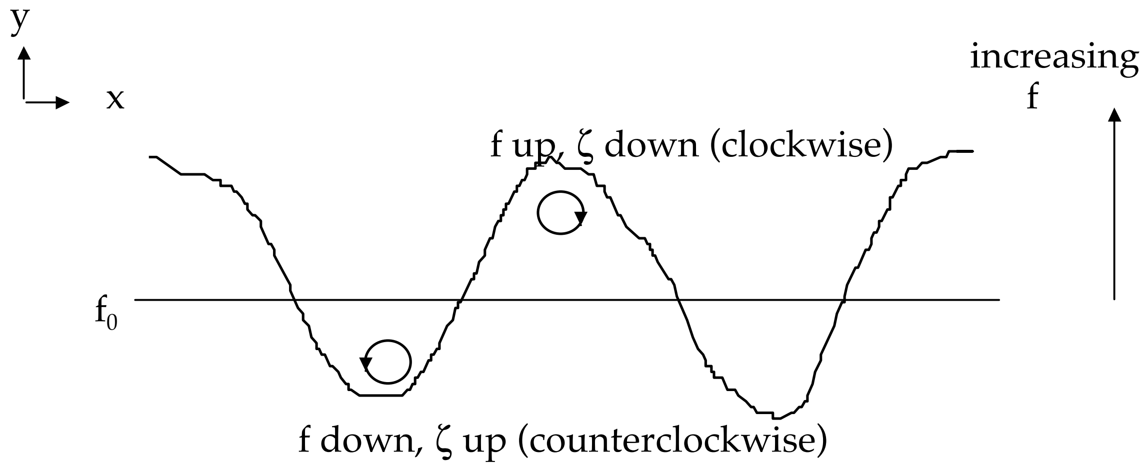
Rossby waves:

<u>Restoring force</u> is the north-south gradient of background potential vorticity (f/H). That gradient can be due to either the variation in f with latitude, or to a variation in H ("topographic Rossby waves" and when trapped near the coast "continental shelf waves").

Note that Rossby waves are tranverse waves, that is the particles move perpendicular to the direction of propagation.

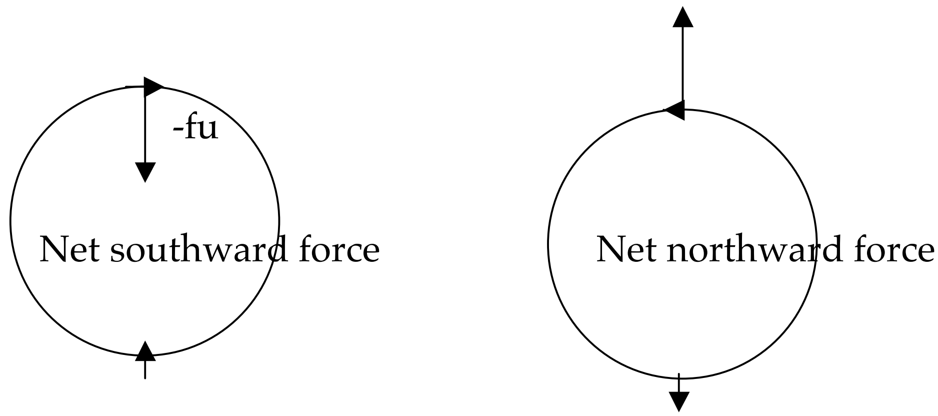
Conserves PV = $(f+\zeta)/H$.

Sketch: northern hemisphere



Imagine that something pushes a water column perpendicular to the planetary vorticity. What will happen? The parcel will conserve total potential vorticity -- $(f+\zeta/H)$. As it moves into a region of higher f, it will tend to decrease relative vorticity -- a tendency to clockwise motion. Likewise if it were displaced towards lower relative vorticity, it will increase relative vorticity -- a tendency towards counterclockwise motion.

Now let's look at the Coriolis forces on the induced circulation pattern



Size of restoring force (for a given displacement) is proportional to $\beta = \partial f/\partial y$

(The same thing will hold true on a slope -- pushed towards shallower water will require decreased relative vorticity and vice versa -- so shallower water is the analog of northward over a flat bottom and there are analogous waves generated on strong topographic slopes called **topographic Rossby waves**)

The Rossby wave solution can be derived from the conditions for a sine/cosine waveform to satisfy the equation for vorticity conservation with relative vorticity, planetary vorticity and stretching vorticity changes all possible:

$$\frac{\partial \varsigma}{\partial t} - f\frac{\partial w}{\partial z} + \beta v = 0$$

Common variable: can relate all these terms to the pressure perturbation associated with the wave, assuming geostrophic velocity and hydrostatic pressure. This is a fairly involved derivation, and we're not going to do that here. But what falls out is a condition on the relationship between frequency and wavenumber in order to satisfy this equation…the dispersion relationship.

$$p' = a(z)\cos(kx+ly-\omega t)$$

$$\omega = \frac{-\beta k}{\frac{1}{R_D^2} + k^2 + l^2}$$

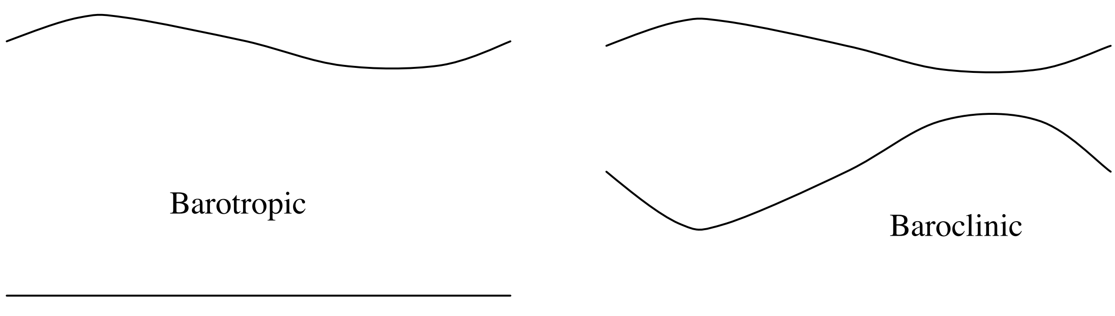
l = north-south wavenumber
k = east-west wavenumber
L_D = Rossby radius (deformation radius)

BAROTROPIC ROSSBY RADIUS

$L_D = \frac{\sqrt{gH}}{f} = \frac{c}{f}$ = gravity wave speed / f

BAROCLINIC ROSSBY RADIUS $L_D = \frac{\sqrt{g'H}}{f}$ = NH/f,

$N = \sqrt{-\frac{g}{\rho_0}\frac{\partial \rho}{\partial z}} \sim \sqrt{\frac{g}{\rho_0}\frac{\Delta\rho}{H}} = \sqrt{\frac{g'}{H}}$, so $NH = \sqrt{g'H}$



Note that in the barotropic motion, the fluid motion is independent of depth. For baroclinic motion, the fluid motion is in the opposite direction in the upper and lower layers.

$$g'\Delta H \sim -g\Delta\eta$$

$$\frac{\Delta H}{\Delta\eta} \sim -\frac{g}{g'} = \frac{\rho_0}{\Delta\rho} \sim \frac{1025}{5} \sim -200$$

Dη ~ a few cm, DH ~ tens of meters.

Notes:

(1) Phase speed. C = (Cx, Cy) Cx = ω/k is always negative -- westward phase propogation. Group velocity dw/dk can is westward for long waves and eastward for short waves.

(2) Important limit: Long waves, k, l << 1

- propagate strictly westward at the speed $c = -\beta R_D^2$
- nondispersive (frequency linear function of wavenumber)
- Rossby wave phase speed decreases moving away from the equator (β decreases). Wave crest is crescent shaped

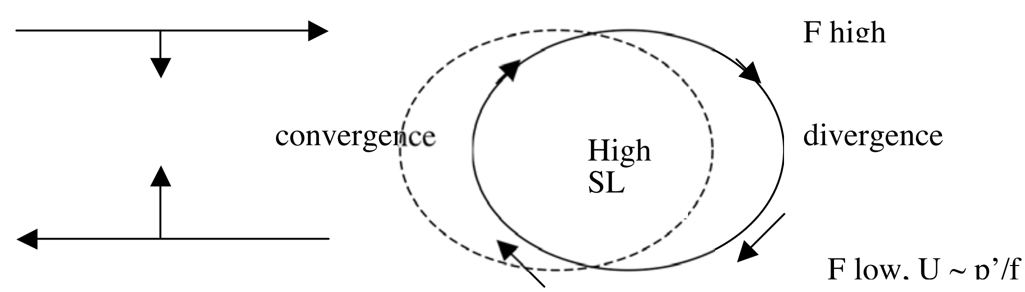
(3) Energy propagation (group velocity) = $d\omega/dk$, can be in either direction. $d\omega/dk < 0$ for long waves (low k) -- westward
$d\omega/dk > 0$ for short waves -- eastward

(4) This function has a maximum frequency that is always < f. Subinertial waves. Generally periods of weeks to months.

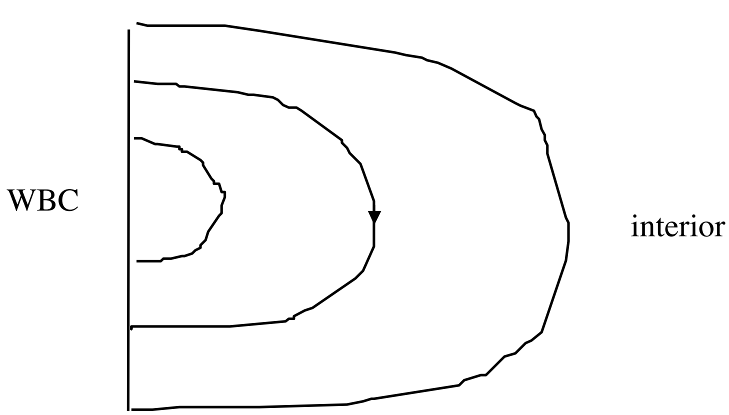
Right near the equator (±5°) … even greater acceleration. The equator acts as a waveguide for Kelvin waves. It has a similar effect on Rossby waves, which are refracted by the strong gradient in f. Bend back towards the equator into a waveguide. Develop a standing wave pattern about the equator that moves westward. Fastest meridional standing wave mode moves westward at c/3.

It is these long waves that we think of as playing an important role in the adjustment of ocean circulation to changing conditions on climate time-scales.

Let’s imagine switching on the wind field over our ocean.



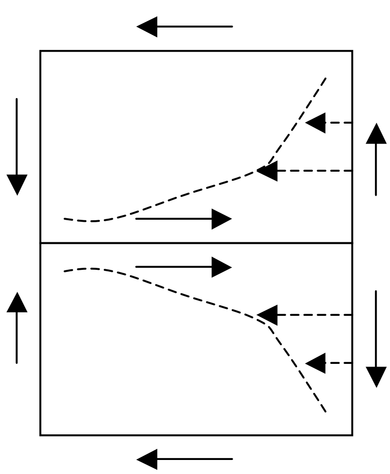
Propagates pressure signal to west, the high ends up near the western boundary.



The end state has a small divergence everywhere in the interior (there is southward flow everywhere) which is exactly balanced by the mass input from the Ekman layer through Ekman pumping. And angular momentum is conserved in the interior as well through the Sverdrup relationship.

What about the western boundary? Here there is a strong northward flow, $\beta v > 0$, representing a gain in planetary vorticity? What is allowing this to occur? In WBC regions, friction can be important and compensates for the planetary vorticity gain. This can only happen because the Rossby waves have shoved the pressure bump up near the western boundary so that there is a strong pressure gradient and strong flow next to the coast, and if the flow is stronger, the frictional torque will be stronger and can impact the potential vorticity balance.

Theoretical spinup pathways



Physical meaning of the Rossby radius:

(1) Length scale at which Coriolis and local acceleration are comparable size for rotating gravity waves
(2) Length scale at which relative vorticity ~ stretching in PV

Deriv. Of (2)

$$\frac{\partial \rho}{\partial t} = w\frac{\partial \rho}{\partial z} = w\frac{\rho_0}{g}N^2$$

$$w = \frac{g}{\rho_0 N^2}\frac{\partial \rho}{\partial t}$$

$$\frac{\partial w}{\partial z} = \frac{g}{\rho_0}\frac{\partial}{\partial t}\left[\frac{\partial}{\partial z}\left(\frac{\rho}{N^2}\right)\right] \qquad \frac{\partial p}{\partial z} = \rho g$$

$$f\frac{\partial w}{\partial z} = \frac{f}{\rho_0}\frac{\partial}{\partial t}\left[\frac{\partial}{\partial z}\left(\frac{1}{N^2}\frac{\partial p}{\partial z}\right)\right]$$

So, scaling the stretching term:

$$fv = \frac{1}{\rho_0}\frac{\partial p}{\partial x} \quad P \sim \rho_0 fUL$$

stretching vorticity rate of change ~ $\frac{\partial}{\partial t}\left(\frac{f^2UL}{N^2H^2}\right)$

compare to relative vorticity rate of change $\frac{\partial}{\partial t}\frac{U}{L}$

are the same size when NH/fL = 1