

# Experimental and Analytical Study of Helical Cross-Flow Turbines for a Tidal Micropower Generation System

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MSME Thesis Defense

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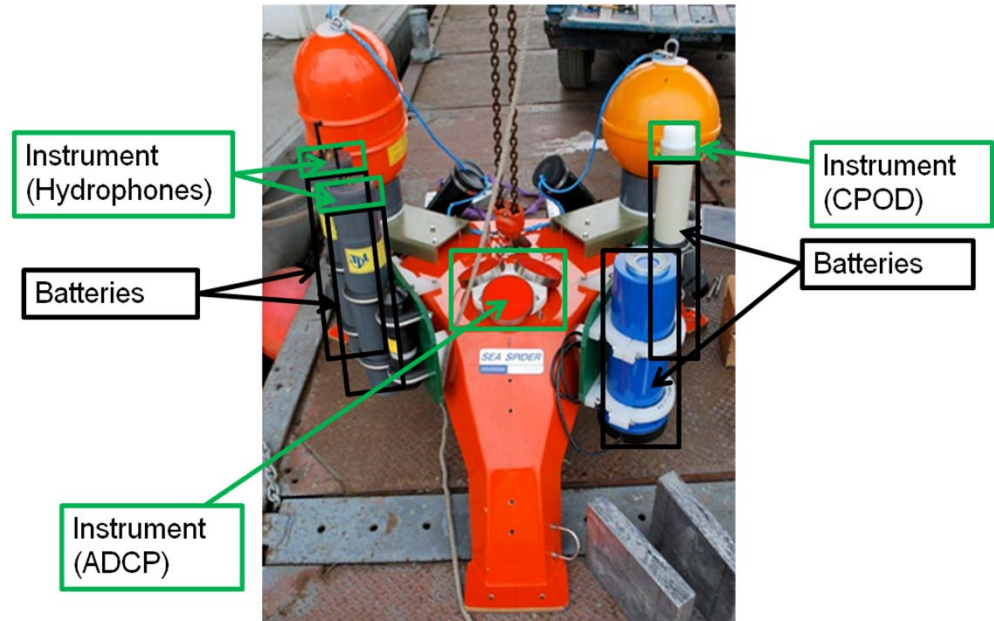
NNMREC

# Outline

- Tidal Micropower System Overview
- Turbine Selection/Design Parameters
- Experimental Testing
- Modeling
- Conclusions and Future Work

# Motivation for Micropower

- Admiralty Inlet – potential site of tidal turbine installation for commercial power
- NNMREC Environmental site characterization monitoring
  - Sea Spider bottom lander tripod
  - Instrumentation requires power
  - Currently battery- powered
  - Grid, solar and wind not feasible



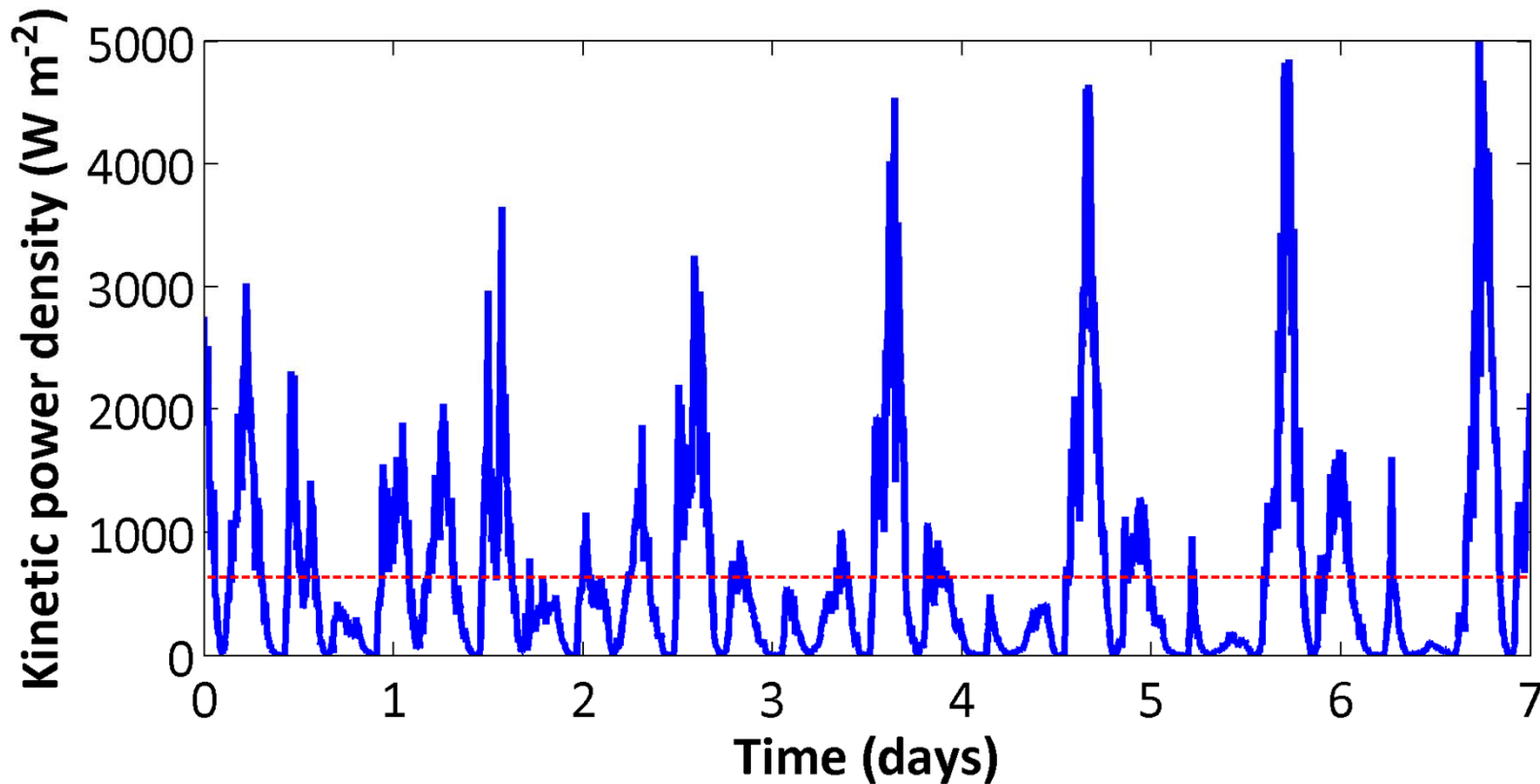
# Motivation for Micropower

	Battery Capacity (W-hr):		165	Cost (\$/W-hr): \$		1.15
Unit	Power Usage (W)	Deployment Duration (days)	Total W-hr	# of batteries Required	Battery Cost	
Doppler Current Profiler	0.9	90	1944	<b>12</b>	<b>\$ 2,280.00</b>	

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Doppler Current Profiler	0.9	<b>180</b>	3888	<b>24</b>	<b>\$ 4,560.00</b>	

# Motivation for Micropower



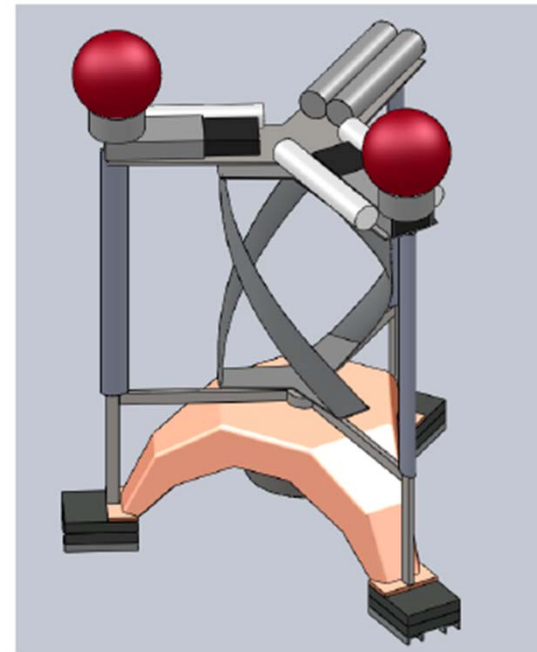
$$KPD = \frac{1}{2} \rho u_0^3$$

$u_0$ : Inflow velocity  
 $\rho$ : Fluid density

# Motivation for Micropower

		Battery Capacity (W-hr):		165	Cost (\$/W-hr): \$		1.15
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Imaging Sonar (20% duty cycle)	20.0	180	86400	<b>524</b>	<b>\$99,560.00</b>		
Imaging Sonar (20% duty cycle)	20.0	180	86400	<b>15</b>	<b>\$ 2,850.00</b>		

# Micropower System Concept



Drawing courtesy of Stelzenmuller et al

Micropower System – A tidal hydrokinetic power generation system to power remote instrumentation on the order of 20 W continuous in 1.5 m/s peak tidal currents.

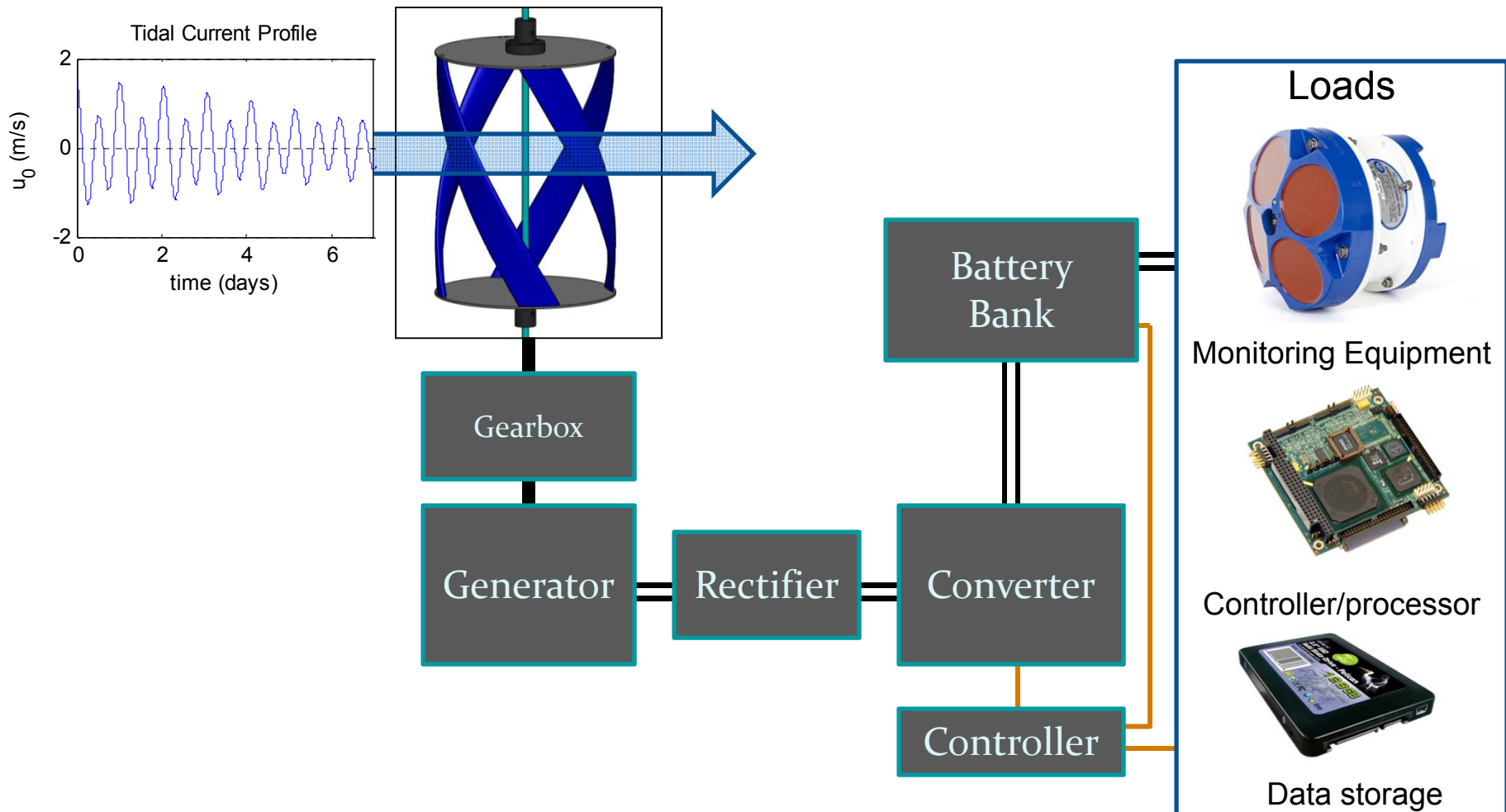


# Research Goals

Provide initial concept for a micropower tidal generation system

Provide detailed design, analysis and experimental testing one of the components: the hydrokinetic turbine

# Micropower System Block Diagram



# Outline

- Micropower System Overview
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# Turbine Performance Metrics

- Efficiency – for power and torque production

$$C_p = \frac{T\omega}{\frac{1}{2}\rho DHu_0^3} \quad C_Q = \frac{T}{\frac{1}{4}\rho D^2 Hu_0^2}$$

- Starting torque magnitude and range – for self-start

$$C_{QS} = \frac{T}{\frac{1}{4}\rho D^2 Hu_0^2} \quad T \text{ vs } \theta$$

- Tip Speed Ratio and Rotation Rate – for generator integration

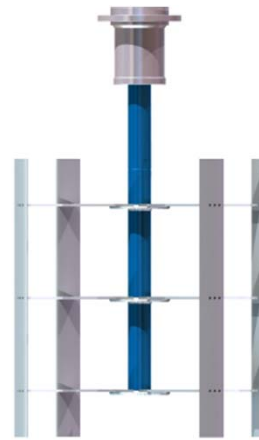
$$\lambda = \frac{\omega R}{u_0}$$

- Performance with respect to inflow orientation – for deployment flexibility

*H: Height*  
*D: Diameter*  
*R: Radius*  
*T: Torque*  
*u<sub>0</sub>: Inflow velocity*  
*ρ: Fluid density*  
*θ: Azimuthal angle*  
*ω: Angular velocity*

# Turbine Design Concepts

- High efficiency (~35%)
- Good self-start capability
- Accepts flow from any horizontal direction



Darrieus  
Turbine



Hybrid  
Turbine



Helical  
Turbine

Sources: Polagye, 2011; [enr.mun.ca/~tariq/jahangiralam.pdf](http://enr.mun.ca/~tariq/jahangiralam.pdf); [newenergycorp.ca](http://newenergycorp.ca);

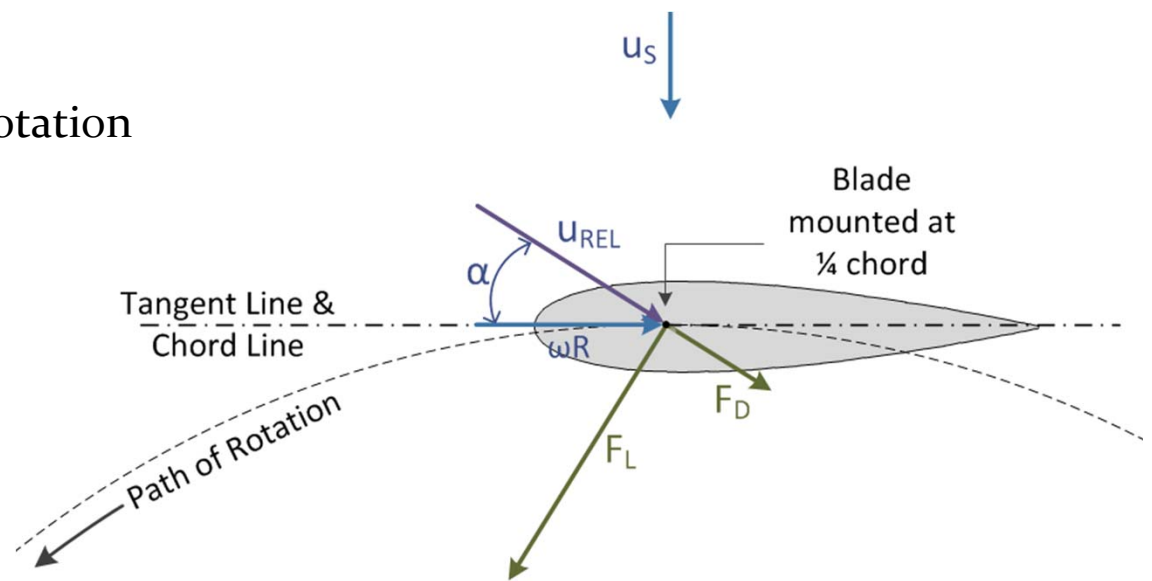
# Blade Element Theory

- Streamwise velocity ( $u_s$ )
- Tip velocity ( $\omega R$ )
  - Induced by turbine's own rotation
- Relative velocity ( $u_{REL}$ )
  - Resultant vector of  $u_s$  and  $\omega R$
- Angle of attack ( $\alpha$ )
  - Angle between  $u_{REL}$  and chord line
- Lift - Drag Forces ( $F_L$ ,  $F_D$ )
  - Generate turbine torque

$$C_L = \frac{F_L}{\frac{1}{2}\rho u_{REL}^2 A_P} = \frac{F_L}{\frac{1}{2}\rho u_{REL}^2 ch}$$

$$C_D = \frac{F_D}{\frac{1}{2}\rho u_{REL}^2 A_P} = \frac{F_D}{\frac{1}{2}\rho u_{REL}^2 ch}$$

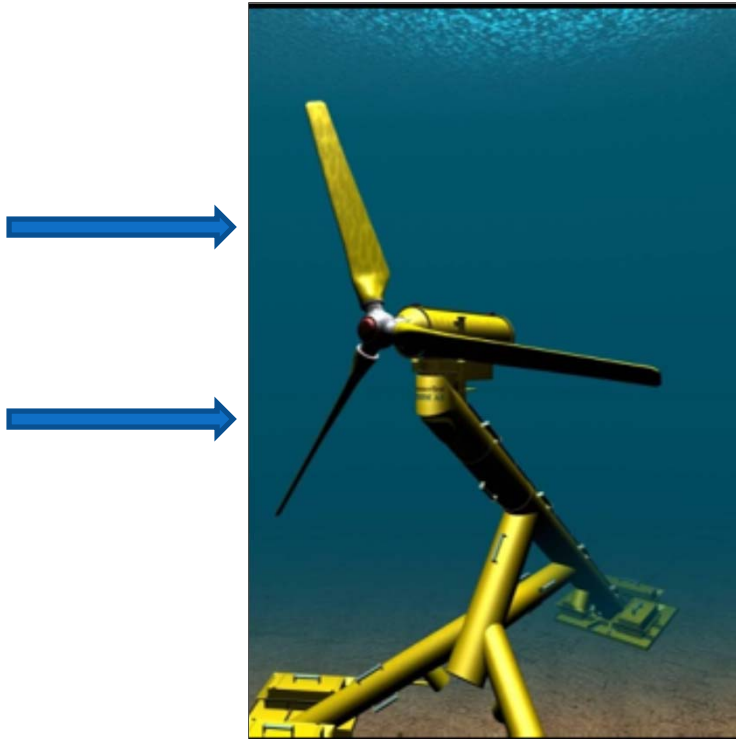
- Best lift-drag ratio when  $-10 < \alpha < 10^\circ$



$c$ : Blade length  
 $h$ : Blade height  
 $A_p$ : Planform area= $ch$

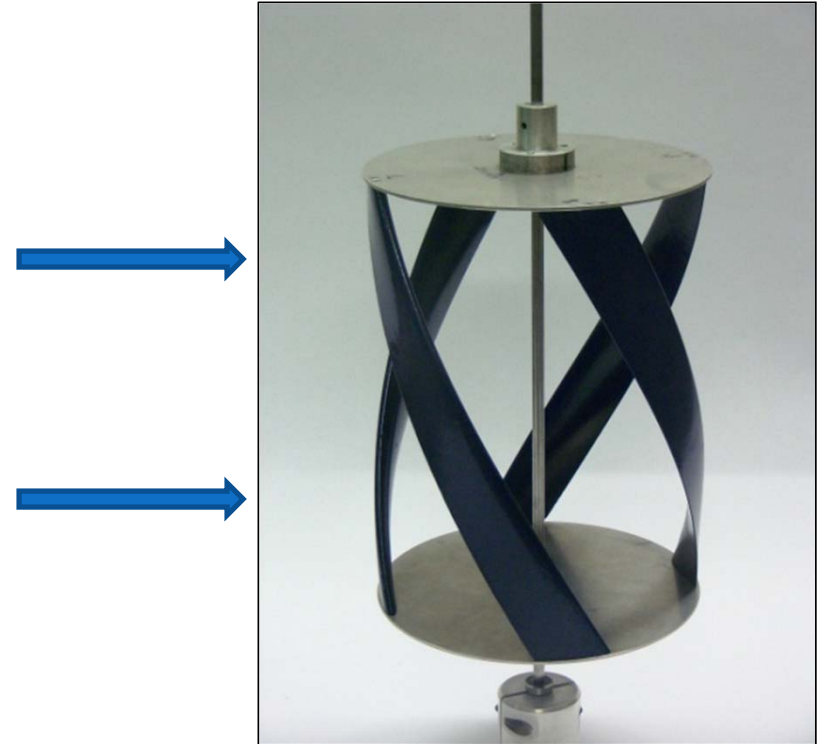
# Axial vs. Cross-Flow Turbines

Axial



Tidal Flow is Parallel to Axis of Rotation

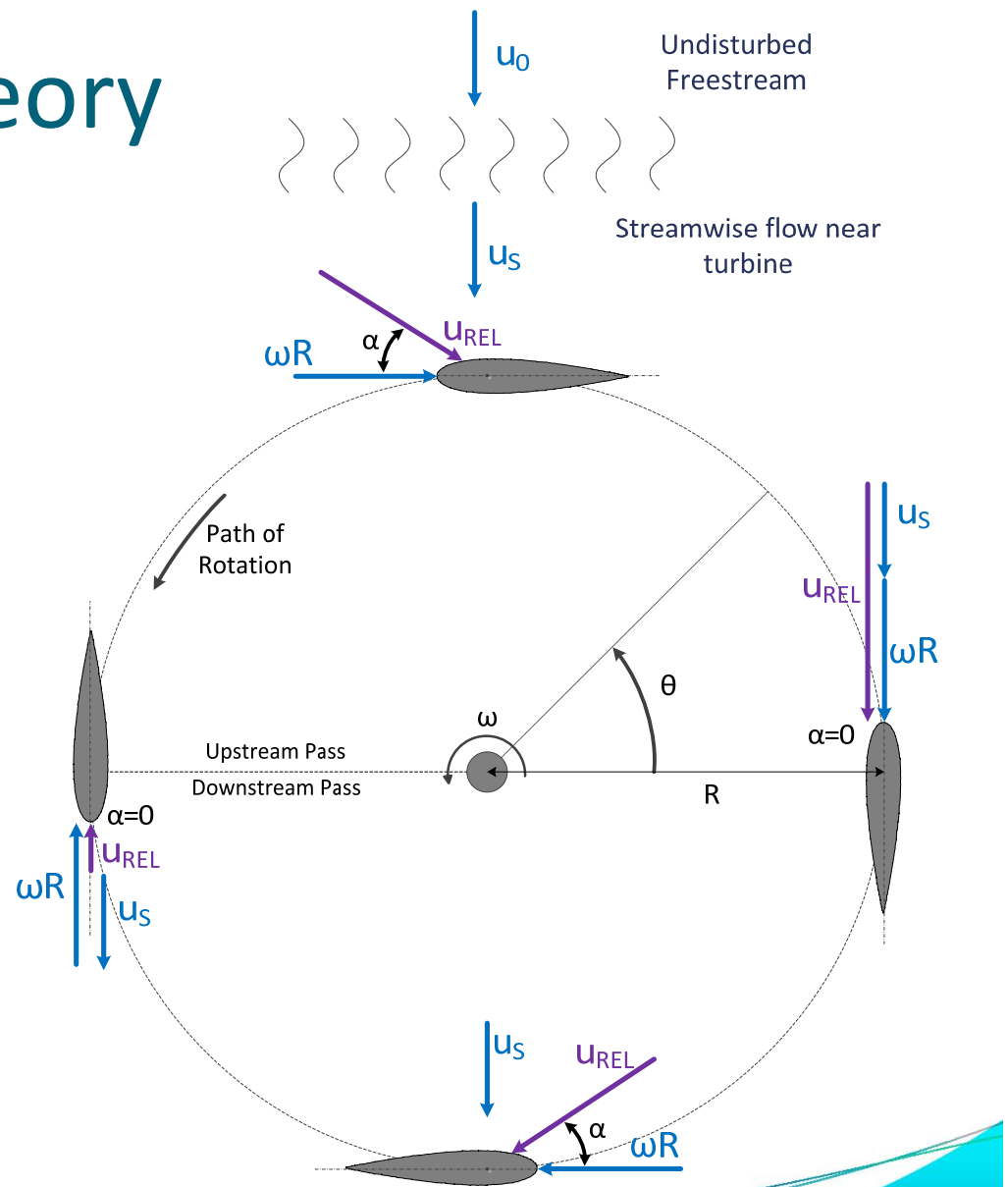
Cross-Flow



Tidal Flow is Perpendicular to Axis of Rotation

# Blade Element Theory

- Angle of attack ( $\alpha$ )
  - Constantly changing as turbine rotates
  - Often exceeds stall angle



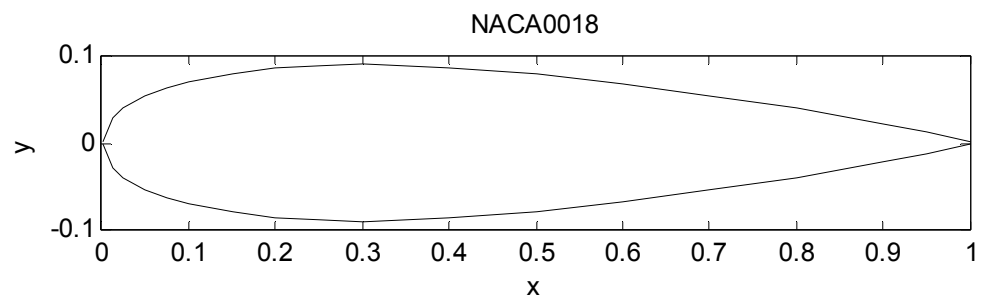
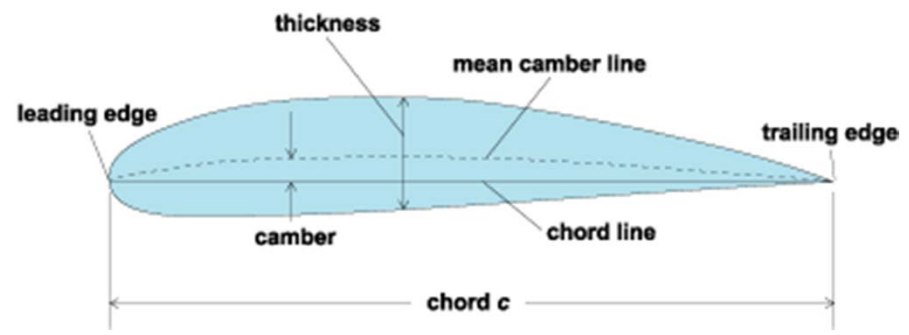


# Helical Turbine Parameters

- Blade Profile
- Blade Pitch
- Helical Pitch
- Aspect Ratio
- Solidity Ratio/ Chord-to-Radius Ratio
- Number of Blades
- Blade Wrap
- Attachment Design

# Helical Turbine Parameters

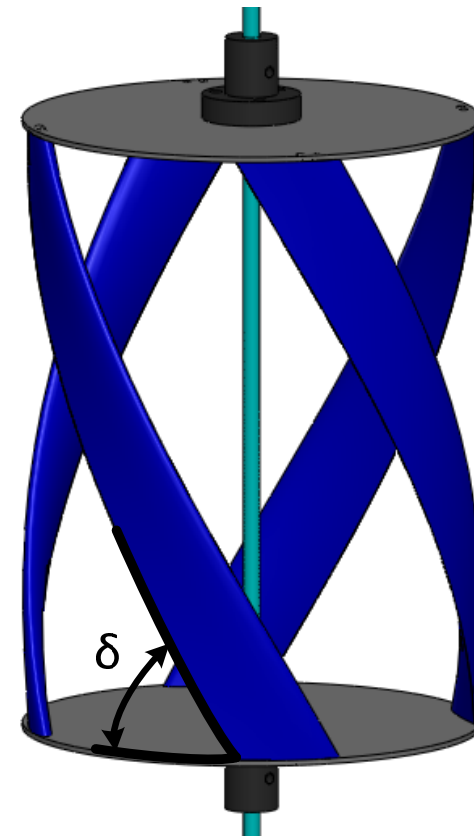
- Blade Profile
- Helical Pitch
- Aspect Ratio
- Solidity Ratio/ Chord-to-Radius Ratio
- Number of Blades
- Attachment Design



Source: answers.com

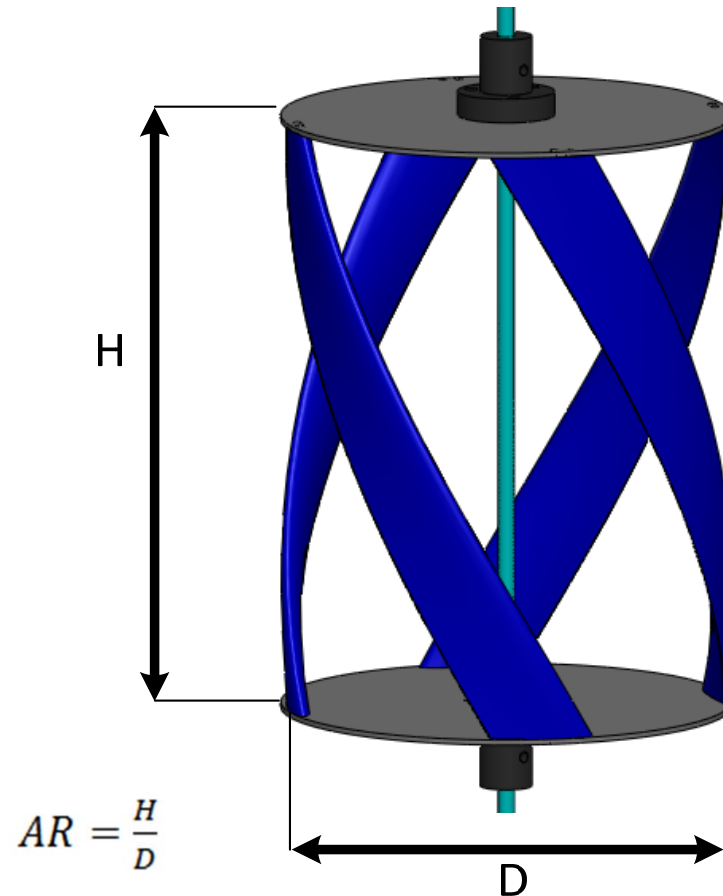
# Helical Turbine Parameters

- Blade Profile
- Helical Pitch
- Aspect Ratio
- Solidity Ratio/ Chord-to-Radius Ratio
- Number of Blades
- Attachment Design



# Helical Turbine Parameters

- Blade Profile
- Helical Pitch
- Aspect Ratio
- Solidity Ratio/ Chord-to-Radius Ratio
- Number of Blades
- Attachment Design
  
- Size limited by deployment constraints



# Helical Turbine Parameters

- Blade Profile
- Helical Pitch
- Aspect Ratio
- Solidity Ratio/ Chord-to-Radius Ratio
- Number of Blades (B)
- Attachment Design

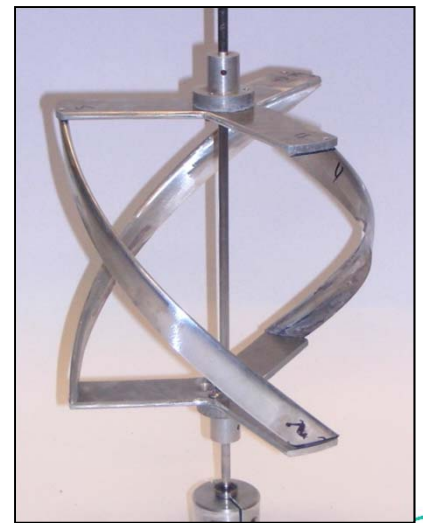
$$\sigma = \frac{Bc}{\pi D}$$

$$\sigma_{C-R} = \frac{c}{R}$$

30% Solidity



15% Solidity

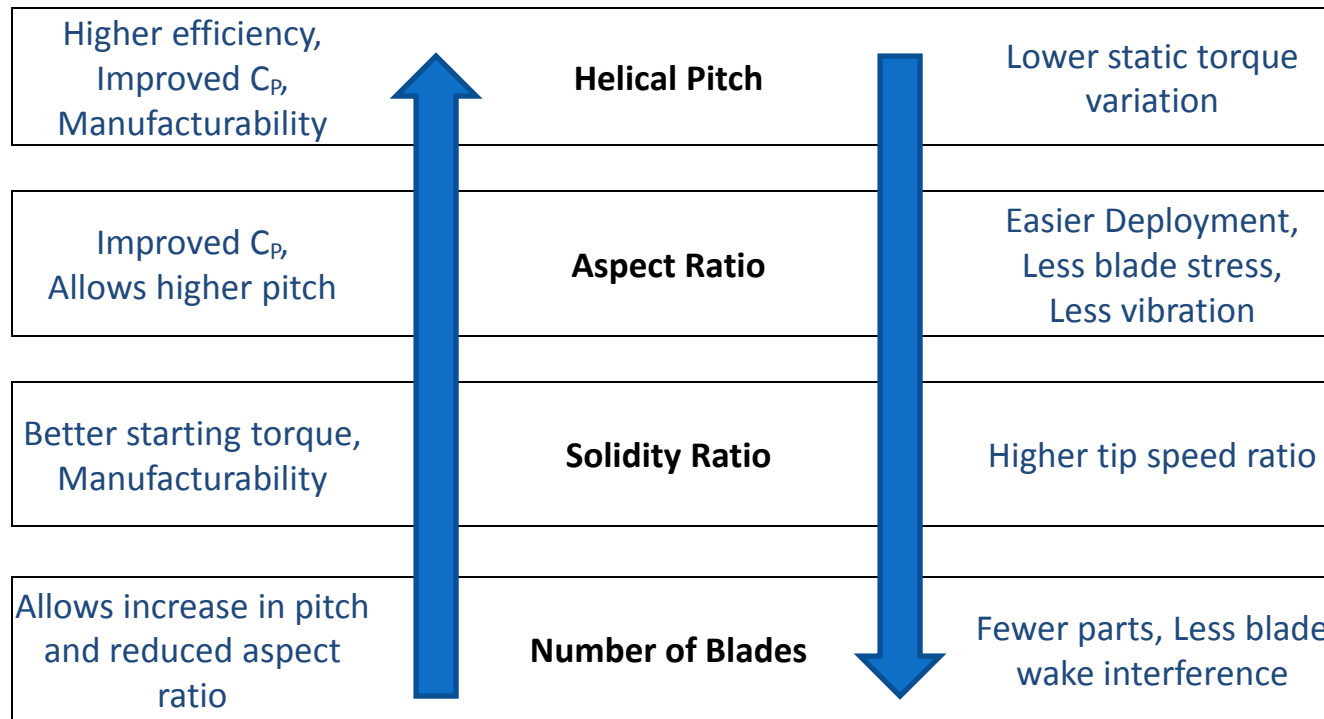


# Helical Turbine Parameters

- Blade Profile
- Helical Pitch
- Aspect Ratio
- Solidity Ratio/ Chord-to-Radius Ratio
- Number of Blades
- Attachment Design



# Parameter Optimization



# Outline

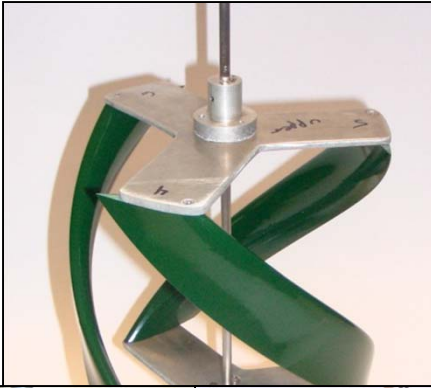
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# Turbine Prototypes

## Baseline Configuration

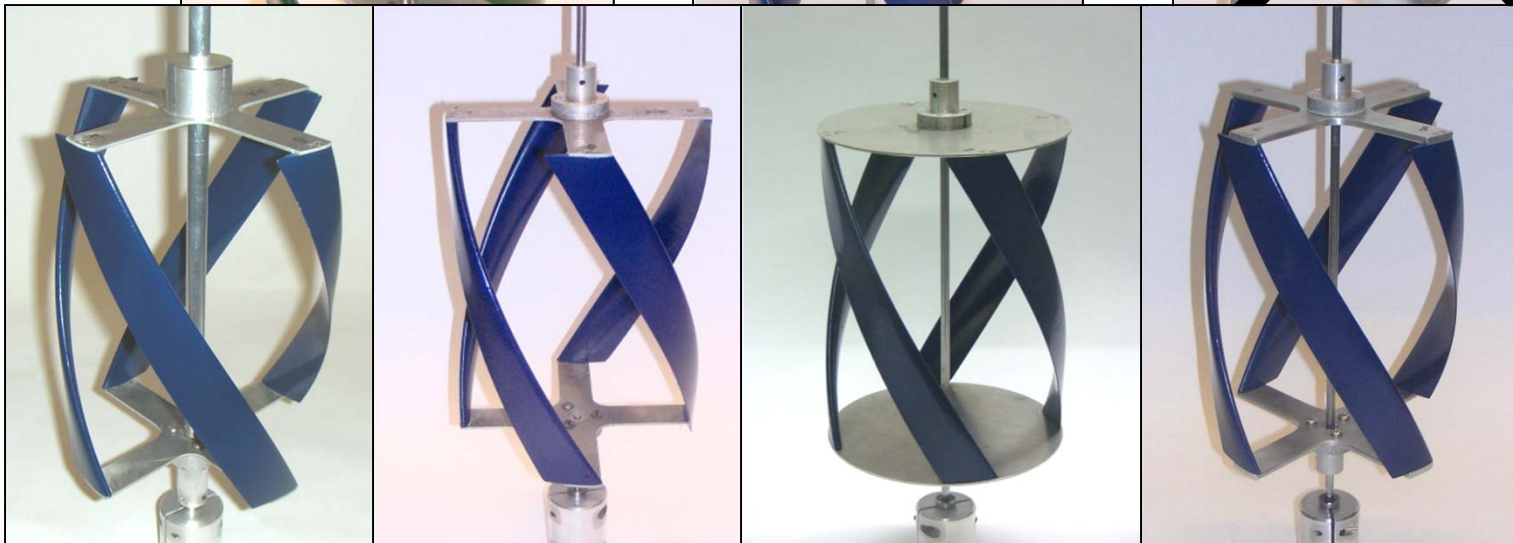
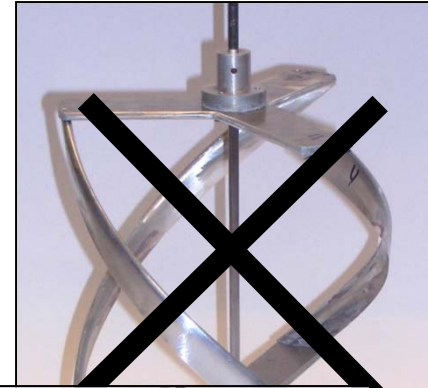
3-Bladed Turbine



4-Bladed Turbine



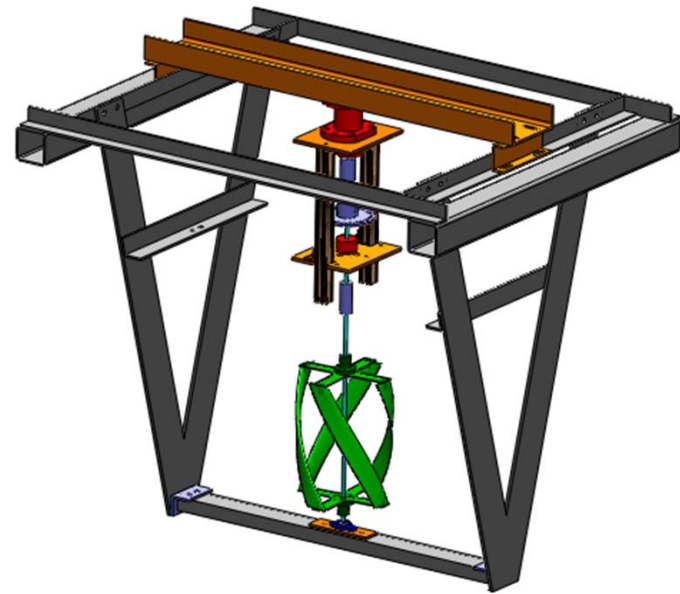
Low Solidity Turbine



Ratio  
Angle

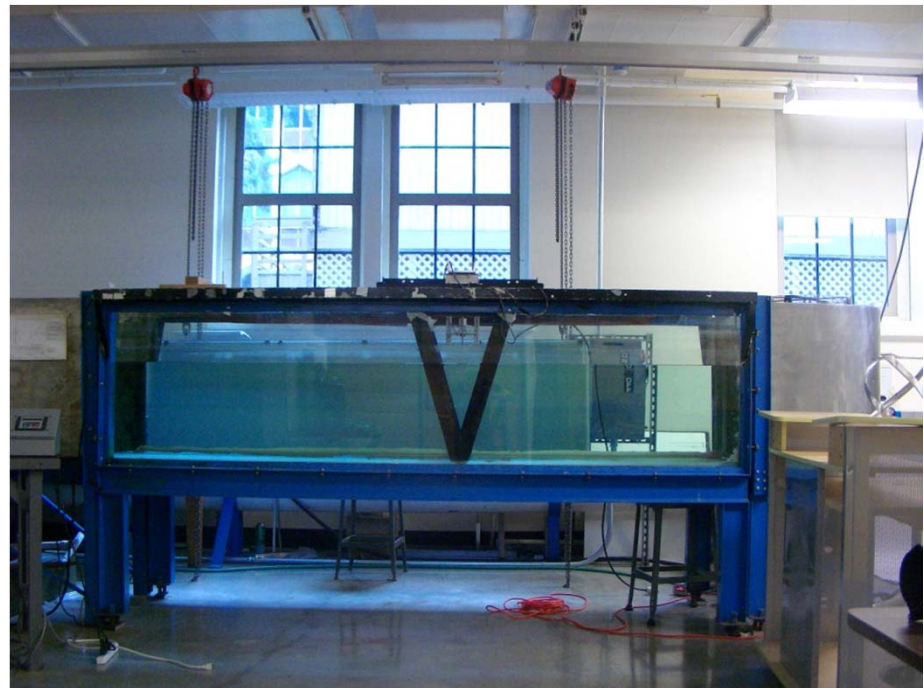
# Tests Performed

- Baseline Configuration
  - Load Performance Test
  - Static Torque Test
  - Tilted Turbine Test
- Design Comparison Tests
  - Helical Pitch/No. of Blades
  - Strut Attachment Design
  - Shaft Design

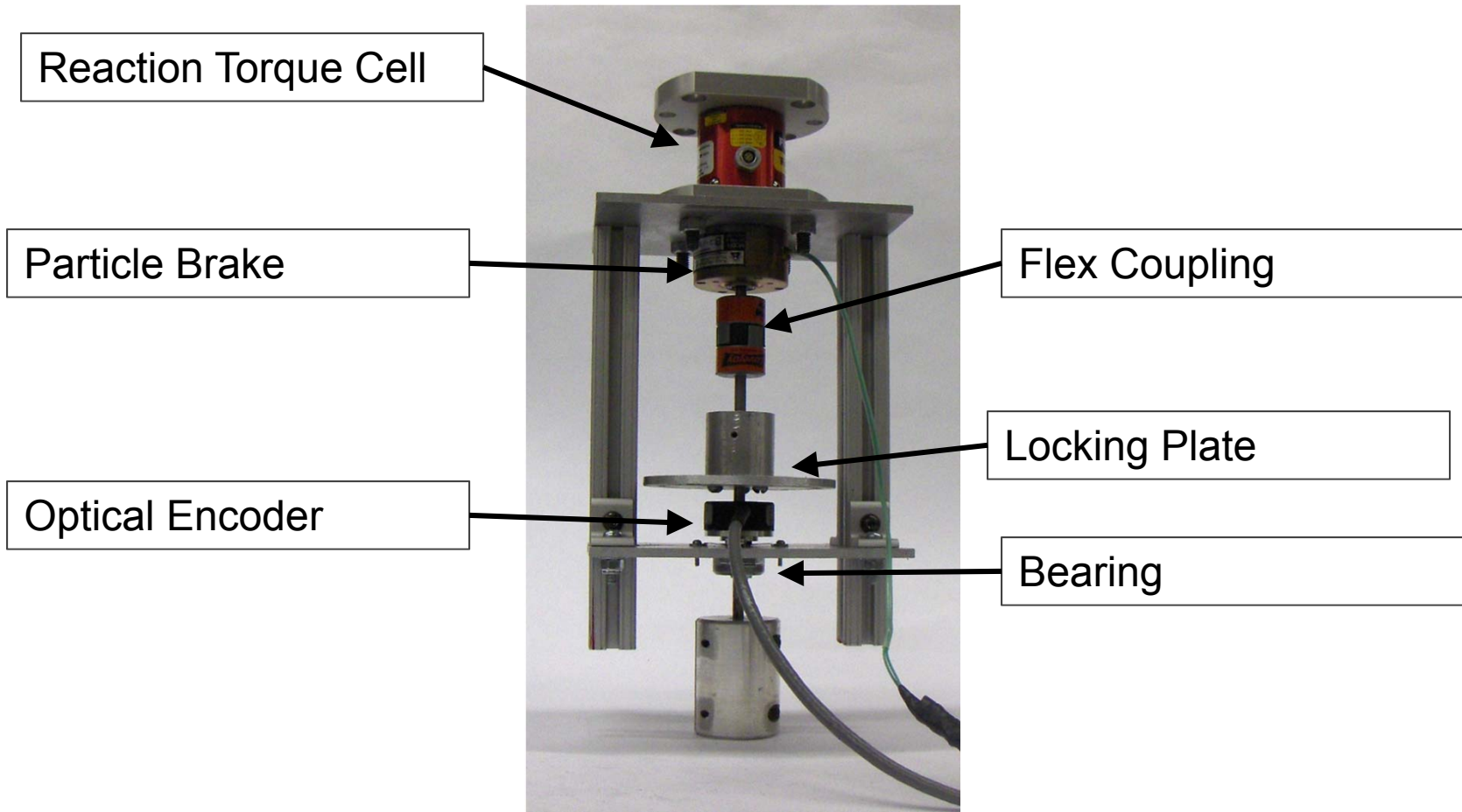


# Flume Test Channel

- 3 meters length
- 72 cm wide
- Maximum 0.8 m/s flow rate
- Turbine located in center of flume width at 1.7 m from test channel inlet
- Acoustic Doppler Velocimeter (ADV) - to measure flume velocity



# Test Instrumentation Module



# Turbine Operation



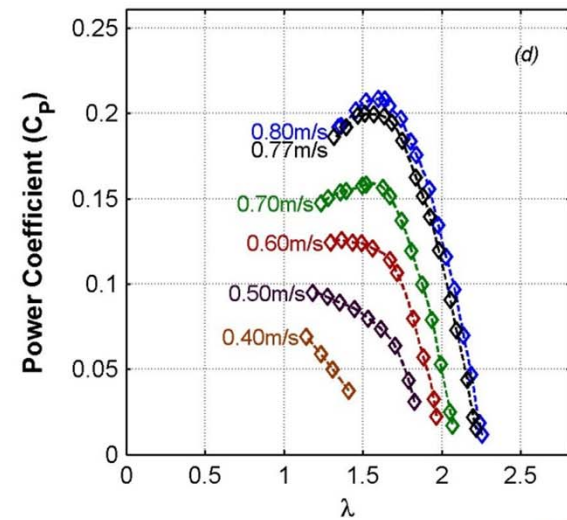
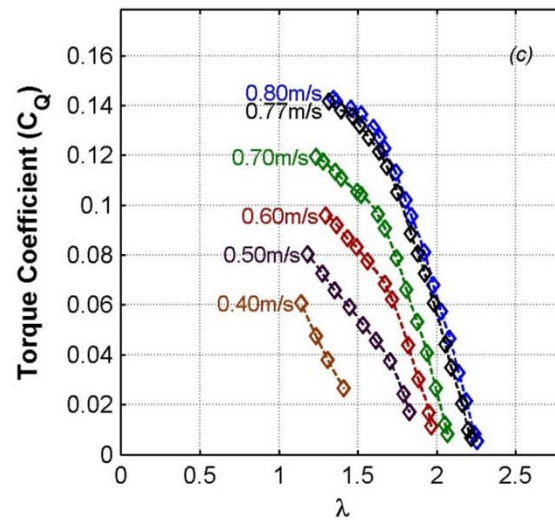
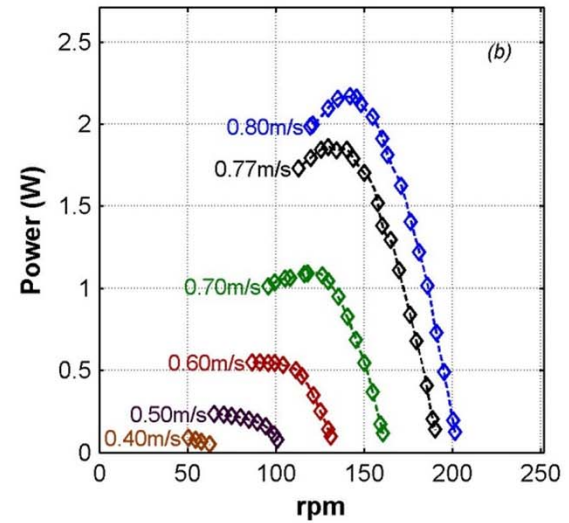
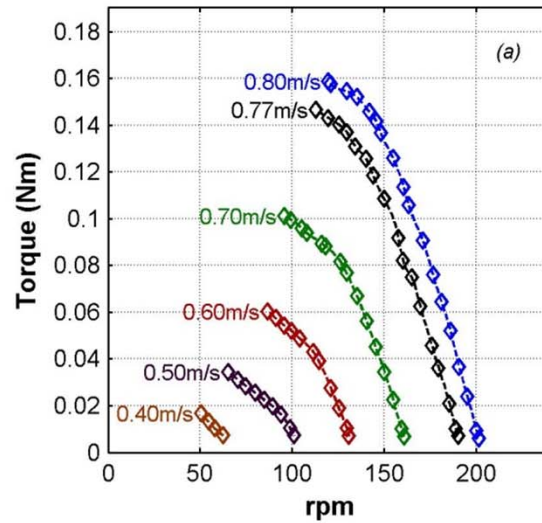
# Load Performance Results

- Baseline Configuration

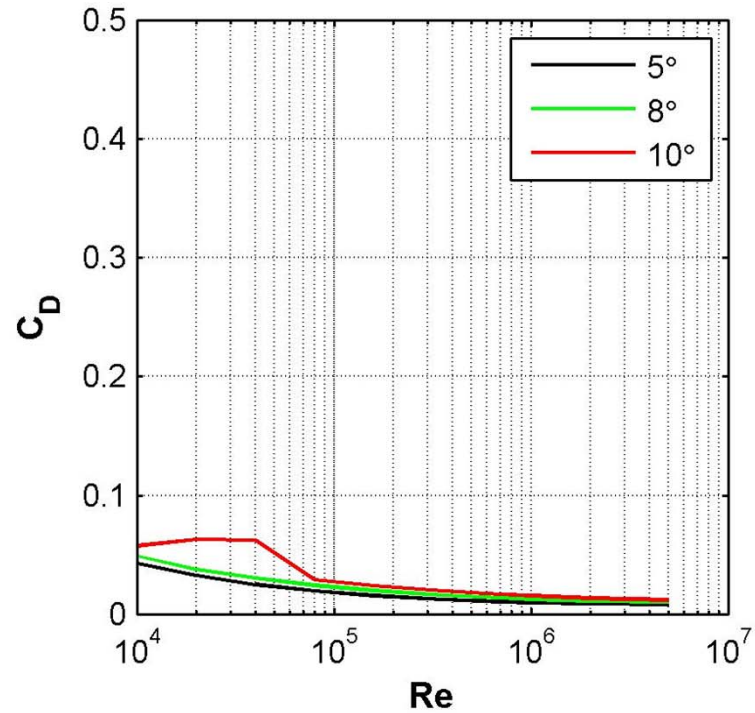
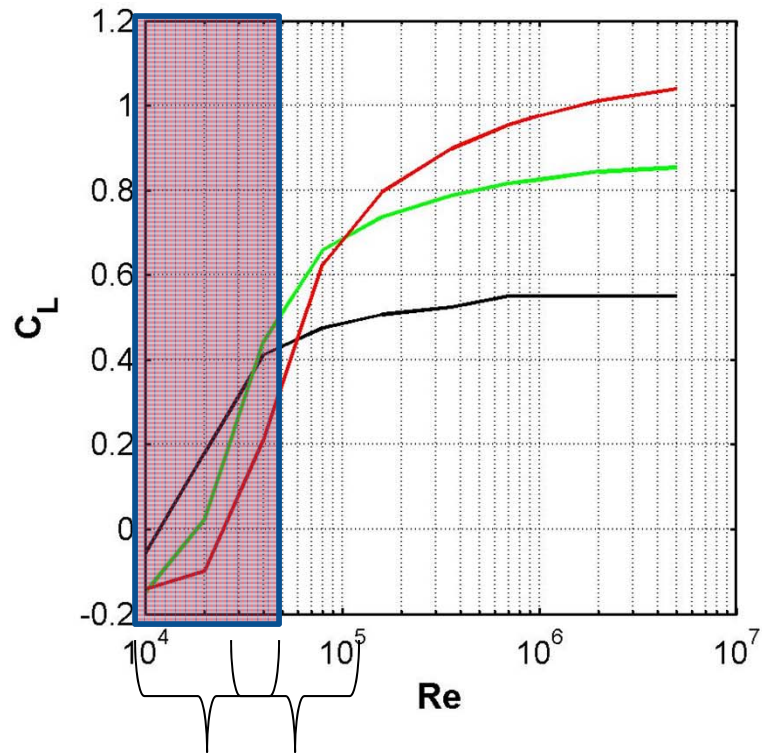


$$C_Q = \frac{T}{\frac{1}{4} \rho D^2 H u_0^2}$$

$$C_P = \frac{T \omega}{\frac{1}{2} \rho D H u_0^3} \quad \lambda = \frac{\omega R}{u_0}$$



# Lift and Drag for different $\alpha$



Region of Turbulence  
 Region of Turbulence  
 Operation at Re/average  $\alpha$

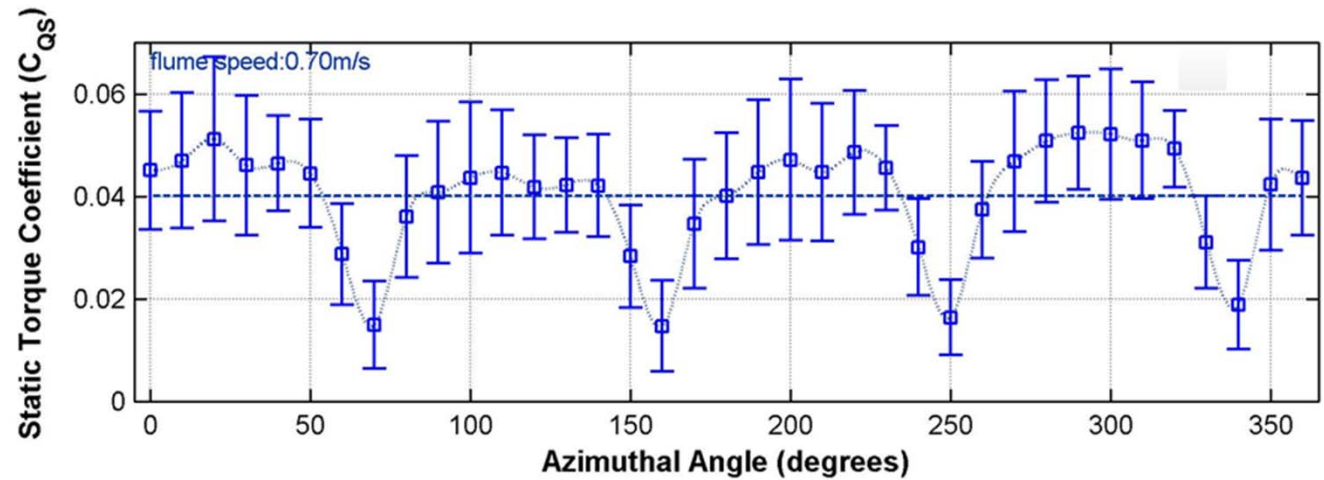
$$Re = \frac{u_{REL} C}{\nu}$$

# Static Torque

- Baseline Configuration
- Flume: 0.7 m/s



0° view,  
looking downstream





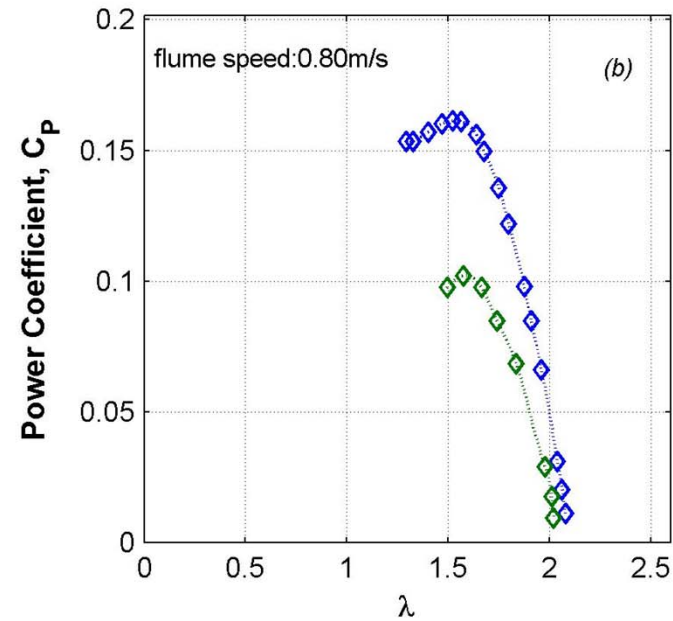
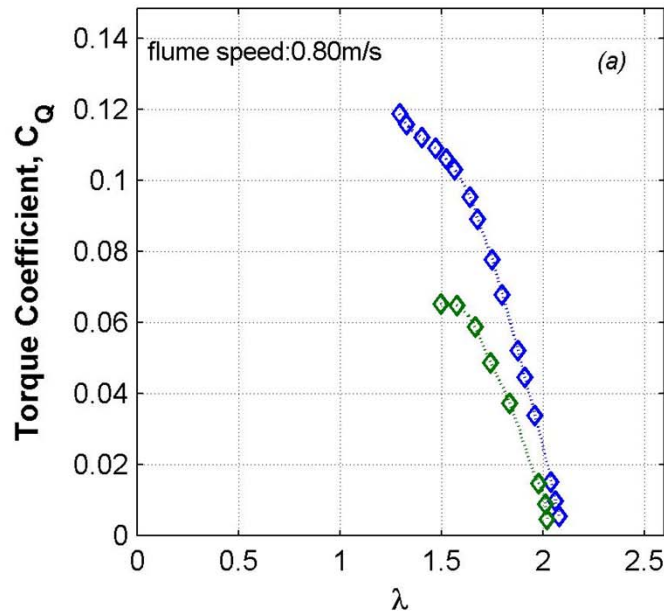
# Turbine Design Comparison

## Load Performance Test Results

3-Bladed Turbine



4-Bladed Turbine



# Strut Design Comparison

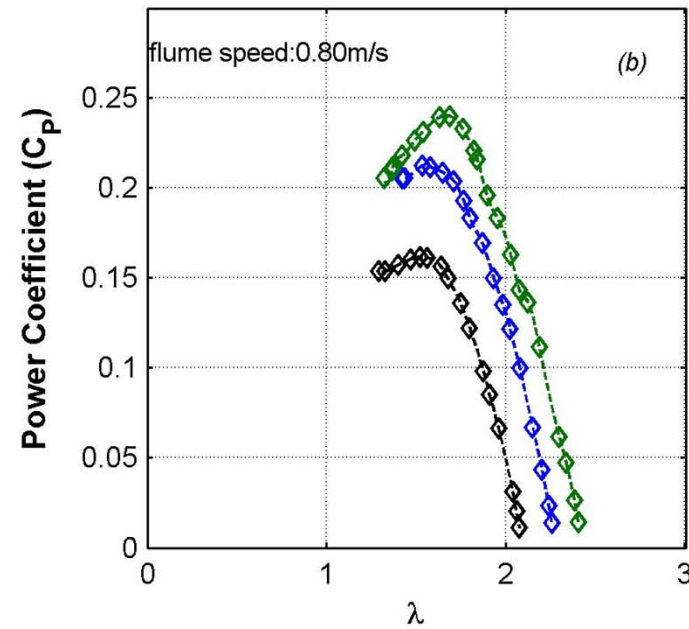
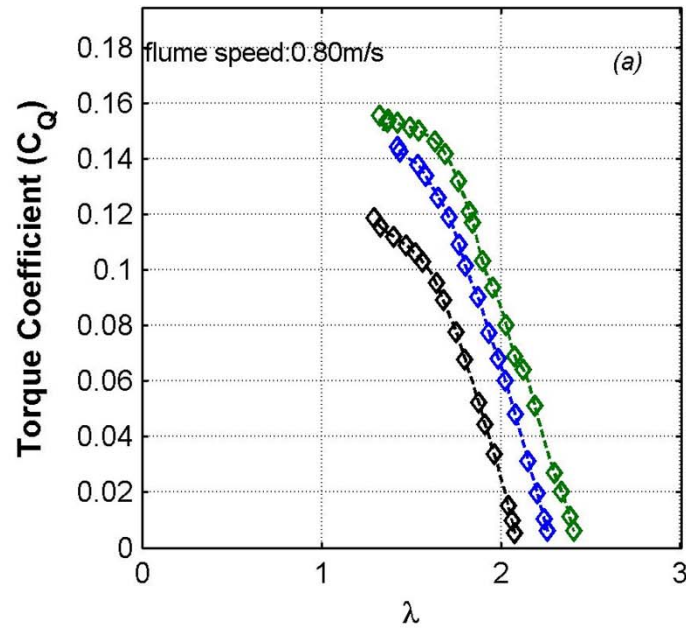
2 mm Spoke



4.8 mm Spoke



Circular Plate



# Shaft Diameter Comparison

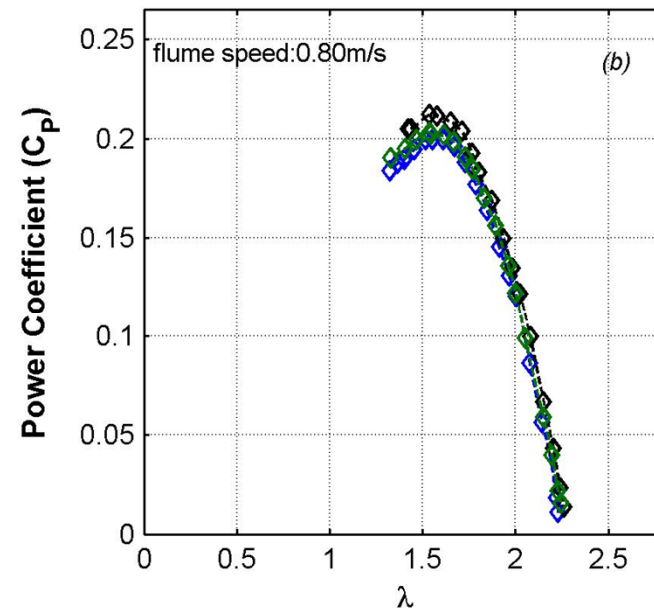
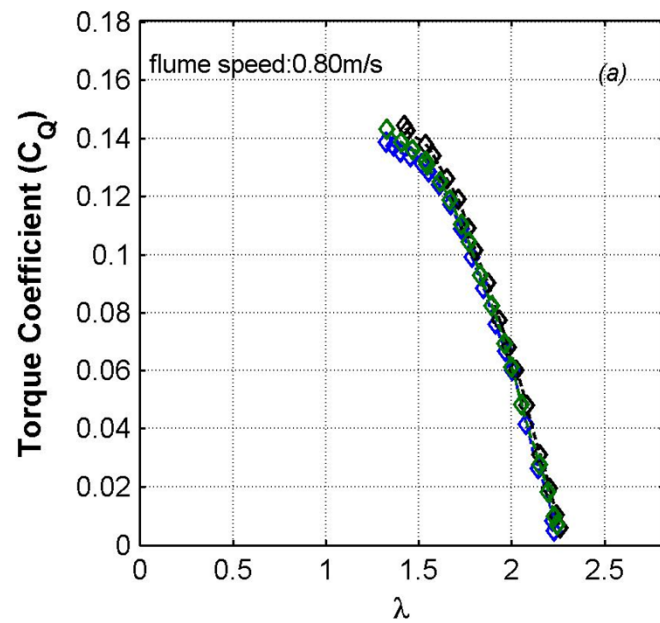
No Shaft



6.4 mm Shaft

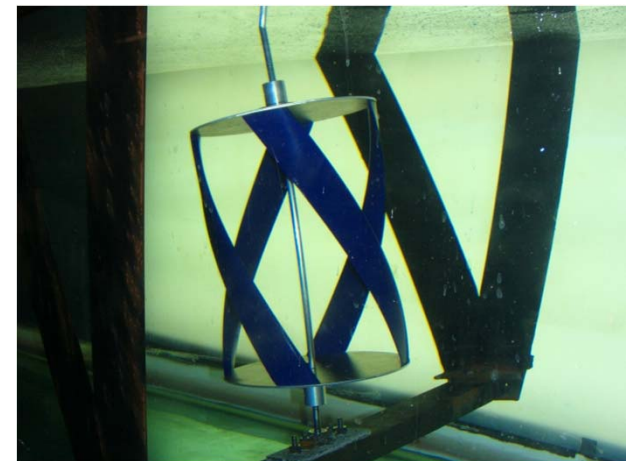
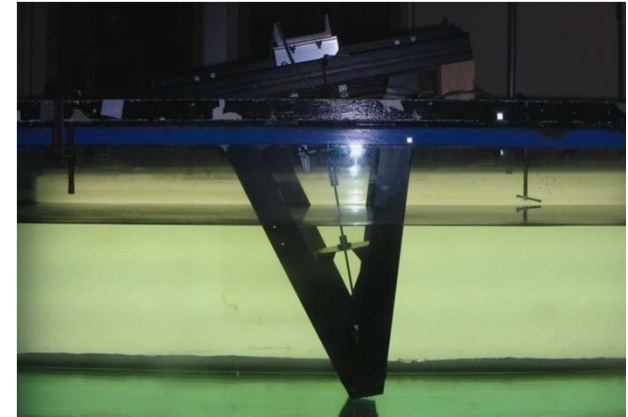


12.7 mm Shaft

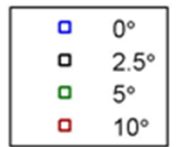
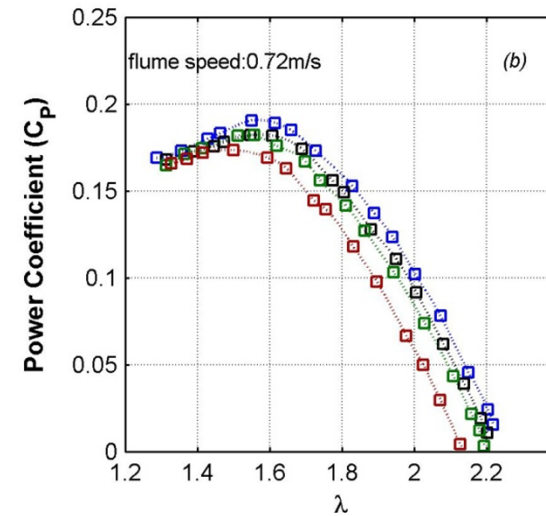
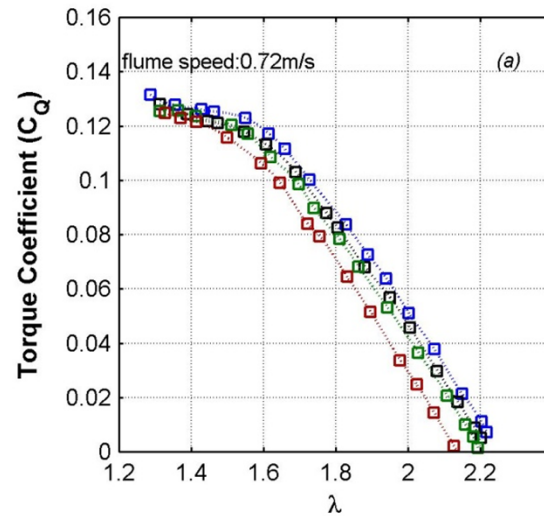
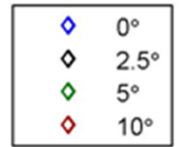
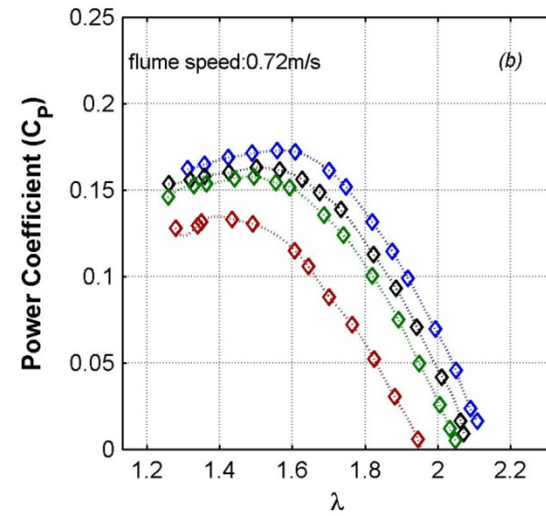
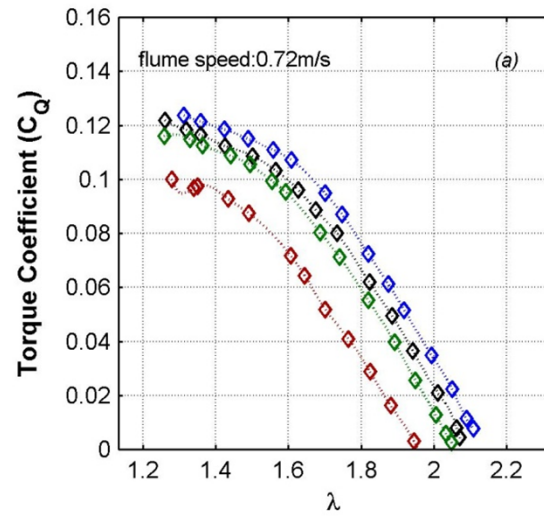


# Tilted Turbine Test

- Downstream end of turbine frame inclined
  - 0, 2.5, 5 and 10 degree inclinations
  - Performance test
  - Baseline spoke design and circular plate configurations



# Tilted Turbine Test



# Test Conclusions

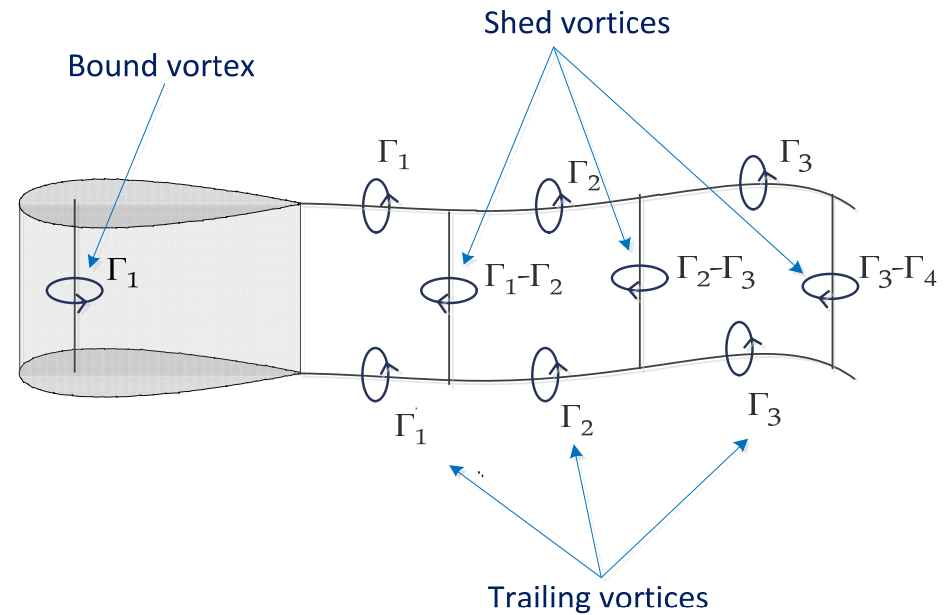
- **Four-bladed turbine** is best design for optimum aspect ratio and performance
- **Good self-start** and performance with **high solidity**
- **Maximize helical pitch angle** to get better max  $C_p$
- **No impact of shaft diameter** on performance
- **Circular plate** – best attachment design - eliminates profile drag, better tilt angle performance
- **Static torque is not uniform**
- **Higher inflow velocity** gives better  $C_p$

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# Vortex Model

- Free-vortex flow model
- Blade divided into several elements
- Blade element performance from  $C_L, C_D$  lookup tables
- Blade elements simulated as vortex generators
- Sum of velocity components gives  $u_{REL}$



$$\Gamma = \int_A (\nabla \times \vec{u}) \cdot d\vec{A}$$

$$\vec{u}_{RELi} = \vec{u}_{0i} + \overline{\omega R}_i + \sum_j (\vec{u}_{Bij} + \vec{u}_{Tij} + \vec{u}_{SHEDij})$$



# Vortex Model

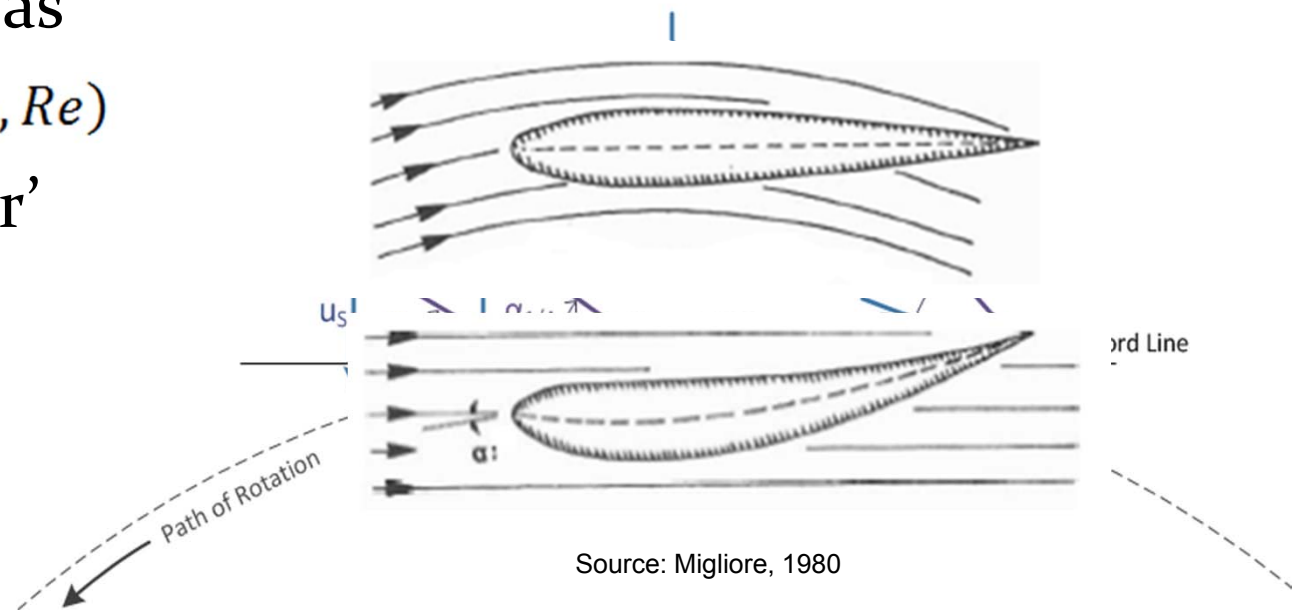
- Comparison to other model types -
  - Less computational time than CFD (days vs. minutes)
  - Compared to momentum models, the vortex model:
    - Can resolve helical geometry without yaw correction factors
    - Better resolves the wake
- Model Modifications
  - Helical geometry
  - Strut drag for circular plate and end spokes
  - Flow curvature effects
- Higher  $\sigma_{C-R}$  than previously modeled

# Flow Curvature

- Change in  $\alpha$  from leading edge to trailing edge
- Important for high chord-to-radius ratio turbines
- Approximated as

$$C_L = f(\alpha + \alpha_C, Re)$$

- ‘Virtual camber’



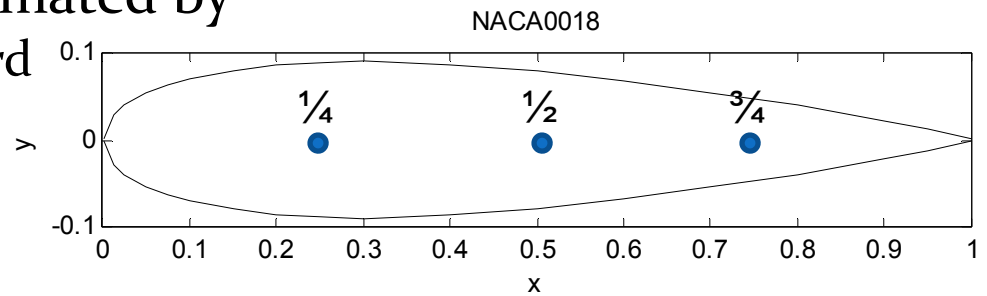
# Pitching and Dynamic Stall

- Lift, drag affected by pitch rate

- $C_{L,D} = f(\alpha, Re) \triangleright C_{L,D} = f(\alpha, \dot{\alpha}, Re)$

- ‘Virtual incidence’ – Approximated by

- $C_L$  calculated from  $\alpha$  at  $3/4$ -chord
    - $C_D$  calculated at  $\alpha$  at  $1/2$ -chord
    - Biases  $\alpha$  positive



- Dynamic stall

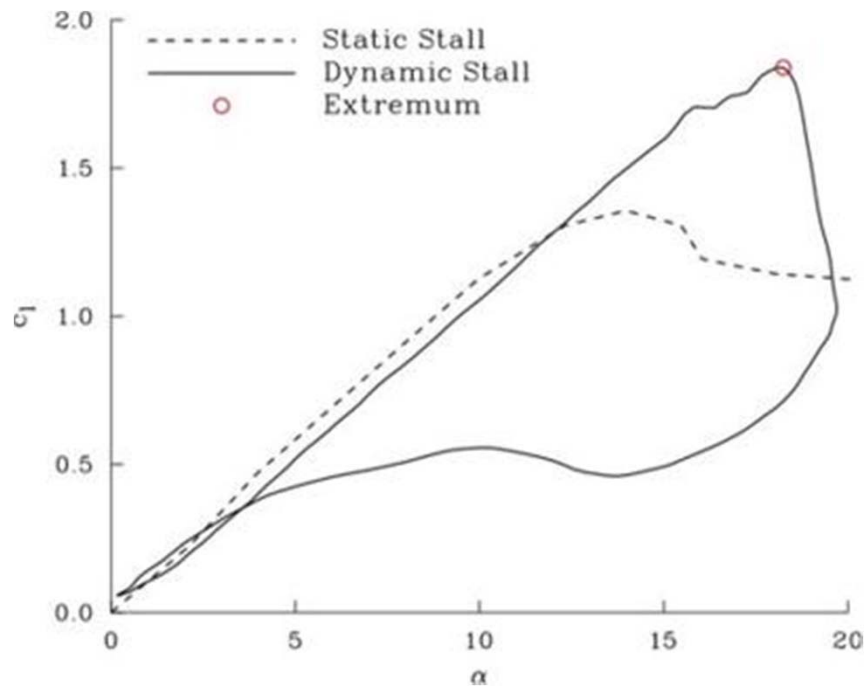
- Vortex forms at the leading edge of hydrofoil and tracks back along the chord
  - Lift is increased during tracking
  - Lift drops as vortex fully sheds



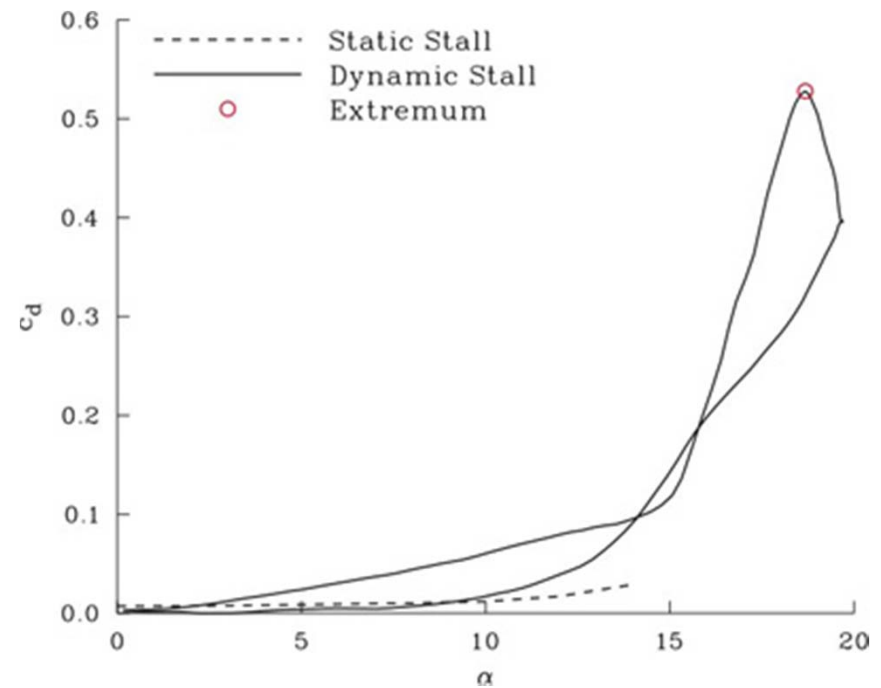
Source: Ekaterinaris & Platzer, 1997

# Dynamic Stall

Lift

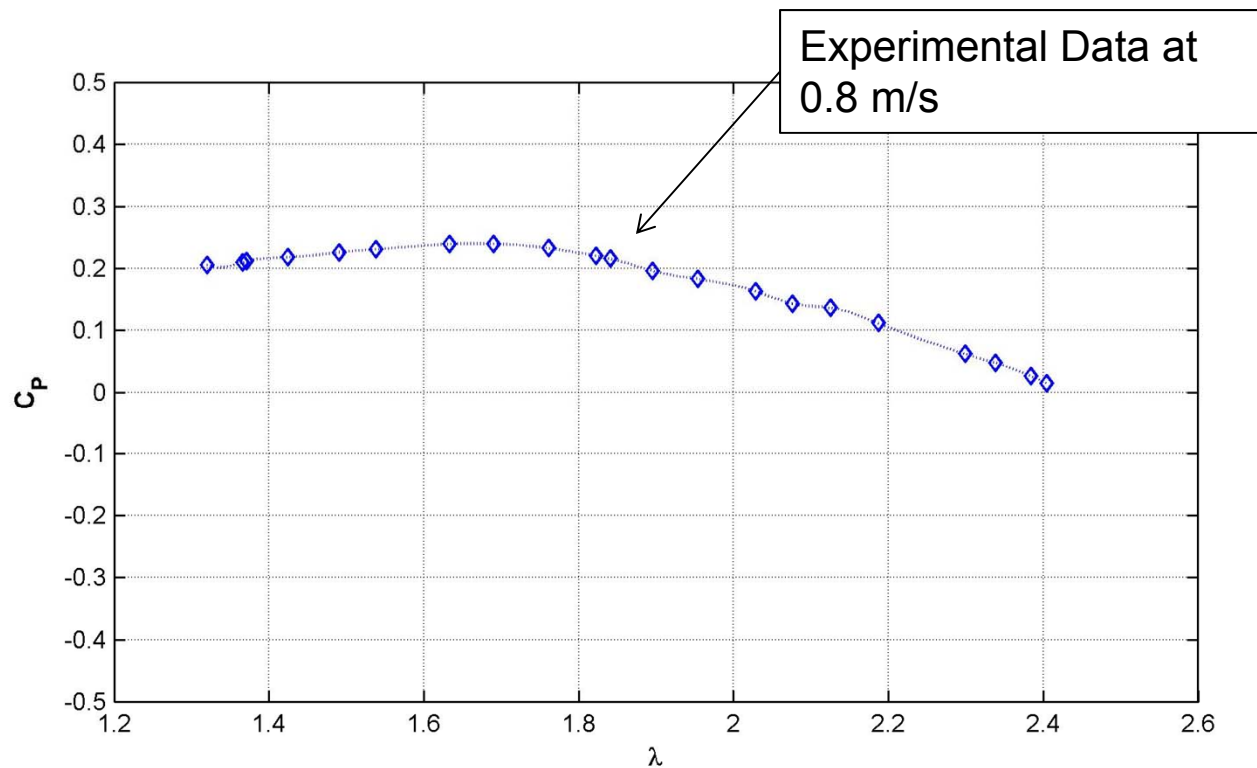


Drag

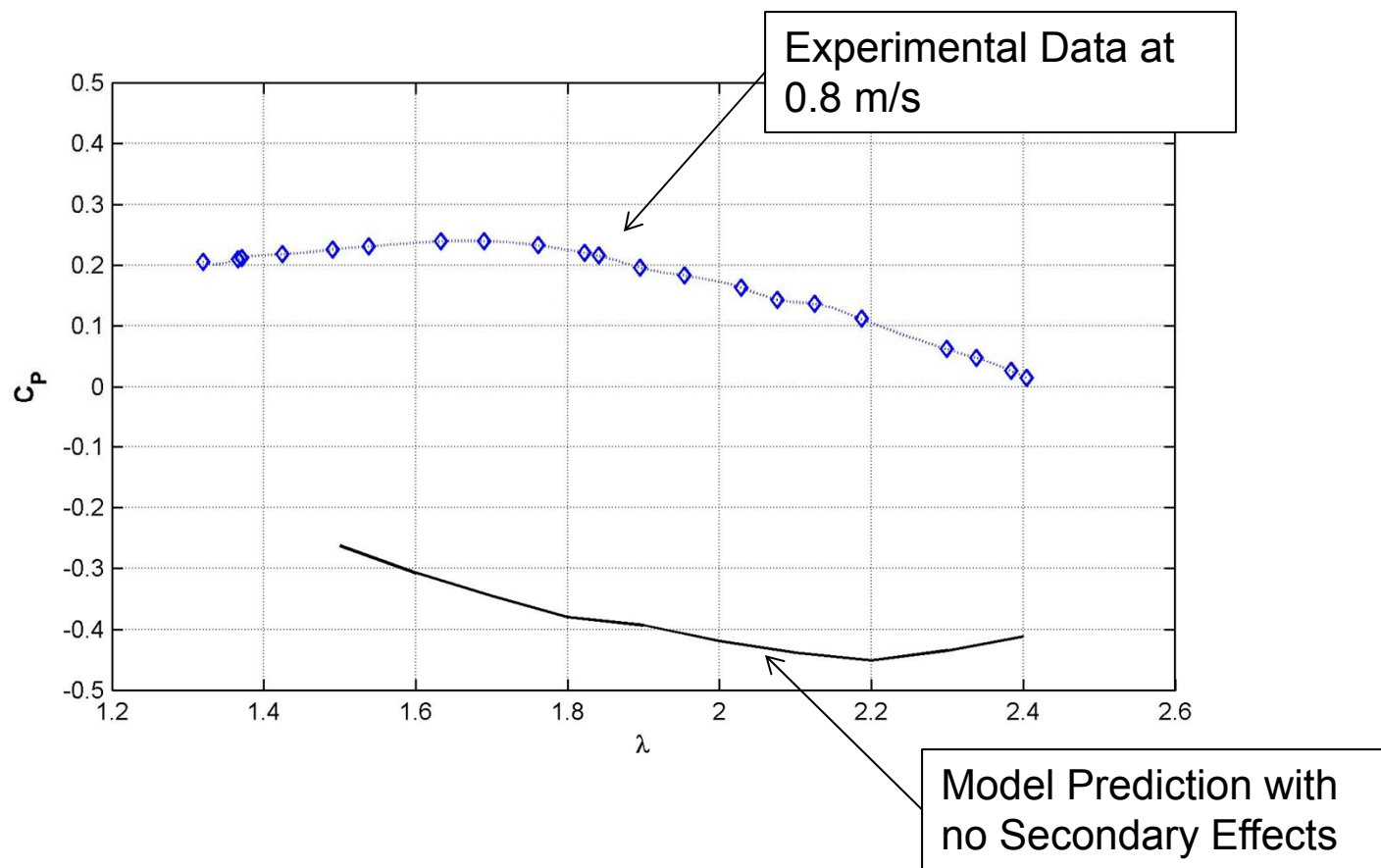


Source: NASA, 2009

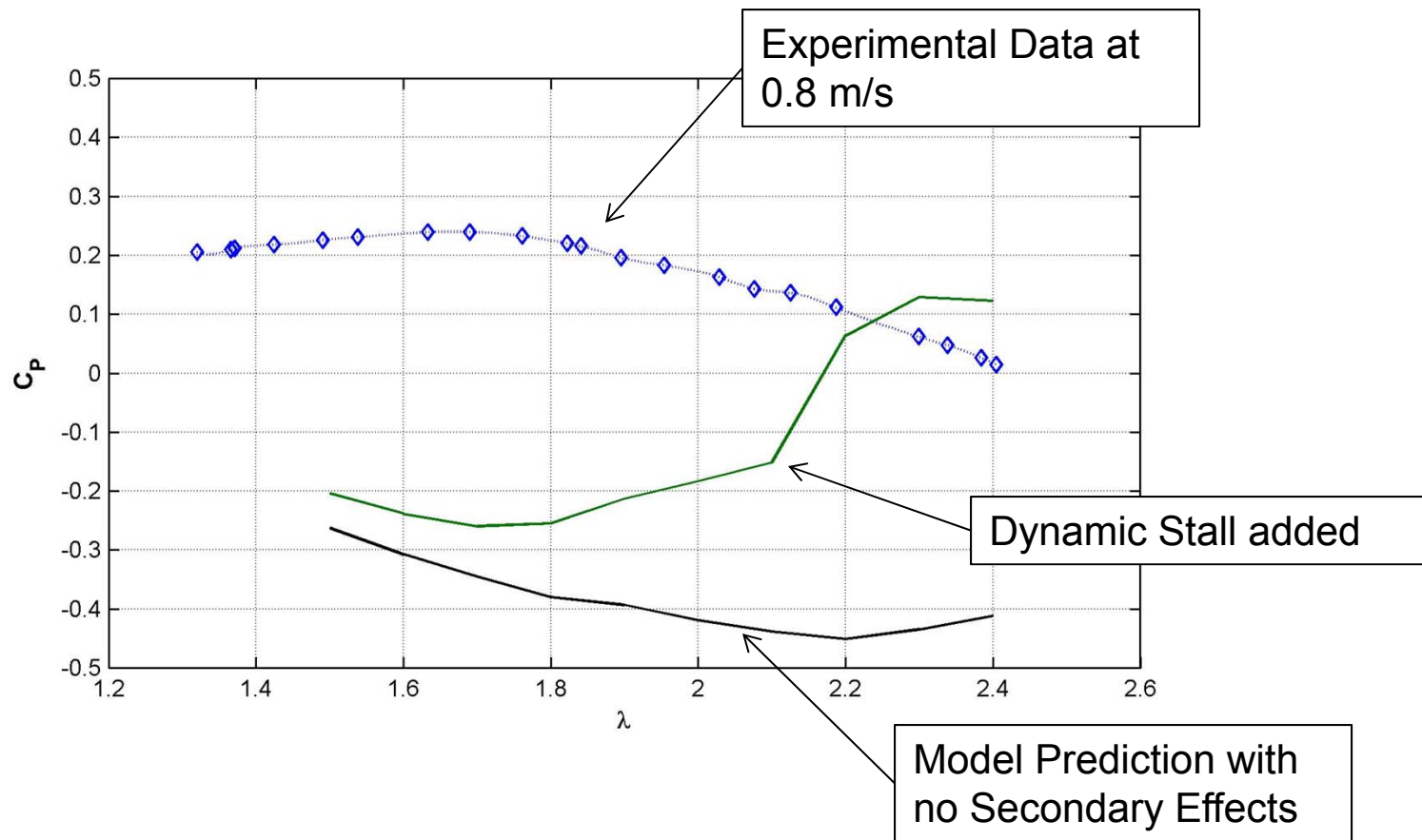
# Implementation of Secondary Effects



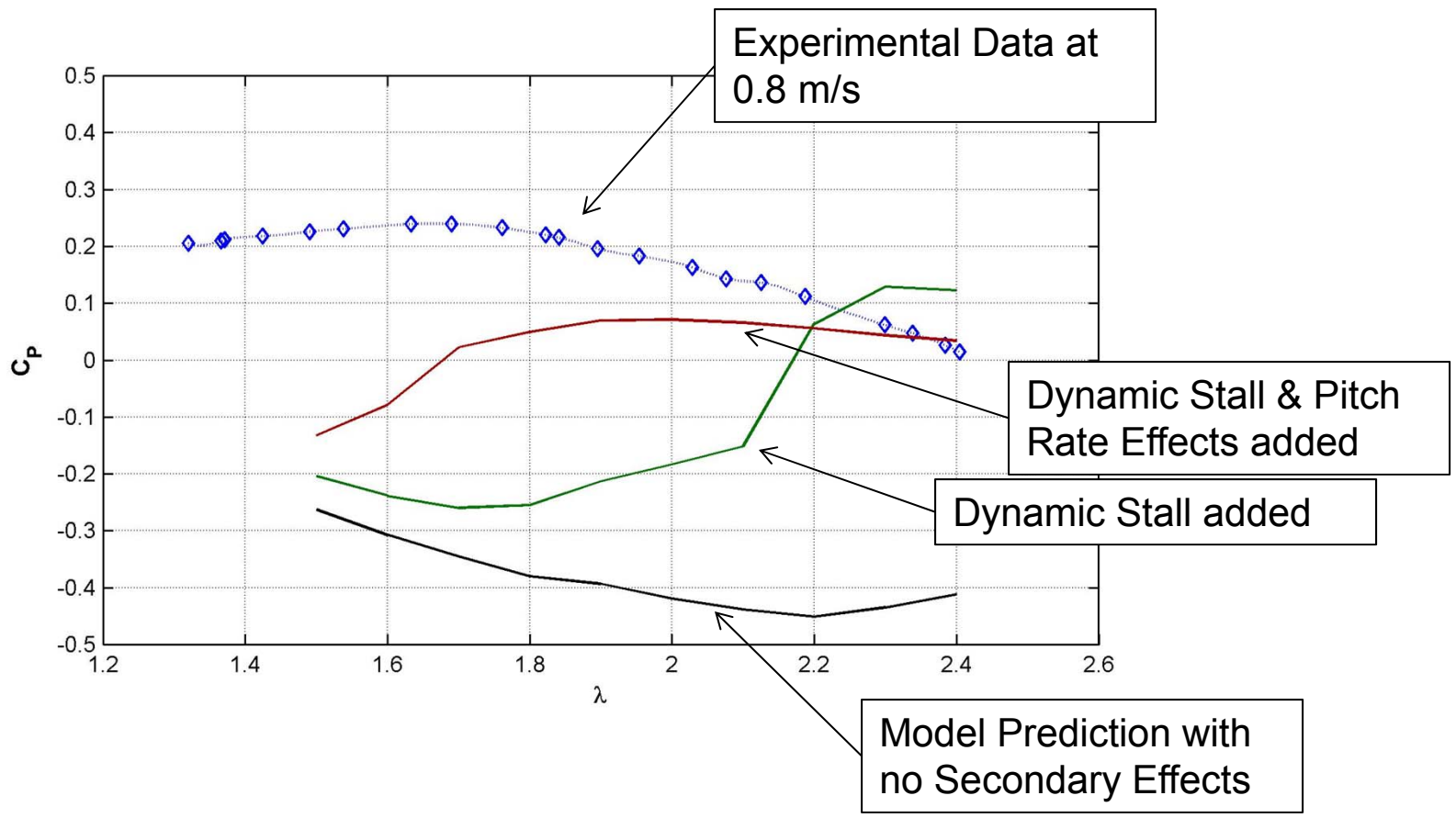
# Implementation of Secondary Effects



# Implementation of Secondary Effects

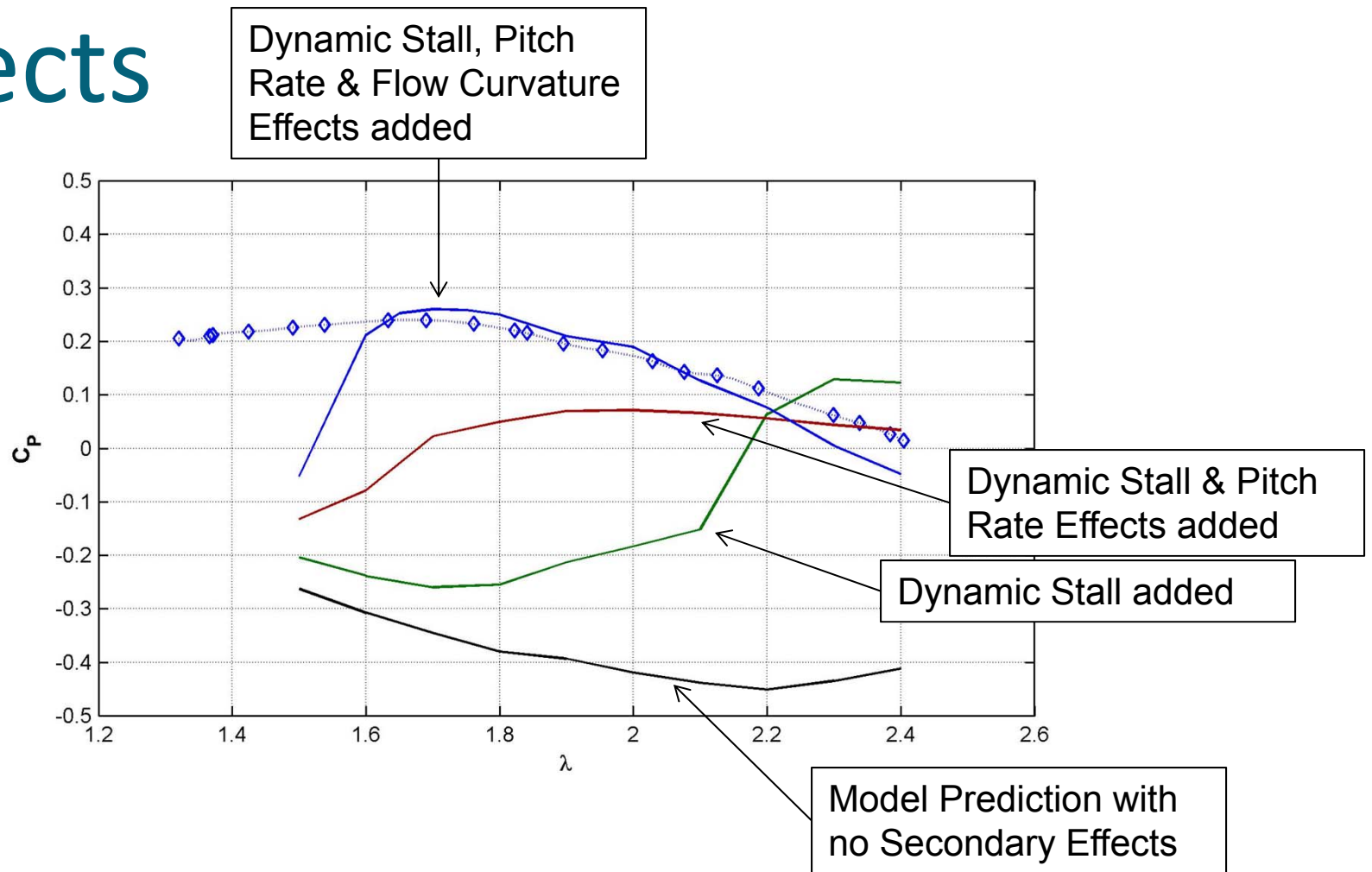


# Implementation of Secondary Effects

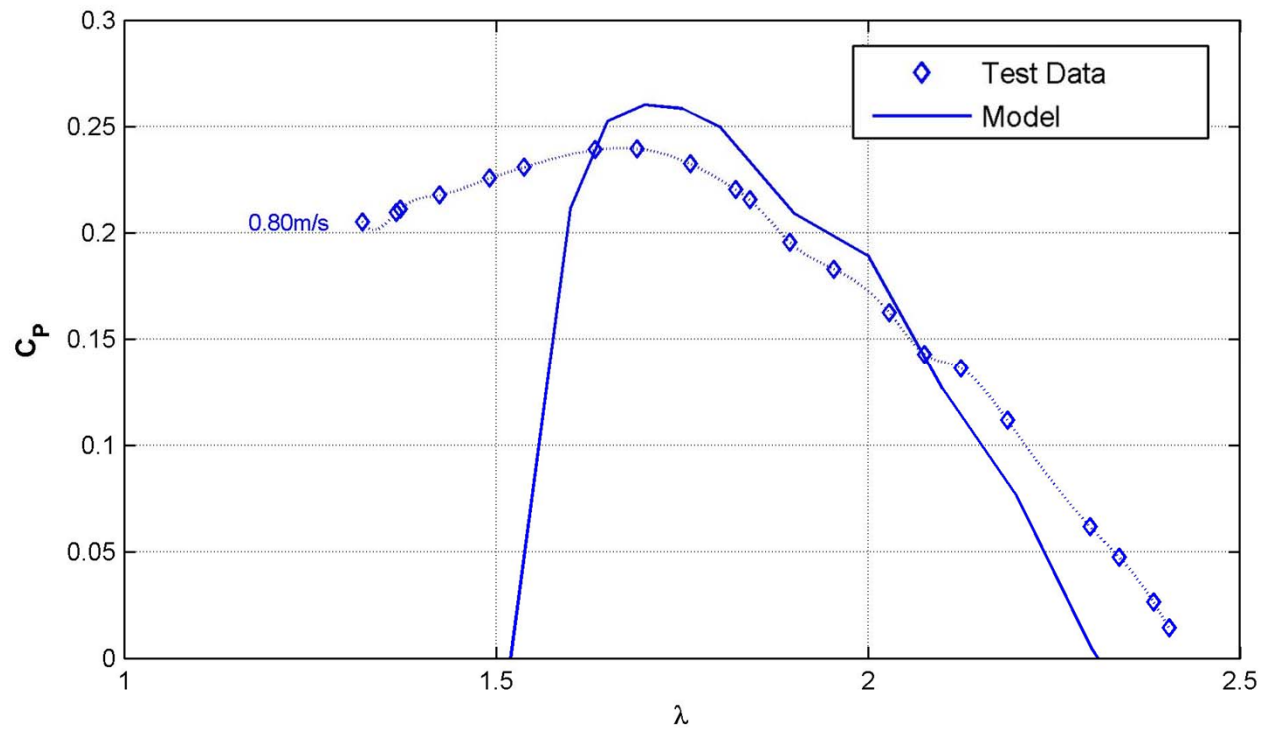




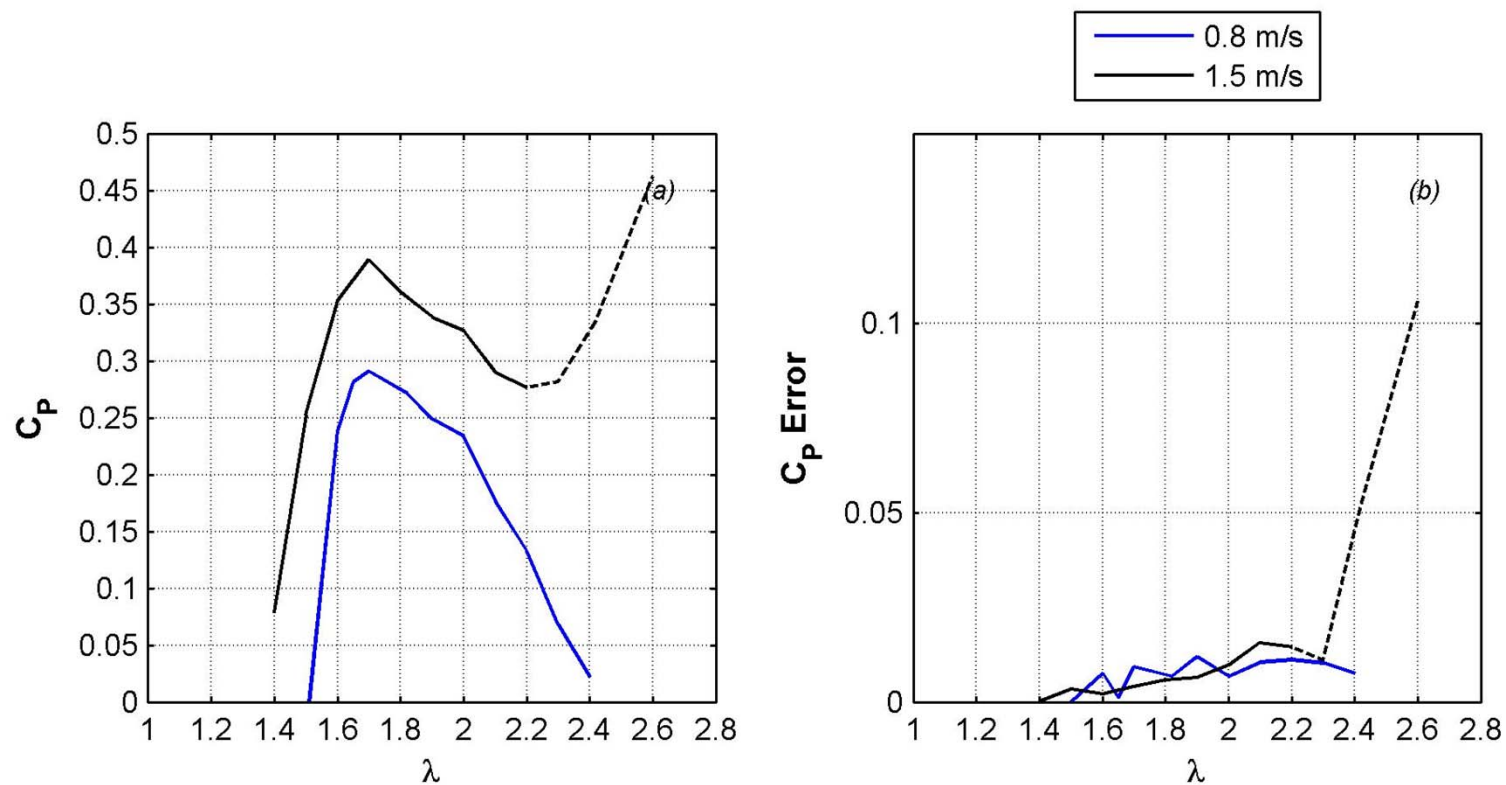
# Implementation of Secondary Effects



# Model Prediction of Inflow Variation



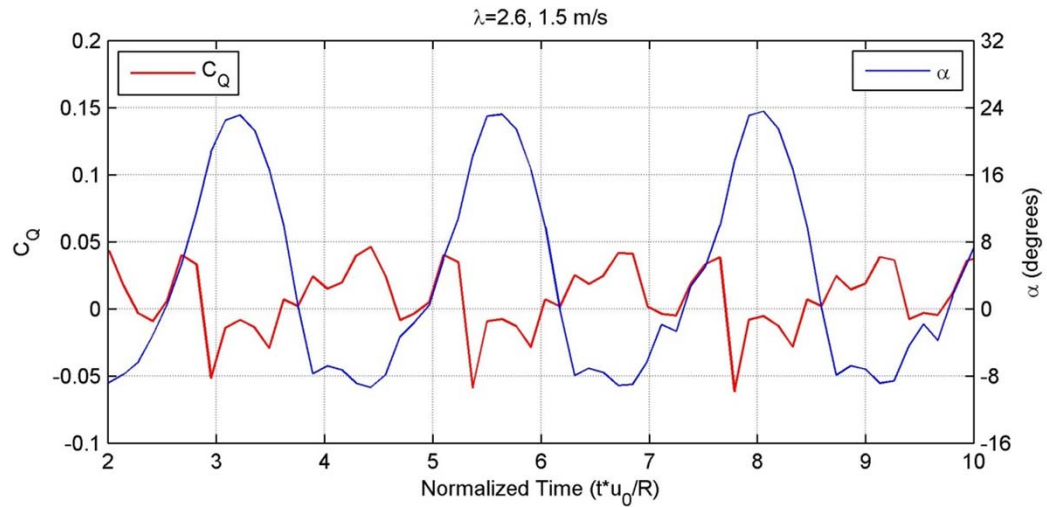
# Extrapolation to 1.5 m/s



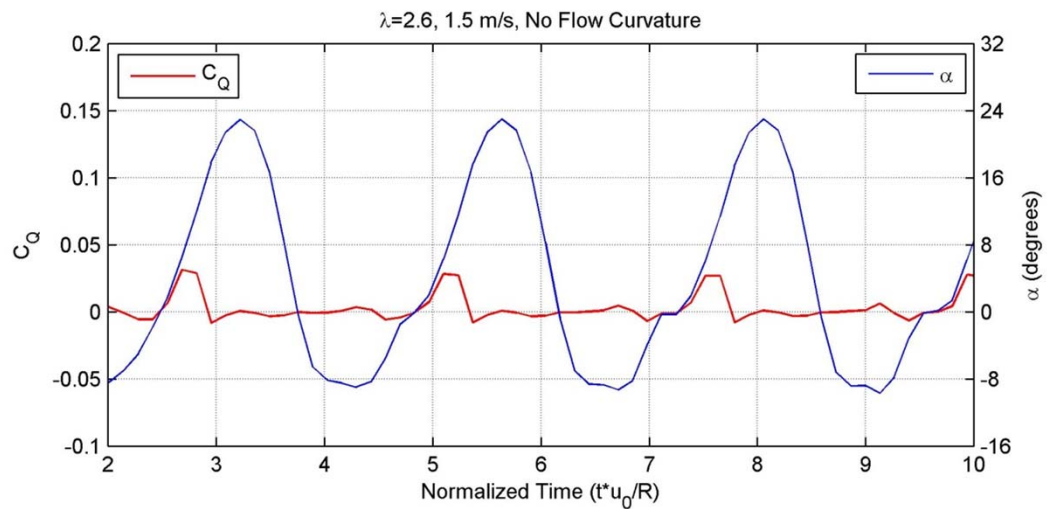
$$C_{P\text{ERROR}} = \max(C_{P(N-4):N}) - \min(C_{P(N-4):N})$$

# Analysis of High Tip Speed Ratio

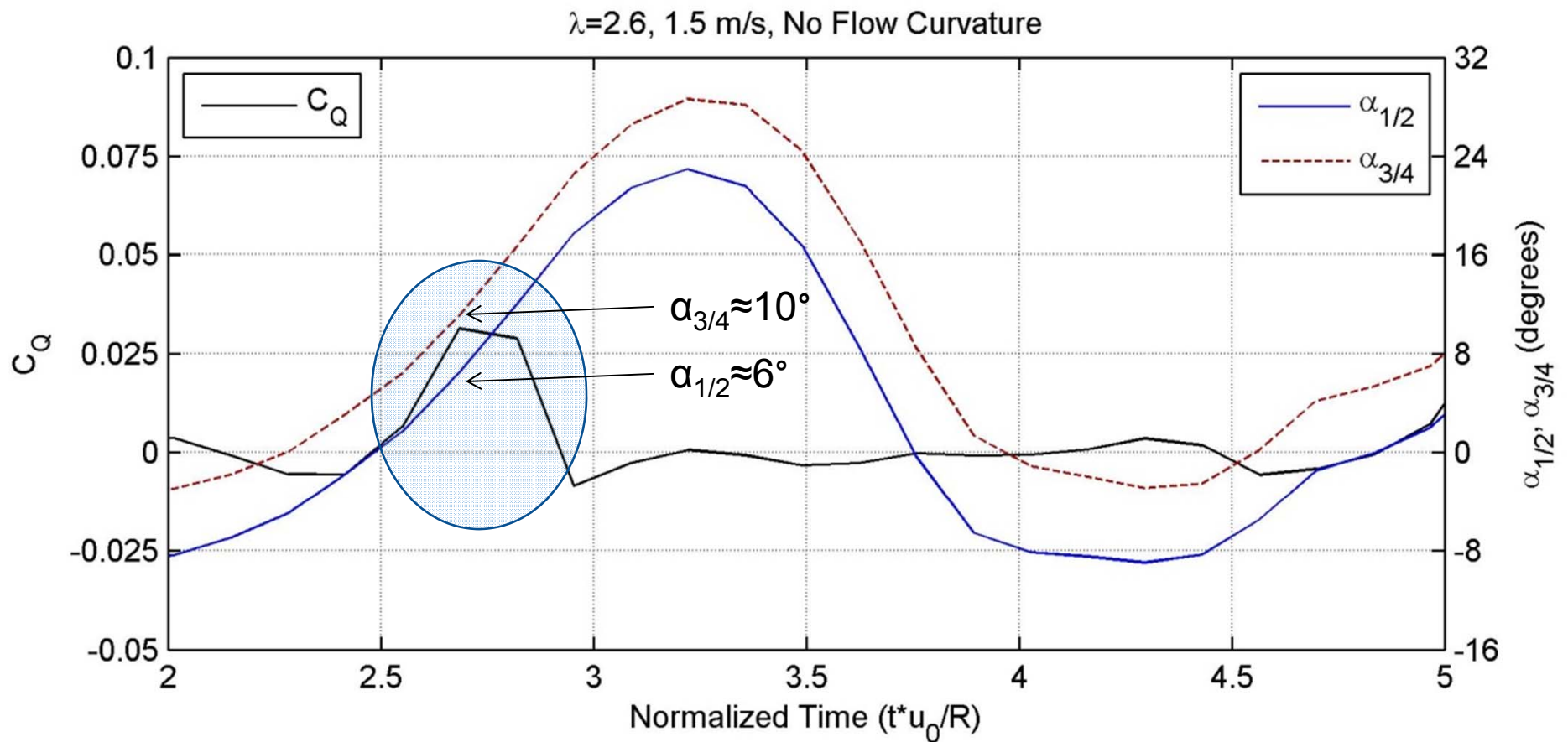
Flow curvature active  
 $C_p=0.46$



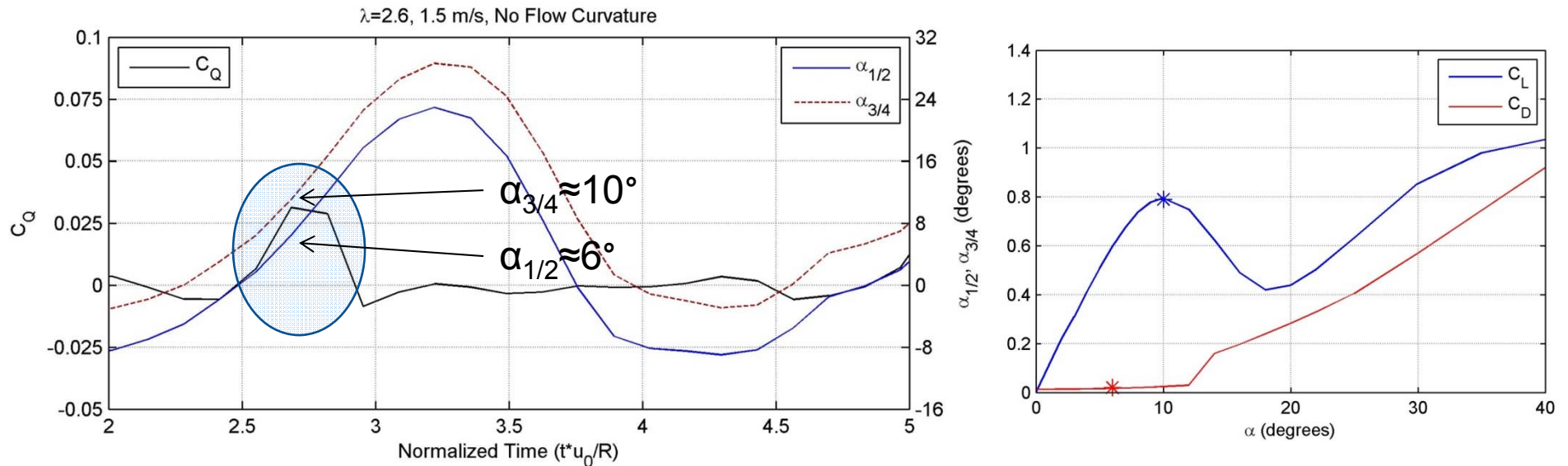
Flow curvature disabled  
 $C_p=0.43$



# Analysis of High Tip Speed Ratio



# Analysis of High Tip Speed Ratio



Due to high chord-to-radius ratio:

- Flow curvature adds instability to model output
- Pitch rate effects and dynamic stall over-amplifying performance
  - This effect has been documented in previous literature (Dai *et al.*, 2011), (Cardona, 1984)

# Project Conclusions

- Demonstrated need for and viability of a tidal micropower system
- Designed a turbine for a micropower system
  - 24% efficient at 0.8 m/s (and increasing with flume speed)
  - Good self-start and flow orientation capabilities
  - Optimized select helical turbine design parameters
- Modified a vortex model to accept helical turbine design
  - Demonstrated importance of secondary effects
  - Demonstrated general trends in performance
  - Need for further study of secondary effects for high chord-to-radius ratio turbines

# Future Work

- Testing of the full-scale turbine
- Design of system drive train and control
  - Turbine-Generator matching
  - Selection of battery type
- Improving turbine efficiency
- Improved turbine modeling
  - Examination and refinement of secondary effects
  - Other model types (CFD, etc.)



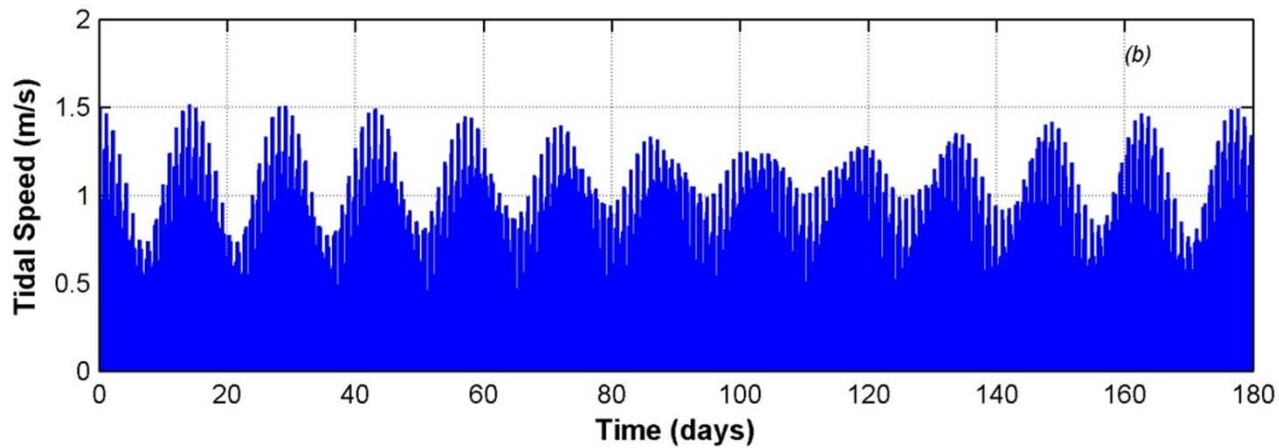
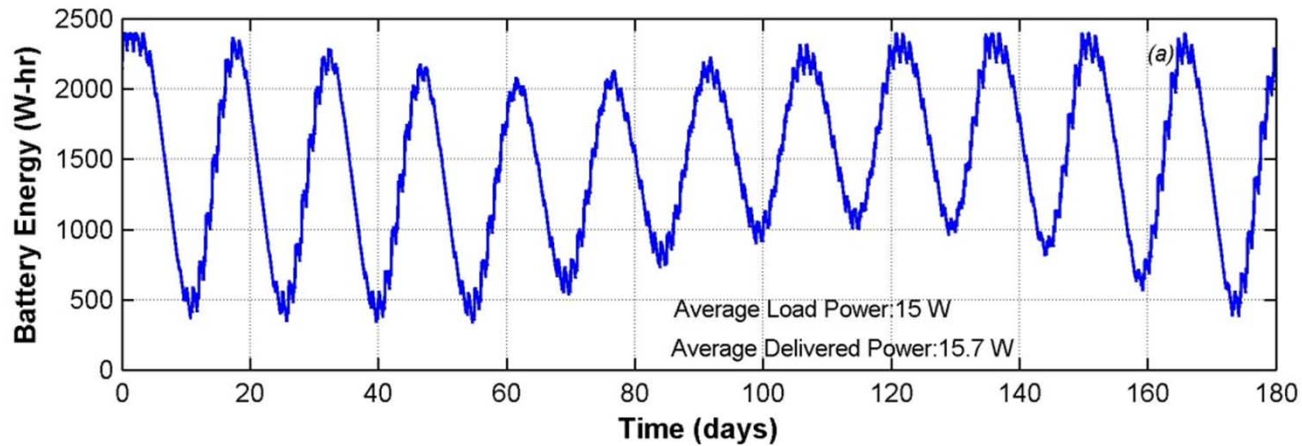
# Acknowledgements

- Dr. Brian Polagye
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- Dr. Philip Malte
- Bill Kuykendall
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- Capstone Design Team:  
Nick Stelzenmuller,  
Bronwyn Hughes, Josh  
Anderson, Celest Johnson,  
Brett Taylor, Leo Sutanto
- NNMREC Organization
- Sandia National Labs
- Marie

Questions?

# Backup Slides

# System Performance



# System Performance

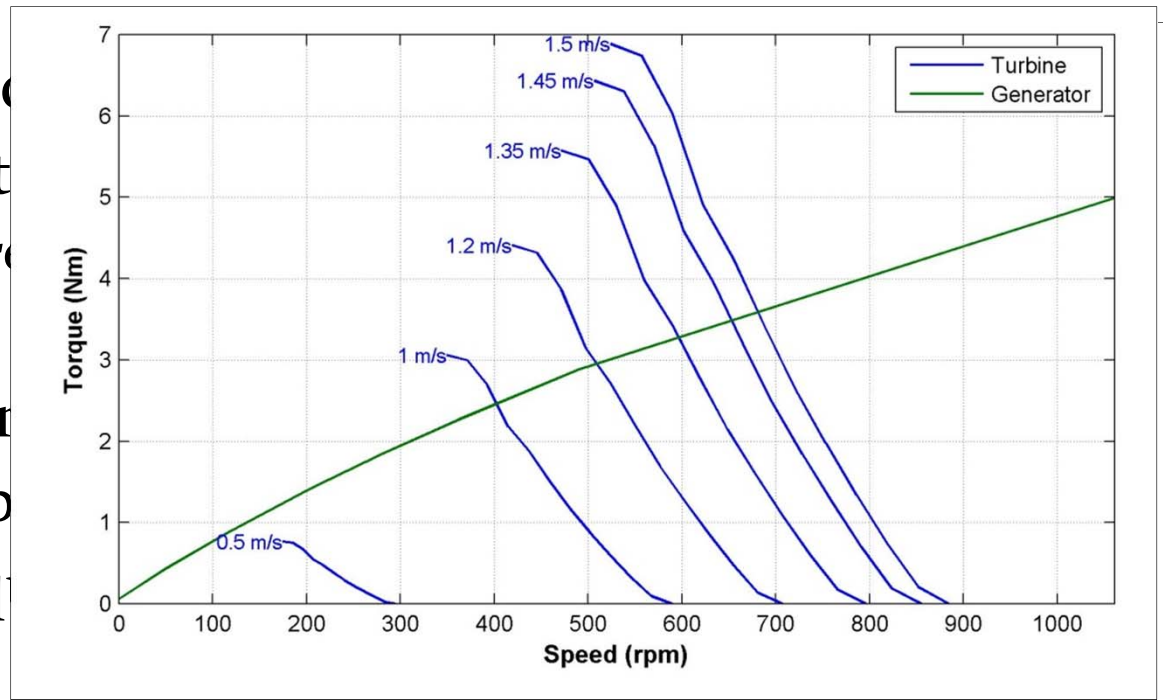
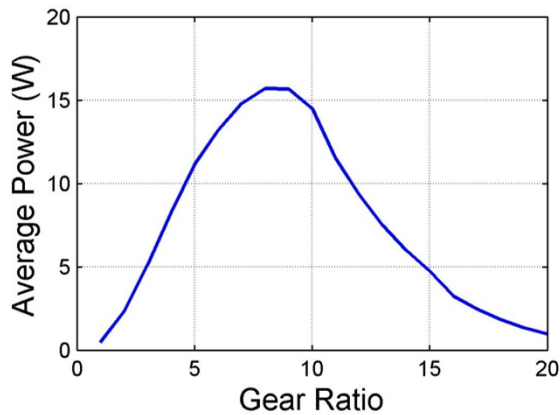
- Turbine-generator matching

$$\omega_{GEN} = G \omega_{TURB}$$

$$T_{GEN} = \frac{\eta}{G} T_{TURB}$$

- Desired generator characteristics

- Low speed operation



# Battery Information

Characteristic	Sealed Lead-Acid	NiMH	NiCd	Lithium-Ion
Energy Density (Wh/L) <sup>[1]</sup>	Low 50-90	High 430	Low 15-100	High 570
Cycle Life (cycles) <sup>[1]</sup>	Low 200-500	Medium 300-1000	High 1500	High 1000+
Memory Effect <sup>[4]</sup>	Low No effect	Medium Less than NiCd	High Periodic discharge required	Low No effect
Charging Time <sup>[2]</sup>	High 8-16 hours	Medium 2-4 hours	Low about 1 hour	Medium 1-4 hours
Cost <sup>[3]</sup> (\$/kWh)	Low \$8.50	Medium \$18.50	Low-Medium \$11.00	High \$24
Toxicity <sup>[2]</sup>	Very high	Low	Very high	Low
Transportation Limitations <sup>[4]</sup>	No	No	No	Yes
Comments <sup>[1,4]</sup>	Slow charging time; low energy density	Generates heat during high rate charge or discharge; tolerant of overcharge overdischarge	Rugged; lowest cost per cycle; some limitations on use due to toxicity	Transportation limitations; requires complex circuit to operate safely; limited availability for high-power applications
Data Source:				
[1] Reddy & Linden (2011) [2] Buchmann (2010d) [3] Buchmann (2011b) [4] Buchmann (2010e)				

# Helical Turbines



- Lift-style device
- Advantages:
  - Easily self-starts
  - Fairly high efficiency (~35%)
  - No torque oscillation
- Disadvantages:
  - Difficult to manufacture
  - Less efficient than straight-blade turbine

# Blade Element Theory

- Lift - Drag Force

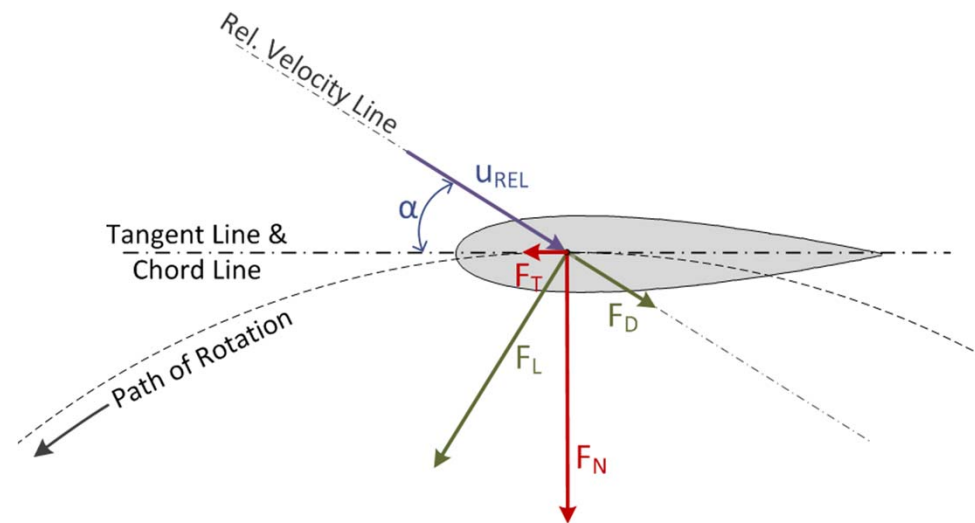
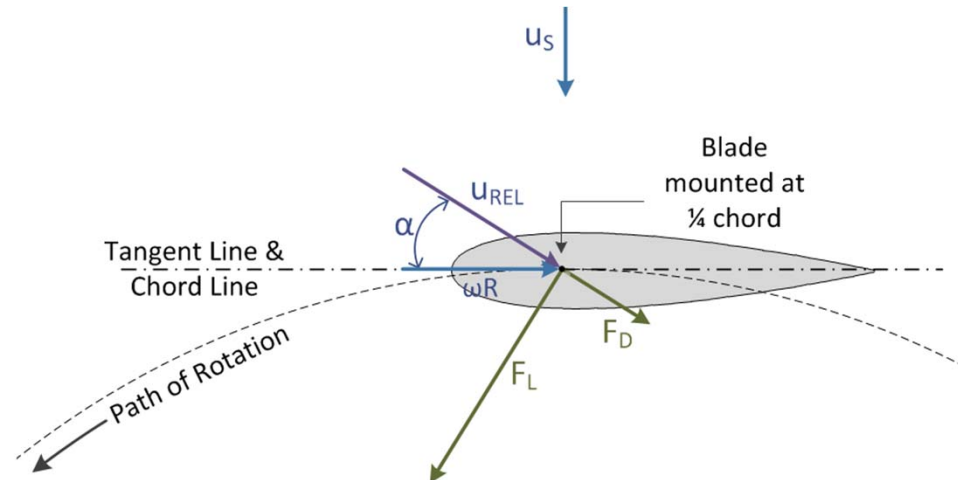
$$C_L = \frac{F_L}{\frac{1}{2}\rho u_{REL}^2 A_P} = \frac{F_L}{\frac{1}{2}\rho u_{REL}^2 ch}$$

$$C_D = \frac{F_D}{\frac{1}{2}\rho u_{REL}^2 A_P} = \frac{F_D}{\frac{1}{2}\rho u_{REL}^2 ch}$$

- Normal - Tangential Force

$$C_T = \frac{F_T}{\frac{1}{2}\rho u_{REL}^2 A_P} = \frac{F_T}{\frac{1}{2}\rho u_{REL}^2 ch}$$

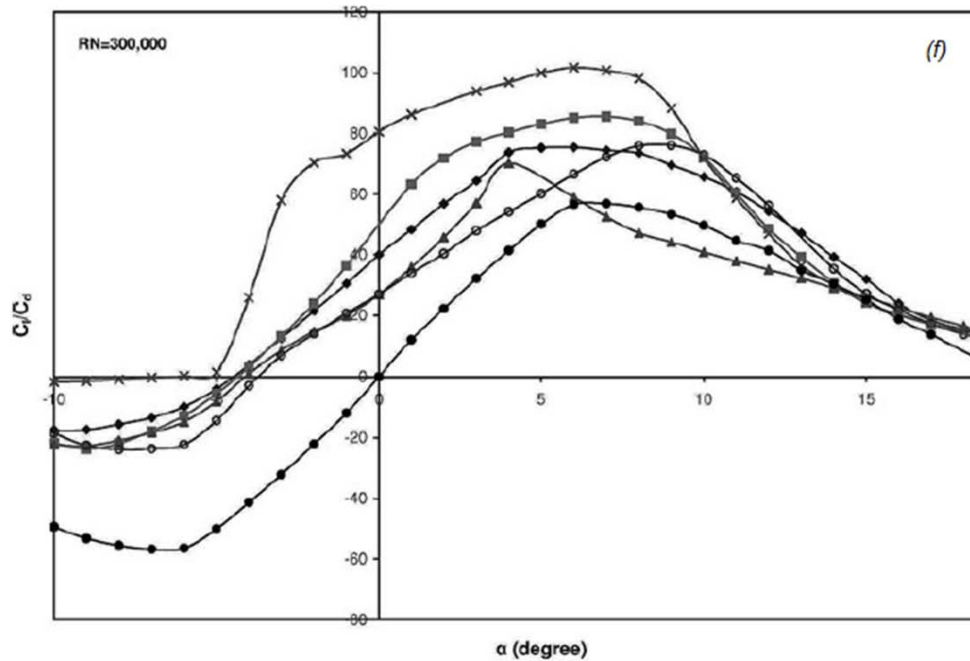
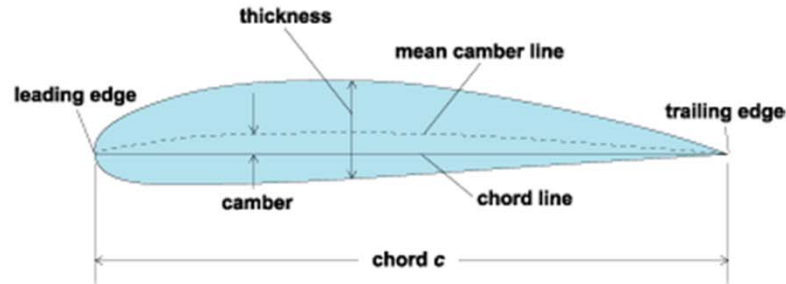
$$C_N = \frac{F_N}{\frac{1}{2}\rho u_{REL}^2 A_P} = \frac{F_N}{\frac{1}{2}\rho u_{REL}^2 ch}$$



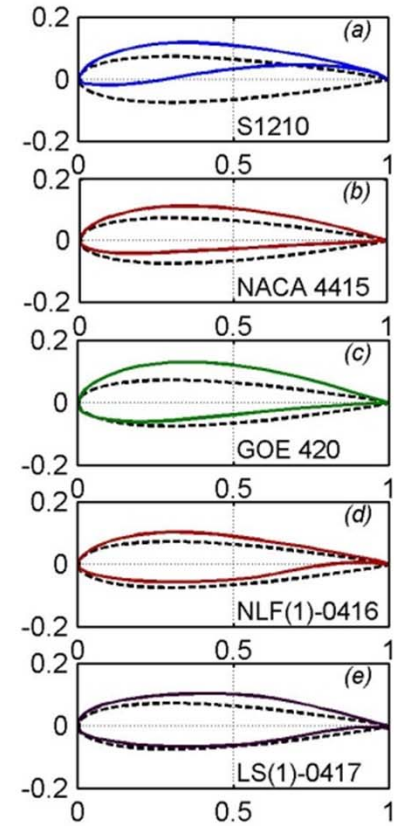


# Helical Turbine Parameters

- Blade Profile

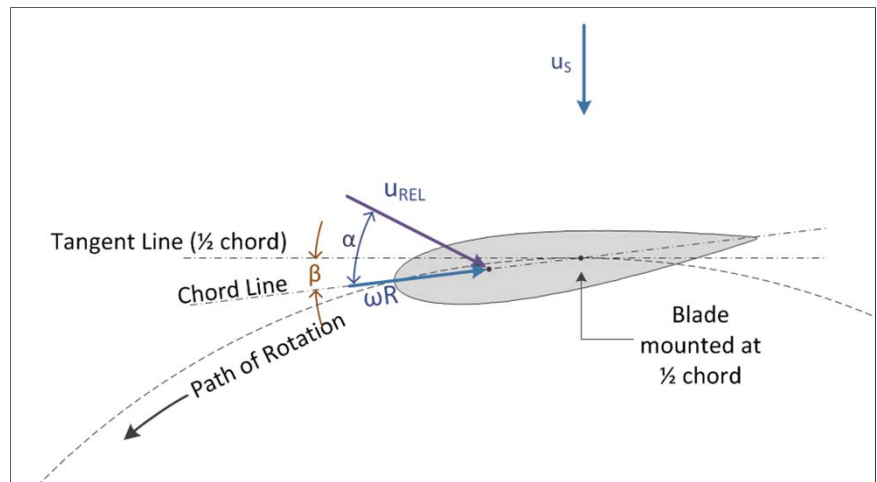
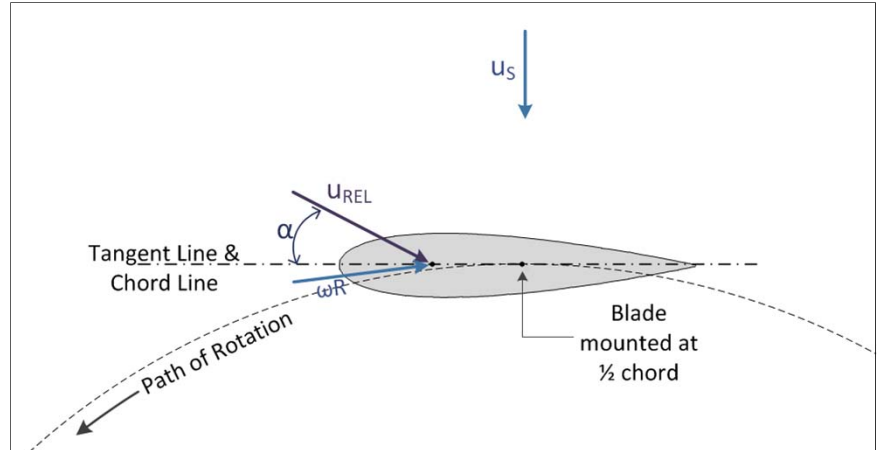


◆ GOE 420    ■ NACA 4415    ▲ NASA LS(1)-0417    ◊ NASA NLF(1)-0416    × S1210    ● NACA 0015



# Helical Turbine Parameters

- Blade Profile
- Blade Pitch
- Helical Pitch
- Aspect Ratio
- Solidity Ratio/ Chord-to-Radius Ratio
- Number of Blades
- Blade Wrap
- Strut/shaft Design

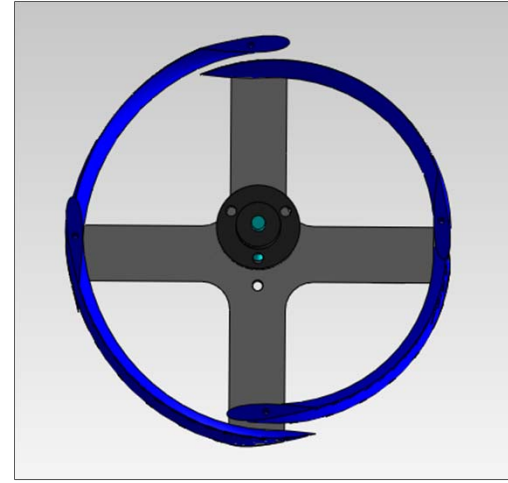


# Helical Turbine Parameters

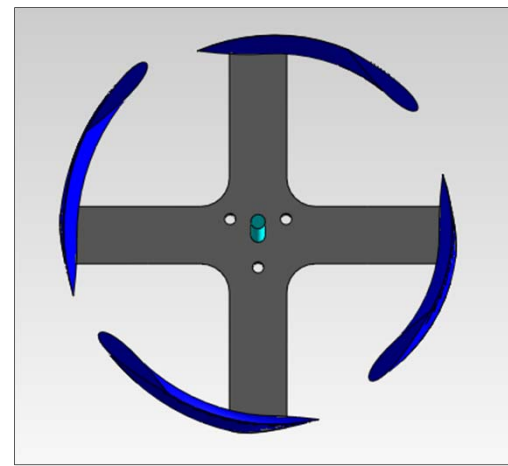
- Blade Profile
- Blade Pitch
- Helical Pitch
- Aspect Ratio
- Solidity Ratio/ Chord-to-Radius Ratio
- Number of Blades
- Blade Wrap
- Strut/shaft Design

$$\varpi = \frac{BH}{\pi D \tan \delta}$$

100% Wrap



50% Wrap



# Test Matrix

Turbine Geometry			Performance test					Static Torque		Dynamic Torque		Startup - Shutdown	Tilted Turbine		Wake Measurement
Flume Velocity (m/s):			0.8	0.7	0.6	0.5	0.4	0.7		0.78	0.7	0-0.7	0.72		0.78
Turbine Design	Strut Design	Shaft Diameter (mm)	--	--	--	--	--	Single blade length	Full Revolution	--	--	--	Load Performance	Static Torque	--
Single Blade, from 4-bladed turbine	Circular Plates	6.4		X					X	X					
4-bladed turbine	4.8 mm, 4-leg spokes	6.4	X	X					X						
	2.0 mm, 4-leg spokes	No shaft	X	X				X							
		6.4	X	X	X	X	X		X	X		X	X	X	X
		12.7	X	X					X						
	Circular Plates	6.4	X	X	X	X	X	X					X	X	
3-bladed turbine	4.8 mm, 3-leg spokes	6.4	X	X	X	X		X		X					
Low Solidity Turbine	4.8 mm, 3-leg spokes	6.4	X	X											

X: Test Completed

# Test Calculations

$$\varepsilon = \frac{A_{TURBINE}}{A_F} = \frac{DH}{w_F(h_0)}$$

$$Fr = \frac{u_0}{\sqrt{gh_0}}$$

$$Re_M = \frac{\rho u_0 D}{\mu}$$

$$Re_B = \frac{\rho u_{REL} C}{\mu}$$

$$C_Q = \frac{T}{\frac{1}{4} \rho D^2 H u_0^2}$$

$$C_{QS} = \frac{T}{\frac{1}{4} \rho D^2 H u_0^2}$$

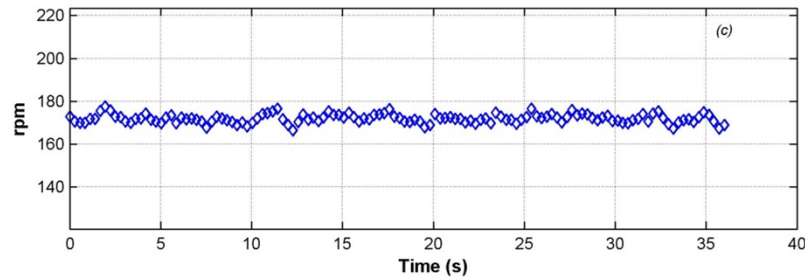
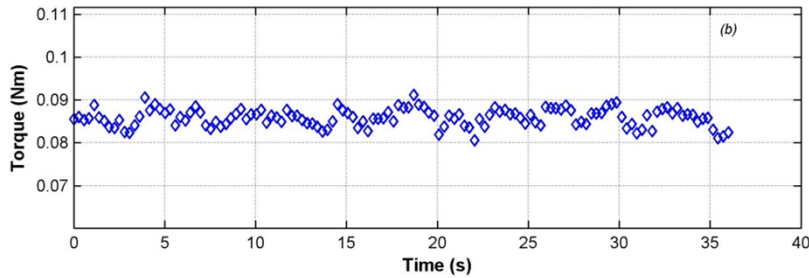
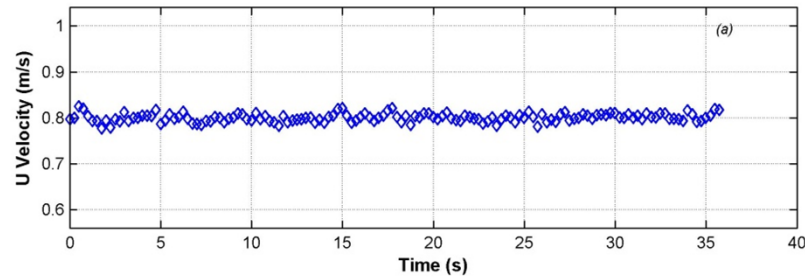
$$C_P = \frac{T \omega}{\frac{1}{2} \rho D H u_0^3}$$

$$\lambda = \frac{\omega R}{u_0}$$

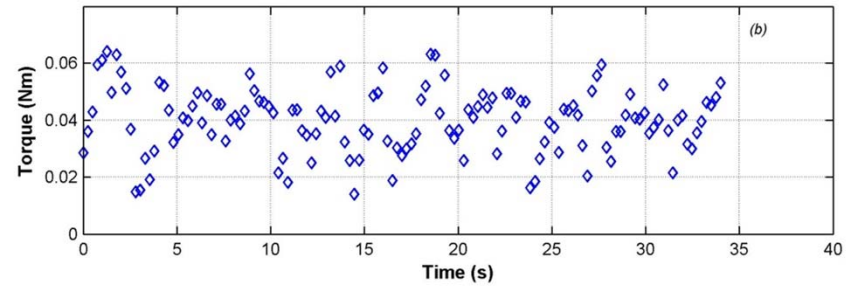
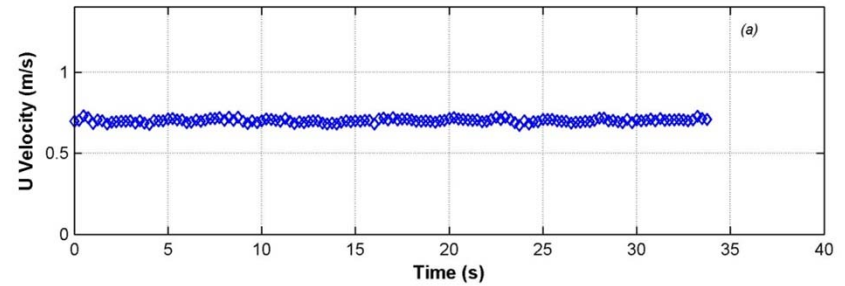
Parameter	Symbol	Data Range	
Flume Speed (m/s)	$u_0$	0.4	0.8
Static Water Height (cm)	$h_{sw}$	50	46
Dynamic Water Height (cm)	$h_0$	49	42
Water Column Cross-Sectional Area (m <sup>2</sup> )	$A_F$	0.371	0.318
Turbine Cross-Sectional Area (m <sup>2</sup> )	$A_C$	0.040	0.040
Blockage Ratio	$\varepsilon$	0.108	0.126
Froude Number	$Fr$	0.182	0.394
Max Blade Reynold's Number (4-Bladed Turbine)	$Re_B$	4.0E+04	9.5E+04
Min Blade Reynold's Number (4-Bladed Turbine)	$Re_B$	3.6E+03	2.2E+04
Machine Reynold's Number (4-Bladed Turbine)	$Re_M$	7.7E+04	1.5E+05

# Typical Test Results

## Load Performance Test

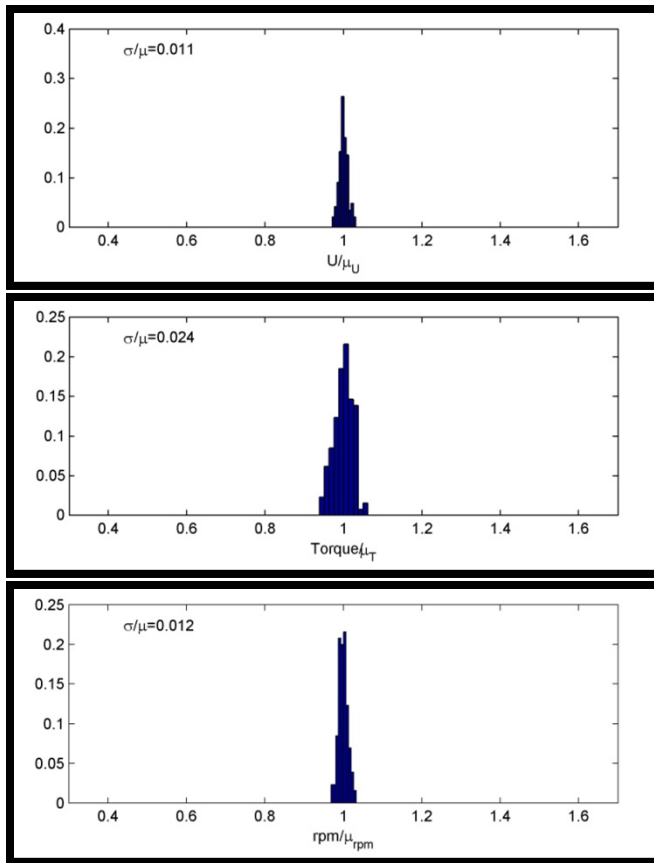


## Static Torque Test

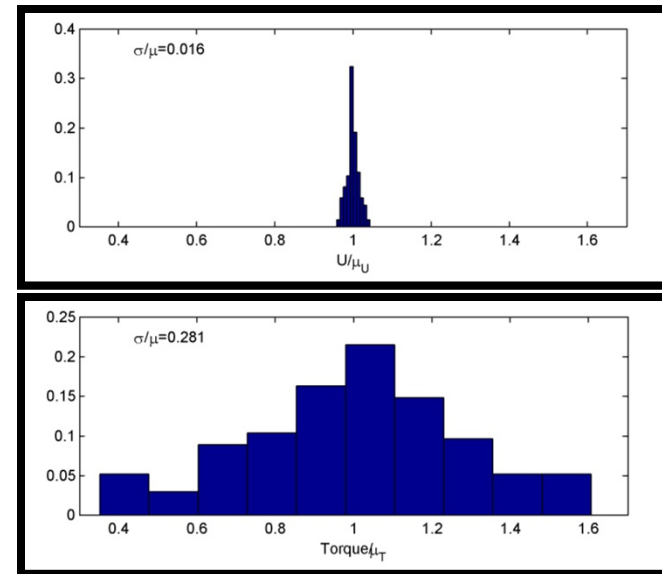


# Typical Test Results

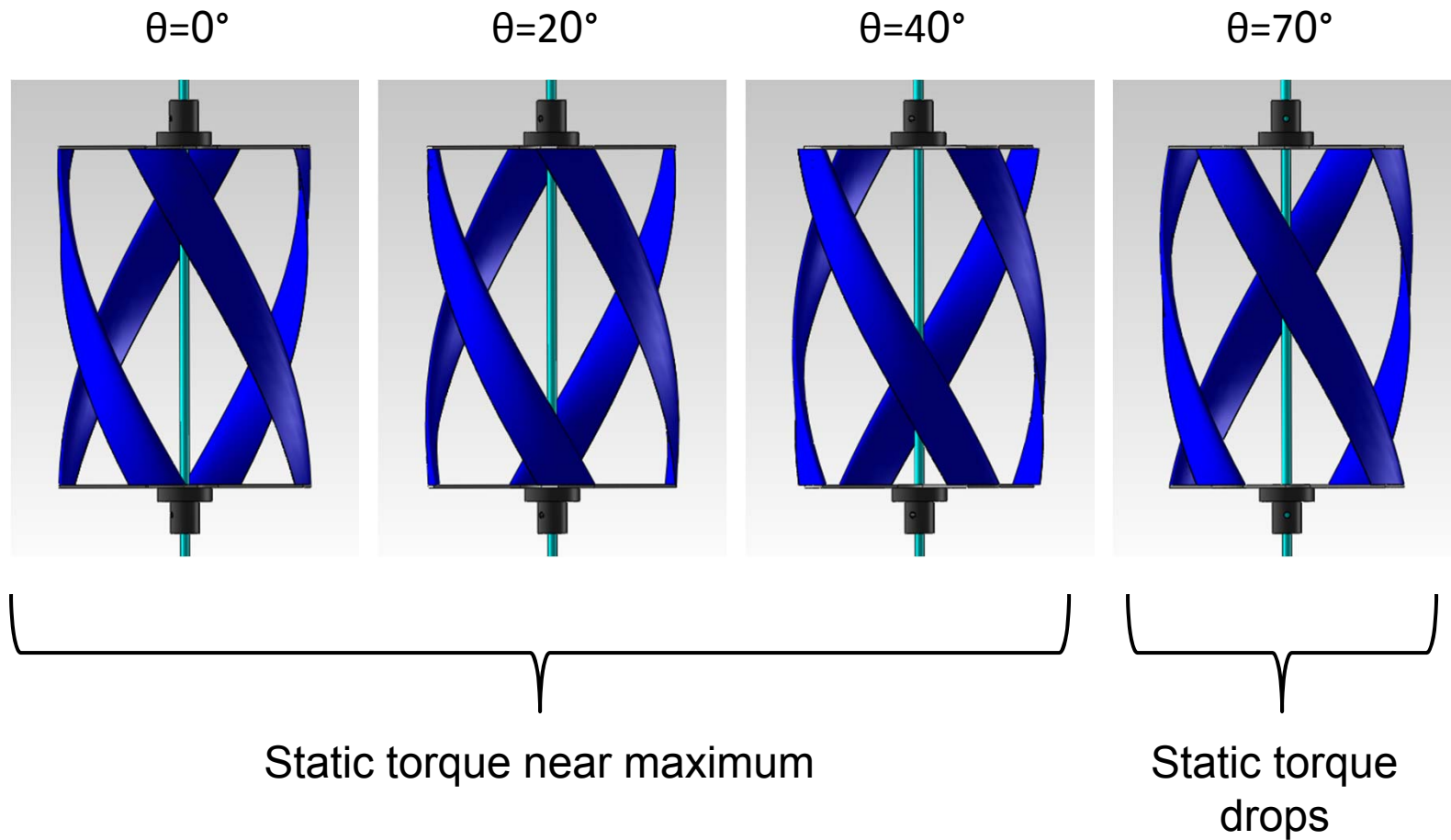
## Load Performance Test



## Static Torque Test

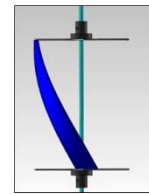
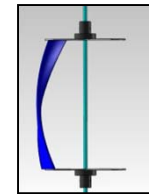
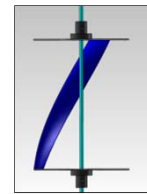
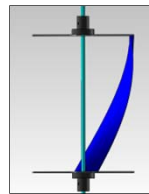
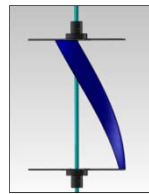
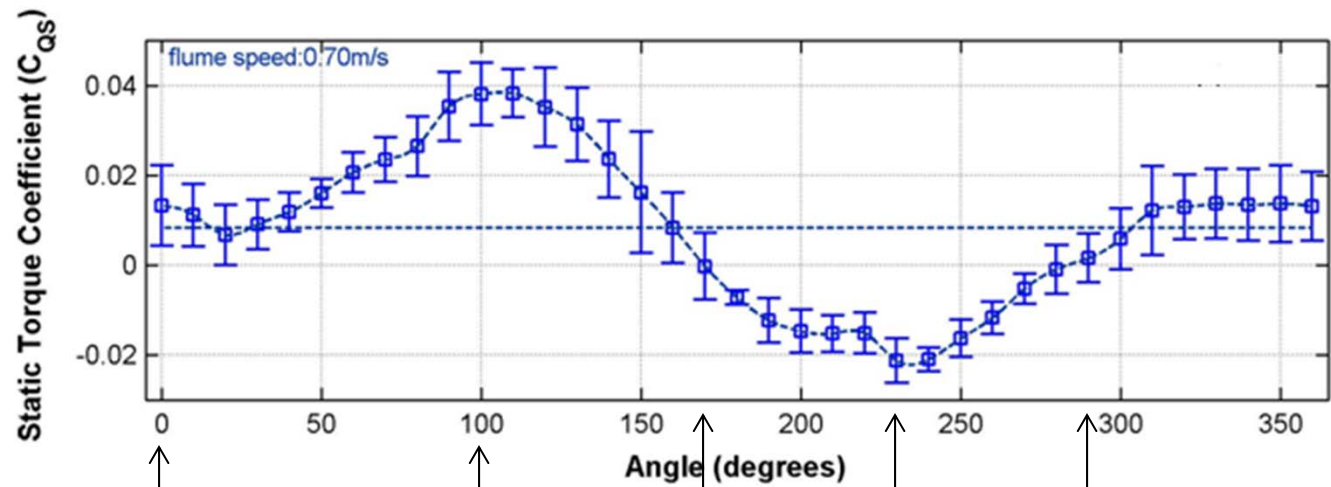


# Streamwise View

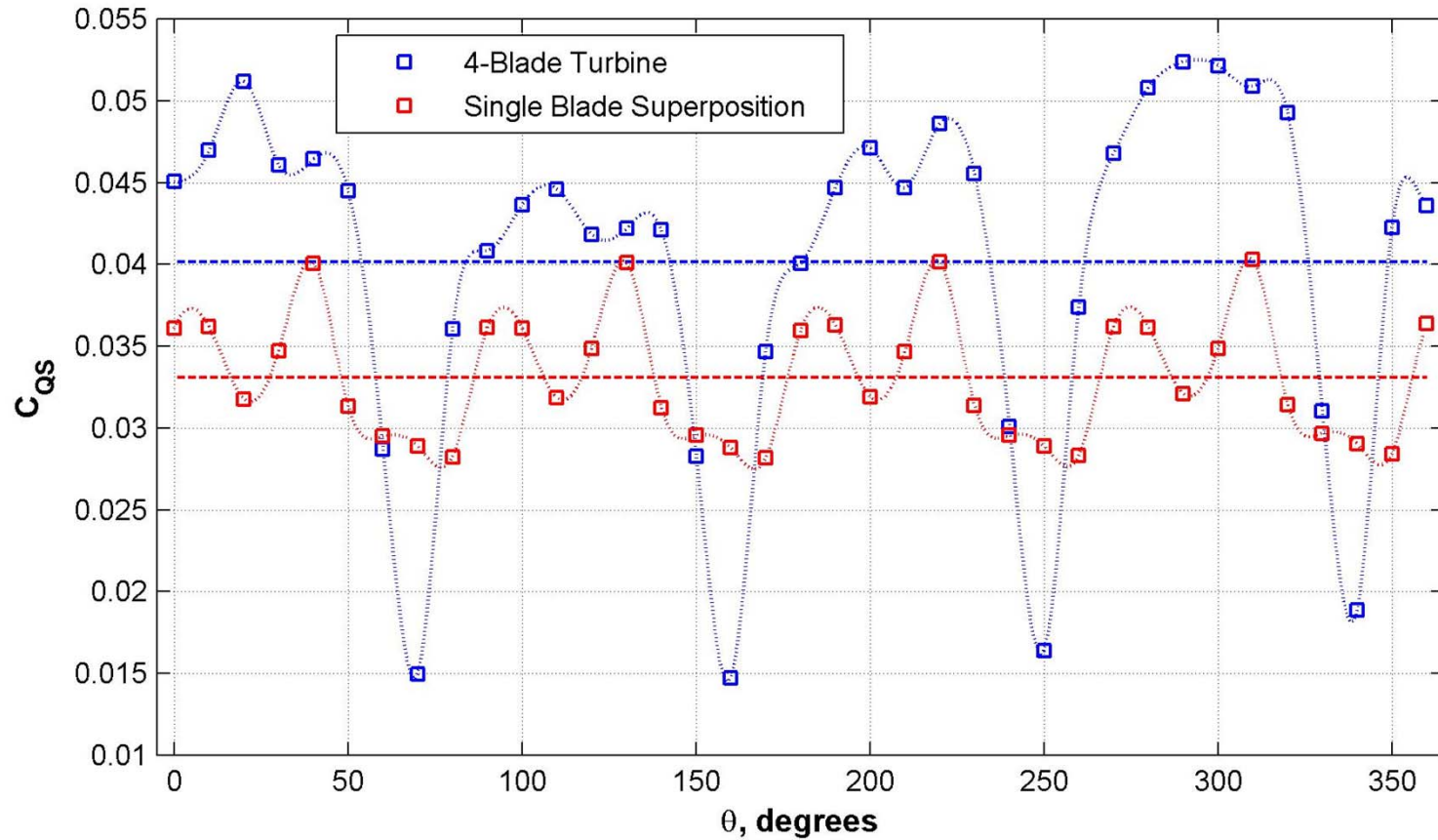




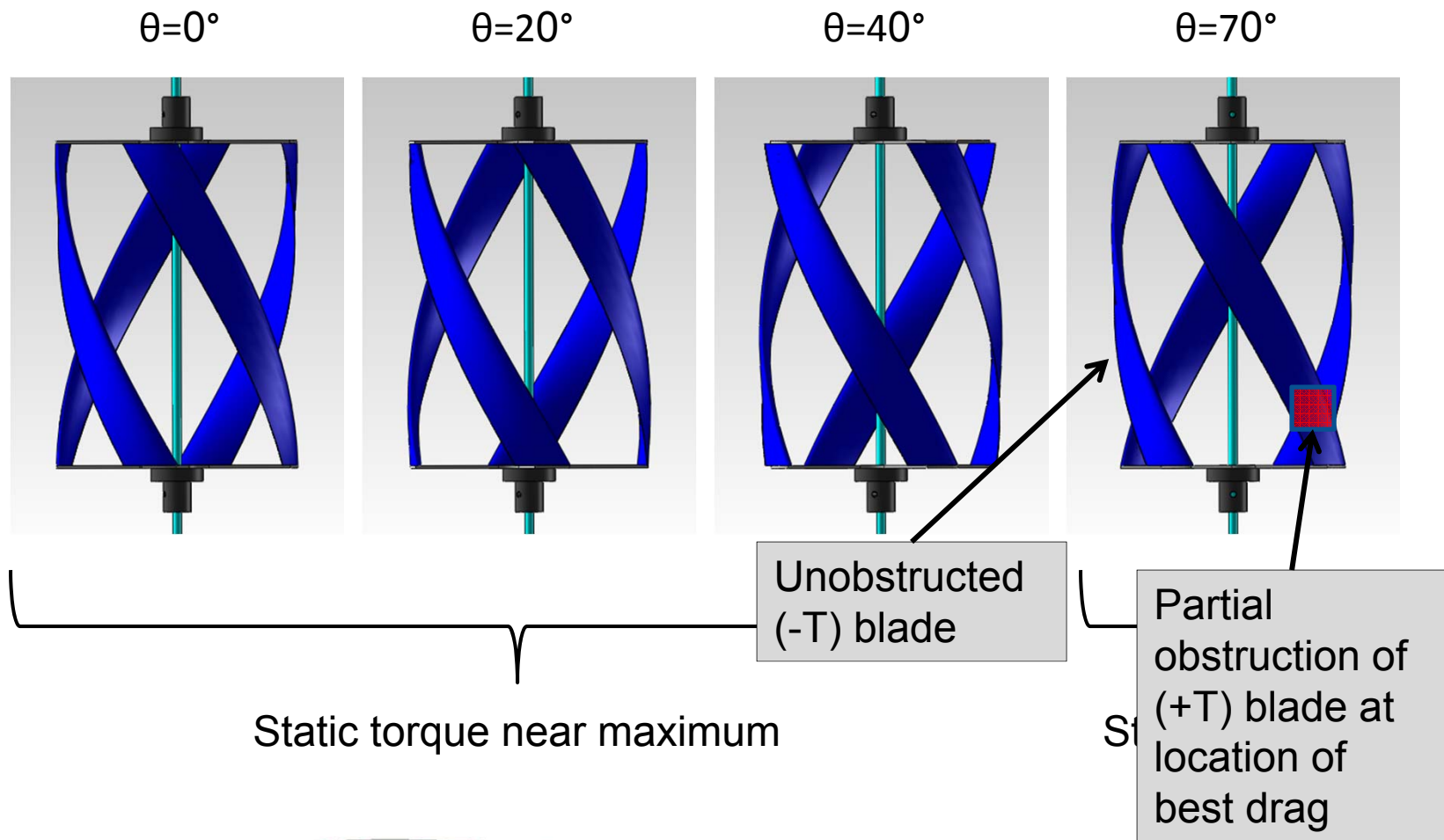
# Single Blade Static Torque



# Single Blade Torque Superposition



# Streamwise View



# Turbine Design Comparison

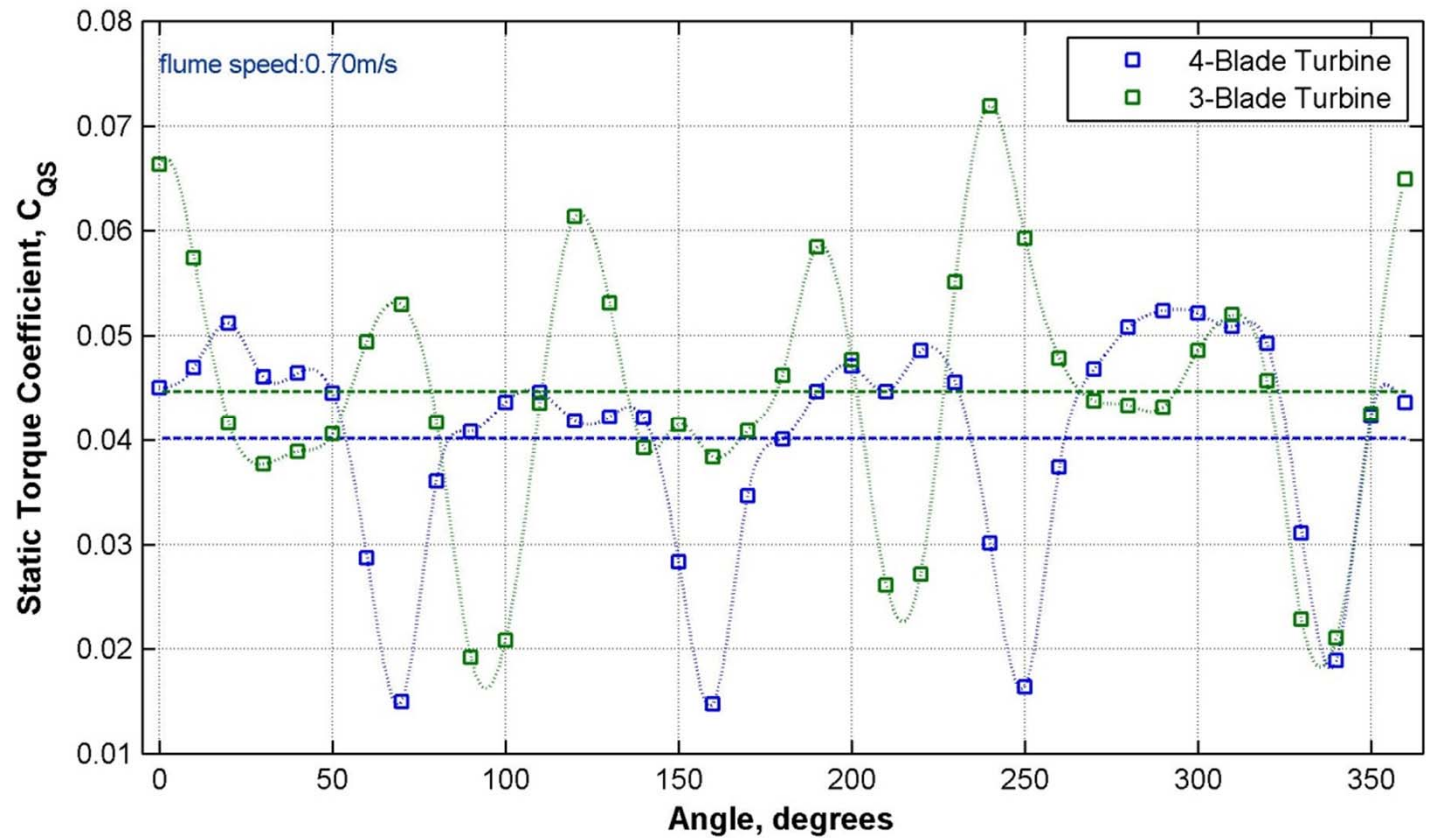
3-Bladed Turbine



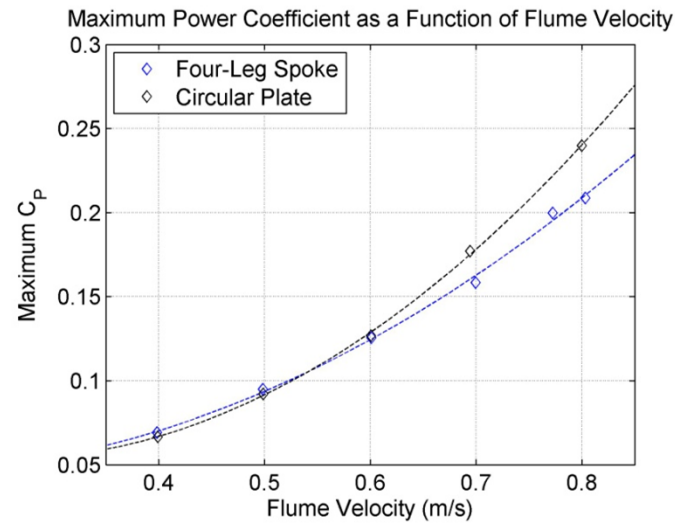
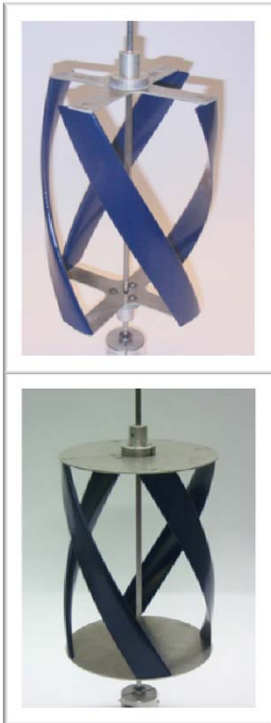
4-Bladed Turbine



Static Torque Test Results



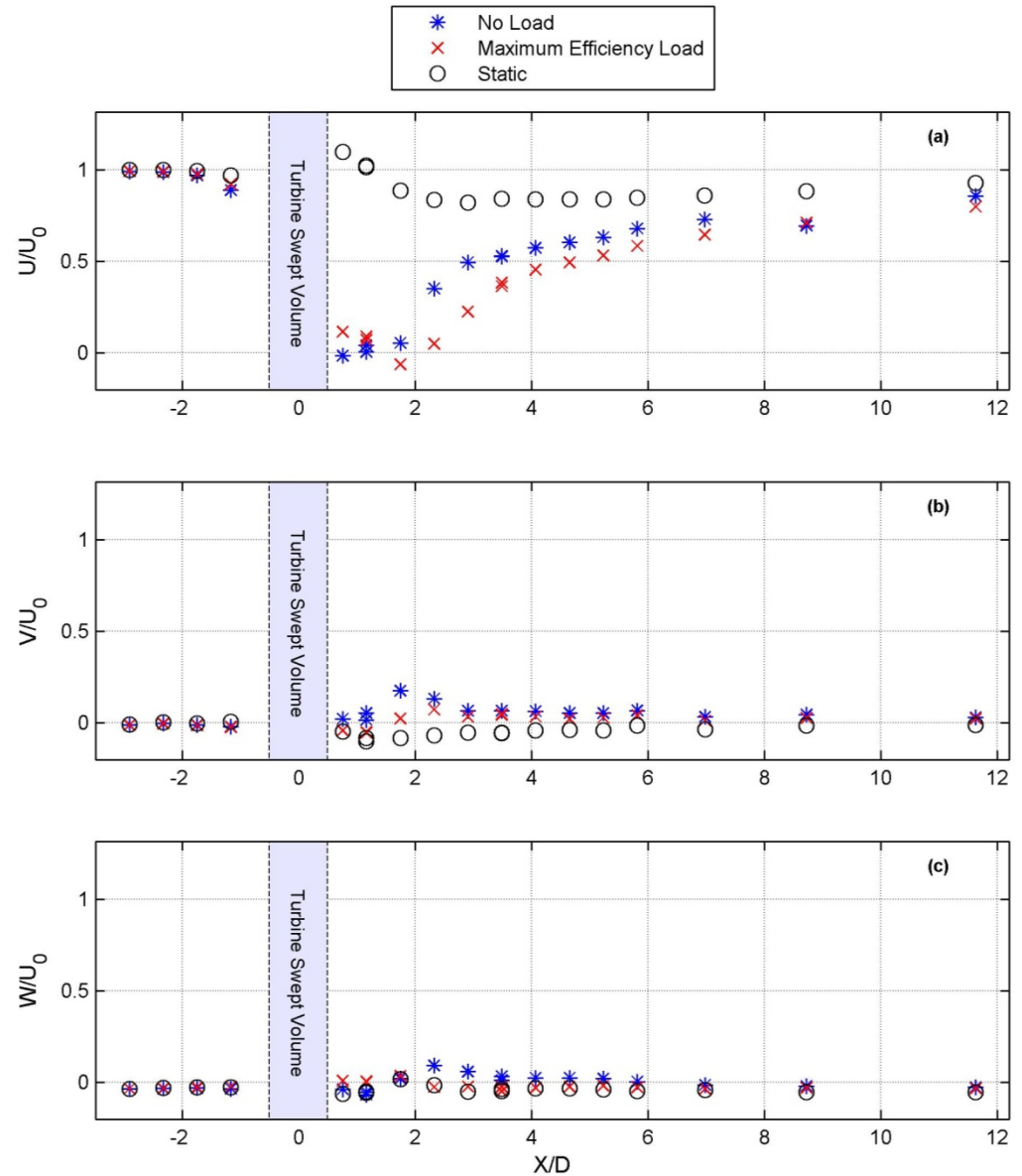
# Tilted Turbine Test



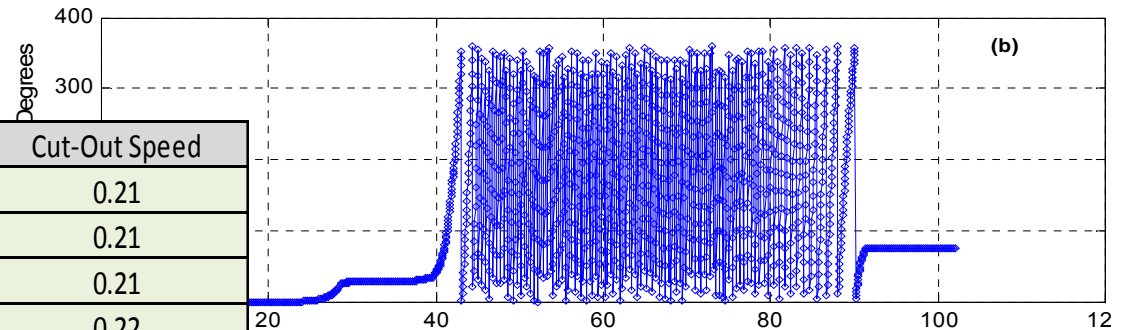
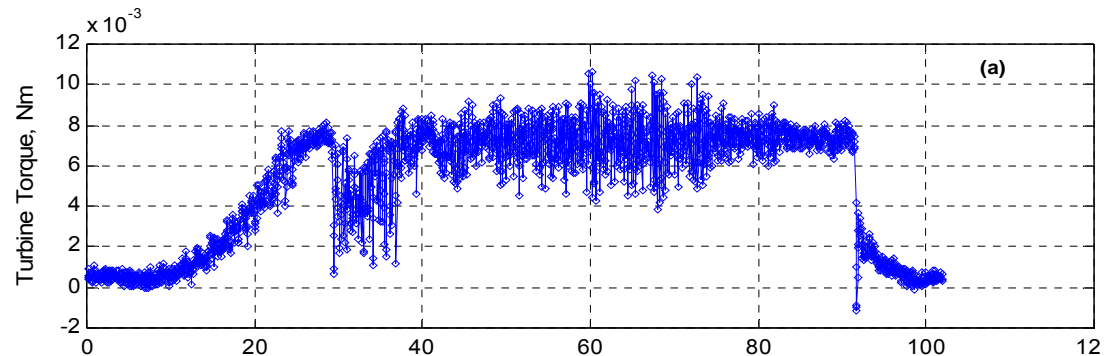
Strut Design	Tilt Angle (degrees)	Tilt Velocity	Predicted $C_p$ Loss from $\cos(\text{Tilt})$	Actual $C_p$ Loss
2 mm, 4-Leg Spoke	0	0.720	0.0%	0.0%
	2.5	0.719	-0.2%	-5.6%
	5	0.717	-0.7%	-8.8%
	10	0.709	-2.8%	-23.0%
2 mm Circular Plate	0	0.720	0.0%	0.0%
	2.5	0.719	-0.2%	-4.5%
	5	0.717	-0.8%	-4.4%
	10	0.709	-3.3%	-8.9%

# Wake Measurement Test

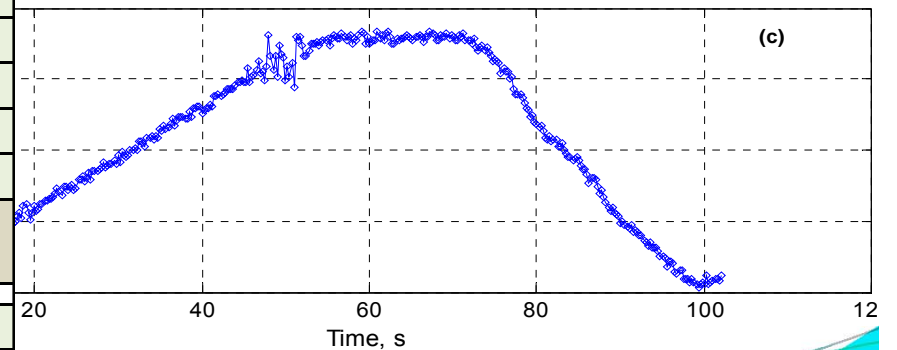
- Baseline Configuration
- Flume: 0.78 m/s
- Wake measurements along turbine centerline



# Start-up Shutdown Test Profile

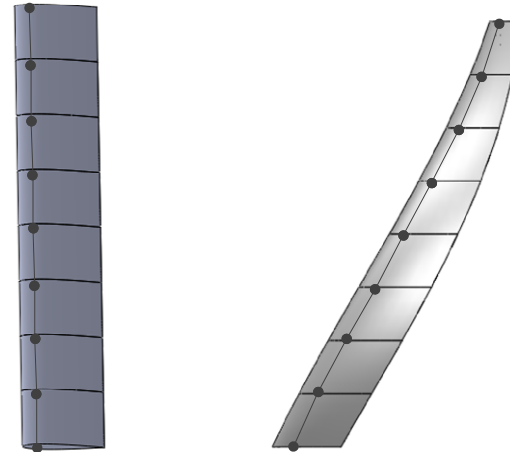


Starting Angle	Load Torque (Nm)	Cut-In Speed (m/s)	Cut-Out Speed
0	0.006	0.47	0.21
10	0.006	0.47	0.21
20	0.006	0.47	0.21
30	0.006	0.58	0.22
40	0.006	0.60	0.20
50	0.006	0.57	0.19
60	0.006	0.60	0.23
70	0.006	0.55	0.18
80	0.006	0.47	0.23
Average cut-in/cut-out speed at 0.006 Nm load torque:		0.53	0.21
0	0.021	0.71	0.38



# Vortex Model

- Blade is divided into segments, revolution into time steps
- 8 elements per blade
- 15 revolutions to establish wake
- Vortex influences the flow of surrounding blades





# Model Solution

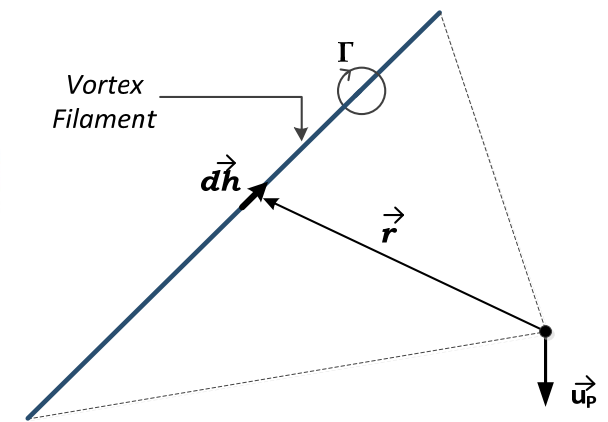
- Sum of velocity components gives  $\vec{u}_{REL}$
- Total change in circulation over time is zero (Kelvin's theorem):
- Vortex-induced velocity at a point is found by Biot-Savart Law:
- Circulation strength is found from Kutta-Joukowski theorem:
- Predictor-corrector to solve system of equations

$$\vec{u}_{RELi} = \vec{u}_{0i} + \vec{u}_{TIPi} + \sum_j (\vec{u}_{Bi,j} + \vec{u}_{Ti,j} + \vec{u}_{SHEDI,j})$$

$$\frac{D\Gamma}{Dt} = 0$$

$$\vec{u}_P = \frac{\Gamma}{4\pi} \int \frac{\vec{r} \times d\vec{h}}{|\vec{r}|^3}$$

$$\Gamma_B = \frac{1}{2} C_L C u_{REL}$$



# Boeing Vertol Dynamic Stall Model

$$\alpha_{REF(L,D)} = \alpha - K_1 \gamma \sqrt{\frac{c\dot{\alpha}}{2u_{REL}}} \text{sign}(\dot{\alpha}). \quad C_L = \left( \frac{C_{LREF}}{\alpha_{REF} - \alpha_{ZL}} \right) \alpha \quad C_D = C_{DREF}$$

# Flow Curvature

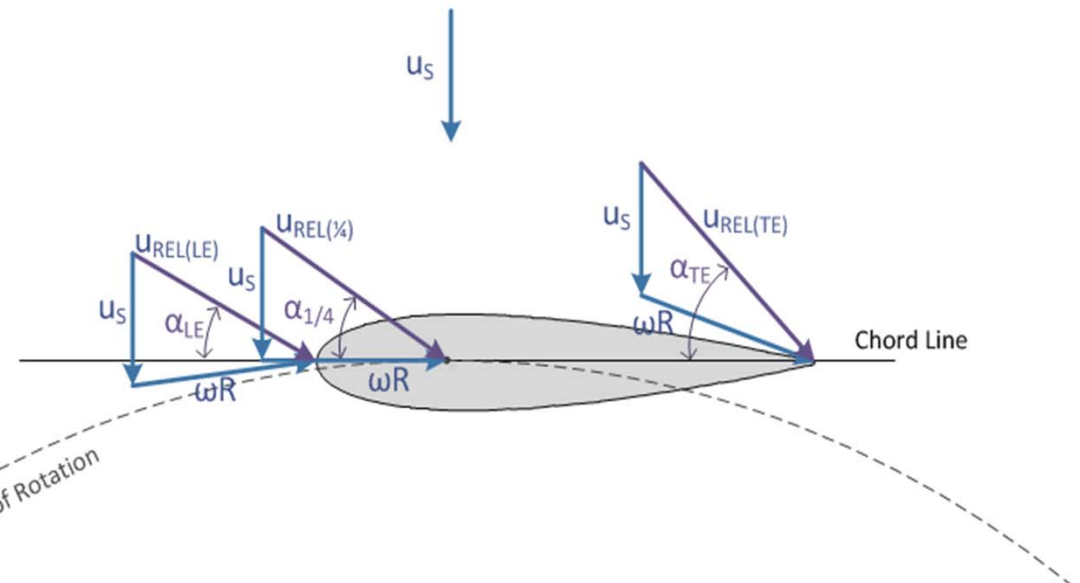
- Change in  $\alpha$  from leading edge to trailing edge
- Important for high chord-to-radius ratio turbines
- Approximated as

$$\tilde{\beta} = \alpha_{TE} - \alpha_{LE}$$

$$\alpha_C = \tan^{-1} \left( \frac{1 - \cos\left(\frac{\tilde{\beta}}{2}\right)}{\sin\left(\frac{\tilde{\beta}}{2}\right)} \right)$$

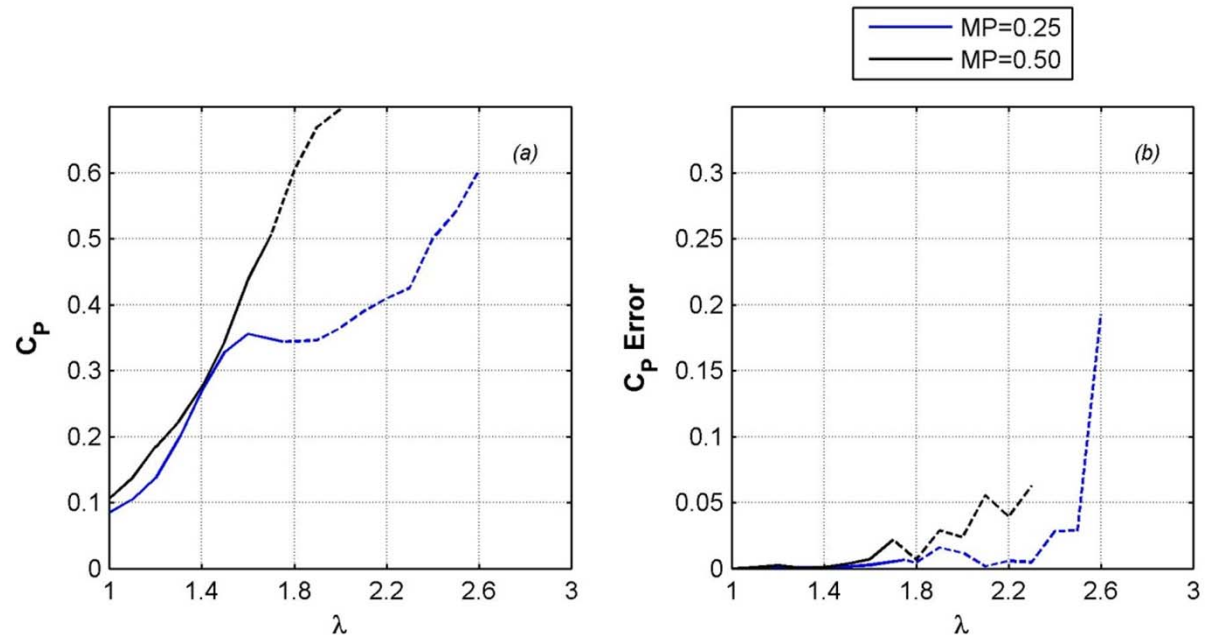
$$C_L = f(\alpha + \alpha_C, Re)$$

- ‘Virtual camber’



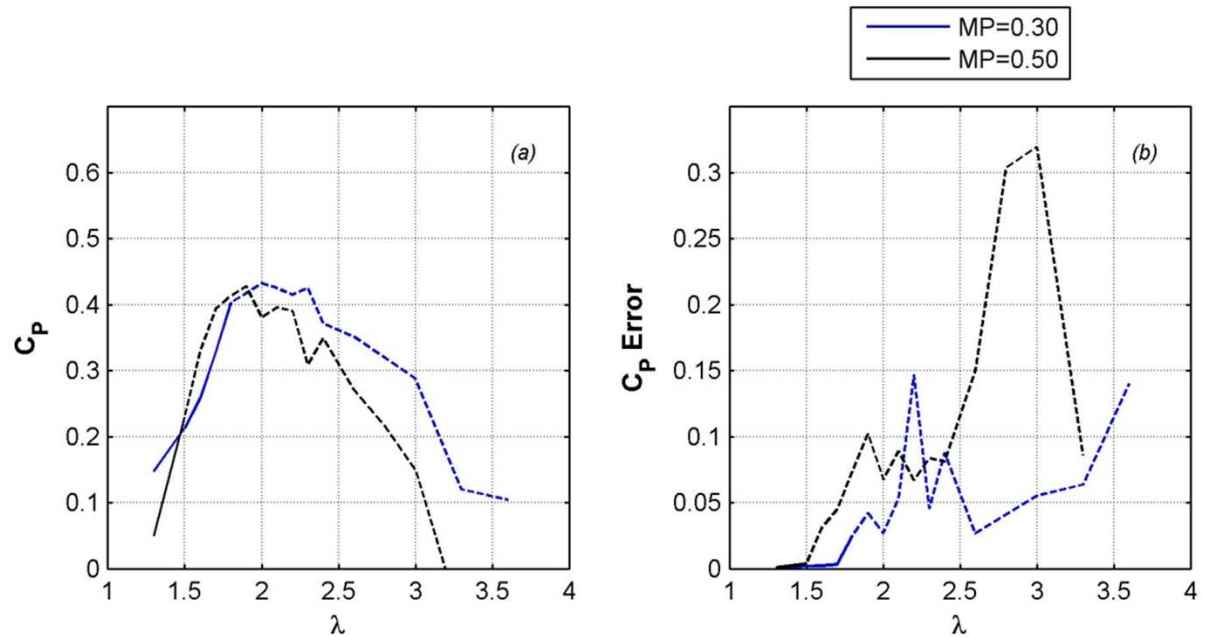
# Full-Scale Turbine Prediction

All Secondary  
Effects Active



# Full-Scale Turbine Prediction

Dynamic Stall  
only



# Model Prediction of Strut Drag

- $u_o = 0.8$  m/s

