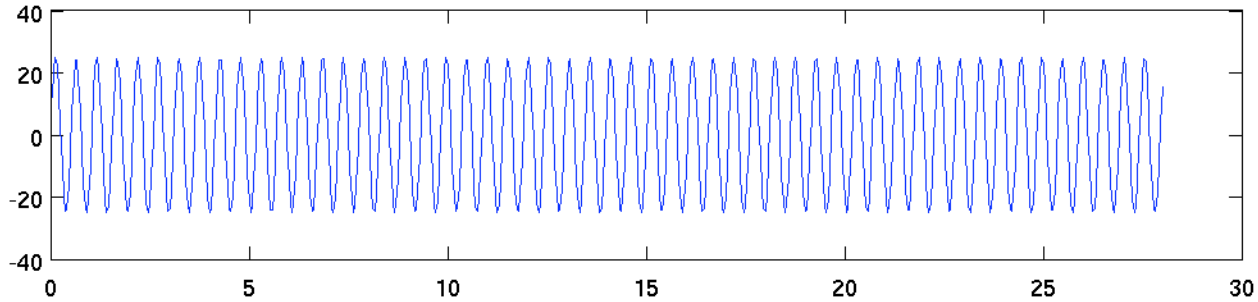
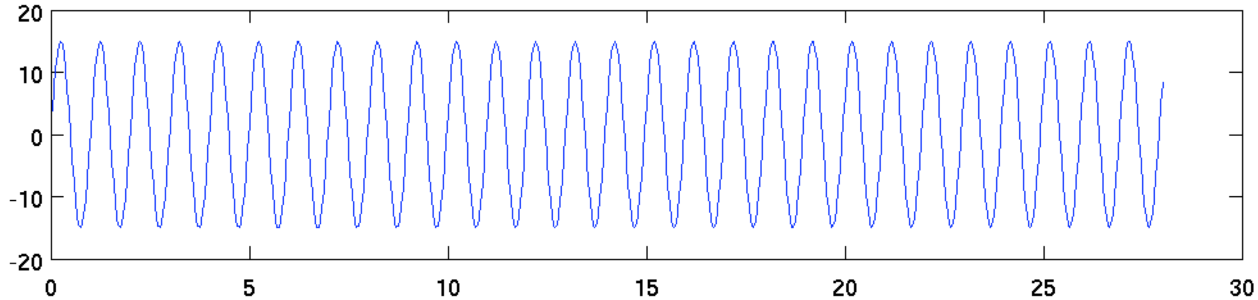
M2


40
20
0
-20
-40
0
5
10
15
20
25
30

K1


20
10
0
-10
-20
0
5
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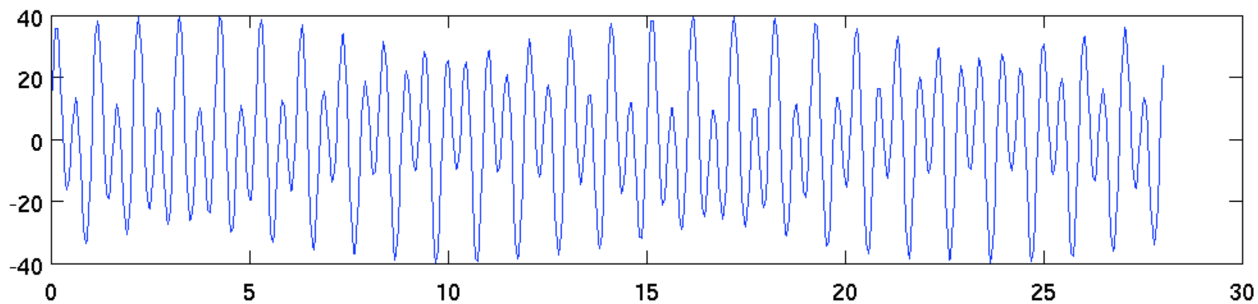
15
20
25
30

K1+M2


40
20
0
-20
-40
0
5
10
15
20
25
30

Drum web site

http://www.kettering.edu/~drussell/Demos/MembraneCircle/Circle.html

Empirical Orthogonal Functions (EOFs): An overview

•What are the dominant patterns of variabililty in time and space?

Mathematical technique which decomposes your data matrix into spatial structures (EOFs) and associated amplitude time series (PCs)

- The EOFs and PCs are constructed to efficiently explain the maximum amount of variance in the data set.
- By construction, the EOFs are orthogonal to each other, as are the PCs.
- In general, the majority of the variance in a data set can be explained with just a few EOFs.
- Provide an 'objective method' for finding structure in a data set, but interpretation requires physical facts or intuition.

•3 “Products” of Principle Component Analysis

•Singular Value Decomposition (SVD)
$X = U\Sigma V^T$

•Some 2-D Data

•(X)

•1) Eigenvectors

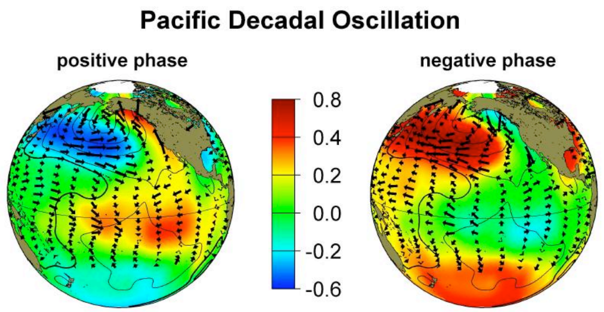
•2) Eigenvalues

•3) Principle Components

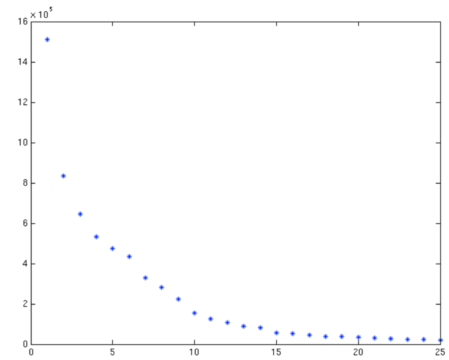
•Eigenanalysis
$XX^T = C; CE = \Lambda E$

•Examples for Today

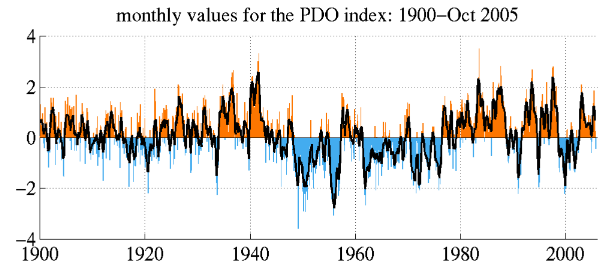
•1) Eigenvectors – Variations explained in space (MAPS)



•2) Eigenvalues - % of Variance explained (spectrum)



•3) Principle Components Variations explained in the time (TIMESERIES)



Eigenvectors, Eigenvalues, PC's

- Eigenvectors explain variance in one dimension (space); Principle components explain variance in the other dimension (time).

- Each *eigenvector* has a corresponding *principle component (time series)*. The PAIR define a mode that explains variance.

- Each eigenvector/PC pair has an associated *eigenvalue* which relates to how much of the total variance is explained by that mode.

EOF's and PC's for geophysical data

- 1st EOF is the spatial pattern which explains the most variance of the data in space and time. The 1st principal component is the time series of the fluctuations of that pattern.

- 2nd EOF is the spatial pattern that explains the most of the remaining variance. 2nd P.C. is the associated time series…

- EOFs are orthogonal to each other (i.e., $\underline{e}_1 . \underline{e}_2 = 0$, where e is vector representing the spatial pattern (such as coskx), and P.C.s are orthogonal to each other (i.e., $\underline{t}_1 . \underline{t}_2 = 0$, where t is vector of time series such as (coswt).

- In general, the majority of the variance in a data set can be explained with just a few EOFs.

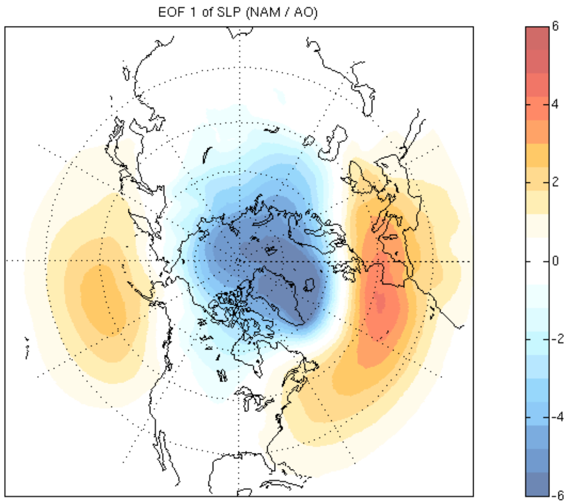
EOF's and PC's for geophysical data

- By construction, the EOFs are *orthogonal* to each other, as are the PCs.

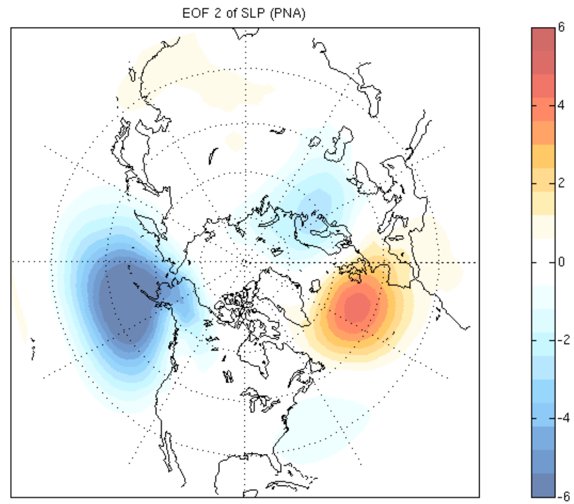
- Provide an *'objective method'* for finding structure in a data set, but interpretation requires physical facts or intuition.

EOFs of Real Data: Winter SLP anomalies

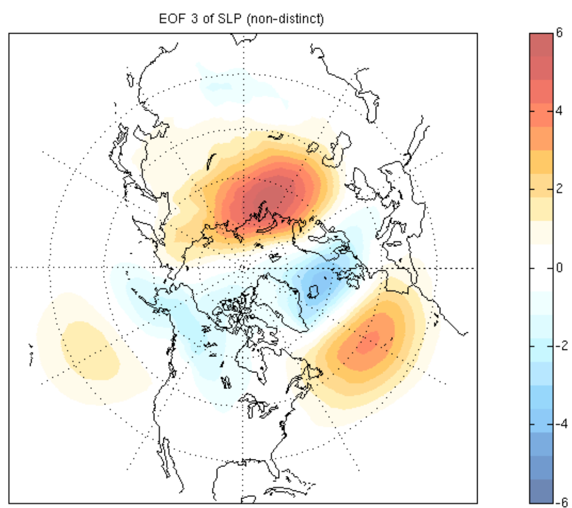
EOF 1: AO/NAM (23% expl).



EOF 2: PNA (13% expl.)

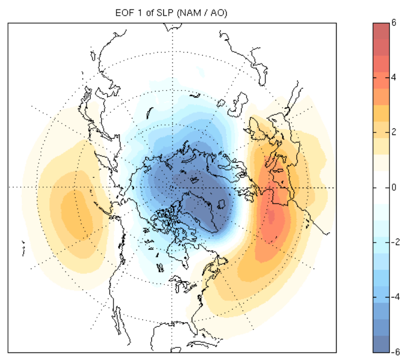


EOF 3: non-distinct(10% expl.)

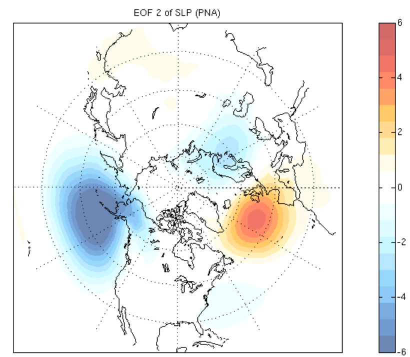


EOFs of seal level pressure in the northern hemisphere

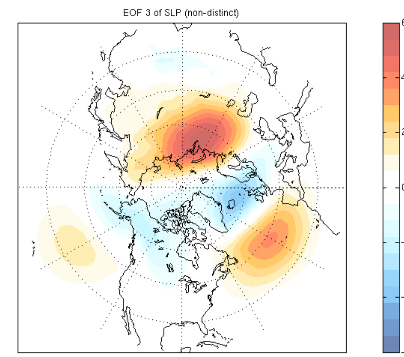
EOF 1 (AO/NAM)



EOF 2 (PNA)



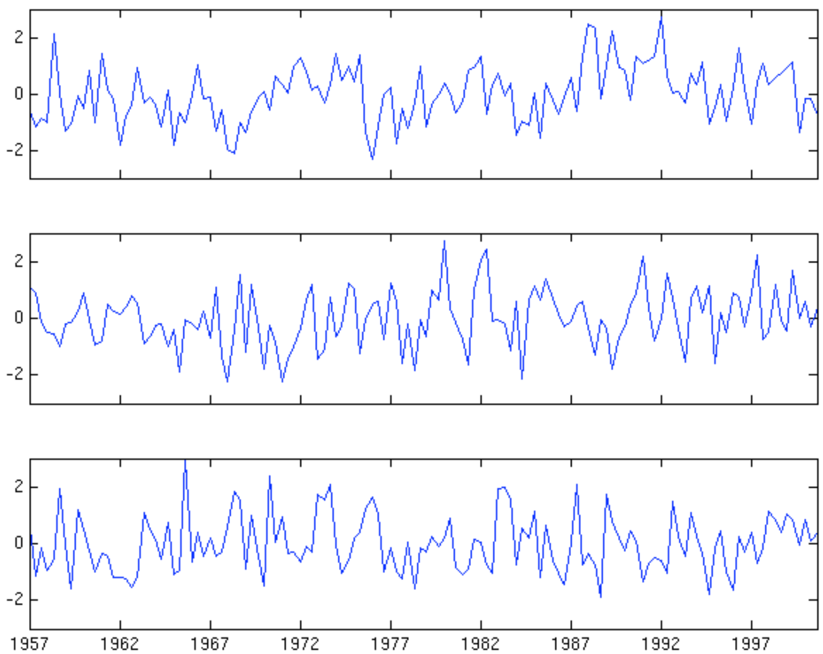
EOF 3 (?)



PC1 (AO/NAM)

PC2 (PNA)

PC3 (?)



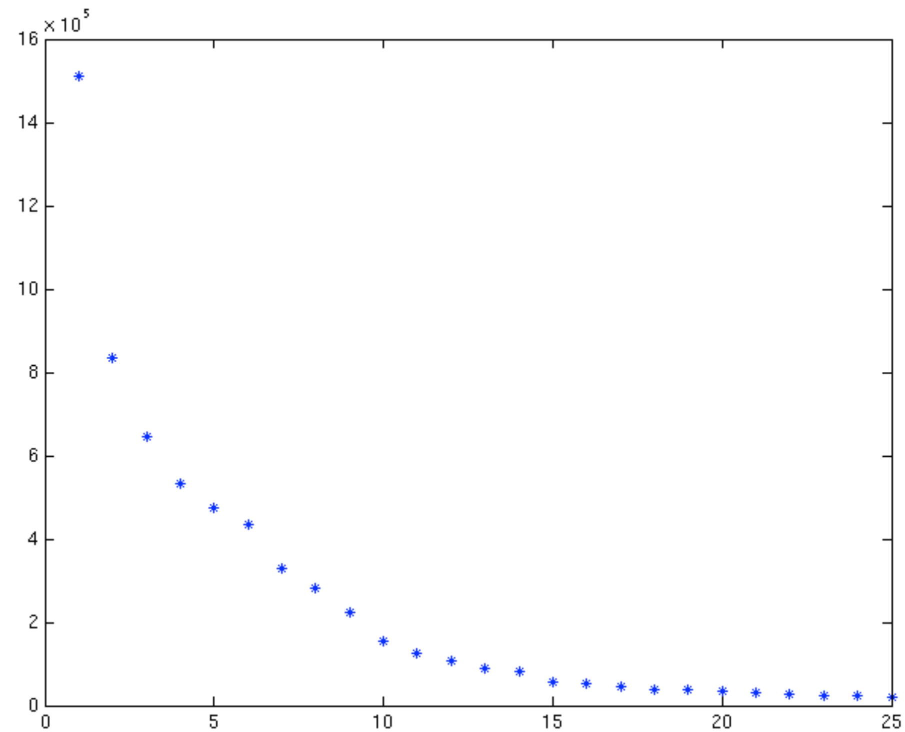
EOF significance

• Each EOF / PC pair comes with an associated *eigenvalue*

• The normalized eigenvalues (each eigenvalue divided by the sum of all of the eigenvalues) tells you the percent of variance explained by that EOF / PC pair.

• Eigenvalues need to be well separated from each other to be considered distinct modes.

First 25 Eigenvalues for DJF SLP



EOFs: Practical Considerations

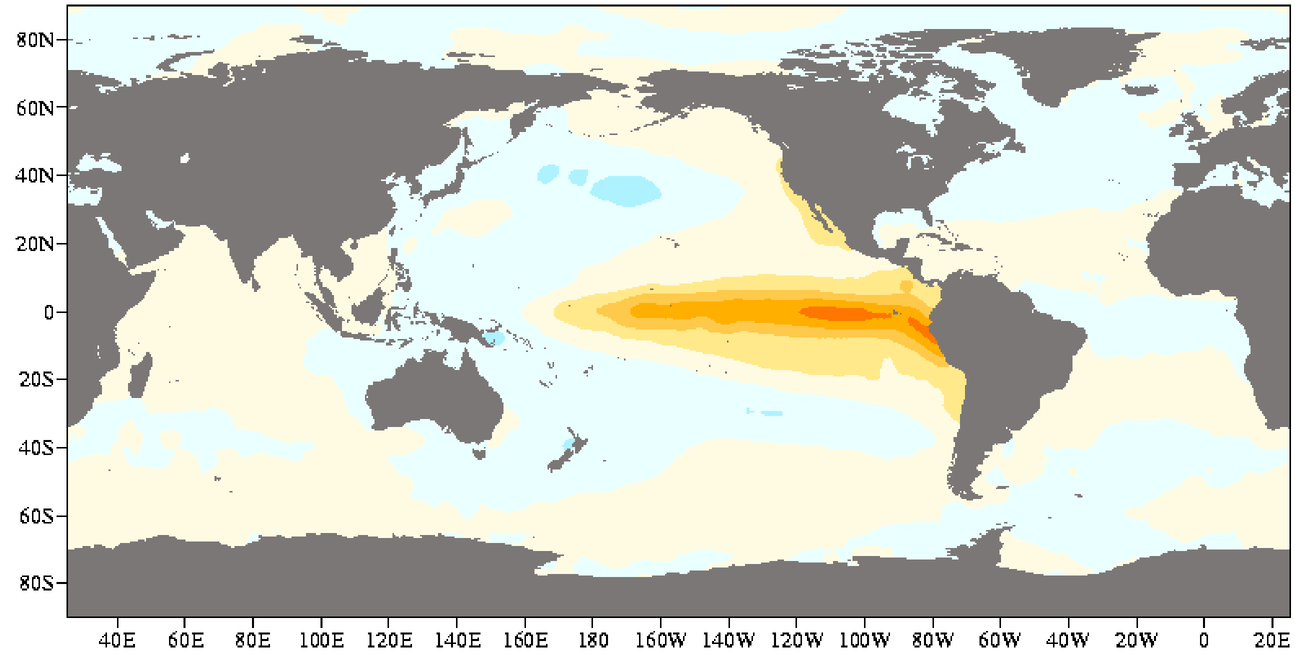
• EOFs are easy to calculate, difficult to interpret. There are no hard and fast rules, physical intuition is a must.

• EOFs are created using linear methods, so they only capture linear relationships.

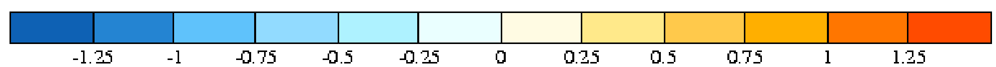
• Due to the constraint of orthogonality (I.e. sinkx and coskx), EOFs tend to create wave-like structures, even in data sets of pure noise. So pretty… so suggestive… so meaningless. Beware of this.

• By nature, EOFs give are fixed spatial patterns which only vary in strength and in sign. E.g., the 'positive' phase of an EOF looks exactly like the negative phase, just with its sign changed. Many phenomena in the climate system don't exhibit this kind of symmetry, so EOFs can't resolve them properly.

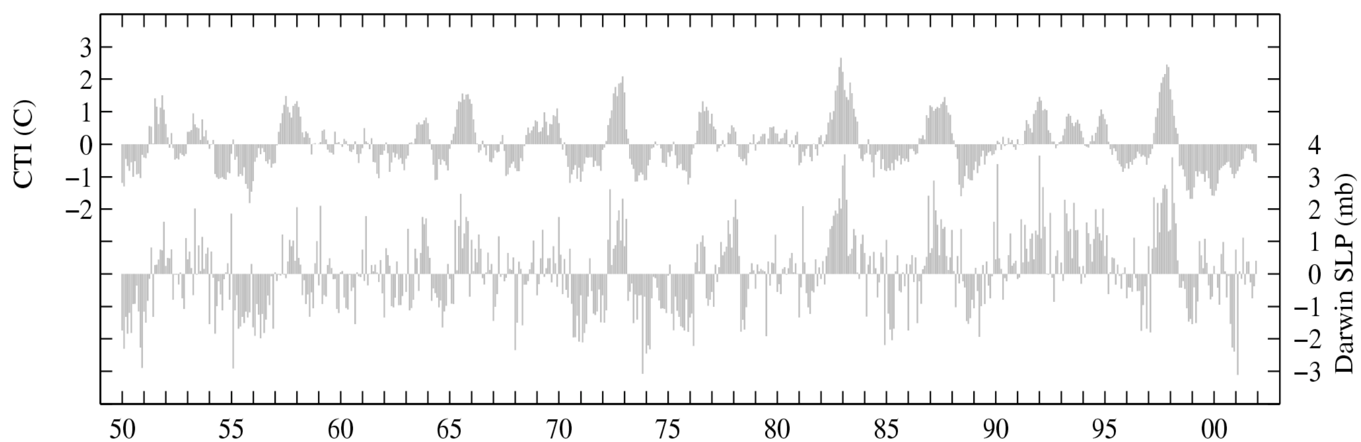
ENSO sea surface temperature anomalies (C)



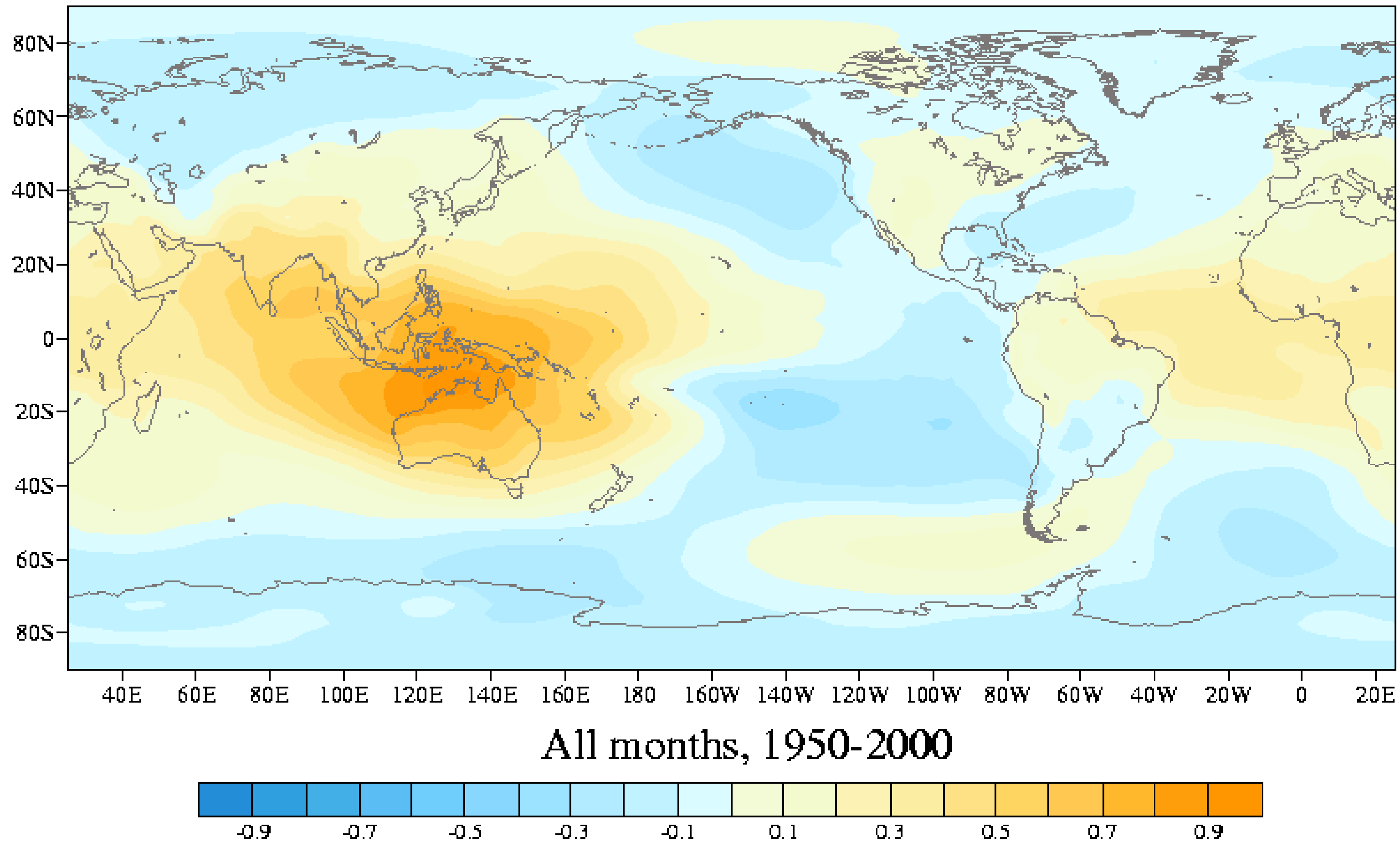
All months, 1950-2000



CTI and Darwin



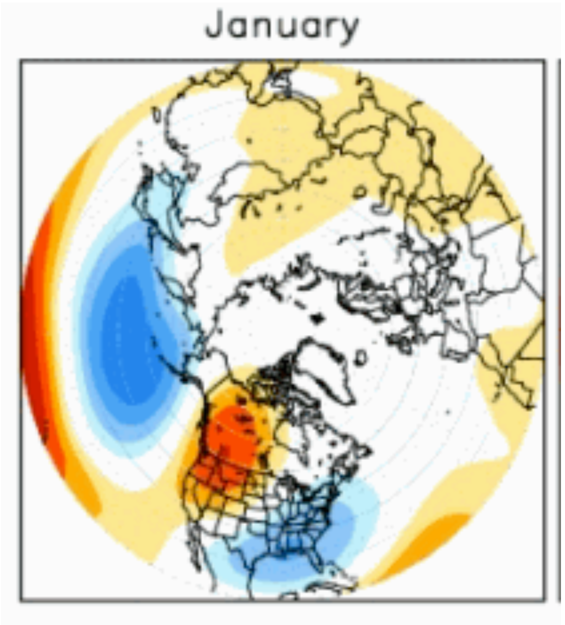
ENSO sea-level pressure correlations

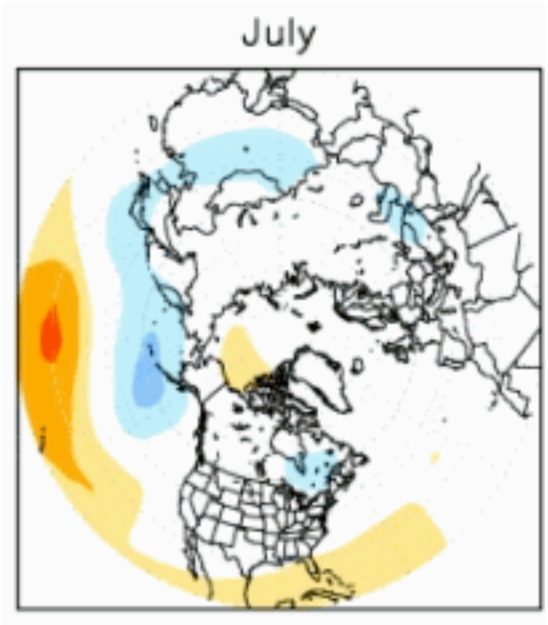


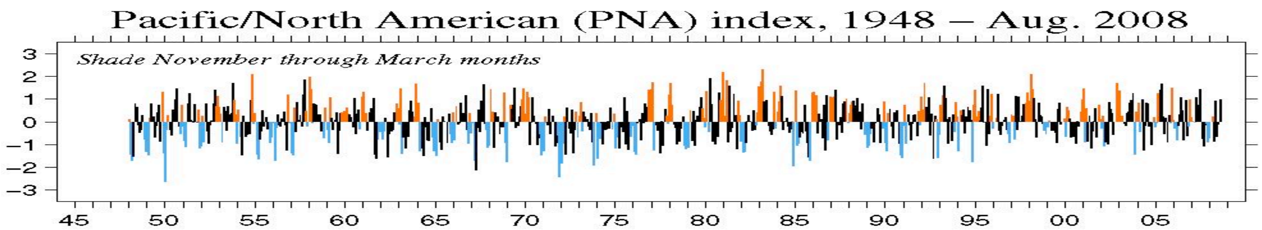
All months, 1950-2000

Correlation maps vs. regression maps

- PNA is a time series of fluctuations in 500 mb heights
- PNA = 0.25 * [Z(20N,160W) - Z(45N,165W) + Z(55N,115W) - Z(30N,85W)]





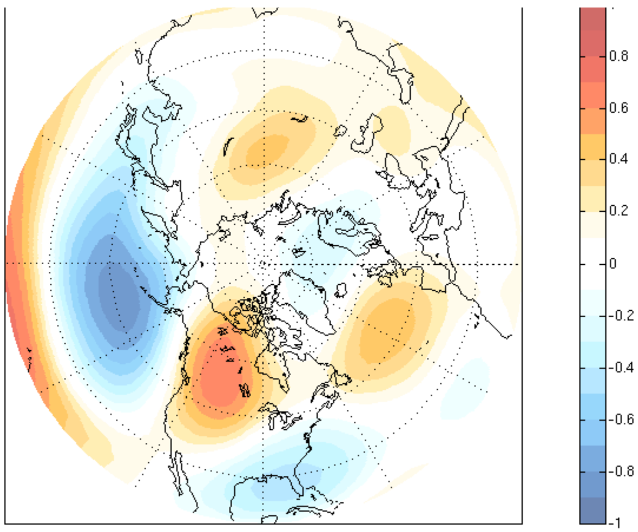


index
vs. time

Correlation maps vs. regression maps

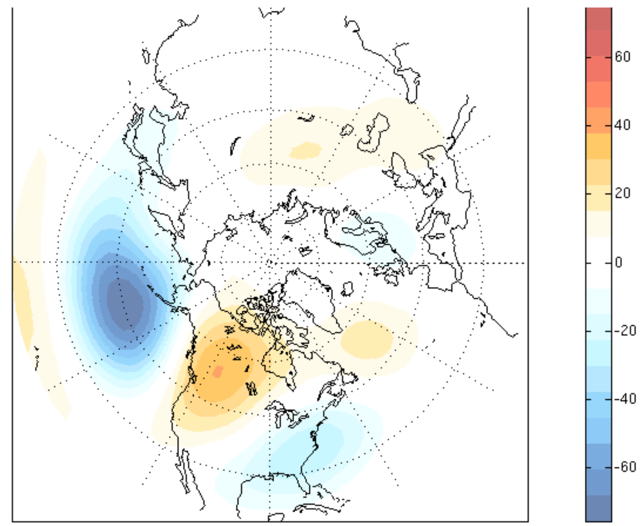
- PNA is a time series of fluctuations in 500 mb heights
- PNA = 0.25 * [Z(20N,160W) - Z(45N,165W) + Z(55N,115W) - Z(30N,85W)]

PNA - Correlation map (r values of each point with index)



$$r = \frac{\overline{x'y'}}{\sigma_x \sigma_y}$$

PNA - Regression map (meters/std deviation of index)



$$r = \frac{\overline{x'y'}}{\sigma_x \sigma_y} \times \sigma_y$$

- Correlation maps put each point on ‘equal footing’
- Regression maps show magnitude of typical variability