Ekman layer

The surface of the ocean is forced by the stress at the surface that comes from the wind. The net movement of the water is to the right of the wind in the Northern Hemisphere (left in the Southern Hemisphere) because of the Coriolis force.

Quantitatively:
The momentum balance in the surface ocean will be given by a balance of the Coriolis force with the frictional stress.

$$-fv = \frac{1}{\rho}\frac{\partial \tau^x}{\partial z} \qquad fu = \frac{1}{\rho}\frac{\partial \tau^y}{\partial z}$$

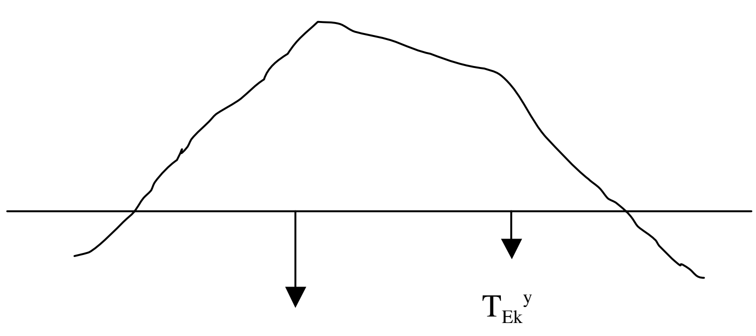
$$\int_{z=-D_E}^{0} v\,dz = T_{Ek}^y - \frac{1}{f\rho}\left[\tau^x(0) - \tau^y(-D_E)\right]$$

$$T_{Ek}^y = -\frac{1}{f\rho}\tau_W^x \qquad T_{Ek}^x = \frac{1}{f\rho}\tau_W^y$$

Note that the net transport in the Ekman layer T_{Ek} does not depend on the structure of the flow in the Ekman layer or the depth D_E (which is typically 50 to 100 m). The units of stress is the same as that as pressure and is given by N/m2.

The wind stress is given by
$\tau_W = \rho_{air} C_D U^2$
where $\rho_{air} = 1.4 kg/m^3$ and $C_D = 1.1 \times 10^{-3}$

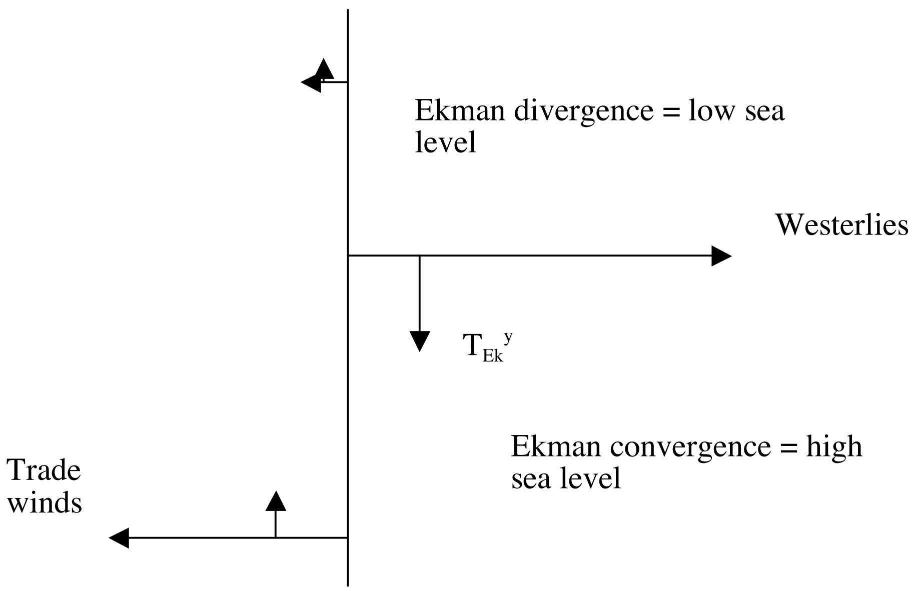
The Ekman transport at a single point in space has units of m2/s. If you want to know the total Ekman transport, say across the width of the ocean, you need to integrate the quantity horizontally.



Total Ekman transport = $\int T_{Ek}^y dx = L\overline{T}_{Ek}^y$

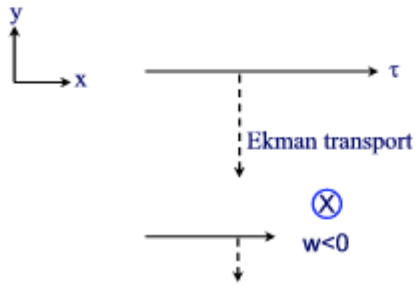
To the extent that t/f varies at all, the Ekman transport will not be uniform horizontally. This has the potential to create convergences and divergences in Ekman transport.

Ekman transport converges and piles up water in the subtropical gyre, creating a high pressure region there. The geostrophic ocean flow is then around this high pressure in a clockwise, anticyclonic sense. The opposite happens in the subpolar gyre:



(i) Ekman Pumping: convergence or divergence of the Ekman transport.

Convergence in Ekman layer = downwelling at base (by conservation of mass)
Divergence in Ekman layer = upwelling at base



Formally:

$$w(-D_E) = \frac{\partial T_{Ek}^{x}}{\partial x} + \frac{\partial T_{Ek}^{y}}{\partial y}$$

$$\begin{aligned} w(-D_E) &= \frac{\partial}{\partial x}\left(\frac{\tau^y}{\rho f}\right) - \frac{\partial}{\partial y}\left(\frac{\tau^x}{\rho f}\right) \\ &= \frac{1}{\rho}\nabla \times \left(\frac{\underline{\tau}}{f}\right) \\ &= \frac{1}{\rho f}\nabla \times \underline{\tau} + \frac{\beta}{\rho f^2}\tau^x \end{aligned}$$

Here,
$\beta = 2\Omega\cos\theta / R_{earth}$
and
$f = 2\Omega\sin\theta$

Sverdrup's theory of wind-driven circulation

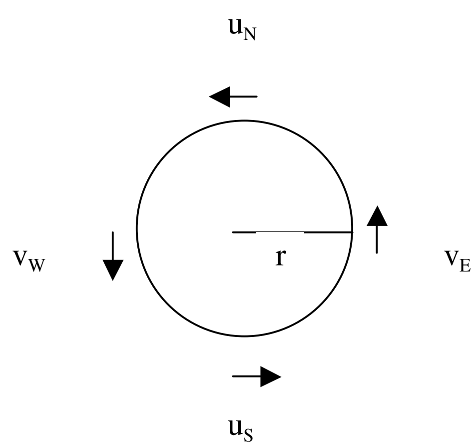
1. **Vorticity**, $\zeta = dv/dx - du/dy$

(Aside: Vorticity is really a 3D vector, $\nabla \times \underline{\mathbf{u}}$, but in large-scale oceanography, generally only the component of spin in the horizontal plane is important.)

A measure of the "spin" of the fluid velocity field, due to (in this case) horizontal shear of the horizontal velocity.

Physical interpretation -- positive vorticity when spin of an ideal vorticity meter tends to be counterclockwise (i.e. in a mathematically positive direction).

Imagine a bit of ocean in solid body rotation:



At any point:
$|\underline{\mathbf{u}}| = \Omega r$

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{v_E - v_W}{2r} - \frac{u_N - u_S}{2r}$$
$$= \frac{\Omega r + \Omega r}{2r} - \frac{-\Omega r - \Omega r}{2r}$$
$$= 2\Omega$$

2. How is the concept of vorticity modified for a rotating earth?

On a rotating earth there are two kinds of vorticity, one associated with the spin of the fluid relative to the Earth, and one associated with the Earth itself.

(1) **Relative vorticity.** A measure of the spin relative to the earth. Relative vorticity = ζ as defined above.

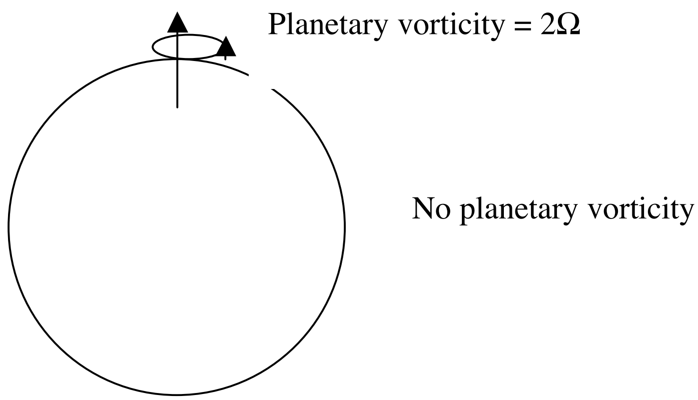
(2) **Planetary vorticity**. The spin due to the rotating Earth. Planetary vorticity = f, the Coriolis parameter. Depends on latitude.

The earth's rotation is described mathematically by a vector that points through the north axis of rotation, perpendicular to the plane in which the rotation occurs and using the right-hand convention.

Planetary vorticity (the Coriolis parameter) is:

$$f = 2\Omega \sin \theta.$$

which can be physically interpreted as the vorticity due to the projection of rotation vector onto local vertical direction. Again, for large-scale flows, the significant term of vorticity is the "spin" in the local horizontal plane.

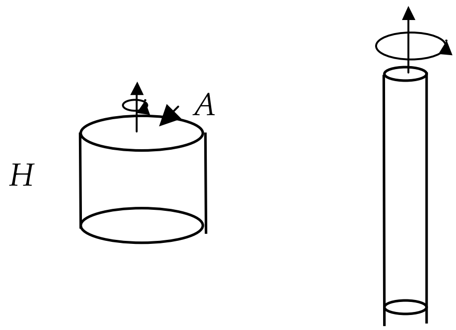


At the equator, the axis of rotation is perpendicular to the local vertical direction -- there is no spin in the local horizontal plane due to the Earth's rotation. As we move northward on the surface of the Earth, the projection of the rotation onto the local horizontal plane increases, until at the poles, it is at a maximum.

If we write an equation for the conservation of angular momentum and mass, what we find is that something called potential vorticity is conserved by a fluid parcel.

What is angular momentum? ~ Spin X area

Think about a column of water of height *H* and horizontal surface area *A*.



Angular momentum conservation:

$(f+\zeta)A = const$ following a fluid parcel

Skater analogy -- if *A* gets smaller, spin increases, if *A* gets larger, spin decreases.

However, because of the continuity equation, the total volume of the fluid parcel must be conserved. If *A* changes, *H* changes in the opposite sense. AH = const. $A \sim 1/H$.