Fast local mode properties in field-reversed configurations

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Local eigenmodes of field-reversed configurations (FRCs) were previously computed using ideal magnetohydrodynamics, including compressibility and double adiabaticity. Here the eigenmodes are compared with earlier analytic models. In equilibria, as initially generated in θ pinches, the rigid displacements of the analytic models are similar to actual eigenmodes in structure and growth rate; moreover, the growth rates are similar to those of global modes. In equilibria that naturally arise later in the quiescent FRC, the analytic models fail to predict features of the eigenmode behavior: ballooning-like structure, and much faster growth rate than global modes. This suggests explanations for the difficulty of forming large FRCs in θ pinches, and for the appearance of characteristic profiles in quiescent FRCs. © *1996 American Institute of Physics.* [S1070-664X(96)02711-5]

In our earlier paper,[1] hereafter called paper I, local eigenmodes in field-reversed configurations (FRCs) were computed using the ideal magnetohydrodynamic (MHD) model including compressibility and double adiabaticity. The following results were found. (a) Unstable eigenmodes are essentially incompressible (Figs. 6 and 7 of paper I). (b) For an elliptical-separatrix, the most unstable eigenmodes are essentially ''radial'' or ''axial'' [Figs. 5(a) and 5(b) of paper I]. For racetrack-like separatrix, the most unstable modes are essentially radial [Figs. 5(c) and 5(d) of paper I]. Although (a) and (b) are reminiscent of Newcomb,[2] hereafter called paper II, a quantitative comparison is cumbersome because paper I used a magnetic-field-based coordinate system (parallel and perpendicular components of the displacement), whereas paper II used a fixed coordinate system. Further, the growth rates found in paper II were given in terms of local quantities, whereas those in paper I used more conventional global parameters.

In this Brief Communication, quantitative comparisons of mode structure and growth rate are made with the so-called co-interchange mode of paper II. The growth rate is also compared with an approximation based on the energy principle analysis of Cary,[3] hereafter called paper III, is also given. The significance of the relative growth rates of local and global modes is also discussed, and related to observations of FRC formation in θ pinches.

The analysis in paper II examined the ideal stability of local incompressible modes near the magnetic axis, adopting a simplified equilibrium (elliptic magnetic flux surfaces, straight magnetic axis). Two classes of modes were found: purely radial ($\xi_r \neq 0$, $\xi_z = 0$) and purely axial ($\xi_r = 0$, $\xi_z \neq 0$). Both are unstable (for unit poloidal mode number) and both are rigid shifts, i.e., ξ_r=const for the radial disturbance; and ξ_z=const for the axial disturbance. Another analysis (paper III) found an upper-bound estimate of the ideal-MHD potential energy based on a rigid-displacement trial function,

$$\xi_T = \text{const}(\mathbf{e}_r \cos\theta_0 + \mathbf{e}_z \sin\theta_0). \tag{1}$$

For $\theta_0 = 0$, $\pi/2$ this corresponds to radial and axial shifts, respectively.

The structure of rigid displacements is compared with actual eigenmodes for two representative equilibria, classified by the shape of internal flux surfaces: (i) elliptical, and (ii) racetrack. *Elliptical* flux surfaces have roughly an elliptical shape (in $r-z$ section), while *racetrack* flux surfaces have a racetrack-like shape, i.e., roughly straight sections (one inside and one outside the magnetic axis) connected by curved sections at each end. The appearance of these types is principally determined by two factors: the current density profile; and the separatrix shape. Elliptical flux surfaces appear in equilibria where (a) the current density (expressed as $j_\theta/r)^4$ is either flat or peaked at the magnetic axis, and (b) the separatrix envelope is nominally ellipsoidal. Racetrack flux surfaces appear in equilibria where (a) the current density is hollow, i.e., j_θ/r has a minimum at the magnetic axis, or (b) the separatrix envelope is also racetrack-like, i.e., its $r-z$ section has roughly constant radius except near the ends of the FRC. Example of these two cases are drawn from Figs. 1(a), and 1(c) of paper I representing elliptical and racetrack examples, respectively.

Elliptical flux surface. Consider first the *elliptical* example in Fig. 1, showing the structure of the radial and axial displacements (vs distance along a field line) for the fastest growing eigenmodes. The parameters used here are the same as used in paper I, Fig. 3: $s=0$ is the symmetry plane on the smaller-radius branch of the field line; and a is the separatrix radius at the symmetry plane. The vertical line marked ''S'' indicates the symmetry plane on the larger-radius branch. The vertical lines marked ''K'' indicate the maximum curvature points. These modes are predominantly axial [Fig. 1(a)] and radial [Fig. 1(b)] shifts. Note that the axial and radial shifts correspond to the *odd* and *even* modes in paper I, Fig. 3, respectively. Their structure is qualitatively the same as the upper two examples in paper I, Fig. 5.

Compare these results with the rigid shifts of papers II

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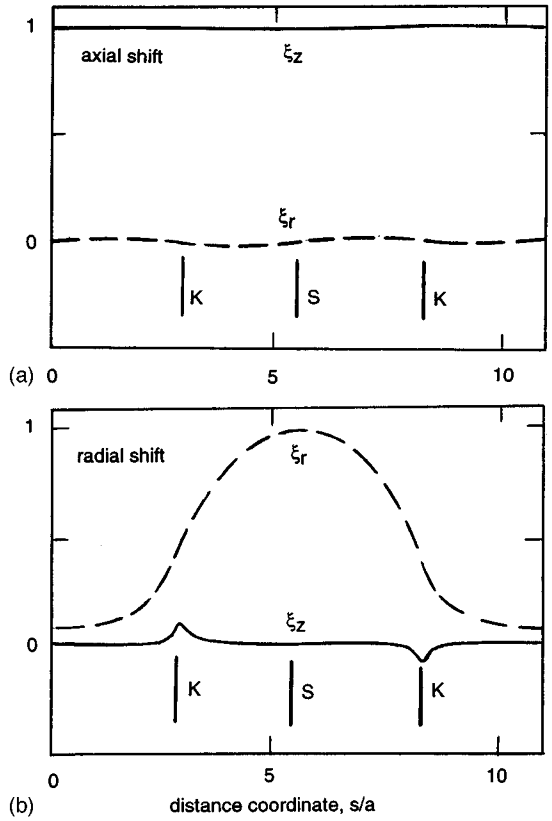

FIG. 1. Radial and axial displacements on an elliptical flux surface. Results are shown for an intermediate flux line $\psi/\psi_0=0.5$ (ψ_0 is the flux function at the magnetic axis). The displacement is normalized by $|\xi|_{\max}$.

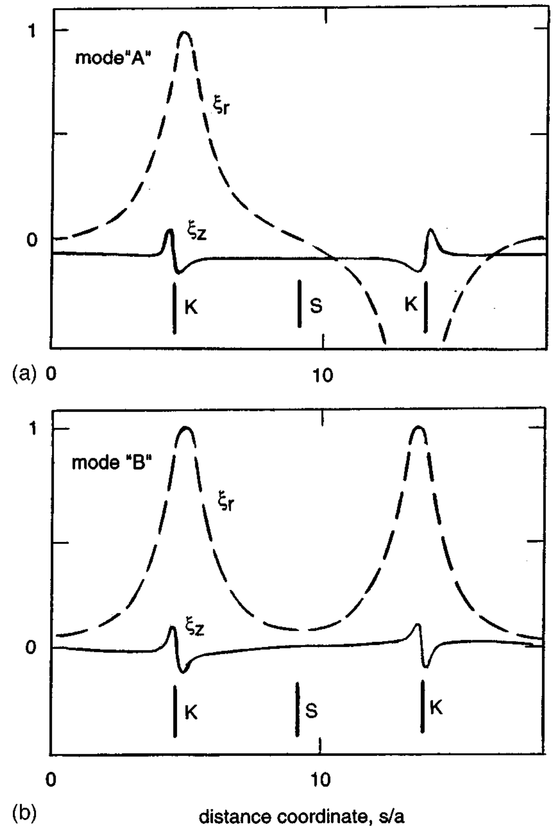

FIG. 2. Radial and axial displacements on a racetrack flux surface. Results are shown for an intermediate flux line $\psi/\psi_0=0.5$. The displacements are normalized by $|\xi|_{\max}$.

and III. The axial mode example [Fig. 1(a)] is exceedingly close to a *rigid* axial shift (ξ_z=const, $\xi_r=0$), even though the particular field line is hardly close to the magnetic axis ($\psi/\psi_0=0.5$, where ψ is the magnetic flux function and ψ_0 is its value at the magnetic axis). This confirms the rigid modes in paper II and III. The radial mode example, however, is hardly rigid and the intermediate flux line in Fig. 1(b) is far from rigid, in contrast to papers II and III. Only quite near the magnetic axis, e.g., $\psi/\psi_0=0.99$ (not shown in Fig. 1) is the radial mode roughly rigid.

Racetrack flux surface. Consider next the racetrack example in Fig. 2, showing the structure of the two most unstable eigenmodes. Modes ''A'' and ''B'' here are identical to the odd and even modes of paper I, Fig. 4, respectively. Their structure is qualitatively the same as the lower two examples in paper I, Fig. 5. In both cases the displacement is predominantly radial, and ballooning-like, i.e., concentrated in the high-curvature regions of the field line. Indeed the disturbances in the two high-curvature regions essentially decouple, so that the only difference between A and B is in the oddness or evenness of ξ_r with respect to the symmetry plane. The mode structure is markedly different from the elliptical example (Fig. 1) and bears no resemblance whatever to the rigid displacements of papers II and III. These insights are clear when viewing the ξ_r, ξ_z structure (as here), but were concealed in the ξ_ψ, ξ_χ portrayal of paper I.

The calculated growth rate in paper II is related to local quantities at the magnetic axis. The flux surfaces in that neighborhood were assumed to be elliptical and the current density uniform [paper II, Eqs. (4), (12)]. These conditions are satisfied near the magnetic axis of a Hill's vortex equilibrium. Adapting the growth rate obtained in paper II, Eq. 16 to the global parameters of a Hill's vortex, it becomes

$$\gamma=\sqrt{2}(v_{A0}/aE), \tag{2}$$

where $v_{A0}=B_0(4\pi M n_0)^{-1/2}$ is the reference Alfvén speed [B_0 is the magnetic field strength at the geometric axis and symmetry plane ($r=z=0$), M is the ion mass, and n_0 is the density at the magnetic axis], and E is the separatrix elongation (length-to-diameter ratio). Note that this growth rate is larger but close to that of the global tilting mode for this equilibrium.[5]

An estimated growth rate can be extracted from the energy-principle analysis of paper III as follows. The square of the mode frequency is

$$\omega^2=W'\{\xi\}/T'\{\xi\}, \tag{3}$$

where $W'\equiv dW/d\psi$ and $T'\equiv dT/d\psi$ are the potential and kinetic energies on a particular flux surface (as is appropriate for local modes), and ψ is the magnetic flux function. If ξ is an eigenvector then Eq. (3) gives the exact eigenvalue. An approximation of the growth rate is found by using estimates of W' and T':

$$\omega^2\approx\frac{1}{4}\frac{\partial}{\partial\psi}\oint dsKrB^2\Big/2\pi\oint\rho\frac{ds}{B}, \tag{4}$$

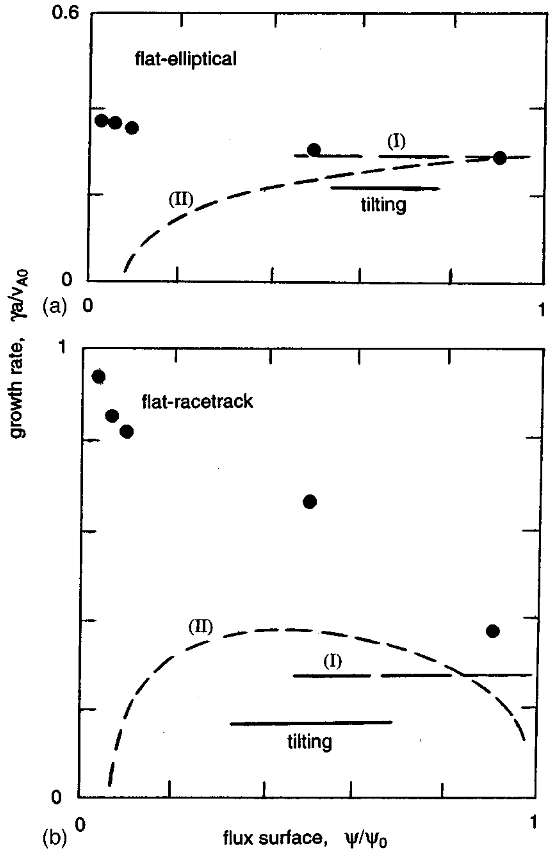

FIG. 3. Growth rates for the most unstable local eigenmodes in the examples in Figs. 1 and 2: the solid symbols are for the eigenmodes; the dashed lines are estimated growth rates using Eq. (2) (paper I), and Eq. (4) (paper II). Also shown are the tilting growth rates [see Eq. (5) of Ref. 6].

where B and K are the magnitude and curvature of the local magnetic field, and ρ is the mass density. The numerator is the upper-bound estimate of W' (from paper III) assuming a rigid displacement [Eq. (1)]; the denominator is the corresponding kinetic energy.

Growth rates for the analytic models and the actual eigenmodes are compared in Fig. 3. The solid circles are computed using the ideal MHD model with compressibility and double adiabaticity. For the elliptical equilibrium [Fig. 3(a)], the estimated growth rates [Eq. (4)] are close to the actual rates, especially near the magnetic axis ($\psi/\psi_0=1$). However for the racetrack equilibrium [Fig. 3(b)] they fall well below the actual values. The difference between analytic rate and the actual rate is smallest near the magnetic axis, but increases steadily toward the separatrix. The difference arises principally from reduced inertia, T', resulting from the concentration of the disturbance in the high-curvature regions.

Also shown in Fig. 3 are the tilting mode growth rates in these equilibria.[6] These are portrayed as horizontal bars since tilting is global and not associated with a particular flux surface. In the elliptical equilibrium the local mode growth is only 30%–40% higher than tilting. In contrast, for the racetrack equilibrium the local mode growth is 3–5 times faster.

Local modes as benign. Local modes enhance transport or simply cause restructuring, while global modes are disruptive. Their relative growth rates strongly influence the subsequent history of the FRC. If the growth rates are comparable (elliptical equilibria) then the FRC is about equally susceptible to global and local modes. Consequently it may be disrupted or suffer a major gyration, depending on the symmetry of the initial state. On the other hand, if local modes are much faster (racetrack equilibria) then benign local modes should dominate. If the consequent restructuring improves stability, as it should, then the stability of the slower global modes will improve accordingly. (Recall that all ideal modes in FRCs, local *and* global, are pressure driven.) A FRC so dominated by fast local modes may never suffer disruption. Of course the conjecture that fast local modes promote restructuring rather than disruption awaits verification by a nonlinear computation.

In this regard we would like to clarify a misunderstood aspect of the proof in paper III that static FRCs are ideally unstable to at least one mode. Since each local mode is more unstable than its corresponding global modes, the conclusion of paper III would more accurately be stated as ''there exists at least one unstable *local* mode.'' There is a common view that this proof applies to global modes as well, which has not been established. The converse, that global modes (e.g., tilting) are stable for certain equilibria[7] remains controversial and has not yet been verified convincingly.

Effect of formation method. In θ-pinch formation of a FRC, the dynamic field-reversal phase concentrates current near the magnetic reversal layer (which becomes the magnetic axis), producing peaked current profile: as mentioned earlier this is accompanied by elliptical flux surfaces. Thus θ-pinch formation is inherently vulnerable to global modes that appear more-or-less simultaneous with formation. Measures which broaden the initial current layer mitigate this tendency, e.g., smaller magnet coil, lower initial filling pressure, lower initial reversed magnetic field, and smaller coil radius.[8] Unfortunately, conditions that might be expected to produce larger, longer-lived FRCs (larger coil, higher pressure, higher initial reversed field) are precisely those which accentuate the problem. This explains why successful creation of larger, longer-lived FRCs has usually been based on more subtle measures, i.e., improved uniformity of the preionized plasma, and methods to soften the field-reversal process.[9]

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