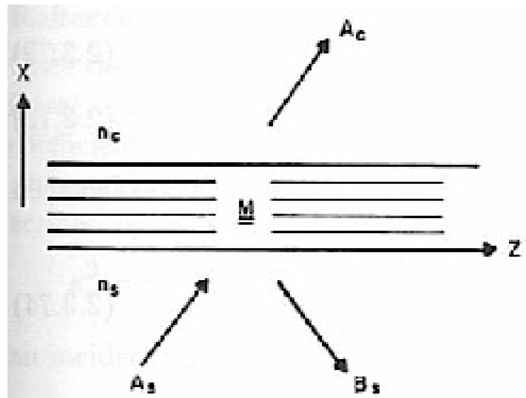

Fig. 2.15. Sketch of a multilayer stack waveguide with substrate index n_s and cover index n_c. The z-axis indicates the direction of mode propagation

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$$\mathbf{M}_i = \begin{bmatrix} \cos(\kappa_i h_i) & \frac{j}{_i}\sin(\kappa_i h_i) \\ j\kappa_i \sin(\kappa_i h_i) & \cos(\kappa_i h_i) \end{bmatrix}$$

$$\begin{bmatrix} U_0 \\ V_0 \end{bmatrix} = \mathbf{M} \begin{bmatrix} U_n \\ V_n \end{bmatrix}$$

$$\mathbf{M} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} = \mathbf{M}_1\ \mathbf{M}_2\ .\ \ \mathbf{M}_n$$

Dispersion Relation for Multilayer Slab Waveguide

Consider guided mode. For substrate and cover,

$$U = A\exp(\gamma x) + B\exp(-\gamma x)$$

$$V = j\gamma[A\exp(\gamma x) - B\exp(-\gamma x)]$$

In the substrate,

$$U_0 = A_s, \quad V_0 = j\gamma_s A_s$$

In the cover,

$$U_n = A_c, \quad V_n = -j\gamma_c A_c$$

Using the multilayer stack matrix theory, we obtain:

$$j(\gamma_s m_{11} + \gamma_c m_{22}) = m_{21} - \gamma_s \gamma_c m_{12}$$

-> ***Dispersion relation for multilayer slab waveguide***

<Example> Four-layer waveguides

Multilayer Stack Theory for TM Modes

$$U = H_y, \quad V = \omega\mu_0 E_z$$

$$U = A\exp(-\ j\kappa x) + B\exp(j\kappa x)$$

$$V = -\ \frac{\kappa}{n^2}[A\exp(-\ j\kappa x) -\ B\exp(j\kappa x)]$$

Therefore,

$$TE \geqslant\ TM \qquad\qquad \kappa\ \text{->} -[\ \ \frac{\kappa}{n^2}\ \ ]$$

Dispersion relation:

$$-\ j(m_{11}\frac{\gamma_s}{n_s^2} + m_{22}\frac{\gamma_c}{n_c^2}) = \ m_{21} -\ \frac{\gamma_s\gamma_c}{n_s^2 n_c^2}m_{12}$$

Characteristic matrix of the i-th layer:

$$\mathbf{M}_i = \begin{bmatrix} \cos(\kappa_i h_i) & -\ j\frac{n_i^2}{{}_i}\sin(\kappa_i h_i) \\ -j\frac{\kappa_i}{n_i^2}\sin(\kappa_i h_i) & \cos(\kappa_i h_i) \end{bmatrix}$$

Rectangular Waveguide Geometries

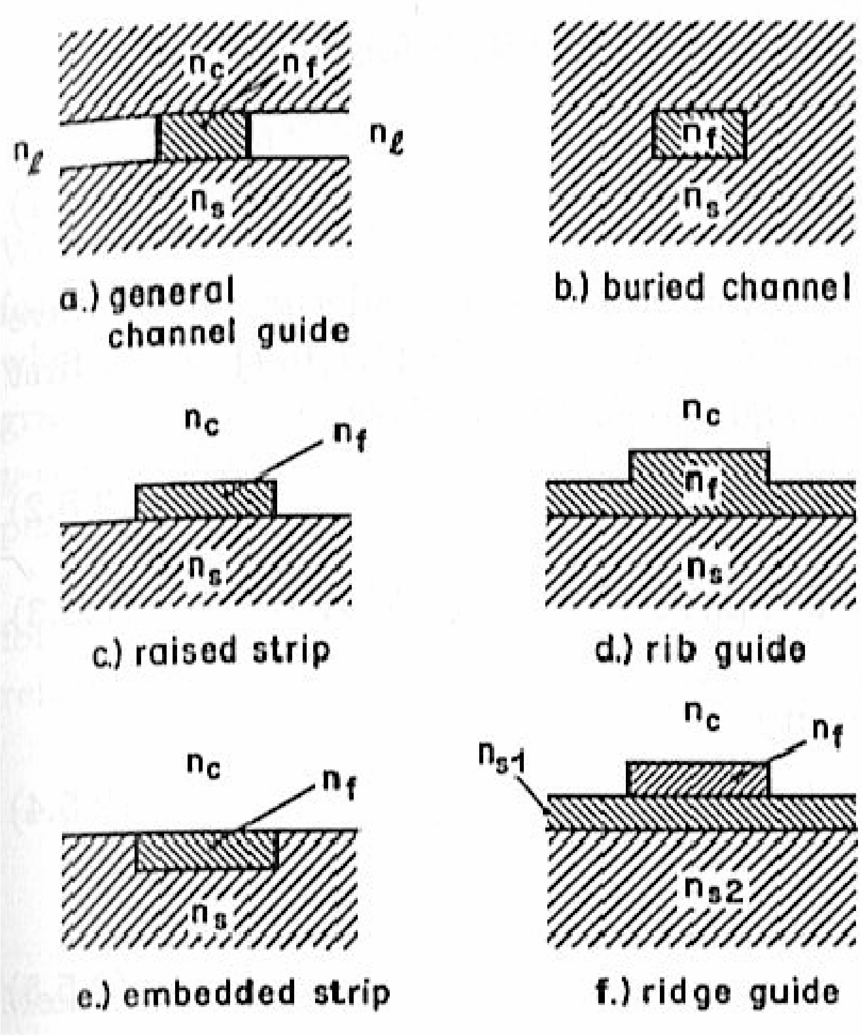


Fig. 2.24a–f. Cross-sections of six channel guide structures

The Method of Field Shadows (I)

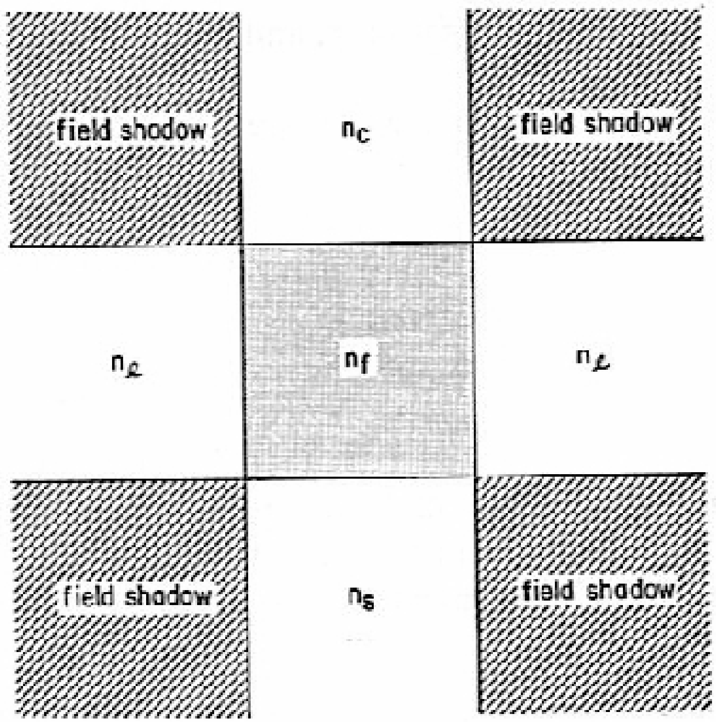


Fig. 2.26. Illustration of the method of field shadows showing the cross-section of a buried channel guide. The method ignores the fields in the shaded "shadow" areas

Ignore the fields and refractive indices in the shaded field shadow regions.

-> Results in separable index profiles.

Works well as long as the fields are well confined in the high index (n_f) region of the waveguide.

-> Not applicable at cut-off.

The Method of Field Shadows (II)

Assuming a buried channel waveguide structure.

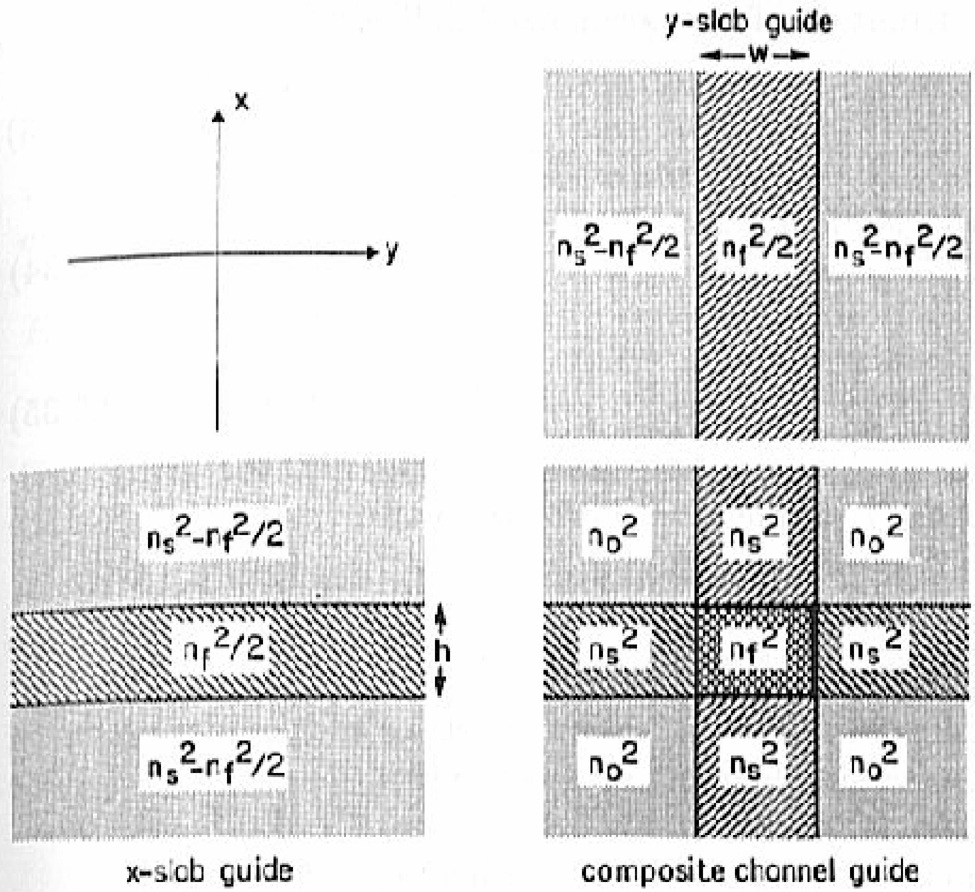


Fig. 2.27. Method of field shadows. The sketch shows the x-y cross-section of a composite guide made up by summing the permittivities (n^2) of an x-slab guide of height h and a y-slab guide of height w. The various n^2 values are indicated

$$E(x, y) = X(x)Y(y)$$

$$\beta^2 = \beta_x^2 + \beta_y^2 \qquad N^2 = N_x^2 + N_y^2$$

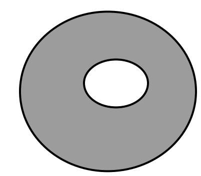
$$V_x = kh\sqrt{n_f^2 - n_s^2}$$

$$V_y = kw\sqrt{n_f^2 - n_s^2}$$

Obtain N_x and N_y, therefore N, by using the dispersion relation chart and

$$b_x = \frac{N_x^2 - n_s^2 + n_f^2/2}{n_f^2 \quad n_s^2}$$

$$b_y = \frac{N_y^2 - n_s^2 + n_f^2/2}{n_f^2 \quad n_s^2}$$



Or instead of solving for N_x and N_y, we can use

$$b = \frac{N^2 - n_s^2}{n_f^2 - n_s^2} = b_x + b_y - 1$$

The Effective-Index Method

(1) Determine the normalized thickness of the channel and lateral guides.

$$V_f = kh\sqrt{n_f^2 - n_s^2}, \quad V_l = kl\sqrt{n_f^2 - n_s^2}$$

(2) Use the dispersion relation chart to determine the normalized guide indices b_f and b_l.

Determine the corresponding effective indices by referring to the Table on **Effective index for rectangular waveguide**

$$N_{f,l}^2 = n_s^2 + b_{f,l}(n_f^2 - n_s^2)$$

(3) Determine the normalized width. $V_{eq} = kw\sqrt{N_f^2 - N_l^2}$

Then determine the normalized guide index b_{eq} using the dispersion relation chart.

(4) The effective index of the waveguide can be determined from

$$b_{eq} = \frac{N^2 - N_l^2}{N_f^2 - N_l^2}$$

$$\Rightarrow \quad N^2 = N_l^2 + b_{eq}(N_f^2 - N_l^2)$$

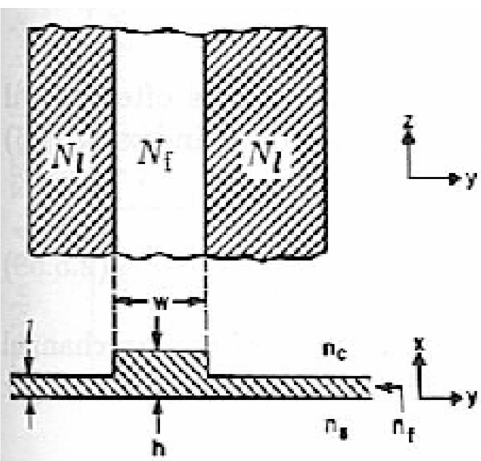
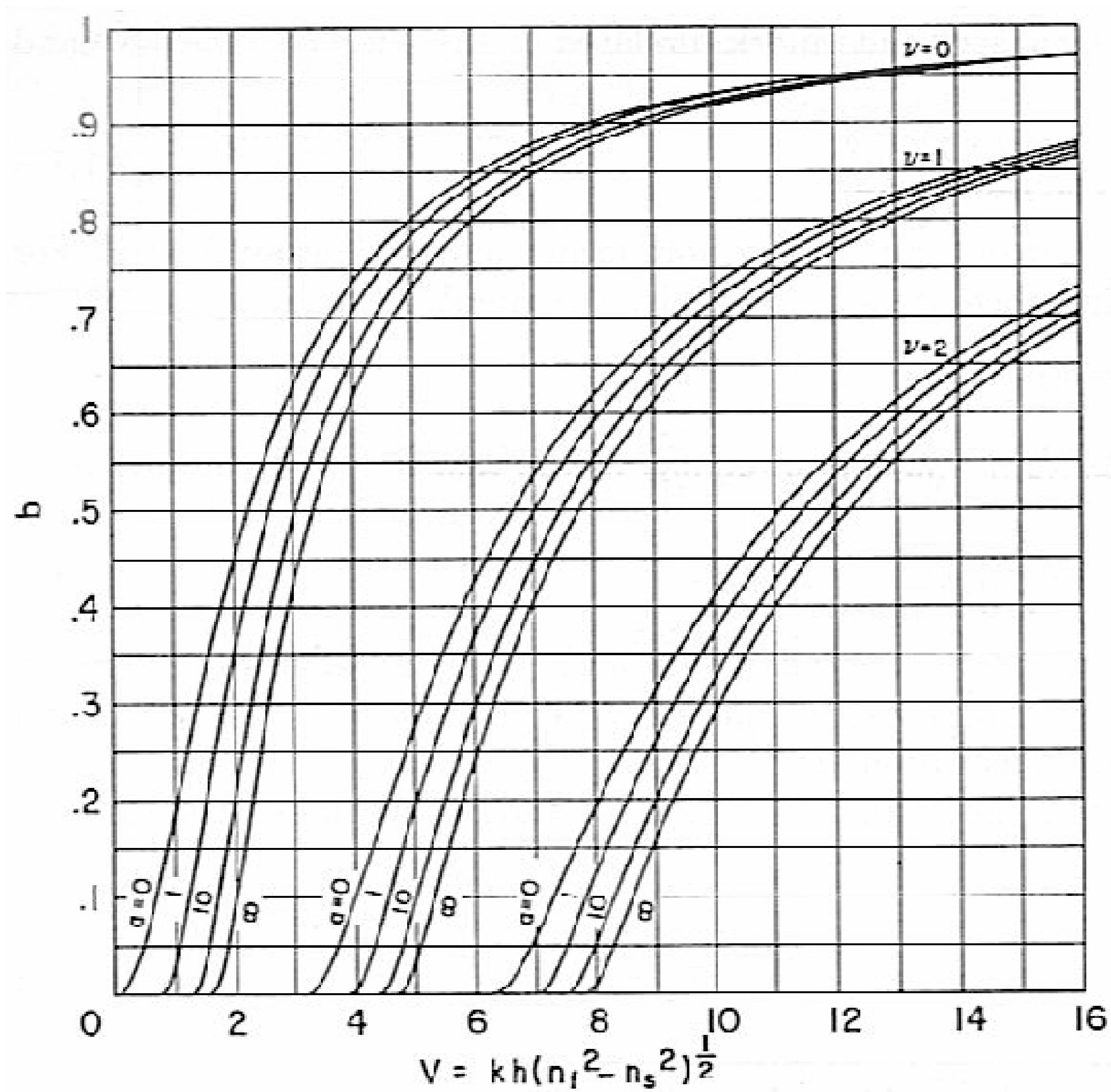


Fig. 2.28. Illustration of the effective-index method showing the top view and the cross section of a rib guide.

Note: For multi-layer waveguide structure, such as ridge waveguides, use the matrix method to determine N_f and N_l, then continue on (3).

<Example> Ti:$LiNbO_3$, λ = 0.8 μm,
n_f = 2.234, n_s = 2.2 14n_c = 1,
h = 1 .8μm, l = " 1μm,

The Dispersion Relation Chart



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Effective Index Parameters for Rectangular Waveguides

Channel structure	Guide height V_f, V_l	Eff. index N_f, N_l	$N_f^2 - N_l^2$	Channel guide index b
a) General	$V_f = kh\sqrt{n_f^2 - n_s^2}$ $V_l = kh\sqrt{n_l^2 - n_f^2}$	$N_f^2 = n_s^2 + b_f(n_f^2 - n_s^2)$ $N_l^2 = n_s^2 + b_l(n_l^2 - n_s^2)$	$b_f(n_f^2 - n_s^2) - b_l(n_l^2 - n_s^2)$	$b_f b_{eq} + b_l(1 - b_{eq})a_{ch}$
b) Buried	$V_f = kh\sqrt{n_f^2 - n_s^2}$	$N_f^2 = n_s^2 + b_f(n_f^2 - n_s^2)$ $N_l = n_s$	$b_f(n_f^2 - n_s^2)$	$b_f b_{eq}$
c) Raised	$V_f = kh\sqrt{n_f^2 - n_s^2}$	$N_f^2 = n_s^2 + b_f(n_f^2 - n_s^2)$ $N_l = n_c$	$(n_s^2 - n_c^2) + b_f(n_f^2 - n_s^2)$	$b_f b_{eq} - (1 - b_{eq})a$
d) Rib	$V_f = kh\sqrt{n_f^2 - n_s^2}$ $V_l = kl\sqrt{n_f^2 - n_s^2}$	$N_f^2 = n_s^2 + b_f(n_f^2 - n_s^2)$ $N_l^2 = n_s^2 + b_l(n_f^2 - n_s^2)$	$(b_f - b_l)(n_f^2 - n_s^2)$	$b_f b_{eq} + b_l(1 - b_{eq})$
e) Embedded	$V_f = kh\sqrt{n_f^2 - n_s^2}$	$N_f^2 = n_s^2 + b_f(n_f^2 - n_s^2)$ $N_l = n_s$	$b_f(n_f^2 - n_s^2)$	$b_f b_{eq}$
f) Ridge	$V_f = kh\sqrt{n_f^2 - n_s^2}$ $V_l = kl\sqrt{n_f^2 - n_s^2}$	$N_f^2 = n_{s1}^2 + b_f(n_f^2 - n_{s1}^2)$ $N_l^2 = n_{s2}^2 + b_l(n_{s1}^2 - n_{s2}^2)$	$(1 - b_l)(n_{s1}^2 - n_{s2}^2) + b_f(n_f^2 - n_{s1}^2)$	$b_{eq}(1 + b_f \cdot a_{ridge}) + b_l(1 - b_{eq})$

Numerical Comparisons Between Different Methods

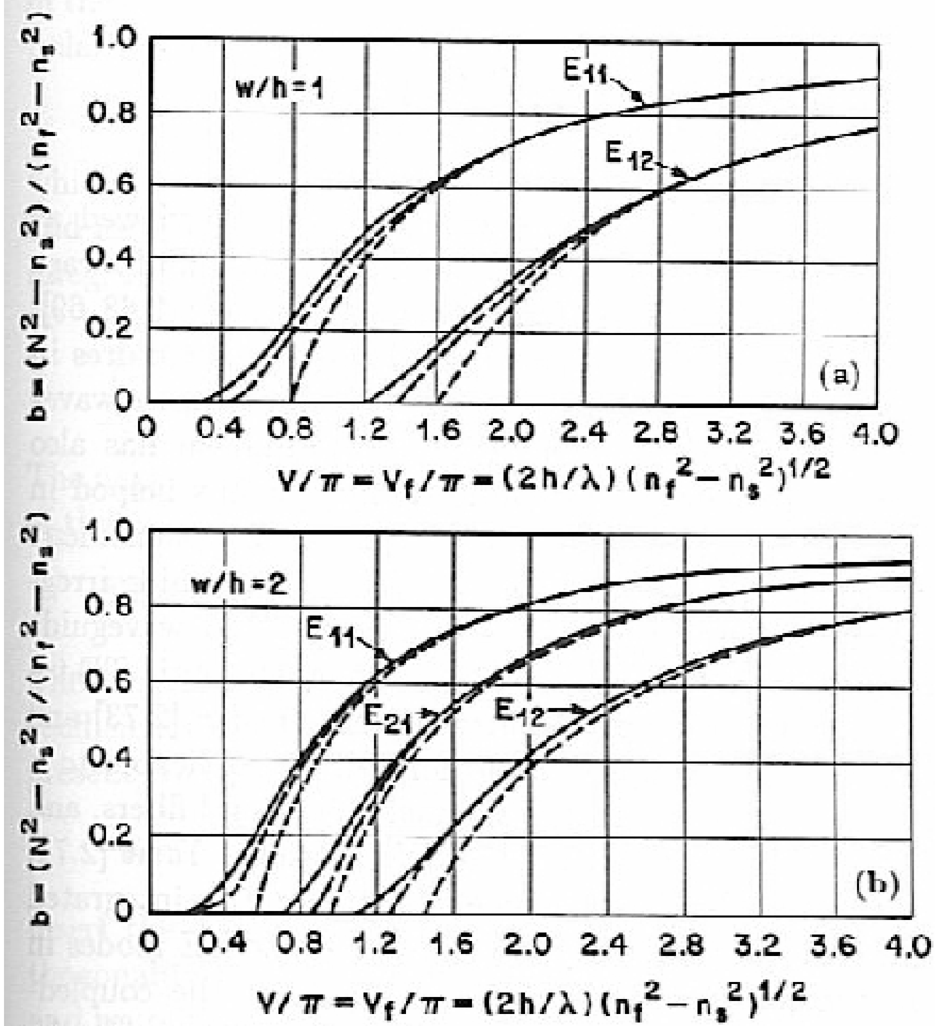
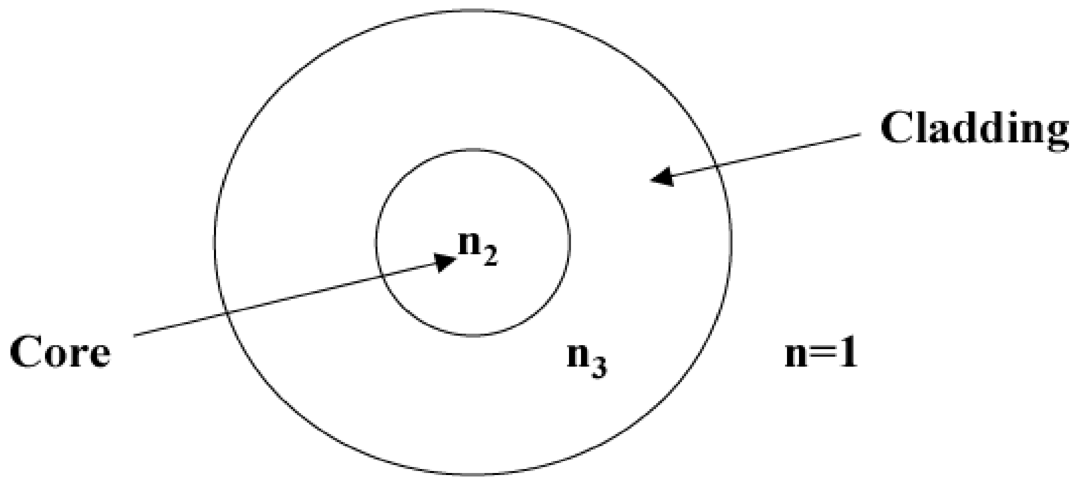


Fig. 2.29a,b. Normalized dispersion curves for a buried channel guide comparing the predictions of the numerical calculations (*dot-dashed lines*), of the effective index method (*solid lines*), and of the field-shadow method (*dashed lines*). Comparisons are shown for the aspect ratios of $w/h = 1$ and $w/h = 2$. (After [2.66])

Effective index method provides good approximation even near cut-off.

Optical Fiber

Silica (S_iO_2) glass



Impurities

a) Increase n of core

OR

b) Decrease n of cladding

Increase	Decrease
Ge	B
F	F
Na-B	

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Wave Analysis:

Cylindrical dielectric waveguide
(step fiber)

assume all fields proportional to $e^{j(\omega t-\beta z)}$

$$E = (E_r, E_\phi, E_z)$$
$$H = (H_r, H_\phi, H_z)$$

E_i and H_i are function of (r, ϕ)

$$\nabla^2 \begin{pmatrix} E \\ H \end{pmatrix} = \omega^* \mu\varepsilon \begin{pmatrix} E \\ H \end{pmatrix}$$

But now need to use cylindrical coordinates:

$$d^2E_z/dr^2 + 1/r\, dE_z/dr + 1/r^2\, d^2E_z/d\phi^2 + (n_1{}^2k^2 - \beta^2)E_z = 0$$

Assume E_z proportional to $E(r)\,h(\phi)$ separation of variables

Since $h(2p + \phi) = h(\phi)$ =>try $h(\phi) =$ $\sin l\,\phi$

$\cos(l\,\phi)$

$e^{jl\phi}$

where l= integers

Substitute back into

$d^2E_z/dr^2 + 1/r\, dE_z/dr + [(n_1{}^2k^2 - \beta^2)-l^2/r^2]E_z = 0$ => Bessel function

Solutions closer to match physical situation.

For guided solutions:

In core, solutions must be finite
In cladding, solutions must approach 0 as r --> ∞

For r < a: $E(r) \propto J_l\,(UR)$ " Bessel function of 1st kind"

For r > a: $E(r) \propto K_l\,(WR)$ " modified Bessel function of 2nd kind"

$$UR = (n_1^2k^2 - \beta^2)^{0.5}\, r = \underbrace{a(n_1^2k^2 - \beta^2)^{0.5}}_{U}\, \underbrace{r/a}_{R}$$

$$WR = (\beta^2 - n_1^2k^2)^{0.5}\, a$$

Let $V^2 = U^2 + W^2 = a^2\,[n_1^2k^2 - \beta^2 + \beta^2 - n_2^2k^2] = a^2k^2[n_1^2 - n_2^2]$

$$\therefore\ V = a \cdot (2\pi/\lambda)\,[n_1^2 - n_2^2]^{0.5} \quad \text{(Normalized frequency)}$$
$$= a \cdot (2\pi/\lambda) \cdot NA$$

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Solution procedure for step-index fiber modes:

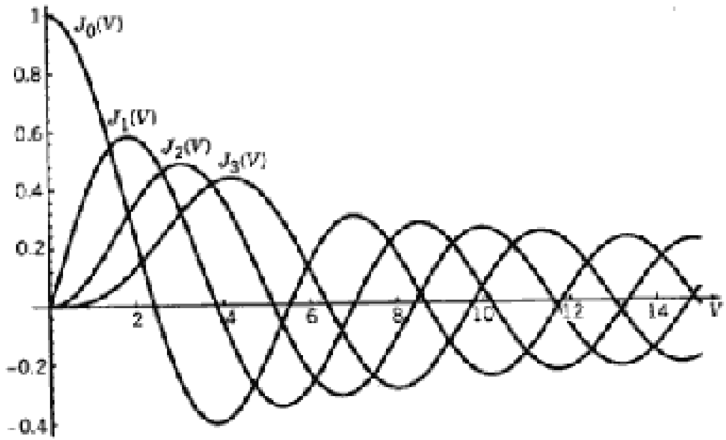
1.

$$\left\{\begin{array}{l} E_z \\ H_z \end{array}\right. = A\, J_l\,(UR)\, e^{jl\phi} e^{j(\omega t-\beta z)} \qquad r < a$$

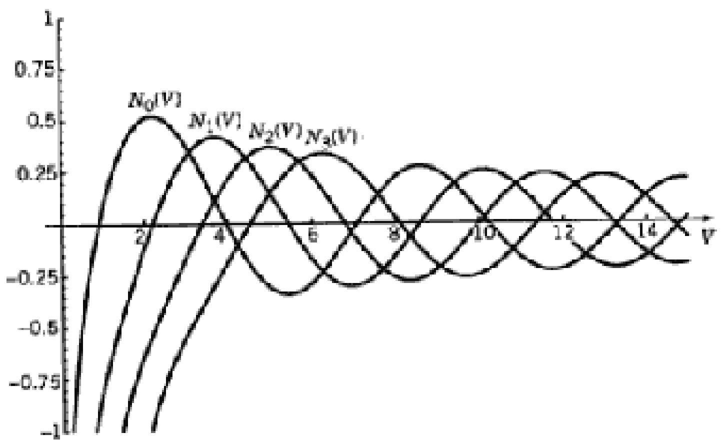
$$= B\, K_l\,(WR)\, e^{jl\phi} e^{j(\omega t-\beta z)} \qquad r > a$$

2. Match E_z and H_z at r = a

3. Use Maxwell's curl equations to find E_θ and H_θ E_z and H_z and E_θ and H_θ must match for r = +a and –a. Solve all four equations simultaneously to yield eigenvalues

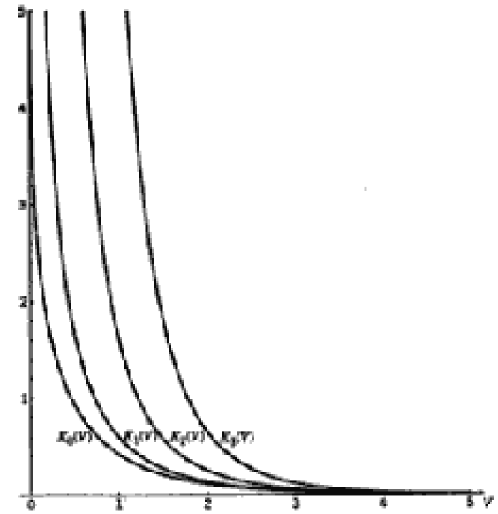


(a)

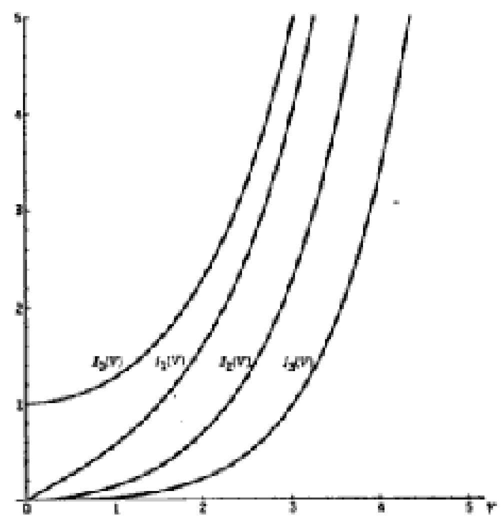


(b)

Figure 3.3. Ordinary Bessel functions.



(a)



(b)

Figure 3.4. Modified Bessel functions.

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A major simplification in math results if $(n_1-n_2)/n_1 << 1$ (weakly-guiding approximation $\Delta << 1$)

The eigenequations reduces to

$$J_{l\pm1}(U) / J_l(U) = \pm (W/U) (K_{l\pm1}(W) / K_l(W) \quad (+ \text{ only for } l = 0)$$

There are *m* possible solutions for each value of *l*

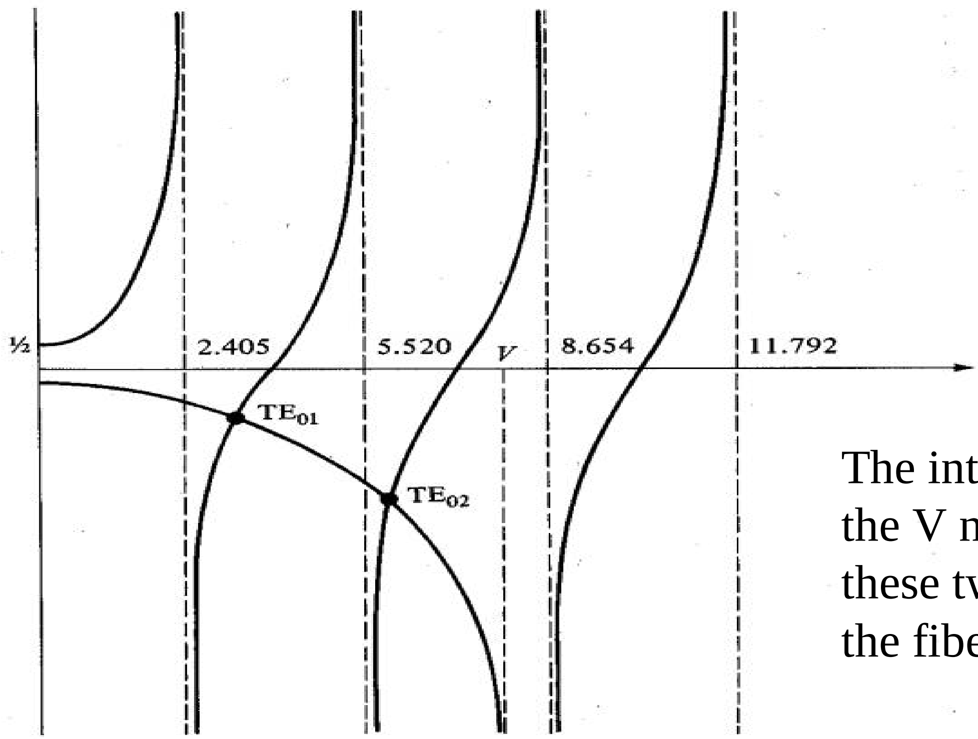
$\therefore$ U_{lm} are solutions

From definition of U, knowing U_{lm} permits calculation of β

$$\beta_{lm} = (n_1^2k^2 - U_{lm})^{0.5}$$

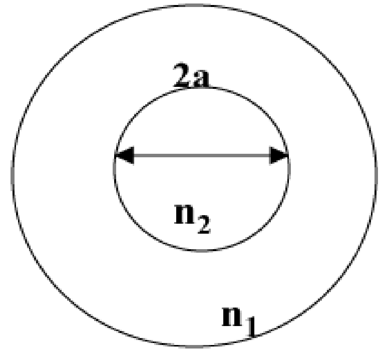
The resulting system of equations can only be solved graphically. The graphical solutions represent the mode cutoffs for the different modes that can propagate in the fiber for any given V, where V is a convenient parameter determined by the properties of the fiber and wavelength of incident light.

$$V = 2*\pi/\lambda*a*NA$$



The intersections represent the V numbers at which these two modes turn on in the fiber.

Normalization Parameters for Cylindrical Waveguides



Normalized frequency

$$V = \frac{2\pi a}{\lambda}\sqrt{n_2^2 - n_1^2}$$

$$\Delta = \frac{n_2^2 - n_1^2}{2n_2^2} \cong \frac{n_2 - n_1}{n_2}$$

$$V \cong \frac{2\pi a}{\lambda} n_2 \sqrt{2\Delta}$$

Normalized propagation constant

$$b = \frac{(\beta / k_0)^2 - n_1^2}{n_2^2 - n_1^2}$$

$$b = \frac{\beta / k_0 - n_1}{n_2 - n_1}$$

$$b_{max} = 1$$

$$b_{min} = 0$$

Same as slab waveguide except a=0 (almost always)

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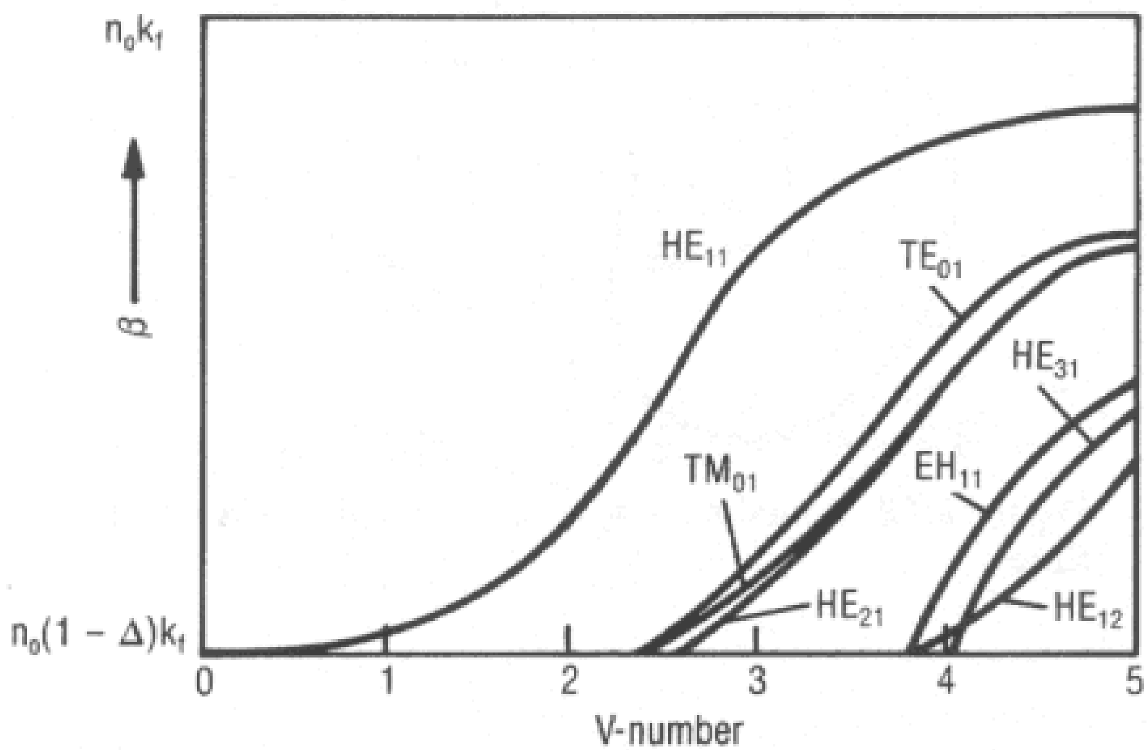

The normalized wave number, or V-number of a fiber is defined as V $= k_f$ a NA. Here k_f, is the free space wave number, $2\pi/\lambda_0$, a is the radius of the core, and NA is the numerical aperture of the fiber, NA = $(n_{core}^2 - n_{cladding}^2)^{1/2} \approx n_{core}(2\Delta)^{1/2}$, with $\Delta = (n_{core}-n_{cladding})/n_{core}$. Many fiber parameters can be expressed in terms of V. The TE and TM modes have non-vanishing cut-off frequencies. The cutoff frequency is found from V = $a\omega(2\Delta)^{1/2}/c = 2.405$. Only the lowest HE mode, HE_{11}, has no cutoff frequency. For $0 < V < 2.405$ it is the only mode that propagates in the fiber.

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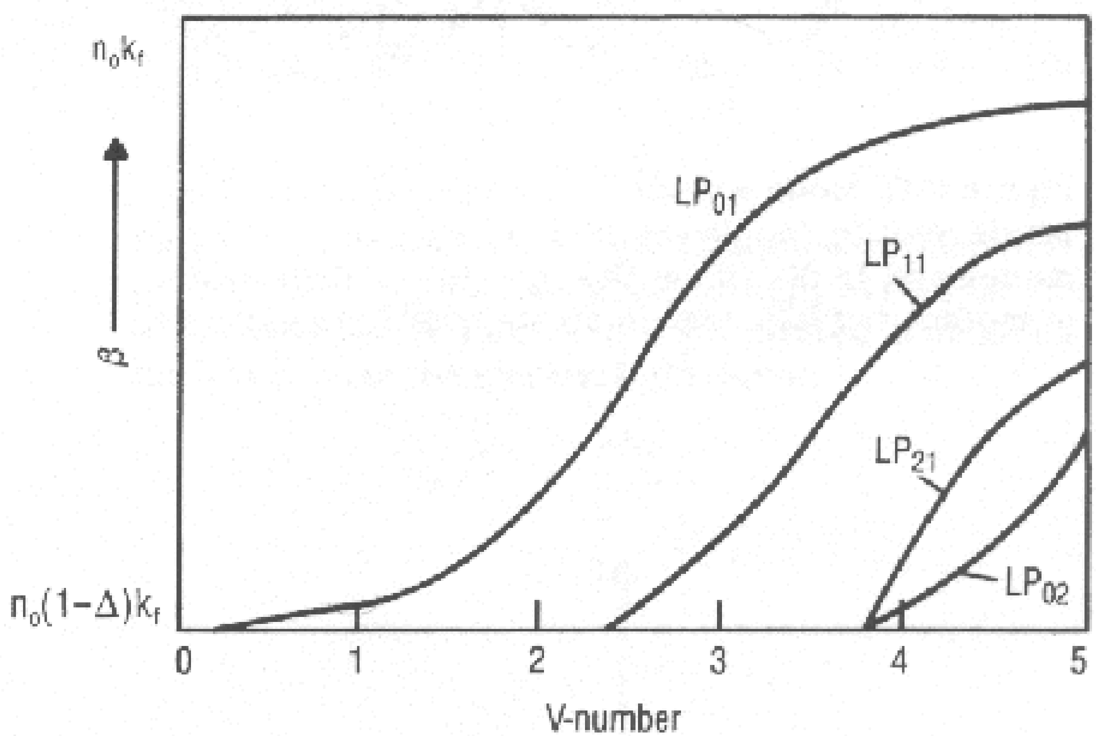
In the weakly-guiding approximation (Δ << 1), the modes propagating in the fiber are linearly polarized (LP) modes characterized by two subscripts, m and n. (The longitudinal components of the fields are small when Δ << 1.) The LP modes are combinations of the modes found from the exact theory of the wave guide. The HE_{11} mode becomes the LP_{01} mode in the weakly-guiding approximation.



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The following table presents the first ten cutoff frequencies in a step-index fiber, as well as their fundamental modes.

V_c	Bessel function	q	Modes	LP desig.
0	–	0	HE_{11}	LP_{01}
2.405	J_0	1	TE_{01}, TM_{01}, HE_{21}	LP_{11}
3.832	J_1	2	EH_{11}, HE_{31}	LP_{21}
3.832	J_{-1}	0	HE_{12}	LP_{02}
5.136	J_2	3	EH_{21}, HE_{41}	LP_{31}
5.520	J_0	1	TE_{02}, TM_{02}, HE_{22}	LP_{12}
6.380	J_3	4	EH_{31}, HE_{51}	LP_{41}
7.016	J_1	2	EH_{12}, HE_{32}	LP_{22}
7.016	J_{-1}	0	HE_{13}	LP_{03}
7.588	J_4	5	EH_{41}, HE_{61}	LP_{51}

Single mode fiber

Single mode (SM) fiber is designed such that all the higher order waveguide modes are cut-off by a proper choice of the waveguide parameters as given below.

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2}$$

where, λ is the wavelength, *a* is the core radius, and n_1 *and* n_2 *are the core and cladding refractive indices, respectively*. *When V < 2.405 single mode condition is ensured. SM fiber is an essential requirement for interferometric sensors. Due to the small core size (~4 μ m) alignment becomes a critical factor.*

Figure 3.10 shows plots of b, calculated using (3.76) and (3.77) in (3.78), for several LP modes as functions of V. As plotted on this scale, these results are essentially indistinguishable from those given by the exact numerical solution [5].

When considering single-mode fibers, the accuracy of the LP_{01} curve in Fig. 3.10 is of increased importance. It was in fact found to be accurate to within 5% over the range of V between 2.0 and 3.0, with the error increasing to around 10% as V decreases to 1.5 [7]. Numerous other approximate formulas exist as alternatives to (3.77) for determining b. The best of these was found by Rudolf and Neumann [8], who recognized that w is a nearly linear function of V over the range $1.3 < V < 3.5$. They were thus able to approximate w over this range by the simple function

$$w \approx 1.1428V - 0.9960 \tag{3.80}$$

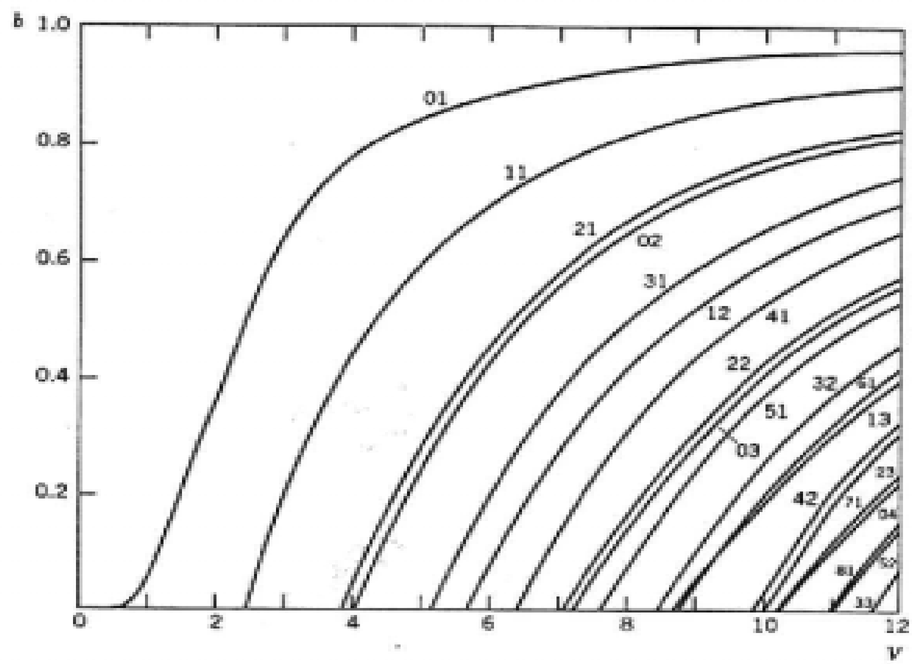
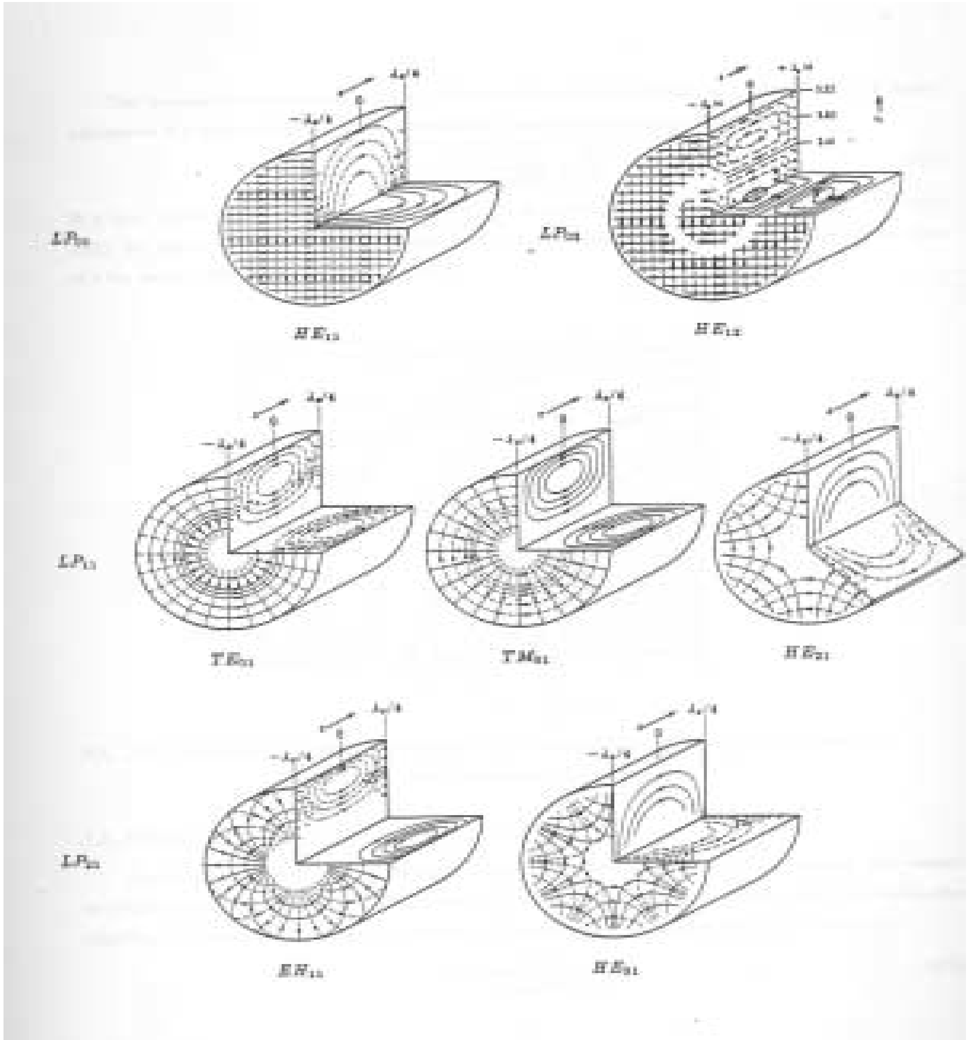
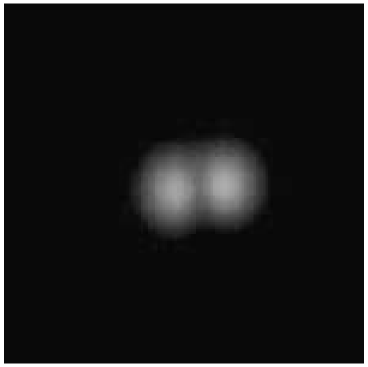


Figure 3.10. Normalized propagation constant, b, for designated LP modes as functions of V. (Adapted from ref. 5.)



Electric and magnetic fields for eight fundamental modes.

LP01



LP11

When the V number is less than 2.405 only the LP_{01} mode propagates. When the V number is greater than 2.405 the next linearly-polarized mode can be supported by the fiber, so that both the LP_{01} and LP_{11}, modes will propagate.

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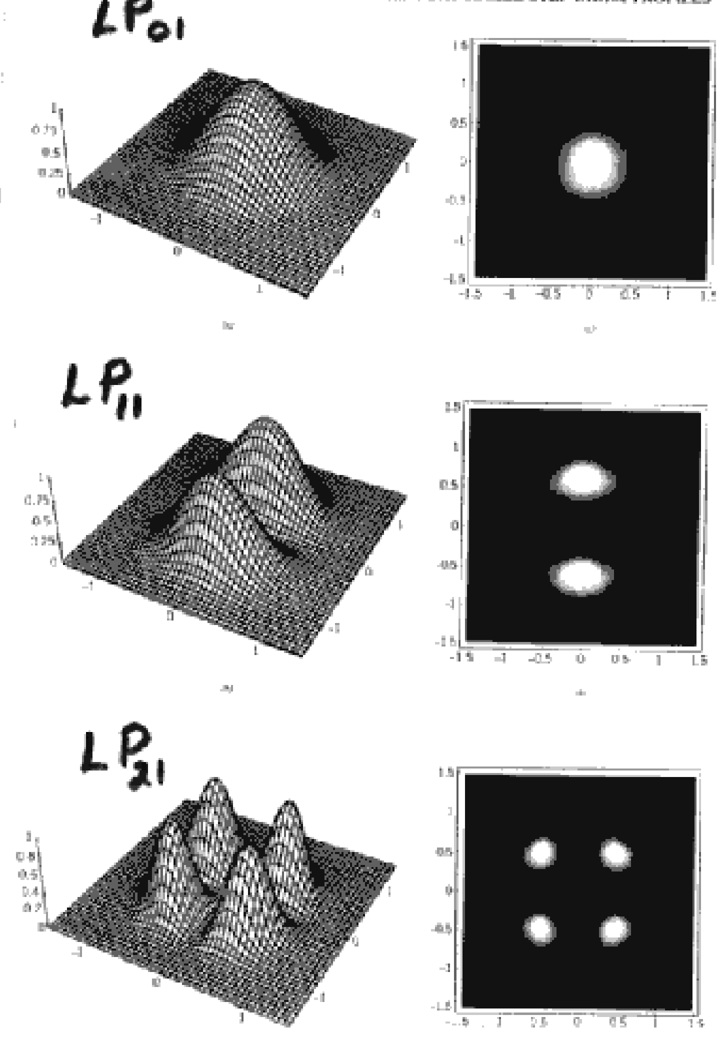


Figure 3.9. Intensity plots for the six LP modes, with $a = 1$. (a) LP_{01}: $v = 2$. (b) LP_{11}: $v = 3$. (c) LP_{21}: $v = 4.5$. (d) LP_{02}: $v = 4.5$. (e) LP_{31}: $v = 5.5$. (f) LP_{12}: $v = 6.3$.

3.7. LP MODES 71

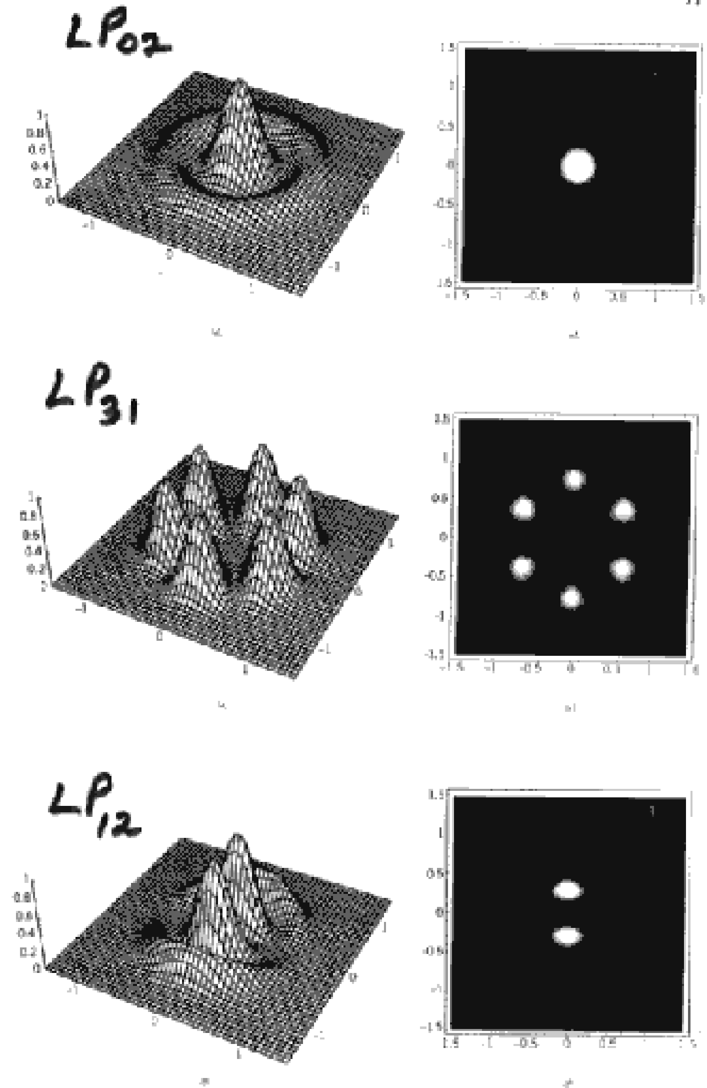
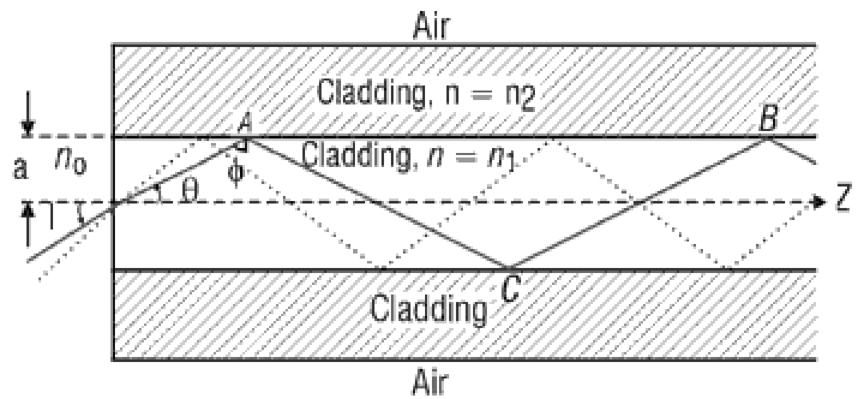


Figure 3.9. *(Continued)*

The SM fiber mentioned above is not truly single mode in that two modes with degenerate polarization states can propagate in the fiber. This can lead to signal interference and noise in the measurement. The degeneracy can be removed and a single mode polarization preserving fiber can be obtained by the use of an elliptical core fiber of very small size or with built in stress. In either case light launched along the major axis of the fiber is preserved in its state of polarization. It is also possible to make a polarizing fiber in which only one state of polarization is propagated. Polarimetric sensors make use of polarization preserving fibers. Thus, multimode fiber, single mode fiber and polarization preserving fiber are the three classes of fibers which are used in the intensity type, the interferometric type and the polarimetric type of sensors, respectively.

While discussing step-index fibers, we considered light propagation inside the fiber as a set of many rays bouncing back and forth at the core-cladding interface. There the angle θ could take a continuum of values lying between 0 and $\cos^{-1}(n_2/n_1)$, i.e.,



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$$0 < \theta < \cos^{-1}(n_2/n_1)$$

For $n_2 = 1.5$ and $\Delta \approx \frac{n_1 - n_2}{n_1} = 0.01$, we would get $n_2/n_1 \simeq$ and $\cos^{-1}\left(\frac{n_1}{n_2}\right) = 8.1°$, so

$$0 < \theta < 8.1°$$

Now, when the core radius (or the quantity Δ) becomes very small, ray optics does not remain valid and one has to use the more accurate wave theory based on Maxwell's equations.

In wave theory, one introduces the parameter

$$V = \frac{2\pi}{\lambda_0} a\sqrt{n_1^2 - n_2^2} = \frac{2\pi}{\lambda_0} a n_1 \sqrt{2\Delta} \approx \frac{2\pi}{\lambda_0} a n_2 \sqrt{2\Delta}$$

where Δ has been defined earlier and $n_1 \simeq n_2$. The quantity V is often referred to as the "*V-number*" or the "*waveguide parameter*" of the fiber. It can be shown that, if

$$V < 2.4045$$

only one guided mode (as if there is only one discrete value of θ) is possible and the fiber is known as a *single-mode fiber*. Further, for a step-index single-mode fiber, the corresponding (discrete) value of θ is approximately given by the following empirical formula

$$\cos\theta \approx 1 - \Delta\left[1 - \left(1.1428 - \frac{0.996}{V}\right)^2\right]$$

We may mention here that because of practical considerations the value Δ of ranges from about 0.002 to about 0.008

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Assignment

Consider a step-index fiber (operating at 1300 nm) with n_2 = 1.447, Δ = 0.003, and a = 4.2 μm. Thus,

$$V = \frac{2\pi}{\lambda_0(\mu m)} \times 4.2(\mu m) \times 1.447 \times \sqrt{0.006} \approx \frac{2.958}{\lambda_0(\mu m)}$$

Thus the fiber will be single moded and the corresponding value of θ—will be about θ = 3.1º. It may be mentioned that for the given fiber we may write

$$V = \frac{2\pi}{1.3(\mu m)} \times 4.2(\mu m) \times 1.447 \times \sqrt{0.006} \approx 2.275$$

Thus, for $\lambda_0 > 2.958/2.4045 = 1.23$ μm

which guarantees that $V < 2.4045$, the fiber will be single moded. The wavelength for which $V = 2.4045$ is known as the *cutoff wavelength* and is denoted by λ_c. In this example, $\lambda_c = 1.23$ μm and the fiber will be single moded for $\lambda_0 > 1.23$ μm.

Assignment

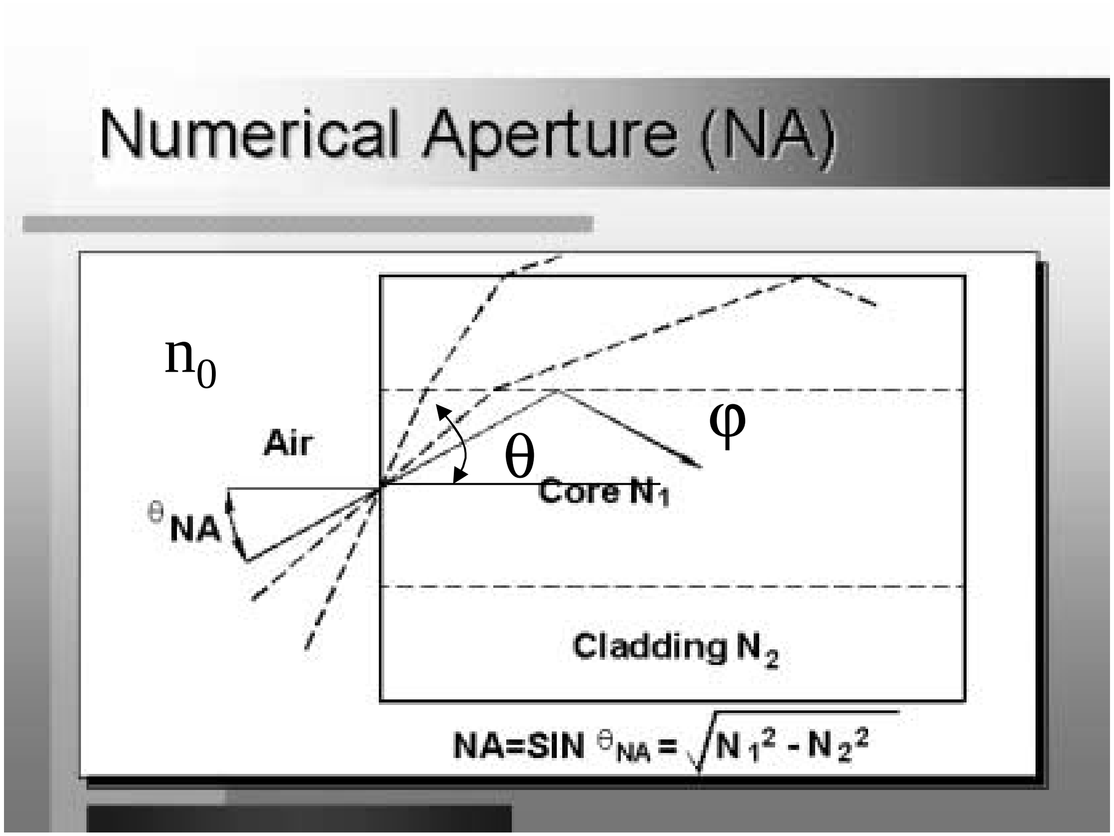
For reasons that will be discussed later, the fibers used in current optical communication systems (operating at 1.55 μm) have a small value of core radius and a large value of Δ. A typical fiber (operating at $\lambda_0 \approx 1.55$ μm) has $n_2 = 1.444$, $\Delta = 0.0075$, and $a = 2.3$ μm. Thus, at $\lambda_0 = 1.55$ μm, the *V*-number is,

$$V = \frac{2\pi}{1.55(\mu m)} \times 2.3(\mu m) \times 1.444 \times \sqrt{0.015} \approx 1.649$$

The fiber will be single moded (at 1.55 μm) with θ = 5.9°. Further, for the given fiber we may write

$$V = \frac{2\pi}{\lambda_0(\mu m)} \times 2.3(\mu m) \times 1.444 \times \sqrt{0.015} \approx \frac{2.556}{\lambda_0(\mu m)}$$

and therefore the cutoff wavelength will be $\lambda_c = 2.556/2.4045 = 1.06$ μm.



- The Numerical Aperture (NA) of a fiber is the measure of the maximum angle (θ_{NA}) of the light entering the end that will propagate within the core of the fiber
- Acceptance Cone = 20NA
- Light rays entering the fiber that exceed the angle θ_{NA} will enter the cladding and be lost
- For the best performance the NA of the transmitter should match the NA of the fiber

NA derivation

We know $\frac{\sin i}{\sin \Theta} = \frac{n_1}{n_0}$ and $\sin\phi(=\cos\Theta) > \frac{n_2}{n_1}$

Since $\sin\Theta = \sqrt{1-\cos^2 \Theta}$ we get $\sin\Theta < \left[1-\left(\frac{n_2}{n_1}\right)^2\right]^{½}$

Assume the θ_{NA} is the half angle of the acceptance cone,

$$\sin\theta_{NA}=(n_1^2-n_2^2)^{1/2} = n_1 \mathrm{sqrt}(\Delta)$$

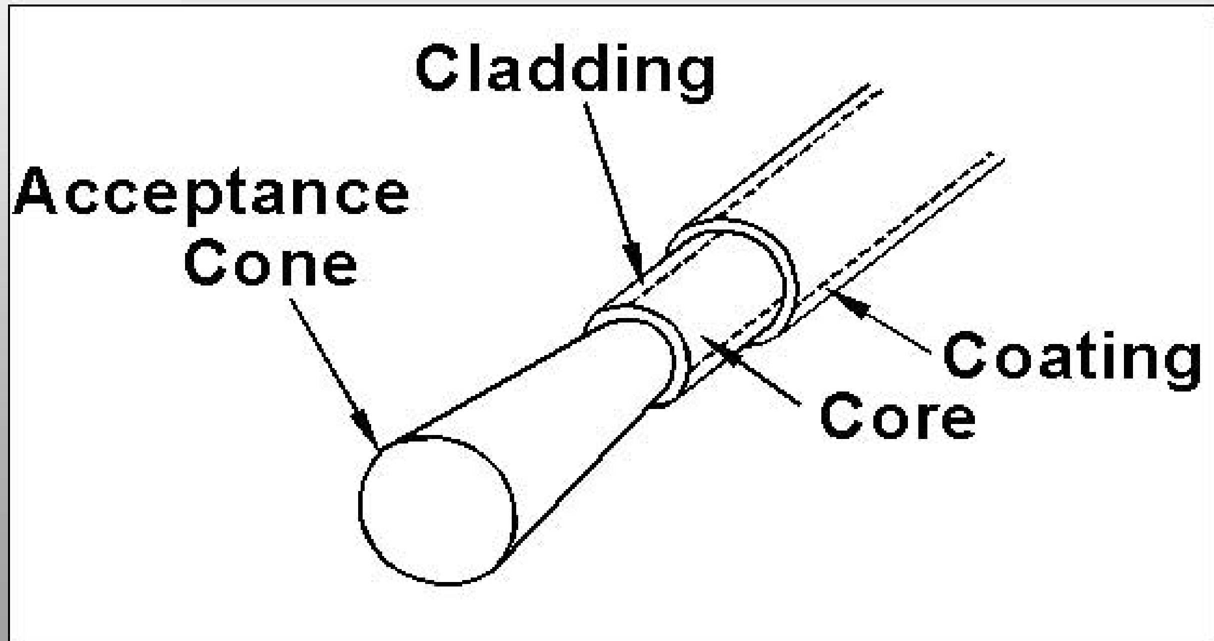
We define a parameter Δ through the following equations.

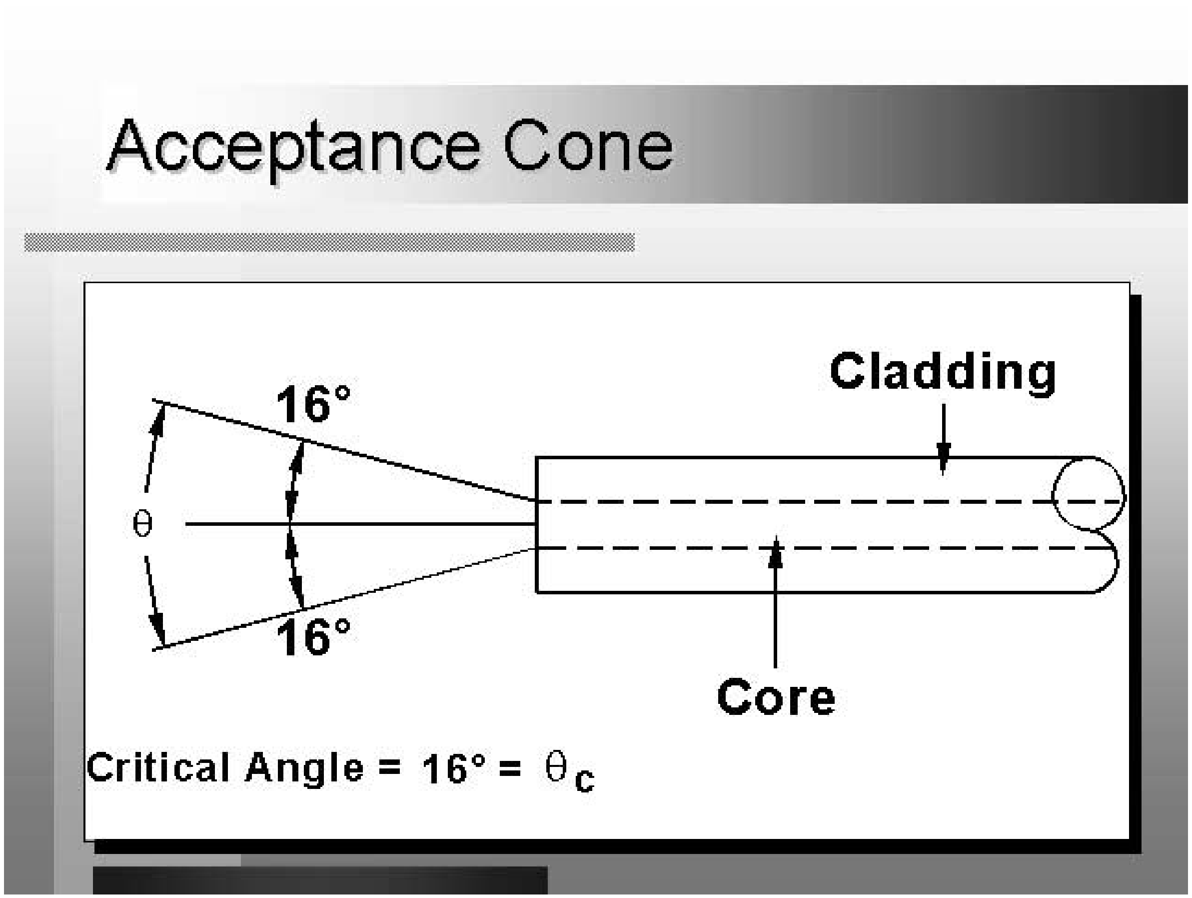
$$\Delta \equiv \frac{n_1^2 - n_2^2}{2n_2^2}$$

When Δ << 1 (as is indeed true for silica fibers where n_1 is very nearly equal to n_2) we may write

$$\Delta = \frac{(n_1 + n_2)(n_1 - n_2)}{2n_1^2} \approx \frac{(n_1 - n_2)}{n_1} \approx \frac{(n_1 - n_2)}{n_2}$$

Acceptance Cone





Single mode fiber critical angle <20^o
Multimode fiber critical angle <60 ^{o}

Example

For a typical step-index (multimode) fiber with $n_1 \approx 1.45$ and $\Delta \approx 0.01$, we get

$$\sin i_m = n_1 \sqrt{2\Delta} = 1.45\sqrt{2 \times (0.01)} = 0.205$$

so that $i_m \approx 12°$. Thus, all light entering the fiber must be within a cone of half-angle 12°.

In a short length of an optical fiber, if all rays between $i = 0$ and i_m are launched, the light coming out of the fiber will also appear as a cone of half-angle i_m emanating from the fiber end. If we now allow this beam to fall normally on a white paper and measure its diameter, we can easily calculate the *NA* of the fiber.